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Research Article

Leaderless Consensus of Semilinear Hyperbolic Multiagent Systems with Semipositive or Seminegative Definite Convection

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This paper deals with a leaderless consensus of semilinear first-order hyperbolic partial differential equation-based multiagent systems (HPDEMASs). A consensus controller under an undirected graph is designed. Dealing with different convection assumptions, two different boundary conditions are presented, one right endpoint and the other left endpoint. Two sufficient conditions for leaderless consensus of HPDEMAS are presented by giving the gain range in the case of the symmetric seminegative definite convection coefficient and the semipositive definite convection coefficient, respectively. Two examples are presented to show the effectiveness of the control methods.

1. Introduction

As one well-known group of dynamical behavior, multiagent systems (MASs) received many researchers' attention in the last few decades [1]. There are a number of applications for MASs in engineering fields, for instance, power engineering [2], artificial intelligence [3], energy optimization [4], security [5, 6], traffic decision [7], and precision agriculture [8].

Consensus control of MASs is to derive agents to do a designated task synchronously[9, 10]. Many meaningful control methods have been presented, such as event-triggered control [11, 12], containment control [13], pinning control [14], impulsive control [15], sampled-data control [16], and adaptive control [17].

To put it another way, the mentioned literature assumed dynamics of MASs depending only on time. In practice, dynamics of all processes depends on both time and space. As a consequence, it is necessary to study spatio-temporal MASs [18]. Qi et al. proposed boundary control for PDE-modeled MASs(PDEMASs) under 3-D space with a control delay [19] and formation control for PDEMASs on a cylindrical surface [20]. An iterative learning algorithm was proposed for the consensus of multiagent system PDEMASs [21]. Yang et al. proposed several control methods for the consensus of semilinear PDEMASs or partial integro-differential equation-based MASs without and with time delays [22–24]. Several iterative learning methods were studied for the consensus of PDEMASs[21, 25–27].

Most of the above references are modeled by parabolic PDE-based MASs, while there are few works considering hyperbolic PDE-based MASs (HPDEMASs). The consensus of HPDEMASs is meaningful and significant, as a result of existence of a number of hyperbolic PDE systems in practice [28, 29], including gas dynamics [30], reactor [31], traffic flow [32], and hyperbolic Hopfield neural networks [33]. There are several important studies about consensus of

HPDEMASs. For example, Fu et al. proposed the containment control method for the consensus of linear parabolic PDEMASs and second-order HPDEMASs [34]. Wang and Huang proposed the boundary control approach for the finite-time consensus of HPDEMASs by assuming the convection coefficient to be 1 [35]. Zhang et al. proposed boundary control for the leader-following consensus of MASs with input delays by assuming the convection coefficient to be a positive definite diagonal matrix[36]. However, there are still technical difficulties in the consensus of semilinear first-order HPDEMASs for the cases of the convection coefficient to be symmetric seminegative definite or semipositive definite, which are motives of this paper.

This paper mainly studies leaderless consensus control of a semilinear HPDEMAS with two sorts of boundary conditions in one-dimensional space. The contribution of this paper contains (1) a class of HPDEMAS models is given, assuming two sorts of conditions, one symmetric seminegative definite convection coefficient and the other semipositive definite convection coefficient. (2) Dealing with different convection assumptions, two different boundary conditions are presented, one right endpoint and the other left endpoint. (3) A consensus controller based on communication is studied to drive HPDEMAS to reach leaderless consensus. (4) Dealing with two sorts of convection coefficients, two sufficient conditions for the consensus of the leaderless HPDEMAS are, respectively, reached.

Notations: Let I denotes the identity matrix with proper order, $\lambda_{\max(\min)}(\cdot)$ denotes the maximum (minimum) eigenvalue of $\cdot$, $\lambda_2(\cdot)$ denotes the smallest nonzero eigenvalue of $\cdot$, and the superscript T denotes the transpose.

