Beam Alignment for Analog Arrays based on Gaussian Approximation

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***Abstract*—Beam alignment is a process for receive and transmit antenna arrays to find the correct beamforming directions. It is typically based on beam scanning and peak-energy searching, which lead to time-consuming beam training process in communication protocols, such as 802.11ad for vehicular networks. In this correspondence, we propose a fast beam alignment method for analog arrays, based on directly estimating the angle-of-arrival (AoA) of the incoming signals. We propose simple and highly efficient AoA estimators, by approximating the power of the array response as a Gaussian function. One estimator is based on the power ratio and can coherently combine multiple measurements scanned at arbitrary intervals and with different beam width. The other two are based on an innovative idea of Gaussian curve fitting with weighted least square techniques, and can even work without knowing the beam width. Simulation results validate the effectiveness of the proposed scheme.**

***Index Terms*—Beam alignment, AoA Estimation, Analog Array, Gaussian Approximation**

I. Introduction

We are seeing booming applications of analog arrays with beam steering capability in wireless communications and radar sensing, for example, in millimeter wave (mmWave) 802.11ad for vehicular networks [1], [2] and future integrated space-air-ground communication networks. In these communication systems with analog arrays, beam alignment, the transmit and receive antenna arrays find the correct beamforming directions, is an essential step for establishing connections. It is typically based on beam scanning and peak-energy searching, and hence is time-consuming, as reflected by, e.g., the multiple beam training sequences and multiple sweeping stages in 802.11ad [2]. Such designs were mainly because of the lack of efficient angle-of-arrival (AoA) estimation techniques for analog antenna arrays.

AoA estimation for analog array is a challenging problem, due to the nonlinear nature of the problem and the availability of only a single measurement at a time. Techniques that have been developed for AoA estimation in analog array may be classified into the following two classes: scanning-based, and direct estimation. The scanning-based techniques are the design basis of the beam sweeping protocols in, e.g., 802.11ad, and they can be further divided into sequential and multi-resolution scanning. The basic idea of *sequential scanning* [3] is using narrow beams of fixed beamwidth to sequentially and exhaustively scan the directions of interest and find the AoA via identifying the one with the largest received power. The basic idea of hierarchical *multi-resolution scanning* [4] is to gradually reduce the width of the scanning beam by narrowing down the possible zone of the AoA to be estimated. The resolution of scanning-based techniques is proportional to the beam width and it may take a long time to find the quantized AoA if a fine resolution is needed. The direct estimation techniques can obtain non-quantized AoA estimates from two or more measurements. These techniques include *compressive sensing* (CS) based estimators [5], conventional spectrum analysis techniques such as MUSIC [6], the auxiliary beam pair (ABP) method in [7], and the virtual-subarray based AoA estimation (ViSA) algorithms in [3]. The first two types of techniques have high complexity. ABP estimates the AoA via comparing the received signal power at two paired scanning beams, based on the sinc function of the beam pattern. However, it lacks flexibility in dealing with different scanning directions and different arrays. The ViSA algorithms can achieve very accurate estimation via exploiting both the phase and power of the received signals, but it requires special training sequences.

In this correspondence, we propose some simple and efficient Gaussian approximation based AoA estimators, which provide great flexibility for beam scanning and enable fast beam alignment. The core idea of these estimators is to use a Gaussian function to approximate the power of the array response, which is typically a sinc function and hard to manipulate. By using the Gaussian approximation, we derive three efficient closed-form estimators. One estimator is based on the power ratio of the measurements, and we can further combine multiple measurements using, e.g., maximal ratio combining (MRC), to get better estimation. This estimator is capable of working with two or multiple scanning measurements at an arbitrary interval. The other two estimators are based on an innovative idea of curve-fitting the Gaussian function with weighted least square techniques. To our best knowledge, this is the first time that the curve fitting technique is introduced to AoA estimation and one of the estimators works even when the beam width is unknown. For these estimators, we demonstrate that one can directly approximate and estimate either the AoA or equivalent AoA, and provide effective techniques to handle the different periodic properties between the power function of array response and its Gaussian approximation. Simulation results are provided and demonstrate that these proposed estimators can achieve much better performance and enable larger beamforming gain and quicker beam alignment, compared to existing scanning-power based schemes.

