Asset Delegation, Career Concerns and Trade Volume

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Abstract

The paper analyzes the effects of career concerns of portfolio managers on their incentives to trade in a order-driven market. We show that career concerns lead portfolio managers to trade even whithout valuable information, and hence even when they expect a negative return from trading. We then analyze how managers reacts to changes in asset volatility and find that uninformed managers facing career concerns trade larger quantities as asset riskiness increases. As a testable empirical implication, the model predicts that increasing levels of institutional ownership in financial markets lead to higher trading volumes that are positively correlated with asset volatility.

1 Introduction

Most of the standard models in the asset pricing literature assign no role to financial institutions. In these models, it is typically assumed that individuals invest their savings directly into stock exchanges, while financial institutions are regarded as a veil between investors and firms with no real effect on market outcomes. However, casual information suggests that financial institutions are assuming increasing importance in modern financial markets and that very often these institutions are guided by incentives that are not fully considered in the standard models of asset pricing.[1]

This paper analyzes the effects of career concerns on the trading strategies of fund managers. The industry of asset management has experienced a remarkable

[1] For a convincing argument in favour of a natural extension of asset pricing models to take into account the role of financial institutions, see Allen (2001).

expansion in the past few decades, and managed funds have arguably become the main vehicle channelling investors' savings into stock exchanges.[2] Thus, it seems natural to conjecture that the behavior of fund managers is likely to affect the features of the financial markets in which they operate. Though fund managers' behavior is certainly driven by the explicit incentives provided by delegation contacts, *implicit* incentives arising from career or reputation concerns are equally crucial, as witnessed by the broad literature on the subject (Trueman (1988), Chevalier and Ellison (1997, 1999), Hu et al. (2009), Dasgupta and Prat (2006, 2008), Malliaris and Yan (2009)).[3]

Building on a market microstructure model proposed by Biais and Rochet (1997), we develop a two-round model of asset delegation where fund managers differ in their abilities to obtain valuable information. Fund managers are risk-neutral and the delegation contract features a fixed fee plus a performace-related component. Career concerns arise as investors use past performance to infer managers' abilities and dismiss those managers whose performance suggests low ability. Introducing asset delegation and career concerns into a market microstructure model allows me to examine the effects of career concerns on the trading strategies of fund managers under different level of asset riskiness and the impact that managers' strategies have on the overall volume of trade. In particular, the analysis leads to the following main findings:

1) Career concerns lead portfolio managers to trade even without information, that is, even when the expected return from trading is negative. This result follows from the fact that informed managers always trade in order to exploit their information. Thus, the absence of trading suggests that the manager is unskilled, leading to his certain termination. On the contrary, trading without information entails that with some positive probability the uninformed manager's choice is correct ex-post. In particular, the trading strategy that maximizes this probability is that of placing orders that are consistent with the optimal strategy of the informed managers. As long as the performance-related component of the delegation contract is relatively

[2] In the U.S., mutual fund holdings of corporate equities have increased from \$2.9 billion in 1950 to \$2,005 billion in 2002 (NYSE Figures&Facts, 2005). From just 1999 to 2005, the number of U.S. households investing in stock mutual funds increased by 9.3 million. By the end of 2005, 90% of all equity-owning households were reported to hold stock mutual funds (ICI and SIA, 2005).

[3] Career or reputation concerns refer to the manager's concern about the impact that performance has on the likelihood of his termination/promotion as well as on his future compensation. In the case of mutual funds, it is worthwhile to notice that though the typical delegation contract of a manager does not include a perfomance-incentive fee, an implicit performance-based compensation arises from the fact that net investment flow into mutual funds varies in a convex fashion as a function of recent performance, and mutual fund managers typically receive a fixed fraction of the value of the assets under management (see Chevalier and Ellison (1997)).

small compared to the fixed component, an uninformed manager is willing to trade-off the negative return from trading with the benefits that this trading might bring about in terms of enhancing the probability of being retained.

2) Career concerns lead to an increase in the expected volume of trade. Since trading with no information has an expected negative return, an uninformed manager never trades in the absence of career concerns. On the contrary, informed managers always trade according to the same trading strategy, independently from the presence of career concerns. Thus, career concerns lead to an increase in the overall expected volume of trade through the contribution of uninformed managers' churning.

3) As the volatility of the asset increases, uninformed managers trade larger quantities. This result follows from the fact that the in equilibrium the strategy of an informed manager is monotonically increasing in the value of the asset. Thus, when asset volatility increases and extreme realizations of the asset values become more likely, it is also more likely that a good manager will place larger orders. Since the uninformed manager mimics the strategy of the informed manager, he will also place larger orders as asset volatility increases.

4) Since under risk neutrality the trading strategy of informed fund managers is not affected by changes in asset volatility, an empirical implication of the previous result is that in the presence of career concerns the expected volume of trade is positively correlated with asset riskiness.

Our paper is closely related to the works of Trueman (1988) and Dasgupta and Prat (2006), who both develop models of asset delegation where career concerns induce managers to trade even when uninformed. While Trueman carries out the analysis within a partial equilibrium framework, Dasgupta and Prat model the financial market as a quote-driven market in the spirit of Glosten and Milgrom (1985). The general equilibrium approach taken by Dasgupta and Prat allows them to fully characterize the conditions for delegation and analyze the feedback that churning has on equilibrium prices and volume, showing that trade by uninformed managers leads to non-fully informative prices and high trade volume.

Our model extends Dasgupta and Prat's (2006) result to an alternative trading mechanism, namely an order-driven market that operates as a call auction, with fund managers able to buy or sell any desired amount of the risky asset at the market-clearing price set by the market maker.[4] Most importantly, the specific modelling

[4]The last decade has witnessed a series of upheavals in the methods exchanges are organized, with many markets moving from dealership systems to auction (or hybrid) systems. In October 1997, following demands from the Securities and Investments Board (in turn due to lobbying from the institutional users of the trading systems), the London Stock Exchange changed its trading system in the most liquid securities from a dealership system (SEAQ) to an auction system of

choice adopted allows us to investigate the relationship between trade volume and asset volatility, an issue that cannot be addressed in Dasgupta and Prat's model, where managers are restricted to trade only one unit of the asset. Truman (1988) finds a positive relation between the probability that an uninformed manager engages in trade and asset volatility. However, his partial equilibrium framework prevents him to discuss the impact on trade volume.

The distorted effects of reputation on the portfolio choices of fund managers are also considered by Huberman and Kandel (1993). They present an adverse selection model where a manager's objective is the sum of the expected investment outcome plus an (expected) reward entailed by the market's inference regarding the manager's ability. Fund managers have to decide which fraction of the portfolio will be invested in a risky asset and make use of the portfolio weights to signal their ability. Huberman and Kandel show that there exists a unique separating equilibrium in which good managers distort their first best investment decision in order to distinguish themselves from bad managers. The distortion can be both in the direction of an overinvestment or of an under-investment in the risky asset. Thus, the model seems to better explain excessive risk taking rather that excessive trading. Furthermore, they frame the analysis in a partial equilibrium setup.

Huddart (1999) extends Huberman and Kandel (1993) to a set-up in which the reward to reputation is endogenous and both advisors and investors are risk-averse. The general conclusion is that either an increase in the degree of risk aversion by the advisor or in the fee makes separation easier to support. Notice that Huddart retains the partial equilibrium approach and the related limitations of Trueman's and Huberman-Kandel's analysis.

Finally, the present contribution bear some relation to the financial literature on the "trade volume puzzle." In standard models of trade with rational agents, individuals trade because of information advantages or because they have been hit by an exogenous shock that forces them to trade (or both). This latter type of trading

limit orders (SETS). London is not alone in changing its trading system. In 1997, the Deutsche Bourse adopted the Xetra electronic order book system. In 2001, the Amsterdam, Brussels, Lisbon, LIFFE and Paris stock exchanges merged to form Euronext, a pan-European exchange, using a single order-driven trading platform based on the French NSC electronic order book system. And NASDAQ, the US technology stock exchange, set up a pan-European technology exchange in 2002 based on the SuperMontage trading platform, a fully integrated central limit order book and quote-driven montage facility and execution system. Most of the on-going debate about dealer systems vs auction systems are in large part attributable to these innovations in trading protocols. In general, theory suggests that multilateral trading systems (such as single-price call auctions) are efficient mechanisms to aggregate diverse information. Consequently, there is interest in how call auctions operate and whether such systems can be used more widely to trade securities.

has been named "noise trading" or "uninformed trading" exactly because it is not driven by any information about the underlying asset that is traded. On strictly theoretical terms, in any model with rational traders, the presence of noise trading is a necessary condition for trading based on information, which would otherwise conflict with the no-trade theorem. The classical justification given by the literature for the presence of noise trade are represented by unexpected liquidity shocks and hedging needs. At an empirical level, there is broad agreement about the fact that only a small fraction of the total volume of trade takes place either for liquidity or hedging purposes. Thus, in keeping with classical explanations, the remaining volume of trade should be due to information. However, it seems highly implausible that a small amount of noise trade can support the remaining large fraction of informative trade.[5,6]

The paper is organized as follows. In section 2, I introduce delegation in the static framework taken from Bias and Rochet (1997) and characterize the equilibrium strategies of investors, fund managers and market makers. In section 3, I extend the analysis to a two-period dynamic setup and compare the equilibrium in which fund managers have career concerns with the equilibrium in which career concerns are absent. In section 4, I analyze how the presence of career concerns affects the trading behavior of fund managers in contexts characterized by different degrees of asset volatility. In section 5, I draw conclusions and discuss the results of the paper.

2 The basic model

In this section, I will first present the way in which the financial market for the risky asset is modelled. Basically, the structure of the market is taken from Rochet and Vila (1994). This is a *discrete and static* variant of Kyle (1985) in which a perfectly informed trader (the insider) trades along with noise traders and market makers, but both the distributions of the liquidation value of the risky asset and of noise trade are assumed to be discrete. I will take this structure and introduce the delegation process between investors and mutual funds. In the next section, I will consider a dynamic version of the model and reputation issues will be introduced as well.

[5] Hence, the appeal to theories based on the irrationality (or limited rationality) of investors.

[6] Dow and Gorton (1997) present a static (one-round-of-trade) model with complete contracts in which fund managers' churning does not originate from career concerns but as a consequence of the optimal contract that is implemented in order to screen good managers from uninformed ones. The optimal contract is designed such that, in equilibrium, only good managers trade. In fact, however, sometimes good managers engage in churning.

2.1 Structure of the market

Consider the financial market for a risky asset. Denote with v the liquidation value of the asset and assume that v can take on values $-2, -1, 1, 2$ with equal probability $\frac{1}{4}$.

There are four classes of agents: noise traders, fund managers, investors and market makers. For simplicity, they are all assumed to be risk neutral.

Noise traders. I denote with u the total order from noise traders and assume that u can take on values 1 and -1 with equal probability $\frac{1}{2}$.[7] Furthermore, u and v are assume to be independently distributed.

Fund managers. I assume that there is a large population of fund managers. There are two types of fund managers in the population, indexed with $i = \{g, b\}$. Types g are perfectly informed about the true liquidation value of the risky asset, while types b are completely uninformed. I assume that a fund manager knows his own type and that this is private information. It is instead common knowledge that $\Pr(i = g) = \theta$. If a fund manager is hired, he decides the order $x \in \mathbb{R}$ to trade.

Investors. Investors are completely uninformed about the true liquidation value of the asset and cannot invest directly.[8] An investor is given the choice either to refrain from trading or to delegate trade to a fund manager.

Market makers. Finally, there is a large population of market makers who compete à la Bertrand to set the price p at which the asset is traded. Market makers observe only the aggregate market order $z = x + u$ and not the single orders x and u. Based on the observation of z, the market maker sets the price p at which trade takes place. The assumption of competition à la Bertrand coupled with that of risk neutrality implies that each market maker will set a semi-strong efficient price:

$$p = E(v|z = x + u) \tag{1}$$

Basically, (1) implies that market makers get zero profits in expectation, reflecting competition à la Bertrand.

All previous agents are assumed to be risk neutral.

The delegation contract. I assume that if an investor chooses to delegate trade, he hires the manager according to the following exogenously specified linear contract. Denote with $R(x) = x(v - p)$ the return that a fund manager delivers

[7] A negative order is to be interpreted as a supply of the asset.

[8] This is done for simplicity and is without loss of generality Giving the investor the possibility of trading directly does not alter our results. Indeed, since the investor is assumed to be completely uninformed, the choice of investing directly produces zero payoffs in expectation, that is the same expected payoff obtained by refraining from trading.

by trading quantity x when the liquidation value of the asset is v and the price set by the market maker is p. Then, the fund manager gets the fraction $\alpha \in (0, 1)$ of the return delivered plus a fixed fee $\beta > 0$. Both α and β are exogenous (i.e. they are parameters of the game). Basically, a fund manager trading x gets $\alpha R(x) + \beta$, while the investor who has hired him gets $(1 - \alpha)R(x) - \beta$.[9] There are a couple of considerations that are worth to be done.

First, in the present setup, the linear contract under consideration does not represent a performance-related compensation scheme. Performance-related fees refers to *relative* performance, that is, they are fees that the manager receives (pays) for over-performing (under-performing) a given *benchmark*. They do not refer to absolute performance, which is instead the case of the linear contract $\alpha R(x) + \beta$ at hand. In fact, this linear contract (with $\alpha > 0$) represents pretty well the form of payment used by large in the industry. Indeed, most of the mutual funds charge a fee that is given by a fixed percentage of the total amount of money under management. Now, let this percentage be τ. Then, if the total funds initially received by the manager are equal to F, after management they will be equal to $F(1+r)$, where r is the percentage return delivered by the manager. Accordingly, the payment from the investor to the manager is equal to $\tau F(1 + r) = \tau F + \tau F r$, from which it is clear that there is a fixed payment (τF) plus a payment that is related to *absolute* performance ($\tau F r$).

Second, in the dynamic extension of the basic model presented here, the fixed component β plays a key role in determining the result that bad managers trade with positive probability when career concerns are at play. Indeed, career concerns are represented by the concern of a manager of being fired and giving up future profits. As it will be clear later, in the present model β represents the only source of future profits for a bad managers. In other words, while the retaining decision of the investor represents the implicit incentive related to having a good reputation, β is the explicit incentive that makes reputation effects be effective.

Timing. At the beginning of the trading session, the following events take place. First, an investor is randomly selected from the pool of all investors and given the choice of refraining from trading or hiring a fund manager. If the fund manager is hired, he observes his own type and then chooses the quantity x he desires to trade on behalf of the investor. At the same time, noise trade u realizes. The fund manager does not observe the realization of u. Once orders x and u have been formed, they are submitted to the market maker who sets the price according to (1) and trade takes place. At the end of the trading session, the true liquidation value of the asset is realized and payoffs are distributed. Of course, payoffs depend on the terms of the

[9] Notice that noise traders get $\pi_n = u(v - p)$, while market makers $\pi_{mm} = -z(v - p)$.

delegation contract between the investor and the manager.

2.2 Equilibrium analysis

In the absence of career concerns, a set of equilibria is described by the following proposition.

