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Online Gyro Bias Estimation from Single Vector Measurements Using Regression Models ⋆

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Abstract: This paper addresses the problem of on-line consistent estimation of gyro bias using the measurements of a single vector and the biased angular velocity – both in the body-fixed frame. We propose two globally convergent gyro bias observers using new regression models, which are capable to deal with the cases of constant and time-varying reference vectors, respectively. Indeed, there are quite a few works discussing the latter case. To address this, we derive a nonlinear regression model, based on which a convexified gradient descent observer is designed, providing globally asymptotically convergent estimates to the gyro bias under some sufficient excitation conditions. The proposed schemes are illustrated by some numerical simulations.

Keywords: Nonlinear systems, observer, parameter identification, robotics

1. INTRODUCTION

The problems of control and on-line estimation of rigid-body attitude are important topics in many robotics applications. For all these tasks, the access to the accurate rotational velocity in the body-fixed frame is essential towards high-performance systems, which, however, is stymied by the velocity bias from gyroscope sensors. This makes gyro bias estimation as an important issue in the fields of robotics and navigation. The most common scenario is that, apart from the measurement of biased triaxial rate gyro, some vector measurements (e.g. from magnetometers, sun sensors, and cameras) are also available, which implicitly contain the information of attitude and gyro bias.

Usually, the gyro bias and the rigid-body attitude are reconstructed concurrently via many sorts of estimators, among which nonlinear observers have become popular and facilitated significant progress in the last two decades, see Mahony et al. (2008); Grip et al. (2011); Martin and Sarras (2018); Batista et al. (2012b). In recent years, it has received increasing interests to investigate the observability and observer design for attitude estimation using a single vector measurement Trumpf et al. (2012); Namvar and Safaei (2013); Grip et al. (2015); Batista et al. (2012a). It is shown in Trumpf et al. (2012) that with only a single complementary vector measurement, it is necessary to require the vector being time-varying to impose observability for attitude estimation. However, this is unnecessary for the observability of gyro bias, which can be asymptotically estimated on-line even for a constant reference vector; see (Metni et al., 2006, Lemma 3.1) for local convergence and the recent work Martin and Sarras (2018) with a global domain of attraction under a persistency of excitation (PE) assumption. Note that those are concerned with on-line gyro bias estimation using constant reference vectors. In contrast, for the time-varying case, the gyro bias estimation has been considered only in a few works on attitude observer design, e.g., Namvar and Safaei (2013); Bahrami and Namvar (2017); and the observer design to estimate gyro bias *solely* has not been well addressed either.

In this paper, we revisit the problem of designing observers to estimate gyro bias online, and propose two regression models for both constant and time-varying vector measurements. The contributions are mainly devoted to the latter case, for which a convexified gradient observer is proposed in terms of the resulting nonlinear regression model. Under some excitation conditions of trajectories, the proposed scheme is able to provide a globally asymptotically convergent estimate to gyro bias. The proposed design is similar to the observers based on gradient descent search, e.g. Ortega et al. (2021), but an elaborated convexification

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mechanism as Malaizé et al. (2012) is adopted to avoid the locality.

The remainder of the paper is organised as follows. In the sequel of this section, all notations used in the paper are introduced, which are followed by the dynamical model and the problem formulation in Section 2. Then, we present our main results on generation of regression models and observers design in Section 3, for the cases with both constant and time-varying reference vector measurements. Some simulation results are shown in Section 4 to illustrate the performance of the proposed scheme. Finally, the paper is wrapped up by some concluding remarks in Section 5.

Notation. Throughout the paper, we use $I_n \in \mathbb{R}^{n\times n}$ to represent the identity matrix of dimension n, and $0_n \in \mathbb{R}^n$ and $0_{n\times m} \in \mathbb{R}^{n\times m}$ denote the zero column vector of dimension n and the zero matrix of dimension $n \times m$, respectively. We use $\mathbb{Z}_+$ to represent the set of all positive integers. For $x \in \mathbb{R}^n$, we denote the Euclidean norm $|x| := \sqrt{x^\top x}$. Given a function $f : \mathbb{R}^n \to \mathbb{R}$ we define the differential operators $\nabla_x f := (\frac{\partial f}{\partial x})^\top$. The n-sphere is defined as $\mathbb{S}^n := \{x \in \mathbb{R}^{n+1} : |x| = 1\}$, and we use $SO(3)$ to represent the special orthogonal group, and $\mathfrak{so}(3)$ is the associated Lie algebra as the set of skew-symmetric matrices satisfying $SO(3) = \{R \in \mathbb{R}^{3\times3} | R^\top R = I_3,\ \det(R) = 1\}$. Given a vector $a \in \mathbb{R}^3$, we define the operator $(\cdot)_\times$ as

$$a_\times := \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \in \mathfrak{so}(3).$$

