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RESEARCH ARTICLE

Multi-Objective Material Generation Algorithm (MOMGA) for Optimization Purposes

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ABSTRACT Optimization is a process of decision-making in which some iterative procedures are conducted to maximize or minimize a predefined objective function representing the overall behavior of a considered system problem. Most of the time, one specific function cannot represent the overall behavior of a system with particular levels of complexity, so the multiple objective functions should be determined for this purpose which requires an algorithm with adaptability to this situation. Multi-objective optimization is a process of decision making in which maximization or minimization of multiple objective functions is considered for reaching the acceptable levels of performance for the considered system problem. In this paper, the multi-objective version of the Material Generation Algorithm (MGA) is proposed as MOMGA, one of the recently developed metaheuristic algorithms for single-objective optimization. To evaluate the overall performance of the MOMGA, the benchmark multi-objective optimization problems of the Competitions on Evolutionary Computation (CEC) are considered alongside the real-world engineering problems. Based on the results, the MOMGA is capable of providing very acceptable results in dealing with multi-objective optimization problems.

INDEX TERMS Material generation algorithm, multi-objective optimization, real-world engineering problems, competitions on evolutionary computation.

I. INTRODUCTION

Optimization is the process of minimizing or maximizing a function that measures the performance of a considered system problem. This procedure ultimately improves the system's overall performance by trying multiple choices for the decision variables, which are utilized for the definition of the considered system problem. The development of computational tools such as gradient-based methods over the past few decades has made it possible for designers to perform designs more quickly through multiple numerical simulations. However, these methods also involve a process of trial and error and, in many cases does not lead to an optimal configuration for the considered system. Hence, the metaheuristic algorithms have been proposed for optimization purposes in recent decades for performance improvement in optimization procedures in which the more optimal solutions can be achieved for the considered system. Metaheuristic algorithms are some sort of searching tools for finding the best optimal solutions in dealing with complex problems in which upper-level techniques are utilized. According to Alatas and Bingöl [1], there are different classes for metaheuristic algorithms containing: Bilogy-based (such as Genetic Algorithm (GA) [2]), Math-based (such as Chaos Game Optimization (CGO) [3]), Swarm-based (such as Particle Swarm Optimizer (PSO) [3] and Ant colony Optimization (ACO) [5]), Music or art-based (such as Stochastic Paint Optimizer (SPO) [6]), Chimistery-based (such as Atomic Orbital Search (AOS) [7] and Material Generation Algorithm (MGA) [8], Optics Inspired Optimization (OIO) [9]), Water-based (such as Flow Direction Algorithm (FDA) [10] and Dynamic Water Strider Algorithm (DWSA) [11]), Social-based (such as Social Network Search (SNS) [12], War Strategy Optimization (WSO) [13]), Physics-based (such as

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Special Relativity Search(SRS) [14], Fusion–Fission Optimization (FuFiO) [15]) and Plant-based (Plant Intelligence Optimization (PIO) [16]). Besides, some improved and hybridized metaheuristics have also been proposed for different applications including the Investigating the optimum design of smart micro-grid problems with metaheuristic algorithms [17], hybrid metaheuristic algorithms for optimum design of constrained optimization problem [18], [19], development of Bluetooth-based indoor localization mechanisms with metaheuristic algorithms [20], metaheuristic optimization algorithms enhanced with levy flight for engineering optimization [21], [22], optimization of civil engineering problems with metaheuristic algorithms [23], combination of metaheuristic and machine learning approaches for slope stability prediction [24], solar energy forecasting by hybridizing neural network and improved metaheuristic algorithms [25], enhanced wind-driven flower pollination for engineering optimization [26], improved butterfly optimization algorithm for engineering design problems [27], dynamic differential annealed optimization for engineering applications [28] and some other [29], [30], [31], [32], [33], [34], [35]. Optimization problems can be divided into two types regarding the number of objective functions and optimization criteria: (1) single-objective optimization and (2) multi-objective optimization. In single-objective optimization problems, the problem is solved by improving a single Performance Index whose minimum or maximum value fully reflects the quality of the response obtained, but in some cases, it is not possible to simply rely on an index in dealing with a complex optimization problem. In this type of problem, we have to define several objective functions or performance indicators and optimize the value of all of them simultaneously. Multi-objective optimization is one of the most active and widely used research fields among optimization topics. Multi-objective optimization is often known as multi-criteria optimization or vector optimization. Many methods have been proposed to solve these problems in recent decades. The Non-Dominated Sorting Genetic Algorithm (NSGA) [36] as the multi-objective version of the GA and the enhanced version of this algorithm (i.e. NSGA-II [37]), are the first multi-objective algorithms in this area. Besides, Coello and Lechuga [38] proposed the multiple objective version of PSO as MOPSO. Zhang and Li [39] developed Multi-Objective Evolutionary Algorithm (MOEA). Alaya *et al.* [40] presented Multi-Objective ACO as MOACO. To solve multi-objective optimization issues, an art-inspired metaheuristic approach is proposed by Khodadadi *et al.*[41] as well. Besides, some other challenges in recent years can be found in the literature such as Multi-Objective Ant Lion Optimizer (MOALO) [42], Multi-Objective Multi-Verse Optimizer (MOMVO) [43], Multi-objective Slap Swarm Algorithm (MSSA) [44], Multi-Objective Crystal Structure Algorithm (MOCryStAl) [45] and some other approaches [46], [47], [48], [49].

The Material Generation Algorithm (MGA) [8] algorithm is a straightforward and effective method of solving optimization problems. Consequently, many different types of optimization problems can benefit significantly from MGA. However, the MGA cannot be used directly to resolve multi-objective problems because it was developed to address single-objective optimization problems. Therefore, the main contribution of this paper is to present a multi-objective MGA approach (MOMGA) for the first time. Although there are different multi-objective algorithms that can be applied to multi-objective problems, since there is currently no algorithm or method that can handle all problems with perfect accuracy, most academics are continually searching for new approaches and techniques with enhanced capabilities. In the other words, the No Free Lunch (NFL) theorem states that no algorithm is capable of solving all problems. As a result, new algorithms can be proposed, or existing ones can be improved using NFL theory. These and other characteristics make it difficult to find a solution to dynamic optimization challenges. Besides, the mechanisms employed may somehow be similar to those employed by MOGWO [50] but the exploration and exploitation phases of MOMGA inherit from the MGA algorithm and are completely different. In this algorithm, the general aspects of the material generation process in chemistry, including the chemical compounds and chemical reactions, are in perspective. For evaluation of the overall performance of the MOMGA, the benchmark multi-objective optimization problems of the Competitions on Evolutionary Computation (CEC) are considered alongside the real-world engineering problems.

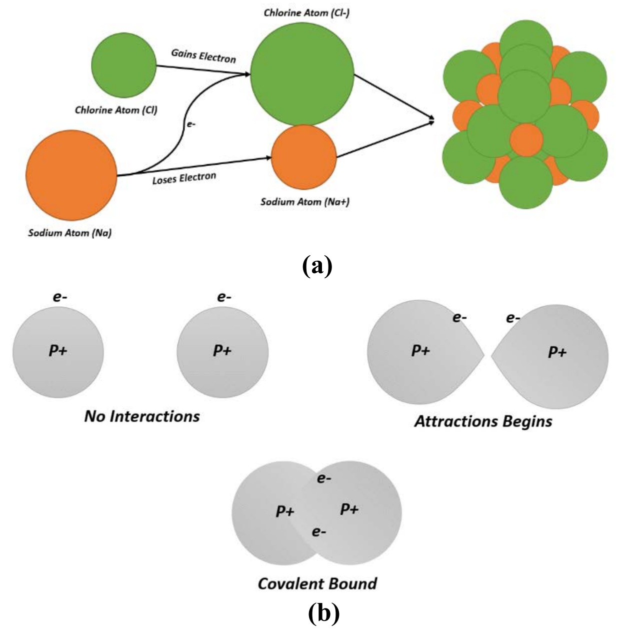


FIGURE 1. Schematic presentation of ionic (a) covalent (b) compounds.

II. MULTI-OBJECTIVE MATERIAL GENERATION ALGORITHM

A. MATERIAL GENERATION ALGORITHM

Material is anything made of matter (physics) or a combination of one or more chemicals. Wood, cement, hydrogen, air, water, and any other ones are all examples of materials. Sometimes the term materials refer more to compounds with the same physical properties as what they make. By this definition, materials are essential components for building other things from buildings to art, airplanes, and computers. As mentioned, the proposed MGA [8] is one of the recently developed metaheuristic algorithms that consider the general aspects of material generation in nature along with the basic and advanced principles of chemistry, including chemical compounds and chemical reactions. In the first step of this algorithm, an initialization process is conducted in which the decision variables are determined as periodic table elements (PTEs), which are utilized for generating materials in nature. These aspects are presented as:

$$Mat = \begin{bmatrix} PTE_1^1 & PTE_1^2 & \cdots & PTE_1^j & \cdots & PTE_1^d \\ PTE_2^1 & PTE_2^2 & \cdots & PTE_2^j & \cdots & PTE_2^d \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ PTE_i^1 & PTE_i^2 & \cdots & PTE_i^j & \cdots & PTE_i^d \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ PTE_n^1 & PTE_n^2 & \cdots & PTE_n^j & \cdots & PTE_n^d \end{bmatrix} \quad (1)$$

$$PTE_i^j(0) = PTE_{i,min}^j + Unif(0,1).\left(PTE_{i,max}^j - PTE_{i,min}^j\right)$$
$$\text{where} \begin{cases} i = 1, 2, \ldots, n. \\ j = 1, 2, \ldots, d. \end{cases} \quad (2)$$

where n is the total number of materials representing solution candidates; d represents the dimension of the considered problem; $PTE_i^j(0)$ is the initial amount of the jth periodic table element in the ith material; $PTE_{i,min}^j$ and $PTE_{i,max}^j$ represent the maximum and minimum allowable amounts of the jth decision variable in the ith solution candidate; and $Unif(0,1)$ is a uniformly distributed random number.

In order to formulate the main loop of the MGA, the concepts of chemical compounds and chemical reactions are utilized for formulating a search algorithm. A chemical compound is a specific chemical substance that is composed of two or more different chemical elements. These elements are bonded together by chemical bonding and can be converted to simple materials by chemical reaction. Each different chemical compound has a uniquely defined chemical structure. In other words, each compound has the same atomic ratio, the atoms of which are arranged in a specific spatial arrangement by chemical bonding. Chemical compounds are categorized into two particular types, ionic compounds and covalent compounds. Ionic composition is a type of chemical compound whose constituent particles are positive and negative ions. The common form of ionic compounds consists of a metal as a cation and a nonmetal as an anion. A covalent compound is a chemical bond in chemistry in which atoms can fill their orbits by sharing electrons and achieving a stable octave arrangement of noble gas after themselves. In Fig. 1, the schematic presentation of ionic and covalent compounds is illustrated.

Changes that occur in reaction to reactants are generally divided into two types: physical and chemical changes. In physical changes, only the physical state of matter changes, not the particle structure of matter; Therefore, changes in all states of matter, such as melting, freezing, evaporation, condensation, sublimation, and density, as well as the dissolution of salts and bases in water, are physical changes. In chemical changes, the bonding of atoms to each other and their electronic arrangement in the reactants change. Of course, in a chemical reaction, atoms do not form or disappear and only combine, decompose or rearrange. A chemical reaction is the expression of a chemical change that may be accompanied by the release of energy in the form of heat, light, or sound, resulting in gas production, scale formation, or discoloration. In other words, a chemical reaction is a process by which the structure of the particles that make up raw materials changes while one or more chemicals are converted to one or more other chemicals. In Fig. 2, the schematic presentation of a chemical reaction is illustrated.

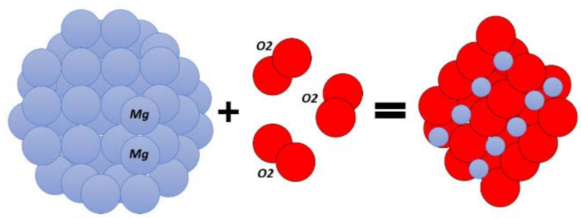


FIGURE 2. Schematic presentation of a chemical reaction.

