Control Design for Interconnected Power Systems with OLTCs via Robust Decentralized Control

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Abstract— **This paper addresses the problem of designing a decentralized control of interconnected power systems, with OLTC and SVCs, under large changes in real and reactive loads that cause large structural changes in the system model. In addition to this, small changes in load are regulated by small disturbance controllers whose gains are adjusted for variations in power system model due to large changes in loads. The only feedback needed by subsystem controllers is the state of the subsystem itself. The design is carried out within a large-scale Markov jump parameter systems framework. In this paper, unlike other control schemes, OLTC transformers are used to damp power-angle oscillations. Simulation results are presented to demonstrate the performance of the designed controller.**

I. Introduction

The primary task of the power system control is to provide reliable and secure electric power supply within a narrow band of voltage and frequency variation. As the demand for electric power is continuously increasing the power system grows in size and complexity. Also to meet the ever increasing demand, the system is forced to operate as close as possible to its maximum limit without sacrificing the reliability. This makes the power system control task more difficult and challenging.

In a multi-machine power system, when the steady state condition is disturbed due to load changes or fault in the system, the rotors of different machines start oscillating with respect to each other, exchanging energy between them. When oscillations are allowed to grow, machines are pulled out of synchronisation. The rotor angle stability is the ability of interconnected synchronous machines to remain synchronised. Small signal disturbances occur in a system continually because of small variations in load and generation. This can produce sustained oscillations in power angles and frequency and may disrupt the service [1]. There are several reasons for the dynamic instability in a power system. Among them, the weak couplings between interconnected systems which are randomly fluctuating, and a small group of machines with relatively low or negative damping against other machines in the system with positive damping, are a couple of important factors which need to be considered. The work mentioned in [1] shows how the systems with above situations can lead to instability created by random fluctuations in the coupling between machines due to variations in load impedances and transmission line reactances. It also shows that such systems will lead to instability for almost every sample path the random variations can take as time goes to infinity. In this paper, the change in the power system model due to the random nature of load variations is modelled using Markov jump parameters.

An interconnected power system is operated by many different utilities. There is an agreement amongst the utilities on performance standards but each utility is free to choose the way in which to maintain the agreed performance. This style of operation necessitates the decentralised control of power system. Since the power system is interconnected in a complex manner, the controller providing damping control of the oscillations may require the knowledge of the states of the other machines connected to the grid in real time. But because of the geographic separation of the location of the generating units, it is not always possible to transfer this information amongst machines in real time. Under these conditions a decentralised controller, which operates strictly based on the information of local states is desirable.

For small-disturbance damping linearised power system models are employed. In many situations the system is further simplified as single machine connected to infinite bus (SMIB), the model is linearised around an operating point and the controllers are designed. But in most situations neither the SMIB nor a single operating mode assumptions is valid as systems undergoes structural changes owing to large changes in load conditions. Also in an interconnected system each machine is affected by the changes happening elsewhere in the system. In this paper we consider the complete interconnected system with changing operating modes.

The robustness problems encountered by the conventional design procedures and the modelling limitations were addressed and improved upon by considerable research work in this area reported in [2]–[6]. In these works, controllers are designed for multi-machine power systems using modern control techniques like H_∞ optimisation, μ-synthesis and LMI approach. These approaches include model uncertainties and control design scheme based on optimising a cost function. The results of these studies suggest that robust control technique can be used to improve the performance and stability of interconnected power systems.

An alternative approach to controlling linearised power system at each operating mode separately one can consider it as an interconnected nonlinear system. Much attention has been given to the design of nonlinear controllers using nonlinear system models [4]–[6]. Even though these controllers do improve the transient stability of the system, their

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practical implementation is quite complicated and difficult due to their structure and often excessive control levels. The work mentioned in [2] explains robust decentralised control for a multi-machine system using a linear controller and the nonlinear interconnections were treated as parametric uncertainties.

In a recent work [7], the power system control is treated as the problem of designing a decentralised robust controller for an interconnected system which is subjected to uncertain disturbances and having randomly changing structure. Structural changes in the system model are assumed to be governed by a continuous-time Markov chain taking values in a finite set. Integral Quadratic Constraints (IQCs) are used to describe modelling errors and unknown systems interconnections. In this paper, it is shown that decentralised power system control using the Markov jump parameters theory [7], with on-load-tap-changing (OLTC) transformer and Static Var Compensators (SVC), is able to provide a fast damping to otherwise persistent low damping modes of power systems.

