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An I&I Adaptive Redesign Approach for Asymptotic Stability without PE

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Abstract: This paper proposes an adaptation redesign approach of the immersion and invariance (I&I) adaptive control to achieve asymptotic stability of the controlled plant and the parameter estimator at the desired equilibrium. The key idea is to employ the technique of generalized parameter estimation-based observer on the parameter estimation error dynamics by applying the indirect I&I adaptive control scheme, yielding a linear regression equation, from which the adaptive law can be redesigned. As a result, it is shown that globally exponential parameter convergence can be guaranteed under an interval excitation (IE) condition, which is much weaker than the conventionally required persistent excitation. Under a stabilizability assumption, an adaptation-redesigned feedback control law can be designed to achieve global asymptotic stability at the desired equilibrium point under the IE condition. The proposed adaptive control approach is applied to a class of parameteric strict-feedback systems without overparameterization, which is needed by the standard I&I adaptive control.

Keywords: Immersion and invariance, parameter estimation, adaptation redesign

1. INTRODUCTION

With the success in the flight control in 1950s, adaptive control has been regarded as an effective and promising tool to handle systems with uncertain parameters Sastry and Bodson (2011); Zhang et al. (2019, 2021). In general, in the adaptive controller design an estimator is usually incorporated, providing an estimate of uncertain parameters of the plant for the indirect approach or the controller for the direct approach. These estimates are then used to drive a feedback control law such that the desired asymptotic tracking/stabilization is achieved. In last decades, several systematic design approaches have been reported in the literature, such as the model reference adaptive control Tao (2003), the Lyapunov-based adaptive control Krstic et al. (1995), and the immersion and invariance (I&I) adaptive control Astolfi et al. (2008). In addition to regulating the plant states to the prescribed points or trajectories asymptotically, an asymptotic parameter estimate may also be desired. In other words, the control object may be to achieve asymptotic stability of the interconnected system of the plant and the parameter estimator. To achieve the parameter convergence, the aforementioned approaches usually require the regression term to satisfy certain persistent excitation (PE) conditions Tao (2003); Wang and Kellett (2019, 2021), which are quite restrictive and may not be satisfied in practical applications.

The PE condition is a necessary and sufficient condition for the *uniform* parameter convergence in adaptive systems Sastry and Bodson (2011). If the uniformity is not pursued, it has been shown in Wang et al. (2021) that the unknown parameters are identifiable if and only if the corresponding regressor is excited over a finite interval, i.e., the interval excitation (IE) condition Chowdhary et al. (2014); Pan and Yu (2015). [1] It is clear that the IE is strictly weaker than the PE by removing the requirement of persistence. Along this line, emerging research attention has recently been devoted to achieving parameter convergence without PE. In Cho et al. (2017); Pan and Yu (2015); Aranovskiy et al. (2017); Bobtsov et al. (2022); Yi and Ortega (2022), the filtering techniques are explored to generate algebraic regression equations, by which new parameter estimators are proposed, guaranteeing parameter convergence with IE. Similar ideas have been extended to handle systems with time-varying parameters in Gaudio et al. (2021). In Wang et al. (2021), for model reference adaptive control of linear systems, the authors firstly propose to establish an gradient-descent estimator, which is then utilized to generate a linear regression equation for the estimator redesign, yielding globally exponential parameter convergence under the IE.

In this paper, we focus on the I&I adaptive control method which has been widely applied in many fields Astolfi et al. (2008); Shao et al. (2021). Given the estimation error dynamics following the I&I adaptive scheme, the technique

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[1] Note that the notion of the finite excitation in Cho et al. (2017) is fundamentally equivalent to the IE.

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of the generalized parameter estimation-based observer (GPEBO) Ortega et al. (2021) is employed to derive a linear regression equation. As a result, a redesign approach of the adaptive law is proposed, guaranteeing a globally exponential parameter convergence with an IE condition. Further, with an ISS-type stabilizability condition on the controlled plant, it is shown that the closed-loop system consisting of the plant and the redesigned estimator can be rendered to be globally bounded, and globally asymptotically stable at the desired equilibrium point if the IE condition is satisfied. To illustrate the effectiveness of the proposed control scheme, we apply it to a class of parametric strict-feedback systems without overparameterization, which is required by the standard I&I adaptive control Astolfi et al. (2008).

The paper is organized as below. The considered problem is formulated in Section 2, and Section 3 presents the proposed adaptive redesign scheme, which is applied to handle a class of parameteric strict-feedback systems in Section 4. A brief conclusion is made in Section 5.