In order to implement the concept of chemical compounds into the mathematical model of the MGA, all of the PTEs are supposed to be in their ground state in which they can be excited easily by photons, magnetic fields, and colliding with other particles. The tendency of PTEs in gaining or losing electrons which represents the formulation of chemical compounds is modeled through a continuous probability distribution which is presented in the following, while the schematic presentation of generating a new material by these aspects is illustrated in Fig. 3.

$$Mat_{new_1} = \left[PTE_{new}^1 PTE_{new}^2 \cdots PTE_{new}^k \cdots PTE_{new}^d \right] \quad (3)$$

$$PTE_{new}^k = PTE_{r_1}^{r_2} \pm e^-, \quad (4)$$

$$f\left(PTE_{new}^k \mid \mu, \sigma^2\right) = \frac{1}{\sqrt{2\pi\sigma^2}}.e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$
$$k = 1, 2, \ldots, d \quad (5)$$

where r_1 and r_2 are random integers distributed uniformly in [1,n] and [1,d], respectively; $PTE_{r_1}^{r_2}$ is selected randomly from the Mat; e^- represents the probabilistic component for determining the process of gaining, losing or even sharing electrons; PTE_{new}^k is the new material; and Mat_{new_1} is the newly generated material.

In order to mathematically implement the chemical reaction aspect of material into the MGA, some of the materials

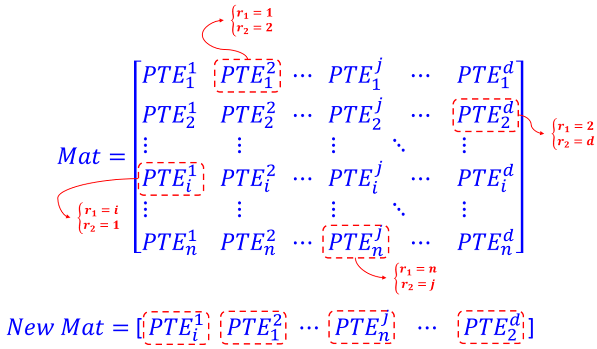


FIGURE 3. Schematic presentation of generating a new material by chemical compounds.

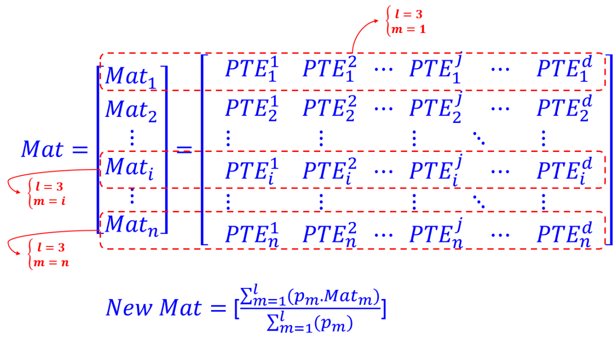


FIGURE 4. Schematic presentation of generating a new material by chemical reactions.

from the search space are selected randomly for this purpose. These materials are combined by means of the following equation, while the schematic presentation of generating a new material by this aspect is illustrated in Fig. 4.

$$Mat_{new_2} = \frac{\sum_{m=1}^{l} (p_m.Mat_{mj})}{\sum_{m=1}^{l} (p_{mj})}, j = 1, 2, \ldots, 1 \qquad (6)$$

where Mat_m represents the *mth* determined random material from search space; p_m denotes on a Gaussian normal distribution; and Mat_{new_2} represents the newly generated material by means of the chemical reaction.

B. MULTI-OBJECTIVE MATERIAL GENERATION ALGORITHM

The MGA [8] was developed to address single-objective optimization issues and cannot be used to solve multi-objective problems directly. As a result, this paper presents a multi-objective version of MGA for solving multi-criterion optimization problems. MGA has three new mechanisms for solving multi-objective optimization.

The first mechanism introduced into MGA is the archive, which serves as a storage facility for storing or restoring the derived Pareto optimal solutions. The archive has a single controller that manages which solutions are added to the archive and when the archive is full. There is a limit to the number of solutions that can be stored in the archive. The residents of the archive are compared to the non-dominated solutions created. Three major scenarios are possible:

I. If there is at least one member in the archive who dominates the new solution, it is not allowed to enter the archive.
II. The new solution may be added to the archive if it dominates at least one solution in the archive by omitting the one already in the archive.
III. If the new and archive solutions do not dominate each other, the new solution is added to the archive.

The grid mechanism, which is included in MGA, is the second effective technique for enhancing non-dominated solutions in the archive. If the archive gets full, the grid technique will be utilized to reorganize the object space's segmentation and find the most populated area in order to eliminate one of the solutions. The additional member should then be included in the least crowded segment to boost the variety of the final approximated Pareto optimal front. As the number of possible solutions in the hypercube expands, the possibility of deleting a solution increases. If the archive is full, the most crowded areas are chosen first, and a solution from one of them is randomly deleted to make way for the new solution. When a solution is placed outside the hypercubes, a special case arises. All segments in this scenario have been expanded to fit the most recent solutions. As a result, the segments of alternative solutions can also be changed.

Due to the Pareto optimality, solutions in a multi-objective search space cannot be compared, hence the Leader Selection Mechanism is the last machine in MGA. As a result, MGA includes a leader selection method to address this problem. The search leaders guide the other search candidates to possible areas of the search space, with the objective of getting a solution that is close to the global optimum. As previously stated, the archive contains just the best non-dominated solutions. The leader selection mechanism chooses the least crowded portions of the search space and presents the best as non-dominated answers. The selection for each hypercube is made using a roulette-wheel approach with the following probability:

$$P_i = \frac{C}{N_i} \qquad (7)$$

where c is a constant number higher than one, as well as N is the variety of acquired Pareto optimal answers in the ith section.

From Eq. (7), less congested hypercubes have a higher probability of suggesting new leaders. The chance of selecting a hypercube from which to select leaders increases as the number of obtained solutions in the hypercube is lowered.

Less crowded hypercubes have a more significant probability of suggesting new leaders, as shown in Eq. (6). When the number of obtained solutions in the hypercube is reduced, the probability of selecting a hypercube to select leaders from them increases. The flowchart of the MGA is illustrated in Fig. 5.

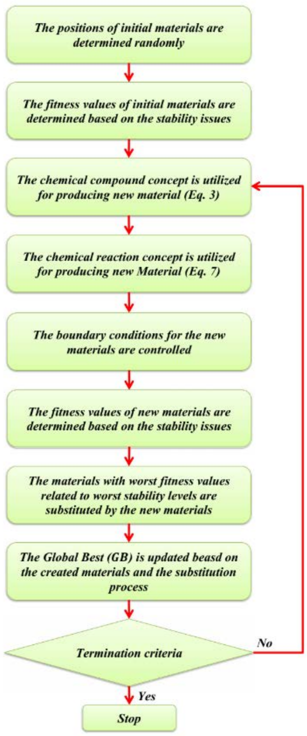


FIGURE 5. Flowchart of the MGA.

The MGA algorithm, of course, provides the MOMGA algorithm with its convergence. If we choose one solution from the archive, the MGA approach will most likely be able to increase its already good consistency. On the other hand, finding the Pareto optimal replies created with a large diversity is challenging.

III. RESULTS AND DISCUSSION

The efficiency of the suggested approach in this part is evaluated using performance measures and case studies, including unconstrained and constrained bi- and tri-objective mathematics (CEC-09) and real-world engineering design problems. The ability of multi-objective optimizers to handle problems with non-convexity and non-linearity is tested using these problems and mathematical functions. The algorithms are programmed by MATLAB R2021a. To have a fair comperation, we solved each problem 30 different runs for each algorithm. The initial condition (initial populations) for all algorithms is the same and they are generated randomly for each run. Other condition such as the maximum number of function evaluation is selected to be the same for all methods as well.

A. PERFORMANCE METRICS

To evaluate the algorithms' results, the following four metrics are used:

1) GENERATIONAL DISTANCE (GD)

This index represents the overall sum of the adjacent distances of solution candidates regarding different achieved sets through multiple algorithms which are known as an intelligent indicator for evaluating the convergence characteristics of metaheuristic algorithms with multiple objectives [51]:

$$GD = \left(\frac{1}{n_{pf}} \sum_{i=1}^{n_{pf}} dis_i^2 \right)^{\frac{1}{2}} \tag{8}$$

TABLE 1. Parameters setting of all algorithms.

Parameters	MOPSO	MOALO	MOMVO	NSGA-II	MSSA	MOGWO	MOMGA
P_w	0.5	-	-	1/D	-	-	-
N_p	100	100	100	100	100	100	100
N_{rep}	100	100	100	100	100	100	100
N_{Grid}	30	30	30	-	30	30	30
C_1	1	-	-	-	-	-	-
C_2	2	-	-	-	-	-	-
w	0.4	-	-	-	-	-	-
Beta	4	4	4	-	4	4	-
Gamma	2	2	2	-	2	2	-
Pc	-	-	-	0.9	-	-	-

2) SPACING (S)

This is an index for measuring the total distance between candidates regarding different achieved sets by means of multiple algorithms [52]:

$$S = \left(\frac{1}{n_{pf}} \sum_{i=1}^{n_{pf}} (d_i - \bar{d})^2 \right)^{\frac{1}{2}}$$
$$Where\ \bar{d} = \frac{1}{n_{pf}} \sum\nolimits_{i=1}^{n_{pf}} d_i \tag{9}$$

3) MAXIMUM SPREAD (MS)

This index represents the spread of candidates among other achieved sets by considering the distinct optimal choices [53]:

$$MS = \left[\frac{1}{m} \sum_{i=1}^{m} \left[\frac{\min\left(f_i^{max}, F_i^{max}\right) - \max(f_i^{min}, F_i^{min})}{F_i^{max} - F_i^{min}} \right]^2 \right]^{\frac{1}{2}} \tag{10}$$

4) INVERTED GENERATIONAL DISTANCE (IGD)

This index is a precise measure for the performance estimation of the Pareto front approximations by means of the results of multiple many-objective optimization algorithms [54]:

$$IGD = \frac{\sqrt{\sum_{i=1}^{n} d_i^2}}{n} \tag{11}$$

5) HYPERVOLUME (HV)

[55]: HV is one of the most commonly used metrics for evaluating the Pareto front (PF) approximations generated by multi-objective evolutionary algorithms. Even so, HV is a resultant of a complex interplay between the PF shape, number of objectives, and user-specified reference points. The HV metric is employed to concurrently examine the uniformity-convergence-spread of the non-dominated set of solutions procured from the computation experiments.

TABLE 2. Bi-objective CEC-09 benchmark functions.