Static var compensators are shunt connected static generators and absorbers whose outputs are varied so as to control specific parameters of the system. Since their first application in the late 1970's the use of SVCs in transmission systems has been increasing steadily. By virtue of their ability to provide continuous and rapid control of reactive power and voltage, SVCs can enhance several aspects of transmission system performance. Applications to date include the control over voltage variations and prevention of voltage collapse and also enhancement of transient stability and enhancement of damping oscillations [8]. In the example power system considered in this paper capacitors are included at specific nodes as SVCs. The use of SVCs enables a weaker interconnection amongst generators by providing reactive power near the load.

Transformers with tap changing facilities constitute an important means of regulating voltage levels in spite of fluctuating load. Usually many OLTC transformers are located throughout the distribution network. The taps on these transformers help to control reactive power flow between subsystems. The control of single transformer will cause changes in voltages at its terminals, it will also influence the reactive power flow through the transformer. The resulting effect on the voltages at other buses will depend on the network configuration and load/generation and distribution. In order to maintain the system voltage at required level the regulators must, however have sufficient capacity to satisfy the needed reactive power requirements [8].

It is important to consider the effect of OLTCs on the power system while designing controllers. The effects of the dynamics of OLTC with respect to voltage collapse, stability and power transfer ability are considered in [9]–[11]. In the works mentioned in [10], [11], the generators feeding the OLTC are assumed as constant voltage sources but the secondary voltage of the OLTC is affected by the changes in the primary voltage as well as the load connected to the OLTC. In this paper a mathematical model is presented which includes the effect of OLTCs on other subsystems in the grid as well as the effect of other subsystems on the OLTC itself.

In most OLTC control schemes [8] the OLTC tap-settings are changed to maintain a constant secondary voltage. In this paper the decentralised control is designed to vary the tap setting not to provide a constant secondary voltage but to adjust it to damp the power-angle oscillations in the power system. A stable OLTC controller is designed using robust control techniques with guaranteed cost. This scheme thus makes use of the OLTC as an effective control element and not as a source of voltage collapse owing to its fixed voltage settings [9]–[11].

In this paper, a numerical example of an interconnected power system jumping between two load profiles are considered. The variations in the load profile is treated as stochastic variations for the controller design. The controller gains designed need to be switched in accordance with the changes in the load. In this example static var compensators and on-load-tap-changers are included for reactive power and voltage control. The robustness and performance of the design is validated through simulation of the system with nonlinear plant model incorporating load variations and controller switchings.

II. Power system model

The main assumptions made in obtaining a linear model for interconnected power system are [7]:

1. All loads are modeled as constant admittances [12].
2. The change in reactive power due to small changes in generator angles is negligible [13].
3. Real power (P_m) and reactive power (Q_m) inputs to the generators and the reference voltage (E_u) to the OLTC are controlled parameters.

The power system has n generators and t OLTCs; load buses are eliminated to obtain the system dynamic model [7].

A. Algebraic Constraints on OLTCs

For the i-th OLTC let the primary voltage $|E_{T_i}|$ an secondary voltage $|E_{S_i}|$ be related as

$$|E_{S_i}| = n_i |E_{T_i}| \quad (1)$$

where n_i is the turns ratio of i-th OLTC. Let the reactive power at OLTC nodes be $Q_T = [Q_{T_1}, \dots, Q_{T_t}]^T$. The reactive power due to the reactor L_{T_i} connected on the secondary side of the OLTC is

$$Q_{T_i} = \frac{|E_{S_i}|^2}{2\pi f L_{T_i}} \quad (2)$$

where f is the system frequency in Hz.

In this paper we consider the effect of tap-change only on reactive part of the load. It is assumed that the real load doesn't change much with the changing tap position. This is a common assumption in the literature [11].

For small variations in $|E_{S_i}|$ expressions (1) and (2) can be written as:

$$\Delta Q_{T_i} = 2\frac{Q^0_{T_i}}{|E^0_{S_i}|}\Delta|E_{S_i}|, \quad (3)$$

$$\Delta|E_{S_i}| = \Delta n_i |E^0_{T_i}| + n^0_i \Delta|E_{T_i}| \tag{4}$$

where superscript 0 is used with variables to denote their equilibrium or steady-state value.

Reactive power equality constraint can be used to obtain an expression for $\Delta|E_T|$ in terms of input reactive power $\Delta|Q_m|$ and Δn, where $\Delta n = [\Delta n_1, \ldots, \Delta n_t]^T$ and $\Delta|E_T| = [\Delta E_{T_1}, \ldots, \Delta E_{T_t}]$.

Let $K_{T_i} = 2\frac{Q^0_{T_i}}{|E^0_{S_i}|}$, substituting (4) in (3) we get

$$\Delta Q_{T_i} = K_{T_i}(\Delta n_i |E^0_{T_i}| + n^0_i \Delta|E_{T_i}|) \tag{5}$$

Let $[\Delta Q_{g_1}, \ldots, \Delta Q_{g_{n+t}}]^T = [\Delta Q_m \;\; \Delta Q_T]^T$ and write

$$\begin{bmatrix} \Delta Q_m \\ \Delta Q_T \end{bmatrix} = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} \Delta|E_g| \\ \Delta|E_T| \end{bmatrix} \tag{6}$$

where subscript g indicates generator and T indicates OLTC.

