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# Saddle-Shaped Pattern Synthesis by Element Rotation and Phase Optimization for Linear Dipole Array

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**Abstract**—The element rotation and phase optimization technique is utilized for synthesizing saddle-shaped pattern in this paper. As is known, the rotation of a dipole changes the co- and cross-polarized patterns in a given cut of the array pattern. Together with suitable phase control, it is feasible to synthesize shaped power patterns. An example of synthesizing a saddle-shaped pattern with reduced sidelobe level (SLL) and restrained cross-polarization level (XPL) is provided. The synthesized result validates the effectiveness of the proposed element rotation based technique.

**Keywords**—Shaped pattern synthesis; cross-polarization level.

## I. INTRODUCTION

The saddle-shaped pattern plays an important role in wireless communication. Many different optimization methods were proposed to obtain the saddle-shaped pattern in the past few decades [1][2]. Most of the existing strategies chose to obtain the desired patterns by finding proper excitations or positions of the elements in linear, planar, or conformal arrays.

As is known, rotating a linearly polarized antenna will provide weight to its co-polarization and cross-polarization pattern at a given cut. Thus the element rotation could be exploited in the array synthesis. The element rotation-based technique has already been applied to several array pattern synthesis [3]–[6], where the obtained results have demonstrated the effectiveness. In this paper, we propose to synthesize a saddle-shaped pattern by jointly optimizing the dipole rotations and excitation phases of the linear array.

Optimizing the rotations and phases of the dipoles is a nonlinear problem. Generally, the stochastic optimization algorithm could be adequate. Here, the dynamic differential evolution (DDE) algorithm is utilized to synthesize the saddle-shaped power patterns. In the DDE, the generated better trial individual will immediately replace the current individual and be utilized in the subsequent process instead of being used in the next generation like that in the conventional DE. Hence, DDE has better performance than the conventional DE. An example of synthesizing saddle-shaped pattern is presented to validate the idea of the element rotation based strategy. Synthesis result shows that the proposed method could obtain relatively low SLL and precise mainlobe shape. Compared to the amplitude modulation methods, optimizing rotation angles and phases of dipoles simplifies the feeding network structure

since there is no need for unequal power dividers.

## II. VECTORIAL PATTERN OF THE DIPOLE-ROTATED ARRAY

Suppose there is a linear array comprised of  $N$  uniformly-spaced dipoles. In this array, each dipole in the array is rotated by an angle  $\xi_n \in [-\pi, \pi]$  with respect to  $z$ -axis, as depicted in Fig. 1. A local coordinate system (LCS)  $x'y'z'$  to facilitate formulations of the array pattern. In the LCS, the vectorial element pattern of this array can be expressed as

$$\vec{E}(\theta', \phi') = \frac{\cos(\pi/2 \cos \theta')}{\sin \theta'} \hat{\theta}' \quad (1)$$

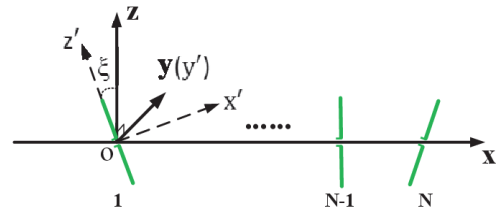


Fig. 1. The rotated dipole array.

To calculate the array pattern, all the vectorial element patterns in the LCSs should be transformed into the global coordinate system (GCS). Firstly, the direction unit vector  $\vec{\theta}'$  should be decomposed and transformed to the GCS

$$\vec{\theta}' = (\cos \xi \cos \theta' \cos \phi' + \sin \xi \sin \theta') \vec{x} + \cos \theta' \sin \phi' \vec{y} + (\sin \xi \cos \theta' \cos \phi' - \cos \xi \sin \theta') \vec{z} \quad (2)$$

Then, we can obtain the element pattern in the GCS by projecting the vector  $\vec{\theta}'$  to  $\hat{\theta}$  and  $\hat{\phi}$  as

$$E_{\theta}(\theta', \phi') = \frac{\cos(\pi/2 \cos \theta')}{\sin \theta'} \vec{\theta}' \cdot \vec{\theta} \quad (3)$$

$$E_{\phi}(\theta', \phi') = \frac{\cos(\pi/2 \cos \theta')}{\sin \theta'} \vec{\theta}' \cdot \vec{\phi} \quad (4)$$

At last, the scalar angles  $\theta'$  and  $\phi'$  in the LCS are required to be transformed to  $\theta$  and  $\phi$  in the GCS. Then, in the principle plane ( $\theta = 90^\circ$ ), the two components of the vectorial element pattern can be obtained as

$$E_\theta(\phi; \xi) = \frac{\cos \xi \cos(\pi / 2 \sin \xi \cos \phi)}{1 - \sin^2 \xi \cos^2 \phi} \quad (5)$$

$$E_\phi(\phi; \xi) = \frac{\sin \xi \sin \phi \cos(\pi / 2 \sin \xi \cos \phi)}{\sin^2 \xi \cos^2 \phi - 1} \quad (6)$$

