1	Decision-making under uncertainty for buildings exposed to environmental hazards
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7	Abstract

Abstract

8 Buildings are exposed to risks from environmental hazards such as earthquakes, windstorms 9 and floods. Substantial uncertainties from various sources are inevitably involved in the risk 10 estimation and decision-making for activities such as design and disaster risk mitigation for 11 buildings. Decision makers seek to achieve economic efficiency while ensure building safety 12 by managing the extreme tail risk that is typically a concern when facing low-probability, highconsequence events. Thus, risk preferences and tolerances play an important role in the 13 14 decision process, which often vary among different decision makers. The conventionally used 15 minimum expected life-cycle cost criterion (MELC) fails to adequately cope with large 16 uncertainty and risk preferences. To this end, this paper presents the application of a set of 17 decision models beyond the MELC to support decision-making under uncertainty for buildings 18 exposed to environmental hazards. The objective is to provide risk-informed decision support 19 for decision-makers with a wide range of risk appetites while taking into account uncertainties 20 involved in the life-cycle cost. The features, strengths and weaknesses of these decision models 21 are discussed from a practical point of view. The application and selection of the decision 22 models are demonstrated by two practical decision problems: (i) seismic design of a high-rise 23 commercial building, and (ii) wind hazard mitigation for a low-rise residential building. These 24 examples illustrate how the decisions for choosing seismic design levels and wind mitigation 25 measures vary when different decision models and model settings are applied.

26 Keywords: Decision-making, life-cycle cost, environmental hazards, uncertainty, risk preferences 27

28 **1. Introduction**

29 Buildings are exposed to risks from natural hazards such as earthquakes, windstorms and 30 floods. The occurrence and intensity of these extreme events, the environmental loads imposed 31 on buildings and the consequent performance, damage and loss of buildings are subjected to 32 significant uncertainties, both aleatory and epistemic. Probabilistic risk assessment (PRA) for 33 buildings provides a systematic way to account for associated uncertainties and estimate risks 34 from environmental hazards. For a certain planning time horizon, the outputs of the PRA are 35 typically probability distributions of life-cycle costs for buildings (e.g. [1-4]). Decision 36 variables such as the net present value and benefit-to-cost ratio can also be obtained from the 37 PRA (e.g. [3] [5-6]). These outputs can inform and support the decision-making for activities 38 such as design and disaster risk mitigation for buildings. The decision-making process aims to 39 achieve economic efficiency while limit extreme tail risks to ensure building safety, and 40 therefore improve building resilience to natural disasters at an optimal cost.

41 Decision-making for buildings is conventionally based on the minimum expected life-cycle 42 cost (MELC) criterion (e.g. [1] [7-8]), which represents a risk-neutral attitude to achieve long-43 term economic efficiency from a societal perspective. Substantial uncertainties are involved in 44 the PRA and the low-probability catastrophic consequences are often the concern of many 45 decision makers. Hence, it is not surprising that risk averseness is found to be prevalent in civil engineering decisions when facing environmental hazards [9]. The MELC fails to fully capture 46 47 the large uncertainty and dispersion involved in the life-cycle cost distribution, and may 48 downplay possible catastrophic losses because only the mean of life-cycle cost is taken into 49 account. Moreover, the MELC is only adequate for risk-neutral decision makers but fails to 50 cope with other risk preferences such as risk aversion.

51 To this end, there is a need for decision models to deal with large uncertainty as well as 52 different risk preferences and tolerances of decision makers. This paper presents the application 53 of a set of decision models to provide decision support for buildings exposed to environmental 54 hazards beyond the decision-making solely based on expected life-cycle cost. The presented 55 decision models in this paper include risk measures such as Value-at-Risk (VaR), Conditional-56 Value-at-Risk (CVaR) [10] and Range-Value-at-Risk (RVaR) [11-12], the utility theory (UT) 57 [13], the stochastic dominance (SD) [14-15] and its extension almost stochastic dominance 58 (ASD) [16]. To the best knowledge of the author, it is the first time that the ASD and RVaR are 59 introduced to decision problems for buildings subjected to environmental hazards. The features, strengths and weaknesses of these decision models are discussed from a practical 60 61 point of view. Two practical decision examples, i.e. seismic design of a high-rise commercial 62 building and wind hazard mitigation for a low-rise residential building, are presented to 63 compare and demonstrate the applicability of these decision models. It is anticipated that the 64 practical examples given in this paper shed some light on the use and selection of decision 65 models for buildings exposed to environmental hazards. The application of these decision models is expected to provide risk-informed decision support for decision-makers with a wide 66 67 range of risk appetites while taking into account uncertainties involved in the life-cycle cost. 68 The remainder of the paper is organized as follows. First a set of decision models are

introduced, and discussions are provided regarding their features, strengths and weaknesses from a practical point of view. Then practical examples of applying these decision models to the seismic design of a high-rise commercial building and wind hazard mitigation for a lowrise residential building are presented. Parametric studies and comparisons of the decisions based on these decision models are also included.