Proposition 1 *Given the previous assumptions and market structure, for any pair* (α, β) *such that* $\alpha \in (0,1)$ *and* $\beta \in \left(0, \frac{3}{4}\theta(1-\alpha)\right]$ *there exists an uncountable family of perfect Bayesian equilibria, indexed by* $q \in \left[\frac{2}{5}, \frac{2}{3}\right)$ *such that:*

i) An investor always hires a fund manager

ii) The trading strategies of good and bad managers are respectively given by:

$$X_{g,q}(v) = \begin{cases} 1+q & \text{when } v = 2 \\ 1-q & \text{when } v = 1 \end{cases}, X^*_{g,q}(-v) = -X^*_{g,q}(v) \tag{2}$$

$$X^*_{b,q} = 0 \tag{3}$$

iii) The pricing strategy of market makers is such that for the set of equilibrium trades $z \in \Psi = \{-2-q, \, -2+q, \, -1, \, -q, \, q, \, 1, \, 2-q, \, 2+q\}$,

$$P_q(z) = \begin{cases} 0 & \text{when } z = 1 \\ \frac{1}{2} & \text{when } z = q \\ 1 & \text{when } z = 2-q \\ 2 & \text{when } z = 2+q \end{cases}, P^*_q(-z) = -P^*_q(z) \tag{4}$$

while for the set of out-of-equilibrium trades $z \in \mathbb{R} \setminus \Psi$,

$$P_q(z) = \begin{cases} 2 & \forall z > 1 \\ 2z & \forall z \in [0,1] \end{cases}, P_q(-z) = -P_q(z) \tag{5}$$

The formal proof of proposition 1 can be found in appendix A. The intuition of the previous result is straightforward. Due to the presence of noise trade, prices do not fully reflect private information. Hence, good managers with complete information trade whenever they are hired by an investor. Being aware of the fact that with positive probability an informed manager is active in the market, market makers set semi-strong efficient prices to protect themselves from the presence of asymmetric information. Consequently, a manager with no information always expects a negative return on his trade and thus never trades. Since bad managers do not trade while good managers always trade and do it under complete information, on average the return delivered by a manager is positive. Under the linear delegation contract described above, the investor enjoys the fraction $(1-\alpha)$ of this average return, while paying a fixed fee equal to β for delegating trade to a manager. Since an investor has the option of staying out of the market, he will delegate trade if and only if the expected payoff of delegation is positive. In the equilibria identified by proposition 1, the average return of the mutual fund industry is equal to $\frac{3}{4}\theta$. Thus, as long as $\beta < \frac{3}{4}\theta(1-\alpha)$, the cost of hiring a manager is lower that its benefits and delegation always takes place.

The bounds on q guarantee that the optimal trading strategy of the good manager is strictly monotonic with respect to his signal and "non-perverse", that is, the good manager buys when observing positive values of the asset and sells when observing negative values. Notice that these two properties would in general be guaranteed by simply imposing that $0 < q < 1$. In fact, the stricter condition required by proposition 1 comes from the incentive compatible constraints of the good manager and thus also guarantees that the good manager has not any incentive to deviate from the prescribed equilibrium strategy.[10] There is a simple intuition for why q must be not too small. In equilibrium, the “scale of trade” does not affect prices in this model: changing q does not change the prices at which orders $1-q$ and $1+q$ are respectively liquidated. However, choosing $1+q$ will lead on average to higher prices than choosing $1-q$. When informed managers observe that the asset is worth 2, they might be tempted to trade $1-q$ in order to induce a lower price and make a higher profit on each unit of the asset. To avoid this occurrence, q must be high enough to make $1+q$ sufficiently large so as to offset the lower margin that informed mangers make on each unit when trading $1-q$. The same logic applies to explain why q must be not too high. Given the price schedule for out of equilibrium aggregate orders, the good manager could be tempted to place very small orders to induce low prices. Since changing q does not change the prices at which orders $1-q$ are liquidated, the size of any order $1-q$ must be large enough to guarantee that equilibrium profits be sufficiently high to make such deviations not appealing.

It is useful to remind that proposition 1 identifies *a* family of perfect Bayesian equilibria of the static game described above. First of all, the specification of the out of equilibrium beliefs and of the consequent out of equilibrium price function is crucial in determining the bounds of q within which the class of equilibria of proposition 1 is defined.[11] Furthermore, proposition 1 focuses on good managers' "symmetric"

[10] See conditions (26) and (32) in the proof of proposition 1

[11] It can be shown that there always exists *a* continuous price function that prevents the good manager from deviating from his equilibrium strategy X_g^* as long as q takes values in subsets of $\left[\frac{1}{3}, \frac{2\sqrt{2}}{3}\right]$. Thus, $\left[\frac{1}{3}, \frac{2\sqrt{2}}{3}\right]$ is also the largest set of values that q can in principle assume in an equilibrium where the good manager trade according to X_g^* (proof available under request).

strategies, that is, strategies represented by (2). If not explicitly specified, in what follows I will focus only on perfect Bayesian equilibria with symmetric strategies for good managers. This keeps the analysis more tractable without losing anything from a qualitative point of view.[12]

3 A Dynamic framework

Let me consider the simplest dynamic extension of the basic model analyzed in the previous section. I will assume that the risky asset can now be traded in two sequential rounds of trade $s = 1, 2$. At the end of each round, the asset pays some dividends, which I denote with v_1 and v_2 respectively. For simplicity, I assume that v_1 can take on values $-2, -1, 1, 2$ with equal probability $\frac{1}{4}$ and that v_1 and v_2 are *i.i.d.*[13] In each round of trade, noise trade can take on values -1, 1 with equal probability $\frac{1}{2}$. Let $u_1$and u_2 denote respectively noise trade at $s = 1$ and 2 and assume that u_1 and u_2 are *i.i.d.* Furthermore, u_1, u_2, v_1 and v_2 are assumed to be independently distributed.

In each round of trade $s = 1, 2$, the trading protocol is the same as that described for the static model presented in the previous section. A fund manager active at round s observes his own type and accordingly chooses the quantity x_s to trade on behalf of the investor who has hired him. At the same time, noise trade u_s realizes (the active manager does not observe the realization of u_s). Then, orders x_s and u_s are submitted to a market maker who observes only the aggregate order $z_s = x_s + u_s$ and accordingly sets a semi-strong efficient price $p_s = E(v_s|z_s)$.[14] At the end of each round of trade $s = 1, 2$, dividends v_s realize.

I assume that at the beginning of the first round of trade, an investor is randomly drawn from the pool of all investors and given the choice to delegate trade to a fund manager or to stay out of the market. The delegation contract is exogenously

[12] See appendix A for more details about equilibria where good managers use "non-symmetric" strategies.

[13] The idea is that between period 1 and 2 there occurs an event unpredictable at the beginning of period 1 and relevant for the financial situation of the company that issued the asset.

[14] In a dynamic model with more than two periods, say $s = 1, 2, ...T$, it is important to specify what the market maker can observe (on top of v_s) before setting prices at $s + 1$. Indeed, the way the market maker sets prices in a given round of trade, say $s + 1$, depends on his belief about the quality of the manager trading at $s + 1$. And this belief is formed based on all the information the manager has collected up to $s + 1$. This assumption is not necessary in a two-period model. Indeed, as we will see soon, the equilibrium is such that only good managers do trade in the second and last period. Therefore, the market maker knows that the manager active in the last round of trade is good (notice that, in fact, the market does not close because of the presence of noise trade u).

specified and given by the linear contract described in the previous section. In order to evaluate the effects produced by the presence of career concerns, I will compare two different contractual setups. In the first setup, the linear delegation contract that is signed at the beginning of $s = 1$ lasts until the end of $s = 2$ (*long-term contracting*). It is apparent that under this contractual arrangement, there is no scope for career concerns since the manager hired at the beginning of the first round of trade is sure to remain in charge in the second round as well.[15] In the second setup, the delegation contract is assumed to last for only one round of trade (*short-term contracting*). Furthermore, it is assumed that at the end of $s = 1$, the investor observes the action taken by the incumbent manager, compares it with the realized value of the dividend and consequently chooses wether to retain the incumbent manager or fire him.[16] If the incumbent manager is retained, he keeps his own type.[17] If instead the incumbent manager is fired, the investor can further choose either to hire a new manager by drawing him from the existing pool of managers or to stay out of the market.

For simplicity, I assume that there is no discounting.

3.1 No career concerns

Assume that the linear delegation contract that is possibly signed between an investor and a fund manager at the beginning of $s = 1$ lasts until the end of $s = 2$ (*long-term contracting*). In particular, *in each round of trade* the investor agrees to pay the manager a fixed fee $\beta > 0$ plus a fraction $\alpha \in (0, 1)$ of the return delivered by the manager. The following proposition characterizes a family of perfect Bayesian equilibria of the dynamic game described above in the case of long-term contracting.

[15] Alternatively, career concerns could have been eliminated by assuming that the decision of an investor to retain or to fire a manager is not affected by manager's performance in the first round, but depends on some other exogenously unspecified reasons. For example, by assuming that the incumbent manager is retained with an exogenous probability equal to $\gamma \in [0, 1]$ (see Dasgupta and Prat 2006).

[16] Remember that at the end of $s = 1$, v_1 is realized and observed by all market participants. Investors can then compare it with the performance delivered by their manager and accordingly form a belief about the quality of the manager. As we will see, I will formally model this updating process by letting investors compare v_1 with the order x_1 submitted by the manager, rather than with his performance. In fact, if we assume that investors can also observe the price that has been paid/recieved by the manager, in equilibrium there is a one-to-one relation between performance and trade order.

[17] The rationale behind this assumption is that if a fund manager is well informed about a given company, then he is likely to receive valuable information whenever a new relevant event that affects the company occurs (remember that in the present model there is only *one* asset). Clearly, this is not without loss of generality. One could easily pick "transition probabilities" on the manager's type to reverse or invalidate career concerns altogether.

Proposition 2 *Under long-term contracting, for any pair (α, β) such that $\alpha \in (0,1)$ and $\beta \in \left(0, \frac{3}{4}\theta(1-\alpha)\right]$ there exists an uncountable family of perfect Bayesian equilibria, indexed by $q \in \left[\frac{2}{5}, \frac{2}{3}\right)$ such that delegation takes place and in each round of trade $s = 1, 2$, managers and market makers behave as prescribed by proposition 1*

I will not formally prove the previous result, since it simply amounts to iteratively apply the prove of proposition 1 by backward induction. Notice that delegation takes as long as β is low enough to guarantee that the costs of hiring a manager through the two rounds of trade are less than the its benefits. It can be easily shown that if the investor hires a good manager, the expected return from his trade is equal to $\frac{3}{4}$ in each round. If instead the manager is bad, the return delivered in each round is 0. Thus, the total return expected from an average manager hired at the beginning of $s = 1$ is equal to $\theta\left(\frac{3}{4} + \frac{3}{4}\right) + (1-\theta)\,0 = \frac{3}{2}\theta$ and the expected benefits from delegation $(1-\alpha)\,\frac{3}{2}\theta$. On the other hand, the costs of hiring a manager for the two rounds of trade are equal to 2β. Thus, as long as $\beta < \frac{3}{4}\theta(1-\alpha)$ delegation takes place.

3.1.1 Trade volume without career concerns

Let me consider the trade volume that is expected in a financial market where career concerns of portfolio managers are absent. In each round of trade, the volume of trade expected from noise traders amounts to 1:

$$E(V_1^u) = E(V_2^u) = \frac{1}{2}\,|1| + \frac{1}{2}\,|-1| = 1$$

Thus, the total volume of trade coming from noise traders is equal to 2. Let me now consider the expected contribution of fund managers in the two rounds of trade. Based on managers' equilibrium strategies outlined in proposition 2, we can easily compute:

$$E(V_1^i + V_2^i) = \theta E(V_1^i + V_2^i | i = G) + (1-\theta)\, E(V_1^i + V_2^i | i = B) = 2\theta$$

where the result comes from the fact that:

$$\begin{aligned} E(V_1^i + V_2^i | i &= G) = 2\left(\frac{1}{4}\,|1+q| + \frac{1}{4}\,|1-q| + \frac{1}{4}\,|-1+q| + \frac{1}{4}\,|-1-q|\right) = 2 \\ E(V_1^i + V_2^i | i &= B) = 2(1-\theta)0 = 0 \end{aligned}$$

Accordingly, the aggregate volume of trade that is expected to take place in the market reads:

$$E(V_1 + V_2) = 2 + 2\theta \tag{6}$$

3.2 Career concerns

Consider now the case in which the delegation contract lasts for only one round of trade (short-term contracting). Furthermore, assume that at the end of the first round of trade, the investor can decide whether to retain the incumbent manager or fire him and possibly hire a new one.

Loosely speaking, the investor makes this decision evaluating the behavior of the incumbent manager in the first round of trade: if at the eyes of the investor the manager has done a good job, he will be retained; if his performance is judged to be poorer than that of an average manager, he is fired and possibly substituted with a new manager.

Formally, I assume that at the end of $s = 1$, the investor can observe both the realized value of the dividend, v_1 and the trade choice of the incumbent manager, x_1. Hence, given these observations and the *equilibrium* strategies of good and bad managers, the investor forms a belief that the incumbent manager is good. Let me denote with $\widehat{\theta}$ this belief, with

$$\widehat{\theta} = \Pr(i = g | v_1, x_1)$$

Since at the end of $s = 1$ the investor has the option of replacing the incumbent manager with a new manager (who has probability θ of being good), the incumbent will be retained if and only if $\widehat{\theta} > \theta$, that is, if and only if according to the investor he is more likely to be good than an average manager[18] Thus, as long as staying in the market for an additional round of trade turns out to be appealing to a manager, this decisional process of the investor creates career concerns on the part of managers. Indeed, any manager trading in the first round is aware of the fact that his trade choice will affect his probability of being retained; accordingly, he will be more inclined to make those trade choices that are likely to increase the probability of being retained. As we will see, in the case of bad managers, these choices are not optimal.

In our simplified world, good managers have complete information about the true liquidation value of the asset. This implies that good managers expect a positive return from trading and thus *always* trade. Furthermore, complete information also implies that they always make the correct choice when trading and accordingly are always retained with probability 1 (we could state that in fact good managers do not face career concerns. On the other hand, bad managers are completely uninformed about the true liquidation value of the asset. Thus, the return that they expect

[18] It is important to stress that this decisional rule is *endogenously* determined in equilibrium.

on trade is always negative and the optimal choice would be that of not trading. However, since good managers always trade, the absence of trade would immediately signal to the investor the fact that the manager is bad and imply hat the manager is fired with probability 1. Therefore, the only chance that a bad manager has to be retained is that of engaging in uninformative trade in the hope of striking a good performance and being perceived as good by the investor.

Notice that a bad manager will in fact engage in uninformative trade if and only if the option of staying in the market for an additional round of trade turns out to be sufficiently attractive. In the second and *last* round of trade there is no scope for career concerns and therefore bad managers never trade. Accordingly, the profits of a bad manager in the second round of trade amount to the fixed fee β. Hence, a bad manager who is active in the first round has to trade off the negative expected return $\alpha R_b^e(x)$ of his uninformative trade with the expected benefits that this trade might bring about in terms of enhancing the probability of enjoying future profits β. It is then clear that career concerns lead to (non-optimal) uninformative trade only for a set of (short-term) delegation contracts characterized by values of α relatively small to those of β.