When clear from context, the arguments and subscripts are omitted.

2. MODEL AND PROBLEM FORMULATION

We consider a rigid body with its attitude state $R \in SO(3)$, which represents the coordinates of the body-fixed frame $\{\mathcal{B}\}$ related to the inertial frame $\{\mathcal{I}\}$. Its dynamics is given by

$$\dot{R} = R\omega'_\times, \quad R(0) = R_0 \tag{1}$$

with the rotational velocity $\omega' \in \mathbb{R}^3$ in the body-fixed coordinate. The measurement $\omega \in \mathbb{R}^3$ contains its true value $\omega' \in \mathbb{R}^3$ and a slowly time-varying but unknown gyro bias $\theta \in \mathbb{R}^3$, i.e.

$$\omega = \omega' + \theta. \tag{2}$$

The gyro bias θ can be viewed as *constant* over time.

Assume there is a reference vector $g \in \mathbb{R}^3$, known in the inertial frame, is measured in the body-fixed frame, and the output is

$$y = R^\top g \tag{3}$$

with $y \in \mathbb{R}^3$, which is known as complementary measurement. Without loss of generality, we assume $|g| = 1$, or equivalently $g \in \mathbb{S}^2$. In the time-varying case, note that the motion of a continuous vector $g \in \mathbb{S}^2$ can be charaterised as

$$\dot{g} = (\Lambda_g)_\times g \tag{4}$$

with $\Lambda_g \in \mathbb{R}^3$ the rotational velocity of g in the inertial frame.

Problem formulation. We are interested in designing an observer to estimate the gyro bias θ from the output $y \in \mathbb{S}^2$, the reference $g \in \mathbb{S}^2$, and the measured rotational velocity $\omega \in \mathbb{R}^3$ asymptotically.

At the end, let us show the dynamics of the output vector, which is useful for observer design in the sequel. If the reference vector g is (piecewisely) smooth, it yields

$$\dot{y} = (-\omega + \theta)_\times y + R^\top \dot{g}. \tag{5}$$

We make the following assumption throughout the paper.

Assumption 1. The rotational velocities ω and Λ_g are bounded.

In order to provide a robust estimate of θ online, we impose the following excitation assumption.

Assumption 2. The measurement vector y satisfies the condition

$$\lambda_2 \left(\int_t^{t+T} y(s)y(s)^\top ds \right) > \delta, \quad \forall t \geq 0 \tag{6}$$

for some $T, \delta > 0$, with λ_2 presenting the second largest eigenvalue of a square matrix.

Note that the above requirement is weaker than the PE property of the signal y, which is defined below.

Definition 3. (*Persistency of excitation*) Given a bounded signal $\phi : \mathbb{R}_+ \to \mathbb{R}^n$, it is persistently excited (PE) if

$$\int_t^{t+T} \phi(s)\phi^\top(s) ds \geq \delta I_n,\ \forall t \geq 0$$

for some $T > 0$ and $\delta > 0$.

3. MAIN RESULTS

In this section, we study the observer design for gyro bias, considering both cases with constant and time-varying reference vector $g \in \mathbb{S}^2$. The main idea we adopt is to generate regression models for the unknown vector θ.

3.1 Single Constant Reference

As a motivating case, let us start with a constant vector g, i.e., $|\Lambda_g| = 0$, which is a relatively easy case and has been widely studied in the literature. Its extension to the non-constant case will be investigated in the next subsection.