Let $K_T = \text{diag}(K_{T_i})$, $\Lambda_{n^0_i} = \text{diag}(n^0_i)$, $\Lambda_{|E^0_{T_i}|} = \text{diag}(|E^0_{T_i}|)$, and $\Lambda_{T_i} = \text{diag}(1/T_i)$ then from (5) and (6) we can write

$$\Delta|E_T| = M_{21}\Delta Q_m + M_{22}\Delta n, \tag{7}$$

where

$$M_{21} = \left(K_T \Lambda_{n^0_i} + N_{21}N_{11}^{-1}N_{12} - N_{22}\right)^{-1} \left(N_{21}N_{11}^{-1}\right),$$

$$M_{22} = -\left(K_T \Lambda_{n^0_i} + N_{21}N_{11}^{-1}N_{12} - N_{22}\right)^{-1} \left(K_T \Lambda_{|E^0_{T_i}|}\right).$$

The above equation (7) gives the algebraic constraint the system must satisfy at all times. In the next section we put together the algebraic constraints derived in this section with the dynamic equations for the generators and the OLTCs to arrive at the interconnected system dynamic equations.

B. The System Dynamic Model

The swing equations which describe the generator dynamics are [12]:

$$m_i \dot{\omega}_i + d_i \omega_i + P_{g_i} = P_{m_i}, \;\; i = 1, \ldots, n \tag{8}$$

and the reactive power constraint equations are:

$$Q_{g_i} = Q_{m_i}, \;\; i = 1, \ldots, n \tag{9}$$

where δ_i is the angle between the generator rotor and a reference frame rotating at the synchronous frequency; ω_i is the rate of change of angle δ_i; P_{m_i} is the real power input and Q_{m_i} is reactive power input of the i-th generator.

The swing equation (8) is linearized about the equilibrium point to obtain a linear model for the interconnected system [1], [13].

The dynamic equation for the tap changing of the i-th transformer is given as

$$\dot{n}_i = \frac{1}{T_i}\left(|E_{u_i}| - |E_{S_i}|\right) \tag{10}$$

and its linearised form is

$$\begin{aligned} \Delta\dot{n}_i &= \frac{1}{T_i}\left(|E^0_{u_i}| + \Delta|E_{u_i}| - |E^0_{S_i}| - \Delta|E_{S_i}|\right) \\ &= \frac{1}{T_i}\left(\Delta|E_{u_i}| - \Delta n_i |E^0_{T_i}| - n^0_i \Delta|E_{T_i}|\right) \end{aligned} \tag{11}$$

under steady state conditions $|E^0_{u_i}| = |E^0_{S_i}|$. Then equation (11) can be collected for all OLTCs and written as

$$\begin{aligned} \Delta\dot{n} = &-(\Lambda_{T_i}\Lambda_{|E^0_{T_i}|} + \Lambda_{T_i}\Lambda_{n^0_i}M_{22})\Delta n - \\ &\Lambda_{T_i}\Lambda_{n^0_i}M_{21}\Delta Q_m + \Lambda_{T_i}\Delta|E_u|. \end{aligned} \tag{12}$$

The linearised power-flow relationship for generators and OLTC (assuming that no real power is supplied at any OLTC bus) can be written (with $\Delta P_{g_i} = 0, i = n+1, \ldots, n+t$ and $\Delta P_g = [\Delta P_{g_1}, \ldots, \Delta P_{g_n}]$ as [7],

$$\begin{bmatrix} \Delta P_g \\ \mathbf{0}_{t\times 1} \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} \Delta\delta_g \\ \Delta\delta_T \end{bmatrix} + \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} \Delta|E_g| \\ \Delta|E_T| \end{bmatrix}. \tag{13}$$

We can eliminate $\Delta\delta_T$ from the above equation (13),

$$\Delta P_g = \left[\, \tilde{R} \,\right] [\Delta\delta_g] + \left[\, \tilde{S}_1, \tilde{S}_2 \,\right] \begin{bmatrix} \Delta|E_g| \\ \Delta|E_T| \end{bmatrix}, \tag{14}$$

where $\tilde{R} = \left[\, R_{11} - R_{12}R_{22}^{-1}R_{21} \,\right]$, $\tilde{S}_1 = \left[\, S_{11} - R_{12}R_{22}^{-1}S_{21} \,\right]$, and $\tilde{S}_2 = \left[\, S_{12} - R_{12}R_{22}^{-1}S_{22} \,\right]$. The swing equation for each generator is

$$\Delta\delta_i = \delta_i - \delta^0_i, \quad \Delta\dot{\delta}_i = \omega_i, \quad \Delta\ddot{\delta}_i = \dot{\omega}_i$$

$$\dot{\omega}_i = -\frac{d_i}{m_i}\omega_i - \frac{1}{m_i}\left(\Delta P_{g_i}\right) + \frac{1}{m_i}\left(\Delta P_{m_i}\right) \tag{15}$$