74 **2. Decision Models**

75 2.1. Risk Measure

A risk measure maps the random variable of interest (e.g. life-cycle cost, benefit-to-cost
 ratio) obtained from the probabilistic risk assessment (PRA) to real numbers, which explicitly

quantifies the risk and provides the basis for choosing between different decision alternatives. Two most common risk measures in financial applications are the Value-at-Risk (VaR) and Conditional-Value-at-Risk (CVaR). For insurance industry and natural disaster management, VaR has also been adopted to represent the probable maximum loss (e.g. [17-18]). Consider that the probability distribution of life-cycle cost (*LCC*) is available from the PRA, then for a certain probability level α (0 < α < 1), VaR_{α} (*LCC*) is

84
$$\operatorname{VaR}_{\alpha}(LCC) = Q_{\alpha}(LCC)$$
 (1)

85 where Q_{α} (*LCC*) is the α -quantile of the random variable *LCC*. CVaR_{α} (*LCC*) can be viewed as 86 an average of quantiles or VaRs for probability levels between α and 1, which is given by [19]

87
$$\operatorname{CVaR}_{\alpha}(LCC) = \frac{1}{1-\alpha} \int_{\alpha}^{1} \operatorname{VaR}_{\gamma}(LCC) d\gamma$$
 (2)

88 The VaR and CVaR above a certain probability level (e.g. $\alpha = 90\%$) can be used to 89 characterize the upper tail of the LCC distribution, which is of primary interest for low-90 probability, high-consequence hazards, and hence account for risk averseness in decision-91 making. The higher the considered probability level α , the higher the degree of risk aversion. The CVaR accounts for possible catastrophic losses in the upper tail exceeding the 92 93 corresponding VaR [19], and hence may be more favoured by risk-averse decision makers in a 94 decision context of extreme environmental events. CVaR also conforms some attractive 95 theoretical axioms such as coherency and regularity [19]. However, compared to VaR, CVaR 96 may be overly sensitive to outliers with extremely small probabilities in the upper tail [11]. In 97 practice, it is often hard to tell if these outliers could really happen in reality or they are just caused by estimation errors of the risk assessment as no PRA models are prefect. Hence, CVaR 98 99 may occasionally lead to costly decisions.

The Range-Value-at-Risk (RVaR) [11-12] offers a compromise between VaR and CVaR,
which is less sensitive to extreme outliers than CVaR while can better capture the critical tail

102 behaviour compared to VaR. At probability levels of α and β ($0 < \alpha \le \beta \le 1$), RVaR_{α,β} (*LCC*) is 103 given by

104
$$\operatorname{RVaR}_{\alpha,\beta}(LCC) = \begin{cases} \frac{1}{\beta - \alpha} \int_{\alpha}^{\beta} \operatorname{VaR}_{\gamma}(LCC) d\gamma & \alpha < \beta \\ \operatorname{VaR}_{\alpha}(LCC) & \alpha = \beta \end{cases}$$
(3)

105 The RVaR is closely connected to VaR and CVaR. When $\alpha = \beta$, RVaR_{$\alpha,\beta} ($ *LCC*) is equivalent $106 to VaR_{<math>\alpha$} (*LCC*). When $0 < \alpha < \beta = 1$, RVaR_{α,β} (*LCC*) equals to CVaR_{α} (*LCC*). Fig. 1 illustrates 107 VaR_{α} (*LCC*), CVaR_{α} (*LCC*) and RVaR_{α,β} (*LCC*) given the cumulative distribution function 108 (CDF) of the life-cycle cost. It is indicated that, at a selected probability level, VaR cannot cope 109 with extreme values of *LCC*s exceeding VaR_{α} (*LCC*). The CVaR_{α} (*LCC*) could be too sensitive 110 to outliers with extremely small probability in the upper tail of life-cycle cost distribution. The 111 RVaR serves as a risk measure that lies between VaR and CVaR.</sub>

The risk measures introduced here can also be used in conjunction with the minimum expected life-cycle cost criterion (MELC) by providing constraints on the maximum tolerable life-cycle cost. In other words, a decision alternative with lower expected life-cycle cost while satisfying the constraints specified by VaR, CVaR and/or RVaR would be preferred in the decision-making. The MELC leads to cost-efficient decisions while the constraints reflect the decision maker's tolerance of extreme losses and costs.

118 2.2. Utility theory

The utility theory (UT) is widely employed to explicitly factor risk preferences into the decision process (e.g. [20-21]), where utility functions are used to subjectively weigh possible consequences. In other words, utility measures the desirability of consequences. The UT provides a normative model that prescribes rational decisions by maximizing the expected utility associated with different risk preferences. For a random variable *x* of the consequence containing *n* outcomes, $x_1, x_2, ..., x_n$, the expected utility (EU) of *x* is given by

125
$$EU(x) = \sum_{i=1}^{n} p_i u(x_i)$$
 (4)

where u(x) is the utility function, and p_i is the probability of x_i . Note that $\sum_{i=1}^{n} p_i = 1$. The utility 126 127 functions are used to reflect decision makers' risk preferences. When a linear utility function 128 is adopted to represent the risk-neutral attitude, the decisions yielded by UT are equivalent to 129 those dictated by MELC. Convex and concave nonlinear utility functions are used to 130 characterize risk-seeking and risk-averse attitudes, respectively. Generally speaking, the more 131 nonlinearity the utility function, the higher the degree of risk proneness or risk aversion. Consider a power utility function $u(x) = -(-x)^l$ where x = -LCC is a negative value to frame 132 133 the life-cycle cost as a loss, and to ensure u(x) is a monotonic and increasing function. In other 134 words, a lower life-cycle cost with a higher utility is preferred. The scaled shape of u(x) with 135 different l values is plotted in Fig. 2, where l = 1, l < 1 and l > 1 stand for risk-neutral, risk-136 seeking and risk-averse attitudes, respectively.

In practice, the elicitation of a widely accepted utility function is often not an easy task, which may hinder the application of UT. For example, decision makers often have difficulties in expressing their risk preferences quantitively via utility functions. It is also common that multiple decision makers with different risk preferences cannot reach an agreement, which could happen even when all decision makers are risk-averse but with varying degrees of riskaverseness.

