Optimising the Length of Doped Polymer Light Mixers

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Abstract

Transparent Refractive Index Matched Micro-particles (TRIMM) in polymer rods are highly efficient light mixers. This paper addresses the problem of estimating the optimum length for the mixing rods, for a given TRIMM-to-matrix refractive index ratio and concentration. Light mixing can be thus maximised and loss minimised, without computer ray tracing simulations. The probability density function and mean angle for a single TRIMM sphere ray deviation are derived, and used in modelling an expression for a critical mixer length for rays of normal incidence. Similar models could be very useful design tools with further development.

Introduction

High performance light mixing using Transparent Refractive Index Matched Micro-particles (TRIMM) doped polymer rods has recently been demonstrated [1,2]. Monte Carlo ray tracing modelling within a mixing rod has also been described [2,3].

Consider a polymer light guide doped with TRIMM at a linear particle density of α particles per metre. A ray passing through a micro-sphere is deviated by a small angle, δ. The sphere to matrix refractive index ratio may be usefully expressed as $(1 + \mu)$, where typically $\mu \approx 0.01$. The Fresnel reflection coefficient from a TRIMM spheres is approximately $(\mu^2/4) << 0.01\%$ [4]. Thus TRIMM homogenise input light with negligible backscatter.

The distribution of light exiting a mixing rod will be a convolution of the source emission function with the point-spread function of the propagating light. As a preliminary estimate of the total angular spread of rays reaching the end of a TRIMM mixer, we have studied the propagation of rays entering a rod at normal incidence. Guidelines for the optimum TRIMM mixer length for a given μ and α to obtain spread of light across the mixer end with minimum side loss are given.

Mean ray deviation by a single TRIMM sphere

As reviewed in fig (1a), in the geometric optic limit, a ray striking a TRIMM sphere at fraction h of a radius will undergo an angular deviation

$$\delta(h) = 2\left[\sin^{-1}(h) - \sin^{-1}\left(\frac{h}{1+\mu}\right)\right] \quad (1)$$

The corresponding probability density function, $f(\delta)$, is derived below and is graphed in fig (1b). Note the high degree of asymmetry and the pronounced tail.

In order to calculate the mean deviation, $\bar{\delta}$, we note that the probability of an incident ray having a fractional radial contact value in the interval h to $h+\partial h$ is $2\pi h\partial h$ (fig (1a)). Thus the mean deviation of a ray striking a single sphere is

$$\bar{\delta} = \frac{\int_0^1 \delta(h) 2\pi h \partial h}{\int_0^1 2\pi h \partial h} = 4\int_0^1 h\left[\sin^{-1}(h) - \sin^{-1}\left(\frac{h}{1+\mu}\right)\right] \tag{2}$$

Strictly speaking Mie theory should be used to calculate the distribution. However, geometric optics is a good approximation provided the ray is more than about a wavelength from the sphere's edge [4]. For a sphere of radius r and light of wavelength λ, the corresponding geometric limit, δ_{geom}, is

$$\delta_{geom} = \delta(1 - \lambda/r) = 2[\sin^{-1}(1 - \lambda/r) - \sin^{-1}(\frac{1 - \lambda/r}{1+\mu})] \tag{3}$$

A typical TRIMM material is 35 μm diameter cross-linked PMMA spheres in a PMMA matrix, which has $\mu = 0.0114$ at $\lambda = 590$ nm [1]. For this material, $\bar{\delta} = 1.80°$ and $\delta_{geom} = 4.5°$.

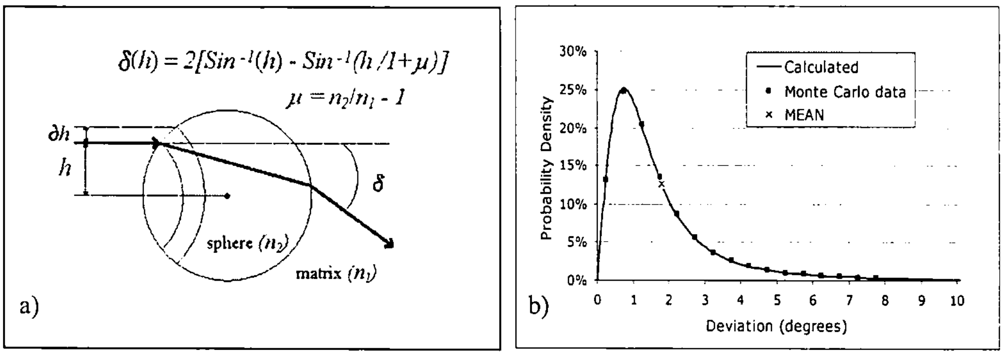

Figure 1: a) Angular deviation of a single ray striking a TRIMM sphere of unit radius.
b) The probability density distribution of the deviation, $f(\delta)$ for $\mu = 0.0114$

