Trajectory Tracking of Autonomous Vehicle That Uses State Feedback Linearization With Ackerman Method and Observer Feedback

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Hasnawiya Hasan
Department of Electrical Engineering
Department of Marine System Engineering
Faculty of Engineering
Hasanuddin University
Makassar, South Sulawesi, Indonesia
Email: hasnahasan@unhas.ac.id

Faizal Arya Samman
Department of Electrical Engineering
Faculty of Engineering
Hasanuddin University
Makassar, South Sulawesi, Indonesia
correspondence email:
faizalas@unhas.ac.id

Muh Anshar
Department of Electrical Engineering
Faculty of Engineering
Hasanuddin University
Makassar, South Sulawesi, Indonesia
Email: anshar@unhas.ac.id

***Abstract*— In recent years, research about vehicle trajectory tracking has become one of the significant fields in autonomous vehicle control development, especially with the presence of some new control technics that can complete the autonomous vehicle technology. The research aim is to develop controllers applied to control the autonomous vehicle that navigates along a predefined path that considers two unique scenarios in real-world driving. This paper contains two parts that define the method of the research. In the first part of the paper, the kinematic model of the vehicle is developed based on some references from previous research, and then a nonlinear controller from the class of state feedback linearization using the Ackerman method is applied to this nonlinear vehicle model to have a new control law that efficient enough in controlling such as model. In the second part of the paper, an observer is added to the state feedback linearization system to generate a steering angle that is a nonlinear vehicle model and to achieve stable tracking performance with minimum lateral error. The simulation results show that the control low can achieve stability, and the vehicle model has a good performance while tracking the predefined trajectory. However, some efforts are still needed to improve the performance of the vehicle system. Therefore, applying a new control method, such as model predictive control combined with a deep learning algorithm, will be preferable in this system.**

Keywords*— Ackerman, autonomous vehicle, nonlinear control, observer, state feedback linearization, trajectory tracking.

I. Introduction

Research about the autonomous vehicle has faced many problems in the last decades. Much work related to motion control of the robot vehicle has been reviewed and analyzed over the past few years. Trajectory tracking control is still a fascinating topic related to the control problems of an autonomous vehicle. Trajectory tracking is a part of other applications in an autonomous car, such as lane keeping, lane changing, and cruise control [1] [2].

A trajectory-tracking system could be designed through these two approaches. The first method is pole placement. Another approach is the state feedback linearization method, a class of nonlinear control design methods [1] [3]. The State Feedback linearization approach significantly increases interest and application in control system design. This approach aims to stabilize a system at an equilibrium point by applying a feedback control law [4]. The approach is possible to implement in a complex nonlinear system due to the model's accuracy.

In the problem of an autonomous vehicle, a method that can keep the vehicle tracking the trajectory almost precisely is essential. The state feedback linearization method can handle a nonlinear dynamic model such as the robot vehicle so that the vehicle can navigate along the desired trajectory while keeping its stability. In addition, a real driving road is sometimes complex or challenging to track, such as a curvature road. Autonomous vehicles usually fall into drifting conditions when tracking a sharp bend, and the vehicle should keep its low speed in this situation[5] [6]. Therefore this method is quite suitable for such as nonlinear model due to its capability to track a complex trajectory while keeping its low speed.

Furthermore, this study aims to meet the need for implementing theoretical nonlinear control systems such as state feedback linearization into a real system such as autonomous vehicles since the method has been popular to control such as nonlinear models. So, the remainder of the paper is organized as follows; in section 2, we discuss related work, a literature review of previous recent research results that closely relate to our research, and the state of the art of this research. While in section 3, we discuss the vehicle model, and section 4 is about designing a control law using the state feedback linearization method. The method consists of two-part; the first is controlling low using the Ackerman method, while the second is state feedback linearization with observer design. Moreover, in section 5, we discuss the simulation result of both approaches.

