Improved Methods for Force and Torque Calculation in Electrical Machines by 3D Finite Element Analysis

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***Abstract* – This paper presents the theoretical background and implementation techniques for improved force or torque calculation by three dimensional magnetic field analyses. The cogging torque of a permanent magnet claw pole machine is calculated and the theoretical results are verified by experiment.**

I. Introduction

Since the operation of electrical machines depends on the forces or torques that act upon either current carrying conductors or ferromagnetic parts, it is essential in electrical machine design to calculate the force or torque as accurately as possible. The finite element method (FEM) has proved to be a powerful tool for design of electromagnetic devices, especially those of complex structures and/or three dimensional (3D) magnetic fields, such as the claw pole and transverse flux machines.

For calculating the force or torque in an electromagnetic device by FEM, the virtual work and the Maxwell stress tensor are two basic methods commonly used. A major difficulty with these methods is that they are very sensitive to the errors of magnetic field computation. This is particularly evident when the force or torque is small.

The virtual work method calculates the force or torque by taking the partial derivative of the total energy or co-energy against the virtual displacement. Theoretically, this method is accurate, but in practice the determination of the non-linear variation of energy or co-energy against the displacement can be difficult as the energy or co-energy usually has close values in two adjacent rotor positions and the accuracy of the energy or co-energy calculation is affected by the round-off errors.

In contrast, the Coulomb virtual work method computes the force or torque on a moving structure by direct, closed form differentiation of the magnetic energy or co-energy of the field in the air gap between the movable and the fixed parts of the system under consideration. It can evaluate the global force from the field solution of one single rotor position, and therefore, the difficulty of choosing suitable position increment can be avoided. However, the direct derivative of the shape function with respect to the displacement may magnify the computation errors of magnetic field.

The Maxwell stress tensor method calculates the force or torque directly from the magnetic field distribution by adding up the force or torque over a surface or path in the air enclosing the part of interest. The final result, however, can be influenced by the selection of the integration surface or path. To improve the accuracy of the Maxwell stress tensor method, the nodal force or torque method was developed. This improved method calculates the total force or torque by adding up the nodal force or torque of all elements in the movable body, while the nodal force or torque in each element is obtained by the Maxwell tensor.

This paper presents the theory and numerical techniques for implementation of improved methods for force or torque calculation by the finite element magnetic field analysis. The cogging torque in a claw pole permanent magnet (PM) motor is predicted by using these methods and verified by experiment.

II. Methods and Implementation Techniques

A. Virtual Work Method

In an ideal loss-less multiphase electrical rotating machine, which can be obtained by excluding all sorts of power losses, such as the copper loss, the core loss, and the mechanical losses, the variation of the total magnetic energy stored in the field dW_{fld} and the mechanical work dW_{mech} done by the electromagnetic torque T acting on the rotor when the rotor rotates for an angular displacement $d\theta$ should balance with the electrical energy dW_{elec} fed into the machine, namely

$$dW_{elec} = dW_{fld} + dW_{mech} \tag{1}$$

where $dW_{elec} = \sum_{k=1}^{m} i_k d\lambda_k$ and $dW_{mech} = Td\theta$, i_k and λ_k ($k=1,2,\ldots,m$) are the currents and the flux linkages of the machine windings, respectively, and m is the number of windings.

Since

$$dW_{fld} = dW_{elec} - dW_{mech} = \sum_{k=1}^{m} i_k d\lambda_k - Td\theta$$

and

$$dW_{fld} = \sum_{k=1}^{m} \frac{\partial W_{fld}}{\partial \lambda_k} d\lambda_k + \frac{\partial W_{fld}}{\partial \theta} d\theta$$

we obtain

$$T = -\frac{\partial W_{fld}}{\partial \theta} \tag{2}$$

Numerically, it can be approximately calculated by

$$T \approx -\frac{W_{fld}(\lambda_1, \lambda_2, \ldots, \lambda_m, \theta + \Delta\theta) - W_{fld}(\lambda_1, \lambda_2, \ldots, \lambda_m, \theta)}{\Delta\theta} \tag{3}$$

The stored magnetic energy W_{fld} can be calculated by

$$W_{fld} = \int_V \int_0^B \mathbf{H} \bullet \mathbf{dB} dV \tag{4}$$

where V is the volume.

