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Varying control intensity of synchronized chaotic system with time delay

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Abstract. Hyperchaotic system is a very useful tool in secure and encrypted communications. But situations arise when engineers and scientists seek to synchronize two hyperchaotic systems. This gives another (error) system. The goal is to minimize the error as much as can be in order to make one system look like the other by synchronization. This is a particularly challenging situation. In this paper, two hyperchaotic systems are synchronized by impulsive control. Also, the condition for uniform asymptotic stability of the synchronized error system was given. Finally, the simulation results to justify the reliability of this method is also presented.

1. Introduction

A typical differential equation which is impulsive is of the form

$$\begin{cases} \dot{x} = f(t, x), & t \neq t_k \\ \Delta x = x(t_k^+) - x(t_k^-) = I_k(x), & t = t_k, k \in \mathbb{N} \\ x(t_0^+) = \theta. \end{cases} \quad (1)$$

So many studies on chaotic control have been done and more information can be found in [1], [2], [3], [4], [5], [6] and [7].

Of particular interest is the work of [8]. It obtained a control matrix. In practice, control should not be continuous, otherwise, it is not impulsive. The intensity of control should reduce with passing time so that the energy of the system is also gradually controlled as control should not suddenly and significantly change the state of the system.



Further to the work of [8], a set of control matrices was obtained to give room for varying and flexible data about interval of impulses and the time at which stabilization is achieved. Also, a completely different sufficient condition for the synchronized error system of Chen to be uniform and asymptotically stable was given.

2. Preliminaries

In what follows, \mathbb{R}_+ represents the nonnegative real numbers and \mathbb{R}^n the Euclidean space. Let $S \in \mathbb{R}^n$ and x_t , a piecewise continuous function.

However, it is important to note that choosing a constant $r > 0$ as the delay system upper bound and open set $D \subset \mathbb{R}^n$, given the functionals $f, I : J \times PC([-r, 0], D) \rightarrow \mathbb{R}^n$, the system

$$\begin{cases} \dot{x}(t) = f(x, x_t), & t \neq \tau_k \\ \Delta x(t) = I(t, x_{t-}), & t = \tau_k \end{cases} \quad (2)$$

where $\Delta x(t) = x(t) - x(t^-)$ with the initial condition

$$x_{t_0} = \theta, \quad (3)$$

where $t_0 \in \mathbb{R}_+$ and $\theta \in PC([-r, 0], \mathbb{R}^n)$, has a trivial solution. This is especially when τ_k , the impulse instances, are assumed to satisfy $0 = \tau_0 < \tau_1 < \tau_2 < \dots$ and $\lim_{k \rightarrow \infty} \tau_k = \infty$.

The solution of (2) which is trivial is stable if for every $\epsilon > 0$ and a non negative real number t_0 , there exists some $\delta > 0$ such that if θ is piecewise continuous on $([-r, 0], D)$ such that $\|\theta\|_r \leq \delta$ and any solution of (2) and (3) of the form $x = x(t_0, \theta)$ implies $x(t, t_0, \theta)$ is defined and $\|x(t, t_0, \theta)\| \leq \epsilon$, for all $t \geq t_0$. Let δ be independent of t_0 , then that trivial solution has a uniform stability. Its stability is uniform and asymptotical if, in addition, there exists some $\eta > 0$ and for every $\gamma > 0$, there is some $F = F(\eta, \gamma) > 0$ such that if $\theta \in PC([-r, 0], D)$ with $\|\theta\|_r \leq \eta$, then $\|x(t, t_0, \theta)\| \leq \gamma$ for $t \geq t_0 + F$.

Take $V : J \times D \rightarrow \mathbb{R}_+$, where J is a clopen interval of the form $[0, \infty)$ in \mathbb{R}_+ . The upper directional derivative of V on the right with respect to system (2) is defined by

$$D^+V(t, \phi(0)) = \lim_{h \rightarrow 0^+} \sup \frac{1}{h} [V(t+h, \phi(0) + hf(t, \phi)) - V(t, \phi(0))], \quad (4)$$

for $(t, \phi) \in J \times PC([-r, 0], D)$.