II. Related Work

Much research has been dedicated to trajectory tracking control for autonomous vehicles. Research by Bacha et al.[7] is almost similar to this research, but they used the input and output state feedback linearization method, while this research used the Ackerman method and observer feedback [8]. Their research also had a good control performance however did not experiment on the sharp bend road. In comparison, Trotta et al. [9] applied a feedback linearization approach to gain adaptive cruise control for an autonomous

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vehicle with the benefit of low computational cost and real-time application. This approach can manage the non-linearities system and guarantee stability around equilibrium [9][10]. Although this approach has been passed on a road test, the research uses a longitudinal vehicle model to design the control architecture, while this research focuses more on the lateral vehicle model to obtain more accurate computation on the ground.

Otherwise, Cao et al.[11] also designed a Linear Time-Varying Model Predictive Control algorithm that performs remarkably in real-time conditions [12]. Although this approach has achieved remarkable tracking control performance and robustness in real-time conditions, the computation is complicated, and the vehicle model has not experimented on the road. In addition, research which is conducted by Wang et al.[13] study about tracking a nonlinear affine using state feedback linearization; although this approach is economical and energy-efficient, however, the approach does not consider nonlinear internal dynamics and disturbances parameters [13] [14]. Tracking is essential for almost all engineering applications, including tracking photons and ranging, which uses a phase-based linearization approach in optical tracking and imaging applications [15] [16].

Based on the literature review, many previous recent research for an autonomous vehicle with the same method only focuses on controlling longitudinal control. However, this research focuses on lateral control, which also has the same significant effect on autonomous vehicle performance [17] [18].

This research contributes to the improvement of controlling autonomous vehicles, which can be summarized as follows :

- The method can improve the performance of the autonomous vehicle by keeping the stability around the equilibrium.
- The vehicle can track curvature trajectory using this method. [19].
- This method has simple computation or is not as complex as other advanced control methods but is reliable enough in controlling highly dynamic models such as autonomous vehicles [20] [21].
- The state feedback linearization method with observer feedback has the advantage which can solve uncertain parameters and disturbances which usually arise from nonlinear models, adding more benefits and ensuring the stability of the system
- Other research or other control advanced methods usually add sensors to measure these uncertain parameters [22] [23], but this method could limit the use of sensors by adding observer feedback.

III. VEHICLE MODEL

A four-wheeled vehicle is considered the model, represented in two-dimensional coordinates (X-Y) in Fig 1. The steer is only applied to the front wheels, with the assumption of rolling without slipping [7]. This model also does not consider inertia. The vehicle state kinematic equation is defined as:

$$\dot{x} = v\cos\theta \tag{1}$$

$$\dot{y} = v\sin\theta \tag{2}$$

$$\dot{\theta} = \frac{v}{L}\tan\phi \tag{3}$$

Where $(x, y) \in R^2$ corresponds to space coordinates, $\theta \in [0, 2\pi]$ is the azimuth angle, and $L \in R$ is the length of the vehicle base. The input controller is defined as $u = [\theta, \phi]$. Where v corresponds to the magnitude of the commanded velocity and ϕ represents the commanded steering angle [24]. The model is only for the simulation of low speeds and low acceleration.

IV. DESIGN CONTROL LAW

A. State Feedback Linearization With Ackerman Formula

The 3 DOF vehicle model is defined in state vectors such as:

$$q = [\dot{x}, \dot{y}, \dot{\theta}] \tag{4}$$

$$\dot{x}_1 = x_2; \tag{5}$$

$$\dot{x}_2 = v\cos\theta = u \tag{6}$$

$$\dot{y}_1 = y_2; \tag{7}$$

$$\dot{y}_2 = v\sin\theta = u \tag{8}$$

$$\dot{\theta}_1 = \theta_2 \tag{9}$$

$$\dot{\theta}_2 = \frac{v}{L}\tan\psi = u \tag{10}$$

Where v is linear velocity, *(x, y)* is vehicle space lateral coordinate, Θ is vehicle heading, L is the length of vehicle base, *and* ϕ is steering angle. The state feedback linearization architecture diagram is shown in Fig 2.

