PID Control for Output Synchronization of Multiple Output Coupled Complex Networks

Lin-hao Zhao, Shiping Wen, *Senior Member, IEEE*, Min Xu, Kaibo Shi, Song Zhu, and Tingwen Huang, *Fellow, IEEE*

***Abstract*—This article attempts to address output synchronization and $\mathcal{H}_\infty$ output synchronization problems for multiple output coupled complex networks (MOCCNs) under proportional-derivative (PD) and proportional-integral (PI) controllers. Firstly, two classes of MOCCNs without and with external disturbances are separately put forward. Secondly, based on the PD and PI control schemes, several output synchronization criteria for MOCCNs are formulated by using the Lyapunov functional method and inequality techniques. Thirdly, $\mathcal{H}_\infty$ output synchronization for MOCCNs is also studied with the help of the PD and PI controllers. Finally, two numerical examples are separately presented to demonstrate the validity of acquired theoretical results.**

***Index Terms*—$\mathcal{H}_\infty$ output synchronization, multiple output coupled complex networks (MOCCNs), output synchronization, proportional-integral-derivative (PID) control.**

I. Introduction

Synchronization, as a ubiquitous and prevalent phenomenon in natural and engineered systems, has seen a growing interest in numerous distinct fields, including biology, economics, and earth sciences [1]. In particular, synchronization for complex networks (CNs), ranging from complex human networks to electric power grids, has received considerable attention, and many advanced results have been published [2]–[13], [53]. In [7], the authors not only presented a class of CNs with fixed coupling model, but also devised a distributed controller to cope with the synchronization problem of an arbitrary subset of the nodes for CNs. Hu et al. [9] considered two types of intermittently coupled complex-valued dynamical networks with heterogeneous or homogeneous adaptive coupling weights, and used some devised adaptive schemes to ensure several synchronization criteria for these two networks by utilizing a direct error approach. As stated in [10], a sufficient condition was established to guarantee the local and global bound synchronization for CNs, and some easy-to-use bound synchronization criteria were also developed, which contained low-dimensional linear matrix inequalities. In addition, to characterize the actual world networks, such as stock transmission systems and inter-city population flow networks, more accuracy and reasonable, numerous authors have borrowed the ideas from single-weighted CNs to multi-weighted CNs (MWCNs). In view of this, a great majority of works related to synchronization for MWCNs have been further reported [14]–[19]. Wang et al. [14] discussed two kinds of MWCNs with undirected and directed topologies, and exploited the Lyapunov functional approach and pinning adaptive control schemes to investigate synchronization for these two networks. In [17], a multiplex network with static and dynamic diffusive couplings was presented, in which the network consists of Chua's circuits, and some sufficient conditions were established to ensure synchronization for the network. Liu et al. [19] not only took synchronization for MWCNs with diagonal inner matrices into consideration by using inequality techniques, but also developed several pinning synchronization criteria for the MWCNs with non-diagonal inner matrices.

Recently, various studies have been related to output synchronization, and many researchers have focused on the output synchronization for CNs [20]–[27]. In [23], the authors explored output synchronization for uncertain CNs with nonidentical nodes based on the neural sliding-mode pinning control strategy. Wei et al. [25] not only presented a class of multi-agent systems composed of heterogeneous individual systems with nonlinear dynamics, but also exploited state feedback and observer-based controllers to address output synchronization problems for the systems. As stated in [26], several sufficient conditions for guaranteeing an optimal control scheme were developed, and output synchronization was also investigated for CNs with partially unknown system dynamics by using the optimal controller. However, few authors also have considered the output synchronization for MWCNs [36], [37], [44]. Wang et al. [36] analyzed output and $\mathcal{H}_\infty$ output synchronization for MWCNs based on inequality techniques and Lyapunov functional, and utilized nodes and edges-based pinning adaptive control methods to guarantee the output and $\mathcal{H}_\infty$ output synchronization for MWCNs.

In addition, due to wide extensive of external disturbance in the real-life world, numerous authors have devoted to paying attention to $\mathcal{H}_\infty$ synchronization and $\mathcal{H}_\infty$ output synchronization for CNs [28]–[32], [34], [35]. Wen et al. [29] not only considered a class of CNs with aperiodic sampled-data communications, but also further used pinning control strategy to address the $\mathcal{H}_\infty$ synchronization issue of this network. As

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L. Zhao and S. Wen are with Australian AI Institute, Faculty of Engineering and Information Technology, University of Technology Sydney, NSW 2007, Australia (linhao.zhao@student.uts.edu.au; shiping.wen@uts.edu.au). M. Xu is with School of Electrical and Data Engineering, Faculty of Engineering and Information Technology, University of Technology Sydney, NSW 2007, Australia (min.xu@uts.edu.au). K. Shi is with School of Information Science and Engineering, Chengdu University, Chengdu, 611040, China, (email: skbs111@163.com). S. Zhu is with School of Mathematics, China University of Mining and Technology, Xuzhou 221116, China (e-mail: songzhu@cumt.edu.cn). T. Huang is with Science Program, Texas A & M University at Qatar, Doha 23874, Qatar (e-mail: tingwen.huang@qatar.tamu.edu).

stated in [30], several $\mathcal{H}_\infty$ synchronization criteria were established for a switched complex network based on the Lyapunov functional, and the $\mathcal{H}_\infty$ synchronization for the network was also discussed by using a switching impulsive controller. Wang et al. [35] put forward four types of undirected and directed coupled neural networks with fixed and adaptive couplings, and formulated several $\mathcal{H}_\infty$ output synchronization criteria for these networks based on adaptive control schemes or the Lyapunov functional. However, few authors have taken $\mathcal{H}_\infty$ synchronization and $\mathcal{H}_\infty$ output synchronization for MWCNs [33], [36], [37]. In [33], the authors analyzed $\mathcal{H}_\infty$ synchronization for MWCNs with fixed and switching topologies by exploiting inequality techniques.

It should be mentioned that for CNs and MWCNs, most of authors mainly studied state coupling of these networks [2]–[11], [14]–[19], [21]–[36], [53], but few works about output coupling have been published [20], [40]–[43]. Chen [41] developed a synchronization criterion for a class of CNs with output coupling by using the Lyapunov functional approach. As stated in [42], the authors presented a class of output coupled CNs with unknown parameters, and used adaptive control strategies to develop several synchronisation criteria for the network. Unfortunately, very few authors also have investigated multiple output coupled complex networks (MOCCNs) [44]. In [44], two kinds of multiple output and output derivative coupled CNs were put forward, and output synchronization problems for these two networks were also dealt with based on the Lyapunov functional and adaptive control schemes.

Proportional-integral-derivative (PID) controller, as a useful and well-known instrument, has played an outstanding role in industry and academics since it is easy and simple to perform and does not rely on a precise model. In view of this, numerous advances works related to the PID controller and its variations have been extensively reported [45]–[49]. Daniel et al. [47] not only developed a distributed multiplex PI control strategy for ensuring the consensus for a heterogeneous network, but also further established several consensus criteria for the network by using the Lyapunov functional method and the multiplex PI controller. Moreover, PID-based synchronization for CNs also has been discussed [50], [51]. As stated in [50], the authors utilized PD controller to investigate synchronization for a class of directed CNs, and developed several synchronization criteria for the CNs with a directed topology and a directed spanning tree based on PI control methods. However, very few authors have considered the synchronization for MWCNs based on PID controller [52]. Motivated by the above discussions, this paper aims to address output and $\mathcal{H}_\infty$ output synchronization problem for MOCCNs based on PID controllers. The main contributions involve the following three aspects.

1) Compare with [34]–[37], we proposed two types of MOCCNs without and with the external disturbance, and further extend the study of [44] to the case of $\mathcal{H}_\infty$ output synchronization.
2) By exploiting devised PD and PI control strategies, several output synchronization criteria are derived for MOCCNs with different dimensions of system state and output. Compared with [50]–[52], we establish some sufficient conditions to guarantee the output synchronization for MOCCNs.
3) We not only address $\mathcal{H}_\infty$ output synchronization problems for MOCCNs based on PD and PI control schemes, but also develop several $\mathcal{H}_\infty$ output synchronization by using inequality techniques.

The rest of this paper is organized as follows: Section II introduces two types of MOCCNs; Section III discusses PID-based output synchronization for MOCCNs; Section IV considers $\mathcal{H}_\infty$ output synchronization for MOCCNs based on PID controllers; Two numerical examples are given in Section V; At last, conclusions are drawn in Section VI.

II. Preliminaries and Network Models

A. Notations

Let $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}^k$ be the space of k real vectors, and $\mathbb{R}^{s\times k}$ be the space of $s \times k$ real matrices. $\mathcal{D} = \{1, 2, \cdots, \mathcal{K}\}$ and $\mathcal{E} \subset \mathcal{D} \times \mathcal{D}$ respectively denote a node set and an undirected edge set in a network. $\varpi_1(\mathcal{X})$ and $\varpi_2(\mathcal{X})$ respectively stand for minimum and maximum eigenvalues of the real symmetric matrix $\mathcal{X}$. The same symbol in this paper indicates the same meaning unless otherwise mentioned.

B. Lemma

Lemma 2.1 (See [55]). The Kronecker product has the following properites:

$$\begin{array}{ll}(1) & (\Gamma\otimes\Psi)^T = \Gamma^T \otimes \Psi^T;\\ (2) & (\Gamma\otimes\Psi)^{-1} = \Gamma^{-1}\otimes\Psi^{-1};\\ (3) & (\beta\Gamma)\otimes\Psi = \Gamma\otimes(\beta\Psi);\\ (4) & (\Gamma+\Phi)\otimes\Psi = \Gamma\otimes\Psi+\Phi\otimes\Psi;\\ (5) & (\Gamma\otimes\Phi)(\Psi\otimes\Xi) = (\Gamma\Phi)\otimes(\Psi\Xi),\end{array}$$

where β is a constant, Γ, Ψ, Φ, Ξ are matrices with suitable dimension.