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II. Gaussian Approximation to Array Response

We consider a uniform linear array (ULA), where the antenna interval is half a wavelength. The received signal at the m-th beam scanning with a single line-of-sight (LOS) path can be represented as

$$y_m = d\mathbf{w}^T(u_m)\mathbf{a}(u)s_m + z_m, \tag{1}$$

where s_m is the transmitted signal, z_m is additive white Gaussian noise (AWGN), d is the path loss, $u = \pi\sin(\theta)$ is the equivalent AoA and θ is the unknown AoA to be estimated, $\mathbf{a}(u) = [1, e^{ju}, \ldots, e^{j(N-1)u}]^T$ is the array steering vector, and

$$\mathbf{w}(u_m) = \mathbf{a}(-u_m) = [1, e^{-ju_m}, \ldots, e^{-j(N-1)u_m}]^T \tag{2}$$

is the beamforming (BF) vector, steering single pencil beam at the equivalent AoA u_m.

It is well-known that the power of the ideal array response for a ULA is given by

$$\rho(u_m, u) \triangleq |\mathbf{w}(u_m)^T\mathbf{a}(u)|^2 = \left|\sum_{n=1}^{N} e^{j(n-1)(u-u_m)}\right|^2 = \frac{\sin^2(\frac{N(u_m-u)}{2})}{\sin^2(\frac{u_m-u}{2})}. \tag{3}$$

The mainlobe of $\rho(u, u_m)$ in (3) can be well approximated by a scaled Gaussian function, given by

$$f(x_m, x, \sigma) = b\, e^{-\dfrac{(x_m-x)^2}{2\sigma^2}}, \tag{4}$$

where b is the scalar. The sidelobes are much smaller compared to the mainlobe and hence can be ignored. Note that such an approximation was also proposed in amplitude-comparison based direction finding techniques [8], which, however, require the use of a pair of squint measurements. Our estimators are substantially different to them in both scanning measurement collection and signal processing techniques.

The Gaussian function can be used to approximate the power of array response with respect to (w.r.t) either AoA θ or equivalent AoA u, represented as

$$\begin{aligned} &\rho(u_m, u) \approx f(u_m, u, \sigma), \text{ appr. w.r.t equivalent AoA;} \\ &\rho(\theta_m, \theta) \approx f(\theta_m, \theta, \sigma(\theta)), \text{ appr. w.r.t to AoA.} \end{aligned} \tag{5}$$

For $f(u_m, u, \sigma)$, the parameter σ is independent of the angle of the incoming signals and the scanning directions, and is fixed for a given array with the BF vector in (2). However, for $f(\theta_m, \theta, \sigma(\theta))$, $\sigma(\theta)$ is varying with the AoA θ of the incoming signal and is unknown. Only when $|\theta|$ is small or its range is small and known, $\sigma(\theta)$ can be approximated as unchanged and known. These are illustrated in Fig. 1, where the Curve Fitting tool in Matlab is used to obtain the parameters of the Gaussian function. From the figure, we can see that in the mainlobe, the Gaussian function generally approximates the beam pattern well, except for $f(\theta_m, \theta, \sigma(\theta))$ with a large $|\theta|$. Although approximation errors in the sidelobes are notable, they are much smaller compared to the power of the mainlobe and can be ignored. The figure also shows that the approximation mismatch of $f(\theta_m, \theta, \sigma(\theta))$ increases with the AoA increasing and becomes quite large