If equation (5) – (10) are written in the form of a state vector, such as follows:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u \tag{11}$$

$$\begin{bmatrix}\dot{y}_1\\ \dot{y}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u \tag{12}$$

$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u \tag{13}$$

$$\text{With } A = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}; B = \begin{bmatrix}0\\ 1\end{bmatrix}; C = [0 \quad 1];$$

Then, design control feedback to satisfy this control law equation such as follows:

$$u = -Kx \tag{14}$$

Where is the state variable feedback matrix such as:

K = [K1, K2]

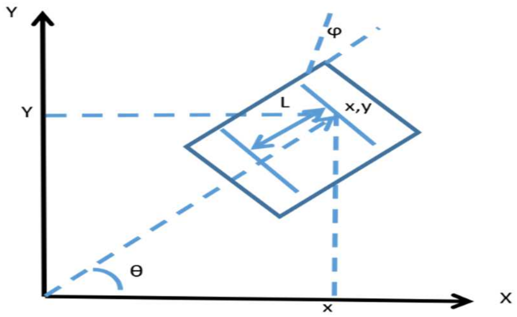

Fig. 1. Vehicle Model Configuration in X-Y Coordinates

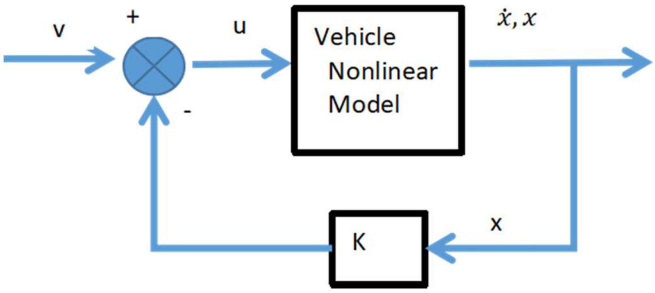

Fig. 2. State Feedback Linearization Block Diagram

Consider also a reference K input (r) so that the control law formula, such as

$$u = -Kx + r \tag{15}$$

Where K is a state variable feedback matrix such as:
K = [K1, K2]
Then, design a characteristic equation such as that

$$\partial^2 + 2\delta\omega_n\partial + \omega_n^2 = 0 \tag{16}$$

Where δ is overshoot and ω_n is a natural frequency

This state feedback linearization approach is designed based on ref [25] and [26]. In this characteristic equation, the system adjusts the state feedback gain to achieve stability in the equilibrium point based on the suitable overshoot and natural frequency.

Ackerman formula can also apply in this 3 DoF vehicle model if the model is controllable [26]. New characteristic equation, then change as follows:

$$L(A) = A^2 + 2\varsigma{\omega_n}^2 A + \omega_n^2 A + \alpha_n I \tag{17}$$

Then, design a state feedback gain matrix such as that:

$$K = [0\ 1]C_M^{-1}L(A) \tag{18}$$

Where C_M is the controllability matrix

Design controller K to satisfy overshoot and natural frequency using Matlab simulation

K = [0.2 0.2] for $\dot{x}, \dot{y}, and\ \dot{\theta}$

Where u is input control and v is reference trajectory.

B. State Feedback Linearization With Observer For Autonomous Vehicle

If we add an observer system in the full state observer system, the state vector is developed such as:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u \tag{19}$$

$$y = [0 \quad 1]x \tag{20}$$

Given an estimated state of the x, such as $\dot{\hat{x}}$

$$\begin{bmatrix}\dot{\hat{x}}_1\\ \dot{\hat{x}}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u + \begin{bmatrix}L_1\\ L_2\end{bmatrix}\hat{y} \tag{21}$$

$$\hat{y} = y - [0 \quad 1]\hat{x} \tag{22}$$

Where C = [0 1]

Where $\dot{\hat{x}}$ denotes the estimates of state x, and L is the observer gain.

The state feedback linearization with observer control architecture is shown in Fig 3.