In the following proposition, I formally characterize a family of perfect Bayesian equilibria in which delegation takes place as a rational choice of investors and bad managers trade without information in the first round as a consequence of career concerns.

Proposition 3 *Assume short-term contracting. Assume further that at the end of* $s = 1$ *the investor can fire the incumbent manager and substitute him with a new one. For any pair of contractual parameters* (α, β) *such that* $\alpha \in \left(0, \frac{5(2-\theta)}{10-\theta}\right)$ *and* $\beta \in \left(\frac{3\alpha\theta^2}{5(2-\theta)}, (1-\alpha)\frac{3}{4}\theta\right)$*, the two-round trade game described above admits an uncountable family of perfect Bayesian equilibria indexed by* $q \in \left[\frac{2}{5}, \frac{2}{3}\right)$ *such that:*

i) delegation takes place in both rounds of trade. At the end of the first round, the investor retains the incumbent manager if and only if

$$\widehat{\theta} > \theta \tag{7}$$

where $\widehat{\theta} = \Pr(i = g|v_1, x_1)$*, otherwise he fires him and hires a new manager.*

ii) In both rounds, good managers trade according to the following strategy:

$$\begin{array}{l} X_{s,g,q}^*(v_s) = \begin{cases} 1+q & \textit{when } v_s = 2 \\ 1-q & \textit{when } v_s = 1 \end{cases}, \; s = 1, 2 \\ X_{s,g,q}^*(-v_s) = -X_{s,g,q}^*(v_2) \end{array} \tag{8}$$

iii) bad managers do not trade in the second round, while they do trade in the first round according to the following (mixed) strategy:

$$X_{1,b,q}^{*} = \begin{cases} 1-q & \textit{with probability } \frac{1}{2} \\ -1+q & \textit{with probability } \frac{1}{2} \end{cases} \tag{9}$$

iv) In the first round, the price strategy of market makers is such that for the set of aggregate equilibrium orders $z_1$reads:

$$\begin{array}{l} P_{1,q}^{*}(z_1) = \begin{cases} 2 & \textit{when } z_1 = 2+q \\ \frac{\theta}{2-\theta} & \textit{when } z_1 = 2-q \\ \frac{\theta}{2} & \textit{when } z_1 = q \end{cases} \\ P_{1,q}^{*}(-z_1) = -P_{1,q}^{*}(z_1) \end{array} \tag{10}$$

In the second round, for equilibrium aggregate orders z_2 market makers set prices according to:

$$\begin{array}{l} P_{2,q}^{*}(z_2) = \begin{cases} 0 & \textit{when } z_2 = 1 \\ \frac{1}{2} & \textit{when } z_2 = q \\ 1 & \textit{when } z_2 = 2-q \\ 2 & \textit{when } z_2 = 2+q \end{cases} \\ P_{2,q}^{*}(-z_2) = -P_{2,q}^{*}(z_2) \end{array} \tag{11}$$

In both rounds of trade market makers reacts to out-of-equilibrium aggregate orders z_s by setting prices according to

$$\begin{array}{l} P_{s,q}(z_s) = \begin{cases} 2 & \forall z_s > 1 \\ 2z_s & \forall z_s \in [0,1] \end{cases} , \; s = 1,2 \\ P_{s,q}(-z_s) = -P_{1,q}^{*}(z_s) \end{array} \tag{12}$$

The formal proof of proposition can be found in appendix A. Good managers have complete information. Their buying and selling strategy is always optimal and makes them sure that they will be retained with probability 1. Accordingly, their trading strategy is not affected by career concerns and indeed is invariant across the two rounds and is exactly the same strategy described in propositions 2 and 1 for the case of no career concerns. Thus, the intuition behind the strategy of good managers and the relative bounds on q is exactly the one given in the discussion of proposition 1. For bad managers', things do in fact change with respect to the case of no career concerns. Their trading behavior is strongly conditioned by the concern of being fired. Since bad managers are completely uninformed, the optimal thing to do for them should be that of not trading. In fact, in the second round, where there is no scope for career concerns, they optimally refrain from trading. However, in the first round they know that the absence of trade would immediately signal their bad type, leading to their dismission. Career concerns pushes them to trade even in the presence of a negative expected return due to the absence of information. The upper bound on α and the lower bound on β make sure that the negative expected return $\alpha R_b^e(x)$ of their uninformative trade is low enough, while future profits β are high enough to make it appealing to the bad manager to engage in uninformative trade in the hope of being retained. Bad managers' equilibrium strategy $X_{1,b}^*$ amounts to mimicking the good manager by randomly placing intermediate orders $1-q$ or $-1+q$, in the hope that the chosen order reveals correct ex-post, inducing the investor to perceive the manager as good. Notice that strategy $X_{1,b}^*$ maximizes the probability of being retained, while minimizing the expected losses from trade. Indeed, placing either order $1+q$ or $-1-q$ leads to same probability $\frac{1}{4}$ that the manager is retained as $X_{1,b}^*$. However, the (negative) return $\alpha R_b^e(x)$ expected by the bad manager is increasing in the order size. Thus, intermediate orders are preferred to large ones.

Notice that the return that good managers expect in the first round is higher than the one they expect in the second round. These expected returns read respectively:

$$\begin{aligned} E(R_{1,g}) &= 1 - \frac{\theta}{4(2-\theta)} + q\left(\frac{\theta}{4(2-\theta)} - \frac{1}{4}\theta\right) \\ E(R_{2,g}) &= \frac{3}{4} \end{aligned}$$

Since $\theta \in (0,1)$ and q is positive, $E(R_{1,g})$ is always greater than $E(R_{2,g})$. This result is not surprising if we consider that the uninformative trade of bad managers in the first round increases the level of noise present in the market and consequently allows good managers to strike better prices. In line with his interpreation, $E(R_{1,g})$ is decreasing in the probability θ of being a good manager: clearly, the higher the fraction of good managers in the population, the smaller that of traders who lose to market makers, which will in turn react more severely to information asymmetry. In particular, notice that

$$\lim_{\theta \to 1} E(R_{1,g}) = E(R_{2,g})$$

As θ approaches 1, market makers post higher prices and $E(R_{1,g})$ decreases to $E(R_{2,g})$ which is indeed the return that good managers expects in the second round, when bad managers do not trade. By proposition 2, we can easily compute that in the absence of career concerns $E(R_{1,g}) = E(R_{2,g}) = \frac{3}{4}$. Thus, good managers enjoys the inefficient behavior of bad managers' uninformative trade brought in by career concerns. It is important to stress that this result is the consequence of market makers reacting more harshly in the framework with no career concerns. Thus, a possible implication of the analysis is that prices are less volatile when a sizable fraction of traders bear career concerns.

Delegation takes always place because the upper bound on β makes sure that in every round of trade the cost of hiring a manager is less than the expected benefits of delegation. The expected benefits of delegation are strictly positive thanks to the presence of good managers who trade under complete information. It is easy to compute the expected returns from delegation in the two rounds of trade, which read:

$$\begin{aligned} ER_1 &= \left(1 - \frac{1}{4}\theta\right)\theta \\ ER_2 &= \frac{3}{4}\theta \end{aligned}$$

Clearly, these returns are increasing in the probability θ that a good manager is selected to trade. It is interesting to notice that average returns from delegation do not depend on the size of trade q. Furthermore, $ER_2 > ER_1$. This is seemingly surprising if we consider that in the second round bad managers behave optimally (by not trading), while they engage in uninformative trade in the first round. The reason why $ER_2 > ER_1$ has to be found in the higher return that good managers can deliver in the first round thanks to the higher level of noise. The positive effect that better prices have on the return of good managers more than offsets the negative impact that bad managers' trade has on first-round average return. Notice further that the average return on delegation is always positive, no matter how big is the fraction of bad manager (as $\theta \to 0$, it approaches zero). This is seemingly surprising and depends on the fact that prices respond endogenously to the level of information present in the market. Thus, the higher the fraction of bad manager, the lower the prices set on average by market makers and the lower the loss sufferred by bad managers. At the limit, when $\theta \to 0$, market makers know that there is no information in the market and set the price of the asset equal to its expected value, thai is zero and the loss of bad managers is reduced to zero.

A word of caution is needed with respect to the generality of the previous two results. First of all, the model assumes that good managers are the only informed traders present in the market. Thus, they enjoy all the increase in noise.[19] If there

[19] This result suggests that in a dynamic model where there is only one insider who does not always have private information, but rather is informed with probability θ in each period, the optimal strategy of the insider could possibly be that of trading also when he does not have information in order to increase the level of noise and enjoy higher returns in periods when he does have information

were other informed traders, these benefits should be shared among more traders and the increase in good managers' return would be lower as well as the average return on delegation, which could possibly become negative. Second, the result of a higher average return on delegation is achieved also because in the specific model at hand, bad managers' uninformative trade is designed as to minimize the negative return that bad managers expect (randomizing over $1-q$ and $-1+q$ rather than on $1+q$ and $-1-q$). As we will see in the next section, this is not always the case. Finally, rember that we are assuming investors' risk neutrality. Thus, in the model under consideration any increase i the average return on delegation is welcome by investors. However, if investors were risk averse, then they would dislike the increase in the variability of managers' returns that occurs with churning.

Staying to our model, it is clear that noise traders are the one who pay for the non optimal behavior of bad managers. It is easy to check that the expected loss of noise traders is higher under career concerns. What if we consider the change in the aggregate return of investors and noise tarders? Focus on the first round of trade. Notice that since market makers xpects zero profits in equilibrium, it must be that

$$E\left(R_{1,u}\right)=-E\left(R_1\right)$$

Investors enjoys $\alpha E\left(R_1\right)$. Thus, the aggregate return of investors and noise traders read

$$-E\left(R_1\right)+\alpha E\left(R_1\right)=-\left(1-\alpha\right)E\left(R_1\right)=-\left(1-\alpha\right)\left(1-\frac{1}{4}\theta\right)\theta$$

What is the situation in the absence of career concerns and churning?

$$-E\left(R_1\right)+\alpha E\left(R_1\right)=-\left(1-\alpha\right)E\left(R_1\right)=-\left(1-\alpha\right)\frac{3}{4}\theta$$

Now, it is apparent that $-\left(1-\alpha\right)\left(1-\frac{1}{4}\theta\right)\theta<-\left(1-\alpha\right)\frac{3}{4}\theta$ and thus, churning makes the class of individuals represented by investors&noiose traders worse.

3.2.1 Trade volume with career concerns

As we did for the case in which there were no career concerns, we can compute the total volume of trade that is expected when career concerns are relevant. As for the model with no career concerns, the volume of trade expected from noise traders is equal to 1 in each round of trade. Let me focus on the expected contribution of the managers across the two rounds of trade. In the first round of trade, both bad and

(of course, this presumably occurs when θ is high enough).

good managers trade and expected volume reads:

$$E(V_1^i) = \theta\left(\frac{1}{4}|1+q| + \frac{1}{4}|1-q| + \frac{1}{4}|-1+q| + \frac{1}{4}|-1-q|\right) +$$
$$+ (1-\theta)\left(\frac{1}{2}|1-q| + \frac{1}{2}|-1+q|\right) = \theta + (1-\theta)(1-q)$$

In the second and last round only good managers trade and the volume of trade expected for in this stage reads:

$$E(V_2^i) = \theta\left(\frac{1}{4}|1+q| + \frac{1}{4}|1-q| + \frac{1}{4}|-1+q| + \frac{1}{4}|-1-q|\right) +$$
$$+ (1-\theta)0 = \theta$$

Hence, the volume of trade that is expected to come from managers is given by:

$$E(V_1^i + V_2^i) = 2\theta + (1-\theta)(1-q)$$

Accordingly, the total expected volume of trade is the sum of noise traders' contribution and managers' contribution over the two rounds of trade, and reads:

$$E(V_1 + V_2) = 2 + 2\theta + (1-\theta)(1-q) \tag{13}$$

Remember that in the absence of career concerns, the expected volume of trade was equal to 2θ. It is apparent that for any value of $\theta \in (0,1)$ and $q \in \left[\frac{2}{5}, \frac{2}{3}\right)$, it is true that $2 + 2\theta + (1-\theta)(1-q) > 2 + 2\theta$. Thus, we can conclude that the presence of career concerns leads to an increase in the expected volume of trade. The additional volume $(1-\theta)(1-q)$ stems exclusively from reputation reasons. In fact, this additional amount of trade comes all from bad managers, who do not possess any valuable information about the liquidation value of the asset.

4 Trade volume and asset volatility

In the present section, I analyze the effects of changes in asset volatily on the trading strategies of managers concerned with their reputation. In order to investigate this issue, consider the two-period trade game described in the previous section and allow for a more general probability distribution of the the liquidation value of the asset traded in the first round v_1. In particular, let $\Pr(v_1 = 2) = \Pr(v_1 = -2) = w$

and $\Pr(v_1 = 1) = \Pr(v_1 = -1) = \frac{1}{2} - w$, with $0 < w < \frac{1}{2}$. Thus, $E(v) = 0$ and $VAR(v) = 6w + 1$. It is apparent that higher values of w are associated to higher values of the variance for the asset traded at $s = 1$.[20]

I will show that as long as α is small enough, when the variance of the asset increases over a given threshold, bad managers increase the volume of their uninformative trade. In fact, the main result is derived under the assumption that $\alpha \to 0$ and $\beta > 0$. Assuming $\alpha \to 0$ allows us to focus on the effects that reputation concerns have on the trading strategies of managers, since in this case the performance of the a manager does not affect his payoff directly, but only through the probability of being retained.

Trueman (1988) shows that the probability that bad managers churn increases with the variance of the asset, predicting that noise trade is more likely to occur for riskier asstes. However, his partial equilibrium framework prevents him to carefully discuss the impact on trade volume. In the present model, the probability that bad managers engage in uninformative trade is fixed at $1 - \theta$. However, the size of their orders is found to be sensible to the volatility of the asset. In particular, when volatility gets above a given threshold, bad managers switch their trading strategy of randomizing over $1 - q$ and $-1 + q$ and start randomizing over orders $1 + q$ and $-1 - q$.