Function	Mathematical formulation	D	Range
UF1	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}\left[x_j - sin(6\pi x_1 + \frac{j\pi}{n})\right]^2,\ f_2 = 1 - \sqrt{x} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}[x_j - sin(6\pi x_1 + \frac{j\pi}{n})]^2$ $J_1 = \{j\lvert j \text{ is odd and } 2 \le j \le n\}, \quad J_2 = \{j\lvert j \text{ is even and } 2 \le j \le n\}$	30	$x_1 \in [0,1]$ $x_i \in [-1,1]$ $i = 1,\dots,D$
UF2	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1} y_j^2, \quad f_2 = 1 - \sqrt{x} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2} y_j^2$ $J_1 = \{j\lvert j \text{ is odd and } 2 \le j \le n\}, \quad J_2 = \{j\lvert j \text{ is even and } 2 \le j \le n\}$ $y_j = \begin{cases} x_j - [0.3x_1^2 cos(24\pi x_1 + \frac{4j\pi}{n}) + 0.6x_1]\, cos(6\pi x_1 + \frac{j\pi}{n}) if j \in J_1 \\ x_j - [0.3x_1^2 cos(24\pi x_1 + \frac{4j\pi}{n}) + 0.6x_1]\, cos(6\pi x_1 + \frac{j\pi}{n}) if j \in J_2 \end{cases}$	30	$x_1 \in [0,1]$ $x_i \in [-1,1]$ $i = 1,\dots,D$
UF3	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}(4\sum_{j\in J_1} y_j^2 - 2\prod_{j\in J_1} cos(\frac{20y_j\pi}{\sqrt{j}}) + 2)$ $f_2 = \sqrt{x_1} + \frac{2}{\lvert J_2\rvert}(4\sum_{j\in J_2} y_j^2 - 2\prod_{j\in J_2} cos(\frac{20y_j\pi}{\sqrt{j}}) + 2)$ J_1 and J_2 are the same as those of UF1, $y_j = x_j - x_1^{0.5\left(1.0+\frac{3(j-2)}{\pi-2}\right)}$, $j = 2,3,\dots,n$	30	$x_i \in [0,1]$
UF4	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1} h(y_j), \quad f_2 = 1 - x_2 + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2} h(y_j)$ J_1 and J_2 are the same as those of UF1, $y_j = x_j - sin(6\pi x_1 + \frac{j\pi}{n}),\ j = 2,3,\dots,n$ $h(t) = \frac{\lvert t\rvert}{1 + e^{2\lvert t\rvert}}$	30	$x_1 \in [0,1]$ $x_i \in [-2,2]$ $i = 1,\dots,D$
UF5	$f_1 = x_1 + (\frac{1}{2N} + \varepsilon)\lvert sin(2N\pi x_1)\rvert + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1} h(y_j)$ $f_1 = 1 - x_1 + (\frac{1}{2N} + \varepsilon)\lvert sin(2N\pi x_1)\rvert + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2} h(y_j)$ J_1 and J_2 are identical to those of UF1 , $\varepsilon > 0, y_j = x_j - sin(6\pi x_1 + \frac{j\pi}{n}),\ j = 2,3,\dots,n$ $h(t) = 2t^2 - cos(4\pi t) + 1$	30	$x_1 \in [0,1]$ $x_i \in [-1,1]$ $i = 1,\dots,D$
UF6	$f_1 = x_1 + max\{0,2(\frac{1}{2N} + \varepsilon)\lvert sin(2N\pi x_1)\rvert\} + \frac{2}{\lvert J_1\rvert}(4\sum_{j\in J_1} y_j^2 - 2\prod_{j\in J_1} cos(\frac{20y_j\pi}{\sqrt{j}}) + 1))$ $f_2 = 1 - x_1 + max\{0,2(\frac{1}{2N} + \varepsilon)\lvert sin(2N\pi x_1)\rvert\}\frac{2}{\lvert J_1\rvert}(4\sum_{j\in J_2} y_j^2 - 2\prod_{j\in J_2} cos(\frac{20y_j\pi}{\sqrt{j}}) + 1))$ J_1 and J_2 are identical to those of UF1, $\varepsilon > 0,\ y_j = x_j - sin(6\pi x_1 + \frac{j\pi}{n}),\ j = 2,3,\dots,n$	30	$x_1 \in [0,1]$ $x_i \in [-1,1]$ $i = 1,\dots,D$
UF7	$f_1 = \sqrt[5]{x_1} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_1} y_j^2, f_2 = 1 - \sqrt[5]{x_1} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2} y_j^2$ J_1 and J_2 are identical to those of UF1, $\varepsilon > 0,\ y_j = x_j - sin(6\pi x_1 + \frac{j\pi}{n}),\ j = 2,3,\dots,n$	30	$[x_1 \in [0,1]$ $x_i \in [-1,1]$ $i = 1,\dots,D$

6) WILCOXON's RANK-SUM TEST (WRT)

The Wilcoxon's rank sum test at a 5% significance level is a non-parametric statistical test that is used to determine whether two or more datasets are from the same distributed population. The Wilcoxon rank-sum test is used to evaluate the algorithm's performance in depth. The null hypothesis states that the mean metrics obtained by the two compared algorithms are the same, which suggests that the performance of the algorithms is the same. The alternative hypothesis states that the mean metrics obtained by the two compared algorithms are not the same. Algorithms proposed in this article are compared to other algorithms using the symbols '-', '+' and '=' that means the algorithm performs poorly, significantly better, and there is no noticeable difference, respectively.

The IGD and GD performance measures quantify the convergence, and the S and MS measure the coverage of Pareto optimal solutions estimated by the algorithms. When evaluating algorithm performance based on the mean value, the Wilcoxon rank-sum test was used, which effectively demonstrated the algorithm's high level of competitiveness and effectiveness.

B. EXPERIMENTAL SETUP

MOPSO, MOALO, NSGA-II, MSSA, MOGWO and MOMVO were compared to MOMGA in order to identify the best figure of a collection of Pareto optimal solutions. The initial parameters of all described algorithms are summarized in Table 1. In this table, Pw is Mutation Probability, *Np* is Population Size, N_{rep} is the size of archive, Number of Adaptive

TABLE 3. Tri-objective CEC-09 benchmark functions.

Function	Mathematical formulation	D	Range
UF8	$f_1 = cos(0.5x_1\pi)\,cos(0.5x_2\pi) + \frac{2}{\|J_1\|}\sum_{j\in J_1}(x_j - 2x_2\,sin(2\pi x_1 + \frac{j\pi}{n})^2)$ $f_2 = cos(0.5x_1\pi)\,sin(0.5x_2\pi) + \frac{2}{\|J_2\|}\sum_{j\in J_2}(x_j - 2x_2\,sin(2\pi x_1 + \frac{j\pi}{n})^2)$ $f_3 = sin(0.5x_1\pi) + \frac{2}{\|J_3\|}\sum_{j\in J_3}(x_j - 2x_2\,sin(2\pi x_1 + \frac{j\pi}{n})^2)$ $J_1 = \{j \| 3 \le j \le n, and j - 1 is a multiplication of 3\}$ $J_2 = \{j \| 3 \le j \le n, and j - 2 is a multiplication of 3\}$ $J_3 = \{j \| 3 \le j \le n, and j is a multiplication of 3\}$	30	$x_1 \in [0,1]$ $x_2 \in [0,1]$ $x_i \in [-2,2]$ $i = 1, \dots, D$
UF9	$f_1 = 0.5[max\{0, (1+\varepsilon)(1 - 4(2x_1 - 1)^2)\} + \frac{2}{\|J_1\|}\sum_{j\in J_1}(x_j - 2x_2\,sin(2\pi x_1 + \frac{j\pi}{n})^2)$ $f_2 = 0.5[max\{0, (1+\varepsilon)(1 - 4(2x_1 - 1)^2)\} + 2x_1]x_2 + \frac{2}{\|J_2\|}\sum_{j\in J_2}(x_j - 2x_2\,sin(2\pi x_1 + \frac{j\pi}{n})^2)$ $f_3 = 1 - x_2 + \frac{2}{\|J_3\|}\sum_{j\in J_3}(x_j - 2x_2\,sin(2\pi x_1 + \frac{j\pi}{n})^2)$ $J_1 = \{j \| 3 \le j \le n, and j - 1 is a multiplication of 3\}$ $J_2 = \{j \| 3 \le j \le n, and j - 2 is a multiplication of 3\}$ $J_3 = \{j \| 3 \le j \le n, and j is a multiplication of 3\}, \quad \varepsilon = 0.1$	30	$x_1 \in [0,1]$ $x_2 \in [0,1]$ $x_i \in [-2,2]$ $i = 1, \dots, D$
UF10	$f_1 = cos(0.5x_1\pi)\,cos(0.5x_2\pi) + \frac{2}{\|J_1\|}\sum_{j\in J_1}[4y_j^2 - cos(8\pi y_j) + 1]$ $f_2 = cos(0.5x_1\pi)\,sin(0.5x_2\pi) + \frac{2}{\|J_2\|}\sum_{j\in J_1}[4y_j^2 - cos(8\pi y_j) + 1]$ $f_3 = sin(0.5x_1\pi) + \frac{2}{\|J_3\|}\sum_{j\in J_3}[4y_j^2 - cos(8\pi y_j) + 1]$ $J_1 = \{j \| 3 \le j \le n, and j - 1 is a multiplication of 3\}$ $J_2 = \{j \| 3 \le j \le n, and j - 2 is a multiplication of 3\}$ $J_3 = \{j \| 3 \le j \le n, and j is a multiplication of 3$	30	$x_1 \in [0,1]$ $x_2 \in [0,1]$ $x_i \in [-2,2]$ $i = 1, \dots, D$

Grid is shown by N_{Grid}. C_1 and C_2 are Personal and global Learning Coefficient, respectively. Inertia weight is shown by w and Crossover probability is as Pc. These parameters of the algorithms were obtained from their main references.

It is worth mentioning that the number of function evaluations is set to 100000 for all methods. Each problem is solved separately 30 times. As illustrated in Tables 2 and 3 and Appendix A, the proposed algorithm was tested in 18 various case studies, including ten unconstrained and restricted mathematical and eight real-world engineering design issues. The authors established an environment that ensures fairness, reliability, and justice among the methods. We compared to avoid any accidental bias toward a better condition for any algorithm. This condition is a constraint in experiments to ensure that the superiorities are not due to the testing advantages. A static penalty method is utilized to handle problem specific constraints. This means that if a design is beyond the limitation of constraint, the objective functions are penalized. For bound constraint a simple fly to boundary method is utilized, in which, the violated value is replaced by the nearest limit.

It is worth noting that finding the exact values of the parameters for MOMGA is not necessary and the required values are selected as the same as other methods.

C. RESULTS AND DISCUSSION OF THE MATHEMATICAL PROBLEMS

In Table 4, the results of different multi-objective optimization algorithms are presented by means of the IGD and GD indices. It is evident that the MOMGA which is proposed as a new algorithm in this paper can outrank the other approaches in most cases. Considering the average and standard divination of the results, almost the all examples, MOMGA finds the best results. Compared to MOMVO which is the closest algorithm to the MOMGA, it can find 8 times better results. For the MOPSO and MOALO, the proposed method can achieve decently better results for all examples.

The results of the MS and S indices as two of the most important ones in the multi-objective optimization field are presented in Table 5. According to Wilcoxon's rank sum test, it is clear that the MOMGA is significantly superior to other methods. From Table 5, it is clear that MOMGA can win at least 9 out of 10 times against other methods considering the MS index. When the S index is investigated, MOMGA is the best one as well for at least 7 out of 10 examples.

In Figs. 6, the true and obtained Pareto front for CEC09 problems are demonstrated as those are obtained by the MOMGA and other alternative algorithms in which the

TABLE 4. The statistical results of mathematical problems for IGD and GD indices.

Functions		IGD				GD			
		MOPSO	*MOALO*	*MOMVO*	*MOMGA*	*MOPSO*	*MOALO*	*MOMVO*	*MOMGA*
UF1	Ave	5.3291E-03	6.1666E-03	**3.6662E-03**	9.1647E-03	2.7430E-02	1.5372E-02	**3.7175E-03**	7.2228E-02
	SD	2.9590E-03	6.0132E-04	5.1992E-04	**4.7993E-04**	2.5390E-02	4.1474E-03	**2.7354E-03**	2.3739E-02
	WRT	+	+	+		+	+	-	
UF2	Ave	4.1586E-03	5.9619E-03	2.8279E-03	**2.7710E-03**	2.0656E-02	1.9644E-02	5.7929E-03	**4.9121E-03**
	SD	4.8128E-04	4.6549E-04	3.2623E-04	**2.1964E-04**	4.6831E-03	4.0405E-03	2.3059E-03	**1.6801E-03**
	WRT	+	+	+		+	+	+	
UF3	Ave	1.7231E-02	1.2781E-02	1.4925E-02	**1.2086E-02**	9.6010E-02	3.9887E-02	5.7714E-02	**2.7605E-02**
	SD	1.0806E-03	1.6226E-03	1.3921E-03	**5.1450E-04**	2.3482E-02	6.2737E-03	1.6501E-02	**2.8687E-03**
	WRT	+	+	+		+	+	+	
UF4	Ave	2.8277E-03	3.5940E-03	3.2478E-03	**2.7269E-03**	9.2957E-03	5.7701E-03	1.0073E-02	**4.7885E-03**
	SD	2.5695E-04	1.0043E-03	1.9370E-04	**1.3836E-04**	9.2862E-04	7.4405E-04	9.4967E-04	**4.0516E-04**
	WRT	+	+	+		+	+	+	
UF5	Ave	3.4235E-01	3.5831E-01	**2.0559E-01**	5.7274E-01	4.2733E-01	**1.7752E-01**	2.1457E-01	6.8870E-01
	SD	1.5317E-01	9.7301E-02	6.4855E-02	**4.6196E-02**	2.9837E-01	**5.7170E-02**	6.2058E-02	1.0501E-01
	WRT	+	+	-		+	-	+	
UF6	Ave	2.9255E-02	2.6411E-02	**1.6393E-02**	4.2876E-02	2.0278E-01	8.6731E-02	**3.3546E-02**	3.8624E-01
	SD	9.1394E-03	6.2357E-03	6.8389E-03	**5.2547E-03**	1.6569E-01	3.0128E-02	**1.5241E-02**	7.7261E-02
	WRT	+	+	-		+	+	-	
UF7	Ave	4.3417E-03	7.8064E-03	**4.3363E-03**	9.3595E-03	1.7938E-02	1.3805E-02	**3.3843E-03**	5.6882E-02
	SD	2.4234E-03	3.6188E-03	6.4896E-03	**1.1857E-03**	1.3813E-02	3.3199E-03	**2.3377E-03**	7.8907E-03
	WRT	+	+	+		+	+	-	
UF8	Ave	9.2776E-03	6.8487E-03	4.9482E-03	**4.7841E-03**	2.6587E-01	4.5825E-02	5.6416E-02	**4.1166E-03**
	SD	1.6147E-03	1.4396E-03	2.2834E-03	**2.9993E-04**	8.2205E-02	2.9935E-02	8.0552E-02	**1.0515E-03**
	WRT	+	+	+		+	+	+	
UF9	Ave	1.2300E-02	6.7235E-03	4.3669E-03	**3.9471E-03**	4.0311E-01	4.5643E-02	9.3547E-02	**4.4914E-02**
	SD	2.9460E-03	1.1738E-03	7.8011E-04	**4.9389E-04**	7.9920E-02	**1.1497E-02**	5.1567E-02	3.3108E-02
	WRT	+	+	+		+	+	+	
UF10	Ave	6.3301E-02	4.0477E-02	1.1965E-02	**1.0921E-02**	1.3828E+00	4.5870E-01	1.9043E-01	**8.9726E-02**
	SD	1.3765E-02	1.2864E-02	5.8420E-03	**8.8693E-07**	2.1017E-01	1.6020E-01	**1.4655E-01**	2.8361E-01
	WRT	+	+	+		+	+	+	
W^{+}/W^{-}		20/0	20/0	16/4		20/0	18/2	14/6	
+/−/=		10/0/0	10/0/0	8/2/0		10/0/0	9/1/0	7/3/0	

ability of MOMGA in producing better solutions with closer distance to the Pareto front is in perspective.