The first step is to obtain a linear regression model with respect to the unknown bias vector $\theta \in \mathbb{R}^3$. For this case, invoking (5) we have

$$\dot{y} = -\omega_\times y - y_\times \theta. \tag{7}$$

Now, applying the stable filter $\frac{\alpha}{\alpha+p}[\cdot]$ with a scalar $\alpha > 0$ and the differential operator $p := \frac{d}{dt}$, one has

$$\frac{\alpha p}{\alpha + p}[y] + \frac{\alpha}{\alpha + p}[\omega_\times y] = -\frac{\alpha}{\alpha + p}[y_\times]\theta. \tag{8}$$

Hence, we have obtained a linear regression model with respect to the unknown bias θ, which does not need the time derivative information of y.

We are now in position to present the gradient gyro bias observer as follows.

Proposition 4. Consider the system (1) with the measurements (2)-(3) and a constant reference vector $g \in \mathbb{S}^2$, and assume that Assumptions 1-2 hold. Then, the gradient observer

$$\begin{aligned} \dot{\xi}_1 &= -\alpha\xi_1 + \alpha\omega_\times y \\ \dot{\xi}_2 &= -\alpha\xi_2 + \alpha^2 y \\ \dot{\Phi} &= -\alpha\Phi + \alpha y_\times \\ \dot{\hat{\theta}} &= \gamma\Phi(Y - \Phi^\top\hat{\theta}) \end{aligned} \tag{9}$$

with the gains $\gamma, \alpha > 0$ and the variable

$$Y := \alpha y - \xi_2 + \xi_1,$$

guarantees global exponential convergence

$$\lim_{t\to\infty} |\hat{\theta}(t) - \theta| = 0. \tag{10}$$

Proof. It is easy to verify that the states ξ_1, ξ_2 and Φ are exactly these filtered signals in (8), i.e.

$$\begin{aligned} \xi_1 &= \frac{\alpha}{\alpha+p}[\omega_\times y] \\ \xi_2 &= \frac{\alpha^2}{\alpha+p}[y] = \alpha y - \frac{\alpha p}{\alpha+p}[y] \\ \Phi &= \frac{\alpha}{\alpha+p}[y_\times]. \end{aligned}$$

Then, the linear regressor (8) can be written in a compact form

$$Y(t) = \Phi^\top(t)\theta + \varepsilon_t, \tag{11}$$

in which ε_t is the exponentially decaying term caused by the initial condition of the filter $\frac{\alpha}{\alpha+p}$. In the following, the regressor Φ will be shown to be PE, and we thus omit the term ε_t in the subsequent convergence analysis, as exponentially vanishing terms do not affect the convergence property of gradient descent algorithms when the regressor is PE (see Sastry and Bodson (2011)).

Now defining the estimation error $\tilde{\theta} := \hat{\theta} - \theta$, the dynamics of which is given by the linear time-varying (LTV) system

$$\dot{\tilde{\theta}} = -\gamma\Phi\Phi^\top\tilde{\theta}. \tag{12}$$

Let us now study the excitation property of the time-varying square matrix $y_\times$. It is clear that

$$y_\times y_\times^\top = -y_\times^2 = I_3 - yy^\top.$$

From (Trumpf et al., 2012, Lemma 2) and Assumption 2, we conclude that the matrix $y_\times$ is PE. Invoking (Sastry and Bodson, 2011, Lemma 2.6.7) – the filtered signal of a PE signal through stable, minimum-phase, rational transfer functions is still PE – we establish that $\Phi \in$PE. This is, indeed, the necessary and sufficient condition of the global exponential stability of the LTV system (12). It completes the proof. □

Remark 5. Once getting the linear regression model, there are a variety of methods to estimate the unknown constant (or slowly time-varying) bias θ on-line. We refer the reader to the recent papers only using an interval excitation (IE) condition to design state observer and on-line parameter estimator Bobtsov et al. (2022, 2021); Wang et al. (2021); Korotina et al. (2022).

3.2 Single Time-varying Reference

In this section, we extend the above results to the more challenging case with a *time-varying* reference vector $g(t)$. The daunting task relies on that, compared to the constant case, the unknown attitude $R \in SO(3)$ appears in the time derivative of output y; see Eq. (5). Thus, we cannot directly adopt the same technical approach in the last subsection.

In order to handle this case, we make the following additional assumption.