The intuition is straightforward. In equilibrium, the strategies of good managers are strictly monotonic with respect to the value of the asset. Since good managers are risk neutral and can perfectly observe the value of the asset, their strategies do not depend on the probability distribution of the values of the asset.[21] Bad managers are completely uninformed about the true value of the asset. Since they know what is the optimal equilibrium strategy of good managers, they submit those orders compatible with the strategy of good managers that are more likely to reveal correct ex post. A *low* variance of the values of the asset arises when the intermediate values of the asset (in the present case 1 and -1) are more likely to occur than the

[20] Assuming a more general distribution function only for v_1, while keeping the same distribution I used in the previous sections for v_2 is done for simplicity and it without loss of generality

[21] Since we are assuming $\alpha \to 0$, one may wonder why good managers should condition their order on the value of the asset they observe. After all, when $\alpha \to 0$, their contract leaves them indifferent to any return they may produce (they get β in any case). In fact, the incentives related to reputation force them to behave in the interest of the investor and follow a trading strategy that maximizes the expected return given the information they possess. Indeed, loosely speaking, the incentives related to reputation are such that if ex post an investor realizes that the manager has adopted an ex-ante non optimal strategy, he fires him. This will formally be translated into equilibrium beliefs by investors about the optimal trading strategy of good managers and a consistent hiring/firing strategy.

extreme values (in our case 2 and -2). Accordingly, from the point of view of our bad managers, it is more likely that good managers submit either order $1-q$ or $-1+q$ than orders $1+q$ or $-1-q$. Therefore, being completely uninformed, bad managers maximize their chances to be correct by randomizing over $1-q$ and $-1+q$. High volatility is associated to extreme values of the asset that are more likely to occur than intermediate ones. In this case, the bad managers maximizes his chances to trade correctly by randomizing over $1+q$ and $-1-q$, since those are the orders that most likely will be placed by a good manager.

Since the optimal strategy of good managers is not affected by the probability distribution of the values of the asset, the aggregate volume of trade is also shown to increase when moving fromstates of low variance to states of high variance. The previous discussion is formalized in the next two propositions. In order to simplify the mathematics, all results have being obtained under the assumption of $\theta = \frac{1}{2}$. It is possible to show that from a qualitative point of view, the result of proposition 4 part a) holds for every $\theta \in (0,1)$, while the result of part b) holds as long as θ is "not too small".

Proposition 4 *Let $\theta = \frac{1}{2}$ and $\alpha \to 0^+$. Then, the following occurs:*

a) When $0 < w \leq \frac{1}{4}$, as long as $0 < \beta \leq \min\left\{\frac{3}{8}, \frac{5}{2}w - 3w^2\right\}$, there exists an uncountable family of Bayesian Nash equilibria indexed by $q \in \left[\frac{2}{5}, \frac{2}{3}\right)$, in which good managers, bad managers and investors behave as described by proposition 3. In particular, in the first period bad managers randomize with equal probability over $1-q$ and $-1+q$.

b) When $\frac{1}{4} < w < \frac{1}{2}$, as long as $0 < \beta \leq \frac{1+4w-12w^2}{4}$, there exists an uncountable family of Bayesian Nash equilibria indexed by $q \in \left[\frac{2}{5}, \frac{2}{3}\right)$ in which good managers and investors still behave as described by proposition 3, while bad managers randomize with equal probability over $1+q$ and $-1-q$ in the first period and do not trade in the last one.[22]

[22] The pricing strategy of market makers in the family of equilibria in 7a is given by $P_1(z_1) = -P_1(-z_1)$, where

$$P_1(z_1) = \begin{cases} 2 & z_1 = 2+q \\ \frac{1-2w}{2(1-w)} & z_1 = 2-q \\ \frac{6w-1}{2} & z_1 = q \end{cases}$$

In the family of equilibria in 7b market makers set prices according to $P_1(z_1) = -P_1(-z_1)$, where

$$P_1(z_1) = \begin{cases} \frac{4w}{1+2w} & z_1 = 2+q \\ 1 & z_1 = 2-q \\ \frac{12w-1}{4} & z_1 = q \end{cases}$$

The proof of this result is detailed in appendix A. Proposition 4) immediately implies the following result.

Proposition 5 *The maximum value of trade volume expected in the equilibrium with low variance is always lower than the minimum value of trade volume expected in the equilibrium with high variance.*

Proof. Let me denote with I the family of equilibria described in part (a) of proposition 4 and with II the family of equilibria described in part (b) of proposition 4. Since the volume of trade expeted from noise traders is the same in both family of equilibria (it sums up to 1 in each round of trade), let me focus on managers' contribution. It is apparent from proposition 1 that the volume of (noise) trade expected from bad managers in equilibrium of type I is lower than the volume of (noise) trade expected from bad managers in equilibrium of type II.[23] Furthermore, it also follows from proposition 1 that when $0 \leq w \leq \frac{1}{4}$ and thus $1 < VAR(w) \leq \frac{5}{2}$, the expected volume of trade reads:

$$\begin{aligned} V_I^e(w,q) &= \frac{1}{2}\left[w\,|1+q| + \left(\frac{1}{2}-w\right)|1-q| + \left(\frac{1}{2}-w\right)|-1+q| + w\,|-1-q|\right] \\ &+ \frac{1}{2}\left(\frac{1}{2}|1-q| + \frac{1}{2}|-1+q|\right) = \\ &= \frac{1}{2}\left[2w(1+q) + (1-2w)(1-q)\right] + \frac{1}{2}(1-q) = \\ &= 2qw + 1 - q \end{aligned}$$

On the other hand, when $\frac{1}{4} < w < \frac{1}{2}$ and thus $\frac{5}{2} < VAR(w) < 4$, the expected

Both in case 7a and 7b, any aggregate order z_1 inconsistent with the equilibrium is met by out of equilibrium beliefs of the market maker that give rise to the following out of equilibrium price function:

$$P_1(z_1) = \begin{cases} 2 & z_1 \geq 1,\ z_1 \neq 2-q, 2+q \\ 2z_1 & -1 \leq z_1 \leq 1,\ z_1 \neq -q, q \\ -2 & z_1 \leq -1,\ z_1 \neq -2-q, -2+q \end{cases}$$

[23] It may be interesting to compare the proportion of noise trade to that of informed trade in the two types of equilibria.

volume of trade reads:

$$V_{II}^{e}(w,q)=\frac{1}{2}\left(w\left|1+q\right|+\left(\frac{1}{2}-w\right)\left|1-q\right|+\left(\frac{1}{2}-w\right)\left|-1+q\right|+w\left|-1-q\right|\right)+$$
$$\frac{1}{2}\left(\frac{1}{2}\left|1+q\right|+\frac{1}{2}\left|-1-q\right|\right)=$$
$$=\frac{1}{2}\left[2w(1+q)+(1-2w)\left(1-q\right)\right]+\frac{1}{2}\left(1+q\right)=$$
$$=2qw+1$$

Equilibrium multiplicity prevents us from analyzing how the expected volume of trade varies with w within each of the two type of equilibria family I and II (indeed, to any given admissible value of w for which an equilibria family holds it is associated an uncountable number of equilibria indexed by q belonging to that family).

However, notice that the highest expected volume of trade in the first family of equilibria reads:

$$\overline{V_{I}^{e}}\equiv V_{I}^{e}(\frac{1}{4},q)=1-\frac{1}{2}q$$

while the lowest expected volume of trade in the second family of equilibria is given by:

$$\underline{V}_{II}^{e}=V_{II}^{e}(w\rightarrow\frac{1}{4},q)=1+\frac{1}{2}q$$

Therefore, since $q\in\left[\frac{2}{5},\frac{2}{3}\right)$, the maximum value of trade volume expected in the equilibrium of type I is always lower than the minimum value of trade volume expected in the equilibrium of type II.[24] ■

5 Discussion and Conclusion

In the present paper, I showed that career concerns induce uninformed fund managers to indulge in excessive trading. This result has been derived in a order-driven market for a risky asset in which investors delegate trade to differently informed fund managers concerned about being perceived as well informed. The financial market has been model through a *discrete* variant of the classic Kyle (1985), in which both the distributions of the liquidation value of the risky asset and of noise trade are

[24] It is important to stress that this result does not depend on the assumption that $\theta=\frac{1}{2}$. Indeed, in the more general case, volumes of trade read: $V_{I}^{e}(w,q)=4\theta qw+1-q$ and $V_{II}^{e}(w,q)=4\theta qw+1+q-2\theta q$. Accordingly, $\overline{V_{I}^{e}}=V_{I}^{e}(w=\frac{1}{4},q)=1-q\left(1-\theta\right)$ and $\underline{V}_{II}^{e}=V_{II}^{e}(w\rightarrow\frac{1}{4},q)=1+q(1-\theta)$, from which it is apparent that $V_{I}^{e}(w=\frac{1}{4},q)<V_{I}^{e}(w=\frac{1}{4},q)$ for every $\theta\in(0,1)$.

assumed to be discrete. This represents an extension of Dasgupta and Prat (2006), who showed that the same result holds in a market modelled à la Glosten and Milgrom (1985). The setup I used allowed me to analyze how the trading strategies of fund managers are affected by career concerns in markets characterized by different asset's volatility. I showed that high volatility is likely to induce bad managers to increase the size of their noise trade. This is a generalization of Trueman (1988) to a general equilibrium framework.

Conditions for delegation take place are explicitly derived. Analogously, conditions for career concerns be effective are characterized. In this respect, it is worth noticing that in the present model, career concerns arise if and only if both the two following conditions take place. First, investors have to rationally allocate their funds to managers that are perceived as being better informed than their competitors. Second, the compensation that managers receive for managing the fund has to exhibit a fixed component. In particular, this fixed component should be high enough relatively to the performance component in order to provide the manager with the incentives to managing the fund without worrying about the return he delivers. Notice that these considerations are drawn under the assumption that the contract between a manager and an investor is exogenously given. It would be interesting to allow for endogenous contracting and check wether under the optimal contract good manager would specify a fixed payment low to prevent bad managers from engaging in *reputation* trade (and eventually kicking them out of the market). On the other hand, notice that all other things being equal, the return delivered by good managers when bad managers trade is higher. This is due to the fact that bad managers' trade is noise trade. The increase of noise in the market allows good managers to strike better prices and thus higher returns. The analysis above has been done under the assumption that the contract between a manager and an investor is exogenously given. It would be interesting to extend the analysis to allow for endogenous contracting and check wether the positive effects that the presence of bad managers have on good managers' profits lead them to optimally choose a contract that does not determine separation.

It is interesting to notice that the (expected) return from delegation is independent from the order size q, and depends only on the exogenous parameters of the model w and θ (see 72 and 66). While it is apparent that the return from delegation is increasing in θ, we cannot draw a definite conclusions about the way in which it varies with respect to w. Indeed, when w gets higher than $\frac{1}{4}$, there is a switch from an equilibrium of low volume to an equilibrium of high volume. It is worth noticing that in both equilibria, the return from delegation is maximized for $w \to \frac{1}{4}$, when there is the highest degree of information asymmetry between the market maker and the good manager. In particular, the highest return expected in the equilibrium with high volatility (and high volume) is lower than the highest return expected in the equilibrium with low volatility (and low volume). This result is driven by the fact that in the equilibrium with high volatility, bad managers trade more aggressively, generating highest losses.

Appendix A

proof of proposition 1. I will first consider the candidate equilibrium q whereby the trading strategy of the good and the bad managers are given by (2) and (3) and compute the prices that would arise for these strategies, using the property that prices are set as conditional expectations given by (1). I will then show that for these prices, strategies (2) and (3) do maximize the expected profits of good and bad managers. Finally, given (2), (3), (4) and (5), I will work out the conditions under which an investor always finds it convenient to hire a fund manager.

Let $\alpha \in (0,1)$ and $\beta > 0$. Let $z = Z(u,v,i)$ denote the aggregate order for the asset, with $i \in \{g,b\}$ indicating manager's type. In order to ease notation, from now on I will drop subscript q from the strategies of the players and let $X_{q,i}(\cdot) \equiv X_i(\cdot)$ and $P_q(\cdot) \equiv P(\cdot)$.

The market maker's problem. A market maker has to set the regret-free price at which to liquidate trade, based on the observation of the aggregate demand for the asset. Given fund managers' equilibrium strategies (2) and (3), and the possible realizations of noise trade and asset value, the equilibrium aggregate demand for the asset is odd and satisfies:

$$\begin{aligned} Z(u,v,i) &= \begin{cases} 2+q & \text{when } \{u=1, v=2, i=g\} \\ 2-q & \text{when } \{u=1, v=1, i=g\} \\ 1 & \text{when } \{u=1, \forall v,\ i=b\} \\ q & \text{when } \{u=-1, v=2, i=g\} \\ & \text{or} \quad \{u=1, v=-1, i=g\} \end{cases} \\ Z(u,v,i) &= -Z(-u,-v,i) \end{aligned}$$

Focus on positive equilibrium orders $z = q, 1, 2-q, 2+q$. Each aggregate order perfectly reveals which type of manager is in the market. When the aggregate order is equal to 1, the market maker infers that a bad manger has been hired and thus that the aggregate order comes all from noise traders. Accordingly, he sets:

$$P(1) \equiv E(v|1) = E(v) = 0$$

In all the other cases ($z = q, 2-q, 2+q$), the market maker realizes that a good manager is trading. Furthermore, when $z = 2-q, 2+q$, he can perfectly recover the information of the good manager. Accordingly, he sets:

$$\begin{aligned} P(2-q) &= 1 \\ P(2+q) &= 2 \end{aligned}$$

On the other hand, when $z = q$, the market maker cannot tell whether the good manager has observed $v = 2$ (and $u = -1$) or $v = -1$ (and $u = 1$). Since the market maker puts equal probability on these two events[25], his optimal price response to aggregate order q reads:

$$P(q) \equiv E(v|q) = \frac{1}{2}2 + \frac{1}{2}(-1) = \frac{1}{2}$$

Since $Z(u,v,i) = -Z(-u,-v,i)$, it is immediate to compute the prices that the market maker sets in response to negative aggregate orders and show that $P(q) = -P(-q)$.

So far, we have shown that the price response of the market maker to equilibrium aggregate orders (i.e., orders that are consistent with strategies (2) and (3) of good and bad managers) is given by (4).

In order to complete the construction of the price function in the candidate equilibrium q, we need to construct the price schedule according to which the market maker sets prices in response to out of equilibrium aggregate orders. In this respect, market maker's out of equilibrium beliefs about information content of trade are crucial. It is important to notice that in the present game, PBE does not imply any restriction on market maker's out of equilibrium beliefs. Thus, the market maker's price response to any out of equilibrium order can be every price belonging to $[-2, 2]$. I assume the following out of equilibrium beliefs:

$$\begin{aligned} &\forall z > 1,\ z \neq 2-a, 2+a,\ \Pr(v = 2|z) = 1 \\ &\forall z \in [0,1],\ z \neq a, 1,\ \Pr(v = 2) = \frac{z+1}{2},\ \Pr(v = -2) = \frac{1-z}{2} \\ &\forall z \in [-1,0],\ z \neq -a, -1,\ \Pr(v = -2) = \frac{z+1}{2},\ \Pr(v = 2) = \frac{1-z}{2} \\ &\forall z < -1,\ z \neq -2+a, -2-a,\ \Pr(v = -2|z) = 1 \end{aligned}$$

[25] This is a consequence of the specific distributional assumptions we have made for v and u, according to which $v = 2$ and $v = -1$ have equal probability to occur and so do $u = -1$ and $u = 1$, with u and v independent.

It is then immediate to show that the out of equilibrium price function is exactly given by (5).[26]

Managers' problem. We have now to show that given (4) and (5), it is indeed optimal for good and bad managers to follow strategies $X_g^*(\cdot)$ and $X_b^*(\cdot)$ respectively. Given the symmetry of the problem (both $P(\cdot)$ and $Z(\cdot)$ are odd functions), it is sufficient to focus on the analysis of the buy side of the problem, that is, on non negative orders.