Furthermore, take $K_1 = \{g(s)\}$ as a set of continuous functions on \mathbb{R}_+ such that $g(s) \neq 0$ for $0 \neq s \in \mathbb{R}_+$. Also, let $\alpha, \beta, a, g \in K_1$ such that g is nondecreasing in s and

$$V : [-r, \infty) \times W(\rho) \rightarrow \mathbb{R}_+,$$

where V is continuous on both $[-r, \tau_0) \times W(\rho)$ and $[\tau_{k-1}, \tau_k) \times W(\rho)$ with the

$$\lim_{(t,y) \rightarrow (\tau_k^-, x)} V(t, y) = V(\tau_k^-, x),$$

for each $x \in W(\rho)$ and $k = 0, 1, 2, 3, \dots$. Then, V is said to be locally Lipschitz in x , if when restricted to $\mathbb{R}_+ \times W(\rho)$, and $q \in PC(\mathbb{R}_+, \mathbb{R}_+)$, the following conditions are satisfied:

(i) $\beta(\|X\|)$ and $\alpha(\|X\|)$ bound $V(t, x)$ below and above, respectively, for all $(t, x) \in [-r, \infty) \times W(\rho)$;

(ii) $D^+V(t, \phi(0))$ is bounded above by $q(t)a(V(t, \phi(0)))$, for all $t \neq \tau_k$ in \mathbb{R}_+ and $\phi \in PC([-r, 0], W(\rho))$ whenever $V(t, \phi(0)) \geq gV(t+s, \phi(s))$ for $s \in [-r, 0]$;

(iii) $V(\tau_k, \phi(0) + I(\tau_k, \phi)) \leq g(V(\tau_k^-, \phi(0)))$, $\forall (\tau_k, \phi) \in \mathbb{R}_+ \times PC([-r, 0], W(\rho_1))$ for which $\phi(0^-) = \phi(0)$; and

$$(iv) \tau = \sup_{k \in \mathbb{Z}} \{\tau_k - \tau_{k-1}\} < \infty, T_1 = \sup_{t \geq 0} \int_t^{t+\tau} q(s) ds < \infty, \text{ and } T_2 = \inf_{p > 0} \int_{g(p)}^p \frac{ds}{a(s)} > T_1.$$

In that case, the solution of (2) which is trivial is uniform and asymptotically stable. More explanations can be found in [5].

Consider the drive system given by

$$\dot{x}(t) = Mx(t) + Nx(t - \tau) + \phi(x(t)) \quad (5)$$

where $x(t) = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$. From the (drive) system (5) and the (response) system (6) below

$$\begin{cases} \dot{x}'(t) = Mx'(t) + Nx'(t - \tau) + \phi(x'(t)), & t \neq t_k \\ \Delta x' = x'(t_k^+) - x'(t_k^-) = J_k(x'(t_k) - x(t_k)), & t = t_k \end{cases} \quad (6)$$

where $\phi(x'(t)) = \begin{pmatrix} 0 \\ -x'z' \\ x'y' \end{pmatrix}$, the error system below can be obtained as

$$\begin{cases} \dot{e}(t) = Me(t) + Ne(t - \tau) + \phi(x(t), x'(t)), & t \neq t_k \\ \Delta e(t_k) = J_k e(t_k^-), & t = t_k \end{cases} \quad (7)$$

3. Main Result

Theorem 3.1 Given an error system (7), if there exist a constant μ and matrix A such that

- (i) $\|\phi(x)\|^2 \leq \mu \|x\|^2$
- (ii) for $\|x\| \leq 1$

$$\nu = \left[\frac{\lambda_x(M^T A + AM) + 2\lambda_x(A^T A)\mu}{\lambda_n(A)} + \frac{2\lambda_x(A^T A)\|N\|^2}{\lambda_x[(I + J_k)^T A(I + J_k)]} \right] > 0$$

and

$$0 < \delta_k < -\frac{\ln(\lambda_x[(I + J_k)^T A(I + J_k)])/\lambda_n(A)}{\nu},$$

where A is a symmetric matrix which is positive definite, $\lambda_x(A)$ is the spectral (or maximum eigenvalue) of matrix A , $\lambda_n(A)$ is that eigenvalue of A which is minimum and δ_k is the interval range of impulse. Then, the stability of (7) is uniform and asymptotic.