Assumption 6. The velocity Λ_g and the time derivative of the vector y are available. Besides, there exist a positive scalar $q > 0$ and a non-decreasing time sequence $\{t_k\}_{k\in\mathbb{Z}^+}$ with $t_k \to \infty$ as $k \to \infty$, such that $\limsup_{k\to\infty} |\Lambda_g(t_k)| = 0$ holds.

By rearranging the equation (5), we have

$$\dot{y} + (\omega - \theta)_\times y = R^\top(\Lambda_g)_\times g.$$

Since the rotation matrix R does not change the range of a vector, we have

$$h(\theta, t) := |\dot{y} + (\omega - \theta)_\times y|^2 - |\Lambda_g \times g|^2 = 0. \tag{13}$$

Hence, we obtain a nonlinear regression model for $t \geq p$

$$H(\theta, t) := Y(t) + \Phi(t)\theta + \theta^\top D(t)\theta = 0 \tag{14}$$

with any $p > 0$ and

$$\begin{aligned} Y(t) &:= \int_{t-p}^{t} |\dot{y}(s) + \omega(s)_\times y(s)|^2 - |\Lambda_g(s)_\times g(s)|^2 ds \\ \Phi(t) &:= \int_{t-p}^{t} 2(\dot{y}(s) + \omega(s)_\times y(s))^\top y(s)_\times ds \\ D(t) &:= -\int_{t-p}^{t} y(s)_\times^2 ds. \end{aligned}$$

Since the regression model is nonlinear in the parameter $\theta \in \mathbb{R}^3$, the standard gradient descent only provides locally convergent estimates. We instead consider the following *convexified* gradient observer, which is motivated by the sensorless observer design for electrical motors in Malaizé et al. (2012). It is capable to provide a global convergence result from the nonlinear regression model (13).

Proposition 7. Consider the system (1) with the outputs (2)-(3), and assume that Assumptions 1-2 and 6 hold. Then, there exists a bound $q > 0$ such that the gyro bias observer

$$\dot{\hat{\theta}} = -\gamma\max\{0, H(\hat{\theta}, t)\} \cdot \nabla_\theta H(\hat{\theta}, t) \tag{15}$$

with the adaptation gain $\gamma > 0$ and $T < q$, guarantees the global convergence (10).

Proof. The proof is motivated by Malaizé et al. (2012). First, we get the Hessian matrix

$$\begin{aligned} \frac{\partial^2 H}{\partial\theta^2}(\theta, t) &= D(t) = -\int_{t-T}^{t} y_\times^2(s) ds \\ &= \int_{t-T}^{t} \Big(I_3 - y(s)y^\top(s)\Big) ds, \end{aligned}$$

which is positive semi-definite. Invoking (Trumpf et al., 2012, Lemma 2) and Assumption 6, we have the following

$$D(t) \geq \delta' I_3 \tag{16}$$

for all $t \geq 0$ and some $\delta' > 0$.

On the other hand, it is easy to obtain the following implication for all $(x_1, x_2) \in \mathbb{R}^3 \times \mathbb{R}^3$

$$H(x_1,t) \ge H(x_2,t) \implies \frac{\partial H}{\partial \theta}(x_1,t)\Delta x \ge \Delta x^\top \frac{\partial^2 H}{\partial \theta^2}\Delta x \ge \delta' |\Delta x|^2 \tag{17}$$

with $\Delta x := x_1 - x_2$.

Now, let us define the estimation error $\tilde{\theta} := \hat{\theta} - \theta$, the dynamics of which is given by

$$\dot{\tilde{\theta}} \;=\; -\gamma \max\{0, H(\hat{\theta},t)\}\nabla_\theta H(\hat{\theta},t). \tag{18}$$