Let me define $R(x) = x(v-p)$ as the ex-post return of a manager trading quantity x. Since $p = P(x+u)$, it is useful to write $R(x) = x\left[v - P(x+u)\right]$ to highlight that the ex-post return delivered by the manager depends on his order x, on the liquidation value of the asset v and on noise u. It is then clear that ex-ante, the return that manager i expects from placing order x depends on the information he has about v and u. This information crucially depends on the type of manager. Let Ω_i denote the information set of manager i. Formally, the expected return of manager i from order x reads:

$$E\left(R(x)|\Omega_i\right) = x\left[E\left(v|\Omega_i\right) - E\left(P(x+u)|\Omega_i\right)\right]$$

Notice that the expected return from order x depends on the expectation that manager i holds about the value of the asset minus the expectation about the price he is going to pay when posting order x. Since $\Omega_g = \{v\}$ and $\Omega_b = \varnothing$, we have that:

$$\begin{aligned} R_g^e\left(x,v\right) &\equiv E\left(R(x)|v\right) = x\left[v - E\left(P(x+u)\right)\right] \\ R_b^e\left(x\right) &\equiv E\left(R(x)\right) = x\left[-E\left(P(x+u)\right)\right] \end{aligned}$$

[26] The general idea behind these out of equilibrium beliefs is that whenever the market maker is sure that a manager has posted a positive order, he takes the harshest belief assigning probability 1 to the event that this positive (negative) order is coming from a good manager who has observed $v = 2$ ($v = -2$). Hence, when aggregate order is greater than 1 (less than -1), the market maker is sure that there is a manager posting a positive order (because noise trade is either 1 or -1) and accordingly sets the harshest price $p = 2$ ($p = -2$).

If the aggregate order is between -1 and 1, the situation is more complex because the market maker cannot tell the direction of manager's trade. Consider for example the case $z \in (0,1)$ (being the case $z \in (-1,0)$ symmetric). Any aggregate order between 0 and 1 can be the result of the following two events:

1) $x \in \left]1,2\right[$ and $u = -1$

2) $x \in \left]-1,0\right[$ and $u = 1$

In the first event, the market maker would set the harshest price $p = -2$. In the second event, he would set the harshest price $p = 2$. Since the market maker cannot distinguish this two events, I assume that when he observes a positive aggregate order $z \in [0,1]$, the probability he assigns to event 1) is increasing in the aggregate order z. Basically, the higher the order size, the more likely it is that it is the result of good manager demanding for the asset.

Notice that good managers can make more accurate forecasts of returns thanks to the fact that they are able to perfectly forecast the true value of the asset. However, they are not better than bad manager in predicting trading prices. Since prices depends only on x and u and no manager has some information about u, both types make the same forecast $E\left(P(x+u)\right)$ about the price they are going to pay by posting order x. To ease notation, let $P^{e}(x) \equiv E(P(x+u))$ and accordingly write:

$$\begin{aligned} R_{g}^{e}\left(x,v\right) &= x\left[v-P^{e}(x)\right] \qquad (14)\\ R_{b}^{e}\left(x\right) &= -xP^{e}(x) \qquad (15)\end{aligned}$$

Notice that since u can take on values 1 and -1 with equal probability, we have that:

$$P^{e}(x) \equiv E(P(x+u)) = \frac{1}{2}P(x+1) + \frac{1}{2}P(x-1) \qquad (16)$$

Bad managers. Let me first consider the case in which a bad manager is selected to trade. Using (15), it is easy to show that under the linear contract specified in section 2.1, the payoff that a bad manager expects from placing order x can be written as follows:

$$\Pi_{b}^{e}(x) = \alpha R_{b}^{e}\left(x\right) + \beta = -\alpha x P^{e}(x) + \beta \qquad (17)$$

Since the equilibrium strategy of the bad manager prescribes to refrain from trading, his expected profits *in equilibrium* read $\Pi_{b}^{e}(0) = \beta$.

In order to show that $X_{b}^{*} = 0$ is the optimal trading strategy of a bad manager, we have to prove the following two different cases:

1) The bad manager has not any incentives in mimicking a good manager by placing any order $x \in \{1-q, 1+q\}$ consistent with good manager's equilibrium strategy. Formally, this amounts to prove that the following two inequalities hold:

$$\begin{aligned} \Pi_{b}^{e}(0) &\geq \Pi_{b}^{e}(1+q) \qquad (18)\\ \Pi_{b}^{e}(0) &\geq \Pi_{b}^{e}\left(1-q\right) \qquad (19)\end{aligned}$$

Using (17), we can write:

$$\begin{aligned} \Pi_{b}^{e}(1+q) &= -\alpha\left(1+q\right)P^{e}(1+q) + \beta \\ \Pi_{b}^{e}\left(1-q\right) &= -\alpha\left(1-q\right)P^{e}(1-q) + \beta \end{aligned}$$

By (16) and (4), we can easily compute:

$$\begin{aligned}
P^e(1+q) &= \frac{1}{2}P(2+q) + \frac{1}{2}P(q) = \frac{5}{4} \\
P^e(1-q) &= \frac{1}{2}P^*(2-q) + \frac{1}{2}P^*(-q) = \frac{1}{4}
\end{aligned}$$

Hence, conditions (18) and (19) can be finally written as follows:

$$\begin{aligned}
\beta &\geq -\alpha\frac{5}{4}(1+q) + \beta \\
\beta &\geq -\alpha\frac{1}{4}(1-q) + \beta
\end{aligned}$$

It is apparent that the previous inequalities are simultaneously satisfied for

$$-1 \leq q \leq 1 \tag{20}$$

2) The bad manager has no incentives to deviate to any other positive order inconsistent with the equilibrium. Let $x \in \mathbb{R}^+ \setminus \{1-q, 1+q\}$ denote an arbitrary positive order inconsistent with the equilibrium. Formally, we have to show that

$$\Pi_b^e(0) = \beta \geq \Pi_b^e(x) = -\alpha x P^e(x) + \beta$$

where

$$P^e(x) = \frac{1}{2}P(x+1) + \frac{1}{2}P(x-1)$$

It is convenient to distinguish the following two cases:

2a) $x \geq 2$. In this case, the aggregate order $z = x + u$ is always greater than 1 for any realization of $u = -1, 1$. Thus, by (5), we have that $P(x+1) = P(x-1) = 2$. Accordingly, $P^e(x) = 2$ and our condition boils down to:

$$\beta \geq -2\alpha x + \beta$$

which is clearly always satisfied in our parameters region (remember that x is positive).

2b) $x \in (0, 2) \setminus \{1-q, 1, 1+q\}$. In this case, when $u = -1$, the aggregate order is $z = x - 1 \in (-1, 1)$; when $u = 1$, the aggregate order is $z = x + 1 > 1$. Hence, by (5), $P(x-1) = 2(x-1)$ and $P(x+1) = 2$. Therefore, $P^e(x) = x$ and our condition can be written as:

$$\beta \geq -\alpha x^2 + \beta$$

which is clearly always satisfied (again, remember that x is positive).

Good managers. Let me now consider the case in which the good manager is selected to trade. Before trading, the good manager observes the true value of the asset v. By (14), the payoff that a good manager expects from placing order x when the asset is worth v can be written as follows:

$$\begin{aligned}\Pi_g^e(x,v) &= \alpha R_g^e(x,v)+\beta= \\ &= \alpha x\left[v-P^e(x)\right]+\beta\end{aligned}$$

Since the symmetry of the problem allows us to focus on the buy side of the market, I will analyze only the cases in which the good manager has observed $v=1$ and $v=2$.

Expected profits of the good manager from following his equilibrium strategy $X_g^*(v)$ read:

$$\begin{aligned}&\Pi_g^e(1-q,1)=\alpha(1-q)\left[1-P^e(1-q)\right]+\beta= \\ &=\frac{3}{4}\alpha(1-q)+\beta\end{aligned}$$

$$\begin{aligned}&\Pi_g^e(1+q,2)=\alpha(1+q)\left[2-P^e(1+q)\right]+\beta= \\ &=\frac{3}{4}\alpha(1+q)+\beta\end{aligned}$$

I will show that $X_g^*(v)$ is the optimal strategy for the good manager by analyzing the following cases:

1) When the good manager observes $v=1$, he trades $X_g^*(1)=1-q$ instead of deviating to any other order consistent with the equilibrium, i.e., $1+q$ or 0. This amount to show that the following inequalities hold:

$$\Pi_g^e(1-q,1) \geq \Pi_g^e(1+q,1) \tag{21}$$
$$\Pi_g^e(1-q,1) \geq \Pi_g^e(0,1) \tag{22}$$

It is easy to check that:

$$\begin{aligned}&\Pi_g^e(1+q,1)=\alpha(1+q)\left[1-P^e(1+q)\right]+\beta= \\ &=-\frac{1}{4}\alpha(1+q)+\beta \\ &\Pi_g^e(0,1)=\beta\end{aligned}$$

so that our conditions (21) and (22) can be written as:

$$\begin{aligned}\frac{3}{4}\alpha(1-q)+\beta &\geq -\frac{1}{4}\alpha(1+q)+\beta \\ \frac{3}{4}\alpha(1-q)+\beta &\geq \beta\end{aligned}$$

Both these inequalities are always satisfied for

$$q \leq 1 \tag{23}$$

2) When the good manager observes $v = 2$, he plays $X_g^*(2) = 1 + q$ instead of deviating to $1 - q$ or 0. Formally, it must be that:

$$\Pi_g^e(1+q,2) \geq \Pi_g^e(1-q,2) \tag{24}$$

$$\Pi_g^e(1+q,2) \geq \Pi_g^e(0,2) \tag{25}$$

which can be written as:

$$\begin{aligned}\frac{3}{4}\alpha(1+q)+\beta &\geq \alpha(1-q)\left(2-\frac{1}{4}\right)+\beta \\ \frac{3}{4}\alpha(1+q)+\beta &\geq \beta\end{aligned}$$

and simplified to:

$$\begin{aligned}3(1+q) &\geq 7(1-q) \\ (1+q) &\geq 0\end{aligned}$$

These inequalities are simultaneously satisfied for

$$q \geq \frac{2}{5} \tag{26}$$

3) Following the equilibrium strategy $X_g^*(v)$ gives the good manager expected profits greater than those he would get by deviating to any positive order inconsistent with the equilibrium. Let $x \in \mathbb{R}^+/\{0, 1-q, 1+q\}$ indicate a positive order inconsistent with the equilibrium. Then, we have to show that the two following

conditions hold:

$$\Pi_g^e(1-q,1) \geq \Pi_g^e(x,1) \tag{27}$$
$$\Pi_g^e(1+q,2) \geq \Pi_g^e(x,2) \tag{28}$$

It is useful to consider the two following cases:

3a) $x \geq 2$. In this case, $z = x + u > 1$ for any realization of u and by (5) $P^e(x) = 2$. Accordingly:

$$\begin{aligned}
&\Pi_g^e(x,1) = \alpha x\left[1 - P^e(x)\right] + \beta = \\
&= \alpha x(1-2) + \beta = -\alpha x + \beta \\
&\Pi_g^e(x,2) = \alpha x\left[2 - P^e(x)\right] + \beta = \\
&= \alpha x(2-2) + \beta = \beta
\end{aligned}$$

Using these results, it is easy to show that conditions (27) and (28) can be written as

$$\begin{aligned}
\frac{3}{4}(1-q) &\geq -x \\
\frac{3}{4}(1+q) &\geq 0
\end{aligned}$$

which are simultaneously satisfied under condition:

$$-1 \leq q \leq \frac{11}{3} \tag{29}$$

3b) $x \in (0,2)\backslash\{1-q, 1, 1+q\}$. We have shown above that in this case $P^e(x) = x$. Thus, we can write:

$$\begin{aligned}
&\Pi_g^e(x,1) = \alpha x\left[1 - P^e(x)\right] + \beta = \\
&= \alpha\left(x - x^2\right) + \beta \\
&\Pi_g^e(x,2) = \alpha x\left[2 - P^e(x)\right] + \beta = \\
&= \alpha\left(2x - x^2\right) + \beta
\end{aligned}$$

After simple algebra it is easy to show that conditions (27) and (28) boil down to:

$$\begin{aligned} \frac{3}{4}(1-q) &\geq x - x^2 \qquad (30) \\ \frac{3}{4}(1+q) &\geq 2x - x^2 \qquad (31) \end{aligned}$$

A necessary and sufficient condition for the first inequality to be satisfied is that $q \leq \frac{2}{3}$, while a necessary and sufficient condition for the second one is that $q \geq \frac{1}{3}$. Hence, (27) and (28) are both satisfied as long as:

$$\frac{1}{3} \leq q \leq \frac{2}{3} \qquad (32)$$

Summing up conditions (23), (26), (29) and (32) for the good manager, we have that $X_g^*(v)$ is an equilibrium strategy as long as:

$$q \in \left[\frac{2}{5}, \frac{2}{3}\right) \qquad (33)$$

Notice that the previous condition is stricter than optimality condition (20) for the bad manager. Thus, under (33) both good and bad managers will play equilibrium strategies $X_g^*(v)$ and $X_b^* = 0$ as optimal responses to $P(z)$.[27]

The investor's problem. Before trade opens, the investor has to decide whether to delegate trade to a fund manager or stay out of the market. Suppose the investor chooses to hire a manager. With probability θ he hires a good manager. The return that he expects in equilibrium from a good manager reads:

$$\begin{aligned} &E\left[R_g^e\left(X_g^*(v), v\right)\right] = \\ &= \frac{1}{4}(1+q)\left[2 - P^e\left(1+q\right)\right] + \frac{1}{4}(1-q)\left[1 - P^e\left(1-q\right)\right] + \\ &+ \frac{1}{4}(-1+q)\left[-1 - P^e\left(-1+q\right)\right] + \frac{1}{4}(-1-q)\left[-2 - P^e\left(-1-q\right)\right] = \frac{3}{4} \end{aligned}$$

With probability $1-\theta$ he hires a bad manager. Since a bad manager does not trade in equilibrium, the return that the investor expects for him in equilibrium is zero. Thus, the average return from delegation expected by the investor in equilibrium is equal to $\frac{3}{4}\theta$. Given the linear contract under consideration, the expected equilibrium

[27] In fact, for $q = \frac{2}{5}$, the good type is indifferent between following strategy $X_{q,g}(\cdot)$ and mimicking the bad type by playing $x = 0$

payoff of delegation reads:

$$\frac{3}{4}\theta(1-\alpha)-\beta$$

If the Investor does not hire any manager, he will get a payoff equal to zero. Therefore, the investor will hire a fund manager if and only if

$$\frac{3}{4}\theta(1-\alpha)-\beta>0$$

which is satisfied for any $\alpha \in (0,1)$ as long as $\beta < \frac{3}{4}\theta(1-\alpha)$. ■

Equilibria multiplicity. It is useful to remind that proposition 1 identifies *a* family of perfect Bayesian equilibria of the static game described above. In particular, proposition 1 focuses on "symmetric" strategies for the good manager, that is, strategies represented by (2). More generally, there exist families of perfect Bayesian equilibria indexed by the pair (q_1, q_2) in which investors delegate trade, bad managers do not trade and good managers behave according to the following more general strategy:

$$X^*_{g,q}(v) = \begin{cases} 1+q_2 & \text{when } v=2 \\ 1-q_1 & \text{when } v=1 \\ -1+q_2 & \text{when } v=-1 \\ -1-q_1 & \text{when } v=-2 \end{cases} \tag{34}$$

with q_1, q_2 taking values in appropriate subsets of the segment $(0,1)$. This result can be easily proved by following the reasoning in the proof of proposition 1. At an intuitive level, what we need is just that (at least in some cases) the good manager be able to disguise his information behind noise trade. Notice that strategy (34) is such that the market maker cannot distinguish ($v=2$, $u=-1$) from ($v=-1$, $u=1$) and ($v=-2$, $u=1$) from ($v=1$, $u=-1$).