2. Problem Formulation

This paper studies a class of semilinear HPDEMASs with time delays

$$\frac{\partial z_i(\zeta,t)}{\partial t} = \Theta\frac{\partial z_i(\zeta,t)}{\partial\zeta} + Az_i(\zeta,t) + f\left(z_i(\zeta,t)\right) + u_i(\zeta,t), \tag{1}$$

where $(\zeta,t)\in t[0,L]n\times q[0,\infty)$ are space and time, respectively. $z_i(\zeta,t), u_i(\zeta,t)\in\mathbb{R}^n$ are the state and control input, respectively. $0<L\in\mathbb{R}$, $i\in\{1,2,\ldots,N\}$, $A,\Theta\in\mathbb{R}^{n\times n}$, and $f(\cdot)$ are a nonlinear function.

The boundary condition is

$$z_i(0,t)=0, \tag{2}$$

or

$$z_i(L,t)=0. \tag{3}$$

The initial condition is

$$z_i(\zeta,t)=z_i^0(\zeta,t). \tag{4}$$

This paper aims to study one controller $u_i(\zeta,t)$ driving HPDEMAS (1) to the leaderless consensus. Let consensus error $e_i(\zeta,t)\triangleq z_i(\zeta,t)-1/N\sum_{j=1}^N z_j(\zeta,t)$ and the controller is designed as follows:

$$u_i(\zeta,t)=c\sum_{j=1}^N g_{ij}\Gamma\left(z_j(\zeta,t)-z_i(\zeta,t)\right), \tag{5}$$

where c is a control gain to be determined and Γ is symmetric positive definite. Assume that the topological structure $G=(g_{ij})_{N\times N}$ is defined $g_{ii}=0$; $g_{ij}=g_{ji}>0\,(i\neq j)$ if the agent i connects to j, otherwise $g_{ij}=0\,(i\neq j)$.

Remark 1. Compared with papers [35, 36] using the information of only one neighbor, this controller (5) considers the whole communication information among all neighbors and takes full advantage of that.

Definition 1. ([35]) HPDEMAS (1) reaches a consensus, if

$$\lim_{t\longrightarrow\infty}\left\|z_i(\zeta,t)-\overline{z}(\zeta,t)\right\|=0, i\in\{1,2,\ldots,N\}, \tag{6}$$

where $\overline{z}(\zeta,t)\triangleq 1/N\sum_{j=1}^N z_j(\zeta,t)$.

Lemma 1. *([37]) For the Laplacian matrix $\mathscr{L}$, symmetric positive definite P and $y\in\mathbb{R}^{Nn}$ such that $1_{Nn}^T y=0$, the following inequality is satisfied:*

$$\lambda_2(\mathscr{L})y^T(I_N\otimes P)y\le y^T(\mathscr{L}\otimes P)y. \tag{7}$$

Assumption 1. ([23]) For any $\zeta_1,\zeta_2\in\mathbb{R}$, there exists $0<\mathscr{X}\in\mathbb{R}$ satisfying

$$\left|f(\zeta_1)-f(\zeta_2)\right|\le\mathscr{X}\left|\zeta_1-\zeta_2\right|. \tag{8}$$

3. Consensus of HPDEMASs with the Seminegative Definite Convection Coefficient

Assumption 2. Assume Θ is symmetric seminegative definite.

Note that Assumption 2 is sort of classical, which is extensively employed in the practice, see, e.g. [38, 39].

The error system of HPDEMAS (1), (2), and (4) can be obtained as follows

$$\begin{cases}\dfrac{\partial\epsilon(\zeta,t)}{\partial t}=\Theta\dfrac{\partial e(\zeta,t)}{\partial\zeta}+(I_N\otimes A)e(\zeta,t)+F(e(\zeta,t))\\ -c(\mathscr{L}\otimes\Gamma)e(\zeta,t),\\ e(0,t)=0,\\ e(\zeta,0)=e^0(\zeta),\end{cases} \tag{9}$$

where $e_i^0(\zeta)\triangleq z_i^0(\zeta)-1/N\sum_{j=1}^N z_j^0(\zeta)$, $e(\zeta,t)\triangleq[e_1^T(\zeta,t), e_2^T(\zeta,t),\ldots,e_N^T(\zeta,t)]^T$, $F(e_i(\zeta,t))\triangleq f(z_i(\zeta,t))-1/N\sum_{j=1}^N f(z_j(\zeta,t))$, $F(e(\zeta,t))\triangleq[F^T(e_1(\zeta,t)),F^T(e_2(\zeta,t)),\ldots, F^T(e_N(\zeta,t))]^T$, $\mathscr{L}=D-G$, $D=\mathrm{diag}\{d_1,d_2,\ldots,d_N\}$, $d_i=\sum_{j=1}^N g_{ij}$, and so $\mathscr{L}$ is a Laplace matrix [37].