These swing equations can be collected and written as a vector equation in terms of states $\Delta\delta$ and Δn and inputs ΔQ_m as follows

$$\begin{aligned} \dot{\omega} = &-\Lambda_d\Lambda_m\omega - \Lambda_m\tilde{R}\Delta\delta_g \\ &- \Lambda_m\left[\tilde{S}_1 \;\; \tilde{S}_2\right] \begin{bmatrix} \Delta|E_g| \\ \Delta|E_T| \end{bmatrix} + \Lambda_m \Delta P_m \end{aligned} \tag{16}$$

From (6), $\Delta|E_g| = N_{11}^{-1}\Delta Q_m - N_{11}^{-1}N_{12}\Delta|E_T|$, and further using the expression for $\Delta|E_T|$ in (7) we can write (16) as

$$\begin{aligned} \dot{\omega} = &-\Lambda_d\Lambda_m\omega - \Lambda_m\tilde{R}\Delta\delta_g - \Lambda_m\left(\tilde{S}_2 - \tilde{S}_1N_{11}^{-1}N_{12}\right)M_{22}\Delta n \\ &-\Lambda_m\left(\tilde{S}_1N_{11}^{-1} - \tilde{S}_1N_{11}^{-1}N_{12}M_{21} + \tilde{S}_2M_{21}\right)\Delta Q_m + \Lambda_m\Delta P_m \end{aligned} \tag{17}$$

We can write the linearised dynamic equations for the entire system in a matrix form as,

$$\begin{bmatrix} \Delta\dot{\delta}_g \\ \dot{\omega} \\ \Delta\dot{n} \end{bmatrix} = \bar{A}\begin{bmatrix} \Delta\delta_g \\ \omega \\ \Delta n \end{bmatrix} + \bar{B}_1\Delta P_m + \bar{B}_2\Delta Q_m + \bar{B}_3\Delta|E_u|, \tag{18}$$

where

$$\bar{A} = \begin{bmatrix} \mathbf{0}_{n\times n} & \mathbf{I}_{n\times n} & \mathbf{0}_{n\times t} \\ -\Lambda_m\tilde{R} & -\Lambda_d\Lambda_m & -\Lambda_m\left(\tilde{S}_2 - \tilde{S}_1N_{11}^{-1}N_{12}\right)M_{22} \\ \mathbf{0}_{t\times n} & \mathbf{0}_{t\times n} & -(\Lambda_{T_i}\Lambda_{|E^0_{T_i}|} + \Lambda_{T_i}\Lambda_{n^0_i}M_{22}) \end{bmatrix},$$

$$\bar{B}_2 = \begin{bmatrix} \mathbf{0}_{n\times n} \\ -\Lambda_m\left(\tilde{S}_1N_{11}^{-1} - \tilde{S}_1N_{11}^{-1}N_{12}M_{21} + \tilde{S}_2M_{21}\right) \\ -\Lambda_{T_i}\Lambda_{n^0_i}M_{21} \end{bmatrix},$$

$$\bar{B}_1 = \begin{bmatrix} \mathbf{0}_{n\times n} \\ \Lambda_m \\ \mathbf{0}_{t\times n} \end{bmatrix}, \bar{B}_3 = \begin{bmatrix} \mathbf{0}_{n\times t} \\ \mathbf{0}_{n\times t} \\ \Lambda_{T_i} \end{bmatrix}, \;\; \begin{matrix} \Lambda_m = \text{diag}(\frac{1}{m_i})_{i=1}^n, \\ \Lambda_d = \text{diag}(\frac{1}{d_i})_{i=1}^n. \end{matrix}$$