143 2.3. Stochastic Dominance and Almost Stochastic Dominance

The stochastic dominance (SD) [14-15] provides an alternative approach for decisionmaking in civil engineering applications, for example, the selection of design levels for buildings and pipelines [2][22]. The SD conforms to the principle of maximum expected utility, and ranks two decision alternatives based on their distributional information without the need to specify any utility functions. In the context of this study, suppose F and G are two design 149 candidates, or two mitigation/retrofit measures. Let F(x) and G(x) denote the cumulative 150 distribution function (CDF) of x where x = -LCC is a random variable associated with the life-151 cycle cost of a design or mitigation/retrofit measure. Then for any decision maker with a non-152 decreasing utility function u(x) (i.e. $u'(x) \ge 0$), F dominates G by the first-degree stochastic 153 dominance (FSD) if and only if $F(x) \leq G(x)$ for all values of x with a strong inequality for at 154 least one value of x [23]. If F dominates G by FSD, F is always preferred to G, and the expected 155 utility associated with F is always no smaller than that of G regardless of the risk preferences 156 of decision makers (i.e. the implicit utility function of x can be linear, concave or convex). In 157 other words, FSD provides a way to rank two decision alternatives for all decision makers who 158 are expected utility maximizers without knowing their exact risk preferences and utility 159 functions. Fig. 3a illustrates the CDFs of F and G when F dominates G by FSD. There are other 160 higher degree SD rules that are narrowly applicable to either risk-averse or risk-seeking 161 decision makers. These higher order SD rules are not included in this paper. See Levy [23] for 162 details.

163 In practice, the SD often fails to fully rank all decision alternatives due to its rigorous rules 164 that are frequently violated by some 'pathological' preferences [23], for example, extremely 165 risk-averse or risk-seeking. Thus, SD is often used in an initial screen to exclude a small number of inefficient or suboptimal alternatives. For example, the FSD adopted by Goda & Hong [2] 166 167 and Zhou & Nessim [22] failed to rank any design candidates, and only a few inferior designs 168 were screened out by assuming risk aversion when higher degree SD rules were adopted. The 169 almost stochastic dominance (ASD) is an extension of SD that provides a relaxation of SD's 170 strict conditions [16][23]. Compared to SD, ASD has an improved capability to rank decision 171 alternatives to satisfy most decision makers who are expected utility maximizers [23].

172 The almost first-degree stochastic dominance (AFSD) allows a relatively small portion of *x* 173 at which the condition of FSD does not hold, i.e. F(x) > G(x). Define S_1 is a subset of *x* that 174 contains *x* values where F(x) > G(x). For $0 \le \varepsilon < 0.5$, F dominants G by AFSD for all values of 175 *x* if and only if [23]

176
$$\int_{S_1} [F(x) - G(x)] dx \le \varepsilon \int |G(x) - F(x)| dx$$
(5)

177 Fig. 3b illustrates the CDFs of F and G when F dominates G by AFSD. The ε value is calculated as the violation area enclosed between F(x) and G(x) when F(x) > G(x) (i.e. 178 $\int_{C} [F(x) - G(x)] dx$ divided by the total area enclosed under the two CDFs (i.e. 179 $\int |G(x) - F(x)| dx$). Note that if there is no violation area ($\varepsilon = 0$), AFSD coincides with FSD. 180 181 If F dominants G by AFSD and ε is sufficiently small, then F is preferred to G, and the expected utility associated with F is no smaller than that of G except for some 'pathological' defined 182 183 utility functions [23], for example, overly convex or concave (i.e. extremely risk-averse or risk-184 seeking). In other words, the rank of alternatives dictated by AFSD would satisfy most expected utility maximizers with non-decreasing utility functions. The ε value in AFSD has a 185 186 broad range, i.e. $0 \le \varepsilon < 0.5$, and therefore caution is needed when selecting the ε value in 187 practice. The smaller the ε value, the stronger the dominance. It may need subjective judgement 188 to determine the largest allowed violation area or ε value which may vary depending on the 189 decision problem. The discretion of ε value in practical civil engineering decision problems is 190 discussed in Section 3. Higher degree ASD rules are not included in this paper. Refer to Leshno 191 & Levy [16] and Tzeng et al. [24] for more details.

192 2.4. Comparison of decision models

Table 1 presents a summary for different decision models in terms of their strengths and weaknesses in practical decision support applications. The application of these decision models are illustrated by two practical decision problems described in the next section.

3. Illustrative Examples

197 3.1. Seismic Design

In this example, the decision analysis was conducted to choose design spectral response acceleration and the corresponding return period for a high-rise commercial building in Vancouver, Canada. The building is described in detail by Goda & Hong [2]. It is a nine-storey office building with a floor area of 9406 m², which is classified as a moderately ductile moment-resisting steel frame.

203 The probabilistic risk assessment (PRA) and life-cycle cost analysis methods by Goda & 204 Hong [2] were adopted considering a building service period of 50 years and a discount rate of 205 5%. The simulation-based PRA and life-cycle cost analysis include seismic hazard analyses, 206 structural damage assessment and cost estimation. The seismic hazard analyses consider twelve 207 seismic source zones including the Cascadia subduction zone that influence Vancouver as 208 described in Adams & Halchuk [25]. The occurrences of seismic events from these source 209 zones are modelled by stochastic processes (i.e. a renewal process for the Cascadia subduction 210 zone and Poisson processes for the other source zones). The seismic magnitudes and ground 211 motion parameters are obtained by relevant magnitude-recurrence relations and attenuation 212 relations. The structural damage assessment simplifies the building as an elastoplastic single-213 degree-of-freedom system (natural vibration period is 1.0s and damping ratio is 5%) to obtain 214 the probability of damage states related to the drift ratio and damage factor. The life-cycle cost 215 includes the initial construction cost of a design candidate (C_0), and the damage and repair costs 216 (C_{DR}) which cover the repair/reconstruction cost of the structural and non-structural 217 components, loss of contents, business interruption and relocation expenses. The cost 218 estimation empirically relates the initial construction costs to design levels, and the damage 219 and repair costs are a function of the damage states (see [2] for details about empirical equations 220 used for cost estimation). Note that injury and fatality costs can be regarded as an externality

and it is assumed that the decision maker is only interested in the proposition of design,
construction and ownership of the building [2][26]. Therefore, injury and fatality costs are not
included in the life-cycle cost.