Then, the design observer gains (L) to satisfy overshoot and natural frequency, such as
L = [16 100] for every tracking x, y, and θ

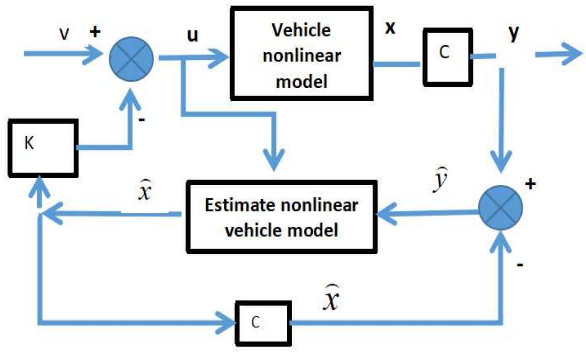

Fig. 3. State Feedback Linearization with Observer

For the desired coordinates $(\dot{x}_d, \dot{y}_d)$, the actual coordinates $(\dot{x}, \dot{y})$ are defined as follows.

$$\dot{x} = \dot{x}_d + k_x x_e \tag{23}$$

$$\dot{y} = \dot{y}_d + k_y y_e \tag{24}$$

Where

$$x_e = x_d - x \tag{25}$$

$$y_e = y_d - y \tag{26}$$

Equations 23 and 24 could be written in the form.

$$\begin{bmatrix}\dot{x}_e\\ \dot{y}_e\end{bmatrix} + \begin{bmatrix}k_x & 0\\ 0 & k_y\end{bmatrix}\begin{bmatrix}x_e\\ y_e\end{bmatrix} = 0 \tag{27}$$

The following Lyapunov function ensures the system stability with a unique equilibrium point at the origin, and it is positive definite [7], as follows :

$$V = \frac{1}{2}e^T, e > 0 \tag{28}$$

Where:

$$e = \begin{bmatrix}x_e\\ y_e\end{bmatrix} \tag{29}$$

$$\dot{e} = -\begin{bmatrix}k_x, x_e\\ k_y, x_e\end{bmatrix} \tag{30}$$

And its derivative is negative definite [7], as follows:

$$\dot{V} = e^T.\dot{e} = -k_x x_e^2 - k_y y_e^2 < 0 \tag{31}$$

The system is asymptotically stable, and the error always converges to zero. The new controller is computed to generate a steering angle so the vehicle can navigate along the desired trajectory.

V. Simulation Results

A. Simulation Result State Feedback Linearization With Ackerman Method

A vehicle model that travels along reference circle and ellipse trajectory shown in Fig 4 and Fig 5 is presented as

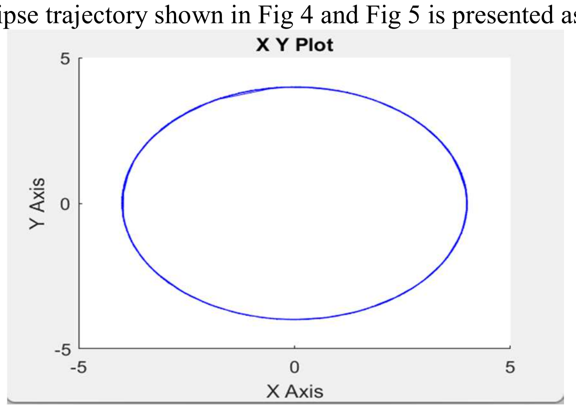

Fig. 4. Reference circular trajectory by Ackerman method

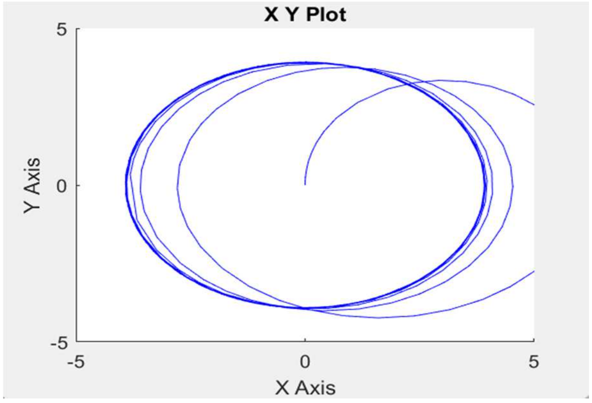

Fig. 5. Actual circular trajectory by Ackerman method

the simulation result. Vehicle fixed velocities is 10 m/s and is set to start at (0,0) in space coordinate (x,y).