III. Subsystem Representation and Controller Design

Each generator and OLTC are considered as subsystems of the interconnected system and can be represented as follows:

$$\begin{aligned} S_i: \quad \dot{x}_i &= A_i(\eta(t))x_i(t) + B_i(\eta(t))u_i(t) \\ &\quad + E_i(\eta(t))\xi_i(t) + L_i(\eta(t))r_i(t), \\ z_i &= C_i(\eta(t))x_i + D_i(\eta(t))u_i, \\ \zeta_i &= H_i(\eta(t))x_i(t) + G_i(\eta(t))u_i(t), \end{aligned} \tag{19}$$

where x_i is state-vector, u_i the control inputs, $\xi_i \in \mathbf{R}^{p_i}$ is the perturbation, $\zeta_i \in \mathbf{R}^{h_i}$ is the uncertainty output and $z_i \in \mathbf{R}^{q_i}$ is the controlled output of the subsystem. The input r_i describes the effect of the subsystems S_j, $j \neq i$, on the subsystem S_i. The input ξ_i describes the effect of local uncertain modeling errors in this subsystem. The variable $\eta(t)$ describes the mechanism of mode switching in the system. It is assumed that $\eta(t)$ is a homogeneous stationary Markov process taking values in a finite state set $\mathbf{K}$. Its state transition rate matrix is $Q := [q_{\nu\mu}]_{\nu,\mu=1}^{k}$ in which $q_{\nu\mu} \geq 0$, $\nu \neq \mu$ and $q_{\nu\nu} = -\sum_{\mu\neq\nu} q_{\nu\mu}$. This process will be assumed to be strictly stationary, its stationary initial distribution $\pi = [\pi_1, \ldots, \pi_k]$ will be assumed to have the property $\pi_j > 0$, $j = 1, \ldots, k$.

In the case of power systems let j indicate different load profiles. The general structure of system matrices and signals for subsystems representing generator and OLTC can be obtained from (18) as follows.

For the generators:

$$A_i(j) = \begin{bmatrix} \bar{A}(i,i) & \bar{A}(i,n+i) \\ \bar{A}(n+i,i) & \bar{A}(n+i,n+i) \end{bmatrix},$$

$$B_i(j) = \begin{bmatrix} \bar{B}_1(i,i) & \bar{B}_2(i,i) \\ \bar{B}_1(n+i,i) & \bar{B}_2(n+i,i) \end{bmatrix},$$

$$L_i(j) = \left[\begin{matrix} \bar{A}(i,l) & \bar{A}(i,n+l) \\ \bar{A}(n+i,n+l) & \bar{A}(n+i,n+l) \end{matrix} \right. \tag{20}$$
$$\left. \begin{matrix} \bar{A}(i,2n+k) & \bar{B}_1(i,l) & \bar{B}_2(i,l) \\ \bar{A}(n+i,2n+k) & \bar{B}_1(n+i,l) & \bar{B}_2(n+i,l) \end{matrix} \right],$$

where the elements for example $\bar{A}(i,l)$ refers to $(i,l)^{th}$ element of the matrix $\bar{A}$ and the signals are defined as,

$$x_i(t) = [\Delta\delta_i, \omega_i]^T, \quad u_i(t) = [\Delta P_{mi}, \Delta Q_{mi}]^T,$$

$$r_i(t) = [\Delta\delta_l, \omega_l, \Delta n_k, \Delta P_{ml}, \Delta Q_{ml}]^T$$

where $l = 1, \ldots, n$, and $j \neq i$; $k = 1, \ldots, t$.

For the OLTC:

$$A_i(j) = \left[\bar{A}(2n+i, 2n+i) \right], \; B_i(j) = \left[\bar{B}_3(2n+i, i) \right],$$

$$L_i(j) = \left[\bar{A}(2n+i,k), \bar{B}_1(2n+i,l), \bar{B}_2(2n+i,l)\right].$$

and the signals are defined as,

$$x_i(t) = [\Delta n_i], \quad u_i(t) = [\Delta |E_{ui}|],$$

$$r_i(t) = [\Delta n_k, \Delta P_{ml}, \Delta Q_{ml}]^T$$

where $l = 1, \ldots, n$; $k = 1, \ldots, t$ and $k \neq i$.

Decentralised controller design for a subsystem represented by the equation (19) is done with the procedures outlined in [7], which makes use of certain assumptions about the magnitude of uncertain perturbations and interconnections between subsystems. Although perturbations and interconnection signals are not known, their magnitude was assumed to satisfy magnitude constraints expressed in terms of time domain Integral Quadratic constraints of the form

$$\mathcal{E} \int_0^{t_l} \left(\|\zeta_i(t)\|^2 - \|\xi_i(t)\|^2 \right) dt \geq -x_{i0}' M_i x_{i0}, \tag{21}$$

$$\mathcal{E} \int_0^{t_l} \left(\sum_{\mu\neq i} \|\zeta_\mu(t)\|^2 - \|r_i(t)\|^2 \right) dt \geq -x_{i0}' \hat{M}_i x_{i0}, \tag{22}$$
$$\forall i = 1, \ldots, N;$$

here $M_i = M_i' > 0$, $\hat{M}_i = \hat{M}_i' > 0$ and $\{t_l\}_{l=1}^{\infty}$, $t_l \to +\infty$, is a sequence of time instants. The sets of admissible uncertainty inputs and admissible interconnection inputs $\xi_i(t), r_i(t)$, satisfying (21) and (22), will be denoted by Ξ, Π respectively.