2. PROBLEM STATEMENT

Consider the following uncertain system

$$\dot{x} = \varphi(t,x)\theta + f(t,x,u) \tag{1}$$

with state $x \in \mathbb{R}^n$, control $u \in \mathbb{R}^m$, uncertain parameter $\theta \in \Theta \subseteq \mathbb{R}^q$, and initial time $t_0 \geq 0$. In this paper, suppose all mappings in (1) are smooth. The control objective is to design an adaptive state-feedback controller of the form

$$\begin{aligned} \dot{x}_c &= g_c(t,x_c,x) \\ u &= h_c(t,x_c,x) \end{aligned} \tag{2}$$

such that the closed-loop (x,x_c)-system is globally asymptotically stable at the equilibrium point $(x,x_c) = (0,x_c^*)$. If the x_c subsystem denotes a parameter estimator of θ, then it is expected that $x_c^* = \theta$.

Following the indirect I&I adaptive control scheme Astolfi et al. (2008), the adaptive law is designed as

$$\dot{\hat{\theta}}_{\rm ii} = -\frac{\partial\beta}{\partial x}(t,x)\big[\varphi(t,x)(\hat{\theta}_{\rm ii}+\beta(t,x))+f(t,x,u)\big]-\frac{\partial\beta}{\partial t}(t,x)\,, \tag{3}$$

where the off-the-manifold function $\beta(t,x)$ can be chosen as a solution of the partial differential equation (PDE)

$$\frac{\partial\beta}{\partial x}(t,x) = \gamma\varphi(t,x)^\top \tag{4}$$

with the design parameter $\gamma > 0$. By denoting the estimation error $\tilde{\theta}_{\rm ii} := \hat{\theta}_{\rm ii} + \beta(t,x) - \theta$, one then can obtain

$$\dot{\tilde{\theta}}_{\rm ii} = -\gamma\varphi(t,x(t))^\top\varphi(t,x(t))\tilde{\theta}_{\rm ii}\,. \tag{5}$$

Thus, let $V_{\rm ii}(\tilde{\theta}_{\rm ii}) = \frac{1}{2}\|\tilde{\theta}_{\rm ii}\|^2$, whose time derivative along (5) is given by

$$\dot{V}_{\rm ii}(\tilde{\theta}_{\rm ii}) = -\gamma\|\varphi(t,x)\tilde{\theta}_{\rm ii}\|^2\,. \tag{6}$$

To further illustrate how the above property can be utilized to design the control law to regulate the closed-loop system, we consider a simple example of (1) with $f(t,x,u) = u$. Then we design the control law as

$$u = -\kappa(x) - x - \varphi(t,x)(\hat{\theta}_{\rm ii} + \beta(t,x))\,,$$

which, with $V_x(x) = \frac{1}{2}\|x\|^2$, yields $\dot{V}_x(x) = -x^\top\kappa(x) - \|x\|^2 - x^\top\varphi(t,x)\tilde{\theta}_{\rm ii}$. As a consequence, by choosing the Lyapunov function $V_{cl}(x,\tilde{\theta}_{\rm ii}) = V_x(x) + \frac{5}{4\gamma}V_{\rm ii}(\tilde{\theta}_{\rm ii})$ for the closed-loop $(x,\tilde{\theta}_{\rm ii})$-system and using the Young's inequality, we can obtain

$$\dot{V}_{cl} \leq -x^\top\kappa(x) - \|\varphi(t,x)\tilde{\theta}_{\rm ii}\|^2\,. \tag{7}$$

By choosing the design function $\kappa(x)$ such that $x^\top\kappa(x) > 0$ for nonzero x, one immediately obtains global stability of the $(x,\tilde{\theta}_{\rm ii})$-system at the zero equilibrium and asymptotic convergence of $x(t)$ and $\varphi(t,x)\tilde{\theta}_{\rm ii}$ to zero from (7). If the regression $\varphi(t,x(t))$ satisfies the PE condition (see Definition 1 below), it is known that the global asymptotic stability of the equilibrium $(x,\tilde{\theta}_{\rm ii}) = (0,0)$ can be concluded. However, it is well-known that the PE condition is indeed rather restrictive, which may fail to be fulfilled in many practical applications. This thus motivates us to develop new adaptive control schemes that can relax the required PE condition for asymptotic stability.

Remark 1. To implement the adaptive law (3), the PDE (4) needs to be solved to derive an off-the-manifold function β. To overcome the obstacle of solving the PDE, as proposed in Dimitrios et al. (2009) one may employ a filter of (1) for an estimate of β satisfying (3) and then the dynamical scaling technique to compensate for the mismatch between the state x and its filtered value. □

Remark 2. In addition to the I&I adaptive control, another class of widely used adaptive control approaches are based on the direct cancellation of uncertain terms appearing in the time derivative of the plant's Lyapunov function $V_x(x)$ (see e.g., Krstic et al. (1995)). For such approaches, one eventually can obtain the global stability at the equilibrium, and $\lim_{t\to\infty} x(t) = 0$, while for the closed-loop asymptotic convergence, one still needs the PE condition on the regressor. □

Instrumental to the forthcoming analysis is the notions of PE and IE below.