In practical application, co-energy

$$W_{fld}' = \int_V \int_0^H \mathbf{B} \bullet \mathbf{dH} dV \tag{5}$$

is more commonly used, and in terms of the co-energy, the torque can be calculated by

$$T = \frac{\partial W_{fld}'}{\partial \theta} \tag{6}$$

or approximately by

$$T \approx \frac{W_{fld}'(i_1, i_2, \ldots, i_m, \theta + \Delta\theta) - W_{fld}'(i_1, i_2, \ldots, i_m, \theta)}{\Delta\theta} \tag{7}$$

In general, this conventional virtual work method is accurate since the energy or co-energy of the whole model is considered. When the force or torque is small or a too small displacement step is chosen, however, the values of the energy or co-energy are very close and the errors of field and energy or co-energy computations may affect the accuracy significantly. Conversely, if the displacement is too large, the calculated force or torque will fall into the average value over the region.

B. *Coulomb Virtual Work Method*

The Coulomb virtual work (CVW) method is also based on the principle of energy conservation and virtual displacement [1]. In contrast to the classical virtual work method, which uses a finite difference to approximate the energy or co-energy derivative, the CVW method calculates the global force acting on a moving structure by a direct, closed form differentiation of the magnetic energy or co-energy in the air gap between the movable and the fixed parts of the system under consideration.

Consider a magnetostatic field described by the magnetic scalar potential $\varphi(x,y,z)$. The magnetic field vector $\mathbf{H}(x,y,z)$ is determined by

$$\mathbf{H} = -\nabla\varphi \tag{8}$$

By dividing the whole solution region into finite elements, at any point within an element, the magnetic scalar potential and the field can be expressed as

$$\varphi(x, y, z) = \sum_{k=1}^{n} \alpha_k(x, y, z)\varphi_k \tag{9}$$

and

$$\mathbf{H}(x, y, z) = -\sum_{k=1}^{n} \varphi_k \nabla \alpha_k(x, y, z) \tag{10}$$

in terms of the shape functions of the element, $\alpha_k(x,y,z)$, and the nodal scalar potential values φ_k ($k=1,2,\ldots,n$) where n is the number of nodes of the element, and the total co-energy can be calculated by

$$W_{fld}' = \sum_{k=1}^{Ne} \int_{Vk} \int_0^H \mathbf{B} \bullet \mathbf{dH} dV \tag{11}$$

where N_e is the number of elements and V_k the volume of the k-th element. Therefore, the electromagnetic torque can be determined by

$$\begin{aligned} T &= \sum_{k=1}^{Ne} \left[\int_{Vk} \frac{\partial}{\partial\theta} \int_0^H \mathbf{B} \bullet \mathbf{dH} dV + \int_{Vk} \int_0^H \mathbf{B} \bullet \mathbf{dH} \frac{\partial}{\partial\theta}(dV) \right] \\ &\approx \sum_{k=1}^{Ne} \left[\int_{Vk} B \frac{\partial H}{\partial\theta} dV + \int_{Vk} \int_0^H \mathbf{B} \bullet \mathbf{dH} \frac{\partial}{\partial\theta}(dV) \right] \end{aligned} \tag{12}$$

where $\mathbf{B}$ is related to $\mathbf{H}$ by the material property and the integrals are calculated numerically.

C. *Maxwell Stress Tensor Method*

According to Faraday's assumption, the space full of electric field and magnetic field is in a special closed state. If a system in electric and magnetic field balance were cut into two parts with an arbitrary closed surface, the total force that one part applies to the other should pass the surface by some ways.

Maxwell proved by strict mathematical derivation that an electromagnetic force, which applies to one part with volume V, can be expressed as the surface force applied to the surrounding surface S of the volume, i.e.

$$\mathbf{F} = \int_V \mathbf{f} dV = \oint_S \mathbf{T}_{str} dS \tag{13}$$

where $\mathbf{T}_{str}$ is the Maxwell stress tensor applied to the surrounding surface S. When the closed surface is in an even and isotropic medium, the stress tensor can be written as

$$\mathbf{T}_{str} = \mu[(\mathbf{n} \bullet \mathbf{H})\mathbf{H} - \frac{H^2}{2}\mathbf{n}] \tag{14}$$

where $\mathbf{n}$ is the unit normal vector of surface S, and μ the medium permeability.

