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A Unified Hybrid Model with Two Level Networks for Peak Current Mode Controlled Buck-Boost Converters Operating in DCM and CCM

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Abstract—This paper presents a unified hybrid model with two level networks for peak current mode controlled buck-boost converters operating in continuous conduction mode (CCM) and discontinuous conduction mode (DCM). It is a network model with two levels in which both a detailed switching model and a small signal average model are included correspondingly. The nonlinearity of some electronic components and the parasitical effects of circuit elements are considered. The nonlinear inductance is calculated by finite element analysis (FEA). The dynamical performance of the buck-boost converter can be obtained from the detailed switching model in the first level network, and then the frequency domain performance can be obtained from the average model in the second level network. As an automatic mechanism of exchanging data between two models is proposed, all the performances can be obtained automatically. By implementing the proposed model in Simulink surrounding, both the large signal transient performance and the frequency domain behavior of converters are obtained efficiently.

I. INTRODUCTION

In order to obtain the stability and dynamic performance of PWM switching DC to DC converter, the detailed switching model is often enrolled to obtain its large-signal (transient) dynamical performance, and the small signal average model is often used for analyzing the frequency domain performance [1-2]. It is well known that the small signal average model is deduced from the method of linearizing the large signal average model around a desired operating point, which is also deduced from the method of averaging the switching model. There are many different operating states which have to be obtained for calculating their corresponding average models, and this will cause heavy work. For that, this paper introduces a unified hybrid model with two level networks for buck-boost converters operating in discontinuous conduction mode (DCM) and continuous conduction mode (CCM), in which the detailed switching model, the small signal average model and their automatic mechanism of exchanging data are included. The nonlinear inductance is calculated by the method of finite element analysis (FEA), and both the nonlinear factors and the parasitical effects of circuit elements are readily considered. By implementing the proposed model in Matlab/Simulink surrounding, both the transient performance and the frequency domain behavior of converters are obtained efficiently.

II. TYPICAL CIRCUIT OF BUCK-BOOST CONVERTER WITH PEAK CURRENT MODEL CONTROLLED

Fig. 1 shows the typical circuit of the buck-boost converter with peak current model controlled which is realized based on UC3842. To effectively analyze the flyback converter, the following assumptions and simplifications are made: (1) the power MOSFET in the ON state is modeled by a zero resistance and in the OFF state by an infinite resistance, R_{INF} , and the output capacitance and inductance of the leading wires are ignored; (2) The diode in the ON state is modeled by a constant voltage source, V_F , with a constant forward resistance, R_F , and in the OFF state by an infinite resistance. The diode junction capacitance is negligible; (3) All the leakage inductances and stray capacitances of the transformer are neglected. (4) Passive components are linear, time invariant, and frequency independent. Then the main equivalent circuit is obtained and shown in Fig. 2.

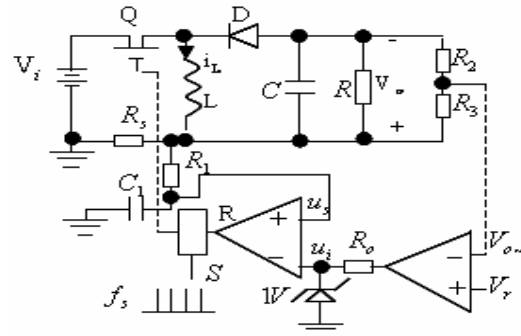


Fig. 1. Buck-boost converter with peak current model controlled

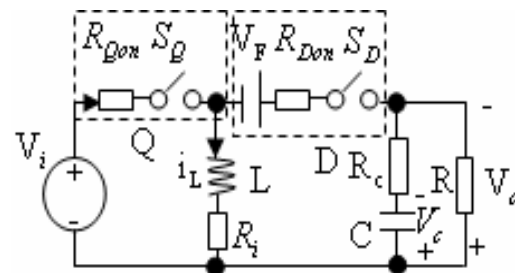


Fig. 2. Main equivalent circuit of buck-boost converter

III. NONLINEAR INDUCTANCE OF TRANSFORMER CONSIDERING THE MAGNETIC SATURATION

In Fig. 2, the inductance, L , of the primary winding of transformer is a differential inductance, which is calculated by using the nonlinear finite element method (FEM), including two steps [3]: (1) For a given winding current, i , conduct a non-linear field analysis to find the flux linkage of windings, ψ_m ; (2) When the flux densities in two consecutive time steps are obtained, the differential inductances can be calculated by