Definition 1. The regressor $\varphi(t,x(t))$ is said to be

(i) PE, if there exist constants $T_0, \delta_0 > 0$ such that

$$\int_t^{t+T_0} \varphi(\tau,x(\tau))^\top\varphi(\tau,x(\tau))d\tau \geq \delta_0 I\,, \quad \forall t \geq t_0\,.$$

(ii) IE, if there exist constants $T_0, \delta_0 > 0$ such that

$$\int_{t_0}^{t_0+T_0} \varphi(\tau,x(\tau))^\top\varphi(\tau,x(\tau))d\tau \geq \delta_0 I\,. \tag{8}$$

The above definitions depend on the system flow $x(t)$, which is closely connected to the uniform PE in Panteley et al. (2001). Since we do not target to achieve uniform stability, there is no need to distinguish the uniformity throughout the paper. Note that in contrast with the PE condition, the IE condition is clearly weaker, which has been shown to be necessary and sufficient for globally exponentially convergent online parameter estimation from linear regression equations Wang et al. (2021). In the following, we will redesign the I&I adaptation law (3), which enables to achieve the global asymptotic stability under the weaker IE condition.

3. I&I ADAPTIVE CONTROLLER REDESIGN

3.1 I&I Adaptation Redesign

In the following, an I&I adaptation redesign approach is developed for an exponential parameter estimation under the IE condition of $\varphi(t, x)$.

We first introduce an auxiliary system of the form

$$\dot{\Phi} = -\gamma\varphi(t, x(t))^\top \varphi(t, x(t))\Phi\,, \qquad \Phi(t_0) = I\,, \quad (9)$$

with $\Phi \in \mathbb{R}^{q\times q}$. The solution $\Phi(t)$ of (9) satisfies the following property, with the proof given in Appendix A.

Lemma 1. Suppose $\varphi(t, x(t))$ is IE in the sense of Definition 1.(ii). Then there exists $1 \geq \epsilon^* > 0$ such that

$$\rho(\Phi(t)) \leq (1 - \epsilon^*)\,, \qquad \forall t \geq t^* := t_0 + T_0 \quad (10)$$

where $\rho(\cdot)$ denotes the spectral radius. □

Bearing this in mind, from (9) we can explicitly express the solution of $\tilde{\theta}_{\rm ii}$ in (5) as

$$\tilde{\theta}_{\rm ii}(t) = \Phi(t)\tilde{\theta}_{\rm ii}(t_0) = \Phi(t)[\hat{\theta}_{\rm ii}(t_0) + \beta(t_0, x(t_0)) - \theta]\,.$$

Without loss of generality, we let $\hat{\theta}_{\rm ii}(t_0) = 0$. Then by recalling $\tilde{\theta}_{\rm ii}(t) := \hat{\theta}_{\rm ii}(t) + \beta(t, x(t)) - \theta$, we have

$$(I - \Phi)\theta = (\hat{\theta}_{\rm ii} + \beta) - \Phi\beta_0 \quad (11)$$

where we have denoted $\beta_0 = \beta(t_0, x(t_0))$ and omitted the arguments of $\Phi, \hat{\theta}_{\rm ii}, \beta$ for simplicity. The above design and analysis indeed are motivated by the idea of GPEBO Ortega et al. (2021). Note that $(I - \Phi(t))$ is nonsingular for $t \geq t^*$ under the IE condition of $\varphi(t, x)$ by Lemma 1.

To this end, we arrive at the linear regression equation (11) with unknown θ, from which online estimation approaches can be applied to design an estimator of θ, yielding a redesigned adaptive law.

We next follow the idea in Aranovskiy et al. (2017) and transform (11) into q scalar equations by multiplying the adjugate matrix of $\Phi - I$, denoted by $\mathrm{adj}(\Phi - I)$, on both sides of (11), yielding

$$\det(I - \Phi)\theta = \Lambda \quad (12)$$

where we define

$$\Lambda := \mathrm{adj}(I - \Phi)\left[\hat{\theta}_{\rm ii} + \beta - \Phi\beta_0\right]\,, \quad (13)$$

which is available for feedback design.

By (12), we propose the following adaptation law as

$$\dot{\hat{\theta}} = -k\Delta\left[\det(I - \Phi)\hat{\theta} - \Lambda\right] \quad (14)$$

with $k > 0$ and

$$\Delta = \frac{\det(I - \Phi)}{1 + |\det(I - \Phi)|^2}\,. \quad (15)$$

We conclude this subsection by the following result.

