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# Low-Complexity Equalisation and Channel Estimation over Fast Fading Channels

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*by*

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the degree of*

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*in*

Engineering

*under the supervision of*

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*to*

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## CERTIFICATE OF ORIGINAL AUTHORSHIP

I, Hongyang Zhang declare that this thesis, is submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the Faculty of Engineering and Information Technology at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This document has not been submitted for qualifications at any other academic institution.

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## ABSTRACT

The next generation wireless communication systems aim to achieve high capacity and low latency with high-mobility scenario as an important channel condition for various new applications. With the significantly increased data rate and Doppler frequency shift, the systems' ability to cope with fast channel variations is of significant importance. This thesis develops effective and efficient solutions to improve the performance of both conventional and emerging modulations over fast fading channels.

The recently proposed orthogonal time frequency space (OTFS) modulation shows outstanding performance over fast fading channels. However, existing research on OTFS is mostly focused on its delay-Doppler domain structure. In this thesis, channel and system models in different signal domains are firstly derived in both continuous and discrete forms, providing the basis for exploiting the full potential of OTFS with low complexity. Particularly, a circular stripe diagonal structure in the frequency-Doppler domain channel matrix for arbitrary multipath delays and Doppler shifts is identified through analyses and simulations, paving the way for low-complexity techniques to be adopted to combat fast channel fading.

Exploiting the circular stripe diagonal nature of the frequency-Doppler channel matrix, a low-complexity frequency-domain minimum mean-square-error (MMSE) equalisation for OTFS systems with long signal frames and fully resolvable Doppler spreads is then formulated. It is also demonstrated that the proposed MMSE equalisation is applicable to conventional modulations with short signal frames and partially resolvable Doppler spreads.

The diversity performance analyses for OTFS are further provided under both maximum likelihood and linear equalisations. Inspired by the frequency-domain precoding structure, an adaptive transmission scheme with frequency-domain precoding matrix composed of the eigenvectors of the channel matrix is proposed to improve the system performance under MMSE equalisation, and its optimised performance is derived with simple analytical expressions. Considering two extreme channel conditions, the lower and upper bounds for the diversity performance of the adaptive transmission scheme are also derived. The derived performance bounds can serve as performance benchmarks for OTFS and other precoded OFDM systems.

Based on the re-formulation of OTFS as precoded-OFDM, three variants of the original OTFS system for low-complexity channel estimation over fast fading channels are finally proposed in this thesis. They enable one-dimensional channel estimation and corresponding equalisation to be applied in either frequency or time domain. Simulation results demonstrate that the proposed frequency-domain pilot aided OTFS scheme is the most effective transmission technique for high-mobility wireless communications in terms of diversity performance, signalling overhead, and power efficiency.

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## TABLE OF CONTENTS

<b>Certificate of Original Authorship</b>	<b>i</b>
<b>Abstract</b>	<b>iii</b>
<b>Acknowledgments</b>	<b>v</b>
<b>List of Publications</b>	<b>vii</b>
<b>List of Figures</b>	<b>xiii</b>
<b>List of Tables</b>	<b>xvii</b>
<b>Abbreviations</b>	<b>xix</b>
<b>Notations</b>	<b>xxiii</b>
<b>List of Symbols</b>	<b>xxv</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Background . . . . .	1
1.1.1 Use Cases for Next Generation Network . . . . .	1
1.1.2 Challenges for Practical Applications . . . . .	2
1.1.3 Affection of Fast Fading Channels . . . . .	4
1.2 Challenges for Fast Fading Channels . . . . .	4
1.3 Thesis Organisation . . . . .	7