Theorem 1. *Suppose that Assumptions 1 and 2 hold. The leaderless HPDEMAS shown in equations (1), (2), and (4) reaches the consensus under the controller (5), if*

$$c > \frac{\lambda_{\max}\left(I_N \otimes A + A^T/2 + \chi I\right)}{\lambda_2(\mathscr{L})\lambda_{\min}(\Gamma)}. \quad (10)$$

Proof. We choose the Lyapunov functional candidate as shown in the following equation:

$$V(t) = 0.5\int_0^L e^T(\zeta,t)e(\zeta,t)d\zeta. \quad (11)$$

Taking the time derivative of $V(t)$, we obtain

$$\begin{aligned}\dot{V}(t) &= \int_0^L e^T(\zeta,t)\frac{\partial e(\zeta,t)}{\partial t}d\zeta \\ &= \int_0^L e^T(\zeta,t)\left(I_N\otimes\Theta\right)\frac{\partial e(\zeta,t)}{\partial\zeta}d \\ &\quad + \int_0^L e^T(\zeta,t)\left(I_N\otimes A - c\mathscr{L}\otimes\Gamma\right)e(\zeta,t)d\zeta\zeta \\ &\quad + \int_0^L e^T(\zeta,t)F(e(\zeta,t))d\zeta.\end{aligned} \quad (12)$$

Since $\mathscr{L}$ is a Laplace matrix and Γ is a symmetric positive definite matrix, using Lemma 1, one has

$$\begin{aligned}&-c\int_0^L e^T(\zeta,t)(\mathscr{L}\otimes\Gamma)e(\zeta,t)d\zeta \\ &\leqslant -c\lambda_2(\mathscr{L})\int_0^L e^T(\zeta,t)\left(I_N\otimes\Gamma\right)e(\zeta,t)d\zeta \\ &\leqslant -c\lambda_2(\mathscr{L})\lambda_{\min}(\Gamma)\int_0^L e^T(\zeta,t)e(\zeta,t)d\zeta,\end{aligned} \quad (13)$$

where $0 = \lambda_1(\mathscr{L}) < \lambda_2(\mathscr{L}) \leq \mathscr{L} \leq \lambda_N(\mathscr{L})$ [40].

For symmetric seminegative definite Θ, employing integrating by parts, one gets

$$\begin{aligned}&\int_0^L e^T(\zeta,t)\left(I_N\otimes\Theta\right)\frac{\partial e(\zeta,t)}{\partial\zeta}d\zeta \\ &= e^T(\zeta,t)\left(I_N\otimes\Theta\right)e(\zeta,t)_{\zeta=0}^{\zeta=L} \\ &\quad -\int_0^L \frac{\partial e^T(\zeta,t)}{\partial\zeta}\left(I_N\otimes\Theta\right)e(\zeta,t) \\ &= e^T(L,t)\left(I_N\otimes\Theta\right)e(L,t) \\ &\quad -\int_0^L e^T(\zeta,t)\left(I_N\otimes\Theta\right)\frac{\partial e(\zeta,t)}{\partial\zeta}d\zeta \\ &\leq -\int_0^L e^T(\zeta,t)\left(I_N\otimes\Theta\right)\frac{\partial e(\zeta,t)}{\partial\zeta}d\zeta,\end{aligned} \quad (14)$$

which implies

$$\int_0^L e^T(\zeta,t)\left(I_N\otimes\Theta\right)\frac{\partial e(\zeta,t)}{\partial\zeta}d\zeta \leq 0. \quad (15)$$