224 Table 2 lists nine seismic design candidates considered in the decision process including 225 their design spectral response acceleration (S_{AEf}), design return period (T_R) and initial 226 construction cost (C_0). The design candidates are from S1 to S9 with an increasing S_{AEf} , and thus increasing C_0 and seismic safety levels. Note that all the costs are presented in 2020 227 228 Canadian dollars (CAD). A total of 50,000 Monte Carlo simulations were conducted, and the 229 random samples of life-cycle costs (LCC) corresponding to different design levels were then 230 obtained for subsequent decision analyses. The major uncertainties involved in the life-cycle 231 cost assessment are from seismic demands and the resulting building damage and losses. The 232 expected damage and repair cost $E[C_{DR}]$, the expected life-cycle cost E[LCC] and standard 233 deviation of life-cycle cost σ_{LCC} derived from the simulation are also listed in Table 2. It 234 suggests that the initial construction cost increases with seismic design levels, whereas the 235 expected damage and repair cost as well as the standard deviation of life-cycle cost decrease 236 with seismic design levels. This is expected as an increased initial expenditure in design and 237 construction reduces potential seismic damage and the uncertainty associated with life-cycle costs. Based on the minimum expected life-cycle cost criterion (MELC), S6 ($T_R = 3,000$ years 238 239 and $S_{AEf} = 0.526$ g, where g is the gravitational acceleration) with the least expected life-cycle 240 cost is the preferred design candidate among S1-S9.

For buildings exposed to seismic hazards, risk-averse decision makers may emphasize on the extreme life-cycle costs. The three risk measures introduced in Section 2.1 can be used to characterize the upper tail of life-cycle costs and provide decision support for risk-averse decision makers. Fig. 4 shows the Value-at-Risk (VaR), Conditional-Value-at-Risk (CVaR) and Range-Value-at-Risk (RVaR) of the life-cycle costs associated with different design candidates 246 at five commonly used probability levels, i.e. 0.50 (median), 0.75, 0.90, 0.95 and 0.99. It 247 suggests that the decision maker prefers a stronger design (i.e. larger T_R and S_{AEf}) as a higher probability level is specified. At the same probability level of α , VaR_{α} (LCC) \leq RVaR_{α, β} (LCC) 248 249 \leq CVaR_{α} (*LCC*) for a given design candidate, and the CVaR yields the most conservative 250 design selection as it fully accounts for the extreme tail risks. For example, at probability levels 251 of 0.90, 0.95 and 0.99, the strongest design S9 is dictated by CVaR as shown in Fig. 4b. S9 is 252 also preferred at probability levels of 0.95 and 0.99 based on VaR, whereas the second strongest 253 design S8 would be preferred at the probability level of 0.90 as shown in Fig. 4a. S6 is preferred 254 for CVaR_{0.50}, which is same with the decision based on MELC. A relatively weaker design S3 255 is dictated by VaR_{0.50}. As suggested by Fig. 4c, the RVaR may provide a middle ground 256 between VaR and CVaR at a given probability level. For example, the seismic design selected 257 based on RVaR0.75, 0.90 is S6, while S5 and S7 are dictated by VaR0.75 and CVaR0.75, 258 respectively.

259 As mentioned in Section 2.1, these risk measures can also be used as constraints in 260 conjunction with the MELC to achieve a balance between economic efficiency and risk 261 aversion. Such constraints reflect decision makers' tolerance of extreme tail risks. For example, 262 the decision criterion may be set as: the best design among S1-S9 is the one with the minimum 263 expected life-cycle cost while also satisfying the condition that VaR_{0.90} (C_{DR}) $\leq C_0/2$. In other 264 words, the VaR of damage and repair cost ($C_{DR} = LCC - C_0$) at a probability level of 0.90 is no 265 greater than 50% of the initial design and construction cost C_0 . Under this constraint, S6 with 266 the minimum expected life-cycle cost among S1-S9 is still the preferred design candidate. If a 267 more risk-averse constraint CVaR_{0.90} (C_{DR}) $\leq C_0/2$ is used, then S7 with the second smallest 268 expected life-cycle cost would be preferred as S6 no longer satisfies the constraint.