For a linear array with  $N$  dipoles lie along  $x$ -axis, the  $\theta$ - and  $\phi$ -polarized components of the array pattern can be given as

$$F_\theta(\phi) = \sum_{n=1}^N E_{n,\theta}(\phi) e^{j(\beta d_n \cos \phi + \alpha_n)} \quad (7)$$

$$F_\phi(\phi) = \sum_{n=1}^N E_{n,\phi}(\phi) e^{j(\beta d_n \cos \phi + \alpha_n)} \quad (8)$$

where  $d_n$  is the the position and  $\alpha_n$  is the excitation phase of the  $n$ th element,  $j$  is the imaginary unit,  $\beta = 2\pi/\lambda$  is the propagation constant,  $\lambda$  is the wavelength.

### III. THE DDE-BASED ROTATION ANGLE AND EXCITATION PHASE OPTIMIZATION

In order to synthesize desired patterns, an appropriate cost function of DDE should be constructed first. Let  $\Gamma_{SLL}$  and  $\Gamma_{XPL}$  denotes the desired SLL and XPL, respectively, and the desired mainlobe shape of the is  $P_t(\phi)$ , the cost function can be set as:

$$f = \frac{1}{A} \sum_{a=1}^A \left\{ |F_\theta(\phi_a)|^2 - P_t(\phi_a) \right\} + \frac{1}{B} \sum_{b=1}^B (X_b + |X_b|)^2 + \frac{1}{C} \sum_{c=1}^C (Y_c + |Y_c|)^2 \quad (9)$$

where

$$X_b = |F_\theta(\phi_b)|^2 - \Gamma_{SLL}, \quad \phi_b \in \text{SLL region} \quad (10)$$

$$Y_c = |F_\phi(\phi_c)|^2 - \Gamma_{XPL}, \quad \phi_c \in \text{XPL region.} \quad (11)$$

In the above,  $\phi_a$  ( $a = 1, 2, \dots, A$ ) and  $\phi_b$  ( $b = 1, 2, \dots, B$ ) are discrete sampling angles in shaped beam region and sidelobe region of the co-polarized pattern  $F_\theta(\phi)$ , and  $\phi_c$  ( $c = 1, 2, \dots, C$ ) are the sampling angles of  $\phi \in [0, \pi]$ . Now, the DDE algorithm is utilized to minimize the cost function through optimizing  $\xi_n$  and  $\alpha_n$  ( $n = 1, 2, \dots, N$ ) for synthesizing the saddle-shaped pattern with controlled SLL and XPL.

### IV. NUMERICAL RESULTS

To illustrate the effectiveness of the proposed method, we consider to synthesis a saddle-shaped pattern with a 22-element linear dipole array. The array has a uniform spacing of  $0.5\lambda$ . Suppose a saddle-shaped in the region of  $\phi \in [78^\circ, 102^\circ]$  with a beamwidth of  $24^\circ$  is desired, and the desired ripple is restricted as  $\pm 0.2$  dB. The sidelobe region is set as  $\phi \in [0^\circ, 72.5^\circ] \cup [107.5^\circ, 180^\circ]$  and the SLL and the XPL is set as  $\Gamma_{SLL} = \Gamma_{XPL} = -12.5$  dB. The parameter settings of DDE in this example are given as follows: the population size is 220, both the mutation intensity and the cross probability are set as

0.8, the maximum generation is 2000, respectively. The co- and cross-polarized patterns obtained by the proposed method are depicted in Fig. 2. It is observed that the synthesized SLL and XPL are -12.47 dB and -12.45 dB, respectively. The ripple of the co-polarized pattern is about  $\pm 0.10$  dB. This example has indicated the effect of the proposed method in synthesizing shaped beam pattern.

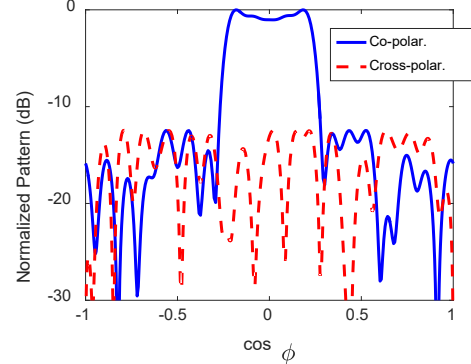


Fig. 2. The obtained saddle-shaped pattern for a 22-element dipole array.

### V. CONCLUSION

Rotating a dipole will change the co- and cross-polarization of its pattern in a fixed observation cut. By optimization of the rotation angles and excitation phases of dipoles in array, it is feasible to obtain desired shaped patterns with restricted SLLs meanwhile constraining the XPLs. A saddle-shaped pattern is synthesized by optimizing the dipole rotations and excitation phases for a linear array using DDE. The obtained results have shown the good capability of the proposed strategy in achieving shaped pattern. The proposed method uses uniform amplitudes, which will result in a simplified feeding network.

### VI. ACKNOWLEDGEMENT

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