C. Networks

Considering two networks composing of $\mathcal{K}$ nodes with dimension k are modeled as

$$\begin{cases} \dot{p}_i(t) &= d(p_i(t)) + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} l_m \mathcal{S}_{ij}^m \mathcal{A}_m w_j(t) \\ & \quad + u_i(t), \\ w_i(t) &= \mathcal{Y} p_i(t), \end{cases} \tag{1}$$

$$\begin{cases} \dot{p}_i(t) &= d(p_i(t)) + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} l_m \mathcal{S}_{ij}^m \mathcal{A}_m w_j(t) \\ & \quad + \varepsilon_i(t) + u_i(t), \\ w_i(t) &= \mathcal{Y} p_i(t), \end{cases} \tag{2}$$

where $i = 1, 2, \cdots, \mathcal{K}; \mathbb{R}^k \ni p_i(t) = (p_{i1}(t), p_{i2}(t), \cdots, p_{ik}(t))^T$ refers to the state of ith node; The function $\mathbb{R}^k \ni d(p_i(t))$ is the continuously differentiable; $\mathbb{R} \ni l_m > 0$

indicates the coupling strength; The inner coupling matrix $\mathbb{R}^{k\times s} \ni \mathcal{A}_m (1 \leqslant s < k)$ is denoted as

$$\mathbb{R}^{k\times s} \ni \mathcal{A}_m = \begin{pmatrix} a_1^m & 0 & 0 & 0 & 0 \\ 0 & a_2^m & 0 & 0 & 0 \\ 0 & 0 & a_3^m & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & a_s^m \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix},$$

in which $\mathbb{R} \ni a_\nu^m > 0, \nu = 1,2,\cdots,s$; $\mathbb{R}^{\mathcal{K}\times\mathcal{K}} \ni \mathcal{S}^m = (\mathcal{S}_{ij}^m)_{\mathcal{K}\times\mathcal{K}}$ stands for the weighted matrix with the following definition:

$$\mathbb{R} \ni \mathcal{S}_{ij}^m = \begin{cases} \mathcal{S}_{ji}^m > 0, & \text{if } (i,j) \in \mathcal{E}, \\ -\sum\limits_{\substack{r=1 \\ r\neq i}}^{\mathcal{K}} \mathcal{S}_{ir}^m, & \text{if } i = j, \\ 0, & \text{otherwise;} \end{cases}$$

$\mathbb{R}^s \ni w_i(t) = (w_{i1}(t), w_{i2}(t), \cdots, w_{is}(t))^T$ refers to the output of ith node; $\mathbb{R}^k \ni u_i(t)$ is the control input;

$$\mathbb{R}^{s\times k} \ni \mathcal{Y} = \begin{pmatrix} y_1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & y_2 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & y_3 & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & y_s & \cdots & 0 \end{pmatrix},$$

where $\mathbb{R} \ni y_\nu, \nu = 1,2,\cdots,s$; $\mathbb{R}^k \ni \varepsilon_i(t) = (\varepsilon_{i1}(t), \varepsilon_{i2}(t), \cdots, \varepsilon_{ik}(t))^T$ stands for the external disturbance with square integrable, which is defined by

$$\int_0^{t_1} \varepsilon_i^T(\rho)\varepsilon_i(\rho)d\rho < +\infty$$

for any $\mathbb{R} \ni t_1 > 0$.

Remark 1. Under some circumstances, the changing of node state may be affected by the output of neighbor nodes in a network. Consequently, the study of CNs with output coupling was firstly proposed in [39]. Up till now, many authors have focused on the synchronization or output synchronization for CNs with output coupling [20], [39]–[43]. Furthermore, considering that node's state is also influenced by various distinct factors, few results have extended to the case of CNs with multiple output couplings [44]. Compared with [44], this paper further investigates $\mathcal{H}_\infty$ output synchronization for MOCCNs based on PID control schemes.

For these two networks (1) and (2), topologies are connected, and all their coupling forms are identical. In addition, $d(\cdot)$ fulfills [54]:

$$\|d(\sigma_1) - d(\sigma_2)\| \leqslant \delta\|\sigma_1 - \sigma_2\|, \tag{3}$$

$$\begin{aligned} & (\sigma_1 - \sigma_2)^T F[d(\sigma_1) - d(\sigma_2) - H(\sigma_1 - \sigma_2)] \\ \leqslant\ & -\alpha(\sigma_1 - \sigma_2)^T(\sigma_1 - \sigma_2) \end{aligned} \tag{4}$$

for any $\sigma_1, \sigma_2 \in \mathbb{R}^\varrho$, in which $\mathbb{R}^{\varrho\times\varrho} \ni F = \mathrm{diag}(f_1, f_2, \cdots, f_\varrho) > 0; \mathbb{R}^{\varrho\times\varrho} \ni H = \mathrm{diag}(h_1, h_2, \cdots, h_\varrho); \mathbb{R} \ni \alpha > 0; \mathbb{R} \ni \delta > 0$.

III. Output Synchronization for MOCCNs

This section respectively uses PD and PI control approaches to address output synchronization problems for MOCCNs, and formulates two criteria of output synchronization for this network based on the Lyapunov functional and inequality techniques.

From network (1), the following equations can be acquired:

$$\begin{cases} \dot{\tilde{p}}_i(t) = \tilde{d}(\tilde{p}_i(t)) + \sum\limits_{m=1}^{n}\sum\limits_{j=1}^{\mathcal{K}} l_m \mathcal{S}_{ij}^m \tilde{\mathcal{A}}_m w_j(t) + \tilde{u}_i(t), \\ w_i(t) = \tilde{\mathcal{Y}}\tilde{p}_i(t), \qquad i = 1,2,\cdots,\mathcal{K}, \end{cases} \tag{5}$$

in which $\mathbb{R}^s \ni \tilde{p}_i(t) = (p_{i1}(t), p_{i2}(t), \cdots, p_{is}(t))^T; \mathbb{R}^s \ni \tilde{d}(\tilde{p}_i(t))$; $\mathbb{R}^{s\times s} \ni \tilde{\mathcal{A}}_m = \mathrm{diag}(a_1, a_2, \cdots, a_s); \mathbb{R}^s \ni \tilde{u}_i(t) = (u_{i1}(t), u_{i2}(t), \cdots, u_{is}(t))^T; \mathbb{R}^{s\times s} \ni \tilde{\mathcal{Y}} = \mathrm{diag}(y_1, y_2, \cdots, y_s)$.

Define $\hat{d}(\eta(t)) = \tilde{d}(\tilde{\mathcal{Y}}^{-1}\eta(t))$ and $\mathbb{R}^s \ni \eta(t)$, it is deduced from (5) that

$$\dot{w}_i(t) = \tilde{\mathcal{Y}}\hat{d}(w_i(t)) + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} l_m \mathcal{S}_{ij}^m \tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m w_j(t) + \tilde{\mathcal{Y}}\tilde{u}_i(t). \tag{6}$$

Denoting $w^*(t) = (w_1^*(t), w_2^*(t), \cdots, w_s^*(t))^T = \frac{1}{\mathcal{K}}\sum\limits_{c=1}^{\mathcal{K}} w_c(t)$, (6) turns into

$$\begin{aligned} \dot{w}^*(t) &= \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}} \dot{w}_c(t) \\ &= \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}} \tilde{\mathcal{Y}}\hat{d}(w_c(t)) + \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}} \tilde{\mathcal{Y}}\tilde{u}_c(t) \\ &\quad + \frac{1}{\mathcal{K}}\sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} l_m \left(\sum_{c=1}^{\mathcal{K}} \mathcal{S}_{cj}^m\right) \tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m w_j(t) \\ &= \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}} \tilde{\mathcal{Y}}\hat{d}(w_c(t)) + \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}} \tilde{\mathcal{Y}}\tilde{u}_c(t). \end{aligned}$$

Letting $\upsilon_i(t) = w_i(t) - w^*(t)$, one acquires

$$\begin{aligned} \dot{\upsilon}_i(t) &= \dot{w}_i(t) - \dot{w}^*(t) \\ &= \tilde{\mathcal{Y}}\hat{d}(w_i(t)) - \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}} \tilde{\mathcal{Y}}\hat{d}(w_c(t)) \\ &\quad + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} l_m \mathcal{S}_{ij}^m \tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m w_j(t) \\ &\quad + \tilde{\mathcal{Y}}\tilde{u}_i(t) - \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}} \tilde{\mathcal{Y}}\tilde{u}_c(t),\ i = 1,2,\cdots,\mathcal{K}, \end{aligned} \tag{7}$$

where $\upsilon(t) = (\upsilon_1^T(t), \upsilon_2^T(t), \cdots, \upsilon_{\mathcal{K}}^T(t))^T$.

Definition 3.1. Network (1) is output synchronized if

$$\lim_{t\to+\infty} \left\| w_i(t) - \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}} w_c(t) \right\| = 0$$

for any $i = 1,2,\cdots,\mathcal{K}$.

For ensuring the output synchronization for network (1), we design a PID controller as follows:

$$\begin{aligned}\tilde{u}_i(t) &= \sum_{m=1}^{n} b_P^m \left(\sum_{j=1}^{\mathcal{K}} \mathcal{Z}_{ij}^m \tilde{\mathcal{A}}_m w_j(t) - w_i(t) + w^*(t)\right) \\ &\quad + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} b_D^m \mathcal{Z}_{ij}^m \tilde{\mathcal{A}}_m \dot{w}_j(t) \\ &\quad + \sum_{m=1}^{n} b_I^m \left(\sum_{j=1}^{\mathcal{K}} \mathcal{Z}_{ij}^m \tilde{\mathcal{A}}_m \int_0^t w_j(\rho)d\rho \right. \\ &\quad \left. - \int_0^t w_i(\rho)d\rho + \int_0^t w^*(\rho)d\rho\right), \end{aligned} \tag{8}$$

in which $i = 1, 2, \cdots, \mathcal{K}; \mathbb{R} \ni b_P^m > 0; \mathbb{R} \ni b_D^m \geqslant 0; \mathbb{R} \ni b_I^m \geqslant 0; \mathbb{R}^{\mathcal{K}\times\mathcal{K}} \ni \mathcal{Z}^m = (\mathcal{Z}_{ij}^m)_{\mathcal{K}\times\mathcal{K}}$;

$$\mathbb{R} \ni \mathcal{Z}_{ij}^m = \begin{cases} \mathcal{Z}_{ji}^m > 0, & \text{if } (i,j) \in \mathcal{E}, \\ -\sum\limits_{\substack{r=1 \\ r\neq i}}^{\mathcal{K}} \mathcal{Z}_{ir}^m, & \text{if } i = j, \\ 0, & \text{otherwise.} \end{cases}$$

Remark 2. To further reinforce the performance of network synchronization, the state of node relies on not only "present" feedback but also "past" information or "future" tendency. Therefore, PD, PI, PID control schemes and their variations have attracted considerable attention, and numerous results have also been published [45]–[52]. Moreover, compared with existing works related to output synchronization and $\mathcal{H}_\infty$ output synchronization [20]–[27], [36], [37], most of authors only considered "present" feedback. Consequently, it is meaningful to discuss PID-based output synchronization and $\mathcal{H}_\infty$ output synchronization for MOCCNs.

A. PD Control for Output Synchronization for MOCCNs

Taking $b_I^m = 0$, it is deduced from (8) that

$$\begin{aligned}\tilde{u}_i(t) &= \sum_{m=1}^{n} b_P^m \left(\sum_{j=1}^{\mathcal{K}} \mathcal{Z}_{ij}^m \tilde{\mathcal{A}}_m \upsilon_j(t) - \upsilon_i(t)\right) \\ &\quad + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} b_D^m \mathcal{Z}_{ij}^m \tilde{\mathcal{A}}_m \dot{\upsilon}_j(t), i = 1, 2, \cdots, \mathcal{K}, \end{aligned} \tag{9}$$

where $\mathbb{R} \ni b_P^m > 0; \mathbb{R} \ni b_D^m > 0$.