Proposition 1. For the adaptive law (3), (9), (14), the resulting trajectories of $(\Phi, \hat{\theta})$ are bounded. Moreover, if the regressor $\varphi(t, x(t))$ is IE, then the $\hat{\theta}$-system (14) is globally exponentially stable at the equilibrium $\hat{\theta} = \theta$. □

Proof. The boundedness of Φ is obvious as it is a state transition matrix of the stable system (5). We denote the redesigned estimation error as $\tilde{\theta} = \hat{\theta} - \theta$. Computing its time derivative along (14) gives

$$\dot{\tilde{\theta}} = -\frac{k|\det(I - \Phi)|^2}{1 + |\det(I - \Phi)|^2}\tilde{\theta}\,. \quad (16)$$

It immediately follows that $\tilde{\theta}$ is stable at the origin and $\hat{\theta}$ are bounded.

If the regression $\varphi(t, x(t))$ is IE, we let $V_{\tilde{\theta}}(\tilde{\theta}) = \frac{1}{2}\|\tilde{\theta}\|^2$, whose time derivative is

$$dV_{\tilde{\theta}}/dt = -k\Delta\|\tilde{\theta}\|^2 \leq 0\,. \quad (17)$$

Moreover, with the IE of $\varphi(t, x)$ and by Lemma 1, we have

$$1 \geq |\det(I - \Phi(t))| \geq \epsilon^{*q}\,, \qquad \forall t \geq t^*\,.$$

As a result, (17) implies

$$dV_{\tilde{\theta}}/dt \leq -2k\frac{\epsilon^{*2q}}{1 + \epsilon^{*2q}}V_{\tilde{\theta}}\,, \qquad \forall t \geq t^*\,.$$

Therefore, the $\tilde{\theta}$-system is globally exponentially stable at the origin if $\varphi(t, x(t))$ is IE, completing the proof. □

3.2 Feedback Control Design

In the following, we elaborate an explicit stabilizability assumption, which induces a feedback control law that together with the redesigned adaptation law (14) can solve the adaptive control problem in question.

Assumption 1. There exist smooth mappings $\Psi : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}^m$ and $\mathcal{T} : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}^n$ such that

(i) for each $(t, \theta) \in \mathbb{R}_+ \times \mathbb{R}^q$, $\mathcal{T}(t, \cdot, \theta)$ is a global diffeomorphism with $\mathcal{T}(t, 0, \theta) = 0$;

(ii) denoting $z = \mathcal{T}(t, x, \hat{\theta})$, the system

$$\begin{aligned}\dot{z} &= \nabla_x\mathcal{T}[\varphi(t, x)\theta + f(t, x, \Psi(t, x, \hat{\theta}))] \\ &\quad + \nabla_t\mathcal{T} + \nabla_{\hat{\theta}}\mathcal{T}\dot{\hat{\theta}}\end{aligned} \quad (18)$$

permits a function $V_z(t, z)$ satisfying

$$\begin{aligned}&\alpha_1(\|z\|) \leq V_z(t, z) \leq \alpha_2(\|z\|) \\ &\dot{V}_z \leq -\alpha_3(\|z\|) + \zeta_1(\|\theta - \hat{\theta}\|) + \zeta_2(\|\dot{\hat{\theta}}\|)\end{aligned} \quad (19)$$

where $\alpha_i \in \mathcal{K}_\infty$, $i = 1, \ldots, 3$, and ζ_1, ζ_2 are nondecreasing positive continuous functions, depending on the unknown parameter θ and vanishing at the origin.

With this assumption, we design the control law as

$$u = \Psi(t, x, \hat{\theta}) \quad (20)$$

where $\hat{\theta}$ is given by (14). As a result, we complete the design of the adaptive controller (2) consisting of the redesigned adaptation law (14) and the feedback control law (20), yielding the following result.

Theorem 1. For the system (1), suppose Assumption 1 holds, and let the adaptive controller (2) consist of the adaptive law (3), (9), (14) and the feedback control law (20). Then the resulting $(x, \hat{\theta})$-system trajectories are globally bounded. Moreover, if the regressor $\varphi(t, x(t))$ is IE, the equilibrium $(x, \hat{\theta}) = (0, \theta)$ of the closed-loop system (1), (14) is globally asymptotically stable. □