D. DESCRIBING THE ENGINEERING PROBLEMS

For treating engineering design problems as a multi-objective problem, it is required to define the objective and design conditions. Traditionally, the cost is known as the main objective and other conditions are considered as constraints; however, as a good alternative it is possible to consider one or some of conditions as the other objectives. It is important to note that the new objective must be independent to the first objective and somehow in the opposite direction of the first objective. This means that by increasing the amount of the variables, if the cost function increases, the other one decreases. In this section, the capability of the proposed multi-objective approach (MOMGA) is evaluated in dealing with real-world engineering design problems, including the four-bar truss design, welded beam design, disk brake design, and speed reducer design problems.

1) THE FOUR-BAR TRUSS DESIGN PROBLEM

The 4-bar truss design is the first engineering problem [56] as shown in Fig. 7. In this example, two objectives (structural volume (f_1) and displacement (f_2)) are considered to be minimized. This problem has four design variables ($x_1 - x_4$) according to the cross-sectional area of members 1, 2, 3, and 4. The equations of this example are written below:

$$Minimize : f_1(x) = 200 \times (2 \times x(1) + sqrt(2 \times x(2)) + sqrt(x(3)) + x(4)) \quad (12)$$

TABLE 5. The statistical results of mathematical problems for MS performance metrics.

Functions		MS				S			
		MOPSO	*MOALO*	*MOMVO*	*MOMGA*	*MOPSO*	*MOALO*	*MOMVO*	*MOMGA*
UF1	**Ave**	9.7019E-01	9.2888E-01	7.7791E-01	**1.721E+00**	1.5649E-02	**3.8318E-03**	3.9366E-03	3.3526E-02
	SD	3.8257E-01	2.3480E-01	**1.7104E-01**	3.2831E-01	9.9871E-03	7.2845E-03	1.6573E-03	**1.1470E-03**
	WRT	+	+	+		+	-	+	
UF2	**Ave**	1.2037E+00	1.1170E+00	9.7535E-01	**1.2638E+00**	1.8565E-02	1.8319E-02	**8.2146E-03**	1.5827E-02
	SD	1.5132E-01	1.6472E-01	1.1893E-01	**4.9696E-02**	3.1440E-03	1.4069E-02	3.5129E-03	**1.1392E-03**
	WRT	+	+	+		+	+	-	
UF3	**Ave**	1.304E+00	8.3938E-01	**1.947E+00**	1.322E+00	2.6957E-02	1.1187E-02	5.6488E-02	**1.0978E-02**
	SD	7.1625E-01	2.4240E-01	8.1063E-01	**1.4888E-01**	1.5713E-02	5.2701E-03	8.4649E-02	**4.4747E-03**
	WRT	+	+	-		+	+	+	
UF4	**Ave**	1.030E+00	8.3622E-01	1.031E+00	**1.038E+00**	1.0182E-02	**3.4636E-03**	8.2315E-03	1.2478E-02
	SD	2.9802E-02	8.1759E-02	1.3932E-02	**1.0666E-02**	1.7583E-03	3.0832E-03	4.7707E-03	**1.6489E-03**
	WRT	+	+	+		+	-	+	
UF5	**Ave**	6.3155E-01	1.725E+00	8.2445E-01	**3.267E+00**	2.5756E-02	1.4277E-02	**6.0884E-03**	1.6935E-01
	SD	7.2675E-01	**4.8531E-01**	1.111E+00	6.5671E-01	3.3090E-02	9.0182E-03	**6.0779E-03**	6.8933E-02
	WRT	+	+	+		+	+	-	
UF6	**Ave**	8.7827E-01	9.8110E-01	4.5000E-01	**2.949E+00**	4.3768E-02	**4.5527E-03**	5.5990E-03	1.2908E-01
	SD	1.209E+00	**3.9658E-01**	4.3436E-01	7.6903E-01	7.4660E-02	**3.6726E-03**	4.1791E-03	3.9041E-02
	WRT	+	+	+		+	-	+	
UF7	**Ave**	1.089E+00	8.8819E-01	6.3955E-01	**1.536E+00**	1.6883E-02	6.1348E-03	**6.1301E-03**	4.1017E-02
	SD	2.5804E-01	3.0321E-01	3.9756E-01	**1.4240E-01**	8.2967E-03	**5.6661E-03**	8.7410E-03	1.9297E-02
	WRT	+	+	+		+	+	-	
UF8	**Ave**	**5.504E+00**	5.7743E-01	1.478E+00	8.6501E-01	2.7757E-01	2.3043E-02	2.4682E-01	**2.1142E-02**
	SD	1.466E+00	4.9527E-01	9.9244E-01	**2.6342E-02**	7.9069E-02	1.6247E-02	4.9334E-02	**6.4219E-03**
	WRT	-	+	+		+	+	+	
UF9	**Ave**	7.618E+00	4.122E-01	1.714E+00	**7.766E+00**	3.7998E-01	1.7566E-02	3.3556E-01	**1.5600E-02**
	SD	1.609E+00	**2.6948E-01**	6.9704E-01	1.232E+00	8.9375E-02	1.1348E-02	3.2153E-02	**1.0047E-02**
	WRT	+	+	+		+	+	+	
UF10	**Ave**	**6.577E+00**	3.150E+00	2.478E+00	3.1794E-01	1.100E+00	2.2214E-01	1.1918E-01	**1.0096E-02**
	SD	2.360E+00	1.107E+00	1.721E+00	**1.005E+00**	1.9502E-01	9.2452E-02	6.6767E-02	**3.1839E-02**
	WRT	-	+	+		+	+	+	
W^+/W^-		18/2	20/0	18/2		20/0	14/6	14/6	
$+/-/=$		9/1/0	10/0/0	9/1/0		10/0/0	7/3/0	7/3/0	

$$Minimize : f_2(x) = 0.01 \times \frac{2}{x(1)} + \frac{2 \times sqrt(2)}{x(2)} - \frac{2 \times sqrt(2)}{x(3)} + \frac{2}{x(1)} \tag{13}$$

$$1 \le x_1 \le 3, \quad 1.4142 \le x_2 \le 3$$
$$1.4142 \le x_3 \le 3, \quad 1 \le x_4 \le 3$$

2) THE WELDED BEAM DESIGN PROBLEM

The welded beam design problem is the second example of this study for engineering problems. This example was tested by Ray and Liew [57] with four constraints and two objectives as the fabrication cost (f_1) and beam deflection (f_2) of a welded beam. Fig. 8 shows the details of this example. This problem has four design variables: the thickness of the weld (x_1), the length of the clamped bar (x_2), the height of the bar (x_3) and the thickness of the bar (x_4).

$$Minimize : f_1(x) = 1.10471 \times x(1)^2 \times x(2) + 0.04811 \times x(3) \times x(4) \times (14 + x(2)) \tag{14}$$

$$Minimize : f_2(x) = 65856000/(30 \times 10^6 \times x(4) \times x(3)^3) \tag{15}$$

$$where : g_1(x) = \tau - 13600 \tag{16}$$

$$g_2(x) = \sigma - 30000 \tag{17}$$

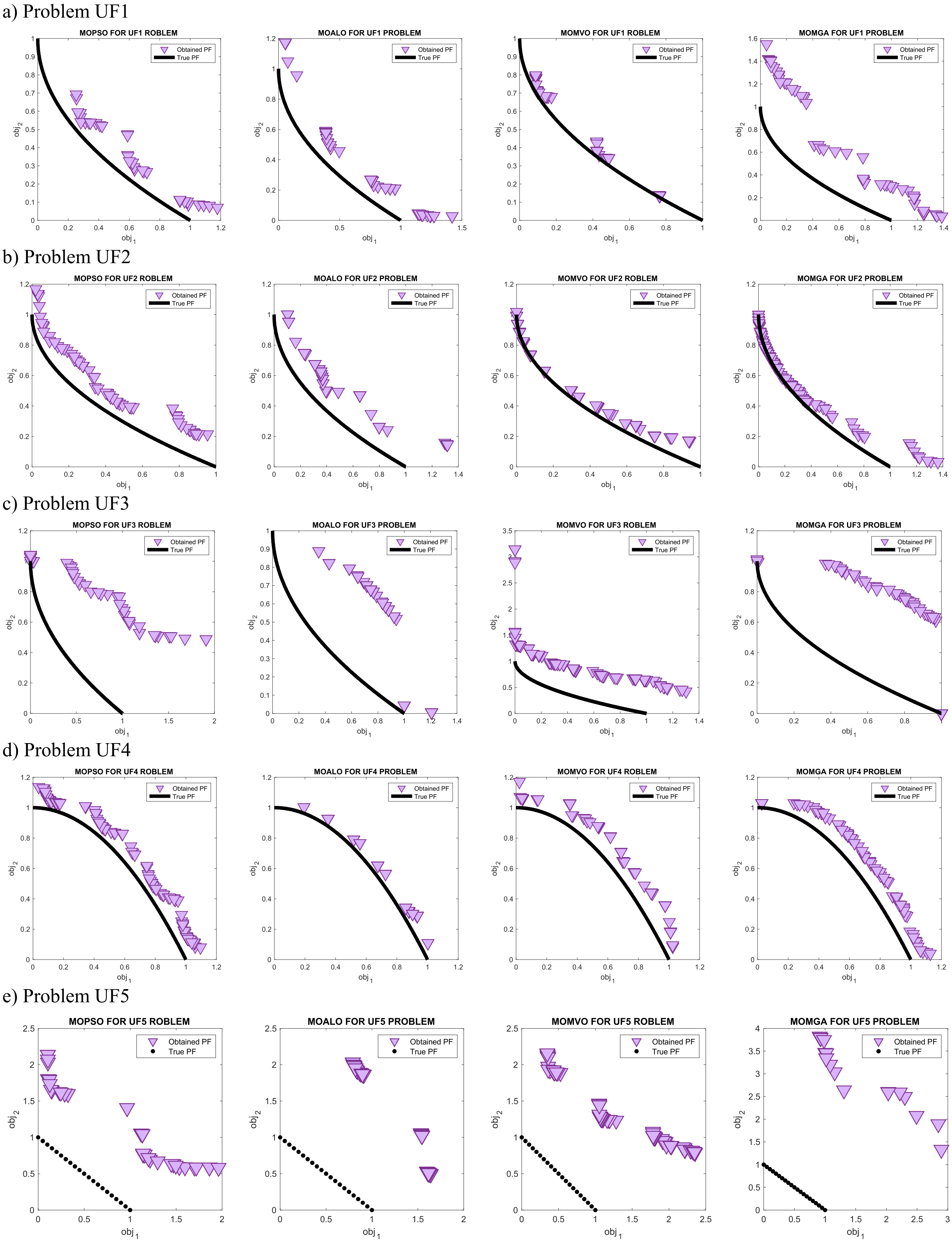
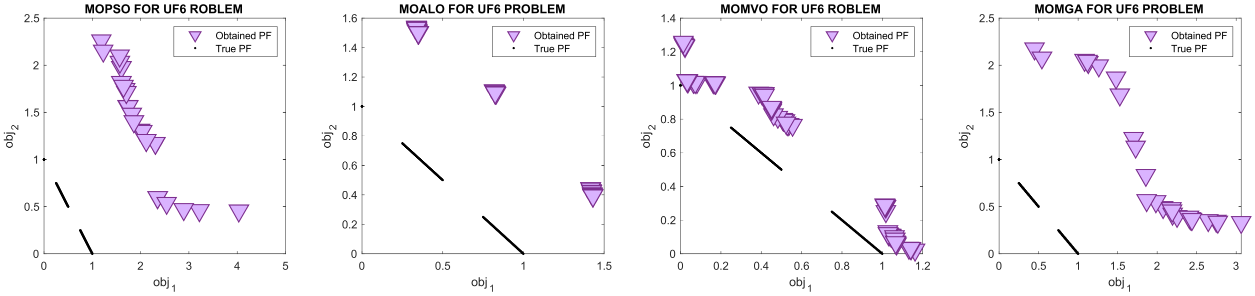
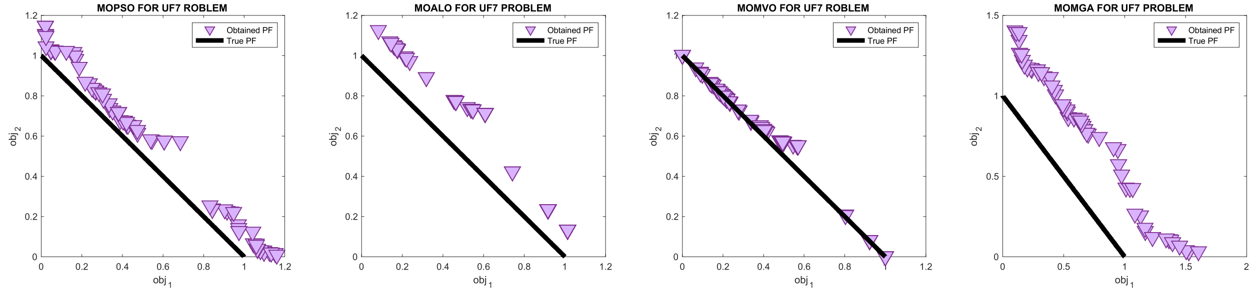