Consider the candidate Lyapunov function

$$V(\tilde{\theta}) = \frac{1}{2}|\tilde{\theta}|^2,$$

and its time derivative is

$$\begin{aligned}
\dot{V} &= -\gamma \max\{0, H(\hat{\theta},t)\}\frac{\partial H}{\partial \theta}(\hat{\theta},t)\tilde{\theta} \\
&= -\gamma \max\{0, H(\hat{\theta},t)\}\frac{\partial H}{\partial \theta}(\hat{\theta},t)\big(\hat{\theta}-\theta\big) \\
&\le -\gamma \max\{0, H(\hat{\theta},t)\}\big(\hat{\theta}-\theta\big)^\top \frac{\partial^2 H}{\partial \theta^2}(\hat{\theta},t)\big(\hat{\theta}-\theta\big) \\
&\le -2\gamma\delta' \max\{0, H(\hat{\theta},t)\}V
\end{aligned}$$

where in the last two inequalities we have used the implication (17) by fixing $x_1 = \hat{\theta}$ and $x_2 = \theta$. Note that if $H(\hat{\theta},t) < 0$, then $\max\{0, H(\hat{\theta},t)\} = 0$, then the third inequality above holds automatically; otherwise, we may apply (17) to obtain the inequality. Hence, one has

$$V(t) \;\le\; \exp(-2\gamma\delta' I(t))V(0)$$

with

$$I(t) := \int_0^t \max\{0, H(\hat{\theta},s)\}ds.$$

We aim to show the convergence $V(t) \to 0$ as $t \to \infty$, which only happens if

$$\limsup_{t\to+\infty} I(t) = +\infty,$$

or equivalently, $\max\{0, H(\hat{\theta},t)\} \notin L_1$. Similarly to (Malaizé et al., 2012, Proposition 1), we may continue the proof by contradiction, assuming $\max\{0, H(\hat{\theta},t)\} \in L_1$. Recalling the error dynamics (18), the boundedness of $\nabla_\theta H$ and the assumption for contradiction, we have $\nabla_\theta H(\hat{\theta},t)\max\{0, H(\hat{\theta},s)\} \in L_1$ as well. As a result $\tilde{\theta}$ converges to some non-zero constant asymptotically, i.e.,

$$\tilde{\theta}(t) = \tilde{\theta}_\star + \epsilon_1(t), \quad |\tilde{\theta}_\star| > 0$$

in which $\epsilon_1(t)$ represents some asymptotically decaying term. On the other hand, in terms of the boundedness of $\dot{\tilde{\theta}}$ and the Barbalat's lemma, we may obtain the convergence of $\max\{0, H(\hat{\theta},t)\}$ to zero, i.e., $\max\{0, H(\hat{\theta},t)\} = \epsilon_2(t)$ with a decaying term $\epsilon_2(t)$. Now, we have

$$\begin{aligned}
\epsilon_2(t) &= \max\{0, H(\hat{\theta}(t),t)\} \\
&\ge H(\hat{\theta}(t),t) \\
&= H\big(\theta + \tilde{\theta}_\star + \epsilon_1(t), t\big).
\end{aligned}$$

Then, we have

$$\begin{aligned}
\epsilon_2(t) &\ge H\big(\theta + r\tilde{\theta}_\star + (1-r)\tilde{\theta}_\star + \epsilon_1, t\big) \\
&\ge H(\theta + r\tilde{\theta}_\star, t) + \frac{\partial H}{\partial \theta}(\theta + r\tilde{\theta}_\star, t)\Big[(1-r)\tilde{\theta}_\star + \epsilon_1\Big] \\
&\quad + \delta' \Big|(1-r)\tilde{\theta}_\star + \epsilon_1\Big|^2.
\end{aligned}$$

Considering the convergence of ϵ_i $(i=1,2)$ to zero, there always exists a moment $t_\star$ after which the following holds

$$\begin{aligned}
0 \;\ge\; & H(\theta + r\tilde{\theta}_\star, t) + (1-r)\frac{\partial H}{\partial \theta}(\theta + r\tilde{\theta}_\star, t)\tilde{\theta}_\star \\
& + \delta'\big|(1-r)\tilde{\theta}_\star\big|^2.
\end{aligned} \tag{19}$$

On the other hand, for the sequence $\{t_k\}_{k\in\mathbb{Z}_+}$, we have $\lim_{k\to\infty}|\dot{g}(t_k)| = 0$, which implies [1]

$$\lim_{k\to\infty}\inf_{\hat{\theta}} h(\hat{\theta}, t_k) = 0, \quad \lim_{k\to\infty}\nabla_\theta h(\theta, t_k) = 0. \tag{20}$$