By (7) and (9), we can obtain

$$\begin{aligned}\dot{\upsilon}_i(t) &= \tilde{\mathcal{Y}}\hat{d}(w_i(t)) - \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}} \tilde{\mathcal{Y}}\hat{d}(w_c(t)) \\ &\quad + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} l_m \mathcal{S}_{ij}^m \tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m \upsilon_j(t) \\ &\quad + \sum_{m=1}^{n} b_P^m \left(\sum_{j=1}^{\mathcal{K}} \mathcal{Z}_{ij}^m \tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m \upsilon_j(t) - \tilde{\mathcal{Y}}\upsilon_i(t)\right) \\ &\quad + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} b_D^m \mathcal{Z}_{ij}^m \tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m \dot{\upsilon}_j(t) \\ &\quad - \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}} \tilde{\mathcal{Y}}\tilde{u}_c(t), \quad i = 1, 2, \cdots, \mathcal{K}. \end{aligned} \tag{10}$$

Theorem 3.1. If the following condition fulfills

$$\begin{aligned} &I_{\mathcal{K}} \otimes \left(\tilde{\mathcal{Y}}H - \alpha I_k - \sum_{m=1}^{n} b_P^m \tilde{\mathcal{Y}}\right) + \sum_{m=1}^{n} (b_P^m \mathcal{Z}^m \\ &+ l_m \mathcal{S}^m) \otimes (\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m) \leqslant 0, \end{aligned} \tag{11}$$

then network (1) is the output synchronized via PD controller (9).

Proof. For network (10), a Lyapunov functional is proposed as follows:

$$\begin{aligned} V_1(t) &= \sum_{i=1}^{\mathcal{K}} \upsilon_i^T(t)\upsilon_i(t) \\ &\quad - \sum_{m=1}^{n} b_D^m \upsilon^T(t)[\mathcal{Z}^m \otimes (\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m)]\upsilon(t). \end{aligned} \tag{12}$$

From (12), one gets

$$\begin{aligned} \dot{V}_1(t) &= 2\sum_{i=1}^{\mathcal{K}} \upsilon_i^T(t)\dot{\upsilon}_i(t) \\ &\quad -2\sum_{m=1}^{n} b_D^m \upsilon^T(t)[\mathcal{Z}^m \otimes (\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m)]\dot{\upsilon}(t) \\ &= 2\sum_{i=1}^{\mathcal{K}} \upsilon_i^T(t)\Bigg[\tilde{\mathcal{Y}}\hat{d}(w_i(t)) - \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}} \tilde{\mathcal{Y}}\hat{d}(w_c(t)) \\ &\quad - \tilde{\mathcal{Y}}\hat{d}(w^*(t)) + \tilde{\mathcal{Y}}\hat{d}(w^*(t)) - \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}} \tilde{\mathcal{Y}}\tilde{u}_c(t) \\ &\quad + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} (b_P^m \mathcal{Z}_{ij}^m + l_m \mathcal{S}_{ij}^m)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m \upsilon_j(t) \\ &\quad - \sum_{m=1}^{n} b_P^m \tilde{\mathcal{Y}}\upsilon_i(t) + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} b_D^m \mathcal{Z}_{ij}^m \tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m \dot{\upsilon}_j(t)\Bigg] \\ &\quad -2\sum_{m=1}^{n} b_D^m \upsilon^T(t)[\mathcal{Z}^m \otimes (\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m)]\dot{\upsilon}(t) \\ &= 2\sum_{i=1}^{\mathcal{K}} \upsilon_i^T(t)\tilde{\mathcal{Y}}\left(\hat{d}(w_i(t)) - \hat{d}(w^*(t))\right) \\ &\quad +2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}} (b_P^m \mathcal{Z}_{ij}^m + l_m \mathcal{S}_{ij}^m)\upsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m \upsilon_j(t) \\ &\quad -2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}} b_P^m \upsilon_i^T(t)\tilde{\mathcal{Y}}\upsilon_i(t) + 2\sum_{i=1}^{\mathcal{K}} \upsilon_i^T(t)\tilde{\mathcal{Y}} \times \\ &\quad \left(\hat{d}(w^*(t)) - \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}} \hat{d}(w_c(t)) - \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}} \tilde{u}_c(t)\right). \end{aligned} \tag{13}$$

Obviously, we have

$$\begin{aligned} &\sum_{i=1}^{\mathcal{K}} \upsilon_i^T(t)\tilde{\mathcal{Y}}\left(\hat{d}(w_i(t)) - \hat{d}(w^*(t))\right) \\ \leqslant &\sum_{i=1}^{\mathcal{K}} \upsilon_i^T(t)(\tilde{\mathcal{Y}}H - \alpha I_k)\upsilon_i(t). \end{aligned} \tag{14}$$

Moreover, since

$$\begin{aligned}\sum_{i=1}^{\mathcal{K}} \upsilon_i(t) &= \sum_{i=1}^{\mathcal{K}} (w_i(t) - w^*(t)) \\ &= \sum_{i=1}^{\mathcal{K}} \left(w_i(t) - \frac{1}{\mathcal{K}} \sum_{c=1}^{\mathcal{K}} w_c(t) \right) \\ &= \sum_{i=1}^{\mathcal{K}} w_i(t) - \sum_{c=1}^{\mathcal{K}} w_c(t) \\ &= 0,\end{aligned}$$

it can be proved that

$$\sum_{i=1}^{\mathcal{K}} \upsilon_i^T(t)\tilde{\mathcal{Y}} \left(\hat{d}(w^*(t)) - \frac{1}{\mathcal{K}} \sum_{c=1}^{\mathcal{K}} \hat{d}(w_c(t)) - \frac{1}{\mathcal{K}} \sum_{c=1}^{\mathcal{K}} \tilde{u}_c(t) \right) = 0. \quad (15)$$

Substituting (14) and (15) into (13), one acquires

$$\begin{aligned}\dot{V}_1(t) &\leqslant 2\sum_{i=1}^{\mathcal{K}} \upsilon_i^T(t) \left(\tilde{\mathcal{Y}}H - \alpha I_k - \sum_{m=1}^{n} b_P^m \tilde{\mathcal{Y}} \right) \upsilon_i(t) \\ &\quad +2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}} (b_P^m \mathcal{Z}_{ij}^m + l_m \mathcal{S}_{ij}^m) \upsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m \upsilon_j(t) \\ &= \upsilon^T(t) \left[2I_{\mathcal{K}} \otimes \left(\tilde{\mathcal{Y}}H - \alpha I_k - \sum_{m=1}^{n} b_P^m \tilde{\mathcal{Y}} \right) \right. \\ &\quad \left. +2\sum_{m=1}^{n} (b_P^m \mathcal{Z}^m + l_m \mathcal{S}^m) \otimes (\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m) \right] \upsilon(t) \\ &\leqslant \varpi_2(F_1)\upsilon^T(t)\upsilon(t), \quad (16)\end{aligned}$$

where $F_1 = 2I_{\mathcal{K}} \otimes \left(\tilde{\mathcal{Y}}H - \alpha I_k - \sum_{m=1}^{n} b_P^m \tilde{\mathcal{Y}} \right) + 2\sum_{m=1}^{n} (b_P^m \mathcal{Z}^m + l_m \mathcal{S}^m) \otimes (\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m)$.

Using (12) and (16), we can get

$$\begin{aligned}\dot{V}_1(t) &\leqslant \varpi_2(F_1)\|\upsilon(t)\|^2, \\ \varpi_1(F_2)\|\upsilon(t)\|^2 &\leqslant V_1(t) \leqslant \varpi_2(F_2)\|\upsilon(t)\|^2, \quad (17)\end{aligned}$$

in which $F_2 = I_{\mathcal{K}k} - \sum_{m=1}^{n} b_D^m [\mathcal{Z}^m \otimes (\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m)]$.

Then, one yields

$$\dot{V}_1(t) \leqslant \frac{\varpi_2(F_1)}{\varpi_2(F_2)} V_1(t).$$

Consequently, we can derive

$$V_1(t) \leqslant V_1(0)\exp^{\frac{\varpi_2(F_1)}{\varpi_2(F_2)}t}. \quad (18)$$

Combining (17) with (18), one obtains

$$\|\upsilon(t)\| \leqslant \sqrt{\frac{\varpi_2(F_2)}{\varpi_1(F_2)}} \|\upsilon(0)\|\exp^{\frac{\varpi_2(F_1)}{2\varpi_2(F_2)}t}.$$

Therefore, network (1) can achieve output synchronization via PD controller (9).

B. PI Control for Output Synchronization for MOCCNs

Letting $b_D^m = 0$, we can acquire from (8) that

$$\begin{aligned}\tilde{u}_i(t) &= \sum_{m=1}^{n} b_P^m \left(\sum_{j=1}^{\mathcal{K}} \mathcal{Z}_{ij}^m \tilde{\mathcal{A}}_m \upsilon_j(t) - \upsilon_i(t) \right) \\ &\quad + \sum_{m=1}^{n} b_I^m \left(\sum_{j=1}^{\mathcal{K}} \mathcal{Z}_{ij}^m \tilde{\mathcal{A}}_m \int_0^t \upsilon_j(\rho)d\rho - \int_0^t \upsilon_i(\rho)d\rho \right) \\ &= \sum_{m=1}^{n} b_P^m \left(\sum_{j=1}^{\mathcal{K}} \mathcal{Z}_{ij}^m \tilde{\mathcal{A}}_m \upsilon_j(t) - \upsilon_i(t) \right) \\ &\quad + \sum_{m=1}^{n} b_I^m \left(\sum_{j=1}^{\mathcal{K}} \mathcal{Z}_{ij}^m \tilde{\mathcal{A}}_m \epsilon_j(t) - \epsilon_i(t) \right), \quad (19)\end{aligned}$$

where $i = 1, 2, \cdots, \mathcal{K}; \mathbb{R} \ni b_P^m > 0; \mathbb{R} \ni b_I^m > 0; \mathbb{R}^s \ni \epsilon_i(t) = (\epsilon_{i1}(t), \epsilon_{i2}(t), \cdots, \epsilon_{is}(t)) = \int_0^t \upsilon_i(\rho)d\rho$.

From (7) and (19), one derives

$$\begin{aligned}\dot{\upsilon}_i(t) &= \tilde{\mathcal{Y}}\hat{d}(w_i(t)) - \frac{1}{\mathcal{K}} \sum_{c=1}^{\mathcal{K}} \tilde{\mathcal{Y}}\hat{d}(w_c(t)) \\ &\quad + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} l_m \mathcal{S}_{ij}^m \tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m \upsilon_j(t) - \frac{1}{\mathcal{K}} \sum_{c=1}^{\mathcal{K}} \tilde{\mathcal{Y}}\tilde{u}_c(t) \\ &\quad + \sum_{m=1}^{n} b_P^m \left(\sum_{j=1}^{\mathcal{K}} \mathcal{Z}_{ij}^m \tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m \upsilon_j(t) - \tilde{\mathcal{Y}}\upsilon_i(t) \right) \\ &\quad + \sum_{m=1}^{n} b_I^m \left(\sum_{j=1}^{\mathcal{K}} \mathcal{Z}_{ij}^m \tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m \epsilon_j(t) - \tilde{\mathcal{Y}}\epsilon_i(t) \right), \\ \dot{\epsilon}_i(t) &= \upsilon_i(t), \quad i = 1, 2, \cdots, \mathcal{K}, \quad (20)\end{aligned}$$

in which $\epsilon(t) = (\epsilon_1^T(t), \epsilon_2^T(t), \cdots, \epsilon_{\mathcal{K}}^T(t))^T$.