Fig 4 until Fig 7 shows that the vehicle navigates along the reference and actual trajectory, circular trajectory, and ellipse trajectory. The actual trajectory tracks the reference trajectory with a small error that validates the developed Ackerman controller. The state feedback linearization approach aims to place desired poles in the left plane of the imaginary plane in a closed-loop system. Therefore, state feedback linearization via the Ackerman method designs its overshoot and natural frequency in this design; overshoot is designed to be low overshoot to meet the desired settling time and to get a rapid response that suits the desired response. However, this condition only can be achieved if the system is controllable. So, the chosen K controller can place the poles in a stable domain [26].

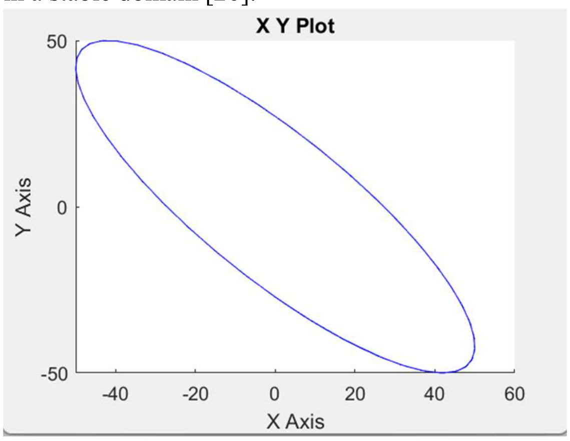

Fig. 6. Reference ellipse trajectory by Ackerman method

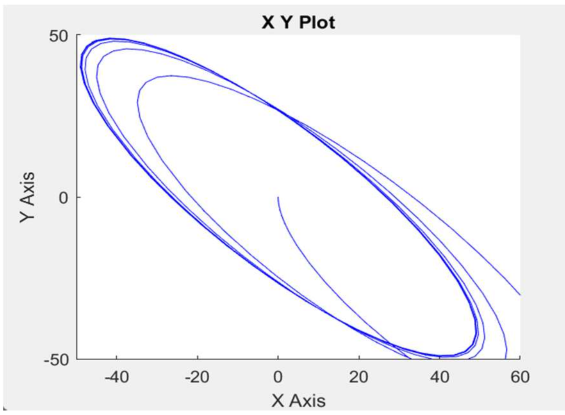

Fig. 7. Actual ellipse trajectory by Ackerman method

Based on the reference and actual trajectory, the lateral error is calculated and represented in Fig 8 and 9, respectively. Because of the overshoot design, the vehicle cannot track the desired trajectory at initial conditions, and it takes time for the design vehicle model to meet the desired reference trajectory. However, this settling time has been designed to track the desired trajectory faster to achieve a stable response. In addition, the overshoot is also designed to be low or almost zero so that the characteristic response still meets the requirement. However, the K controller can still be adjusted to get better performance. The lower overshoot and faster settling time can result in better control performance.

The lateral error of both circular and ellipse trajectories using the Lyapunov function is set to asymptotically stable. So that even though the error exists in Fig 8 and 9, it converges to zero.

B. Simulation Result of State Feedback Linearization with Observer Feedback

A full control law requires all states to be available for measurable and feedback responses. However, many methods only have a sub of the state which is ready for measurement. It means the system needs to provide a sensor or a complex sensor architecture to measure unreadable states from the system. While providing many sensors is not economical and also complex design. So the design of an observer which can measure those states reduces the system's cost design [26].

Design an observer design a state estimation that the feedback cannot measure. The requirement is that the system should be observable so that the observer gain L can be found. Observer L is designed to meet the desired characteristic response so that the poles can be placed in the left half plane. Therefore the tracking error can be asymptotically stable.

Simulation result of state feedback linearization with the observer, the reference trajectory is still the same circular and ellipse trajectory, as shown in Figures 10 and 12. The actual trajectory is then presented in Fig 11 and 13.