FIGURE 6. True and obtained Pareto front for mathematical problems.

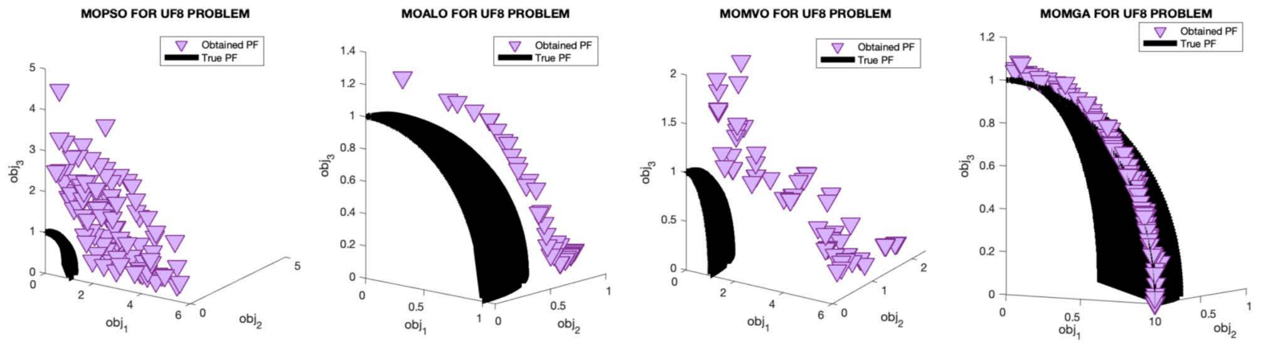
f) Problem UF6



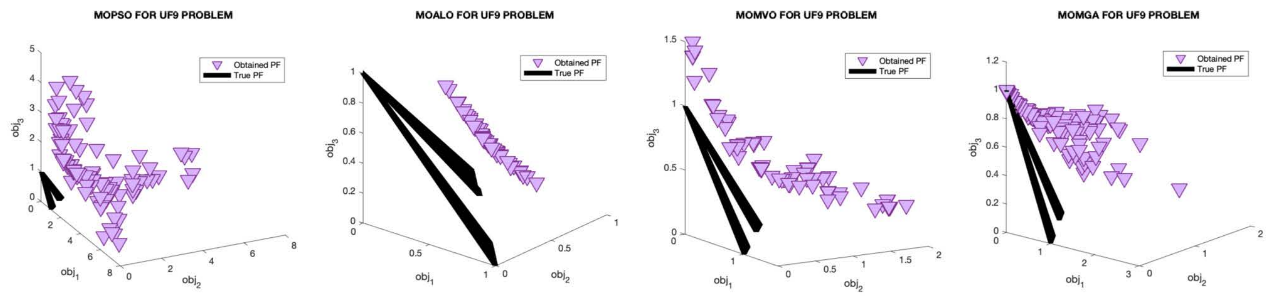
g) Problem UF7



h) Problem UF8



i) Problem UF9



j) Problem UF10

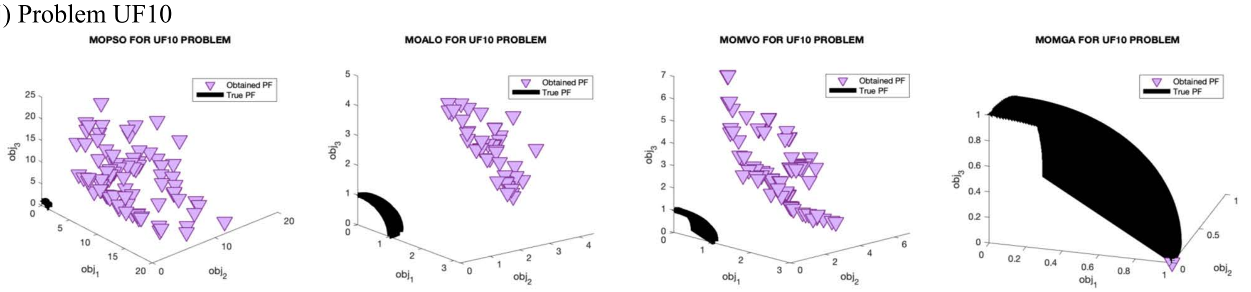


FIGURE 6. ***(Continued.)*** **True and obtained Pareto front for mathematical problems.**

TABLE 6. The statistical results of engineering problems for GD performance.

Functions		MOPSO	MOALO	MOMVO	NSGA-II	MSSA	MOGWO	MOMGA
P1: BNH	Ave	3.3894E-02	5.3758E-02	8.5875E-02	3.0657E-02	6.4575E-02	6.2465E-02	**2.9677E-02**
	SD	2.1392E-03	1.7132E-02	4.0751E-02	3.9887E-02	1.5035E-02	1.5645E-02	**1.9306E-03**
	WRT	+	+	+	+	+	+	
P2: CONSTR	Ave	**8.3098E-04**	1.5711E-03	9.2588E-04	3.9804E-03	1.6878E-03	8.9220E-04	8.9014E-04
	SD	3.3369E-05	7.7806E-04	2.4850E-04	8.7867E-02	5.8950E-04	1.0565E-04	**2.5599E-05**
	WRT	-	+	+	+	+	+	
P3: DISK BRAKE	Ave	2.3045E-03	4.1887E-02	1.5855E-01	7.7875E-01	6.2545E-01	4.3311E-01	**2.2434E-03**
	SD	5.6273E-03	5.4951E-03	2.4372E-01	9.6574E-02	3.3423E-03	**1.8074E-03**	**8.2734E-02**
	WRT	+	+	+	+	+	+	
P4: 4-BAR TRUSS	Ave	1.410E+01	2.590E+00	1.102E+01	8.769E+01	7.797E+01	1.170E+01	**1.052E+01**
	SD	**5.1580E-01**	2.216E+00	4.655E+00	1.008E+00	3.549E+00	2.334E+00	1.201E+00
	WRT	+	+	+	+	+	+	
P5: WELDED BEAM	Ave	1.1946E-02	4.4399E-03	1.5031E-02	2.8970E-02	6.3467E-02	3.9207E-02	**3.9704E-03**
	SD	1.9956E-03	**6.9956E-04**	4.7477E-03	2.5463E-03	**1.0983E-03**	3.8097E-02	2.0982E-03
	WRT	+	+	+	+	+	+	
P6: OSY	Ave	3.577E+00	**7.0760E-01**	8.4098E-01	**1.002E+00**	9.4637E-01	1.274E+00	1.608E+00
	SD	2.525E+00	2.4932E-01	**1.4591E-01**	2.5649E-01	1.9612E-01	4.0179E-01	7.5758E-01
	WRT	+	-	+	-	+	+	
P7: SPEED REDUCER	Ave	8.252E+01	7.682E+00	9.261E+00	**6.221E+00**	7.866E+00	8.289E+00	**1.406E+01**
	SD	9.870E+01	3.470E+00	**1.672E+00**	1.3427E+00	3.6189E+00	1.6129E+00	1.937E+00
	WRT	+	+	+	-	+	+	
P8: SRN	Ave	3.1617E-02	1.5798E-02	3.3373E-01	1.4537E-02	4.1566E-02	1.6743E-02	**1.1811E-02**
	SD	1.0695E-02	**3.2892E-03**	1.4098E-01	5.6750E-03	2.4971E-02	**3.9924E-03**	2.2195E-02
	WRT	+	+	+	+	+	+	
W^+/W^-		14/2	14/2	14/0	12/4	16/0	16/0	
$+/-/=$		7/1/0	7/1/0	7/0/0	6/2/0	8/0/0	8/0/0	

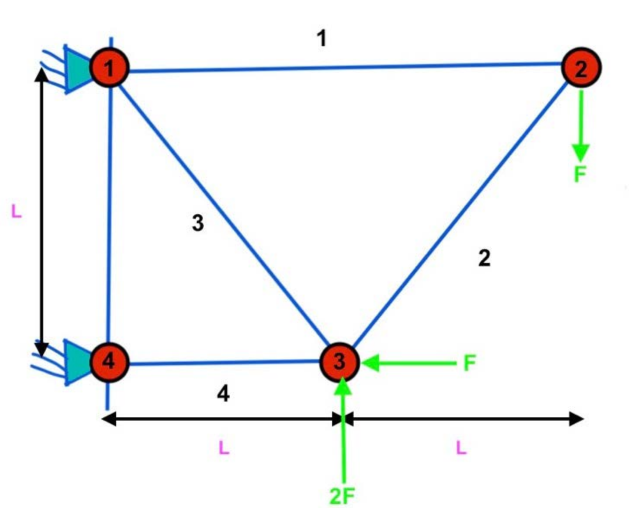


FIGURE 7. The schematic view of the four-bar truss.

$$g_3(x) = x(1) - x(4) \tag{18}$$

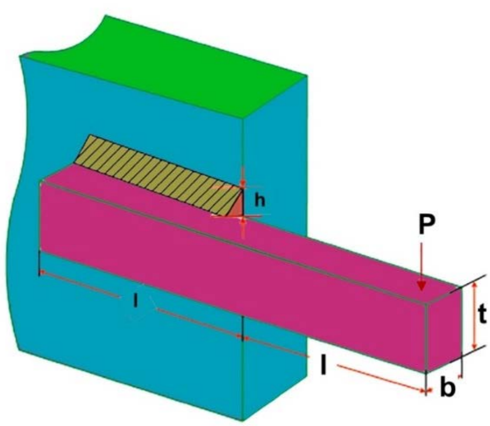


FIGURE 8. The details of the welded beam.

$$g_4(x) = 6000 - P \tag{19}$$

$$0.125 \le x_1 \le 5, 0.1 \le x_2 \le 10$$
$$0.1 \le x_3 \le 10, 0.125 \le x_4 \le 5$$

TABLE 7. The statistical results of engineering problems for IGD performance.