Proof. With the controller (3), (9), (14) and (20), by Assumption 1, we have

$$\dot{V}_z \leq -\alpha_3(\|z\|) + \zeta_1(\|\tilde{\theta}\|) + \zeta_2(\|\dot{\hat{\theta}}\|) \quad (21)$$

with $z = \mathcal{T}(t, x, \hat{\theta})$. By Lemma 1, it can be verified that $|\det(I - \Phi(t))| \leq 1$ for all $t \geq t_0$. Thus, it follows that $\|\dot{\hat{\theta}}\| \leq k\Delta\|\tilde{\theta}\| \leq \frac{k}{2}\|\tilde{\theta}\|$. Substituting this to (21) implies

$$\dot{V}_z \leq -\alpha_3(\|z\|) + \alpha_4(\|\tilde{\theta}\|)$$

with $\alpha_4 \in \mathcal{K}$ such that $\alpha_4(\|\tilde{\theta}\|) \geq \zeta_1(\|\tilde{\theta}\|) + \zeta_2(\frac{k}{2}\|\tilde{\theta}\|)$, yielding that the z-system is uniformly ISS with respect to input $\tilde{\theta}$ and state z. In combination with Proposition 1, we then can conclude that z is bounded, and the zero equilibrium $(z, \tilde{\theta}) = 0$ is globally asymptotically stable if the IE of $\varphi(t, x(t))$ is satisfied. This, by recalling that $\mathcal{T}(t, \cdot, \theta)$ is a global diffeomorphism and $\mathcal{T}(t, 0, \theta) = 0$ for all $(t, \theta) \in \mathbb{R}_+ \times \mathbb{R}^q$, completes the proof. □

Remark 3. Different from the conventional stabilizability conditions in Krstic et al. (1995); Astolfi et al. (2008) that are established on the original coordinate x, Assumption 1 implies the existence of a change of coordinate z defined by the mapping $\mathcal{T}$ such that the resulting z-system with control (20) is ISS with respect to the inputs $\theta - \hat{\theta}$ and $\dot{\hat{\theta}}$.

4. AN ILLUSTRATIVE EXAMPLE

In this section, we apply the previously established scheme to the stabilization problem of the parametric strict-feedback systems of the form

$$\begin{aligned} \dot{x}_1 &= x_2 + \varphi_1(t, x_1)\theta \\ \dot{x}_2 &= u + \varphi_2(t, x)\theta \end{aligned} \tag{22}$$

with state $x = \text{col}(x_1, x_2)$, control u and uncertain parameter $\theta \in \mathbb{R}^q$. Suppose all mappings are smooth, and $\varphi_1(t, 0)\theta = 0$ for all $t \in \mathbb{R}_+$ and $\theta \in \mathbb{R}^q$, which guarantees that the zero-equilibrium stabilization problem of the system (22) is feasible. Note that the subsequent design and analysis can be adapted to more general parametric strict-feedback systems with state dimension $n > 2$.

Firstly, the redesigned I&I adaptation law (1), (9), (14) can be directly applied with $f(t, x, u) = \text{col}(x_2, u)$ and $\varphi(t, x) = \text{col}(\varphi_1(t, x_1), \varphi_2(t, x))$. In view of this, the controller design remains to design the feedback control to satisfy Assumption 1, for which we adopt the backsteping-based approach Krstic et al. (1995), consisting of the following two steps.

Step 1. Let

$$z_1 = x_1\,, \quad z_2 = -v_1 + x_2 + \varphi_1(t, x_1)\hat{\theta} \tag{23}$$

with the virtual control v_1 to be determined later. Along (22), we compute the time-derivative of z_1, yielding

$$\dot{z}_1 = v_1 + z_2 + \ell_1(t, x_1, \tilde{\theta}) \tag{24}$$

where the function ℓ_1 is defined by

$$\ell_1(t, x_1, \tilde{\theta}) = -\varphi_1(t, x_1)\tilde{\theta}\,.$$

Choose $V_1(z_1) = \frac{1}{2}z_1^2$, whose time derivative along (24) is given by

$$\dot{V}_1 \leq z_1\big(v_1 + \frac{1}{2}z_1 + \frac{1}{4}z_1\|\varphi_1\|^2\big) + \frac{1}{2}z_2^2 + \|\tilde{\theta}\|^2\,.$$

Thus, let

$$v_1(t, x_1, z_1) = -(k_1 + \frac{1}{2})z_1 - \frac{1}{4}z_1\|\varphi_1\|^2\,, \quad k_1 > 0 \tag{25}$$

which implies

$$\dot{V}_1 \leq -k_1 z_1^2 + \frac{1}{2}z_2^2 + \|\tilde{\theta}\|^2\,. \tag{26}$$

It is noted that with (23) and (25), the mapping $\mathcal{T}$ from x to $z := \text{col}(z_1, z_2)$ is a global diffeomorphism, and for all $t \in \mathbb{R}_+$ and $\hat{\theta} \in \mathbb{R}^q$, $z = 0$ if and only if $x = 0$, indicating that Assumption 1.(i) is satisfied.