Basically, (34) is derived by considering that the optimal strategy of the good manager must be monotonic with respect to the manager's signal and must satisfy the following *camouflage* conditions:

$$\begin{aligned} X^*_g(2)-1 &= X^*_g(-1)+1 \\ X^*_g(-2)+1 &= X^*_g(1)-1 \end{aligned}$$

In order to obtain (34), let

$$\begin{aligned} X^*_{g,q}(2)-1 &= X^*_{g,q}(-1)+1 = q_2 \in \mathbb{R} \\ X^*_{g,q}(-2)+1 &= X^*_{g,q}(1)-1 = q_1 \in \mathbb{R} \end{aligned}$$

Optimality requires strictly monotonicity, which restricts q_1 and q_2 to be positive. Optimality also implies that q_1 and q_2 be less than one (values of q_1 and q_2 greater than one implies that the manager is selling when observing positive values of the dividends and buying when observing negative values of the dividends, thus delivering negative returns).[28]

Proof of proposition 3. The prices that the manager expects to pay in the equilibria identified by propositio 3 read:

$$P_2^e(1+q) = -P_1^e(-1-q) = \frac{5}{4} \tag{35}$$

$$P_2^e(1-q) = -P_1^e(-1+q) = \frac{1}{4} \tag{36}$$

$$P_1^e(1+q) = -P_2^e(-1-q) = 1 + \frac{1}{4}\theta \tag{37}$$

$$P_1^e(1-q) = -P_2^e(-1+q) = \frac{\theta}{2(2-\theta)} - \frac{\theta}{4} \tag{38}$$

The returns that a good manager expects to deliver in equilibrium read:

$$\begin{aligned} & R_{2,g}^e\left(X_{2,g}^*(v_2), v_2\right) = \\ & = \begin{cases} (1+q)\left(2 - P_2^e(1+q)\right) = \frac{3}{4}(1+q) & \text{if } v_2 = 2 \text{ or } v_2 = -2 \\ (1-q)\left(1 - P_2^e(1-q)\right) = \frac{3}{4}(1-q) & \text{if } v_2 = 1 \text{ or } v_2 = -1 \end{cases} \end{aligned} \tag{39}$$

$$\begin{aligned} & R_{1,g}^e\left(X_{1,g}^*(v_1), v_1\right) = \\ & = \begin{cases} (1+q)\left[2 - P_1^e(1+q)\right] = (1+q)\left(1 - \frac{1}{4}\theta\right) & \text{if } v_1 = 2 \text{ or } v_1 = -2 \\ (1-q)\left[1 - P_1^e(1-q)\right] = (1-q)\left(1 - \frac{\theta}{2(2-\theta)} + \frac{1}{4}\theta\right) & \text{if } v_2 = 1 \text{ or } v_2 = -1 \end{cases} \end{aligned} \tag{40}$$

The returns that a bad manager expects to deliver in equilibrium read:

$$R_{2,b}^e\left(X_{2,b}^*\right) = 0 \tag{41}$$

[28] Another camouflage case arises when:

$$\begin{aligned} X_{g,q}^*(2) - 1 &= X_{g,q}^*(-2) + 1 \\ X_{g,q}^*(-1) + 1 &= X_{g,q}^*(1) - 1 \end{aligned}$$

however, the strategy satisfying this conditions is not strictly monotonic and thus non optimal.

$$
\begin{aligned}
&R^e_{1,b}\left(X^*_{1,b}\right) = \\
&= \tfrac{1}{2}\left(1-q\right)\left[-P^e_1\left(1-q\right)\right] + \tfrac{1}{2}\left(-1+q\right)\left[-P^e_1\left(-1+q\right)\right] = \\
&= -\left(\tfrac{\theta}{2(2-\theta)} - \tfrac{\theta}{4}\right)\left(1-q\right)
\end{aligned} \tag{42}
$$

Let $\alpha \in (0,1)$, $\beta > 0$ and $q \in (0,1)$. I will prove proposition 3 by backward induction.

Second round of trade. Since the second round of trade is also the last one, managers do not face career concerns. Thus, the second round of trade is equivalent to the basic financial market of section 2. Accordingly, investors, managers and market makers' equilibrium behavior for the second round of trade is the one described in proposition 1: investors always hire a manager, bad managers do not trade, good managers trade according to (2) and market makers set prices according to (4) and (5). Furthermore, we know from proposition 1 that this behavior constitutes an equilibrium as long as:

$$
\frac{2}{5} < q < \frac{2}{3} \tag{43}
$$

$$
\beta < (1-\alpha)\frac{3}{4}\theta \tag{44}
$$

Investor's retaining rule at the end of the first round of trade. The *equilibrium* average return that the investor expects to be delivered by a bad manager in the second round is clearly equal to zero; on the other hand, that expected from a good manager reads:

$$
E\left[R^e_{2,g}\left(X_{2,g}\left(v_2\right), v_2\right)\right] = \frac{1}{4}\left[\frac{3}{4}(1+q)\right] + \frac{1}{4}\left[\frac{3}{4}(1-q)\right] + \\
\frac{1}{4}\left[\frac{3}{4}(1+q)\right] + \frac{1}{4}\left[\frac{3}{4}(1+q)\right] = \frac{3}{4}
$$

Let $\widehat{\theta} = \Pr(i = g|v_1, x_1)$ denote the investor's believe that the incumbent manager is good. Accordingly, the equilibrium payoff that the investor expects in the second round from retaining the incumbent manager reads:

$$
(1-\alpha)\frac{3}{4}\widehat{\theta} - \beta
$$

Alternatively, the equilibrium payoff that the investor expects in the second round from hiring a new manager reads:

$$
(1-\alpha)\frac{3}{4}\theta - \beta
$$

Thus, the investor retains the old manager if and only if:

$$(1-\alpha)\frac{3}{4}\widehat{\theta}-\beta>(1-\alpha)\frac{3}{4}\theta-\beta$$

or equivalently,

$$\widehat{\theta}>\theta$$

Notice that under condition (44), this is a necessary and sufficient condition for the old manager to be retained. Indeed, $\widehat{\theta}>\theta$ also implies that $\widehat{\theta}(1-\alpha)\frac{3}{4}-\beta>\theta(1-\alpha)\frac{3}{4}-\beta>0$, which ensures that the investor will in fact prefer to retain the incumbent manager than refraining from trading (which would give him zero profits).

Investor's belief $\widehat{\theta}$. Based on the observation of x_1 and v_1, the investor updates his belief about the fact that the manager is good. Formally, the investor computes (whenever possible) $\widehat{\theta}\equiv\Pr(i=g|v_{1,}x_1)$, where

$$\Pr(i=g|v_1,x_1)=\frac{\Pr(i=g)\Pr(x_1|v_1,i=g)}{\Pr(i=g)\Pr(x_1|v_1,i=g)+\Pr(i=b)\Pr(x_1|v_1,i=b)}$$

Let $\Phi=\{-1-q,-1+q,1-q,1+q\}$ denote the set of managers' equilibrium orders. Given the equilibrium strategies $X^*_{1,g}$ and $X^*_{1,b}$, it is easy to show that for any $x_1\in\Phi$, the following holds true:

$$\Pr(i=g|v_1,x_1)=\begin{cases}1 & \text{if } (v_1=2,x_1=1+q) \text{ or} \\ & \quad (v_1=-2,x_1=-1-q) \\ \frac{2\theta}{1+\theta} & \text{if } (v_1=1,x_1=1-q) \text{ or} \\ & \quad (v_1=-1,x_1=-1+q) \\ 0 & \text{if } v_1\in\{-2,-1,1,2\} \text{ and} \\ & \quad x_1\neq X^*_{1,g}(v_1)\end{cases} \tag{45}$$

On the other hand, for orders off the equilibrium path, Bayesian perfection imposes no restrictions on $\Pr(i=g|v_{1,}x_1)$, which could in principle take any value in $[0,1]$. I thus assume that:

$$\Pr(i=g|v_1,x_1)=0,\ \forall x_1\in\mathbb{R}\setminus\Phi \tag{46}$$

Managers' expected probability of being retained. A manager knows that he is retained if $\widehat{\theta} > \theta$. He also knows (45) and (46), from which he can conclude that

$$\widehat{\theta} > \theta \begin{cases} \text{if } (v_1 = 2, x_1 = 1+q) \text{ or } (v_1 = -2, x_1 = -1-q) \\ \text{or } (v_1 = 1, x_1 = 1-q) \text{ or } (v_1 = -1, x_1 = -1+q) \end{cases} \tag{47}$$

$$\widehat{\theta} < \theta \text{ otherwise} \tag{48}$$

Given (47) and (48), a manager of type i can easily compute the probability of being retained when trading order x_1. In particular, since a good manager observes v_1, he knows that:

$$\Pr(\widehat{\theta} > \theta | x_1, v_1) = \begin{cases} 1 \text{ if } x_1 = X^*_{1,g}(v_1) \\ 0 \text{ if } x_1 \neq X^*_{1,g}(v_1) \end{cases} \tag{49}$$

A bad manager does not observe v_1. Therefore, in his case we have that

$$\Pr(\widehat{\theta} > \theta | x_1) = \begin{cases} \frac{1}{4} \text{ if } x_1 = \{-1-q, -1+q, 1-q, 1+q\} \\ 0 \text{ otherwise} \end{cases} \tag{50}$$

Good managers' strategy in the first round of trade.[29] Let me define the total profits that a good manager expects from trading order $x_1$at the beginning of $s = 1$ (when he observes v_1) as follows:

$$\Pi^e_{tot,g}(x_1, v_1) = \Pi^e_{1,g}(x_1, v_1) + \Pr(\widehat{\theta} > \theta | x_1, v_1)\Pi^*_{2,g} \tag{51}$$

where:

$$\Pi^e_{1,g}(x_1, v_1) = \alpha R^e_{1,g}(x_1, v_1) + \beta = \alpha x_1 [v_1 - P^e_1(x_1)] + \beta$$

are the profits that the good manager expects to get in the first round of trade from trade x_1. $\Pr(\widehat{\theta} > \theta | x_1, v_1)$ is the probability of being retained if trading x_1. $\Pi^*_{2,g}$ are the *equilibrium* profits that the good manager expects to gain in the second round if he is retained (these are the profits expected at the beginning of $s = 1$ when the manager does not know v_2 yet). We know that the average *equilibrium* return delivered by a good manager in the second round is $\frac{3}{4}$. Thus

$$\Pi^*_{2,g} = \frac{3}{4}\alpha + \beta$$

In order to show that $X^*_{1,g}(\cdot)$ is the equilibrium strategy, I have to show that when the true state is observed to be v_1, order $X^*_{1,q,g}(v_1)$ maximizes (51). Again, given the

[29] Remember that I am assuming that the type of manager does not change over the two rounds of trades.

symmetry of the problem, it is sufficient to focus only on the cases in which the good manager observes $v_1 = 1, 2$ and only on the buy side of the problem (i.e. positive orders). Using this fact and expression (51), our task amounts to show that:

$$\text{for } v_1 = 1, 2 \text{ and } \forall x_1 \geq 0, \quad \Pi^e_{tot,g}(X^*_{1,g}(v_1), v_1) \geq \Pi^e_{tot,g}(x_1, v_1) \tag{52}$$

Using (40) and (49), the two previous expressions can be written as:

$$\Pi^e_{tot,g}\left(X^*_{1,g}(v_1), v_1\right) = \begin{cases} \alpha(1-q)\left(1 - \frac{\theta}{2(2-\theta)} + \frac{1}{4}\theta\right) + \frac{3}{4}\alpha + 2\beta & \text{if } v_1 = 1 \\ \alpha(1+q)\left(1 - \frac{1}{4}\theta\right) + \frac{3}{4}\alpha + 2\beta & \text{if } v_1 = 2 \end{cases}$$

I will proof (52) by considering the following three cases:

a) When the good manager observes $v_1 = 1, 2$, he will follow the equilibrium strategy and play $X^*_{1,g}(v_1)$ instead of refraining from trading. Formally:

$$\text{for } v_1 = 1, 2, \ \Pi^e_{tot,g}\left(X^*_{1,g}(v_1), v_1\right) \geq \Pi^e_{tot,g}\left(0, v_1\right) \tag{53}$$

From (51) and (49) we have that $\Pi^e_{tot,g}\left(0, v_1\right) = \beta$. Therefore, (53) can be written as:

$$\alpha(1-q)\left(1 - \frac{\theta}{2\left(2-\theta\right)} + \frac{1}{4}\theta\right) + \frac{3}{4}\alpha + \beta \geq 0 \tag{54}$$

$$\alpha(1+q)\left(1 - \frac{1}{4}\theta\right) + \frac{3}{4}\alpha + \beta \geq 0 \tag{55}$$

It is apparent that inequalities (54) and (55) are always satisfied in our parameters region.

b) When the good manager observes $v_1 = 1, 2$, he is better off playing $X^*_{1,g}(v_1)$ instead of either deviating to any other order that could in principle arise in equilibrium, i.e. $x_1 \in \{1+q, 1-q\}$. Formally:

$$\Pi^e_{tot,g}\left(X^*_{1,g}(1), 1\right) \geq \Pi^e_{tot,g}\left(X^*_{1,g}(2), 1\right) \tag{56}$$

$$\Pi^e_{tot,g}\left(X^*_{1,g}(2), 2\right) \geq \Pi^e_{tot,g}\left(X^*_{1,g}(1), 2\right) \tag{57}$$

which can be written as:

$$\alpha(1-q)\left(1-\frac{\theta}{2\,(2-\theta)}+\frac{1}{4}\theta\right)+\frac{3}{4}\alpha+\beta\geq-\frac{1}{4}\alpha\theta(1+q) \tag{58}$$

$$\alpha(1+q)\left(1-\frac{1}{4}\theta\right)+\frac{3}{4}\alpha+\beta\geq\alpha(1-q)\left(2-\frac{\theta}{2\,(2-\theta)}+\frac{1}{4}\theta\right) \tag{59}$$

Inequality (58) is clearly always satisfied in our parameters region. Since $\beta > 0$, a sufficient condition for (59) is that:

$$(1+q)\left(1-\frac{1}{4}\theta\right)+\frac{3}{4}\geq(1-q)\left(2-\frac{\theta}{2\,(2-\theta)}+\frac{1}{4}\theta\right)$$

which is satisfied as long as:

$$q\geq\frac{2+\theta-2\theta^2}{2(12-7\theta)}$$

Notice that for every value of $\theta\in(0,1)$, the RHS of the previous inequality is lower than $\frac{2}{5}$ and thus the previous condition is weaker than condition (43) and can be ignored.

c) Finally, when the good manager observes $v_1 = 1, 2$, he is better off playing $X^*_{1,q,g}(v_1)$ instead of deviating to any positive order off the equilibrium path.