$$L_m(i) = \frac{d\psi_m(k)}{di(k)} = An \frac{B_m(k) - B_m(k-1)}{i(k) - i(k-1)} \quad (1)$$

where A is the cross-sectional area, n_l is the number of turns of the primary winding, and subscripts (k) and $(k-1)$ refer to the k th and $(k-1)$ th steps, respectively.

Fig. 3(a) shows the 2D magnetic force lines of an E25 transformer with air gap. Both the differential inductance and apparent inductance of transformer are shown in Fig. 3(b). The major data of the E25 transformer with PC40 ferrite core include: number of turns of primary winding, $n_1=58$, number of turns of secondary winding, $n_2=12$, and length of air gap, $g=0.16$ mm. Fig. 2 is the actual equivalent circuit of a flyback converter with input voltage of 102 V, 15V/1.2A rated output, and PWM frequency of 100 KHz.

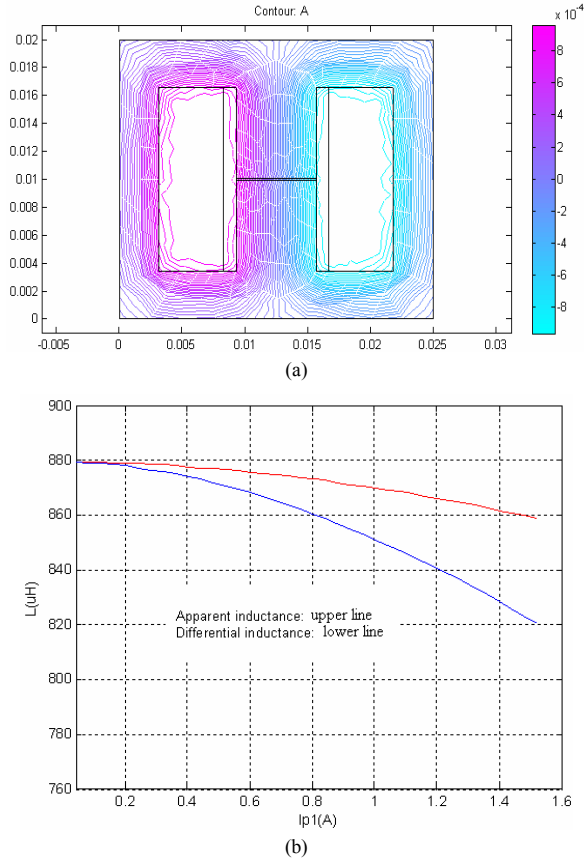


Fig. 3. Calculation of nonlinear inductance: (a) magnetic force lines, and (b) apparent and differential inductances vs. primary current.

IV. DETAILED SWITCHING MODEL OF BUCK-BOOST CONVERTER WITH PEAK CURRENT MODEL CONTROLLED

According to Figs. 1 and 2, the current flowing through the inductance may be in continuous conduction mode (CCM) or in discontinuous conduction mode (DCM). The typical current waveform flowing through the inductance in DCM is shown in Fig. 4, where d_1 , d_2 and d_3 are duty ratios of PWM duty cycle, respectively. When $d_3=0$, CCM can be seen as a special condition of DCM. Fig. 5 shows the current in CCM.