Step 2. With (25) and letting

$$\psi_1(t, x_1, z_1, \hat{\theta}) = -v_1 + \varphi_1^\top\hat{\theta} \tag{27}$$

we have $z_2 = x_2 + \psi_1$, whose time derivative is given by

$$\begin{aligned} \dot{z}_2 &= u + \nabla_t\psi_1 + \nabla_{z_1}\psi_1 \cdot (v_1 + z_2) + \nabla_{x_1}\psi_1 \cdot x_2 \\ &\quad + (\varphi_2 + \nabla_{x_1}\psi_1 \cdot \varphi_1)^\top\theta + \nabla_{z_1}\psi_1 \cdot \ell_1 + \nabla_{\hat{\theta}}\psi_1 \cdot \dot{\hat{\theta}}\,. \end{aligned} \tag{28}$$

Here for convenience, for mapping $\psi(s_1, s_2, s_3, s_4)$ we denote $\nabla_{s_i}\psi = \frac{\partial\psi}{\partial s_i}(s_1, s_2, s_3, s_4)$, $i = 1, \ldots, 4$. Then let

$$\begin{aligned} u &= -v_2 - \nabla_t\psi_1 + \nabla_{z_1}\psi_1 \cdot (v_1 + z_2) \\ &\quad + \nabla_{x_1}\psi_1 \cdot x_2 + (\varphi_2 + \nabla_{x_1}\psi_1 \cdot \varphi_1)^\top\hat{\theta} \end{aligned} \tag{29}$$

with v_2 to be determined later. Thus, the time derivative of z_2 can be rewritten by

$$\dot{z}_2 = v_2 + \ell_2 + \rho_2\dot{\hat{\theta}} \tag{30}$$

where

$$\begin{aligned} \ell_2(x, z_1, \tilde{\theta}, \hat{\theta}) &= -\left(\nabla_{x_1}\psi_1 \cdot \varphi_1 + \varphi_2\right)^\top\tilde{\theta} + \nabla_{z_1}\psi_1 \cdot \ell_1 \\ \rho_2(x_1) &= \nabla_{\hat{\theta}}\psi_1\,. \end{aligned}$$

With $\hat{\theta} = \tilde{\theta} + \theta$, it can be verified that there exist positive functions h_2, κ_2, and a class $\mathcal{K}_\infty$ function $c_{\theta,2}(\cdot)$ such that

$$\begin{aligned} \|z_2\ell_2(x, z_1, \tilde{\theta}, \theta)\| &\leq z_2^2 h_2(x, z_1) + c_{\theta,2}(\|\tilde{\theta}\|) \\ \|z_2\rho_2(x_1)\dot{\hat{\theta}}\| &\leq z_2^2\kappa_2(x_1) + \|\dot{\hat{\theta}}\|^2\,. \end{aligned} \tag{31}$$

Thus, let

$$v_2(x, z) = -(k_2 + \frac{1}{2})z_2 - z_2 h_2(x, z_1) - z_2\kappa_2(x_1) \tag{32}$$

with $k_2 > 0$, and choose $V_2(z_2) = \frac{1}{2}z_2^2$, whose time derivative along (30) is given by

$$\begin{aligned} \dot{V}_2 &= z_2 v_2 + z_2\ell_2 + z_2\rho_1\dot{\hat{\theta}} \\ &\leq z_2 v_2 + \frac{z_2^2}{2} + z_2^2 h_2 + c_{\theta,2}(\|\tilde{\theta}\|) + z_2^2\kappa_2 + \|\dot{\hat{\theta}}\|^2 \\ &\leq z_2\big[v_2 + \frac{1}{2}z_2 + z_2 h_2 + z_2\kappa_2\big] + c_{\theta,2}(\|\tilde{\theta}\|) + \|\dot{\hat{\theta}}\|^2 \\ &\leq -k_2 z_2^2 + c_{21}\|\tilde{\theta}\|^2 + c_{22}\|\tilde{\theta}\|^4 + \|\dot{\hat{\theta}}\|^2\,. \end{aligned}$$

To this end, by letting $V_z = V_1 + V_2$, we have

$$\dot{V}_z \leq -k_1 z_1^2 - k_2 z_2^2 + (c_{21} + 1)\|\tilde{\theta}\|^2 + c_{22}\|\tilde{\theta}\|^4 + \|\dot{\hat{\theta}}\|^2\,,$$

which indicates that Assumption 1 is satisfied.

In summary, the proposed approach is applicable to handle the adaptive control problem of (22). To further illustrate the effectiveness, we consider a numerical example with $\varphi_1 = \text{col}(x_1, x_1^2)^\top$ and $\varphi_2 = \text{col}(x_2, 1)^\top$. By choosing the parameters $\gamma = 3$ and $k = k_1 = k_2 = 10$, the simulation results are presented in Fig. 1 and 2, where the trajectory of the spectral radius $\rho(\Phi)$ is less than one and both states x_1, x_2 and the parameter estimation errors $\tilde{\theta}$ asymptotically converge to zero, demonstrating the effectiveness of Lemma 1 and Theorem 1, respectively. In comparison, we follow the Lyapunov-based adaptive control approach with tuning function Krstic et al. (1995) and the I&I adaptive control with overparameteration Astolfi et al. (2008) to design adaptive controller for (22), respectively. The corresponding simulation results are given in Figs. 3 and 4, respectively, where the states x_1, x_2 asymptotically converge to zero while there is no