Since $\sum_{i=0}^N\int_0^L e_i^T(\zeta,t)[f(\overline{y}(\zeta,t)) - 1/N \quad \sum_{j=0}^N f(y_j(\zeta,t))]dx = 0$, under Assumption 1, we can get

$$\begin{aligned}&\int_0^L e^T(\zeta,t)F(e(\zeta,t))d\zeta \\ &= \sum_{i=0}^N\int_0^L e_i^T(\zeta,t)\left(f(z_i(\zeta,t)) - \frac{1}{N}\sum_{j=1}^N f\left(z_j(\zeta,t)\right)\right)d\zeta \\ &= \sum_{i=0}^N\int_0^L e_i^T(\zeta,t)\Big(f(z_i(\zeta,t)) - f(\overline{z}(\zeta,t)))d\zeta \leq \chi\int_0^L e^T(\zeta,t)e(\zeta,t)d\zeta.\end{aligned} \quad (16)$$

Substitution of (12), (14), (15) into (11), we obtain

$$\dot{V}(t) \leqslant \int_0^L e^T(\zeta,t)\Psi e(\zeta,t)d\zeta, \quad (17)$$

where $\Psi \triangleq I_N\otimes A + A^T/2 + \chi I - c\lambda_2(\mathscr{L})\lambda_{\min}(\Gamma)I$.

It is obvious that (9) implies

$$\Psi < 0. \quad (18)$$

Substitution of (17) into (16), we obtain $\dot{V}(t) \leqslant -\lambda\|\widetilde{e}(\cdot,t)\| \leqslant -\lambda\|e(\cdot,t)\|$ for all nonzero $e(\zeta,t)$, implying consensus of HPDEMAS (1). □

4. Consensus of HPDEMASs with the Symmetric Semipositive Definite Convection Coefficient

Assumption 3. Assume Θ is symmetric semipositive definite.

Note that Assumption 3 is sort of classical, which is extensively employed in practice, see, e.g. [35, 36].

The error system of the HPDEMAS (1), (3), and (4) can be obtained as follows:

$$\begin{cases}\dfrac{\partial\epsilon(\zeta,t)}{\partial t} = \Theta\dfrac{\partial e(\zeta,t)}{\partial\zeta} + \left(I_N\otimes A\right)e(\zeta,t) \\ +F(e(\zeta,t)) - c(\mathscr{L}\otimes\Gamma)e(\zeta,t), \\ e(L,t) = 0, \\ e(\zeta,0) = e^0(\zeta).\end{cases} \quad (19)$$

Theorem 2. *Suppose that Assumptions 1 and 3 hold. The leaderless HPDEMAS shown in equations (1), (3), and (4) reaches the consensus under controller (5) if (9) holds.*

Proof. We choose the same Lyapunov functional candidate as in (10). Taking the time derivative of $V(t)$, we obtain (11). For the symmetric semipositive definite $\Theta > 0$, employing integrating by parts, one has