Theorem 3.2. If the following conditions fulfill

$$\begin{pmatrix} I_s & I_s \\ I_s & \hat{\mathcal{Z}} \end{pmatrix} > 0 \quad (21)$$

$$\begin{pmatrix} F_1 + (2+\delta^2)I_{\mathcal{K}k} & F_3 \\ F_3^T & F_4 \end{pmatrix} \leqslant 0 \quad (22)$$

where $\hat{\mathcal{Z}} = \sum_{m=1}^{n} b_P^m \tilde{\mathcal{Y}} - \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} b_I^m \mathcal{Z}_{ij}^m \tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m, F_3 = \sum_{m=1}^{n} (b_P^m \mathcal{Z}^m + l_m \mathcal{S}^m) \otimes (\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m), F_4 = I_{\mathcal{K}} \otimes (\tilde{\mathcal{Y}}^2 - 2\sum_{m=1}^{n} b_I^m \tilde{\mathcal{Y}}) + 2\sum_{m=1}^{n} b_I^m \mathcal{Z}^m \otimes (\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m)$, then network (1) can achieve the output synchronization under PI controller (19).

Proof. For network (20), a Lyapunov functional is formulated as follows:

$$V_2(t) = \sum_{i=1}^{\mathcal{K}} \tilde{\upsilon}_i^T(t) \begin{pmatrix} I_s & I_s \\ I_s & \hat{\mathcal{Z}} \end{pmatrix} \tilde{\upsilon}_i(t), \quad (23)$$

in which $\tilde{\upsilon}_i(t) = (\upsilon_i^T(t), \epsilon_i^T(t))^T$.

According to (23), we can obtain

$$\begin{aligned}
\dot{V}_2(t) &= 2\sum_{i=1}^{\mathcal{K}} \tilde{\upsilon}_i^T(t) \begin{pmatrix} I_s & I_s \\ I_s & \hat{\mathcal{Z}} \end{pmatrix} \dot{\tilde{\upsilon}}_i(t) \\
&= 2\sum_{i=1}^{\mathcal{K}} \Big(\upsilon_i^T(t)\dot{\upsilon}_i(t) + \upsilon_i^T(t)\dot{\epsilon}_i(t) + \epsilon_i^T(t)\dot{\upsilon}_i(t) \\
&\quad +\epsilon_i^T(t)\hat{\mathcal{Z}}\dot{\epsilon}_i(t)\Big) \\
&= 2\sum_{i=1}^{\mathcal{K}}(\upsilon_i^T(t) + \epsilon_i^T(t))\Big[\tilde{\mathcal{Y}}\hat{d}(w_i(t)) - \tilde{\mathcal{Y}}\hat{d}(w^*(t)) \\
&\quad -\frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\hat{d}(w_c(t))+\tilde{\mathcal{Y}}\hat{d}(w^*(t))-\frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\tilde{u}_c(t) \\
&\quad +\sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}}(b_P^m\mathcal{Z}_{ij}^m + l_m\mathcal{S}_{ij}^m)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t) \\
&\quad -\sum_{m=1}^{n} b_P^m\tilde{\mathcal{Y}}\upsilon_i(t) + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} b_I^m\mathcal{Z}_{ij}^m\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\epsilon_j(t) \\
&\quad -\sum_{m=1}^{n} b_I^m\tilde{\mathcal{Y}}\epsilon_i(t)\Big] + 2\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\upsilon_i(t) \\
&\quad +2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}} b_P^m\epsilon_i^T(t)\tilde{\mathcal{Y}}\upsilon_i(t) \\
&\quad -2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}} b_I^m\mathcal{Z}_{ij}^m\epsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t) \\
&= 2\sum_{i=1}^{\mathcal{K}}(\upsilon_i^T(t) + \epsilon_i^T(t))\Big(\tilde{\mathcal{Y}}\hat{d}(w_i(t)) - \tilde{\mathcal{Y}}\hat{d}(w^*(t))\Big) \\
&\quad +2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}}(b_P^m\mathcal{Z}_{ij}^m+l_m\mathcal{S}_{ij}^m)\upsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t) \\
&\quad +2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}}(b_P^m\mathcal{Z}_{ij}^m+l_m\mathcal{S}_{ij}^m)\epsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t) \\
&\quad +2\sum_{i=1}^{\mathcal{K}}\left(\upsilon_i^T(t)\upsilon_i(t) - \sum_{m=1}^{n} b_P^m\upsilon_i^T(t)\tilde{\mathcal{Y}}\upsilon_i(t)\right) \\
&\quad +2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}} b_I^m\mathcal{Z}_{ij}^m\epsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\epsilon_j(t) \\
&\quad -2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}} b_I^m\epsilon_i^T(t)\tilde{\mathcal{Y}}\epsilon_i(t) \\
&\quad +2\sum_{i=1}^{\mathcal{K}}(\upsilon_i^T(t) + \epsilon_i^T(t))\tilde{\mathcal{Y}}\Bigg(\hat{d}(w^*(t)) \\
&\quad -\frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\hat{d}(w_c(t)) - \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{u}_c(t)\Bigg) \\
&\leqslant 2\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\left[\tilde{\mathcal{Y}}H - (\alpha-1)I_k - \sum_{m=1}^{n} b_P^m\tilde{\mathcal{Y}}\right]\upsilon_i(t) \\
&\quad +2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}}(b_P^m\mathcal{Z}_{ij}^m+l_m\mathcal{S}_{ij}^m)\upsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t) \\
&\quad +2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}}(b_P^m\mathcal{Z}_{ij}^m+l_m\mathcal{S}_{ij}^m)\epsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t) \\
&\quad +\sum_{i=1}^{\mathcal{K}}\epsilon_i^T(t)\tilde{\mathcal{Y}}^2\epsilon_i(t) + \delta^2\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\upsilon_i(t) \\
&\quad +2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}} b_I^m\mathcal{Z}_{ij}^m\epsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\epsilon_j(t) \\
&\quad -2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}} b_I^m\epsilon_i^T(t)\tilde{\mathcal{Y}}\epsilon_i(t) \\
&= \upsilon^T(t)\left[F_1 + (2+\delta^2)I_{\mathcal{K}k}\right]\upsilon(t) + 2\epsilon^T(t)F_3\upsilon(t) \\
&\quad +\epsilon^T(t)F_4\epsilon(t) \\
&= \tilde{\upsilon}^T(t)\begin{pmatrix} F_1 + (2+\delta^2)I_{\mathcal{K}k} & F_3 \\ F_3^T & F_4 \end{pmatrix}\tilde{\upsilon}(t) \\
&\leqslant \varrho_1\tilde{\upsilon}^T(t)\tilde{\upsilon}(t), \qquad (24)
\end{aligned}$$

where $\varrho_1 = \varpi_2\left(\begin{pmatrix} F_1 + (2+\delta^2)I_{\mathcal{K}k} & F_3 \\ F_3^T & F_4 \end{pmatrix}\right), \tilde{\upsilon}(t) = (\upsilon^T(t), \epsilon^T(t))^T$.

Similar to the proof of Theorem 3.1, it is easy to obtain that

$$\begin{aligned}
\varpi_1(F_5)\|\tilde{\upsilon}(t)\|^2 &\leqslant V_2(t) \leqslant \varpi_2(F_5)\|\tilde{\upsilon}(t)\|^2, \\
V_2(t) &\leqslant V_2(0)\exp^{\frac{\varrho_1}{\varpi_2(F_5)}t},
\end{aligned}$$

in which $F_5 = \begin{pmatrix} I_s & I_s \\ I_s & \hat{\mathcal{Z}} \end{pmatrix}$.

Accordingly, one yields

$$\|\upsilon(t)\| \leqslant \|\tilde{\upsilon}(t)\| \leqslant \sqrt{\frac{\varpi_2(F_5)}{\varpi_1(F_5)}}\|\tilde{\upsilon}(0)\|\exp^{\frac{\varrho_1}{2\varpi_2(F_5)}t}. \qquad (25)$$

Therefore, network (1) under PI controller (9) realizes output synchronization. □

IV. $\mathcal{H}_\infty$ Output Synchronization for MOCCNs

In this section, we not only investigate PD-based $\mathcal{H}_\infty$ output synchronization for MOCCNs, but also establish several sufficient conditions for guaranteeing the $\mathcal{H}_\infty$ output synchronization for MOCCNs by exploiting a PI control strategy.

From network (2), the following equations can be derived:

$$\begin{cases} \dot{\tilde{p}}_i(t) = \tilde{d}(\tilde{p}_i(t)) + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} l_m\mathcal{S}_{ij}^m\tilde{\mathcal{A}}_m w_j(t) \\ \qquad\quad +\tilde{\varepsilon}_i(t) + \tilde{u}_i(t), \\ w_i(t) = \tilde{\mathcal{Y}}\tilde{p}_i(t), \qquad i = 1, 2, \cdots, \mathcal{K}, \end{cases} \qquad (26)$$

where some parameters $\tilde{p}_i(t), \tilde{d}(\tilde{p}_i(t)), \tilde{\mathcal{A}}_m, \tilde{u}_i(t), \tilde{\mathcal{Y}}$ are identical as those parameters of the Section III and $\mathbb{R}^s \ni \tilde{\varepsilon}_i(t) = (\varepsilon_{i1}(t), \varepsilon_{i2}(t), \cdots, \varepsilon_{is}(t))^T$.

According to (6) and (26), we can get

$$\begin{aligned}
\dot{w}_i(t) &= \tilde{\mathcal{Y}}\hat{d}(w_i(t)) + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} l_m\mathcal{S}_{ij}^m\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m w_j(t) \\
&\quad +\tilde{\mathcal{Y}}\tilde{\varepsilon}_i(t) + \tilde{\mathcal{Y}}\tilde{u}_i(t), \qquad (27)
\end{aligned}$$

Defining $w^*(t) = (w_1^*(t), w_2^*(t), \cdots, w_s^*(t))^T = \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}} w_c(t)$, (27) turns into

$$\begin{aligned}\dot{w}^*(t) &= \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\hat{d}(w_c(t)) + \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\tilde{u}_c(t) + \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\tilde{\varepsilon}_c(t) \\ &\quad + \frac{1}{\mathcal{K}}\sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} l_m\left(\sum_{c=1}^{\mathcal{K}}\mathcal{S}_{cj}^m\right)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m w_j(t) \\ &= \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\hat{d}(w_c(t)) + \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\tilde{u}_c(t) + \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\tilde{\varepsilon}_c(t).\end{aligned}$$

Denoting $\upsilon_i(t) = w_i(t) - w^*(t)$, one yields

$$\begin{aligned}\dot{\upsilon}_i(t) &= \tilde{\mathcal{Y}}\hat{d}(w_i(t)) - \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\hat{d}(w_c(t)) \\ &\quad + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} l_m\mathcal{S}_{ij}^m\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m w_j(t) + \tilde{\mathcal{Y}}\tilde{u}_i(t) \\ &\quad - \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\tilde{u}_c(t) + \tilde{\mathcal{Y}}\tilde{\varepsilon}_i(t) - \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\tilde{\varepsilon}_c(t), \quad (28)\end{aligned}$$

in which $i = 1, 2, \cdots \mathcal{K}; \upsilon(t) = (\upsilon_1^T(t), \upsilon_2^T(t), \cdots, \upsilon_{\mathcal{K}}^T(t))^T$.