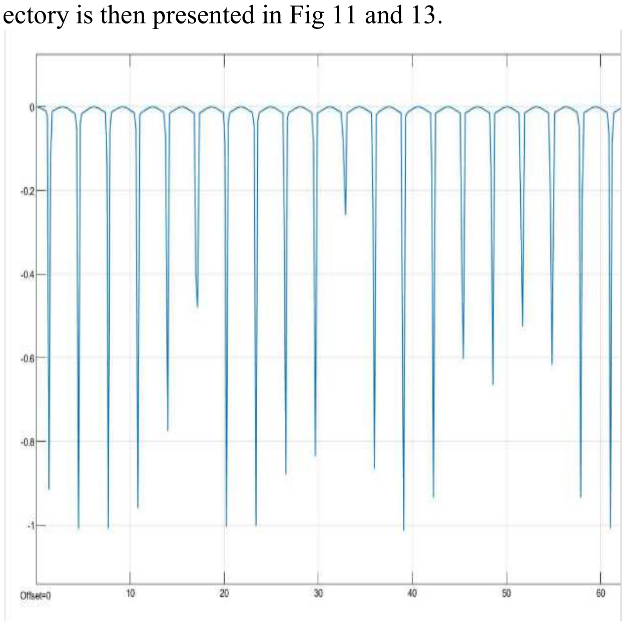

Fig. 8. Trajectory error circular path by Ackerman method

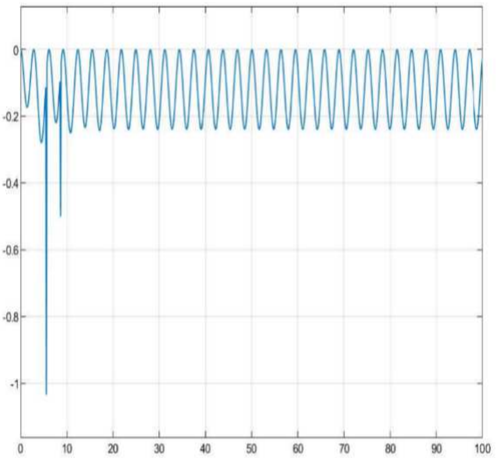

Fig. 9. Trajectory error ellipse path by Ackerman method

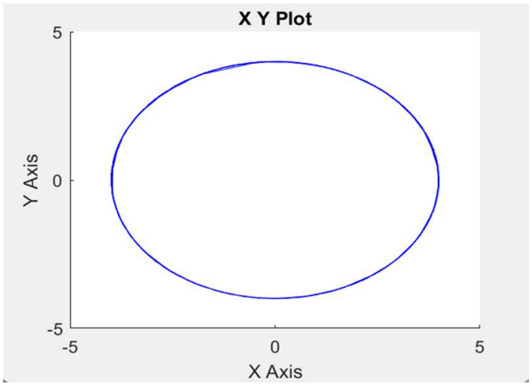

Fig. 10. Reference circular trajectory by state feedback linearization with observer method

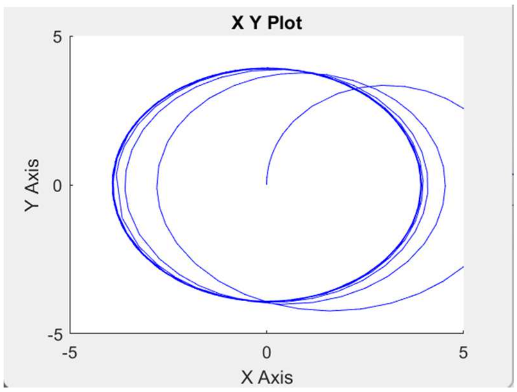

Fig. 11. Actual circular trajectory by state feedback linearization with observer method

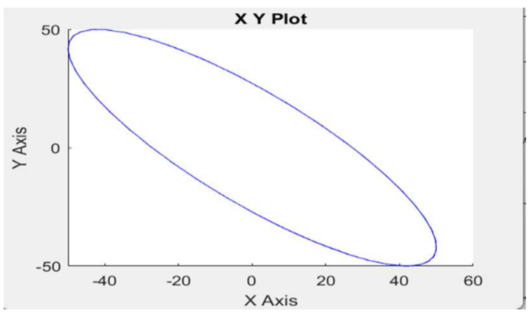

Fig. 12. Reference ellipse trajectory by state feedback linearization with observer method