Considering (20), the fact $H(\theta,t) = 0$, and the relation

$$H(\theta,t) = \int_{t-T}^t h(\theta,t)ds,$$

for given $\tilde{\theta}_\star$ and a small $r > 0$ we have

$$|H(\theta + r\tilde{\theta}_\star, t)| \;\le\; \beta_1(r), \quad \forall t \ge 0 \tag{21}$$

$$\limsup_{k\to\infty}|\nabla_\theta H(\theta, t_k)| \;\le\; \beta_2(T) \tag{22}$$

for some class $\mathcal{K}$ functions β_1, β_2. Clearly, the last term in (19) satisfies

$$\delta'\big|(1-r)\tilde{\theta}_\star\big|^2 \;=\; \delta'(1-r)|\tilde{\theta}_\star|^2 \;>\; 0.$$

For the second term, the coefficient $(1-r) > 0$ and

$$\begin{aligned}
\frac{\partial H}{\partial \theta}(\theta + r\tilde{\theta}_\star)\tilde{\theta}_\star &= \Big[\Phi + (\theta + r\tilde{\theta}_\star)^\top D\Big]\tilde{\theta}_\star \\
&= r\tilde{\theta}_\star^\top D\tilde{\theta}_\star + \frac{\partial H}{\partial \theta}(\theta,t)\tilde{\theta}_\star \\
&\ge r\delta'|\tilde{\theta}_\star|^2 + \frac{\partial H}{\partial \theta}(\theta,t)\tilde{\theta}_\star,
\end{aligned} \tag{23}$$

and

$$\limsup_{k\to\infty}\frac{\partial H}{\partial \theta}(\theta + r\tilde{\theta}_\star, t_k)\tilde{\theta}_\star \;\ge\; r\delta'|\tilde{\theta}_\star|^2 - \beta_2(T)|\tilde{\theta}_\star|.$$

Combining the above inequalities as $k \to \infty$, we conclude that for the sequence $\{t_k\}_{k\in\mathbb{Z}_+}$, we have

$$0 \ge \delta'|\tilde{\theta}_\star|^2 - \beta_1(r) - \beta_2(T)|\tilde{\theta}_\star|.$$

Since the parameter $r \in (0,1)$ can be selected arbitrarily, the above clearly yields a contradiction for a small $T > 0$. Hence, we have that the variable $\max\{0, H(\hat{\theta},t)\} \notin L_1$, and as a consequence, $\hat{\theta} \to \theta$ as $t \to \infty$. It completes the proof. □

Remark 8. The gyro bias observer in Proposition 7 imposes the same excitation (i.e. Assumption 2) as required for the constant vector measurement in Proposition 4. This is also required in many other works on gyro bias estimation and/or on-line attitude determination; see for example Martin and Sarras (2018), (Trumpf et al., 2012, Sec. IV) and (Grip et al., 2015, Proposition 3).

Remark 9. When considering time-varying vector measurements, for the purposes of observer design, we additionally require the technical assumption on the existence of a time sequence $\{t_k\}_{k\in\mathbb{Z}_+}$ verifying $\limsup_{k\to\infty}|\Lambda_g(t_k)| = 0$. This is a relatively mild assumption with many sorts of trajectories satisfied, e.g., the simplest case with a constant vector g (i.e. $\Lambda_g = 0_3$), and the case that the velocity norm $|\Lambda_g|$ continuously approaches to zero over time.

[1] Roughly speaking, the function $h(\cdot, t_k)$ along the sequence $\{t_k\}_{k\in\mathbb{Z}_+}$ becomes a quadratic form asymptotically as $k \to \infty$.

Remark 10. Let us look at the solutions to the regression equation $H(\theta, t) = 0$. For a single time instance $t \geq 0$, apart from $\theta = 0_3$, there exist some other feasible solutions. However, we need to solve the equation $H(\theta, t) = 0$ over time, i.e., finding a feasible solution to a set of algebraic equations

$$\begin{cases} H(\theta, t_1) &= 0 \\ &\vdots \\ H(\theta, t_n) &= 0 \end{cases} \tag{24}$$

Indeed, there are infinite number of equations defined as above over time. Assumptions 2 and 6 implicitly make the set of feasible solutions over $[0, +\infty)$ only contain the true gyro bias $\theta \in \mathbb{R}^3$.