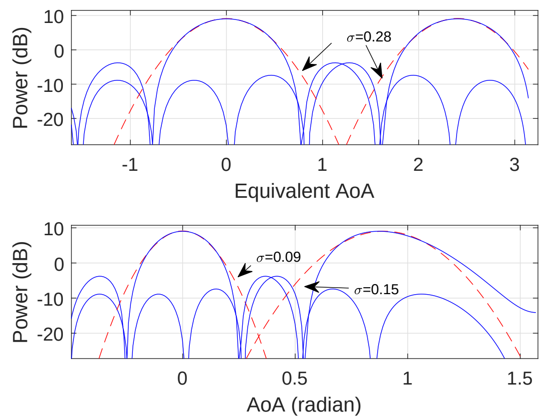

Fig. 1. Beampattern of an 8-element ULA and its Gaussian approximation, w.r.t the equivalent AoA (top) and AoA (bottom), for incoming signals at two different AoAs (0 and 50 degrees). Red dashed curves are for Gaussian approximation and blue solid lines are for the power of the array response.

with $|\theta|$ approaches $\pi/2$. There is one important issue on the periodic properties of these functions, which we leave to address in Section III-C.

The two approximations have different performances, despite there is a one-to-one mapping between the AoA θ and equivalent AoA u. On one hand, $f(u_m, u, \sigma)$ can lead to uniform estimation performance for u over $u \in [-\pi, \pi]$, while $f(\theta_m, \theta, \sigma(\theta))$ will see increasingly degraded performance for estimating both u and θ due to the enlarged approximation mismatch with an increasing θ and varying $\sigma(\theta)$. On the other hand, for estimating θ, the estimation error with $f(u_m, u, \sigma)$ will increase with $|\theta|$ increasing. This is because $\mathrm{d}u = \pi\cos(\theta)\,\mathrm{d}\theta$ where d denotes the differentiation operator, and for the same $\mathrm{d}u$, $\mathrm{d}\theta$ is larger when $|\theta| \in [0, \pi/2]$ is larger. In addition, using $f(\theta_m, \theta, \sigma(\theta))$ to estimate θ saves the operation of $\theta = \mathrm{asin}(u/\pi)$, which is needed in using $f(u_m, u, \sigma)$ and requires considerable resource and/or time in real-time implementation. We will summarize the choices of the two approximations, by considering these factors, together with the simulation results in Section IV.

III. Beam Alignment based on Gaussian Estimators

The beam alignment process can be separated into transmit and receive beam scanning stages, as in the 802.11ad standard. In each stage, the angle-of-departing (AoD) or AoA can be estimated, respectively. Below, we describe the AoA estimation process based on the Gaussian estimators to be proposed.

S1: Fix the transmit beamforming with a large mainlobe. The receiver scans M directions θ_m over the range of interest $[\theta_s, \theta_e]$, using the BF vector $\mathbf{w}(u_m) = \mathbf{a}(-u_m)$. Generally, the directions of scan should be uniformly distributed over either the AoA or equivalent AoA domain, but this is not essential for our proposed estimators. To ensure that the AoA is covered in the mainlobe of at least one scanning beam, the condition $M \geq (\theta_e - \theta_s)/(2\pi/N)$ needs to be satisfied;

S2: Compute the power of the received signal for each scan. Pick up a candidate set $\mathcal{S}$ of K power measurements;

S3: With $\mathcal{S}$, estimate the AoA based on one of the two methods to be presented;

S4: If needed, a refined (narrowed) range of scan can be formed based on the estimate, and the process is repeated. Measurements from different rounds of scans can be combined to obtain improved estimation.

The above beam scanning process resembles those defined in standards, such as 802.11ad, but requires much shorter training sequences. Estimating AoD is almost the inverse process of estimating AoA, with beam scanning being implemented at the transmitter instead of the receiver. The scanning range can be refined after the first stage. As will be seen, our proposed Gaussian estimators can achieve accurate estimation with only a small number of measurements, and hence beam alignment can be quickly achieved. Next, we introduce the Gaussian estimators.

Let $x_m, m = 1, ..., M$ denote either the AoAs or equivalent AoAs corresponding to the scanning directions, and x denote the unknown AoA or equivalent AoA. Let the corresponding measurements be y_m. Note that the magnitude of y_m is affected by the pathloss, array gain, and the magnitude of the transmitted signals, while only the array gain contains the information of AoAs. In order to estimate the AoA, we need to assume that the transmitted signals $s_m, m = 1, \ldots, M$ have the same magnitude across the measurements, i.e., $|s_m|^2$ are constant, but s_m $(\forall m)$ can have different phases. This can generally be satisfied in practical systems.