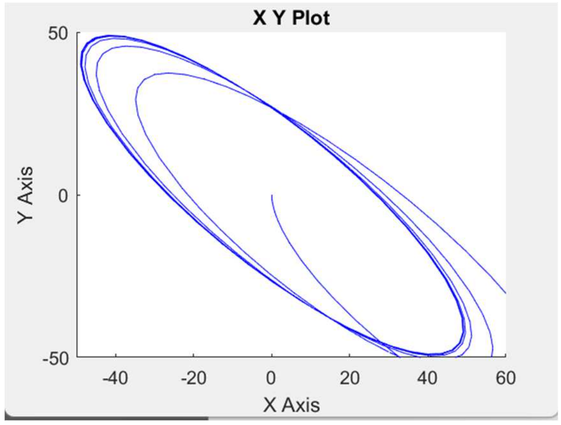

Fig. 13. Actual ellipse trajectory by state feedback linearization with observer method.

The result shows that the actual and desired trajectory error is even more minor than the previous Ackerman controller. Because the design of the observer, which can measure states for feedback, eventually results in a minor error. However, the system cannot achieve stability immediately due to the existing overshoot and desired settling time designed in the Ackerman method. Fig 14 and 15 show trajectory error of state feedback linearization with the observer for both circular and ellipses paths.

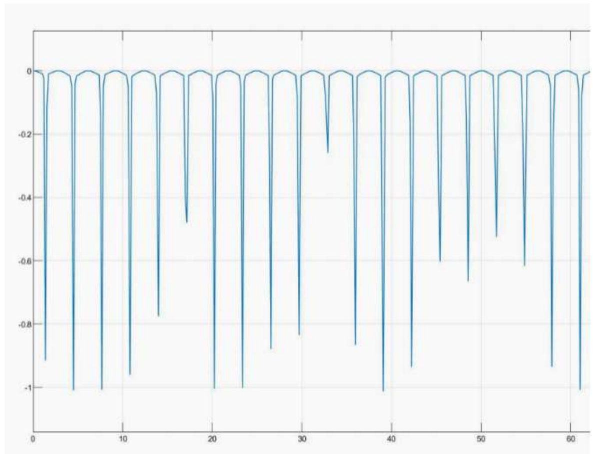

Fig. 14. Trajectory error circular path by state feedback linearization with observer method

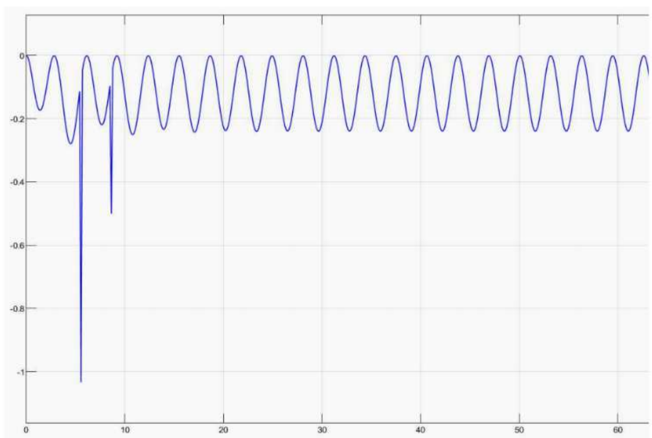

Fig. 15. Trajectory error ellipse path by state feedback linearization with observer method

In addition, Fig 16 shows the steering angle of the vehicle model simulation; it shows that for the first few seconds, the steering angle turns a few angles to track the desired trajectory and then reaches stability because the trajectory is circular. Hence, the remaining steering angle is constant. While Fig 17 shows the steering angle of the ellipse trajectory. It shows that the model turns a few angles too in the first few seconds to follow the reference trajectory, then it quite stables around 0; the curve is less stable than the circular trajectory due to the ellipse trajectory.