The decision analysis was further conducted by comparing the expected utility associated with the nine design candidates considering a variety of risk preferences. The power utility

function described in Section 2.2 was used, i.e. $u(x) = -(-x)^l$, where x = -LCC to ensure u(x)271 272 is a monotonic and increasing function. The expected utilities were then evaluated for the l273 value ranging from 0.1 to 5.0 with an increment of 0.1 to represent a wide variety of risk 274 preferences. For the risk-neural attitude (i.e. l = 1.0), the rank of design candidates based on 275 the expected utility is equivalent to those based on the expected life-cycle cost (i.e. a design 276 with a higher expected utility or lower life-cycle cost is preferred). Table 3 lists the preferred 277 design among S1-S9 with different *l* values. A stronger design would be generally preferred as 278 *l* increases. Fig. 5 shows the comparison of expected utility for l = 0.1, 1.7, 3.0, 4.0 and 5.0. 279 The numerical value for utility in the vertical axis is not shown in the figure because only the 280 rank of expected utility is of concern. It suggests that only for risk-seeking decision-makers 281 with an extremely convex utility function (i.e. l = 0.1), a weaker design (i.e. S5) than that 282 dictated by the MELC (i.e. S6) would be preferred. S6 is the preferred design for most risk-283 seeking decision-makers ($0.2 \le l < 1.0$), mildly risk-averse decision-makers ($(1.0 < l \le 1.7)$) and 284 risk-neutral decision-makers (l = 1.0). For decision-makers with higher levels of risk aversion 285 $(1.8 \le l \le 5.0)$, a stronger design S7, S8 or S9 would be preferred. Recall the preferred design 286 S7, S8 or S9 dictated by VaR and CVaR at relatively high probability levels (i.e. 0.90 or higher 287 for VaR and 0.75 or higher for CVaR), which is consistent with that dictated by highly concave 288 utility functions, and thus represents a high level of risk aversion.

Without specifying utility functions, the first-degree stochastic dominance (FSD) was used to select among the design candidates. Fig. 6 depicts the cumulative distribution functions (CDF) of x = -LCC corresponding to the nine design candidates S1-S9. The CDF curves intersect with each other, and therefore no FSD relation is found for any designs. Besides a visual check, the FSD relation between any two design candidates can also be determined by the algorithm presented in Appendix A. The almost first-degree stochastic dominance (AFSD) with relaxed conditions was then employed. The ε value characterizing the area of violation 296 can be assessed by the algorithm given in Appendix A. Table 4 shows if a design candidate in 297 the first column dominates those in the first row by AFSD, and the corresponding ε value if 298 AFSD relation exists. As shown in Table 4, S6 dominates the other design candidates by AFSD. 299 However, in practice, S6 may not be viewed as the best design among S1-S9 that satisfies most 300 decision makers because the ε values for some AFSD relations are very close to the maximum 301 allowed value 0.5, which indicates a weak dominance relation. For example, $\varepsilon = 0.48$ for the 302 AFSD relation between S6 and S7, and $\varepsilon = 0.45$ for the AFSD relation between S6 and S8. This 303 is consistent with the utility analysis that S6 is not preferred by most decision makers that a 304 considerable portion of risk-averse decision makers would choose S7 or S8. If the maximum 305 allowed ε value is subjectively set as 0.30 in practice, then one design candidate would be 306 chosen over the other only when the former dominates the later by AFSD with ε no greater 307 than 0.30. In this case, S6 and S7 would be chosen over S1-S4, and S1 is screened out as most 308 designs (S3-S9) would be chosen over S1. In this example, the AFSD fails to practically 309 determine the best design option among S5-S9 but only screens out some inferior design 310 candidates (S1-S4). If the selected threshold for ε is reduced, then less inferior design 311 candidates will be screened out. On the other hand, increase of threshold for ε can better rank 312 design candidates. However, caution should be exercised when increasing the threshold for ε 313 as the resulting decision rank may not be preferred by a majority of decision makers who are 314 utility maximizers.

315 3.2. Wind Hazard Mitigation

The second example presents a decision problem of wind hazard mitigation for a low-rise residential building in Brisbane, Australia. The building is a one-storey contemporary house with a complex hip-end roof. It has timber roof and wall frames. The exterior of wall is brick veneer. The roof cladding is corrugated metal sheeting attached to metal top-hat battens. This building is designed and constructed to a wind classification of N2 (a flat site with suburban terrain and partial shielding) according to AS 4055 [27] and AS 1684.2 [28]. The replacement
values of the building and contents are estimated to be \$375,000 and \$75,000 Australian dollars
(AUD) according to Australian housing cost guides ([29]), respectively. Refer to Qin [30] for
more details about this low-rise residential building.

325 Brisbane is primarily under the threat of non-cyclonic windstorms such as severe 326 thunderstorms and east coast lows. The economic losses for the contemporary house mainly 327 arise from direct wind damage to the building envelope (e.g. roof cladding and windows) and 328 subsequent rainwater intrusion through the damaged building envelope. Three mitigation 329 measures are considered to reduce wind damage risks including (i) improvement of roof 330 resistance by strengthening roof connections (RF), (ii) installation of window shutters (WS), 331 and (iii) improvement of the design strength of windows (WR). See Qin & Stewart [6] for 332 details of these mitigation measures and their influences on structural performance. Apart from 333 these three mitigation measures, another decision alternative is to keep 'business as usual' with 334 no mitigation (BAU). The life-cycle cost (LCC) includes a one-off cost for adopting a 335 mitigation measure, and the damage, repair and replacement cost (inclusive of relocation and 336 additional living cost) caused by severe windstorms. The base construction cost of the building 337 is the same for the four decision alternatives and hence is excluded from the life-cycle cost. 338 The mitigation measures applied at the initial design and construction stage are RF and WR, 339 and the window shutters (WS) are also assumed to be installed at the beginning of the building 340 service period. The mitigation cost additional to the base construction cost is 0%, 0.56%, 1.60% 341 and 0.32% of the building replacement value, respectively, for BAU, RF, WS and WR. The 342 damage, repair and replacement costs associated with the four decision alternatives are 343 obtained from the probabilistic risk assessment (PRA) by Qin & Stewart [31] and Qin [30] for 344 a building service period of 50 years and a discount rate of 4%. This simulation-based risk 345 assessment consists of four major components, i.e. i) hazard modelling for extreme wind and associated rainfall, ii) reliability-based wind damage assessment for roof and windows, iii)
evaluation of rainwater intrusion, and iv) loss estimation. See details in Qin & Stewart [31] and
Qin [30].