Definition 4.1. Network (2) realizes $\mathcal{H}_\infty$ output synchronization if

$$\sum_{i=1}^{\mathcal{K}}\int_0^{t_\rho}\upsilon_i^T(\rho)\upsilon_i(\rho)d\rho \leqslant \mathcal{W}(0) + \phi^2\sum_{i=1}^{\mathcal{K}}\int_0^{t_\rho}\tilde{\varepsilon}_i^T(\rho)\tilde{\varepsilon}_i(\rho)d\rho$$

for any $\mathbb{R}^n \ni t_\rho > 0$; $\mathbb{R} \ni \phi > 0$, and $\mathcal{W}(\cdot)$ refers to the non-negative function.

For guaranteeing the $\mathcal{H}_\infty$ output synchronization for network (2), a PID controller can be developed as follows:

$$\begin{aligned}\tilde{u}_i(t) &= \sum_{m=1}^{n}\tilde{b}_P^m\left(\sum_{j=1}^{\mathcal{K}}\tilde{\mathcal{Z}}_{ij}^m\tilde{\mathcal{A}}_m w_j(t) - w_i(t) + w^*(t)\right) \\ &\quad + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}}\tilde{b}_D^m\tilde{\mathcal{Z}}_{ij}^m\tilde{\mathcal{A}}_m\dot{w}_j(t) \\ &\quad + \sum_{m=1}^{n}\tilde{b}_I^m\left(\sum_{j=1}^{\mathcal{K}}\tilde{\mathcal{Z}}_{ij}^m\tilde{\mathcal{A}}_m\int_0^t w_j(\rho)d\rho \right. \\ &\quad \left. - \int_0^t w_i(\rho)d\rho + \int_0^t w^*(\rho)d\rho\right), \quad (29)\end{aligned}$$

where $i = 1, 2, \cdots, \mathcal{K}; \mathbb{R} \ni \tilde{b}_P^m > 0; \mathbb{R} \ni \tilde{b}_D^m \geqslant 0; \mathbb{R} \ni \tilde{b}_I^m \geqslant 0; \mathbb{R}^{\mathcal{K}\times\mathcal{K}} \ni \tilde{\mathcal{Z}}^m = (\tilde{\mathcal{Z}}_{ij}^m)_{\mathcal{K}\times\mathcal{K}}$;

$$\mathbb{R} \ni \tilde{\mathcal{Z}}_{ij}^m = \begin{cases}\tilde{\mathcal{Z}}_{ji}^m > 0, & \text{if } (i,j) \in \mathcal{E}, \\ -\sum\limits_{\substack{r=1 \\ r\neq i}}^{\mathcal{K}}\tilde{\mathcal{Z}}_{ir}^m, & \text{if } i = j, \\ 0, & \text{otherwise.}\end{cases}$$

A. PD Control for $\mathcal{H}_\infty$ Output Synchronization for MOCCNs

Taking $\tilde{b}_I^m = 0$, it is deduced from (29) that

$$\begin{aligned}\tilde{u}_i(t) &= \sum_{m=1}^{n}\tilde{b}_P^m\left(\sum_{j=1}^{\mathcal{K}}\tilde{\mathcal{Z}}_{ij}^m\tilde{\mathcal{A}}_m\upsilon_j(t) - \upsilon_i(t)\right) \\ &\quad + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}}\tilde{b}_D^m\tilde{\mathcal{Z}}_{ij}^m\tilde{\mathcal{A}}_m\dot{\upsilon}_j(t), \quad (30)\end{aligned}$$

in which $i = 1, 2, \cdots, \mathcal{K}; \mathbb{R} \ni \tilde{b}_P^m > 0; \mathbb{R} \ni \tilde{b}_D^m > 0$.

From (28) and (30), we can get

$$\begin{aligned}\dot{\upsilon}_i(t) &= \tilde{\mathcal{Y}}\hat{d}(w_i(t)) - \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\hat{d}(w_c(t)) \\ &\quad + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} l_m\mathcal{S}_{ij}^m\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t) + \tilde{\mathcal{Y}}\tilde{\varepsilon}_i(t) \\ &\quad + \sum_{m=1}^{n}\tilde{b}_P^m\left(\sum_{j=1}^{\mathcal{K}}\tilde{\mathcal{Z}}_{ij}^m\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t) - \tilde{\mathcal{Y}}\upsilon_i(t)\right) \\ &\quad + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}}\tilde{b}_D^m\tilde{\mathcal{Z}}_{ij}^m\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\dot{\upsilon}_j(t) \\ &\quad - \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\tilde{u}_c(t) - \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\tilde{\varepsilon}_c(t), \quad (31)\end{aligned}$$

where $i = 1, 2, \cdots, \mathcal{K}$.

Theorem 4.1. If the following condition fulfills

$$\begin{aligned}& I_{\mathcal{K}} \otimes \left[\tilde{\mathcal{Y}}H - \left(\alpha - \frac{1}{2}\right)I_k - \sum_{m=1}^{n}\tilde{b}_P^m\tilde{\mathcal{Y}} + \frac{\tilde{\mathcal{Y}}^2}{2\phi^2}\right] \\ & + \sum_{m=1}^{n}(\tilde{b}_P^m\tilde{\mathcal{Z}}^m + l_m\mathcal{S}^m) \otimes (\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m) \leqslant 0, \quad (32)\end{aligned}$$

then network (2) can realize the $\mathcal{H}_\infty$ output synchronization under PD controller (30).

Proof. For network (31), a Lyapunov functional is presented as follows:

$$V_3(t) = \sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\upsilon_i(t) - \sum_{m=1}^{n}\tilde{b}_D^m\upsilon^T(t)\left[\tilde{\mathcal{Z}}^m \otimes (\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m)\right]\upsilon(t).$$

Accordingly, one obtains

$$\begin{aligned}\dot{V}_3(t) &= 2\sum_{i=1}^{\mathcal{S}}\upsilon_i^T(t)\left[\tilde{\mathcal{Y}}\hat{d}(w_i(t)) - \tilde{\mathcal{Y}}\hat{d}(w^*(t))\right. \\ &\quad - \frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\hat{d}(w_c(t)) + \tilde{\mathcal{Y}}\hat{d}(w^*(t)) \\ &\quad + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}} l_m\mathcal{S}_{ij}^m\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t) + \tilde{\mathcal{Y}}\tilde{\varepsilon}_i(t) \\ &\quad + \sum_{m=1}^{n}\tilde{b}_P^m\left(\sum_{j=1}^{\mathcal{K}}\tilde{\mathcal{Z}}_{ij}^m\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t) - \tilde{\mathcal{Y}}\upsilon_i(t)\right) \\ &\quad + \sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}}\tilde{b}_D^m\tilde{\mathcal{Z}}_{ij}^m\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\dot{\upsilon}_j(t)\end{aligned}$$

$$-\frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\tilde{u}_c(t)-\frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\tilde{\varepsilon}_c(t)\Bigg]$$
$$-2\sum_{m=1}^{n}\tilde{b}_D^m\upsilon^T(t)[\tilde{\mathcal{Z}}^m\otimes(\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m)]\dot{\upsilon}(t)$$
$$\begin{aligned}\leqslant\ & 2\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\left(\tilde{\mathcal{Y}}H-\alpha I_k-\sum_{m=1}^{n}\tilde{b}_P^m\tilde{\mathcal{Y}}\right)\upsilon_i(t)\\ &+2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}}(\tilde{b}_P^m\tilde{\mathcal{Z}}_{ij}^m+l_m\mathcal{S}_{ij}^m)\upsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t)\\ &+2\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\varepsilon}_i(t). \end{aligned}\tag{33}$$

From (33), we can derive

$$\begin{aligned}&\dot{V}_3(t)+\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\upsilon_i(t)-\phi^2\sum_{i=1}^{\mathcal{K}}\tilde{\varepsilon}_i^T(t)\tilde{\varepsilon}_i(t)\\ \leqslant\ & 2\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\left(\tilde{\mathcal{Y}}H-\alpha I_k-\sum_{m=1}^{n}\tilde{b}_P^m\tilde{\mathcal{Y}}\right)\upsilon_i(t)\\ &+2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}}(\tilde{b}_P^m\tilde{\mathcal{Z}}_{ij}^m+l_m\mathcal{S}_{ij}^m)\upsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t)\\ &+2\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\varepsilon}_i(t)+\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\upsilon_i(t)\\ &-\phi^2\sum_{i=1}^{\mathcal{K}}\tilde{\varepsilon}_i^T(t)\tilde{\varepsilon}_i(t)\\ \leqslant\ & 2\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\left[\tilde{\mathcal{Y}}H-\left(\alpha-\frac{1}{2}\right)I_k-\sum_{m=1}^{n}\tilde{b}_P^m\tilde{\mathcal{Y}}\right]\upsilon_i(t)\\ &+2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}}(\tilde{b}_P^m\tilde{\mathcal{Z}}_{ij}^m+l_m\mathcal{S}_{ij}^m)\upsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t)\\ &+\frac{1}{\phi^2}\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\tilde{\mathcal{Y}}^2\upsilon_i(t)\\ =\ & \upsilon^T(t)\Bigg\{2I_{\mathcal{K}}\otimes\left[\tilde{\mathcal{Y}}H-\left(\alpha-\frac{1}{2}\right)I_k-\sum_{m=1}^{n}\tilde{b}_P^m\tilde{\mathcal{Y}}+\frac{\tilde{\mathcal{Y}}^2}{2\phi^2}\right]\\ &+2\sum_{m=1}^{n}(\tilde{b}_P^m\tilde{\mathcal{Z}}^m+l_m\mathcal{S}^m)\otimes(\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m)\Bigg\}\upsilon(t).\end{aligned}\tag{34}$$

In view of (32) and (34), one gets

$$\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\upsilon_i(t)\leqslant-\dot{V}_3(t)+\phi^2\sum_{i=1}^{\mathcal{K}}\tilde{\varepsilon}_i^T(t)\tilde{\varepsilon}_i(t).$$

Accordingly, we can acquire

$$\begin{aligned}\sum_{i=1}^{\mathcal{K}}\int_0^{t_\rho}\upsilon_i^T(\rho)\upsilon_i(\rho)d\rho\ &\leqslant\ V_3(0)-V_3(t_\rho)\\ &\quad+\phi^2\sum_{i=1}^{\mathcal{K}}\int_0^{t_\rho}\tilde{\varepsilon}_i^T(\rho)\tilde{\varepsilon}_i(\rho)d\rho\\ &\leqslant\ V_3(0)+\phi^2\sum_{i=1}^{\mathcal{K}}\int_0^{t_\rho}\tilde{\varepsilon}_i^T(\rho)\tilde{\varepsilon}_i(\rho)d\rho\end{aligned}$$

for any $\mathbb{R}\ni t_\rho>0$.