## TABLE OF CONTENTS

---

<b>2</b>	<b>Analyses on Channel and System Models</b>	<b>9</b>
2.1	Introduction and Literature Review . . . . .	9
2.2	Fast Fading Channel Representations . . . . .	10
2.3	Signal Models in Time and Frequency Domains . . . . .	12
2.3.1	Continuous Signal Models . . . . .	13
2.3.2	Discrete Signal Models . . . . .	14
2.4	Simulation Results . . . . .	22
2.5	Conclusions . . . . .	24
<b>3</b>	<b>Low-Complexity Equalisation Techniques over Fast Fading Channels</b>	<b>27</b>
3.1	Introduction and Literature Review . . . . .	27
3.1.1	Conventional Modulation over Fast Fading Channels . . . . .	27
3.1.2	OTFS over Fast Fading Channels . . . . .	28
3.1.3	Existing Equalisation Techniques of OTFS . . . . .	28
3.1.4	Chapter Structure . . . . .	30
3.2	Low-Complexity MMSE Equalisation . . . . .	30
3.2.1	MMSE Equalisation with Fully Resolvable Doppler Spread . . . . .	30
3.2.2	MMSE Equalisation with Partially Resolvable Doppler Spread . . . . .	33
3.3	Equalisation Performance Analysis . . . . .	35
3.3.1	Input-Output Relationships . . . . .	35
3.3.2	Output SNR Analysis . . . . .	37
3.4	Simulation Results . . . . .	40
3.5	Conclusions . . . . .	44
<b>4</b>	<b>Adaptive Transmission with Frequency-Domain Precoding</b>	<b>47</b>
4.1	Introduction and Literature Review . . . . .	47
4.1.1	Existing Diversity Study of OTFS . . . . .	47
4.1.2	Chapter Structure . . . . .	48

4.2	OTFS as Precoded OFDM . . . . .	49
4.3	Diversity with ML Equalisation . . . . .	52
4.4	Adaptive Transmission and BER Bounds . . . . .	53
4.4.1	Adaptive Transmission . . . . .	54
4.4.2	BER Bounds and Analysis . . . . .	56
4.5	Simulation Results . . . . .	59
4.5.1	Adaptive Transmission Performance and Comparison . . . . .	59
4.5.2	BER Bound Validation . . . . .	63
4.6	Conclusions . . . . .	65
<b>5</b>	<b>Low-Overhead OTFS Transmission with Frequency or Time Domain</b>	
	<b>Channel Estimation</b>	<b>67</b>
5.1	Introduction and Literature Review . . . . .	67
5.1.1	Existing Channel Estimation techniques of OTFS . . . . .	67
5.1.2	Chapter Structure . . . . .	68
5.2	OTFS Variants and Channel Estimation . . . . .	69
5.2.1	Frequency-Domain Pilot-Aided Scheme . . . . .	69
5.2.2	Time-Domain Pilot-Aided Scheme . . . . .	72
5.2.3	Time-Domain Training Sequence Scheme . . . . .	75
5.3	Performance Analysis . . . . .	77
5.3.1	MSE Analysis for FD-PA-OTFS . . . . .	78
5.3.2	MSE Analysis for TD-PA-OTFS . . . . .	79
5.3.3	MSE Analysis for TD-TS-OTFS . . . . .	79
5.4	Simulation Results . . . . .	81
5.5	Conclusions . . . . .	85
5.6	Appendix . . . . .	86
<b>6</b>	<b>Conclusions and Future Work</b>	<b>89</b>

## TABLE OF CONTENTS

---

6.1	Summary of Contributions . . . . .	89
6.2	Future Work . . . . .	91
6.2.1	Extension to MIMO Systems . . . . .	92
6.2.2	Application in Joint Radar Communication (JRC) . . . . .	92
6.2.3	Further System Complexity Reduction . . . . .	93
	<b>Bibliography</b>	<b>95</b>
	<b>Appendix</b>	<b>113</b>
	Appendix A . . . . .	113
	Appendix B . . . . .	114

## LIST OF FIGURES

<b>FIGURE</b>	<b>Page</b>
2.1 Relationships among different continuous channel representations. . . . .	12
2.2 Construction of delay-time domain channel matrix. . . . .	17
2.3 Construction of frequency-Doppler domain channel matrix. . . . .	19
2.4 An example demonstrating the process of forming the delay-time domain and frequency-Doppler domain channel matrices. . . . .	20
2.5 Relationships among channel matrices in different domains. . . . .	23
2.6 Power delay profiles of TDL-A (a) and TDL-D (b) channel models. . . . .	25
2.7 Frequency-time domain representation of fast fading channels. . . . .	25
2.8 Frequency-Doppler domain channel matrix over fast fading channels. . . . .	26
3.1 Comparison between long and short signal frames. . . . .	33
3.2 Performance comparison in LOS channel. . . . .	41
3.3 Performance comparison in NLOS channel. . . . .	42
3.4 BER versus velocity variation rate for detection with outdated CSI in LOS channel at SNR = 15 dB. . . . .	43
3.5 Performance comparison between MMSE and MP equalisations. . . . .	44
3.6 Performance comparison with LDPC coding. . . . .	45
4.1 Time domain transmitted signal frame of original OTFS system. . . . .	50
4.2 Frequency domain transmitted signal frame of OTFS as precoded OFDM form. . . . .	51