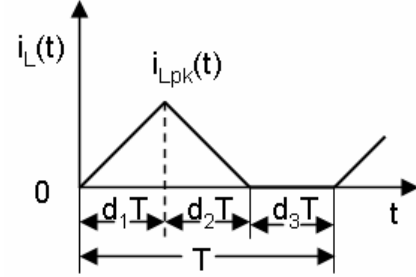


Fig. 4. Typical inductor current waveform in DCM

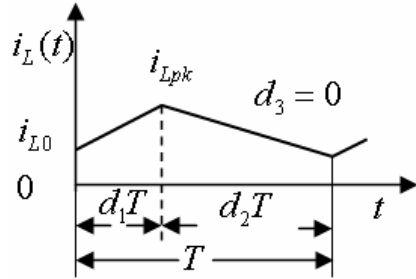


Fig. 5. Typical inductor current waveform in CCM

In order to describe the converter easily, considering that a complete converter is constituted by two parts: one is the main topology, and the other is the feedback circuits, and the duty ratios d_1 , d_2 and d_3 can be seen as variables, then the general control scheme of PWM switching DC to DC converter is obtained and shown in Fig. 6. In other words, not only the current, $i_L(t)$ flowing through the inductance and the voltage, $V_c(t)$, across the capacitance, but also the duty ratios d_1 , d_2 and d_3 are considered as the system state variables. The input voltage, V_i , and the constant voltage source of diode, V_F , are the input variables of system, and the output voltage, V_o , is the output variable of system.

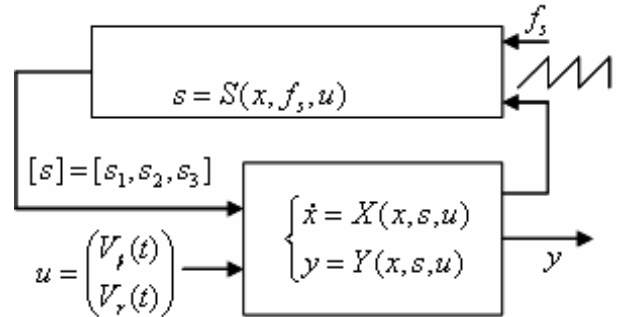


Fig. 6. General control scheme of PWM switching DC to DC converter

According to the KVL and KCL circuit laws, the detailed switching dynamical model of buck-boost converter can be obtained and shown in Table I with a set of state space differential equations. By omitting the control delay in feeding loop, the states of the converter, s_1 , s_2 and s_3 , corresponding to the duty cycles, d_1 , d_2 and d_3 respectively, are also decided by the corresponding equations shown in the table.

TABLE I
DETAILED SWITCHING MODEL OF THE BUCK-BOOST CONVERTER

t or S	Model
$0 \leq t < d_1 T$ or $S1=1,$ $S2=0,$ $S3=0$	$\begin{cases} \frac{di_L}{dt} = \frac{1}{L}[-(R_{Qon} + R_i)i_L + V_i] \\ \frac{dV_c}{dt} = -\frac{1}{(R+R_c)C}V_c \end{cases}$ $V_o = \frac{R}{R + R_c}V_c$ $s_1^k = \bar{s}_1^{k-1} * (u_i < u_s) * cp$ $+ s_1^{k-1} * (u_i \leq u_s)$
$d_1 T \leq t < (d_1 + d_2) T$ or $S1=0,$ $S2=1,$ $S3=0$	$\begin{cases} \frac{di_L}{dt} = \frac{1}{L}[-(R_{Don} + R_i + \frac{RR_c}{R+R_c})i_L - \frac{R}{R+R_c}V_c - V_F] \\ \frac{dV_c}{dt} = \frac{1}{C}[\frac{R}{(R+R_c)}i_L - \frac{1}{R+R_c}V_c] \end{cases}$ $V_o = \frac{RR_c}{R + R_c}i_L + \frac{R}{R + R_c}V_c$ $s_2^k = \bar{s}_1^k * (i_L > 0)$
$(d_1 + d_2) T \leq t < T$ or $S1=0,$ $S2=0,$ $S3=1$	$\begin{cases} i_L = 0 \\ \frac{dV_c}{dt} = -\frac{1}{(R+R_c)C}V_c \end{cases}$ $V_o = \frac{R}{R + R_c}V_c$ $s_3^k = \bar{s}_1^k * \bar{s}_2^k * (i_L \leq 0)$