Formally, let $x_1 \in \mathbb{R}\backslash\{1+q,\,1-q\}$. Then, it must be that:

$$\text{for } v_1=1,2,\ \Pi^e_{tot,g}\left(X^*_{1,g}(v_1),v_1\right)\geq\Pi^e_{tot,g}\left(x_1,v_1\right)$$

Focusing on the buy side of the problem, we have to consider the two separated cases in which $x_1 \in (0, 2)$ and $x_1 \geq 2$.

$\mathbf{x_1 \geq 2}$. Given market makers' first-round price strategy for out of equilibrium aggregate orders (12), we have that $P^e_1(x_1) = 2$ and thus:

$$E(\Pi_{1,g}\left(x_1\right)|v_1)=\alpha x_1\left[v_1-P^e_1(x_1)\right]+\beta=\alpha x_1\left[v_1-2\right]+\beta$$

Furthermore, given (49), it is also true that for $v_1 = 1, 2$, $\Pr(\widehat{\theta} > \theta|x_1, v_1) = 0$. Thus, total expected profits from deviating to any $x_1 \geq 2$ are equal to $\alpha x_1 [v_1 - 2] + \beta$. Notice that $\alpha x_1 [v_1 - 2] \leq 0$ (remember that we are focusing on $v_1 = 1, 2$). Therefore, deviations to any $x_1 \geq 2$ are (weakly) dominated by the choice of not trading, which we have shown to be non optimal.

$\mathbf{x} \in (0,2)$. Again, given (12) we can compute:

$$P_1^e(x_1) = \frac{1}{2}P_1(x_1+1) + \frac{1}{2}P_1(x_1-1) = \frac{1}{2}2 + \frac{1}{2}2(x_1-1) = x_1$$

and accordingly

$$E(\Pi_{1,g}(x_1)|v_1) = \alpha x\left[v_1 - P_1^e(x_1)\right] + \beta = \alpha x_1(v_1 - x_1) + \beta$$

Again, given (49), we have that for $v_1 = 1, 2$, $\Pr(\widehat{\theta} > \theta|x, v_1) = 0$. Thus, when the true value of the asset is observed to be $v_1 = 1$, the total expected profits from deviating to any $x_1 \in (0,2)$ are equal to:

$$E(\Pi_{1,g}(x_1)|v_1 = 1) = \alpha x_1\left[1 - x_1\right] + \beta$$

On the other hand, when the true vale of the asset is observed to be $v_1 = 2$, the total expected profits from deviating to any $x_1 \in (0,2)$ are equal to

$$E(\Pi_{1,g}(x_1)|v_1 = 2) = \alpha x_1\left[2 - x_1\right] + \beta$$

Using the previous results, the conditions for the equilibrium can be written as:

$$\begin{aligned} \alpha(1-q)\left(1 - \frac{\theta}{2(2-\theta)} + \frac{1}{4}\theta\right) + \frac{3}{4}\alpha + 2\beta &\geq \alpha x_1\left[1 - x_1\right] + \beta \\ \alpha(1+q)\left(1 - \frac{1}{4}\theta\right) + \frac{3}{4}\alpha + 2\beta &\geq \alpha x_1\left[2 - x_1\right] + \beta \end{aligned}$$

and are always satisfied in our parameters region (remember that here we are focusing on $x_1 \in (0,2)$).

Bad managers' strategy in the first round of trade. A bad manager does not observe v_1. Accordingly, let me define the total profits that he expects from trading order $x_1$at the beginning of $s = 1$ as follows:

$$\Pi_{tot,b}^e(x_1) = \Pi_{1,g}^e(x_1) + \Pr(\widehat{\theta} > \theta|x_1)\Pi_{2,b}^*$$

where

$$\Pi_{1,b}^e(x_1) = \alpha R_{1,b}^e(x_1) + \beta = -\alpha x_1 P_1^e(x_1) + \beta$$

are the profits that the good manager expects to get in the first round of trade from trade x_1. $\Pr(\widehat{\theta} > \theta|x_1)$ is the probability of being retained if trading x_1. $\Pi_{2,g}^*$ are the *equilibrium* profits that the good manager expects to gain in the second round if he is retained. Since a bad manager does not trade in the second round, $\Pi^*_{2,b} = \beta$. Thus, we can write:

$$\Pi^e_{tot,b}(x_1) = -\alpha x_1 P^e_1(x_1) + \beta + \frac{3}{4}\beta \tag{60}$$

In order to show that the mixed strategy $X^*_{1,b}$ is an equilibrium strategy, we have to show that $\forall x_1 \geq 0$ the following condition holds:

$$\Pi^e_{tot,b}(X^*_{1,b}) \geq \Pi^e_{tot,b}(x_1) \tag{61}$$

Notice that using (42) and (50), we have that

$$\Pi^e_{tot,b}(X^*_{1,b}) = \alpha R^e_{1,b}\left(X^*_{1,b}\right) + \beta + \Pr(\widehat{\theta} > \theta | X^*_{1,b})\beta =$$
$$-\alpha\left(1-q\right)\left(\frac{\theta}{2\left(2-\theta\right)} - \frac{\theta}{4}\right) + \frac{5}{4}\beta$$

I will prove (61) by considering the following different cases:

a) Playing $X^*_{1,b}$ is better than deviating to any other order that is consistent with the equilibrium. Notice that

$$\Pi^e_{tot,b}(X^*_{1,b}) = \frac{1}{2}\Pi^e_{tot,b}(1-q) + \frac{1}{2}\Pi^e_{tot,b}(-1+q)$$

It is trivial to show that $\Pi^e_{tot,b}(1-q) = \Pi^e_{tot,b}(-1+q)$ and thus that

$$\Pi^e_{tot,b}(X^*_{1,b}) = \Pi^e_{tot,b}(1-q) = \Pi^e_{tot,b}(-1+q)$$

In fact the bad manager is indifferent between $X^*_{1,b}$ and playing either the pure strategy $1-q$ or $-1+q$. It is also trivial to show that $\Pi^e_{tot,b}(1+q) = \Pi^e_{tot,b}(-1-q)$. Therefore, we just have to check that

$$\Pi^e_{tot,b}(X^*_{1,b}) \geq \Pi^e_{tot,b}(1+q)$$

which can be written as:

$$\left(1-q\right)\left(\frac{\theta}{2\left(2-\theta\right)} - \frac{\theta}{4}\right) \leq \left(1 + \frac{1}{4}\theta\right)\left(1+q\right) \tag{63}$$

It is immediate to check that (63) is always satisfied in our parameters region. Notice that the bad manager prefers $X^*_{1,b}$ to any other mixed strategy $X_{1,b}$ that randomizes over $\{1+q,\, 1-q,\, -1+q,\, -1-q\}$. Indeed, notice that the expected payoff of any mixed strategy $X_{1,b}$ is a convex combination of $\Pi^e_{tot,b}(1-q)$ and $\Pi^e_{tot,b}(1+q)$. That is,

$$\Pi^e_{tot,b}\left(X_{1,b}\right) = h\Pi^e_{tot,b}(1-q) + (1-h)\Pi^e_{tot,b}(1+q), \text{ with } h \in [0,1]$$

But since we have shown that $\Pi^e_{tot,b}(X^*_{1,b}) > \Pi^e_{tot,b}(1+q)$ and that $\Pi^e_{tot,b}(X^*_{1,b}) = \Pi^e_{tot,b}(1-q)$, it immediately follows that $\Pi^e_{tot,b}(X^*_{1,b}) > \Pi^e_{tot,b}(X_{1,b})$.

b) $X^*_{1,b}$ is better than refraining from trading. Formally:

$$\Pi^e_{tot,b}(X^*_{1,b}) \geq \Pi^e_{tot,b}(0)$$

Using (60) and (50), we find that $\Pi^e_{tot,b}(0) = \beta$. Thus, the previous inequality can be written as:

$$-\alpha\left(1-q\right)\left(\frac{\theta}{2\left(2-\theta\right)} - \frac{\theta}{4}\right) + \frac{5}{4}\beta \geq \beta$$

and simplified to:

$$\beta \geq \alpha(1-q)\frac{\theta^2}{2-\theta} \tag{64}$$

c)$X^*_{1,b}$ is better than deviating to any order off the equilibrium path. Let $x_1 \in \mathbb{R}^+ \setminus \{-1-q,\, -1+q,\, 0,\, 1+q,\, 1-q\}$ and consider the two following cases:

$\mathbf{x}_1 \geq \mathbf{2}$. Given (12), $P^e(x_1) = 2$. Given ((50), $\Pr(\widehat{\theta} > \theta|x_1) = 0$. Thus:

$$\Pi^e_{tot,b}(x_1) = -2\alpha x_1 + \beta$$

Notice that $-2\alpha x_1 < 0$. Therefore, deviations to $x_1 \geq 2$ are strictly dominated by the choice of not trading, which we have just shown to be non optimal.

$\mathbf{x} \in (0,2)$. In this case $P^e(x_1) = x_1$ and $\Pr(\widehat{\theta} > \theta|x_1) = 0$. Thus:

$$\Pi^e_{tot,b}(x_1) = -\alpha x_1^2 + \beta$$

Again, $-\alpha x_1^2 < 0$ and consequently also deviations to $x_1 \in (0,2)$ are strictly dominated by the choice of not trading. Thus, a bad managers will never deviate to out of equilibrium orders.

We can then conclude that under condition (64), $X^*_{1,q,b}$ is the equilibrium first-round strategy of the bad managers.

Therefore, we have that good an bad managers follow their equilibrium strategies $X^*_{1,q,g}(\cdot)$, $X^*_{1,q,b}$ and $X^*_{2,q,g}(\cdot)$, $X^*_{2,q,b}$ as long as conditions (43) and (64) are satisfied, that is:

$$\frac{2}{5} \leq q < \frac{2}{3}$$
$$\beta \geq \alpha(1-q)\frac{\theta^2}{2-\theta}$$

This implies that a sufficient condition on β, expressed in terms of the parameters of the game, can be written as follows:

$$\beta \geq \frac{3\alpha\theta^2}{5\,(2-\theta)} \tag{65}$$

Market makers' strategy in the first round of trade. Given bad and good managers' equilibrium strategies (9) and (8), aggregate order $Z_1(u,v,i)$ satisfies:

$$\begin{aligned} Z_1(u,v,i) &= \begin{cases} 2+q & \text{if } \{u=1, v=2, i=g\} \\ 2-q & \text{if } \{u=1, v=1, i=g\} \text{ or } \{u=1, i=b \text{ with } x_{1,b}=1-q\} \\ q & \text{if } \{u=-1, v=2, i=g\} \text{ or } \{u=1, v=-1, i=g\} \\ & \text{or } \{u=1, i=b \text{ with } x_{1,b}=-1+q\} \end{cases} \\ Z_1(u,v,i) &= -Z_1(-u,-v,i) \end{aligned}$$

Focus on positive aggregate orders. When a market maker observes $z_1 = 2+q$ he immediately infers that the good type is trading and that the value of the asset is $v_1 = 2$. When he observes $z_1 = 2-q$, he updates the distribution of v_1 as follows:

$$\begin{aligned} \Pr(v &= 2|2-q) = \Pr(v=-1|2-q) = \Pr(v=-2|2-q) = \frac{1-\theta}{2(2-\theta)} \\ \Pr(v &= 1|2-q) = \frac{\theta+1}{2(2-\theta)} \end{aligned}$$

and accordingly sets $P(2-q) \equiv E(v|2-q) = \frac{\theta}{2-\theta}$. Analogously, when he observes

$z_1 = q$, he computes:

$$\begin{aligned}
\Pr(v &= 2|q) = \frac{1}{4}(1+\theta) \\
\Pr(v &= 1|q) = \frac{1}{4}(1-\theta) \\
\Pr(v &= -1|q) = \frac{1}{4}(1+\theta) \\
\Pr(v &= -2|q) = \frac{1}{4}(1-\theta)
\end{aligned}$$

and sets $P(q) \equiv E(v|q) = \frac{\theta}{2}$. Therefore, for the set of positive aggregate orders that can arise in equilibrium, we obtain the following market maker's price strategy:

$$P_1(z_1) = \begin{cases} 2 & \text{if } z_1 = 2+q \\ \frac{\theta}{2-\theta} & \text{if } z_1 = 2-q \\ \frac{\theta}{2} & \text{if } z_1 = q \end{cases}$$

Following the same logic, it is easy to show that for negative aggregate orders $z_1 = -q, -2+q, -2-q$ we have that

$$P_1(z_1) = -P_1(-z_1)$$

This proves that for equilibrium aggregate orders, the price strategy of the market maker is indeed given by (10).

In order to complete the construction of the price function in the candidate equilibrium q, we need to construct the price schedule according to which the market maker sets prices in response to out of equilibrium aggregate orders. In this respect, market maker's out of equilibrium beliefs about information content of trade are crucial. It is important to notice that in the present game, PBE does not imply any restriction on market maker's out of equilibrium beliefs. Thus, the market maker's price response to any out of equilibrium order can be every price belonging to $[-2, 2]$. Accordingly, I arbitrarily assume that market makers price out of equilibrium aggregate orders according to (12).

Investor's decision at the beginning of the first round. An investor is completely uninformed about the liquidation value of the asset. He has to decide whether to delegate trade to a fund manager or stay out of the market. Suppose the investor chooses to hire a manager. With probability θ he hires a good manager.