proof Consider the Lyapunov function

$$V(t, e(t)) = e^T A e$$

such that $\lambda_n(A)\|e\|^2 \leq V(t, e(t)) \leq \lambda_x(A)\|e\|^2$. For $t = t_k$

$$\begin{aligned} V(e(t)) &= e(t_k)^T A e(t_k) = e(t_k^-)^T [(I + J_k)^T A(I + J_k)] e(t_k^-) \\ &\leq \lambda_x[(I + J_k)^T A(I + J_k)] \|e(t_k^-)\|^2 \\ &\leq \frac{\lambda_x[(I + J_k)^T A(I + J_k)]}{\lambda_n(A)} V(t_k^-, e(t_k^-)) \\ &\leq g V(t_k^-, e(t_k^-)), \end{aligned} \quad (8)$$

where $g = \frac{\lambda_x[(I + J_k)^T A(I + J_k)]}{\lambda_n(A)}$.

If $V(t, e(t)) \geq gV(t+i, e(t+i))$ for $-\tau \leq i \leq 0$, then, $V(t, e(t)) \geq g\lambda_n(A)\|e(t+i)\|^2$ and $\|e(t+i)\|^2 \leq \frac{V(t+i, e(t+i))}{\lambda_n(A)} \leq \frac{V(t, e(t))}{g\lambda_n(A)}$.

For $t \neq t_k$

$$\begin{aligned}
D^+(V(e)) &= 2[Me + Ne(t-\tau) + \phi(e)]^T Ae \\
&= e^T(M^T A + AM)e + 2e^T ANe(t-\tau) + 2e^T A\phi(e) \\
&\leq \frac{\lambda_x[M^T A + AM]}{\lambda_n(A)}V(e) + 2\|Ae\|^2\|N\|^2\|e(t-\tau)\|^2 + 2\|Ae\|^2\|\phi(e)\|^2 \\
&\leq \frac{\lambda_x[M^T A + AM]}{\lambda_n(A)}V(e) + \frac{2\lambda_x(A^T A)\|e\|^2\|N\|^2}{g\lambda_n(A)}V(e) + 2\lambda_x(A^T A)\|e\|^2\|\phi(e)\|^2 \\
&\leq \frac{\lambda_x[M^T A + AM]}{\lambda_n(A)}V(e) + \frac{2\lambda_x(A^T A)\|e\|^2\|N\|^2}{g\lambda_n(A)}V(e) + 2\lambda_x(A^T A)\|e\|^2\mu\|e\|^2 \\
&\leq \frac{\lambda_x[M^T A + AM]}{\lambda_n(A)}V(e) + \frac{2\lambda_x(A^T A)\|e\|^2\|N\|^2}{g\lambda_n(A)}V(e) + \frac{2\lambda_x(A^T A)\|e\|^2\mu}{\lambda_n(A)}V(e) \quad (9) \\
&\leq \left[\frac{\lambda_x[M^T A + AM]}{\lambda_n(A)} + \frac{2\lambda_x(A^T A)\|N\|^2}{g\lambda_n(A)} + \frac{2\lambda_x(A^T A)\mu}{\lambda_n(A)} \right] V(e) \\
&\leq \left[\frac{\lambda_x[M^T A + AM] + 2\lambda_x(A^T A)\mu}{\lambda_n(A)} + \frac{2\lambda_x(A^T A)\|N\|^2}{g\lambda_n(A)} \right] V(e) \\
&\leq \left[\frac{\lambda_x[M^T A + AM] + 2\lambda_x(A^T A)\mu}{\lambda_n(A)} + \frac{2\lambda_x(A^T A)\|N\|^2\lambda_n(A)}{\lambda_x[(I + J_k)^T A(I + J_k)]\lambda_n(A)} \right] V(e) \\
&= \left[\frac{\lambda_x[M^T A + AM] + 2\lambda_x(A^T A)\mu}{\lambda_n(A)} + \frac{2\lambda_x(A^T A)\|N\|^2}{\lambda_x[(I + J_k)^T A(I + J_k)]} \right] V(e) \\
&= q(t)V(t, e),
\end{aligned}$$

where $q(t) = \frac{\lambda_x[M^T A + AM] + 2\lambda_x(A^T A)\mu}{\lambda_n(A)} + \frac{2\lambda_x(A^T A)\|N\|^2}{\lambda_x[(I + J_k)^T A(I + J_k)]} = \mathcal{V}$.