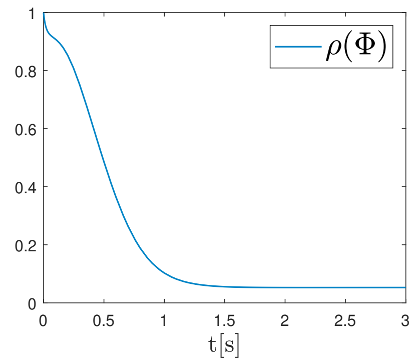

Fig. 1. Trajectory of the spectral radius $\rho(\Phi)$

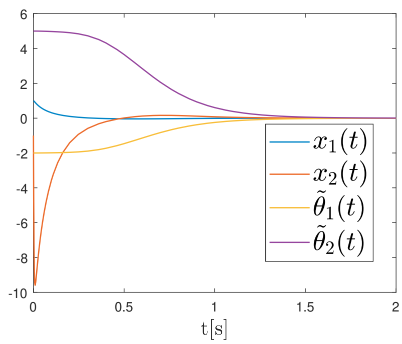

Fig. 2. Trajectories of states and estimation errors via the proposed approach

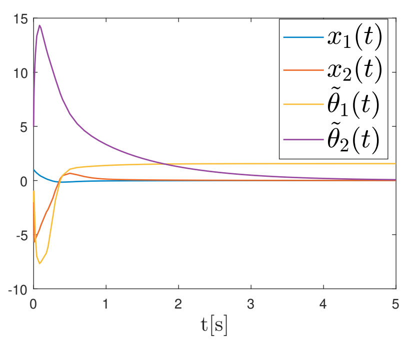

Fig. 3. Trajectories of states and estimation errors via the Lyapunov-based adaptive control Krstic et al. (1995)

guarantee of asymptotic convergence of the parameter estimation errors to zero.

Remark 4. We remark that the I&I adaptive control method has been applied to handle the system (22) in Astolfi et al. (2008), where the overparameterization is required to overcome the mismatch between the control input and the uncertain parameter. In contrast, by applying our proposed approach in Theorem 1, the overparameterization can be removed as in Krstic et al. (1995) where the technique of tuning function is used. □

5. CONCLUSION

In this paper, we proposed a new I&I adaptive controller design paradigm, where the GPEBO was employed to redesign the I&I adaptive law in such a way that parameter

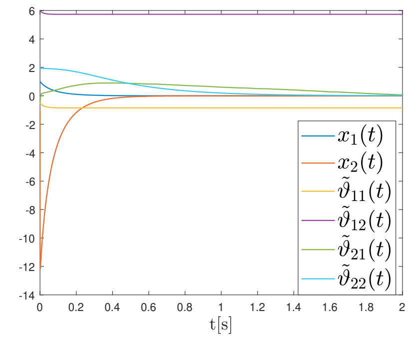

Fig. 4. Trajectories of states and estimation errors via the I&I adaptive control Astolfi et al. (2008)

estimation errors were steered to zero exponentially under a rather weak interval excitation (IE) condition. Under the IE condition and an ISS-type stabilizability assumption, it was shown that an adaptation-redesigned control law could be designed to achieve the globally asymptotic stability at the desired equilibrium point. The proposed adaptive control approach was applied to a class of parameteric strict-feedback system without overparameterization, which is needed by the standard I&I adaptive control.

It is known that for the standard I&I adaptive control Astolfi et al. (2008), the asymptotic convergence of the regression error $\|\varphi(t,x)\tilde{\theta}_{\text{ii}}\|$ can be concluded from (6), under which the state x may still be regulated to vanish asymptotically without the IE condition. In view of this, one may combine the I&I adaptation law (3) and the redesigned adaptation law (14), leading a composite adaptive law as

$$\hat{\theta} = \lambda(\hat{\theta}_1 + \beta(t,x)) + (1-\lambda)\hat{\theta}_2\,, \qquad \lambda \in (0,1)\,, \tag{33}$$

where β is given by (4), and $\hat{\theta}_1, \hat{\theta}_2$ satisfy

$$\begin{aligned} \dot{\hat{\theta}}_1 &= -\gamma\varphi(t,x)^\top\left[\varphi(t,x)\hat{\theta} + f(t,x,u)\right] - \nabla_t\beta(t,x) \\ \dot{\hat{\theta}}_2 &= -k\Delta\left[\det(I-\Phi)\hat{\theta} - \Lambda\right]\,. \end{aligned} \tag{34}$$

As a result, it can be easily verified that the regression error $\varphi(t,x)\tilde{\theta}_{\text{ii}}$ converges to zero, even if there is no IE condition. This thus preserves the benefits of the both design approaches, enabling to deal with the adaptive control problem of more general uncertain systems.