By using the discrete method, the discrete model for the buck-boost converter with the peak current mode control is obtained from Table 1 as the following equations.

$$\begin{cases} i_L^k = i_L^{k-1} + \frac{T}{L}(s_1^{k-1} * V_1^{k-1} + s_2^{k-1} * V_2^{k-1} + s_3^{k-1} * V_3^{k-1}) \\ V_c^k = V_c^{k-1} + \frac{T}{C}[\frac{R}{R+R_c}i_L^{k-1} * (s_2^{k-1} + s_3^{k-1}) - \frac{1}{R+R_c}V_c^{k-1}] \end{cases} \quad (2)$$

$$\begin{cases} V_o^k = \frac{RR_c}{R+R_c}i_L^k * (s_2^{k-1} + s_3^{k-1}) + \frac{R}{R+R_c}V_c^k \\ \begin{cases} V_1^k = -(R_{Qon} + R_i)i_L^k + V_i^k \\ V_2^k = -(R_{Don} + R_i)i_L^k - V_o^k - V_F \\ V_3^k = 0 \end{cases} \end{cases} \quad (3)$$

where the inductance is nonlinear and is the function of the current flowing through the primary winding in transformer. As

the time of each step is very small, the inductance can be seen as a constant during each step, and can be obtained by the method of lookup table.

V. AVERAGE MODEL OF BUCK-BOOST CONVERTER WITH PEAK CURRENT MODEL CONTROLLED

A. Large Signal Average Model

Suppose that the state vector \mathbf{x} is defined by the inductor current and capacitor voltage as follows:

$$\mathbf{x} = [i_L \quad v_c]^T \quad (4)$$

When the duty ratio $d_3=0$, CCM can be seen as a special condition of DCM. In DCM, switching interval T_s is divided into three subintervals ($k=1, 2, 3$) for (t_0, t_1) , (t_1, t_2) and (t_2, t_3) , and the corresponding duty ratios are d_1 , d_2 and d_3 , respectively. Regardless of the topology circuit in each operation state, the system state model within each k -th subinterval can be expressed by the system matrices \mathbf{A}_k , \mathbf{B}_k , and \mathbf{C}_k .

The corresponding state models are

$$\dot{\mathbf{x}}_1(t) = \begin{cases} \mathbf{A}_1 \mathbf{x}(t) + \mathbf{B}_1 \mathbf{u}, & 0 < t < t_1 \\ \mathbf{0} \end{cases} \quad (5)$$

$$\dot{\mathbf{x}}_2(t) = \begin{cases} \mathbf{A}_2 \mathbf{x}(t) + \mathbf{B}_2 \mathbf{u}, & t_1 < t < t_2 \\ \mathbf{0} \end{cases} \quad (6)$$

$$\dot{\mathbf{x}}_3(t) = \begin{cases} \mathbf{A}_3 \mathbf{x}(t) + \mathbf{B}_3 \mathbf{u}, & t_2 < t < t_3 \\ \mathbf{0} \end{cases} \quad (7)$$

From Fig. 3(b), it is found that the differential inductance does not change much within the range of current, so it can be processed by its average value for building the average model. By taking the average of (5)-(7), the local average model is produced as the following:

$$\begin{aligned} \dot{\bar{\mathbf{x}}}_k &= \frac{1}{T_s} \int_0^{d_k T_s} (\mathbf{A}_k \mathbf{x}(\tau) + \mathbf{B}_k \mathbf{u}) d\tau \\ &= d_k \mathbf{A}_k \bar{\mathbf{x}} + d_k \mathbf{B}_k \mathbf{u} \end{aligned} \quad (8)$$