Functions		MOPSO	MOALO	MOMVO	NSGA-II	MSSA	MOGWO	MOMGA
P1: BNH	Ave	9.6868E-04	1.2198E-02	3.2191E-03	9.8498E-04	7.5631E-03	3.3817E-03	**9.6566E-04**
	SD	1.7147E-04	3.6455E-03	3.3695E-04	**5.0065E-05**	4.5344E-03	1.1513E-03	**1.1270E-04**
	WRT	+	+	+	+	+	+	
P2: CONSTR	Ave	**5.1838E-04**	2.5041E-03	1.8516E-03	5.6591E-03	2.1706E-03	7.4460E-04	6.4940E-04
	SD	**5.5618E-05**	9.6528E-04	3.6526E-04	2.4357E-04	7.3799E-04	1.7697E-04	6.4604E-05
	WRT	-	+	+	+	+	+	
P3: DISK BRAKE	Ave	5.8831E-04	1.7399E-03	1.3515E-03	4.5649E-03	2.5113E-03	7.2091E-04	**4.4814E-04**
	SD	**5.5995E-05**	7.7421E-04	2.1737E-04	8.6759E-04	1.1615E-03	1.1206E-04	7.0569E-05
	WRT	+	+	+	+	+	+	
P4: 4-BAR TRUSS	Ave	2.0010E-02	2.2004E-02	2.1020E-02	2.2001E-02	2.1434E-02	2.1409E-02	**2.0007E-02**
	SD	3.9632E-05	1.1024E-03	4.6879E-04	3.5498E-04	3.8292E-04	1.5445E-04	**2.4101E-05**
	WRT	+	+	+	+	+	+	
P5: WELDED BEAM	Ave	5.9705E-04	6.1770E-03	2.1075E-03	2.7680E-03	4.8242E-03	1.3441E-03	**5.4137E-04**
	SD	**4.6341E-05**	2.8159E-03	4.2017E-04	3.6578E-03	3.5852E-03	3.9377E-04	1.0025E-04
	WRT	+	+	+	+	+	+	
P6: OSY	Ave	1.4663E-02	8.1016E-03	**5.8358E-03**	8.7768E-03	7.7532E-03	7.6220E-03	6.4819E-03
	SD	8.6917E-03	2.2613E-04	7.8106E-04	5.9760E-04	1.2021E-03	6.4006E-04	**1.5356E-04**
	WRT	+	+	-	+	+	+	
P7: SPEED REDUCER	Ave	6.0305E-02	1.7737E-02	8.7559E-03	1.564E+00	1.493E+00	1.424E+00	**8.3785E-03**
	SD	7.2130E-02	3.3878E-03	2.1149E-03	2.0631E-03	5.7981E-03	3.2032E-03	**1.2641E-03**
	WRT	+	+	+	+	+	+	
P8: SRN	Ave	4.5146E-04	6.2137E-03	1.1274E-03	3.6578E-03	3.2308E-03	3.4823E-03	**3.6186E-04**
	SD	1.1656E-04	1.8810E-03	2.5526E-04	**3.6574E-05**	1.5266E-03	1.0614E-03	**1.0199E-04**
	WRT	+	+	+	+	+	+	
W^+/W^-		14/2	16/0	14/2	16/0	16/0	16/0	
$+/-/=$		7/1/0	8/0/0	7/1/0	8/0/0	8/0/0	8/0/0	

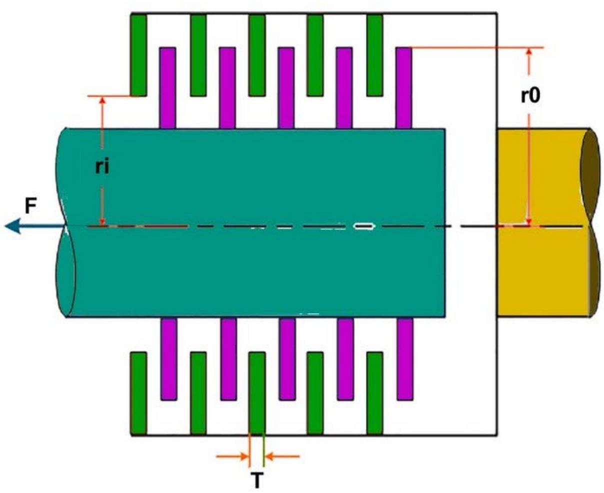


FIGURE 9. The schematic view of the disk brake.

$$where : q = 6000 * \left(14 + \frac{x\,(2)}{2}\right); \tag{20}$$

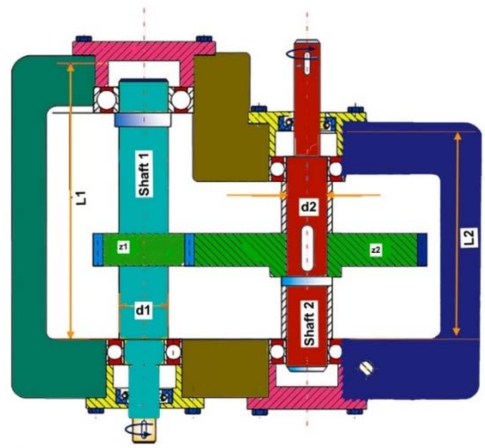


FIGURE 10. The schematic view of the Speed reducer.

$$D = sqrt\left(\frac{x(2)^2}{4} + \frac{(x(1)+x(3))^2}{4}\right)$$

$$J = 2 * (x\,(1) * x\,(2) * sqrt\,(2)$$

TABLE 8. The statistical results of engineering problems for MS performance.

Functions		*MOPSO*	*MOALO*	*MOMVO*	*NSGA-II*	*MSSA*	*MOGWO*	*MOMGA*
P1: BNH	Ave	**1.000E+00**	5.4090E-01	9.6908E-01	**1.262E+00**	7.6222E-01	8.7156E-01	1.000E+00
	SD	**0.000E+00**	1.0692E-01	2.7128E-02	2.1449E-02	1.3378E-01	6.6957E-02	**0.000E+00**
	WRT	=	+	+	-	+	+	
P2: CONSTR	Ave	9.9384E-01	7.7822E-01	9.7543E-01	8.8769E-01	9.0536E-01	9.7790E-01	**9.9474E-01**
	SD	**6.8603E-03**	7.6343E-02	1.7403E-02	5.8261E-02	4.8380E-02	2.0113E-02	**1.6125E-02**
	WRT	+	+	+	+	+	+	
P3: DISK BRAKE	Ave	9.9928E-01	9.1779E-01	1.323E+00	**1.048E+00**	7.9510E-01	9.9582E-01	1.000E+00
	SD	1.1417E-03	2.0904E-01	4.6277E-01	2.769E+00	1.3013E-01	1.2807E-02	**1.0642E-03**
	WRT	+	+	-	-	+	+	
P4: 4-BAR TRUSS	Ave	1.488E+00	8.4793E-01	1.401E+00	1.454E+00	1.202E+00	1.379E+00	**1.544E+00**
	SD	**5.4502E-04**	1.2454E-01	9.2764E-02	**3.9374E-02**	1.8532E-01	4.6120E-02	8.7530E-02
	WRT	+	+	+	+	+	+	
P5: WELDED BEAM	Ave	1.007E+00	6.2463E-01	1.055E+00	**1.836E+01**	7.9085E-01	1.111E+00	1.071E+00
	SD	**6.1053E-02**	6.9354E-02	9.0721E-02	1.128E+01	1.4645E-01	1.0194E-01	1.2073E-01
	WRT	+	+	+	-	+	+	
P6: OSY	Ave	3.2390E-01	5.7530E-01	7.1746E-01	6.0422E-01	6.2048E-01	6.6490E-01	**8.1700E-01**
	SD	3.4284E-01	2.6351E-02	3.8964E-02	5.7308E-02	8.0930E-02	**2.5118E-02**	**2.5281E-02**
	WRT	+	+	+	+	+	+	
P7: SPEED REDUCER	Ave	2.3456E-01	6.6868E-01	8.0454E-01	6.1007E-01	7.2004E-01	7.6707E-01	**8.8453E-01**
	SD	2.9012E-02	6.0939E-02	3.9588E-02	1.2080E-01	6.9034E-02	2.4182E-02	**1.6080E-02**
	WRT	+	+	+	+	+	+	
P8: SRN	Ave	9.0900E-01	3.9177E-01	9.2751E-01	9.8524E-01	7.0583E-01	7.0014E-01	**9.7667E-01**
	SD	4.5463E-02	8.2165E-02	4.4769E-02	8.8439E-02	1.5109E-01	8.0177E-02	**1.9187E-02**
	WRT	+	+	+	+	+	+	
W^+/W^-		14/0	16/0	14/2	10/6	16/0	16/0	
$+/-/=$		7/1/1	8/0/0	7/1/0	5/3/0	8/0/0	8/0/0	

$$* \left(\frac{x(2)^2}{12} + \frac{(x(1)+x(3))^2}{4}\right)\Bigg) \tag{21}$$

$$\alpha = \frac{6000}{sqrt(2) * x(1) * x(2)} \tag{22}$$

$$\beta = Q * \frac{D}{J} \tag{23}$$

3) DISK BRAKE DESIGN PROBLEM

Ray and Liew [57] proposed the disc brake design issue, which has five constraints. Stopping time (f_1) and brake mass (f_2) for a disc brake are the two objectives to be minimized. The details of the disk brake are shown in Fig. 9. This problem has five design variables: the inner radius of the disc (x_1), the outer radius of the disc (x_2), the engaging force (x_3), and the number of friction surfaces (x_4). The equations of this example are written below:

$$\begin{aligned} Minimize: f_1(x) &= 4.9 \times (10)^{(-5)} \times (x(2)^{(2)} \\ &\quad - x(1)^{(2)}) \times (x(4) - 1) \end{aligned} \tag{24}$$

$$\begin{aligned} Minimize: f_2(x) &= (9.82 \times (10)^{(6)})) \times (x(2))^{(2)} \\ &\quad - x(1)^{(2)}))/((x(2))^{(3)} \\ &\quad - x(1)^{(3)}) \times x(4) \times x(3)) \end{aligned} \tag{25}$$

$$g_1(x) = 20 + x(1) - x(2) \tag{26}$$

$$g_2(x) = 2.5 + (x(4) + 1) - 30 \tag{27}$$

$$\begin{aligned} g_3(x) &= (x(3))/(3.14 \\ &\quad \times \left(x(2)^2 - x(1)^2\right)^2) - 0.4 \end{aligned} \tag{28}$$

$$\begin{aligned} g_4(x) &= (2.22 \times (10)^{(-3)} \times x(3) \\ &\quad \times \left(x(2)^3 - x(1)^3\right))/(\left(x(2)^2 - x(1)^2\right) \\ &\quad ^2) - 1 \end{aligned} \tag{29}$$

$$\begin{aligned} g_5(x) &= 900 - (2.66 \times (10)^{(-2)} \times x(3) \\ &\quad \times x(4) \\ &\quad \times \left(x(2)^3 - x(1)^3\right)) \\ &\quad /(\left(x(2)^2 - x(1)^2\right)^2) \end{aligned} \tag{30}$$

$$55 \le x_1 \le 80,\ 75 \le x_2 \le 110$$
$$1000 \le x_3 \le 3000,\ 2 \le x_4 \le 20$$

TABLE 9. The statistical results of engineering problems for S performance.