The probability density distribution $f(\delta)$ can be calculated analytically. The integrated probability density, $P(h)$, is

$$P(h) = h^2 = \int_0^{\delta(h)} f(\delta)\partial\delta \tag{4}$$

and so

$$f(\delta) = \frac{\partial P}{\partial \delta} = \frac{\partial P}{\partial h}\cdot\frac{\partial h}{\partial \delta} = \frac{h}{\frac{1}{\sqrt{1-h^2}} - \frac{1}{\sqrt{(1+\mu)^2 - h^2}}} \tag{5}$$

Figure (1b) shows $f(\delta)$ as calculated using (4) and (5), and $f(\delta)$ as derived using Monte Carlo data for ~100,000 deviations. The Monte Carlo data is sorted into 0.5° bins, hence the slight inaccuracy of the curve in the 0-1° range.

Angular spread after multiple TRIMM deviations

To optimise design parameters to obtain uniform light output, we wish to determine the angular distribution of light at a certain propagation distance along a mixing rod. A ray's path along a TRIMM mixer, as it deviates with every sphere interaction, can be described as a random walk. Multiple interactions will tend to randomly deviate the ray. In a path of length l a light ray will encounter $a = \alpha l$ particles. We call a the axial particle number of the TRIMM rod. Because $\bar{\delta}$ is small, for rays reaching the guide end, l is approximately equal to the mixing rod length, L. The ray undergoes approximately a interactions of average deviation $\bar{\delta}$, so the Central Limit suggests that the mean half cone angular spread, $\bar{\Sigma}$, of light in the cross-sectional plane when it reaches the end of a TRIMM doped rod can be approximated by

$$\bar{\Sigma} \approx \bar{\delta}\sqrt{a} \qquad (6)$$

Now the probability distribution of deviation by a single sphere is such that there will be a number of events with a deviation significantly higher than $\bar{\delta}$ (fig (1b)). In addition, the number of particle interactions per ray is actually greater than a, since $l > L$. It is also desirable to derive a model in terms of μ rather than $\bar{\delta}$, for ease of design. Evaluation of (2) shows that $\bar{\delta}$, when expressed as a multiple of μ varies slowly with μ, so $\bar{\delta} \approx \text{constant} \times \mu$. Consequently, we have modelled a best fit for a independent additions of deviations given by equations (1) and (4) by

$$\bar{\Sigma} = k\mu\, a^{p} \qquad (7)$$

The optimisation range was chosen as $0.01 < \mu < 0.02$, and $50 < a < 130$, to match the typical experimental range of these parameters.

Critical length and mixer design

A convenient parameter for TRIMM mixer design is the critical length, L_{crit}. This is defined as the length, for a given μ and α, where $\bar{\Sigma}$ is equal to θ_{crit}, the ray angle with respect to the rod axis where rays will escape out of the walls due to the critical condition. For meridional rays in a circular light guide, or for a guide with square walls, $\theta_{crit} = \pi/2 - Sin^{-1}\left(1/n_1\right)$, assuming the medium outside the mixer is air. From (6) it can be seen that the critical length, assuming a guassian distribution of $\bar{\delta}$, is

$$L_{crit} = \frac{\theta_{crit}^2}{\alpha(\bar{\delta})^2} \qquad (8)$$

Results

The optimised fit for (7) is

$$\Sigma \approx 3.39\mu\, a^{0.514} \qquad (9)$$

The error in the sum of the squares of the residuals varied from 1.1% - 6.2% over the optimised range of μ and a. From (6) and (9) a general formula for the critical length was derived:

$$L_{crit} = \frac{\theta_{crit}^{1.946}}{\alpha(3.39\mu)^{1.946}} \qquad (10)$$

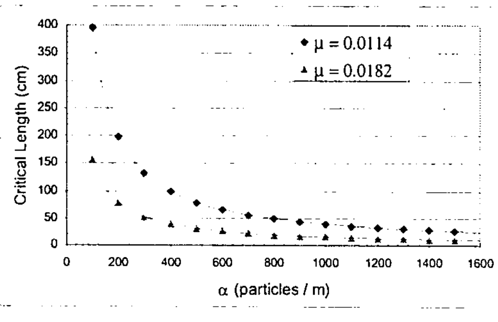

Figure 2: critical length vs TRIMM Concentration for $\mu = 0.0114$ and $\mu = 0.0182$