V. CONCLUSION

A trajectory-tracking solution for an autonomous vehicle is proposed in this paper. The vehicle lateral kinematic motion is controlled using a state feedback linearization technique, divided into two parts: an Ackerman method and a system with an observer method. The new controller is designed to have a steering angle that can control the vehicle to track along the predefined path with minimal error. The simulation result shows the satisfactory performance of the vehicle trajectory tracking simulation due to the stability and minimized error between reference tracking and actual tracking both in a circular path and ellipse path scenario. State feedback linearization with observer shows better performance than the nonlinear controller with the Ackerman method due to is possible to estimate the states that are not directly measured. This method can generate a stable steering angle that suits the nonlinear model to reach minimum error in the system. In the future, the autonomous vehicle needs to be improved its control response performance, particularly for the complex driving scenario. So, advanced control approaches such as Model Predictive Control are expected to give good control solutions for higher nonlinear vehicle models. The approach can be combined with deep learning, which works remarkably for control-based computer vision.

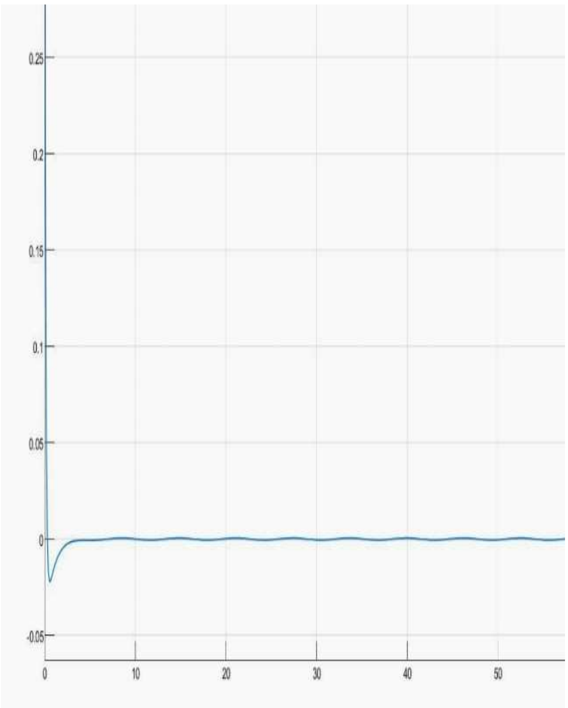

Fig. 16. The steering angle is generated from the circular desired trajectory.

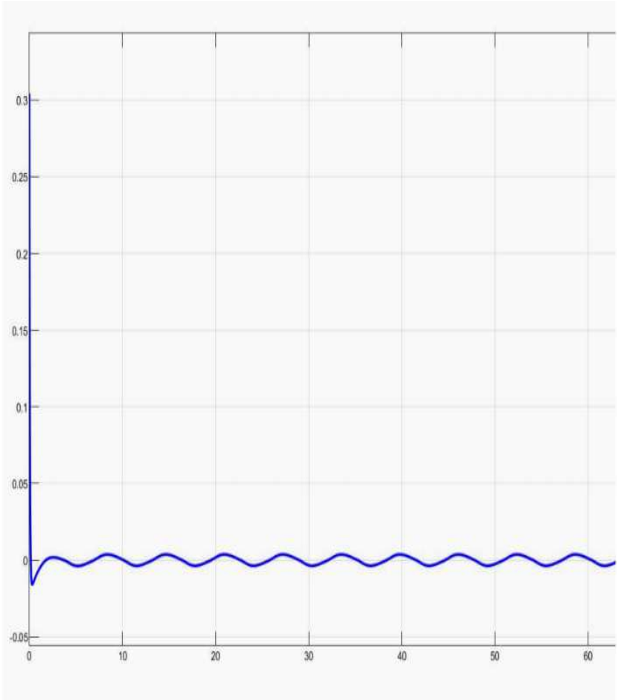

Fig. 17. steering angle generates from ellipse desired trajectory

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