On surface S, the stress tensor can be resolved into the normal and tangential components. The normal component of the stress tensor

$$\mathbf{n} \bullet \mathbf{T}_{str} = \mu(\mathbf{n} \bullet \mathbf{H})^2 - \frac{\mu}{2} H^2 \tag{15}$$

is a pressure force per unit area and does not contribute to the overall torque. However, the tangential component

$$\mathbf{t} \bullet \mathbf{T}_{str} = \mu(\mathbf{n} \bullet \mathbf{H})(\mathbf{t} \bullet \mathbf{H}) \tag{16}$$

where $\mathbf{t}$ is the unit tangential vector of surface S, is a shearing force per unit area and contributes to the torque.

In a rotating electrical machine, it is convenient to choose the cylindrical surface in the middle of the air gap as the Maxwell integration surface, and therefore, in terms of the cylindrical coordinates, the overall electromagnetic torque can be expressed as

$$T = \oint_S \mathbf{t} \bullet \mathbf{T} r dS = \oint_S \mu_0 H_r H_\theta r dS \tag{17}$$

where r is the radius of the cylindrical surface, and H_r and H_θ are the radial and circumferential components of the magnetic field intensity on the surface, respectively.

D. Nodal Force or Torque Method

To improve the accuracy of the Maxwell tress tensor method, the nodal force or torque method is developed to avoid the problem of sensitivity on the selection of integration surface. This method calculates the total force or torque by adding up the force or torque of each nodal force or torque of all the elements in the movable body, while the nodal force or torque in each element is obtained by the Maxwell tensor. This method can find the spatial distribution of electromagnetic force. This can be very useful if one wants to solve for the mechanical deformation of the machine, or vibration and noise.

In the nodal force method, the nodal force distribution for a given finite element mesh can be obtained by the principal of virtual work and the expression of the Maxwell stress tensor [2]. The nodal force at node k can be given by

$$\mathbf{f}^k = f_x^k \mathbf{u}_x + f_y^k \mathbf{u}_y + f_z^k \mathbf{u}_z \tag{18}$$

and

$$f_x^k = -\sum_{e=1}^{Ne} \int_{Ve} \left(\frac{\partial \alpha_k^e}{\partial x} T_{xx}^e + \frac{\partial \alpha_k^e}{\partial y} T_{xy}^e + \frac{\partial \alpha_k^e}{\partial z} T_{xz}^e \right) dV$$
$$f_y^k = -\sum_{e=1}^{Ne} \int_{Ve} \left(\frac{\partial \alpha_k^e}{\partial x} T_{yx}^e + \frac{\partial \alpha_k^e}{\partial y} T_{yy}^e + \frac{\partial \alpha_k^e}{\partial z} T_{yz}^e \right) dV$$
$$f_z^k = -\sum_{e=1}^{Ne} \int_{Ve} \left(\frac{\partial \alpha_k^e}{\partial x} T_{zx}^e + \frac{\partial \alpha_k^e}{\partial y} T_{zy}^e + \frac{\partial \alpha_k^e}{\partial z} T_{zz}^e \right) dV$$

where T_{xx}^e, T_{xy}^e, T_{xz}^e, T_{yx}^e, T_{yy}^e, T_{yz}^e, T_{zx}^e, T_{zy}^e, and T_{zz}^e are components of the Maxwell stress tensor, and α_k^e is the shape function at the node k in element e.

III. Cogging Torque of a PM Claw Pole Machine

To examine the above theories and implementation techniques, the cogging torque of a PM claw pole machine is calculated by the Coulomb virtual work method and nodal torque method. As discussed above, these two improved methods can provide more reliable results and are easier to use than the traditional energy method and the Maxwell stress tensor method. Fig.1 illustrates the geometrical solution region for the field analysis of the claw pole PM motor [3]. The outer rotor consists of a mild steel cylinder, 20 surface mounted NdFeB magnets and two aluminium end plates (not shown in the figure). The stator consists of two claw pole pieces of soft magnetic composite material and a steel shaft. A single concentrated winding is housed between the two claw pole pieces. The winding is not shown in the figure for clarity. Because of the symmetry of the motor structure, it is only required to analyse the magnetic field in one pole pitch.