Applying the Gaussian approximation and ignoring the noise, we can construct a series of equations from the received signal as

$$\begin{aligned} |y_m|^2 &= |ds_m|^2 \left|\mathbf{w}(x_m)^T \mathbf{a}(x)\right|^2 \\ &\approx b|ds_m|^2 \, e^{-\dfrac{(x_m - x)^2}{2\sigma^2}}, m = 1, \cdots, M, \end{aligned} \tag{6}$$

where x is to be estimated, with σ either known or unknown, as discussed in Section II.

Assume the set of power measurements $\mathcal{S}$ is selected for now. We propose the following two methods for AoA estimation, with $m \in \mathcal{S}$.

A. Power Ratio (PR) based Method

We consider the case when σ is known in this subsection. From $\mathcal{S}$, we pick up the one with the highest power as a reference, denoted as $|y_{m'}|^2$, and compute the power ratio between the $K-1$ measurements and $|y_{m'}|^2$.

Referring to the Gaussian approximation in (6), we can obtain

$$\ln\left|\frac{y_m}{y_{m'}}\right|^2 = \frac{1}{\sigma^2}\left((x_m - x_{m'})x + \frac{x_{m'}^2 - x_m^2}{2}\right).$$

Then

$$(x_m - x_{m'})x = \frac{x_m^2 - x_{m'}^2}{2} + \sigma^2 \ln\left|\frac{y_m}{y_{m'}}\right|^2. \tag{7}$$

We can then combine all the log-ratios using maximal ratio combining (MRC). That is, multiply the conjugate of $(x_m - x_{m'})$ to both sides of (7), and then sum them up over all K results. This generates

$$x = \frac{\sum_m (x_m - x_{m'})^* \left(0.5(x_m^2 - x_{m'}^2) + \sigma^2 \ln\left|\frac{y_m}{y_{m'}}\right|^2\right)}{\sum_m |x_m - x_{m'}|^2}. \tag{8}$$

From (7), we can also see that even for results obtained with scanning beams of different width in e.g., multi-resolution scanning [4], i.e., different σ's, MRC can be similarly applied to combine these results and obtain an estimator by replacing σ in (8) with different σ's.

B. Weighted Least Square (WLS) Method

The PR method is simple and easy to implement, but computing the ratio may cause increased noise and degrade estimation performance. There may also be cases that the beam width and hence σ are unknown. Instead of removing the unknown amplitude term $b|ds_m|^2$, we now propose an alternative method to estimate all the parameters based on the idea of fitting a Gaussian function with the WLS method [9].

Applying the $\ln(\cdot)$ function to (6), we can obtain

$$r_m \triangleq \ln|y_m|^2 = \alpha + \beta x_m + \gamma x_m^2, \tag{9}$$

where $\alpha = \ln(b|ds_m|^2) - x^2/(2\sigma^2)$, $\beta = x/\sigma^2$, $\gamma = -1/(2\sigma^2)$.

Considering the noise in measurements, the basic idea of the WLS method is to find $\{\alpha, \beta, \gamma\}$ that minimize the weighted mean square function

$$\sum_m g_m \left(r_m - (\alpha + \beta x_m + \gamma x_m^2)\right)^2, \tag{10}$$

where g_m are the weights and can be selected as $g_m = |y_m|^{2p}$, with p typically being 1, or 2.