349 A total of 100,000 Monte Carlo simulations were conducted, and the random samples 350 of life-cycle costs corresponding to the four decision alternatives were then obtained for 351 subsequent decision analyses. The major uncertainties involved in the life-cycle cost 352 assessment are from wind hazards, building damage and loss modelling. Note that all the costs 353 are presented in 2020 Australian dollars. The expected life-cycle costs for the 50-year building 354 service period are \$8,609, \$10,218, \$6,285 and \$4,992 AUD for BAU, RF, WS and WR, 355 respectively. Based on the minimum expected life-cycle cost criterion (MELC), implementing 356 mitigation measures WR and WS would be preferred to 'do nothing' (BAU), and WR is the 357 most cost-effective mitigation measure. Fig. 7 plots the histograms of the random samples of 358 life-cycle costs corresponding to the four decision alternatives. It is indicated that mitigation 359 measures WR and WS considerably reduce the extreme tail risks of the life-cycle cost 360 distribution. Although WS is less cost-effective than WR according to MELC, it provides a 361 significant reduction of uncertainty and extreme losses, which may be favoured by many risk-362 averse decision-makers. RF is not a cost-effective mitigation measure.

363 Wind mitigation decisions based on risk measures are then examined. The Value-at-Risk 364 (VaR), Conditional-Value-at-Risk (CVaR) and Range-Value-at-Risk (RVaR) of the life-cycle 365 costs associated with the four decision alternatives at five commonly used probability levels, 366 i.e. 0.50 (median), 0.75, 0.90, 0.95 and 0.99 are compared in Fig. 8. RF is still not a viable 367 mitigation measure based on all considered risk measures. WS would be preferred based on CVaR for all the five probability levels. This is expected as WS significantly reduces the 368 369 extreme tail risks, though the expected life-cycle cost with WS is higher than that 370 corresponding to WR. For mitigation decisions based on VaR, WR would be preferred at

371 probability levels of 0.50 and 0.75, which is the same with the decision dictated by MELC and 372 less conservative than that dictated by CVaR. The decision based on RVaR_{α,β} is getting closer 373 to that dictated by CVaR_{α} as β increases. For example, at a probability level $\alpha = 0.50$, the 374 preferred mitigation decision for RVaR_{0.50}, 0.75 is WR, which is the same with that for VaR_{0.50}, 375 while the mitigation decision dictated by RVaR_{0.50}, 0.90 is WS, same with that dictated by 376 CVaR_{0.50}.

377 The decision analysis was further conducted by comparing the expected utility associated with the four decision alternatives considering a variety of risk preferences. Same as the seismic 378 design problem, the power utility function was used, i.e. $u(x) = -(-x)^l$, where x = -LCC. The 379 expected utilities were then evaluated for the *l* value ranging from 0.1 to 5.0 with an increment 380 381 of 0.1. It suggests that WS is preferred to BAU for most decision-makers except those with an 382 extremely convex utility function (i.e. l = 0.1 for risk-seeking). WR is preferred to BAU for all 383 the utility functions considered (i.e. $0.1 \le l \le 5.0$), whereas RF is not preferred to BAU. WR is 384 preferred to WS for all considered risk-seeking, risk-neutral and a portion of risk-averse 385 decision-makers (i.e. $1.0 < l \le 1.5$), which is consistent with the decision dictated by VaR_{0.50}, 386 VaR_{0.75} or RVaR_{0.50, 0.75}. WS is preferred to WR for a considerable portion of risk-averse 387 decision-makers (i.e. $1.6 \le l \le 5.0$), which is the same decision with that based on CVaR at all the five probability levels as shown in Fig. 8b. This again shows CVaR is a more conservative 388 389 390 1.6 and 4.0 is summarized in Table 5 to show the above findings.

The first-degree stochastic dominance (FSD) was then used for the mitigation decisionmaking. Using the FSD algorithm given in Appendix A, BAU, WS and WR are all found to dominate RF by FSD. It means that, for all decision makers with a non-decreasing utility function, RF is not preferred regardless of the decision makers' risk preferences. This is consistent with the decision based on the three risk measures and the utility analysis which 396 concludes RF is not a viable mitigation measure. BAU, WS and WR cannot be determined by 397 FSD. The almost first-degree stochastic dominance (AFSD) was further adopted to identify the mitigation measure that would be preferred by most decision makers. Given the random 398 399 samples of x (x = -LCC) associated with the mitigation options, the ε value for one dominating 400 the other by AFSD can be calculated by Eq. (6) in Appendix A. Table 6 shows if a mitigation 401 measure in the first column dominates those in the first row by AFSD, and the corresponding 402 ε value if AFSD relation exists. Table 6 suggests that WR and WS dominate BAU by AFSD 403 with $\varepsilon = 0.02$ and 0.21, respectively. These ε values are deemed to be small enough for most 404 decision-makers to choose WR and WS over BAU. This is consistent with the decision analysis 405 based on expected utility that WR is preferred to BAU for a wide range of risk preferences, and 406 WS is preferred to BAU by most decision-makers except for those who are extremely risk-407 seeking. For these two viable mitigation measures, WR dominates WS by AFSD with $\varepsilon = 0.36$. 408 This ε value may not be small enough to choose WR over WS for most decision-makers, which 409 is consistent with the utility analysis that WS would be preferred to WR by a considerable 410 portion of risk-averse decision-makers. By specifying a threshold for ε value (i.e. the maximum 411 allowed ε value), the AFSD can be used to rank the mitigation options for most decision makers 412 if the corresponding ε value does not exceed the threshold. This threshold value is somewhat 413 determined by subjective judgement in practice and can vary depending on the decision 414 problem of interest. For this wind mitigation problem, a threshold value around 0.25 might be 415 appropriate. However, for the seismic design problem, it fails to find the best design candidate 416 even with a larger threshold value (e.g. 0.30).