4.3	OTFS system block diagram in precoded OFDM form: (a) transmitter and (b) receiver. S/P and P/S stand for serial-to-parallel and parallel-to-serial conversions respectively, and CP stands for cyclic prefix. . . . .	52
4.4	Adaptive transmission system block diagram: transmitter (a) and receiver (b).	55
4.5	Comparison of various modulation schemes in LOS channels using 4-QAM. .	60
4.6	Comparison of various modulation schemes with and without channel estimation error in NLOS channels using 4-QAM. . . . .	61
4.7	Comparison of various modulation schemes with 16-QAM under both LOS and NLOS channels. . . . .	62
4.8	Lower bounds of MMSE equalisation performance under different multipath diversity orders without Doppler frequency shifts. The curves from right to left correspond to $P = 1, 2, 4, 8, 16, 32, 64, 128, 1024$ , and asymptotic $\infty$ . . . . .	64
4.9	Upper bounds of MMSE equalisation performance under different Doppler frequency diversity orders with $P = M \times N$ . The curves from left to right correspond to $K_{\max} = 1, 4, 64$ , and asymptotic $\infty$ . . . . .	65
4.10	BER comparison under adaptive transmission when $K_{\max} = 1, 4, 16, 64$ in high SNR region. . . . .	66
5.1	FD-PA-OTFS frequency-domain transmission frame construction with pilots.	70
5.2	FD-PA-OTFS channel estimation process. . . . .	71
5.3	Time-domain transmission frame constructions of (a) TD-PA-OTFS and (b) TD-TS-OTFS. . . . .	74
5.4	Channel estimation performance comparison among FD-PA-OTFS, TD-PA-OTFS and TD-TS-OTFS. . . . .	83
5.5	BER performance comparison among FD-PA-OTFS, TD-PA-OTFS and TD-TS-OTFS with estimated CSIs. . . . .	84
5.6	BER performance comparison between FD-PA-OTFS and original OTFS. . . .	85



5.7	BER performance of FD-PA-OTFS under different maximum speeds. . . . .	86
5.8	BER performance of TD-TS-OTFS under different maximum speeds. . . . .	87
1	Illustration of Gaussian elimination. . . . .	114



## LIST OF TABLES

<b>TABLE</b>	<b>Page</b>
1 Signals, Channel Representations and System Parameters . . . . .	xxiii
2 Signals, Channel Representations and System Parameters . . . . .	xxv
3 Matrices and Vectors . . . . .	xxvii
2.1 Simulation Parameters . . . . .	24
5.1 Overhead of Different Transmission Schemes . . . . .	82



## ABBREVIATIONS

1G - first generation

2D - two-dimensional

3GPP - 3rd generation partnership project

4G - fourth generation

5G - fifth generation

6G - sixth generation

ADC - analog-to-digital converter

ADS - autonomous driving system

AR - augmented reality

AWGN - additive white Gaussian noise

BER - bit error rate

CDSC - continuous-Doppler-spread channel

CFO - carrier frequency offset

CP - cyclic prefix

CSI - channel state information

DAC - digital-to-analog converter

DFS - Doppler frequency shift

DFT - discrete Fourier transform

## ABBREVIATIONS

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eMBB - enhanced mobile broadband