This problem can be solved by differentiating the function w.r.t α, β, γ, respectively, and setting the resultant expressions to be zeros. We can then obtain three linear equations represented in the form of matrix operation as

$$\begin{aligned} &\begin{pmatrix} \sum_m g_m & \sum_m g_m x_m & \sum_m g_m x_m^2 \\ \sum_m g_m x_m & \sum_m g_m x_m^2 & \sum_m g_m x_m^3 \\ \sum_m g_m x_m^2 & \sum_m g_m x_m^3 & \sum_m g_m x_m^4 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \\ &= \begin{pmatrix} \sum_m g_m r_m \\ \sum_m g_m x_m r_m \\ \sum_m g_m x_m^2 r_m \end{pmatrix}. \end{aligned} \tag{11}$$

Solving these equations, we can not only obtain x, but also σ and $b|ds_m|^2$. Thus we can see an attractive aspect of this method: *the beam width or even the beam shape does not need to be known in order to estimate AoA, as long as the beam can be well approximated by a Gaussian function*. This can be very useful for some applications. For example, we may use a single antenna at the receiver to estimate the angle of departure for signals from the transmitter array, without requiring to know the type of array, only if the transmitter array steers beams in a sequence of known directions.

When σ is known, from (11) we can obtain a simplified estimator for x as

$$x = \frac{\left(\left(\sum_m g_m x_m\right)\left(\sum_m v_m\right) - \left(\sum_m g_m\right)\left(\sum_m v_m x_m\right)\right)\sigma^2}{\left(\sum_m g_m x_m\right)^2 - \left(\sum_m g_m\right)\left(\sum_m g_m x_m^2\right)}, \tag{12}$$

where $v_m \triangleq g_m\left(r_m + x_m^2/(2\sigma^2)\right)$. It is interesting to see that, for $M = 2$, (12) becomes

$$x = \frac{1}{x_1 - x_2}\left(\frac{x_1^2 - x_2^2}{2} + \sigma^2(r_1 - r_2)\right), \tag{13}$$

which is independent of the weighting factor g_m and has the same expression with (8).

C. Selection of Candidate Set of Measurements

One remained problem is how to determine the set $\mathcal{S}$ from the M measurements. The measurements in the sidelobes do not contribute much to improving the performance, because (1) the measurements may contain more noise than effective signals, and (2) the Gaussian approximation at sidelobes is inaccurate. So we shall mainly use measurements in the mainlobe to construct $\mathcal{S}$. Intuitively and based on simulation results, the two methods are found to have different preferences.

We first define a common pre-selection process for both methods. Compute the mean power of K_1 ($K_1 \geq 2$) neigbhouring scans over all the M measurements, and then choose the group that has the highest mean power. The AoA is assumed to be covered by this group. From these K_1 measurements, we further identify the one with the maximal power, and denote its index as p. Using $K_1 \geq 2$ can reduce false detection probability, particularly at lower SNRs. We then define two different follow-up processes for the two methods next.

For the PR method, from (7) and (8), we can see that the term $y_m/y_{m'}$ is noisy, and hence we shall avoid to use a measurement that may introduce more noise than effective signal. Thus we propose the following approach: we first select $y_m, m = [p - \text{floor}(K_2/2), p + \text{floor}(K_2/2)]$, where floor$(x)$ denotes rounding x towards minus infinity; if K_2 is even, we then drop the one with the least power; finally we select the $K \geq 2$ measurements with power larger than $w_p|y_p|^2$, where $0 < w_p < 1$ is a preset threshold. The last step is to ensure that we only keep "effective" measurements in the estimation.

For the WLS method, the set of measurements are used to characterize the curve shape, hence even small measurements at the bottom of the mainlobe can play an important role in curve fitting, particularly when the number of measurements is limited. Thus we select and keep $K = K_2$ measurements $y_m, m = [p - \text{floor}(K_2/2), p + \text{floor}(K_2/2)]$, with $K_2 \geq 3$ generally being an odd number. Note that in conventional curve fitting applications, a large number of random measurements are present [9]. In our problem here, however, we find that the AoA estimation performance is surprisingly accurate when we only use as few as two measurements for (12) and three for (11) from M uniform measurements, as will be shown in Section IV.