The final average system-equation is obtained from (8) as

$$\begin{cases} \dot{\bar{\mathbf{x}}} = \sum_{k=1}^3 \dot{\bar{\mathbf{x}}}_k = \sum_{k=1}^3 (d_k \mathbf{A}_k \bar{\mathbf{x}} + d_k \mathbf{B}_k \mathbf{u}) \\ \mathbf{y} = \sum_{k=1}^3 d_k \mathbf{C}_k \bar{\mathbf{x}} \end{cases} \quad (9)$$

B. Small Signal Average Model

Suppose that d_1 , d_2 , d_3 , $\mathbf{x}(t)$, $\mathbf{y}(t)$, and $\mathbf{u}(t)$ are transient parameters, D_1 , D_2 , D_3 , \mathbf{X} , \mathbf{Y} , and \mathbf{U} represent their stable ones, and $\hat{d}_1, \hat{d}_2, \hat{d}_3, \hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{u}}$ represent their corresponding

disturbance ones, respectively. By using the method of linearizing the large signal average model around a desired local operating point, the small signal average model is deduced as the follows.

$$\begin{cases} \dot{\hat{\mathbf{x}}} = \sum_{k=1}^3 [(D_k \mathbf{A}_k \hat{\mathbf{x}} + D_k \mathbf{B}_k \hat{\mathbf{u}}) + (\mathbf{A}_k \bar{\mathbf{X}} + \mathbf{B}_k \mathbf{U}) \hat{d}_k] \\ \hat{\mathbf{y}} = \sum_{k=1}^3 (D_k \mathbf{C}_k \hat{\mathbf{x}} + \mathbf{C}_k \bar{\mathbf{X}} \hat{d}_k) \end{cases} \quad (10)$$

and

$$\begin{cases} \sum_{k=1}^3 D_k = 1 \\ \sum_{k=1}^3 \hat{d}_k = 0 \end{cases} \quad (11)$$

By using the Laplace transform, one can obtain

$$\begin{cases} \hat{\mathbf{x}}(s) = (s\mathbf{I} - \sum_{k=1}^3 D_k \mathbf{A}_k)^{-1} \sum_{k=1}^3 [D_k \mathbf{B}_k \hat{\mathbf{u}} + (\mathbf{A}_k \bar{\mathbf{X}} + \mathbf{B}_k \mathbf{U}) \hat{d}_k] \\ \hat{\mathbf{y}}(s) = \sum_{k=1}^3 [D_k \mathbf{C}_k \hat{\mathbf{x}}(s) + \mathbf{C}_k \bar{\mathbf{X}} \hat{d}_k(s)] \end{cases} \quad (12)$$

By using (12), some small signal transfer functions can be obtained. For example, G_{vd_k} can be derived as

$$\mathbf{G}_{vd_k} = \frac{\hat{\mathbf{y}}(s)}{\hat{d}_k(s)} = \sum_{k=1}^3 (\mathbf{C}_k \bar{\mathbf{X}}) (s\mathbf{I} - \sum_{k=1}^3 D_k \mathbf{A}_k)^{-1} \sum_{k=1}^3 (\mathbf{A}_k \bar{\mathbf{X}} + \mathbf{B}_k \mathbf{U}) \quad (13)$$

Through the transfer functions, some frequency domain behaviors of the converter can be obtained.