Remark 11. The proposed observer is motivated by Malaizé-Praly-Henwood's convexified gradient descent design in Malaizé et al. (2012), which has been proven to be effective to the sensorless control of electrical motors Bernard and Praly (2018); Ortega et al. (2021). However, we cannot apply (Malaizé et al., 2012, Proposition 1) directly to the gyro bias estimation problem studied in the paper due to the following points.

- In Malaizé et al. (2012), a two-dimensional dynamical model is considered, and we have a higher dimensional system.
- Second but more importantly, for the given model we cannot verify the excitation condition A3 therein, i.e., for any unit vector $v \in \mathbb{S}^2$ there exists a sequence $\{t_k\}_{k\in\mathbb{Z}_+}$ such that

$$\frac{\partial H}{\partial \theta}(\theta, t_k) \cdot v > 0. \tag{25}$$

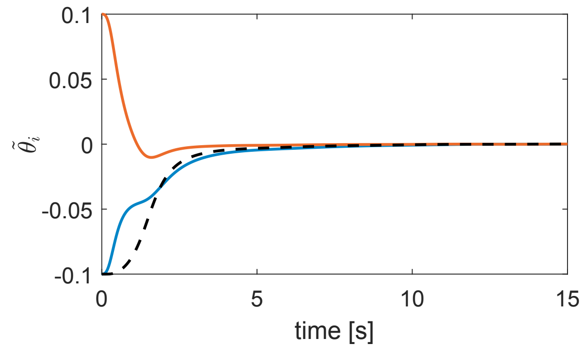
In our proof, we consider a similar inequality (23) at $\theta + r\tilde{\theta}_\star$ rather than the true value θ in (25).

4. SIMULATIONS

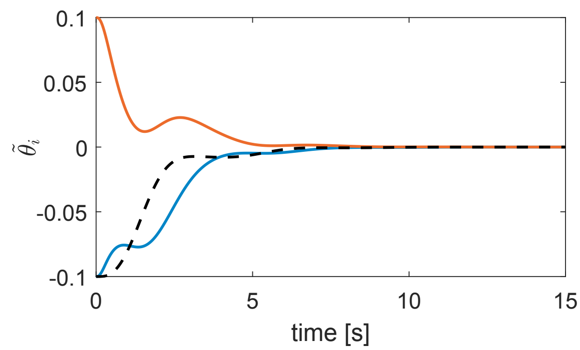
In this section, we provide some simulation results to show the efficiency of the proposed observer. Throughout this section, all the units are SI units. We evaluate the performance of the observer in Proposition 7, which is capable to deal with both the constant and time-varying vector cases.

We consider the attitude from $R(0) = I_3$ – equivalently three Euler angles being (0,0,0) – with the rotational velocity $\omega := [1, -1, 2]^\top$, and set the gyro bias as $\theta = [0.05, 0.06, 0.07]^\top$. First, a constant vector $g; = [0, 0, 1]^\top$, mimicking the gravity direction, is used to compare the performance of the observer in Proposition 7 and that in Martin and Sarras (2018). It is easy to verify the excitation condition in Assumption 2 for this given scenario. The initial conditions of them are both set as $\hat{\theta}(0) = 0_3$, and we select the gain $\gamma = 500$ for the observer in Proposition 7 and the gains $k_\alpha = l_\alpha = 10$ for the other. Simulation results can be found in Fig. 1. For this case, the estimation errors converge to zero asymptotically after a short transient period.

Second, we consider the case with a time-varying reference vector g, which is governed by the dynamics (4). We adopt the same scenario as the constant vector case with the same gains and initialisation. The rotational velocity



(a) The convexified gradient observer in Proposition 7



(b) The observer in (Martin and Sarras, 2018, Thm. 1)