ETSI - European telecommunications standards institute

FBMC - filter bank multicarrier

FD-PA - frequency-domain pilot aided

FT - Fourier transform

Gbps - Gigabit per second

GFDM - generalized frequency division multiplexing

H2H - human to human

HD - high-definition

IAI - inter antenna interference

ICI - inter-carrier interference

IDFT - inverse discrete Fourier transform

IDI - inter Doppler interference

IFT - inverse Fourier transform

ISFFT - inverse symplectic finite Fourier transform

ISTNs - integrated space and terrestrial networks

JRC - joint radar communication

LDPC - low-density parity-check

LOS - line-of-sight

LTE - long term evolution

M2M - machine to machine

MAP - maximum A Posteriori probability

MCMC - Markov chain Monte Carlo

MIMO - multiple-input multiple-output

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ML - maximum-likelihood

MLSE - maximum likelihood sequence estimation

MMSE - minimum mean square error

mMTC - massive machine type communication

mmWave - millimeter-wave

MP - message passing

MSE - mean-square-error

NLOS - non-line-of-sight

OFDM - orthogonal frequency division multiplexing

OTFS - orthogonal time frequency space

PA - power amplifiers

PAPR - peak to average power ratio

PDF - probability density function

QAM - quadrature amplitude modulation

RHS - right-hand-side

SC-FDE - single carrier frequency domain equalisation

SFFT - symplectic finite Fourier transform

SISO - single-input single-output

SNR - signal-to-noise ratio

TDL - tapped delay line

TD-PA - time-domain pilot aided

TD-TS - time-domain training sequence

TF - time-frequency

UAV - unmanned aerial vehicle

## ABBREVIATIONS

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UMa - urban macrocell

uRLLC - ultra-reliable and low latency communication

V2V - vehicle-to-vehicle

VR - virtual reality

ZC - Zadoff Chu

ZF - zero forcing



## NOTATIONS

Table 1: Signals, Channel Representations and System Parameters

<b>Expressions</b>	<b>Definitions</b>
<b>Bold letters</b>	Matrices and vectors signal
$(\cdot)^{\mathbf{T}}$	Transpose operation
$(\cdot)^*$	Conjugate operation
$(\cdot)^{\mathbf{H}}$	Conjugate transpose operation
$\mathbf{X}_{m \times n}$	$m$ by $n$ matrix
$\mathbf{I}_m$	$m$ by $m$ identity matrix
$\mathbf{F}_m$	$m$ -point normalised DFT matrix
$\mathbf{1}_{m \times n}$	$m$ by $n$ matrices with all 1 elements
$\mathbf{0}_{m \times n}$	$m$ by $n$ matrices with all 0 elements
$vec(\cdot)$	Vectorisation operation
$diag\{\mathbf{X}\}$	Extracting the diagonal elements from matrix $\mathbf{X}$
$diag\{\mathbf{x}\}$	Forming a diagonal matrix with vector $\mathbf{x}$
$\odot$	Hadamard product
$\otimes$	Kronecker product
$(\cdot)_M$	Modulo $M$ operation
$\lceil \cdot \rceil$	Ceiling operation
$\lfloor \cdot \rfloor$	Flooring operation
$\mathbf{X}(i, :)$	$i$ -th row of $\mathbf{X}$
$\mathbf{X}(:, j)$	$j$ -th column of $\mathbf{X}$



## LIST OF SYMBOLS

Table 2: Signals, Channel Representations and System Parameters

<b>Symbols</b>	<b>Definitions</b>
$s(t)$	Time domain continuous transmitted signal
$r(t)$	Time domain continuous received signal
$w(t)$	Time domain continuous noise
$S(f)$	Frequency domain continuous transmitted signal
$R(f)$	Frequency domain continuous received signal
$W(f)$	Frequency domain continuous noise
$s[i]$	Time domain discrete transmitted signal
$r[i]$	Time domain discrete received signal
$w[i]$	Time domain discrete noise
$S[i]$	Frequency domain discrete transmitted signal
$R[i]$	Frequency domain discrete received signal
$W[i]$	Frequency domain discrete noise
$h_i, \tau_i, \text{ and } \nu_i$	The path gain, delay and Doppler shift of the $i$ -th path in sparse $P$ -path channel model
$P$	Number of multipaths in sparse P-path channel model.
$\tau$	Delay
$\nu$	Doppler frequency shift
$t$	Time variables
$f$	Frequency variables
$h(\tau, \nu)$	Continuous delay-Doppler channel representation
$h_t(\tau, t)$	Continuous delay-time channel representation
$H_\nu(f, \nu)$	Continuous frequency-Doppler channel representation
$H(f, t)$	Continuous time-frequency channel representation
$M$	Number of subcarriers
$N$	Number of OFDM/SC-FDE Symbols

LIST OF SYMBOLS

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<b>Symbols</b>	<b>Definitions</b>
$M_1$	Number of pilot sections in frequency domain for FD-PA-OTFS
$N_1$	Number of precoded data symbols in frequency domain in each pilot section for FD-PA-OTFS
$M_2$	Number of precoded data symbols in time domain in each pilot or training sequence section for TD-PA-OTFS or TD-TS-OTFS
$N_2$	Number of pilot or training sequence sections in time domain for TD-PA-OTFS or TD-TS-OTFS
$d_r$	Delay resolution
$f_r$	Doppler resolution
$T$	Duration of OFDM symbol
$L$	Length of channel impulse response
$L_{\max}$	Maximum number of resolvable multipaths
$K_{\max}$	Maximum number of resolvable Doppler frequency shifts
$L_{cp}$	Length of CP
$T_{cp}$	Duration of CP
$\sigma_x^2$	Time domain data symbol power
$\sigma_s^2$	Time domain signal power (the same as $\sigma_x^2$ )
$\sigma_w^2$	Time domain noise power
$\sigma_S^2$	Frequency domain signal power
$\sigma_W^2$	Frequency domain noise power
$\gamma_{in}$	Input SNR before equalisation
$\gamma_{out}$	Output SNR after equalisation