Consequently, network (2) via PD controller (30) can realize the $\mathcal{H}_\infty$ output synchronization.

B. PI Control for $\mathcal{H}_\infty$ Output Synchronization for MOCCNs

Letting $\tilde{b}_D^m=0$, we can derive from (29) that

$$\begin{aligned}\tilde{u}_i(t)\ =\ &\sum_{m=1}^{n}\tilde{b}_P^m\left(\sum_{j=1}^{\mathcal{K}}\tilde{\mathcal{Z}}_{ij}^m\tilde{\mathcal{A}}_m\upsilon_j(t)-\upsilon_i(t)\right)\\ &+\sum_{m=1}^{n}\tilde{b}_I^m\left(\sum_{j=1}^{\mathcal{K}}\tilde{\mathcal{Z}}_{ij}^m\tilde{\mathcal{A}}_m\epsilon_j(t)-\epsilon_i(t)\right),\end{aligned}\tag{35}$$

where $i=1,2,\cdots,\mathcal{K};\mathbb{R}\ni\tilde{b}_P^m>0;\mathbb{R}\ni\tilde{b}_I^m>0;\mathbb{R}^n\ni\epsilon_i(t)=(\epsilon_{i1}(t),\epsilon_{i2}(t),\cdots,\epsilon_{is}(t))=\int_0^t\upsilon_i(\rho)d\rho$.

By (28) and (35), one obtaines

$$\begin{aligned}\dot{\upsilon}_i(t)\ =\ &\tilde{\mathcal{Y}}\hat{d}(w_i(t))-\frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\hat{d}(w_c(t))\\ &+\sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}}l_m\mathcal{S}_{ij}^m\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t)+\tilde{\mathcal{Y}}\tilde{\varepsilon}_i(t)\\ &+\sum_{m=1}^{n}\tilde{b}_P^m\left(\sum_{j=1}^{\mathcal{K}}\tilde{\mathcal{Z}}_{ij}^m\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t)-\tilde{\mathcal{Y}}\upsilon_i(t)\right)\\ &+\sum_{m=1}^{n}\tilde{b}_I^m\left(\sum_{j=1}^{\mathcal{K}}\tilde{\mathcal{Z}}_{ij}^m\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\epsilon_j(t)-\tilde{\mathcal{Y}}\epsilon_i(t)\right)\\ &-\frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\tilde{u}_c(t)-\frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\tilde{\varepsilon}_c(t),\\ \dot{\epsilon}_i(t)\ =\ &\upsilon_i(t),\ i=1,2,\cdots,\mathcal{K},\end{aligned}\tag{36}$$

in which $\epsilon(t)=(\epsilon_1^T(t),\epsilon_2^T(t),\cdots,\epsilon_{\mathcal{K}}^T(t))^T$.

Theorem 4.2. If the following conditions fulfill

$$\begin{pmatrix} I_s & I_s\\ I_s & \tilde{\hat{\mathcal{Z}}}\end{pmatrix}>0,\tag{37}$$

$$\begin{pmatrix} F_5 & F_3\\ F_3^T & F_6\end{pmatrix}\leqslant 0,\tag{38}$$

where $\tilde{\hat{\mathcal{Z}}}=\sum_{m=1}^{n}\tilde{b}_P^m\tilde{\mathcal{Y}}-\sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}}\tilde{b}_I^m\tilde{\mathcal{Z}}_{ij}^m\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m, F_5=2I_{\mathcal{K}}\otimes\left[\tilde{\mathcal{Y}}H-\left(\alpha-\frac{3}{2}-\delta^2\right)I_k-\sum_{m=1}^{n}\tilde{b}_P^m\tilde{\mathcal{Y}}\ +\frac{\tilde{\mathcal{Y}}^2}{\phi^2}\right], F_6=I_{\mathcal{K}}\otimes\left(\tilde{\mathcal{Y}}^2-2\sum_{m=1}^{n}\tilde{b}_I^m\tilde{\mathcal{Y}}+\frac{2\tilde{\mathcal{Y}}^2}{\phi^2}\right)+2\sum_{m=1}^{n}\tilde{b}_I^m\tilde{\mathcal{Z}}\otimes(\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m)$, then network (2) can achieve the $\mathcal{H}_\infty$ output synchronization under PI controller (35).

Proof. For network (36), a Lyapunov functional is selected as follows:

$$V_4(t)=\sum_{i=1}^{\mathcal{K}}\tilde{\upsilon}_i^T(t)\begin{pmatrix} I_s & I_s\\ I_s & \tilde{\hat{\mathcal{Z}}}\end{pmatrix}\tilde{\upsilon}_i(t).$$

Then, we have

$$\begin{aligned}
\dot{V}_4(t) &= 2\sum_{i=1}^{\mathcal{K}}(\upsilon_i^T(t)+\epsilon_i^T(t))\Bigg[\tilde{\mathcal{Y}}\hat{d}(w_i(t))-\tilde{\mathcal{Y}}\hat{d}(w^*(t)) \\
&\quad -\frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\hat{d}(w_c(t))+\tilde{\mathcal{Y}}\hat{d}(w^*(t))+\tilde{\mathcal{Y}}\tilde{\varepsilon}_i(t) \\
&\quad +\sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}}(\tilde{b}_P^m\tilde{\mathcal{Z}}_{ij}^m+l_m\mathcal{S}_{ij}^m)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t) \\
&\quad -\sum_{m=1}^{n}\tilde{b}_P^m\tilde{\mathcal{Y}}\upsilon_i(t)+\sum_{m=1}^{n}\sum_{j=1}^{\mathcal{K}}\tilde{b}_I^m\tilde{\mathcal{Z}}_{ij}^m\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\epsilon_j(t) \\
&\quad -\sum_{m=1}^{n}\tilde{b}_I^m\tilde{\mathcal{Y}}\epsilon_i(t)-\frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\tilde{u}_c(t)-\frac{1}{\mathcal{K}}\sum_{c=1}^{\mathcal{K}}\tilde{\mathcal{Y}}\tilde{\varepsilon}_c(t)\Bigg] \\
&\quad +2\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\upsilon_i(t)+2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\tilde{b}_P^m\epsilon_i^T(t)\tilde{\mathcal{Y}}\upsilon_i(t) \\
&\quad -2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}}\tilde{b}_I^m\tilde{\mathcal{Z}}_{ij}^m\epsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t) \\
&\leqslant 2\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\left[\tilde{\mathcal{Y}}H-(\alpha-1)I_k-\sum_{m=1}^{n}\tilde{b}_P^m\tilde{\mathcal{Y}}\right]\upsilon_i(t) \\
&\quad +2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}}(\tilde{b}_P^m\tilde{\mathcal{Z}}_{ij}^m+l_m\mathcal{S}_{ij}^m)\upsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t) \\
&\quad +2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}}(\tilde{b}_P^m\tilde{\mathcal{Z}}_{ij}^m+l_m\mathcal{S}_{ij}^m)\epsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t) \\
&\quad +2\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\varepsilon}_i(t)+2\sum_{i=1}^{\mathcal{K}}\epsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\varepsilon}_i(t) \\
&\quad +\sum_{i=1}^{\mathcal{K}}\epsilon_i^T(t)\tilde{\mathcal{Y}}^2\epsilon_i(t)+\delta^2\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\upsilon_i(t) \\
&\quad +2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}}\tilde{b}_I^m\tilde{\mathcal{Z}}_{ij}^m\epsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\epsilon_j(t) \\
&\quad -2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\tilde{b}_I^m\epsilon_i^T(t)\tilde{\mathcal{Y}}\epsilon_i(t). \qquad (39)
\end{aligned}$$

According to (39), one derives

$$\begin{aligned}
&\dot{V}_4(t)+\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\upsilon_i(t)-\phi^2\sum_{i=1}^{\mathcal{K}}\tilde{\varepsilon}_i^T(t)\tilde{\varepsilon}_i(t) \\
\leqslant\ & 2\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\left[\tilde{\mathcal{Y}}H-\left(\alpha-\frac{3}{2}\right)I_k-\sum_{m=1}^{n}\tilde{b}_P^m\tilde{\mathcal{Y}}\right]\upsilon_i(t) \\
&+2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}}(\tilde{b}_P^m\tilde{\mathcal{Z}}_{ij}^m+l_m\mathcal{S}_{ij}^m)\upsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t) \\
&+2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}}(\tilde{b}_P^m\tilde{\mathcal{Z}}_{ij}^m+l_m\mathcal{S}_{ij}^m)\epsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t) \\
&+2\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\varepsilon}_i(t)+2\sum_{i=1}^{\mathcal{K}}\epsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\varepsilon}_i(t) \\
&+\sum_{i=1}^{\mathcal{K}}\epsilon_i^T(t)\tilde{\mathcal{Y}}^2\epsilon_i(t)+\delta^2\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\upsilon_i(t) \\
&+2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}}\tilde{b}_I^m\mathcal{Z}_{ij}^m\epsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\epsilon_j(t) \\
&-2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\tilde{b}_I^m\epsilon_i^T(t)\tilde{\mathcal{Y}}\epsilon_i(t)-\phi^2\sum_{i=1}^{\mathcal{K}}\tilde{\varepsilon}_i^T(t)\tilde{\varepsilon}_i(t) \\
\leqslant\ & 2\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\left[\tilde{\mathcal{Y}}H-\left(\alpha-\frac{3}{2}-\delta^2\right)I_k-\sum_{m=1}^{n}\tilde{b}_P^m\tilde{\mathcal{Y}}\right]\upsilon_i(t) \\
&+2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}}(\tilde{b}_P^m\tilde{\mathcal{Z}}_{ij}^m+l_m\mathcal{S}_{ij}^m)\upsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t) \\
&+2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}}(\tilde{b}_P^m\tilde{\mathcal{Z}}_{ij}^m+l_m\mathcal{S}_{ij}^m)\epsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\upsilon_j(t) \\
&+\sum_{i=1}^{\mathcal{K}}\epsilon_i^T(t)\tilde{\mathcal{Y}}^2\epsilon_i(t)-2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\tilde{b}_I^m\epsilon_i^T(t)\tilde{\mathcal{Y}}\epsilon_i(t) \\
&+2\sum_{m=1}^{n}\sum_{i=1}^{\mathcal{K}}\sum_{j=1}^{\mathcal{K}}\tilde{b}_I^m\tilde{\mathcal{Z}}_{ij}^m\epsilon_i^T(t)\tilde{\mathcal{Y}}\tilde{\mathcal{A}}_m\epsilon_j(t) \\
&+\frac{2}{\phi^2}\sum_{i=1}^{\mathcal{K}}\upsilon_i^T(t)\tilde{\mathcal{Y}}^2\upsilon_i(t)+\frac{2}{\phi^2}\sum_{i=1}^{\mathcal{K}}\epsilon_i^T(t)\tilde{\mathcal{Y}}^2\epsilon_i(t) \\
=\ & \tilde{\upsilon}^T(t)\begin{pmatrix} F_5 & F_3 \\ F_3^T & F_6 \end{pmatrix}\tilde{\upsilon}(t).
\end{aligned}$$

By (38) and the similar proof of Theorem 3.2, it can be easily proved that

$$\sum_{i=1}^{\mathcal{K}}\int_0^{t_\rho}\upsilon_i^T(\rho)\upsilon_i(\rho)d\rho\leqslant V_4(0)+\phi^2\sum_{i=1}^{\mathcal{K}}\int_0^{t_\rho}\tilde{\varepsilon}_i^T(\rho)\tilde{\varepsilon}_i(\rho)d\rho$$

for any $\mathbb{R}\ni t_\rho>0$.