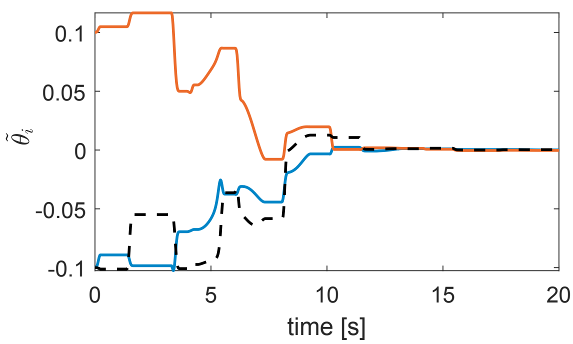
Fig. 1. [Constant vector] Evolution of the estimation error $\tilde{\theta}_i$ $(i = 1, 2, 3)$ for the observers in Proposition 7 and in (Martin and Sarras, 2018, Thm. 1)

Λ_g for the reference vector is set as $\Lambda_g = (0.5\sin(\pi t) + 0.2) \cdot [1, 1, 1]^\top$, for which there exists a sequence $\{t_k\}_{k\in\mathbb{Z}_+}$ satisfying Assumption 6. In Fig. 2, we show the estimation errors for both the convexified gradient observer in Proposition 7 and in Martin and Sarras (2018). As expected, the proposed scheme provides an asymptotically convergent estimate to the unknown gyro bias. Since the method in Martin and Sarras (2018) was developed for constant reference vectors, it fails to provide satisfactory estimates in this case.

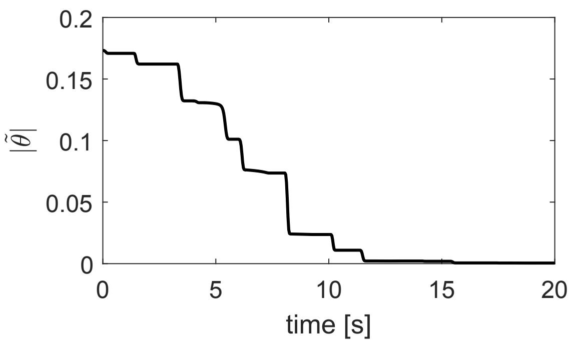
5. CONCLUSION

In this paper, we studied the on-line gyro bias estimation problem using a single vector measurement in the body-fixed frame. To this end, we propose a linear regression model and a nonlinear one for constant and time-varying reference vectors, respectively, based on which two gradient observers were proposed to provide globally asymptotically convergent estimates under some excitation conditions. Indeed, the latter case has been discussed only in a few works, and for that we additionally require that the rotational velocity Λ_g of the reference vector is continuously approaching zero in a subsequence $\{t_k\}_{k\in\mathbb{Z}_+}$. The performance of the proposed schemes has been illustrated by means of numerical simulations. Some problems that need to be further studied include the following:

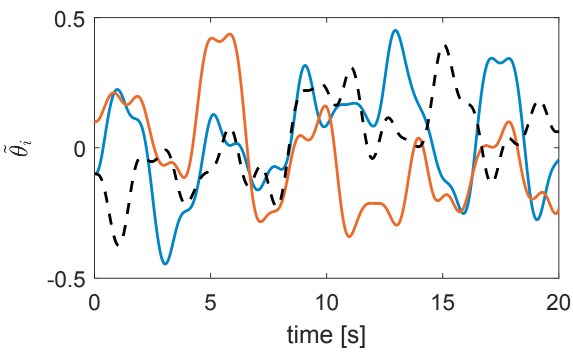
- The convexified observer in Proposition 7 needs the time derivative information $\dot{y}$, which needs to be calculated via some numerical tools. It would be



(a) The convexified gradient observer in Proposition 7



(b) Error norm of the observer in Proposition 7



(c) The observer in (Martin and Sarras, 2018, Thm. 1)

Fig. 2. [Time-varying vector] Evolution of the estimation error $\tilde{\theta}_i$ $(i = 1, 2, 3)$ for the observers in Proposition 7 and in (Martin and Sarras, 2018, Thm. 1)

interesting to study how to overcome such an issue by introducing filters.

- The proposed observer is based on the nonlinear regression model $H(\theta, t) = 0$ in (14). It is promising to use the regression model $h(\theta, t) = 0$ directly to simplify the observer design. Though the matrix $y_\times^2$ is not full rank at any single moment, it may "span" the full space under Assumption 2.
- It is of practical interests to relax the conditions in Assumption 6 in order to extend the realm of applicability.

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