One important issue that shall be highlighted is that the power function of array response $\rho(u_m, u)$ is periodic with the period 2π, $\rho(\theta_m, \theta)$ does not have a period π, whereas the Gaussian function is not periodic. This causes problems when the above measurement selection involves beyond-edge points. For $f(u_m, u, \sigma)$, we offer the solution of periodic extension, for example, if $p \leq M - \text{floor}(K_2/2)$, we select $y_m, m = [1, \cdots, K_2 - \text{floor}(K_2/2), M - \text{floor}(K_2/2) + 1, \cdots, M]$ measurements, and update u_m with $[u_1, \cdots, u_{K_2 - \text{floor}(K_2/2)}, u_{M - \text{floor}(K_2/2)+1} - 2\pi, \cdots, u_M - 2\pi]$. However, for $f(\theta_m, \theta, \sigma(\theta))$, we cannot do such an extension and thus select the measurements up to the edge, for example, $y_m, m = [1, \cdots, K_2]$ when $p \leq M - \text{floor}(K_2/2)$. This is another disadvantage of using $f(\theta_m, \theta, \sigma(\theta))$.

IV. Simulation Results

In this section, we provide simulation results to validate the performance of our proposed scheme. We simulate a ULA with $N = 8$ antenna elements. The equivalent AoA u is randomly and uniformly distributed in $[-0.7\pi, 0.7\pi]$. The M scanning directions are uniformly distributed over the equivalent AoA of $(-\pi, \pi]$. The SNR is defined at the antenna level, i.e., as the ratio between the received signal power and the noise power at each antenna. We use "PR", "WLS1", and "WLS2" to denote the power ratio based method, the WLS methods based on (11) with unknown σ and (12) with known σ, respectively. Unless stated otherwise, $K_1 = K_2 = 2$ and $w_p = 0.2$ for PR, and $K_1 = 2$, $K = K_2 = 3$, and $g_m = |y_m|^4$ for WLS1 and WLS2. We also use the functions $f(u_m, u, \sigma)$ and $f(\theta_m, \theta, \sigma(\theta))$ to represent approximations w.r.t the equivalent AoA and AoA, respectively. When being assumed known, $\sigma(\theta) = \sigma(0) = 0.09$, and $\sigma = 0.28$, as shown in Fig. 1.

Fig. 2 plots the mean square error (MSE) of estimated equivalent AoA u, with $f(u_m, u, \sigma)$. For comparison, we also plot the results for the conventional sequential scanning method (denoted as "Scan"), where the direction with the maximum received power from M uniformly scanned directions is identified as the estimate, the ABP scheme in [7], and the ViSA-MSS method in [3]. Note that the ViSA-MSS method requires special training sequence so that a coherent estimation can be conducted by using both amplitude and phase of the array response, and its performance can serve as a lower bound for the scanning-power based schemes including our Gaussian estimators. The figure clearly shows that our proposed three Gaussian estimators achieve similar performance with PR slightly inferior, and they significantly outperform the ABP and "Scan" methods when slightly increasing the number of scans. When $M = 10$, their performance already approaches that of ViSA-MSS, with the advantages of much simpler implementation and system requirements. It is surprising to see that the performance of the WLS1 estimator, assuming an unknown σ, is only slightly degraded, compared to WLS2 with σ known. Notably, this is achieved with only using $K = 3$ measurements for WLS1 and WLS2.

Fig. 3 compares the estimation performance of AoA for $f(\theta_m, \theta, \sigma(\theta))$ and $f(u_m, u, \sigma)$, when the range of equivalent AoA u changes. The figure demonstrates that using

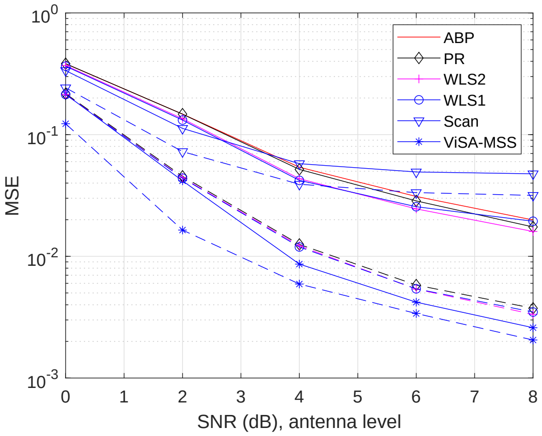

Fig. 2. MSE of equivalent AoA estimates versus SNR for $M = 8$ (solid curves) and $M = 10$ (dashed curves), with $f(u_m, u, \sigma)$.