417 **4. Conclusions**

This paper presents the application of a set of decision models for buildings exposed to environmental hazards beyond the minimum expected life-cycle cost criterion (MELC), which can cope with large uncertainty and different risk preferences involved in decision problems 421 for low-probability, high-consequence natural hazards. The decision models in this paper 422 include risk measures such as Value-at-Risk (VaR), Conditional-Value-at-Risk (CVaR) and 423 Range-Value-at-Risk (RVaR), the utility theory (UT), the stochastic dominance (SD) and its 424 extension almost stochastic dominance (ASD), whereby the RVaR and ASD are newly 425 introduced to a decision context for buildings exposed to environmental hazards. The features 426 of these decision models were discussed from a practical point. The risk measures well capture 427 extreme tail risks that are often of concern to risk-averse decision makers. The UT provides a 428 full rank of decision alternatives for decision-makers with all possible risk preferences (risk 429 aversion, risk proneness, risk neutral) that are encoded in utility functions. The SD and ASD 430 have the advantage that a specific utility function is not required for decision-making as it is 431 often difficult to elicit a widely accepted utility function in practice.

432 With the consideration of decision makers' possible risk preferences and tolerances, the 433 selection and application of these decision models were illustrated by two practical engineering 434 decision examples, i.e. seismic design of a high-rise commercial building and wind hazard 435 mitigation for a low-rise residential building. It was found that, the decision based CVaR tends 436 to yield a stronger seismic design or a more effective wind mitigation measure (e.g. installing 437 window shutters) with high initial expenditures. CVaR is the most conservative risk measure that may suit decision makers with relatively high levels of risk averseness. The RVaR may 438 439 give a decision in the middle ground between VaR and CVaR. The risk measures can also be 440 used in conjunction with the MELC as a constraint for decision makers' risk tolerances. The 441 seismic design example suggests that a higher design level may be selected when using a CVaR 442 constraint rather than a VaR constraint. The utility analyses were conducted for a wide range 443 of nonlinear utility functions. For most concave utility functions representing risk averseness, 444 the seismic design and wind mitigation decisions yielded by UT are comparable to those based 445 on risk measures at many commonly used probability levels. The first-degree stochastic

446 dominance (FSD) and its extension the first-degree almost stochastic dominance (AFSD) with 447 an improved capacity to rank decision alternatives were also adopted in these two examples. 448 They failed to find the best seismic design candidate, and only a few inferior designs were 449 screened out. For the wind mitigation problem, AFSD successfully selected mitigation 450 measures that are worth adopting and satisfy most decision makers who are expected utility 451 maximizers. The wind mitigation decisions by AFSD are consistent with those by the utility 452 analysis.

453 The application and selection of decision models to adequately address the uncertainty 454 involved in the decision problem as well as various risk preferences and tolerances can be 455 challenging, and on a case-by-case basis depending on a particular decision context, whereby 456 expert judgement is often required. This study attempts to make the decision-making process 457 as objective as possible by applying quantitative decision models, however, a certain level of 458 subjectivity is unavoidable in practical decision-making problems because decision makers 459 will have different preferences and tendencies. It is anticipated that the two practical decision 460 problems given in this paper shed some light on the application of decision models for buildings 461 exposed to environmental hazards. The application of these decision models is expected to 462 better support decision-making by providing decision options that meet different needs and risk appetites of decision makers while taking into account uncertainties involved in the life-cycle 463 464 cost. The decision models introduced in this paper may also be extended to other civil structures 465 and infrastructure systems subjected to low-probability, high-consequence events.

466 Appendix A: Algorithms for FSD and AFSD

467 Suppose F and G are two design candidates, or two mitigation/retrofit measures. *x* and *y*, 468 each containing *n* random samples, are the decision variables (e.g. – *LCC*) corresponding to F 469 and G, respectively. Arrange the samples of *x* and *y* in a non-descending order (i.e. $x_1 \le x_2 \le ...$ 470 $\le x_n$; $y_1 \le y_2 \le ... \le y_n$), and assign a probability of 1/*n* to each sample.

- 471 The algorithm for FSD [23]: F dominates G by FSD if and only if $x_i \ge y_i$ for all *i* values (*i* = 472 1, 2, ..., *n*), and $x_i > y_i$ for at least one *i* value.
- 473 The algorithm for AFSD [31]: The ε value characterizing the area of violation is calculated 474 as

475
$$\varepsilon = \frac{\sum_{i:y_i > x_i} (y_i - x_i)}{\sum_{i=1}^n |y_i - x_i|}$$
 (A1)

476 If $\varepsilon < 0.5$, then F dominates G by AFSD.

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Tables

	Decision model		Description	Strength	Weakness
	VaR		Quantile value of the decision variable (e.g. life-cycle cost)	Easy to use	May not well capture extreme tail risks
	Risk Measure	CVaR	Average of quantiles beyond a certain probability level	Better capture extreme tail risks	May be too sensitive to outliners in the tail of probability distribution
		RVaR	Average of quantiles between two probability levels	Offer combined features of VaR and CVaR	More effort needed to determine the two probability levels
	UT		Decision-making based on maximum expected utility.	Explicitly factor risk preferences into the utility function that measures the desirability of consequences	Elicitation of a widely accepted utility function is often not an easy task
	SD		Rank two decision alternatives based on their distributional information without knowing risk attitudes; conform the principle of maximum expected utility.	No need to subjectively specify utility functions	Often fail to rank decision alternatives due to its rigorous rules that are frequently violated by some 'pathological' preferences
	ASD		Select decision alternatives accepted by most decision makers; extension of SD.	No need to subjectively specify utility functions; a relaxation of SD's strict conditions; allow some extreme risk preferences as exemptions	May still not fully rank decision alternatives in practice
547					
549					
550					
551					