The magnetic scalar potential is used to solve the 3D magnetic field distribution and the half periodical boundary condition

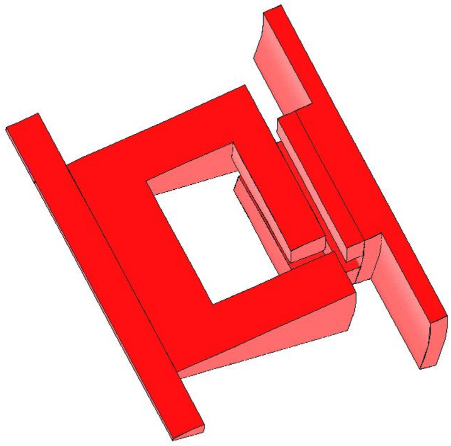

Fig.1 Geometry and field solution region of a claw pole PM motor

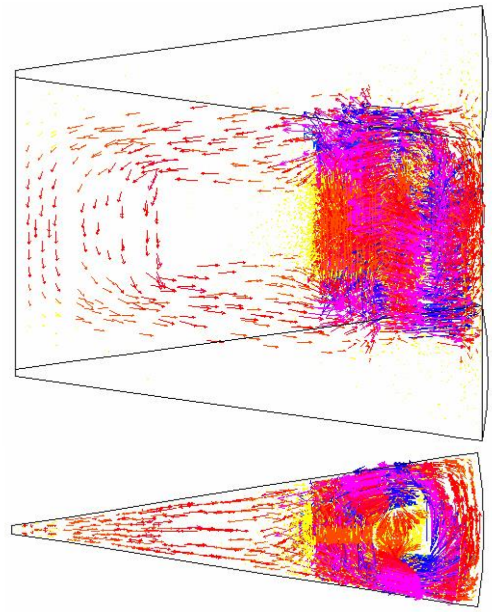

Fig.2 Magnetic flux density vectors at no-load

$$\varphi_m(r,\theta,z) = -\varphi_m(r,-\theta,-z) \tag{19}$$

is applied such that the magnetic flux densities in the two half-periodical boundary surfaces are related by

$$B_{r,\theta}(r,\theta,z) = -B_{r,\theta}(r,-\theta,-z) \tag{20}$$

and

$$B_z(r,\theta,z) = B_z(r,-\theta,-z) \tag{21}$$

The whole solution region is divided into 14,908 ten-node second order tetrahedral elements and 23,283 nodes. The tetrahedral element is used since it is suitable for the complex structure of a claw pole permanent magnet motor.

In Fig.2, it can be seen that the major path for the magnetic flux of the permanent magnets is along one of the permanent magnets, the main air gap, one of the claw pole stator core piece, the stator yoke or the shaft, another claw pole stator core piece, main air gap, another permanent magnet and then the rotor yoke to form a closed loop. There is also a considerable amount of leakage flux through the gaps between the side and end surfaces of the claw poles of the two separated pieces.

Fig.3 compares the predicted cogging torque by the Coulomb virtual work and the nodal torque methods and the experimental results. As illustrated, the theoretical and experimental results match very well.

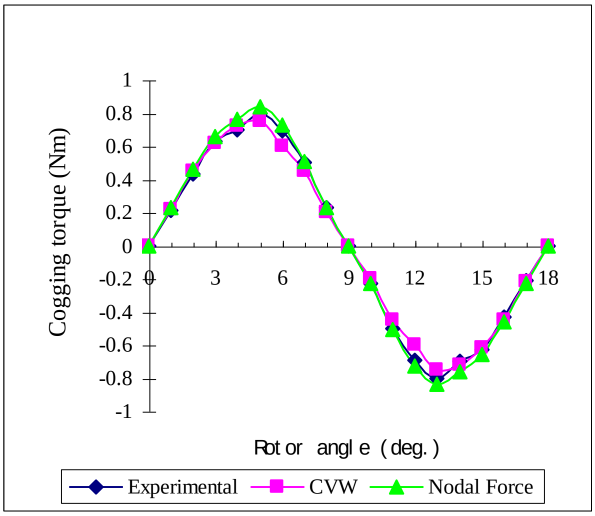

Fig.3 Cogging torque versus rotor angle

IV. Conclusions

The theory and the numerical techniques for implementation of various methods for calculating electromagnetic torque in rotating electrical machines using 3D FEM are described. The Coulomb virtual work method can calculate force or torque from one magnetic field solution and hence can avoid the difficulty of choosing suitable displacement step in the traditional energy method. The nodal force/torque method can avoid the difficulty of selecting the integration surface and/or path in the Maxwell stress tensor method. Therefore, the Coulomb virtual work method and the nodal force/torque method can provide more accurate results and are more convenient to use.

References

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