VI. MECHANISM FOR EXCHANGING DATA BETWEEN THE DETAILED SWITCHING MODEL AND THE AVERAGE MODEL

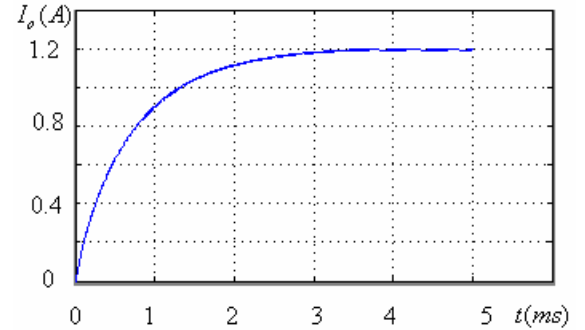
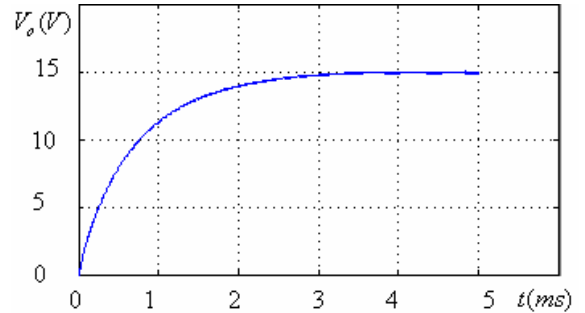
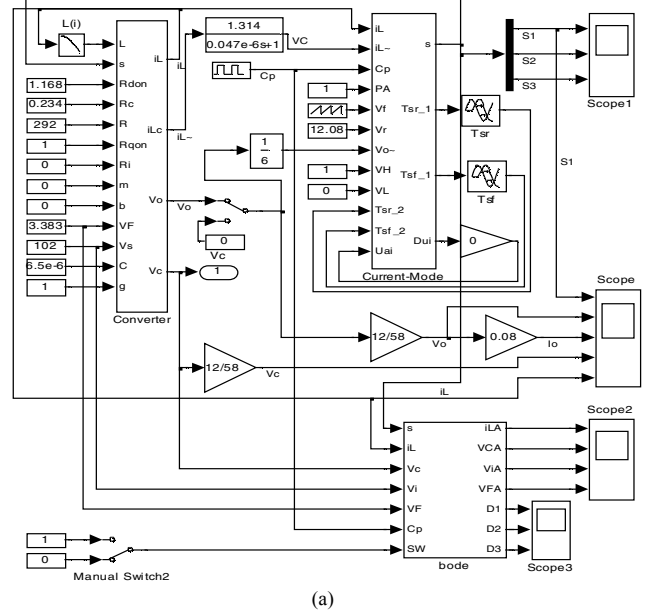
In (12), all the system matrices, \mathbf{A}_k , \mathbf{B}_k and \mathbf{C}_k , are constants which can be preset before running the system model. Other parameters D_k , $\bar{\mathbf{X}}$, \mathbf{U} can also be preset. However, the relation among D_k , $\bar{\mathbf{X}}$, \mathbf{U} is not unique, so it is preferred to set them from the detailed switching model in each PWM cycle automatically. There are many methods for executing the small signal model. The manual method, in which a switching is used to control the execution of the small signal model, is adopted in this paper.

VII. MODEL IMPLEMENTATION

In Fig. 6, the complete simulation model is composed of two parts: one is the block of the converter topology, and the other is the block of the peak current mode control. According to Table I, the simulation model for the block of the converter topology is obtained and shown in the converter block of Fig. 7(a), where the nonlinear differential inductance is calculated in advance and stored into a look-up table. The simulation model for the

block of the closed-loop current mode control is shown in the current mode control block of Fig. 7(a), in which the system switching states, s_1 , s_2 and s_3 , are calculated. By connecting these two blocks, the complete simulation model of buck-boost converter is obtained and shown in Fig. 7(a).

By inputting the parameters of parts into the proposed model shown in Fig. 7(a) and running it in MATLAB/Simulink surrounding, all the system output and system state variables can be obtained and shown in Fig. 7(b)-(e), which are in agreement with the rated specification of the converter.



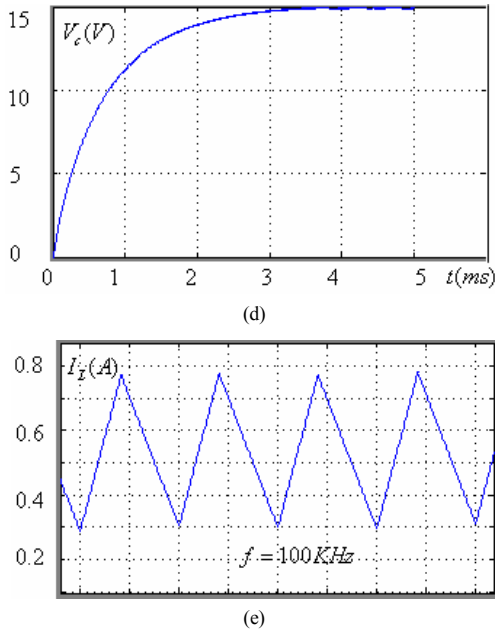


Fig. 7. Implementation of buck-boost converters with peak current mode control: (a) complete simulation model, (b) output voltage, (c) output current, (d) voltage across the capacitor, and (e) current flowing through the inductor.