Table 1. Comparison of decision models in practical application.

design	n T_R	SAEf	cost statistics (million CAD)				
candidate	(year)	(g)	C_0	$E[C_{DR}]$	E[LCC]	σ_{LCC}	
S 1	250	0.178	19.5	9.3	28.8	12.7	
S 2	500	0.252	19.8	5.4	25.2	9.5	
S 3	750	0.303	20.1	3.9	24.0	8.2	
S 4	1000	0.343	20.3	3.2	23.5	7.4	
S 5	1500	0.405	20.6	2.4	23.0	6.1	
S 6	3000	0.526	21.3	1.4	22.7	5.0	
S 7	5000	0.635	21.8	1.1	22.9	4.0	
S 8	7500	0.736	22.4	0.7	23.1	3.4	
S 9	10000	0.829	22.9	0.6	23.5	3.1	

Table 2. Seismic design levels and statistical cost information.

555	Table 3. Preferre	d seismic	design	dictated	by	expected	utility.
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	l = 0.1	$0.2 \le l \le 1.7$	$1.8 \le l \le 3.1$	$3.2 \le l \le 4.6$	$4.7 \le l \le 5.0$
preferred design candidate	S5	S6	S7	S 8	S9

Table 4. AFSD relations for the nine design candidates.

design candidate	S 1	S2	S 3	S4	S5	S6	S 7	S 8	S 9
S1	\mathbf{N}^{a}	Ν	Ν	Ν	Ν	Ν	Ν	Ν	Ν
S2	$\varepsilon = 0.33$	Ν	Ν	Ν	Ν	Ν	Ν	Ν	Ν
S 3	$\varepsilon = 0.26$	$\varepsilon = 0.42$	Ν	Ν	Ν	Ν	Ν	Ν	Ν
S 4	$\varepsilon = 0.22$	$\varepsilon = 0.37$	$\varepsilon = 0.40$	Ν	Ν	Ν	Ν	Ν	Ν
S5	$\varepsilon = 0.18$	$\varepsilon = 0.28$	$\varepsilon = 0.32$	$\varepsilon = 0.42$	Ν	Ν	Ν	Ν	$\varepsilon = 0.42$
S 6	$\varepsilon = 0.16$	$\varepsilon = 0.22$	$\varepsilon = 0.24$	$\varepsilon = 0.28$	$\varepsilon = 0.33$	Ν	$\varepsilon = 0.48$	$\varepsilon = 0.45$	$\varepsilon = 0.31$
S 7	$\varepsilon = 0.16$	$\varepsilon = 0.25$	$\varepsilon = 0.27$	$\varepsilon = 0.30$	$\varepsilon = 0.39$	Ν	Ν	$\varepsilon = 0.42$	$\varepsilon = 0.34$
S 8	$\varepsilon = 0.17$	$\varepsilon = 0.27$	$\varepsilon = 0.30$	$\varepsilon = 0.35$	$\varepsilon = 0.42$	Ν	Ν	Ν	$\varepsilon = 0.36$
S 9	$\varepsilon = 0.19$	$\varepsilon = 0.36$	$\varepsilon = 0.41$	$\varepsilon = 0.49$	Ν	Ν	Ν	Ν	Ν

559 Notes: ^a N means no AFSD relation.

	-			
		rank based on	expected utility	
	No.1	No.2	No.3	No.4
l = 0.1	WR	BAU	WS	RF

561 **Table 5.** Rank of decision alternatives based on the expected utility.

WR

WS

WS

562

l = 1.0

l = 1.6

l = 4.0

563 **Table 6.** AFSD relations for the four mitigation decision alternatives.

mitigation option	BAU	RF	WS	WR
BAU	Ν	$\varepsilon = 0$ (FSD)	Ν	Ν
RF	Ν	Ν	Ν	Ν
WS	$\varepsilon = 0.22$	$\varepsilon = 0$ (FSD)	Ν	Ν
WR	$\varepsilon = 0.02$	$\varepsilon = 0$ (FSD)	$\varepsilon = 0.36$	Ν

WS

WR

WR

BAU

BAU

BAU

RF

RF

RF





Fig. 1. Illustration of VaR, CVaR and RVaR using the CDF of life-cycle cost *LCC*.











Fig. 3. Graphical interpretation of stochastic dominance: (a) FSD; (b) AFSD.







578 Fig. 4. Comparison of risk measures of life-cycle costs associated with the nine design
579 candidates at different probability levels: (a) VaR; (b) CVaR; (c) RVaR.







Fig. 5. Comparison of expected utility for the nine design candidates: (a) l = 0.1; (b) l = 1.7;

587 (c) l = 3.0; (d) l = 4.0; (e) l = 5.0.



Fig. 6. CDFs of x = -LCC corresponding to design candidates S1-S9.



593 Fig. 7. Histograms of life-cycle costs corresponding to four wind mitigation decisions: (a)
594 BAU; (b) RF; (c) WS; (d) WR.







Fig. 8. Comparison of risk measures of life-cycle costs associated with the four mitigation
decisions at different probability levels: (a) VaR; (b) CVaR; (c) RVaR.