Consequently, network (2) via PI controller (35) can realize the $\mathcal{H}_\infty$ output synchronization.

Remark 3. More recently, a great deal of attention has been drawn to $\mathcal{H}_\infty$ and $\mathcal{H}_\infty$ output synchronization for CNs, with many profound results established [28]–[32], [34], [35]. Unfortunately, very few authors have considered $\mathcal{H}_\infty$ output synchronization for MWCNs [33], [36], [37]. It is worth pointing out that the results in this paper have been extended to the case that $\mathcal{H}_\infty$ output synchronization of MOCCNs under PID control. In this section, several sufficient conditions are established for ensuring $\mathcal{H}_\infty$ output synchronization by using inequality techniques, and $\mathcal{H}_\infty$ output synchronization problem for MOCCNs is also dealt with on the basis of PID control strategies [see Theorems 4.1-4.2].

V. Numerical Examples

To illustrate the validity of theoretical results, two numerical examples are given here.

$$\begin{aligned}
b_P^1 &= 3.8933, b_P^2 = 3.8933, b_P^3 = 3.8933,\\
Z^1 &= \begin{pmatrix} -2.8007 & 0.9952 & 0 & 0.9543 & 0.8511 & 0\\ 0.9952 & -2.8013 & 0.9516 & 0 & 0 & 0.8546\\ 0 & 0.9516 & -1.9204 & 0 & 0 & 0.9688\\ 0.9543 & 0 & 0 & -1.9132 & 0.9589 & 0\\ 0.8511 & 0 & 0 & 0.9589 & -2.8084 & 0.9984\\ 0 & 0.8546 & 0.9688 & 0 & 0.9984 & -2.8218 \end{pmatrix},\\
Z^2 &= \begin{pmatrix} -1.3882 & 0.4944 & 0 & 0.5288 & 0.3650 & 0\\ 0.4944 & -1.3697 & 0.5200 & 0 & 0 & 0.3553\\ 0 & 0.5200 & -1.0315 & 0 & 0 & 0.5115\\ 0.5288 & 0 & 0 & -1.0564 & 0.5277 & 0\\ 0.3650 & 0 & 0 & 0.5277 & -1.3826 & 0.4898\\ 0 & 0.3553 & 0.5115 & 0 & 0.4898 & -1.3566 \end{pmatrix}, \qquad (40)\\
Z^3 &= \begin{pmatrix} -0.9274 & 0.3745 & 0 & 0.3321 & 0.2208 & 0\\ 0.3745 & -0.9450 & 0.3290 & 0 & 0 & 0.2415\\ 0 & 0.3290 & -0.6832 & 0 & 0 & 0.3542\\ 0.3321 & 0 & 0 & -0.6749 & 0.3427 & 0\\ 0.2208 & 0 & 0 & 0.3427 & -0.9340 & 0.3705\\ 0 & 0.2415 & 0.3542 & 0 & 0.3705 & -0.9662 \end{pmatrix},
\end{aligned}$$

$$\begin{aligned}
b_P^1 &= 13.9410, b_P^2 = 13.9410, b_P^3 = 13.9410,\\
Z^1 &= \begin{pmatrix} -6.1986 & 3.0962 & 3.1023 & 0 & 0\\ 3.0962 & -9.2651 & 3.0235 & 0 & 3.1454\\ 3.1023 & 3.0235 & -9.2639 & 3.1381 & 0\\ 0 & 0 & 3.1381 & -6.4449 & 3.3068\\ 0 & 3.1454 & 0 & 3.3068 & -6.4523 \end{pmatrix},\\
Z^2 &= \begin{pmatrix} -5.7356 & 2.8636 & 2.8720 & 0 & 0\\ 2.8636 & -8.5673 & 2.7875 & 0 & 2.9162\\ 2.8720 & 2.7875 & -8.5678 & 2.9083 & 0\\ 0 & 0 & 2.9083 & -5.9678 & 3.0595\\ 0 & 2.9162 & 0 & 3.0595 & -5.9757 \end{pmatrix}, \qquad (41)\\
Z^3 &= \begin{pmatrix} -6.0175 & 3.0028 & 3.0147 & 0 & 0\\ 3.0028 & -8.9820 & 2.9134 & 0 & 3.0658\\ 3.0147 & 2.9134 & -8.9840 & 3.0559 & 0\\ 0 & 0 & 3.0559 & -6.2599 & 3.2040\\ 0 & 3.0658 & 0 & 3.2040 & -6.2698 \end{pmatrix}.
\end{aligned}$$

Example 5.1. Consider the following network composing of six Chua's circuits [20]:

$$\begin{aligned}
\dot{p}_i(t) &= d(p_i(t)) + 1.3\sum_{j=1}^{\mathcal{K}} \mathcal{S}_{ij}^1 \mathcal{A}_1 w_j(t) + 0.6\sum_{j=1}^{\mathcal{K}} \mathcal{S}_{ij}^2 \mathcal{A}_2 w_j(t)\\
&\quad + 0.9\sum_{j=1}^{\mathcal{K}} \mathcal{S}_{ij}^3 \mathcal{A}_3 w_j(t) + u_i(t),\\
w_i(t) &= \mathcal{Y} p_i(t), \qquad (42)
\end{aligned}$$

in which $i = 1, 2, \cdots, 6$,

$$\mathcal{A}_1 = \begin{pmatrix} 0.1 & 0\\ 0 & 0.3\\ 0 & 0 \end{pmatrix}, \mathcal{A}_2 = \begin{pmatrix} 0.3 & 0\\ 0 & 0.4\\ 0 & 0 \end{pmatrix},$$

$$\mathcal{A}_3 = \begin{pmatrix} 0.2 & 0\\ 0 & 0.5\\ 0 & 0 \end{pmatrix}, \mathcal{Y} = \begin{pmatrix} 2.5 & 0 & 0\\ 0 & 3.5 & 0 \end{pmatrix},$$

$$\mathcal{S}^1 = \begin{pmatrix} -0.6 & 0.2 & 0 & 0.1 & 0.3 & 0\\ 0.2 & -0.6 & 0.1 & 0 & 0 & 0.3\\ 0 & 0.1 & -0.4 & 0 & 0 & 0.3\\ 0.1 & 0 & 0 & -0.3 & 0.2 & 0\\ 0.3 & 0 & 0 & 0.2 & -0.7 & 0.2\\ 0 & 0.3 & 0.3 & 0 & 0.2 & -0.8 \end{pmatrix},$$

$$\mathcal{S}^2 = \begin{pmatrix} -0.4 & 0.1 & 0 & 0.2 & 0.1 & 0\\ 0.1 & -0.5 & 0.2 & 0 & 0 & 0.2\\ 0 & 0.2 & -0.3 & 0 & 0 & 0.1\\ 0.2 & 0 & 0 & -0.4 & 0.2 & 0\\ 0.1 & 0 & 0 & 0.2 & -0.4 & 0.1\\ 0 & 0.2 & 0.1 & 0 & 0.1 & -0.4 \end{pmatrix},$$

$$\mathcal{S}^3 = \begin{pmatrix} -0.5 & 0.2 & 0 & 0.2 & 0.1 & 0\\ 0.2 & -0.7 & 0.2 & 0 & 0 & 0.3\\ 0 & 0.2 & -0.4 & 0 & 0 & 0.2\\ 0.2 & 0 & 0 & -0.4 & 0.2 & 0\\ 0.1 & 0 & 0 & 0.2 & -0.4 & 0.1\\ 0 & 0.3 & 0.2 & 0 & 0.1 & -0.6 \end{pmatrix},$$

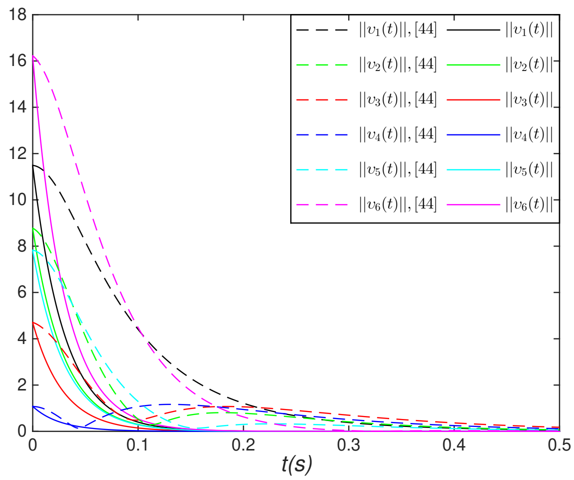

Fig. 1. Evolutions of $\|\upsilon_i(t)\|$, $i = 1, 2, \cdots, 6$, in network (1) under PD controller (9).

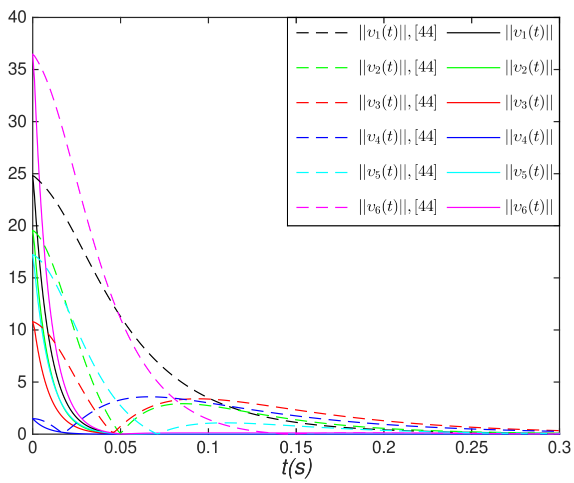

Fig. 2. Change curves of $\|\upsilon_i(t)\|$, $i = 1, 2, \cdots, 6$, in network (1) under PI controller (19).

$$d(p_i(t)) = \begin{pmatrix} 10(-p_{i1}(t) + p_{i2}(t) - \psi(p_{i1}(t))) \\ p_{i1}(t) - p_{i2}(t) + p_{i3}(t) \\ -14.87p_{i2}(t) \end{pmatrix},$$

with $\psi(p_{i1}(t)) = -0.68p_{i1}(t) + 0.5(-1.27 + 0.68)(|p_{i1}(t) + 1| - |p_{i1}(t) - 1|)$.