The controllers considered are decentralized linear state feedback controllers of the form

$$\begin{aligned} \dot{x}_{c,i} &= A_{c,i}(\eta(t))x_{c,i}(t) + B_{c,i}(\eta(t))x_i(t); \\ u_i &= K_{c,i}(\eta(t))x_{c,i}(t), \end{aligned} \tag{23}$$

where $x_{c,i} \in \mathbf{R}^{n_{c,i}}$ is the ith controller state vector.

Having the uncertainties, controller structure and Markov jump parameters defined, we can find the decentralised controllers as follows: Let $\tau_i > 0$, $\theta_i > 0$, $i = 1, \ldots, N$, be given constants, and $\bar{\theta}_i = \sum_{j=1, j\neq i}^{N} \theta_j$. We consider a collection of the game-type algebraic Riccati equations

$$\begin{aligned} & A_i(j)'X_i(j) + X_i(j)A_i(j) + \bar{C}_i(j)'\bar{C}_i(j) \\ & -X_i(j)[B_i(j)R_i^{-1}(j)B_i(j)' - \bar{B}_{2,i}(j)\bar{B}_{2,i}'(j)]X_i(j) \\ & + \sum_{\nu=1}^{k} q_{j\nu} X_i(\nu) = 0, \qquad j = 1, \ldots, k, \end{aligned} \tag{24}$$

where $R_i(j) = \bar{D}_i'(j)\bar{D}_i(j)$,

$$\bar{C}_i(j) = \begin{bmatrix} C_i(j) \\ (\tau_i + \bar{\theta}_i)^{1/2} H_i(j) \end{bmatrix}, \bar{D}_i(j) = \begin{bmatrix} D_i(j) \\ (\tau_i + \bar{\theta}_i)^{1/2} G_i(j) \end{bmatrix},$$
$$\bar{B}_{2,i}(j) = \begin{bmatrix} \tau_i^{-1/2} E_i, \; \theta_i^{-1/2} L_i \end{bmatrix}.$$

Associated with the Riccati equations (24) is a collection of decentralized static state feedback controllers of the form

$$u_i(t) = K_i(\eta(t))x_i(t), \tag{25}$$

$$K_i(j) = -R_i^{-1}(j)B_i(j)'X_i(j), \tag{26}$$

and the corresponding systems

$$\dot{x}_i = (A_i(\eta(t)) + B_i(\eta(t))K_i(\eta(t)))x_i(t). \tag{27}$$

Furthermore, consider a set of vectors $\mathcal{T} = \{\{\, \tau_i \;\, \theta_i \,\}_{i=1}^{N} \in \mathbf{R}^{2N}, \tau_1 > 0, \theta_i > 0\}$ satisfying the condition: For all $i = 1, \ldots, N$, equation (24) has a solution $\{X_i(j) = X_i'(j) > 0,\; j \in \mathbf{K}\}$, such that the jump parameter systems (27) are mean-square stable.

When the set $\mathcal{T}$ is not empty, then the worst case performance achievable via decentralised controller (23) is,

$$\begin{aligned} & \inf_{u_i,\; i=1,\ldots,N} \sup_{\Xi,\Pi} \mathcal{E} \int_0^\infty \sum_{i=1}^{N} \|z_i\|^2 dt \\ & = \inf_{\mathcal{T}} \sum_{i=1}^{N} x_{i0}' \left[\sum_{j=1}^{k} \pi_j X_i(j) + \tau_i M_i + \theta_i \hat{M}_i \right] x_{i0}. \end{aligned} \tag{28}$$

Suppose the infimum on the right-hand side of (28) be attained at $\tau_i^*, \theta_i^*,\ i = 1, \ldots, N$. Then, the minimax optimal controller of the optimal worst-case control problem on the left-hand side of (28) is given by the decentralized controller (25), (26) in which $\tau_i = \tau_i^*$, $\theta_i = \theta_i^*$, $i = 1, \ldots, N$.

IV. Example: Control design for a nine-bus power system

To demonstrate the design, a nine bus power grid system consisting of 3 generator buses, 3 load buses, 2 SVSs and one OLTC is considered here. One-line diagram of the example system is shown in Figure 1. The OLTC node has a purely inductive load connected to it. Each machine is considered a subsystem interconnected by two weak tie-lines to the other two machines.