According to (10)-(13), the small signal simulation model can be easily built and shown in Fig. 8. The average value of D_k , \bar{X} and U in each PWM cycle is calculated by the function of umocav and transferred to the function of bplot. These two functions are programmed according to (10)-(13). Fig. 9 shows the Bode plot of G_{vdi} when the converter operates with the rated condition.

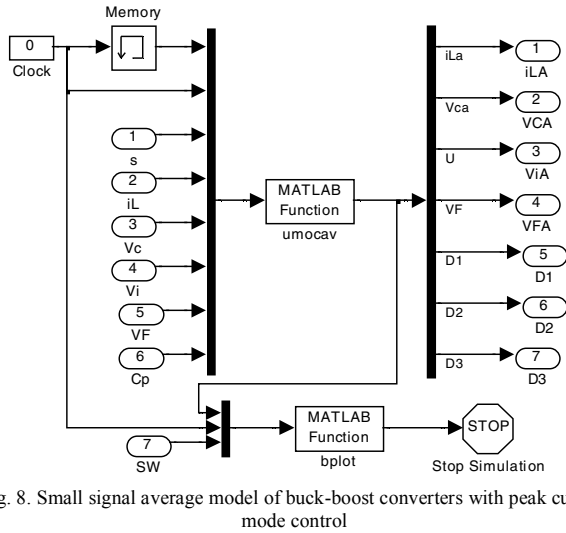


Fig. 8. Small signal average model of buck-boost converters with peak current mode control

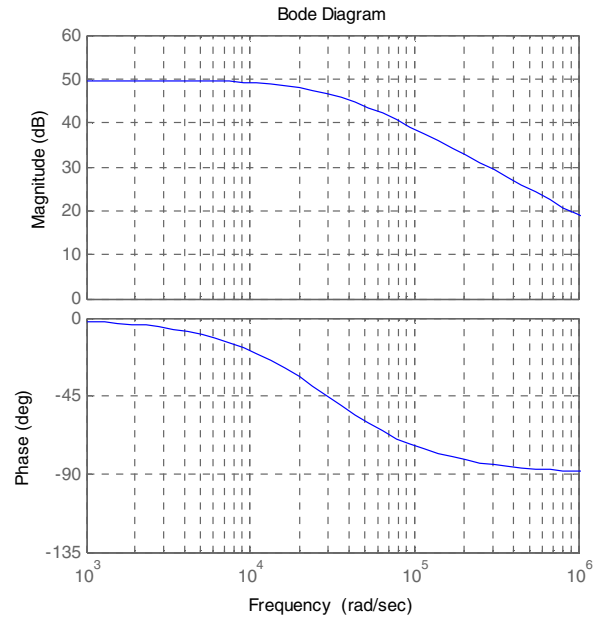


Fig. 9. Bode plot of G_{vdi}

VIII. CONCLUSION

In this paper, a unified hybrid model is proposed for peak current mode controlled buck-boost converters operating in both continuous conduction mode and discontinuous conduction mode. The model consists of two level networks including a detailed switching model and a small signal average model, respectively. Nonlinearity such as the nonlinear inductance is considered by using magnetic field finite element analysis.

The hybrid model has been implemented in Matlab/Simulink for simulating the transient performance and frequency domain characteristics of a buck-boost converter in different conduction modes. Theoretical analysis and simulation results show that comparing with the traditional method, the proposed model is more feasible and more efficient.

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