Critical lengths were calculated for $\alpha = 2000$ TRIMM particles per metre, using two values of μ from previous experiments [1,2];

1) $\mu = 0.0114$, $a \sim 400$, giving $L_{crit} = 19.7$ cm,
2) $\mu = 0.0182$, $a \sim 160$, $L_{crit} = 7.8$ cm.

Monte Carlo ray tracing for 100,000 rays was performed for each system using a light guide with a 2 cm square cross-section, with L ~ L_{crit}. Figure (3a) shows the percentage of initial rays that have exited through the walls as a function of fraction of critical length. Figures (3b) and (3c) show ray traces (100 rays) for each system.

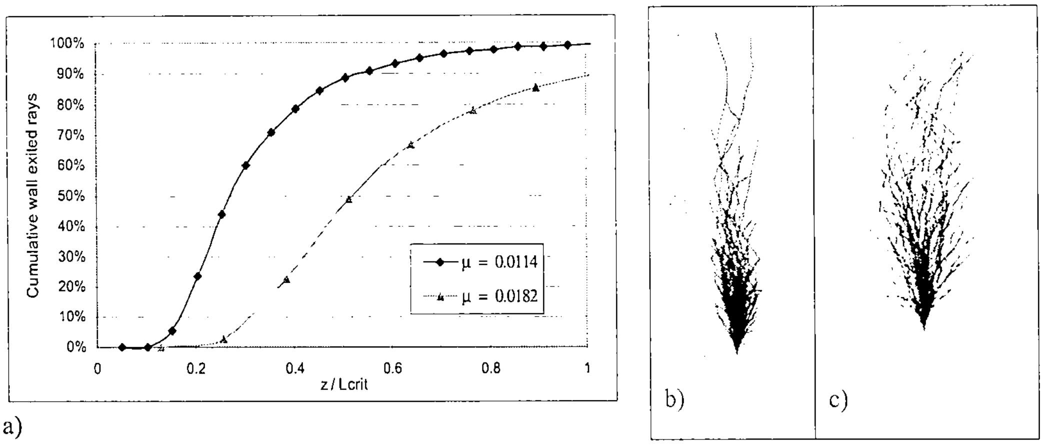

Figure 3: a) Monte Carlo modelled wall exited rays as a function of z / L_{crit} (fraction of total guide length) for two μ values. b) Ray trace for $\mu = 0.0114$. c) Ray trace for $\mu = 0.0182$. ($\alpha = 2000$).

Discussion

The purpose of modelling a critical length was to aid in the choice of optimum parameters when designing TRIMM rod light mixers. It would be useful to have an equation to estimate, for example, the length required to have 50% of the light reaching the end of the light guide. For $\mu = 0.182$, this length is $\sim L_{crit} / 2$. It can be seen that the critical length for $\mu = 0.0114$ is overestimated by using (10). This could be because the parameters in this region are not well optimised. In further work we could investigate calculating a critical length based on, say, a certain fraction of θ_{crit}.

We have considered a simplified propagation model for rays at normal incidence. $\bar{\Sigma}$ is spread about the incident ray direction, so $\bar{\Sigma}$ should be convolved with the source angular distribution for actual light sources.

Conclusion

Probability density function for a single TRIMM deviation has been calculated, and agrees with previously published distributions derived using Monte Carlo ray tracing. Mean deviation for TRIMM spheres has been calculated. It was found that the total angular spread at the end of a mixing rod for normally incident rays increases slightly faster than the square root of the axial particle number. This is in part due to the highly asymmetrical nature of the probability density distribution. A preliminary optimised fit for TRIMM concentration and material parameters shows that such models will be useful for calculating lengths of TRIMM mixers to obtain uniform light mixing and minimum loss.

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