Appendix A. PROOF OF LEMMA 1

It is clear that the proof is done if there exists an $\epsilon^* \in (0,1]$ such that

$$\|\Phi(t)\| \leq 1 - \epsilon^*$$

holds for all $t \geq t^* := t_0 + T_0$, with T_0 given in (8). In this respect, we now use contradiction to prove the above statement, and thus assume that for any small enough $\epsilon > 0$, there always exists $T \geq t^*$ such that $\|\Phi(T)\| > 1-\epsilon$. The contradiction is formed by the following two steps.

(S1) We show that the assumed statement implies $\|\Phi(t)\| = 1$ for all $t \in [t_0, t^*]$;

(S2) We show that $\|\Phi(t)\| = 1$ for all $t \in [t_0, t^*]$ contradicts with the IE condition.

Let $h \in \mathbb{R}^q$ be any vector such that $\|h\| = 1$, and denote $z(t) = \Phi(t)h$ and $\varphi_t = \varphi(t, x(t))$. Thus, with (9) we have

$$\dot{z} = -\varphi_t^\top \varphi_t z\,, \qquad z(t_0) = h\,. \tag{A.1}$$

By letting $V_z = \|z(t)\|^2$, it immediately follows that

$$\dot{V}_z = -\|\varphi_t z\|^2 \leq 0\,. \tag{A.2}$$

Thus, we can obtain that $\|z(t)\| := \|\Phi(t)h\|$ decreases as t increases, implying for all $t \geq t_0$, $\|\Phi(t)h\| \leq 1$ and thus $\|\Phi(t)\| \leq 1$ as h is an arbitrary unit vector.

We now complete proof of the (S1) by contradiction. From the continuity of $\|\Phi(t)\|$, we suppose that there exist $t' \in [t_0, t^*)$ and $\epsilon' \in (0,1)$ such that $\|\Phi(t')\| = 1 - \epsilon'$. This implies that $\|\Phi(t')h\| \leq 1 - \epsilon'$ for each $\|h\| = 1$. As for any $\|h\| = 1$, $\|\Phi(t')h\|$ decreases as t increases, we have $\|\Phi(t)h\| \leq 1 - \epsilon'$ and thus $\|\Phi(t)\| \leq 1 - \epsilon'$ for all $t \geq t'$. This clearly contradicts with the assumption that for arbitrarily small ϵ, there always exists $T \geq t^* > t'$ such that $\|\Phi(T)\| \geq 1 - \epsilon$. Hence, it can be concluded that $\|\Phi(t)\| = 1$ for all $t \in [t_0, t^*]$. Then fix any $\|h\| = 1$ such that $\|z(t^*)\| = \|\Phi(t^*)h\| = 1$. It is clear from (A.2) that $\|z(t^*)\| \leq \|z(t)\| \leq \|z(t_0)\| = 1$ for all $t^* \geq t \geq t_0$, and thus $\|z(t)\| = 1$ for all $t \in [t_0, t^*]$.

Next, we proceed to prove the (S2), and represent (A.1) in polar coordinates by setting $z(t) = r(t)y(t)$ where

$$r(t) := \|z(t)\|\,, \qquad y(t) := z(t)/\|z(t)\|\,.$$

Note that $r(t) = 1$ for all $t \in [t_0, t^*]$, and the polar coordinate representation $z(t) = r(t)y(t)$ is well-defined for $t \in [t_0, t^*]$. We then have

$$\begin{aligned} \dot{r} &= -\|\varphi_t y\|^2 r \\ \dot{y} &= -\varphi_t^\top \varphi_t y + \|\varphi_t y\|^2 y \end{aligned}\,, \quad t \in [t_0, t^*]\,. \tag{A.3}$$

Recalling that $r(t) = 1$ for all $t \in [t_0, t^*]$, we thus obtain $\varphi_t y(t) = 0$ for all $t \in [t_0, t^*]$, yielding $y(t) = y(t_0) = h$.

This, on the other hand, implies $\dot{r} = -\|\varphi_t h\|^2 r$ for all $t \in [t_0, t^*]$, rendering

$$r(T) = \exp\left(-\int_{t_0}^{t^*} h^\top \varphi_\tau^\top \varphi_\tau h d\tau\right) r(0) \leq e^{-\delta_0} < 1$$

where the inequality is obtained by invoking the IE condition (8) and $\|h\| = r(t_0) = 1$. This clearly contradicts with $r(t) := \|z(t)\| = 1$ for all $t \in [t_0, T]$. This then proves the (S2) and thus completes the proof of the lemma.

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