Table 3: Matrices and Vectors

<b>Symbols</b>	<b>Definitions</b>	<b>Dimensions</b>
$\mathbf{r}$	Time domain received sequence	$MN$ by 1
$\mathbf{s}$	Time domain transmitted sequence	$MN$ by 1
$\mathbf{w}$	Time domain noise sequence	$MN$ by 1
$\mathbf{H}_t$	Delay-time channel matrix	$MN$ by $MN$
$\mathbf{R}$	Frequency domain received sequence	$MN$ by 1
$\mathbf{S}$	Frequency domain transmitted sequence	$MN$ by 1
$\mathbf{W}$	Frequency domain noise sequence	$MN$ by 1
$\mathbf{H}_v$	Frequency-Doppler channel matrix	$MN$ by $MN$
$\mathbf{h}$	Discrete version of $h(\tau, \nu)$	$MN$ by $MN$
$\mathbf{H}$	Discrete version of $H(f, t)$	$MN$ by $MN$
$\mathbf{h}_t$	Discrete version of $d_r h_t(\tau, t)$	$MN$ by $MN$
$\mathbf{h}_v$	Discrete version of $f_r H_v(f, \nu)$	$MN$ by $MN$
$\hat{\mathbf{s}}$	Estimate of $\mathbf{s}$	$MN$ by 1
$\hat{\mathbf{S}}$	Estimate of $\mathbf{S}$	$MN$ by 1
$\mathbf{G}_t$	Time domain MMSE matrix	$MN$ by $MN$
$\mathbf{G}_v$	Frequency domain MMSE matrix	$MN$ by $MN$
$\mathbf{X}$	Data symbol matrix for original OTFS	$M$ by $N$
$\mathbf{x}$	Data symbol vector	$MN$ by 1
$\mathbf{y}$	Received signal after MMSE equalisation	$MN$ by 1
$\mathbf{X}_1$	Data symbol matrix for FD-PA-OTFS	$M_1$ by $N_1$
$\mathbf{X}_2$	Data symbol matrix for TD-PA-OTFS and TD-TS-OTFS	$M_2$ by $N_2$
$\mathbf{S}_{data}$	Precoded data matrix in frequency domain for FD-PA-OTFS	$M_1$ by $N_1$
$\mathbf{s}_{data}$	Precoded data matrix in time domain for TD-PA-OTFS and TD-TS-OTFS	$M_2$ by $N_2$

LIST OF SYMBOLS

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<b>Symbols</b>	<b>Definitions</b>	<b>Dimensions</b>
<b><math>\Theta</math></b>	Frequency domain interpolation matrix	$MN$ by $N_1$
<b><math>\Psi</math></b>	Time domain interpolation matrix	$N_2$ by $MN$
<b><math>\Phi</math></b>	Doppler domain phase shifting matrix	$L_{max} + 1$ by $MN$
<b><math>E</math></b>	Doppler domain phase shifting matrix composed of columns from $\Phi$	$L_{max} + 1$ by $N_2$
<b><math>z</math></b>	ZC sequence	$L_{max} + 1$ by 1
<b><math>q</math></b>	Toeplitz-form matrix composed of ZC sequence	$L_{max} + 1$ by $L_{max} + 1$
<b><math>Q</math></b>	Unitary matrix composed of eigenvectors of $\mathbf{H}_v^H \mathbf{H}_v$	$MN$ by $MN$
<b><math>\mathbb{Q}</math></b>	Diagonal matrix composed of rows from $q$	$L_{max} + 1$ by $(L_{max} + 1)^2$
<b><math>\Lambda</math></b>	Diagonal matrix composed of eigenvalues of $\mathbf{H}_v^H \mathbf{H}_v$	$MN$ by $MN$
<b><math>V</math></b>	Modulation matrix in general	Dimensions vary for different modulations