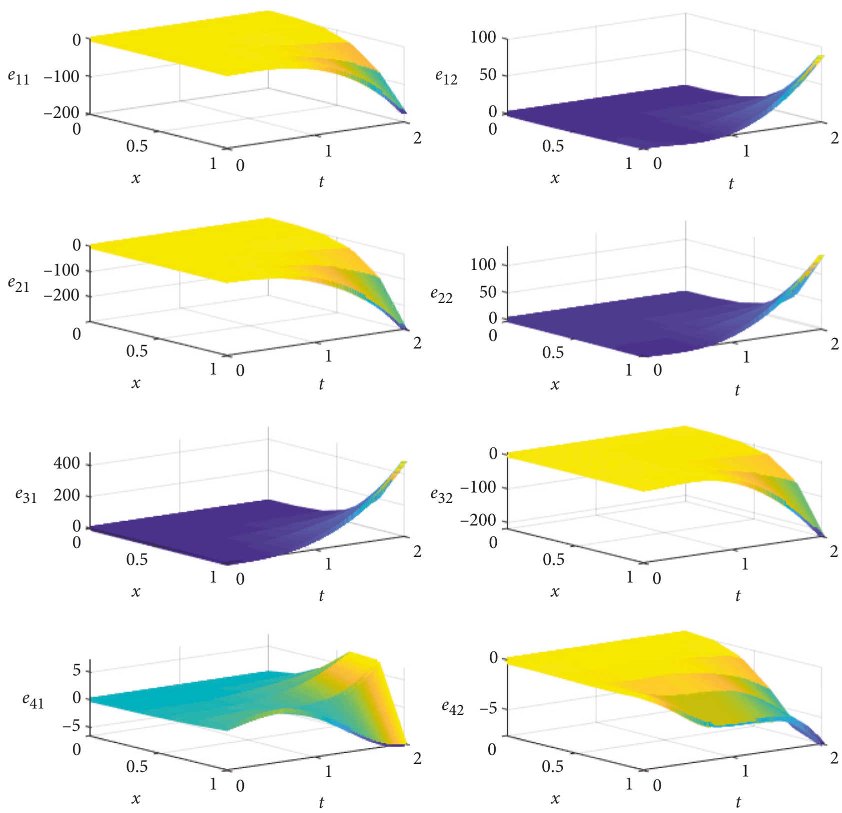

FIGURE 1: The open-loop profile of $z(\zeta, t)$.

$$\begin{aligned}
&\int_0^L e^T(\zeta,t)(I_N\otimes\Theta)\frac{\partial e(\zeta,t)}{\partial\zeta}d\zeta \\
&= e^T(\zeta,t)(I_N\otimes\Theta)e(\zeta,t)\Big|_{\zeta=0}^{\zeta=L} \\
&\quad-\int_0^L\frac{\partial e^T(\zeta,t)}{\partial\zeta}(I_N\otimes\Theta)e(\zeta,t) \\
&= -e^T(0,t)(I_N\otimes\Theta)e(0,t) \\
&\quad-\int_0^L e^T(\zeta,t)(I_N\otimes\Theta)\frac{\partial e(\zeta,t)}{\partial\zeta}d\zeta \\
&\le -\int_0^L e^T(\zeta,t)(I_N\otimes\Theta)\frac{\partial e(\zeta,t)}{\partial\zeta}d\zeta,
\end{aligned} \tag{20}$$

which implies

$$\int_0^L e^T(\zeta,t)(I_N\otimes\Theta)\frac{\partial e(\zeta,t)}{\partial\zeta}d\zeta\le 0. \tag{21}$$

Substitution of (12), (15), (19) into (11), we obtain (16). It is obvious that (9) implies

$$\Psi<0. \tag{22}$$

The later part of the proof is similar to that of Theorem 2, and so it is omitted. □

Remark 2. Different from the control design for consensus of parabolic PDEMASs in [41, 42], this paper deals with the consensus of a class of HPDEMASs.

Remark 3. Consensus of HPDEMASs has been studied by assuming the convection coefficient to be 1 in [35] and to be a positive definite diagonal matrix in [36]. Different from these results, this paper assumes the convection coefficient to be symmetric seminegative and semipositive definite.

5. Numerical Simulation

Example 1. This example considers one HPDEMAS (1) as follows:

$$\begin{cases}
\dfrac{\partial z_i(\zeta,t)}{\partial t}=\begin{bmatrix}-0.8 & 0\\ 0 & -1.6\end{bmatrix}\dfrac{\partial z_i(\zeta,t)}{\partial\zeta} \\
+\begin{bmatrix}5 & 2.6\\ -1.2 & 3.9\end{bmatrix}z_i(\zeta,t)+\tanh\left(z_i(\zeta,t)\right)+u_i(\zeta,t), \\
z_i(0,t)=0, \\
z_i(\zeta,t)=z_i^0(\zeta,t),
\end{cases} \tag{23}$$