The jump parameter three-machine system switches between two load profiles. The load profiles are given in Table I. The two load profiles are chosen to have an open-loop system with small damping. Between the two load profiles there is a big change in both the real and reactive loads. The P_{L_i} are in MW and Q_{L_i} are in MVars. The system jumps between the above two load conditions in a random way, described by the Markov chain parameter $\eta(t)$ whose transition probabilities are determined by the parameters $q_{12} = q_{21} = 0.1$, and the initial distribution is $\pi = [0.5\ 0.5]'$.

The numerical value of system parameters is as follows: line impedances are: $z_{15} = \jmath 0.0576$ pu, $z_{27} = \jmath 0.0625$ pu, $z_{39} = \jmath 0.0586$ pu, $z_{45} = (0.5 + \jmath 4.3) \times 10^{-3}$ pu, $z_{56} = (0.9 + \jmath 4.6) \times 10^{-3}$ pu, $z_{47} = (1.6 + \jmath 8) \times 10^{-3}$ pu, $z_{78} = (0.4 + \jmath 3.6) \times 10^{-3}$ pu, $z_{89} = (0.6 + \jmath 5) \times 10^{-3}$ pu, and $z_{69} = (0.2 + \jmath 8.5) \times 10^{-3}$ pu. All the three machines are assumed to have unity damping, i.e., $d_i = 1$ s/rad, $i = 1, 2, 3$; the inertias of the generators are: $m_1 = 8.1$ s^2/rad, $m_2 = 10.39$ s^2/rad, $m_3 = 8.59$ s^2/rad and the synchronous impedances are: $x_{d_1} = 0.146$ pu, $x_{d_2} = 0.155$ pu, $x_{d_3} = 0.13$ pu (on a 100 MVA base).

Bus	Profile 1	Profile 2
1	$23 + \jmath 0.14$	$185 + \jmath 110$
2	$31 + \jmath 13.8$	$100 + \jmath 77$
3	$21 + \jmath 14$	$60 + \jmath 78$
4	$0 - \jmath 10$	$0 - \jmath 15$
5	$-25 - \jmath 10$	$-125 - \jmath 40$
6	$-20 - \jmath 15$	$-100 - \jmath 45$
7	$0 + \jmath 4$	$0 + \jmath 5$
8	$-30 - \jmath 10$	$-120 - \jmath 35$
9	$0 + \jmath 3$	$0 + \jmath 4$

TABLE I
Load Profiles (MW, MVars)

A. Controller

The interconnected power system model in (18) for the system in Figure 1 is written as subsystems S_i given in (19). Numerical values of matrices $E_i(j)$, $C_i(j)$, $D_i(j)$, $H_i(j)$ and $G_i(j)$ used for this design for generator subsystems are

$$[C_i(j)\ D_i(j)] = 0.1 * \left[\begin{array}{c|c} \mathbf{I}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{I}_2 \end{array}\right],$$

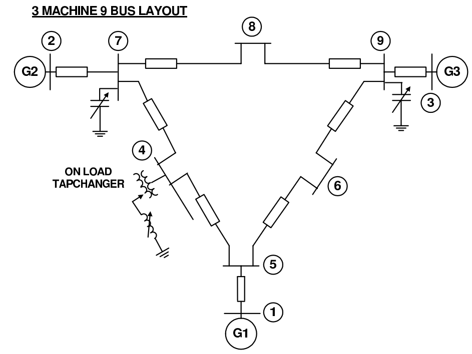

Fig. 1. One-Line Diagram on Nine-Bus Three-Generator System

$$E_i(j) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, [H_i(j)\ G_i(j)] = 0.2 * \left[\begin{array}{c|c} \mathbf{I}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{I}_2 \end{array}\right],$$

and for OLTC subsystems are

$$[C_i(j)\ D_i(j)] = \left[\begin{array}{c|c} 0.1 & 0.0 \\ 0.0 & 0.1 \end{array}\right],$$

$$E_i(j) = \begin{bmatrix} 0.1 \end{bmatrix}, [H_i(j)\ G_i(j)] = \left[\begin{array}{c|c} 0.2 & 0.0 \\ 0.0 & 0.2 \end{array}\right].$$

Also, we let $M_i = \hat{M}_i = \mathbf{I}_2$, and $x_{i,0} = [0.1\ 0.1]'$ for the generators; $M_i = \hat{M}_i = 1$ and $x_{i,0} = 0.141$ for the OLTC. The controller is obtained by solving the Riccati equations (24) by using the convex algorithm given in [16] and the cost is globally minimised. With the above chosen parameter values, it was found that the function (28) attained a minimum at: $\tau_1^* = 0.0220$, $\tau_2^* = 0.0308$, $\tau_3^* = 0.0249$, $\tau_4^* = 0.0001$, $\theta_1^* = 0.0132$, $\theta_2^* = 0.0163$, $\theta_3^* = 0.0093$, $\theta_4^* = 0.0001$, resulting in the minimax value of the performance cost 0.0098. These values of the parameters were then used in (25), (26) to obtain a controller as follows:

$$[K_1(1)\ K_1(2)] = \left[\begin{array}{cc|cc} -0.1196 & -1.2244 & -0.1583 & -1.3515 \\ 0.0679 & 0.6950 & 0.1261 & 1.0766 \end{array}\right],$$

$$[K_2(1)\ K_2(2)] = \left[\begin{array}{cc|cc} -0.1240 & -1.4628 & -0.1551 & -1.4820 \\ 0.0941 & 1.1101 & 0.0813 & 0.7769 \end{array}\right],$$

$$[K_3(1)\ K_3(2)] = \left[\begin{array}{cc|cc} -0.1138 & -1.2856 & -0.1399 & -1.2652 \\ 0.0591 & 0.6675 & 0.0493 & 0.4455 \end{array}\right],$$

$$[K_4(1)\ K_4(2)] = \left[\begin{array}{c|c} -0.9362 & -0.9303 \end{array}\right].$$

B. Simulation Results

Simulation is performed for the nonlinear power system dynamics, given by equations (8) and (10), with the decentralised linear controller obtained above. The following initial conditions were used for the simulation:

$$\Delta\delta_i = 0.1, \Delta\omega_i = 0.01, i = 1, 2, 3;\ \Delta n = 0.005.$$

The units for $\Delta\delta$'s and $\Delta\omega$'s are radians and radians/second respectively. The load conditions and the corresponding controllers are switched at t = 40s, 80s and 120s. Simulations show that the disturbance created by initial conditions and due to load variations are contained rapidly. Figures 2 and 3 compares the responses of generators and OLTC tap-setting with and without controller. From the

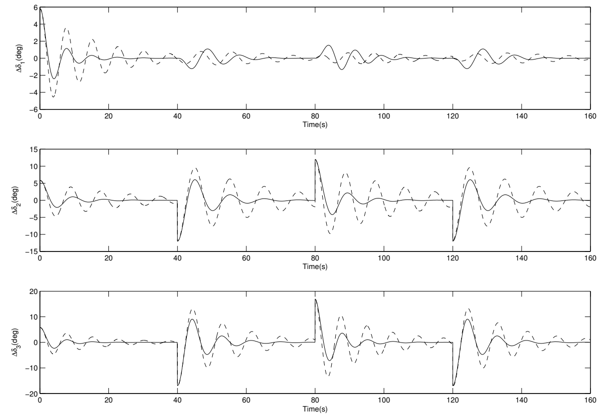

Fig. 2. Nonlinear simulation Generator 1 to 3 rotor angles ($\Delta\delta_1$ to $\Delta\delta_3$). The solid line indicates the closed loop response (with controller) and the dash line indicates the open loop response.

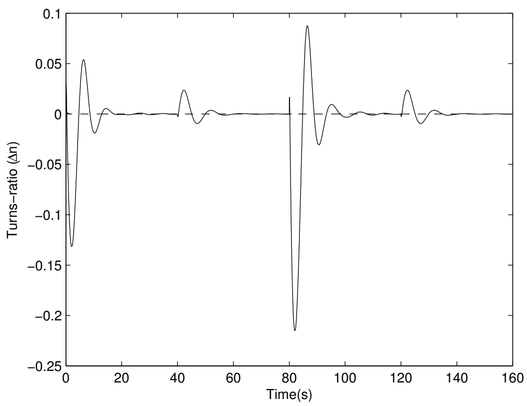

Fig. 3. OLTC tap position (Δn) - thick line indicates the response with controller and dotted without controller

figures it can be seen that the oscillations are damped much faster than their natural damping time.

In most power systems OLTC controllers are set to achieve a constant secondary voltage but in this decentralised controller design the set voltage of the OLTC is varied to damp the oscillations. This can be seen in Figure 3. The OLTC voltage setting changes by a considerable amount just when the oscillation magnitude peaks showing the effect of the decentralised control of the OLTC tap-setting.

Further to evaluate the robustness of the system under load perturbations, the load and generation in Profile 1 is reduced by -20% and Profile 2 by +20%. Simulations are carried out with the controllers designed for the nominal system and the nonlinear dynamics of the perturbed grid conditions. The simulation shows that there is very little degradation in performance proving the robustness of the designed controller.

V. Conclusion

In this paper power system model incorporating the dynamics of generators and OLTC is presented along with a method to design robust controller using Markov jump parameter systems framework. This paper demonstrates the effectiveness of a decentralised robust controller for a system whose load profiles vary and give rise to lightly damped oscillation modes. The simulations results show that the controller, with the use of OLTC, is capable of quickly damping out the disturbances induced in the power system. The simulation shows that the switching transients of the controllers when the load profile changes are also contained.

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