Case 1. Apparently, denote $H = 11I_2$ and $\alpha = 1$, the function $\hat{d}(\cdot) = (d_1(\cdot), d_2(\cdot))^T$ fulfills (4). By virtue of MATLAB YALMIP Toolbox, the following parameters [see (40) at the top of page 10] can be acquired such that (11) holds. From Theorem 3.1, network (42) can realize output synchronization under PD controller (9). Letting $b_D^1 = 0.09, b_D^2 = 0.1, b_D^3 = 0.07$, the evolutions of $\upsilon_i(t)$ $(i = 1, 2, \cdots, 6)$ are displayed in Fig. 1.

Case 2. Obviously, take $H = 11I_2$ and $\alpha = 1$, the function $\hat{d}(\cdot) = (d_1(\cdot), d_2(\cdot))^T$ fulfills (4). Moreover, we can easily demonstrate that

$$\|\hat{d}(\sigma_1) - \hat{d}(\sigma_2)\| \leqslant 10\|\sigma_1 - \sigma_2\|$$

for any $\sigma_1, \sigma_2 \in \mathbb{R}^\varrho$. Define $\mathcal{Z}^1 = S^1, \mathcal{Z}^2 = 1.1 * S^2, \mathcal{Z}^3 = 0.8 * S^3$ and on the basis of MATLAB YALMIP Toolbox, the following parameters:

$$b_P^1 = 8.8876, b_P^2 = 14.9866, b_P^3 = 11.9341,$$
$$b_I^1 = 3.3112, b_I^2 = 3.8919, b_I^3 = 5.9665,$$

can be obtained such that (21) and (22) hold. Based on Theorem 3.2, network (42) can achieve output synchronization under PI controller (19). The changing curves of $\upsilon_i(t)$ $(i = 1, 2, \cdots, 6)$ are shown in Fig. 2.

Example 5.2. Consider the following network [53]:

$$\begin{aligned} \dot{p}_i(t) &= d(p_i(t)) + 0.5\sum_{j=1}^{\mathcal{K}} \mathcal{S}_{ij}^1 \mathcal{A}_1 w_j(t) + 0.9\sum_{j=1}^{\mathcal{K}} \mathcal{S}_{ij}^2 \mathcal{A}_2 w_j(t) \\ &\quad + 0.7\sum_{j=1}^{\mathcal{K}} \mathcal{S}_{ij}^3 \mathcal{A}_3 w_j(t) + \varepsilon_i(t) + u_i(t), \\ w_i(t) &= \mathcal{Y} p_i(t), \end{aligned} \tag{43}$$

in which $i = 1, 2, \cdots, 5$,

$$\mathcal{A}_1 = \begin{pmatrix} 0.6 & 0 \\ 0 & 0.5 \\ 0 & 0 \end{pmatrix}, \mathcal{A}_2 = \begin{pmatrix} 0.6 & 0 \\ 0 & 0.7 \\ 0 & 0 \end{pmatrix},$$

$$\mathcal{A}_3 = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.8 \\ 0 & 0 \end{pmatrix}, \mathcal{Y} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1.5 & 0 \end{pmatrix},$$

$$\mathcal{S}^1 = \begin{pmatrix} -0.5 & 0.2 & 0.3 & 0 & 0 \\ 0.2 & -1.4 & 0.5 & 0 & 0.7 \\ 0.3 & 0.5 & -1.2 & 0.4 & 0 \\ 0 & 0 & 0.4 & -0.5 & 0.1 \\ 0 & 0.7 & 0 & 0.1 & -0.8 \end{pmatrix},$$

$$\mathcal{S}^2 = \begin{pmatrix} -0.6 & 0.3 & 0.3 & 0 & 0 \\ 0.3 & -0.9 & 0.4 & 0 & 0.2 \\ 0.3 & 0.4 & -0.8 & 0.1 & 0 \\ 0 & 0 & 0.1 & -0.2 & 0.1 \\ 0 & 0.2 & 0 & 0.1 & -0.3 \end{pmatrix},$$

$$\mathcal{S}^3 = \begin{pmatrix} -0.4 & 0.2 & 0.2 & 0 & 0 \\ 0.2 & -0.6 & 0.1 & 0 & 0.3 \\ 0.2 & 0.1 & -0.5 & 0.2 & 0 \\ 0 & 0 & 0.2 & -0.3 & 0.1 \\ 0 & 0.3 & 0 & 0.1 & -0.4 \end{pmatrix},$$

$$d(p_i(t)) = \begin{pmatrix} -p_{i1}(t) + p_{i2}^2(t) \\ -2p_{i2}(t) \\ -3p_{i3}(t) + p_{i2}(t)p_{i3}(t) \end{pmatrix}.$$

Case 1. Taking $H = \text{diag}\,(2, 3)$ and $\alpha = 1$, the function $\hat{d}(\cdot) = (d_1(\cdot), d_2(\cdot))^T$ fulfills (4). Select $\phi = 1.2$ and by virtue of MATLAB YALMIP Toolbox, the following parameters [see (41) at the top of page 10] can be acquired such that (32) holds. From Theorem 4.1, network (43) can realize $\mathcal{H}_\infty$ output synchronization under PD controller (30). Letting $b_D^1 = 0.01, b_D^2 = 0.01, b_D^3 = 0.02$, and $\varepsilon_i(t) = (1.8i\sin(12\pi t), 1.7i^2\sin(12\pi t), 1.6\sqrt{i}\sin(12\pi t))^T$, the evolutions of $\upsilon_i(t)$ and $\varepsilon_i(t)$ $(i = 1, 2, \cdots, 5)$ are displayed in Fig. 3.

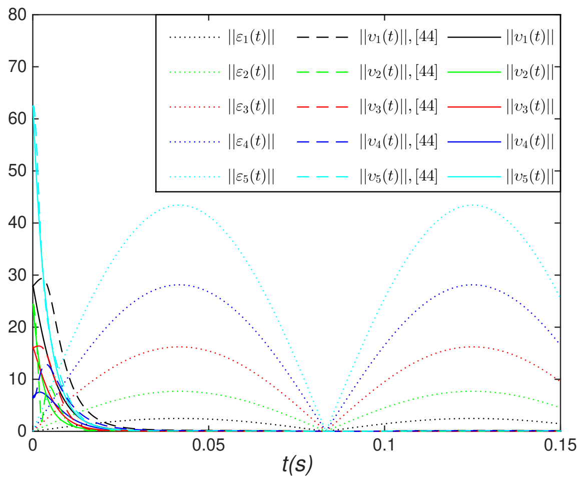

Fig. 3. Evolutions of $\|\upsilon_i(t)\|, \|\varepsilon_i(t)\|,\ i = 1, 2, \cdots, 5,$ in network (2) under PD controller (30).

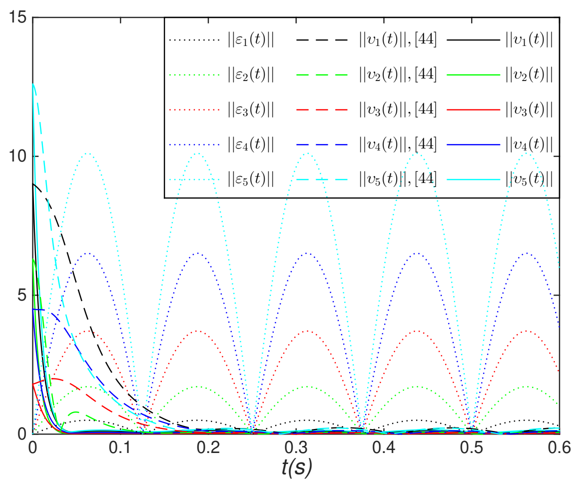

Fig. 4. Change curves of $\|\upsilon_i(t)\|, \|\varepsilon_i(t)\|,\ i = 1, 2, \cdots, 5,$ in network (2) under PI controller (35).

Case 2. Letting $H =$ diag $(2, 3)$ and $\alpha = 1$, the function $\hat{d}(\cdot) = (d_1(\cdot), d_2(\cdot))^T$ fulfills (4). In addition, it can be easily verified that

$$\|\hat{d}(\sigma_1) - \hat{d}(\sigma_2)\| \leqslant 2\|\sigma_1 - \sigma_2\|$$

for any $\sigma_1, \sigma_2 \in \mathbb{R}^{\varrho}$. Define $\phi = 0.4, \mathcal{Z}^1 = 0.5 * S^1, \mathcal{Z}^2 = 0.6 * S^2, \mathcal{Z}^3 = 0.7 * S^3$ and on the basis of the MATLAB YALMIP Toolbox, the following parameters:

$$b_P^1 = 3.5259, b_P^2 = 9.1904, b_P^3 = 17.4043,$$
$$b_I^1 = 4.4479, b_I^2 = 7.8420, b_I^3 = 12.7134,$$

can be obtained such that (37) and (38) hold. Based on Theorem 4.2, network (43) can achieve $\mathcal{H}_\infty$ output synchronization under PI controller (35). Denoting $\varepsilon_i(t) = (0.3i\sin(8\pi t), 0.4i^2\sin(8\pi t), 0.5\sqrt{i}\sin(8\pi t))^T$, the changing curves of $\upsilon_i(t)$ and $\varepsilon_i(t)$ $(i = 1, 2, \cdots, 5)$ are shown in Fig. 4.

Remark 4. In order to compare the performances of PD/PI control strategies, we also execute the control schemes in [44] with this paper network parameters. The convergence trends for norms of synchronization errors in network (42) under PD/PI control approaches and [44] control approaches are displayed in Figs. 1-2, respectively. Furthermore, Figs 3-4 are shown the evolutions of $\|\upsilon_i(t)\|$ and $\|\varepsilon_i(t)\|, (i = 1, 2, \cdots, 5)$ of network (43) under these control methods. From Fig. 1 to Fig. 4, it can be seen that performances of PD/PI control strategies are better than [44] control schemes.

VI. Conclusion

In this article, we have not only utilized PD control schemes to tackle output and $\mathcal{H}_\infty$ output synchronization problems for MOCCNs, but also established several output and $\mathcal{H}_\infty$ output synchronization criteria for MOCCNs based on PI controllers. By using the PD control approaches and Lyapunov functional, some sufficient conditions for guaranteeing the output and $\mathcal{H}_\infty$ output synchronization for MOCCNs have been acquired. Furthermore, we also have been developed the PI control strategies to investigate the output and $\mathcal{H}_\infty$ output synchronization for MOCCNs based on inequality techniques. At last, several obtained results have been allowed to verify by adopting two numerical examples, which illustrate their effectiveness. In the future, we will combine the PID controller with the adaptive control strategy to investigate finite-time synchronization for MWCNs.

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