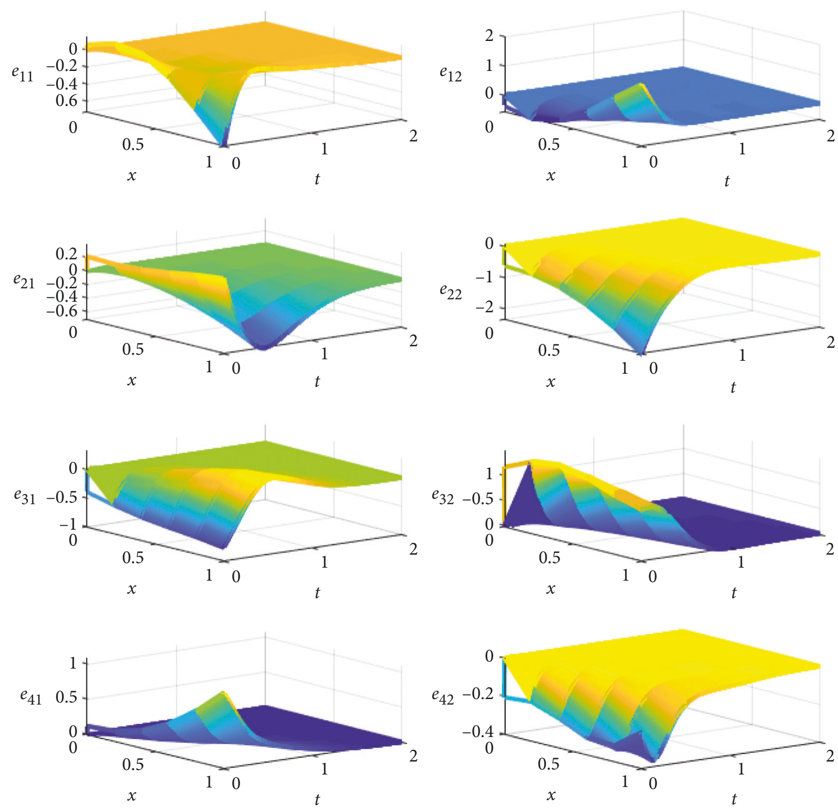

FIGURE 2: The closed-loop profile of $z(\zeta, t)$.

with random initial conditions. We get the following parameters:

$$\begin{aligned} \Theta &= \begin{bmatrix} -0.8 & 0 \\ 0 & -1.6 \end{bmatrix}, \\ A &= \begin{bmatrix} 5 & 2.6 \\ -1.2 & 3.9 \end{bmatrix}, \\ L &= 1, f(\cdot) = \tanh(\cdot). \end{aligned} \tag{24}$$

The controller (5) is used with the following parameters:

$$\begin{aligned} \Gamma &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ g_{ij} &= 1, \text{for } i, j = 1, 2, 3, 4 \text{ and } i \neq j. \end{aligned} \tag{25}$$

From Figure 1, it can be seen that HPDEMAS (1) cannot reach the consensus without control. With Theorem 1, solving (9) by Matlab, $c = 1.59$ is obtained. Figure 2 shows that the HPDEMAS (1) reaches the consensus under controller (5) with $c = 1.59$. Controller (5) with the feedback gain $c = 1.59$ is shown in Figure 3.

Example 2. This example considers one HPDEMAS (1) as follows:

$$\begin{cases} \dfrac{\partial z_i(\zeta, t)}{\partial t} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix} \dfrac{\partial z_i(\zeta, t)}{\partial \zeta} + \begin{bmatrix} 5 & 2.6 \\ -1.2 & 3.9 \end{bmatrix} z_i(\zeta, t) \\ + \tanh(z_i(\zeta, t)) + u_i(\zeta, t), \\ z_i(L, t) = 0, \\ z_i(\zeta, t) = z_i^0(\zeta, t), \end{cases} \tag{26}$$

with random initial conditions.

We get $\Theta = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix}$, and the other parameters are the same as (22). The parameters Γ and g_{ij} of the controller (5) are the same as (23).

From Figure 4, it can be seen that HPDEMAS (1) cannot reach the consensus without control. With Theorem 2, solving (9) by Matlab, $c = 1.59$ is obtained. Figure 5 shows that the HPDEMAS (1) reaches the consensus under controller (5) with $c = 1.59$. The controller (5) with the feedback gain $c = 1.59$ is shown in Figure 6.