The first-round return that he expects in equilibrium from a good manager reads:

$$
\begin{aligned}
&E\left[R_{1,g}^{e}\left(X_{1,g}^{*}(v_1),v_1\right)\right]=\\
&=\frac{1}{4}(1+q)\left[2-P_1^e\left(1+q\right)\right]+\frac{1}{4}(1-q)\left[1-P_1^e\left(1-q\right)\right]+\\
&+\frac{1}{4}(-1+q)\left[-1-P_1^e\left(-1+q\right)\right]+\frac{1}{4}(-1-q)\left[-2-P_1^e\left(-1-q\right)\right]=\\
&=\frac{1}{2}(1+q)\left(1-\frac{1}{4}\theta\right)+\frac{1}{2}(1-q)\left(1-\frac{\theta}{2\left(2-\theta\right)}+\frac{1}{4}\theta\right)=\\
&q\left(\frac{\theta}{4\left(2-\theta\right)}-\frac{1}{4}\theta\right)+1-\frac{\theta}{4\left(2-\theta\right)}
\end{aligned}
$$

On the other hand, with probability $1-\theta$ the investor hires a bad manager. The first-round return that he expects in equilibrium from a bad manager reads:

$$
\begin{aligned}
E\left[R_{1,b}^{e}\left(X_{1,b}^{*}\right)\right] &= \frac{1}{2}(1-q)\left(-\frac{\theta}{2\left(2-\theta\right)}+\frac{1}{4}\theta\right)+\frac{1}{2}(-1+q)\left(\frac{\theta}{2\left(2-\theta\right)}-\frac{1}{4}\theta\right)=\\
&= -\frac{1}{2}(1-q)\left(\frac{\theta}{2-\theta}-\frac{1}{2}\theta\right)
\end{aligned}
$$

Thus, the *equilibrium* average return from delegation expected by the investor in the first round is equal to

$$
\begin{aligned}
E\left(R_1^e\right) &= \theta E\left[R_{1,g}^{e}\left(X_{1,g}^{*}(v_1),v_1\right)\right]+(1-\theta)E\left[R_{1,b}^{e}\left(X_{1,b}^{*}\right)\right]= \\
&= \left(1-\frac{1}{4}\theta\right)\theta
\end{aligned}
\tag{66}
$$

and the *equilibrium* payoff that the investor expects from the average manager in the first round is given by:

$$
(1-\alpha)\left(1-\frac{\theta}{4}\right)\theta-\beta
$$

Since the investor has the option of staying out of the market, he will hire a manager at the beginning of the first round of trade if and only if the payoff of delegation is positive. This is guaranteed when

$$
\beta<(1-\alpha)\left(1-\frac{\theta}{4}\right)\theta \tag{67}
$$

Notice that in our parameters region it is always true that

$$\frac{3}{4}\theta < \left(1 - \frac{\theta}{4}\right)\theta$$

Thus, condition (67) is weaker than condition (44), and can then be ignored.

Summing up, the relevant conditions on β are represented by (44) and (65), which can be written as:

$$\frac{3\alpha\theta^2}{5\,(2-\theta)} \leq \beta < (1-\alpha)\frac{3}{4}\theta$$

Notice that for the previous inequality to be meaningful, it must be that $\frac{3\alpha\theta^2}{5(2-\theta)} < (1-\alpha)\frac{3}{4}\theta$. It is easy to show that this inequality is guaranteed as long as the following condition on α holds:

$$\alpha < \frac{5\,(2-\theta)}{10-\theta}$$

Notice that the RHS of the previous condition is positive and less than 1 for every value of $\theta \in (0,1)$, which guarantees that α takes values in its proper range $(0,1)$. ■

Proof of proposition 4. I will prove only part a) of the proposition, since the proof of part b) is based on the same logic. Let $\alpha \to 0^+$ and $\theta = \frac{1}{2}$ and proceed by backward induction.

Second round of trade. The last round of the trade game described in section 4 is equivalent to that of the trade game described in section 3.2. Hence, by following the proof of proposition 3, it can be easily shown that as long as the two following conditions are met:

$$\beta \in \left(0, \frac{3}{8}\right) \tag{68}$$

and

$$q \in \left[\frac{2}{5}, \frac{2}{3}\right) \tag{69}$$

there exists a family of equilibria for the second round of trade where good managers follow

$$X_{2,g}^{*}(v_2) = \begin{cases} 1+q & \text{when } v_2 = 2 \\ 1-q & \text{when } v_2 = 1 \end{cases}$$
$$X_{2,g}^{*}(-v_2) = -X_{2,g}^{*}(v_2)$$

and bad managers do not trade, as prescribed by proposition 3.[30]

[30] Note that for $\alpha = 0$, both the good and the bad manager would in fact be indifferent about any trading strategy in the last round. Indeed, in the last round of trade, good and bad managers'

Investor's retaining rule at the end of the first round of trade and investors' belief $\widehat{\theta}$. By following the arguments in the proof of proposition 3, it is easy to show that the investor retains the incumbent manager as long as $\widehat{\theta} > \theta$, with $\widehat{\theta}$ given by (45) and (46).

First round of trade. In the first round of trade, aggregate order for the asset reads:

$$Z(u_1, v_1, i) = \begin{cases} 2+q & \text{when } \{u_1 = 1, v_1 = 2, i = g\} \\ 2-q & \text{when } \{u_1 = 1, v_1 = 1, i = g\} \\ & \text{or } \{u_1 = 1, i = b \text{ with } x_{1,b} = 1-q\ \} \\ q & \text{when } \{u_1 = -1, v_1 = 2, i = g\} \\ & \text{or } \{u_1 = 1, v_1 = -1, i = g\} \\ & \text{or } \{u_1 = 1, i = b \text{ with } x_{1,b} = -1+q\ \} \end{cases}$$

By following the usual reasoning (see proof of proposition 3), it is easy to show that given bad and good managers' equilibrium strategies in the first round, market maker's pricing strategy in the first round of trade is described by the odd function $P_1(z_1) = -P_1(-z_1)$, with

$$P_1(z_1) = \begin{cases} 2 & z_1 = 2+q \\ \frac{1-2w}{2(1-w)} & z_1 = 2-q \\ \frac{6w-1}{2} & z_1 = q \end{cases} \tag{70}$$

As for the out of equilibrium aggregate orders, we assume that the market maker's pricing strategy is the usual one, that is:

$$P_1(z_1) = \begin{cases} 2 & z_1 \geq 1,\ z_1 \neq 2-q, 2+q \\ 2z_1 & -1 \leq z_1 \leq 1,\ z_1 \neq -q, q \\ -2 & z_1 \leq -1,\ z_1 \neq -2-q, -2+q \end{cases} \tag{71}$$

---

expected profits from placing order x_2 read respectively:

$$\begin{aligned} E(\Pi_{2,g}(x_2)|v_2) &= \alpha R^e_{2,g}(x_2) + \beta = \alpha x_2 \left[v_2 - P^e_2(x_2)\right] + \beta \\ E(\Pi_{2,b}(x_2)) &= \alpha R^e_{2,b}(x_2) + \beta = -\alpha x_2 P^e_2(x_2) + \beta \end{aligned}$$

Therefore, for $\alpha = 0$, any order x_2 delivers the same profit β, both to good and bad managers. However, it also true that for any positive (whatever small) α, the good manager follows $X^*_{2,G}(v)$ and the bad manager does not trade. The case $\alpha \to 0^+$ is meant to represent this situation in which the good manager's indifference is broken in favour of the strategy that maximizes investor's expected return (From a technical point of view, notice that both $E(\Pi_{2,g}(x_2)|v_2)$ and $E(\Pi_{2,g}(x_2))$ are continuous in $\alpha \in (0,1)$).

Before going on, notice that given this market makers' pricing strategy, we can compute the prices expected by a manager when he places equilibrium orders $1+q$, $1-q$, $-1+q$ and $-1-q$:

$$
\begin{aligned}
P_1^e(1+q) &= -P_1^e(-1-q) = 1+\frac{1}{4}(6w-1) \\
P_1^e(1-q) &= -P_1^e(-1+q) = \frac{1}{4}\left(\frac{1-2w}{1-w}\right)-\frac{1}{4}(6w-1)
\end{aligned}
$$

Good managers' strategy in the first round. Proposition 4 prescribes that good managers trade according to

$$
\begin{aligned}
X_{1,g}^*(v_1) &= \begin{cases} 1+q & \text{when } v_1 = 2 \\ 1-q & \text{when } v_1 = 1 \end{cases} \\
X_{1,g}^*(-v_1) &= -X_{1,g}^*(v_1)
\end{aligned}
$$

Let good manager's total expected profits from order x_1 at the beginning of the first round be given by:

$$
\Pi_{tot,g}^e(x_1, v_1) = \Pi_{1,g}^e(x_1, v_1) + \Pr\left(\widehat{\theta} > \theta | x_1, v_1\right)\Pi_{2,g}^*
$$

For $\alpha \to 0^+$, order x_1 does not affect good manager's payoff directly, and $\Pi_{1,g}^e(x_1, v_1) = \Pi_{2,g}^* = \beta$. Therefore, we can write:

$$
\Pi_{tot,g}^e(x_1, v_1) = \left[1 + \Pr\left(\widehat{\theta} > \theta | x_1, v_1\right)\right]\beta
$$

In order to show that $X_{1,g}^*(v_1)$ is optimal for the good manager in the first round, we have to show that for every $v_1 = -2, -1, 1, 2$ and every $x_1 \in \mathbb{R}$, the following holds true:

$$
\left[1 + \Pr\left(\widehat{\theta} > \theta | X_{1,g}^*(v_1), v_1\right)\right]\beta \geq \left[1 + \Pr\left(\widehat{\theta} > \theta | x_1, v_1\right)\right]\beta
$$

Given investor's beliefs (45) and (46), for a good manager it is true that:

$$
\Pr(\widehat{\theta} > \theta | x_1, v_1) = \begin{cases} 1 \text{ if } x_1 = X_{1,g}^*(v_1) \\ 0 \text{ if } x_1 \neq X_{1,g}^*(v_1) \end{cases}
$$

Thus, the optimality condition for $X_{1,g}^*(v_1)$ is always trivially satisfied as long as $\beta > 0$.

In the present context, it important to note that since $\alpha \to 0^+$, the specific order placed by a good manager does not affect his payoff directly, and good manager's and investor's incentives are not obviously aligned. Order x_1 affects the payoff by affecting good manager's probability of being retained. As we know, this probability crucially depends on investor's beliefs about the strategy that a good manager follows in equilibrium. Here, we are (reasonably) focusing on the class of equilibria in which the investor conjectures that in equilibrium good managers do follow the strategy that maximizes the expected return from trade, $X^*_{1,g}(v_1)$.

Remark 6 *For $\alpha \to 0^+$, the good manager follows his equilibrium strategies $X^*_{1,g}(v_1)$ and $X^*_{2,g}(v_2)$ as long as $\beta > 0$ and condition (69) is satisfied, that is $q \in \left[\frac{2}{5}, \frac{2}{3}\right)$.*

Bad managers' strategy in the first round. We know that in equilibrium the bad managers does not trade in the second round. Furthermore, the bad manager does not observe v_1. Accordingly, we can write his total expected profits from order x_1 at the beginning of the first round as follows:

$$\Pi^e_{tot,g}(x_1) = \left[1 + \Pr\left(\widehat{\theta} > \theta | x_1\right)\right]\beta$$

In order to show that strategy $X^*_{1,b}$ of randomizing with equal probability over $1-q$ and $-1+q$ is the optimal strategy for the bad manager in the first round, we have to show that for every $x_1 \in \mathbb{R}$, the following holds true:

$$\left[1 + \Pr\left(\widehat{\theta} > \theta | X^*_{1,b}\right)\right]\beta \geq \left[1 + \Pr\left(\widehat{\theta} > \theta | x_1\right)\right]\beta$$

Again, given investors' beliefs (45) and (46), we have that for a bad manager

$$\Pr(\widehat{\theta} > \theta | x_1) = \begin{cases} \frac{1}{2} - w & \text{if } x_1 = 1-q, -1+q \\ w & \text{if } x_1 = 1+q, -1-q \\ 0 & \text{otherwise} \end{cases}$$

and accordingly

$$\Pr(\widehat{\theta} > \theta | X^*_{1,b}) = \frac{1}{2} - w$$

Thus, it must be true that for every $x_1 \in \mathbb{R}$,

$$\left(1 + \frac{1}{2} - w\right)\beta \geq \left[1 + \Pr\left(\widehat{\theta} > \theta | x_1\right)\right]\beta$$

The previous condition is always trivially satisfied for orders $x_1 \neq 1+q$, $1-q$, $-1+q$,

$-1-q$, for which $\Pr(\widehat{\theta} > \theta|x_1) = 0$.[31] Let $X_{1,b}^{+}$ denote a mixed strategy that consists in randomizing over the equilibrium orders $1+q$ and $-1-q$. In this case

$$\Pr(\widehat{\theta} > \theta|X_{1,b}^{+}) = w$$

and the equilibrium condition must now satisfy

$$\left(\frac{1}{2} - w\right)\beta \geq w\beta$$

which is always satisfied as long as $\beta > 0$ and $w \in \left(0, \frac{1}{4}\right]$. Note that when w gets greater than $\frac{1}{4}$, the bad manager has an incentive to deviate to from the equilibrium mixed strategy $X_{1,b}^{*}$ to $X_{1,b}^{+}$. Hence, the conjecture at the base of part b) of proposition 4 that for $w \in \left(\frac{1}{4}, \frac{1}{2}\right)$ there exist equilibria in which bad managers randomizes over $1+q$ and $-1-q$.

Remark 7 *For $\alpha \to 0^{+}$, the bad managers follows his mixed strategy of randomizing over $1-q$ and $-1+q$ in the first round and does not trade in the second round as long as $\beta > 0$ and $w \in \left(0, \frac{1}{4}\right]$*

The investor's problem. An investor hires a fund manager at the beginning of the first round if the profits he expects to gain from delegating trade are higher than those he would get by staying out of the market. When $\alpha \to 0^{+}$, the investor finds it convenient to hire a fund manager in the first round of trade as long as:

$$E\left(R_1^e\right) - \beta \geq 0$$

where

$$E\left(R_1^e\right) = \frac{1}{2}E\left(R_{1,g}^e\right) + \frac{1}{2}E\left(R_{1,b}^e\right)$$

and

$$\begin{aligned} E\left(R_{1,g}^e\right) &= 2w(1+q)\left[1 - \frac{1}{4}(6w-1)\right] + (1-2w)(1-q)\left[1 - \frac{1}{4}\frac{1-2w}{1-w} + \frac{1}{4}(6w-1)\right] \\ E\left(R_{1,b}^e\right) &= -(1-q)\left[\frac{1}{4}\frac{1-2w}{1-w} - \frac{1}{4}(6w-1)\right] \end{aligned}$$

[31] Clearly, also any mixed strategy $X_{1,b}$ that randomizes over orders $x_1 \in \mathbb{R}\backslash\{1+q,\ 1-q,\ -1+q,\ -1-q\}$ gives rise to $\Pr(\widehat{\theta} > \theta|X_{1,b}) = 0$ and thus is dominated by $X_{1,b}^{*}$

After a bit of algebra, one can show that

$$E\left(R_1^e\right) = \frac{5}{2}w - 3w^2 \tag{72}$$

Therefore, delegation occurs as long as $E\left(R_1^e\right) - \beta \geq 0$, or equivalently:

$$\beta \leq \frac{5}{2}w - 3w^2$$

Notice that $E\left(R_1^e\right)$ is strictly increasing in $w \in \left(0, \frac{1}{4}\right]$. Furthermore, it is easy to compute that $\lim_{w \to 0^+} E\left(R_1^e\right) = 0^+$, while for $w = \frac{1}{4}$, $E\left(R_1^e\right) = \frac{7}{16}$. Hence, $E\left(R_1^e\right) \in \left(0, \frac{7}{16}\right]$.

Remark 8 *When $w \in \left(0, \frac{1}{4}\right]$ and $\alpha \to 0^+$, for any $\beta \leq \frac{5}{2}w - 3w^2$ an investor finds it convenient to hire a fund manager at the beginning of the first round of trade.*

Note that condition (68) implies $\beta \leq \frac{3}{8}$. Since $\frac{5}{2}w - 3w^2 \in \left(0, \frac{7}{16}\right]$, we can safely conclude that a manager is hired in both periods as long as $\beta \leq \min\left(\frac{3}{8}, \frac{5}{2}w - 3w^2\right)$
■

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