Functions		*MOPSO*	*MOALO*	*MOMVO*	*NSGA-II*	*MSSA*	*MOGWO*	*MOMGA*
P1: BNH	Ave	1.090E+00	**8.8402E-01**	1.049E+00	**1.0043+00**	1.255E+00	1.748E+00	1.069E+00
	SD	**1.3631E-01**	4.4648E-01	7.7663E-01	**4.7685E-02**	3.9410E-01	4.7352E-01	1.9835E-01
	WRT	+	-	+	-	+	+	
P2: CONSTR	Ave	5.8740E-02	6.9071E-02	5.0134E-02	4.9809E-02	5.5908E-02	5.4894E-02	**4.4786E-02**
	SD	7.8936E-03	1.8623E-02	2.7727E-02	**4.3448E-03**	1.4886E-02	8.3095E-03	**6.0912E-03**
	WRT	+	+	+	+	+	+	
P3: DISK BRAKE	Ave	1.1452E-01	1.4457E-01	2.7320E-01	1.880E+00	1.2768E-01	1.3331E-01	1.1434E-01
	SD	1.3022E-02	1.1754E-02	3.9542E-01	2.657E+00	7.0776E-02	1.9321E-02	**1.0836E-02**
	WRT	+	+	+	+	+	+	
P4: 4-BAR TRUSS	Ave	5.361E+00	**4.699E+00**	4.825E+00	4.300E+01	6.116E+00	**5.905E+00**	7.649E+00
	SD	**2.6169E-01**	1.172E+00	3.358E+00	2.667E+02	1.661E+00	1.746E+00	8.5322E-01
	WRT	+	-	+	+	+	-	
P5: WELDED BEAM	Ave	2.3432E-01	2.2431E-01	2.0967E-01	7.760E+01	**1.8912E-01**	4.4716E-01	**2.0909E-01**
	SD	2.5702E-02	1.3595E-01	1.1261E-01	6.448E+01	8.7588E-02	2.1585E-01	**2.4605E-02**
	WRT	+	+	+	+	-	+	
P6: OSY	Ave	1.128E+00	1.438E+00	1.799E+00	**8.5570E-01**	1.350E+00	1.628E+00	**1.114E+00**
	SD	1.468E+00	4.8498E-01	**3.4639E-01**	**1.4536E-01**	6.2967E-01	4.7312E-01	4.822E+00
	WRT	+	+	+	-	+	+	
P7: SPEED REDUCER	Ave	3.453E+01	3.672E+01	**2.250E+01**	1.413E+01	**1.395E+01**	2.062E+01	4.997E+01
	SD	4.438E+00	9.362E+00	4.723E+00	6.547E+00	9.820E+00	**3.186E+00**	**1.052E+01**
	WRT	+	+	-	+	-	+	
P8: SRN	Ave	2.240E+00	1.739E+00	2.742E+00	3.652E+00	2.272E+00	2.649E+00	**1.684E+00**
	SD	4.7324E-01	8.6872E-01	1.041E+00	**1.0044E-01**	9.3573E-01	1.256E+00	**3.5416E-01**
	WRT	+	+	+	+	+	+	
W^{+}/W^{-}		16/0	12/4	16/0	12/4	12/4	14/2	
$+/-/=$		8/0/0	6/2/0	8/0/0	6/2/0	6/2/0	7/1/0	

TABLE 10. The statistical results of engineering problems for the HV performance metric.

Functions		MOPSO	MOALO	MOMVO	NSGA-II	MSSA	MOGWO	MOMGA
P1: BNH	Ave	8.0086E-01	8.8012E-01	9.6758.E-01	7.9858E-01	8.5462E-01	8.5498E-01	**7.8796E-01**
	SD	2.3409E-03	1.2311E-02	3.4536E-02	2.7685E-03	4.5467E-03	1.4657E-03	**1.0098E-03**
P2:CONSTR	Ave	**3.0003E-01**	6.6579E-01	3.9087E-01	3.2348E-01	2.9870E-01	8.5478E-01	3.1134E-01
	SD	8.7675E-03	2.3324E-02	3.4538E-01	**1.0787E-03**	1.1123E-01	3.5469E-03	1.2319E-03
P3:DISK BRAKE	Ave	5.3324E-01	5.5489E-01	5.4189E-01	5.7864E-01	5.2156E-01	5.3345E-01	**5.1088E-01**
	SD	3.0984E-03	2.0047E-02	3.6574E-01	3.5656E-03	6.5876E-03	8.5543E-03	**2.7868E-03**
P4:4-BAR TRUSS	Ave	8.6246E-01	8.6583E-01	8.7437E-01	8.6432E-01	8.6832E-01	8.6758E-01	**8.5417E-01**
	SD	**2.0505E-03**	7.8710E-03	2.1653E-02	3.0035E-03	5.0645E-03	4.0779E-03	3.8780E-03
P5:WELDED BEAM	Ave	8.6564E-01	8.6547E-01	9.6542E-01	8.4906E-01	8.5467E-01	8.8796E-01	**8.4356E-01**
	SD	1.0802E-03	1.8070E-02	3.9853E-02	1.0099E-03	4.2069E-03	1.0401E-03	**9.8761E-04**
P6:OSY	Ave	4.2341E-01	5.5463E-01	**4.0547E-01**	4.1091E-01	5.0948E-01	4.6580E-01	4.2311E-01
	SD	**9.9593E-04**	4.3427E-02	3.4364E-02	1.1876E-02	1.3246E-02	1.0965E-02	1.0983E-03
P7:SPEED REDUCER	Ave	1.2765E+01	1.2743E+01	1.7437E+01	1.3477E+01	1.4478E+01	1.2035E+01	**1.1893E+01**
	SD	6.4487E-03	2.1651E-02	3.1003E-02	6.0325E-03	6.1756E-02	1.0074E-02	**4.0678E-03**
P8:SRN	Ave	1.8005E+01	1.7874E+01	1.9637E+01	1.6711E+01	1.7401E+01	1.8765E+01	**1.6636E+01**
	SD	1.6107E-02	4.1763E-01	4.1983E-01	2.7699E-02	2.4359E-02	**5.7891E-03**	1.8777E-02

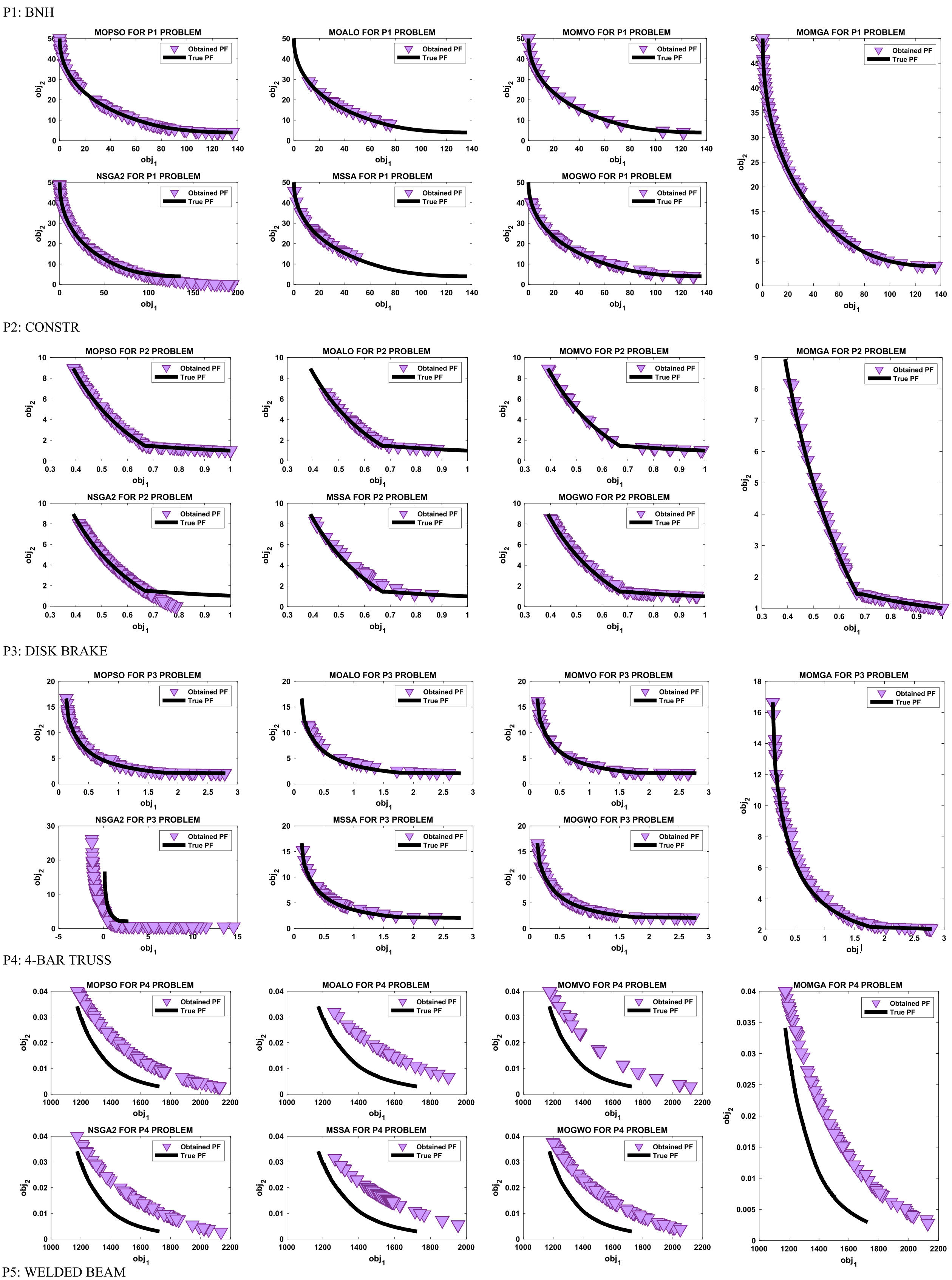
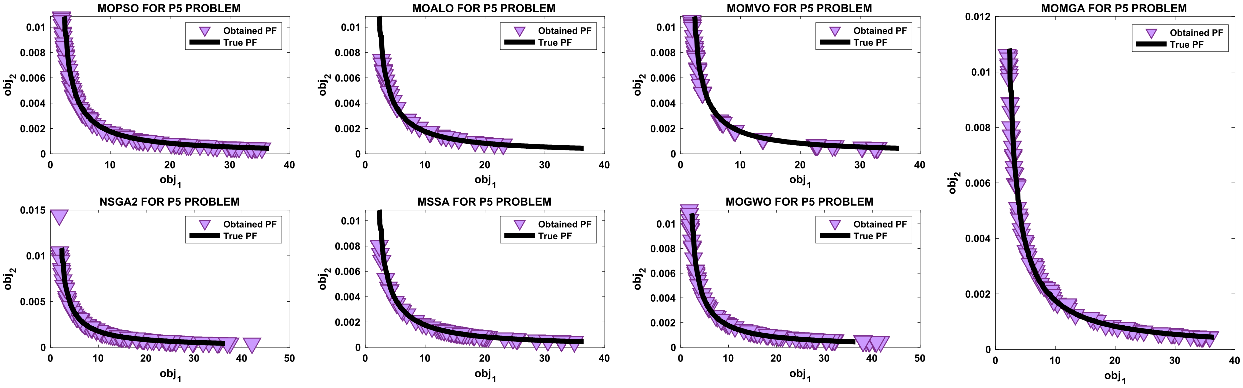
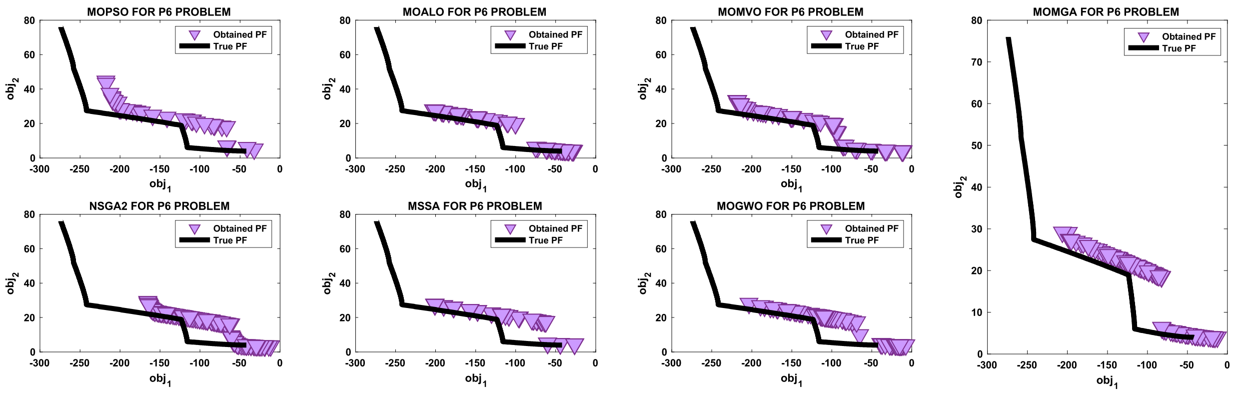


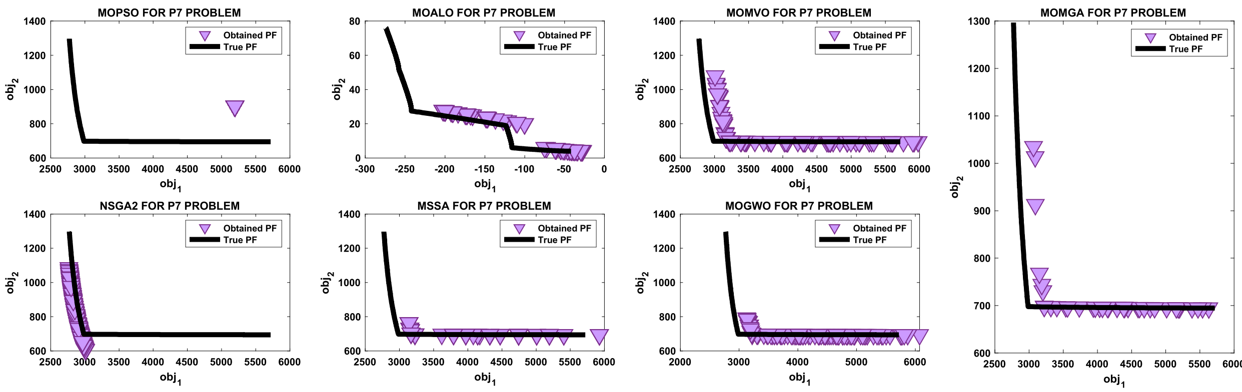
FIGURE 11. True and obtained Pareto front for engineering design problems.