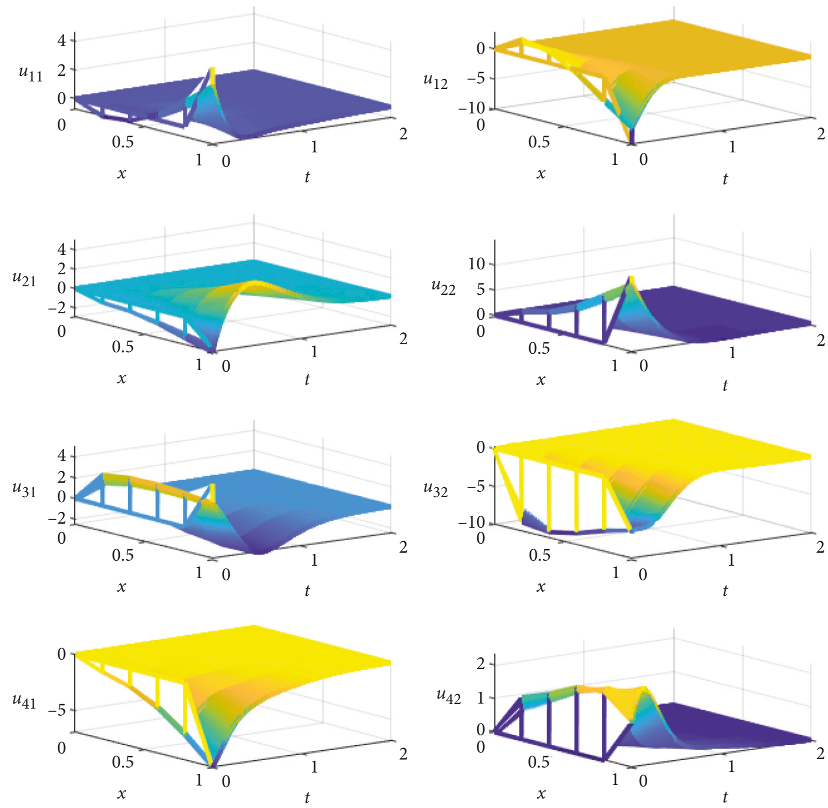

Figure 3: The profile of $u(\zeta, t)$.

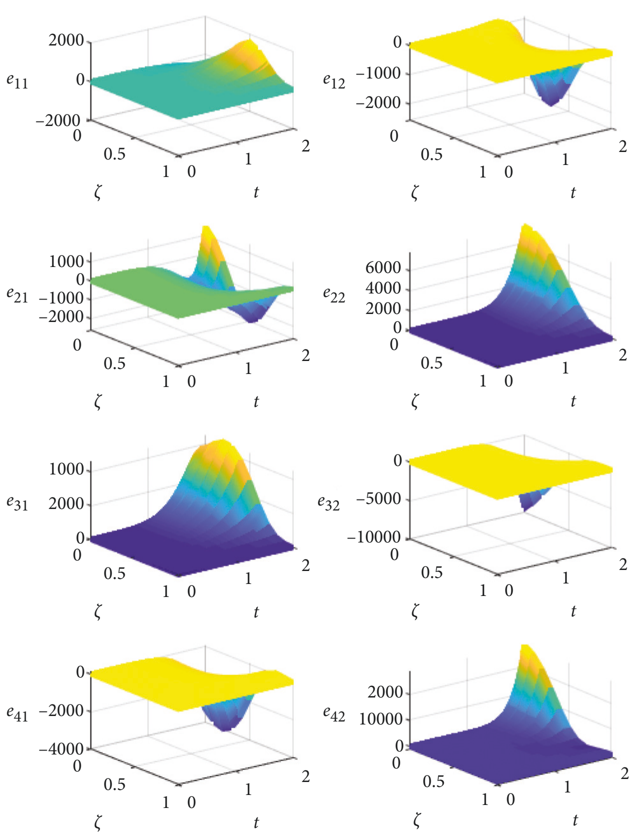

Figure 4: The open-loop profile of $z(\zeta, t)$.