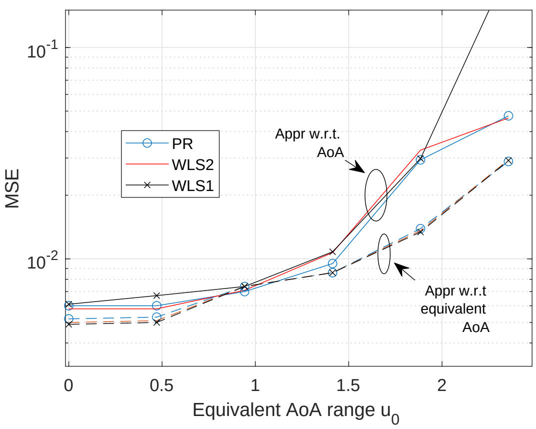

Fig. 3. MSE of AoA estimates for different and unknown ranges of equivalent AoA with $f(\theta_m, \theta, \sigma(\theta))$ and $f(u_m, u, \sigma)$, respectively. For each x-axis value u_0, the simulated equivalent AoA u is uniformly distributed over the range of $[u_0, u_0 + 0.25\pi]$.

$f(u_m, u, \sigma)$ almost always achieves a lower MSE than using $f(\theta_m, \theta, \sigma(\theta))$; however, when $\theta < \text{asin}(0.7) \approx 45^o (u_0 = 0.45\pi)$, the MSE gap between these two approximations is quite small, which indicates that using $f(\theta_m, \theta, \sigma(\theta))$ could be a better option for estimating small AoAs, if simpler system implementation is preferred.

We also compare the beamforming gain when the estimated AoA is applied to generate beamforming weights, as shown in Fig. 4. It is clear that the proposed three methods achieve significantly higher beamforming gains than the conventional "Scan" method. With $M = 10$, their gains are even higher than the latter with $M = 24$. Therefore, using the proposed methods with marginally increased computational complexity, the beam training sequences can be significantly reduced.

V. Conclusions

Using the Gaussian function to approximate the power of array response, we have proposed three simple yet highly efficient AoA estimators, which enable accurate and fast beam alignment. The PR estimator can estimate the AoA without any particular requirement on the directions of scans and the scanning beam widths, and hence it can be conveniently used in conjunction with flexible scanning strategies, such as those proposed in [3]. The WLS1 estimator can work even with unknown beam width, which is attractive in vehicular networks where diverse arrays may be used. All estimators can estimate either the equivalent AoAs or the AoA directly. Simulation results demonstrate that these proposed methods can achieve significantly improved AoA estimation accuracy and beamforming gain with only a small number of beam scans, compared to conventional methods.

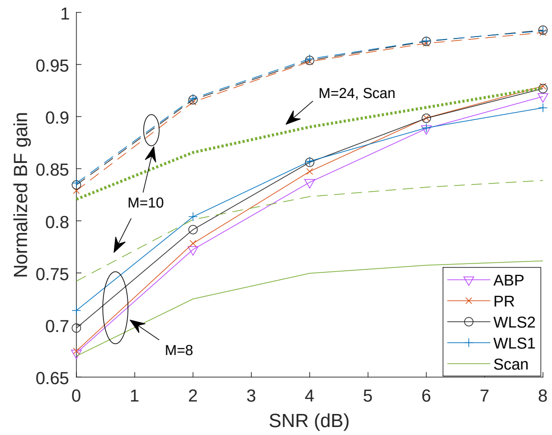

Fig. 4. Beamforming gain, normalized to the ideal one with the perfect AoA estimation, for $M = 8$ (solid curves), $M = 10$ (dashed curves), and $M = 24$ (dotted curve, for the "Scan" method only.).

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