P6:OSY



P7: SPEED REDUCER



P8: SRN

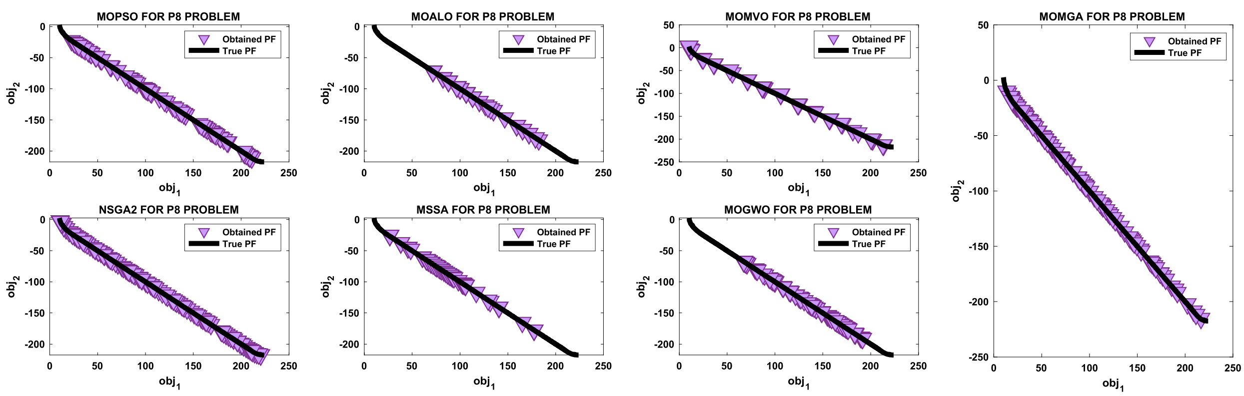


FIGURE 11. ***(Continued.)*** **True and obtained Pareto front for engineering design problems.**

4) SPEED REDUCER DESIGN PROBLEM

The speed reducer design is the last engineering problem in the field of mechanical engineering [56], [58] (see Fig. 10). In this example, two objectives (weight (f_1) and stress (f_2)) are considered to be minimized. There are seven design variables: gear face width (x_1), teeth module (x_2), a number of teeth of a pinion (x_3 integer variable) the distance between bearings 1 (x_4), the distance between bearings 2 (x_5), the diameter of shaft 1 (x_6), and diameter of shaft 2 (x_7) as well as eleven constraints. The equations of this example are written below:

$$\begin{aligned} Minimize: f_1(x) = {} & 0.7854 \times x(1) \times x(2)^2 \\ & \times (3.3333 \times x\left(3)^2 + 14.9334 \times x(3)\right) \\ & \ldots - 43.0934) - 1.508 \\ & \times .x(1) \times (x(6)^2 + x(7)^2 \end{aligned} \tag{31}$$

$$\begin{aligned} \text{Minimize}: f_2(x) = {} & ((sqrt(((745 * x(4))/x(2) * x(3)))^2 \\ & +19.9e6)/(0.1 * x(6)^3)) \end{aligned} \tag{32}$$

$$\begin{aligned} where: g_1(x) = {} & 27/(x(1) \\ & \times x\left(2)^2 \times x(3)\right) - 1 \end{aligned} \tag{33}$$

$$\begin{aligned} g_2(x) = {} & 397.5/(x(1) \\ & \times x(2)^2 \times x\left(3)^2\right) - 1 \end{aligned} \tag{34}$$

$$\begin{aligned} g_3(x) = {} & (1.93 \times (x\left(4)^3\right)/(x(2) \\ & \times x(3) \times x\left(6)^4\right) - 1 \end{aligned} \tag{35}$$

$$\begin{aligned} g_4(x) = {} & (1.93 \times (x\left(5)^3\right)/(x(2) \times x(3) \\ & \times x\left(7)^4\right) - 1 \end{aligned} \tag{36}$$

$$\begin{aligned} g_5(x) = {} & ((sqrt((745 \times x(4))/x(2) \\ & \times x\left(3)))^2 + 16.9e6\right))/(110 \\ & \times x\left(6)^3\right)) - 1 \end{aligned} \tag{37}$$

$$\begin{aligned} g_6(x) = {} & ((sqrt((745 \times x(4))/x(2) \\ & \times x\left(3)))^2 + 157.5e6\right))/ \\ & (85 \times x\left(7)^3\right)) - 1 \end{aligned} \tag{38}$$

$$g_7(x) = ((x(2) \times x(3))/40)1 \tag{39}$$

$$\tau = sqrt\left(\alpha^2 + 2 \times \alpha \times \beta \times \frac{x(2)}{2 \times D} + \beta^2\right) \tag{40}$$

$$\sigma = \frac{504000}{x(4) \times x(3)^2} \tag{41}$$

$$tmpf = 4.013 \times \frac{30 \times 10^6}{196} \tag{42}$$

$$\begin{aligned} P = {} & tmpf \times sqrt\left(x(3)^2 \times \frac{x(4)^6}{36}\right) \\ & \times \left(1 - x(3) \times \frac{sqrt(\frac{30}{48})}{28}\right) \end{aligned} \tag{43}$$

5) RESULTS OF THE ENGINEERING PROBLEMS

The potential of the suggested multi-objective method, MOMGA, to deal with engineering problems is evaluated in this section compared to MOPSO, MOALO, MOMVO, NSGA-II, MSSA and MOGWO. In Table 6, the results of the GD index for MOMGA and the other alternatives are presented in which the proposed methods outrank the others in most cases. At least in 7 out of 8 examples, the MOMGA perform superior compared to other methods. In addition, the results of the IGD index, which is presented in Table 7 are very competitive for the MOMGA. Also, the results show that MOMGA outperforms for at least 7 out of 8 problems. It is interesting to point out that none of algorithms can win MOMGA more than once for engineering problems considering Wilcoxon rank sum test.

Regarding the MS and S indices, results are presented in Tables 8 and 9, respectively. MOMGA can provide outstanding results in most cases. The other approach that has close competition with MOMGA is NSGA-II which can overcome MOMGA for 3 examples while for the rest 5 ones, MOMGA is the winner. It is obvious, based on the results of the Wilcoxon rank sum test presented in Tables 7 and 8, that the MOMGA is much more effective than the other approaches.

Table 10 presents the statistical results of engineering problems for the HV performance index. According to the table, the proposed algorithm can find better results for 10 (out of total of 16 sets) for engineering problems considering HV index.

The True and obtained Pareto front for the considered engineering design problems obtained by different multi-objective algorithms is presented in Fig. 11 in which the MOMGA represents more exact and closer results to the parent front.

The superior results of the new method is because of some reasons: firstly, the good balance between exploration and exploitation plays a significant role in the good performance of algorithms. Since here, the chemical compounds and chemical reactions are considered as two main bases of the algorithm, this matter is controlled carefully. Secondly, a good distribution of pareto front is important for solving multi-objective problems and here, we utilized the grid mechanism in MOMGA. In addition, this method applies the leader selection mechanism in which directs the other search candidates to possible areas of the search space to reach a global solution. Finally, archive mechanism is utilized to save the best results as pareto front.

IV. CONCLUSION AND FUTURE WORKS

In this paper, the multi-objective version of the Material Generation Algorithm (MGA) is proposed as MOMGA, which is one of the recently developed metaheuristic algorithms for single-objective optimization. For evaluation of the overall performance of the MOMGA, the benchmark multi-objective optimization problems of the Competitions on Evolutionary Computation (CEC) are considered alongside the real-world engineering problems. Based on the results, the MOMGA can provide very acceptable results in dealing with multi-objective optimization problems. MOMGA is capable of outranking the other multi-objective methods considering different indices in the considered CEC-09 problems regarding the fact that other algorithms also provide very competitive results. Regarding engineering design problems, the MOMGA can provide very acceptable results in dealing with these complex problems. By considering the true and obtained Pareto fronts, it is concluded that the proposed MOMGA can create better solutions with a closer distance from the Pareto front. Despite this encouraging performance, there are some disadvantages for the MOMGA. This method needs a suitable constraint handling method that for complex problems, choosing a proper method that works well alongside the MOMGA can be a challenging task. Also, for very high dimensional problems, it seems that MOMGA should be equipped with some special tools. Finally, in some cases, the MOMGA cannot find the best result and it means that this method can be improved.

Future works are advised to use MOMGA for various other engineering design problems such as truss structures and developing the structural health assessment. Also, since the definition and application of multi-objective problems are completely different from single-objective ones specially for constrained large-scale examples, the newly proposed method can be applied to solving such problems. Developing a suitable constraint handling method alongside the MOMGA opens an interesting research area. Besides, the MOMGA can be modified and adapted for problems with special properties containing special strategies to be solved.

Code aviabliy: The matlab codes of this paper is avaible in mathwork, as:

https://www.mathworks.com/matlabcentral/fileexchange/118335-multi-objective-material-generation-algorithm-momga

APPENDIX A: CONSTRAINED MULTI-OBJECTIVE TEST PROBLEMS

CONSTR

There are two constraints and two design variables in this problem, which have a convex Pareto front.

$$Minimize : f_1\ (x) = x_1 \quad (A.1)$$
$$Minimize : f_2\ (x) = (1 + x_2)/x_1 \quad (A.2)$$
$$where : g_1\ (x) = 6 - (x_2 + 9x_1) \quad (A.3)$$
$$g_2\ (x) = 1 + (x_2 - 9x_1) \quad (A.4)$$

$$0.1\ lex_1 \leq 1, 0 \leq x_2 \leq 5$$

SRN:

Srinivas and Deb [36] suggested a continuous Pareto optimal front for the next problem as follows:

$$Minimize : f_1(x) = +(x_1 2)^2 + (x_2 1)^2 \quad (A.5)$$
$$Minimize : f_2(x) = 9x_1(x_2 1)^2 \quad (A.6)$$
$$where : g_1(x) = x_1^2 + x_2^2 - 255 \quad (A.7)$$
$$g_2\ (x) = x_1 - 3x_2 + 10$$
$$-20 \leq x_1 \leq 20, -20 \leq x_2 \leq 20 \quad (A.8)$$

BNH

Binh and Korn [59] were the first to propose this problem as follows:

$$Minimize : f_1(x) = 4x_1^2 + 4x_2^2 \quad (A.9)$$
$$Minimize : f_2(x) = (x_1 - 5)^2 + (x_2 - 5)^2 \quad (A.10)$$
$$where : g_1(x) = (x_1 - 5)^2 + x_2^2 - 25 \quad (A.11)$$
$$g_2(x) = 7.7 - (x_1 - 8)^2 - (x_2 + 3)^2 \quad (A.12)$$

$$0 \leq x_1 \leq 5, 0 \leq x_2 \leq 3$$

OSY

Osyczka and Kundu [60] proposed five distinct regions for the OSY test issue. There are also six constraints and six design variables to consider as below:

$$Minimize : f_1(x) = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 \quad (A.13)$$
$$\text{Minimize } : f_2(x) = [25(x_1 - 2)^2 + (x_2 - 1)^2 + (x_3 - 1) + (x_4 - 4)^2 + (x_5 - 1)^2] \quad (A.14)$$
$$\text{Where } : g_1\ (x) = 2 - x_1 - x_2) \quad (A.15)$$
$$g_2(x) = -6 + x_1 + x_2 \quad (A.16)$$
$$g_3(x) = -2 - x_1 + x_2 \quad (A.17)$$
$$g_4(x) = -2 + x_1 - 3x_2 \quad (A.18)$$
$$g_5(x) = -4 + x_4 + (x_3 - 3)^2 \quad (A.19)$$
$$g_6(x) = 4 - x_6 - (x_5 - 3)^2 \quad (A.20)$$
$$0 \leq x_1 \leq 10, 0 \leq x_2 \leq 10, 1 \leq x_3 \leq 5 \quad (A.21)$$

$$0 \leq x_4 \leq 6, 1 \leq x_5 \leq 5, 0 \leq x_6 \leq 10$$

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