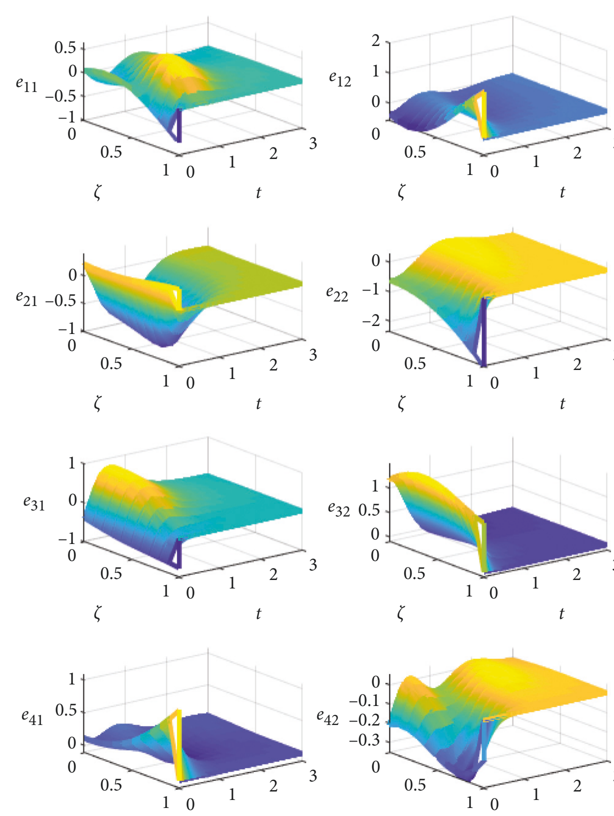

Figure 5: The closed-loop profile of $z(\zeta, t)$.

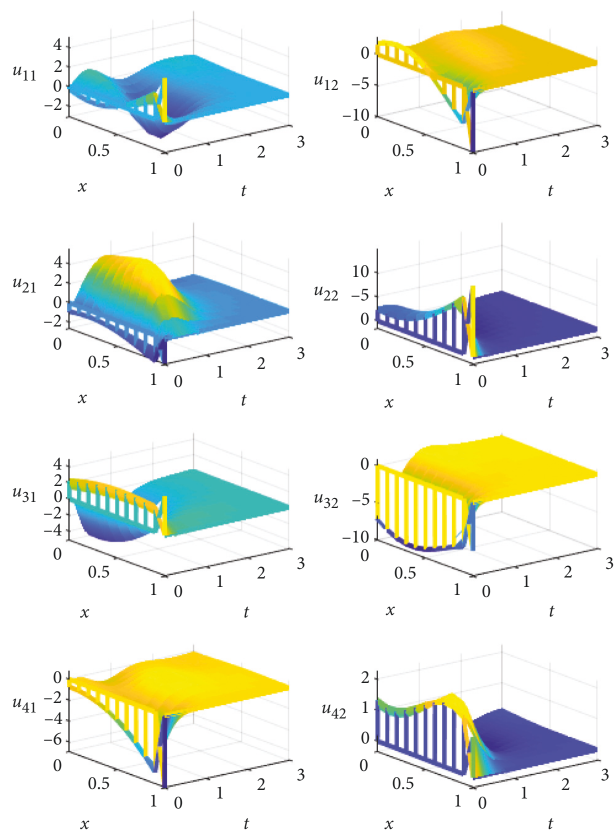

Figure 6: The profile of $u(\zeta, t)$.

6. Conclusion

This paper has dealt with leaderless consensus control of a class of semilinear HPDEMASs. One consensus controller of HPDEMASs under the structure of undirected graphs, making use of communication among agents, was established. Firstly, for the case of the symmetric seminegative definite convection coefficient, the boundary condition of the right endpoint was given. For the case of the symmetric semipositive definite convection coefficient, the boundary condition of the left endpoint was given. Two sufficient conditions for the consensus of HPDEMASs were obtained. Two examples illustrated the effectiveness of developed theoretical results. In future work, containment control, event-triggered control, and many other factors will be studied.

Data Availability

All the data in the simulation are included within this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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