Assume $a(s) = s$, from item (iv) of the hypothesis,

$$\begin{aligned}
T_2 - T_1 &= \inf_{p>0} \int_{g(p)}^p \frac{ds}{a(s)} - \sup_{t \geq 0} \int_t^{t+\delta} q(s)ds > 0 \\
&= \inf_{p>0} [\ln p - \ln g(p)] - \sup_{t \geq 0} [q(t+\delta) - q(t)] > 0 \\
&= -\ln g(p) - ((\mathcal{V} + \delta\mathcal{V}) - \mathcal{V}) > 0 \\
&= -\ln g(p) - \delta\mathcal{V} > 0 \\
&= -\ln \frac{\lambda_x[(I + J_k)^T A(I + J_k)]}{\lambda_n(A)} - \delta_k \mathcal{V} > 0
\end{aligned}$$

Hence,

$$0 < \delta_k < -\frac{\ln(\lambda_x[(I + J_k)^T A(I + J_k)]/\lambda_n(A))}{\mathcal{V}}.$$

According to the hypothesis, the error system (7) is uniform and asymptotically stable.

4. Simulation of numerical example

The following is a system of Chen [3]

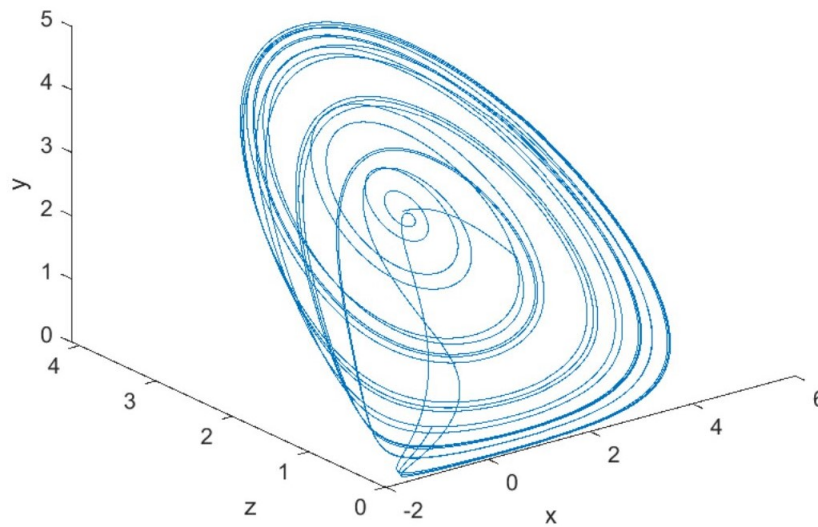


Figure 1. Chen System's hyperchaotic phenomenon when $c = 18.35978$

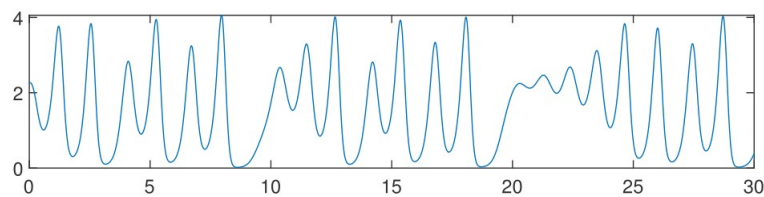


Figure 2. x against t when $c = 18.35978$

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = (c - a)x - xz + cy \\ \dot{z} = xy - bz + K(z - z(t - \tau)) \end{cases} \quad (10)$$

where $a = 35, b = 3, K = 2.85, \tau = 0.3, x(0) = \begin{pmatrix} 2.27 \\ 2.27 \\ 1.72 \end{pmatrix}$ and $z(t) = 0$ for $t \in [-0.3, 0)$.

This system exhibits a single-scroll attractor when $c = 18.35978$ as in [2]. This same system is hyperchaotic as shown in **Figure 1**. The chaos diagrams along x, y and z axes against t axis are respectively shown in **Figure 2, Figure 3** and **Figure 4**.

From (10),

$$M = \begin{pmatrix} -35 & 35 & 0 \\ -16.64 & 18.35978 & 0 \\ 0 & 0 & -0.15 \end{pmatrix},$$

$$N = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2.85 \end{pmatrix},$$

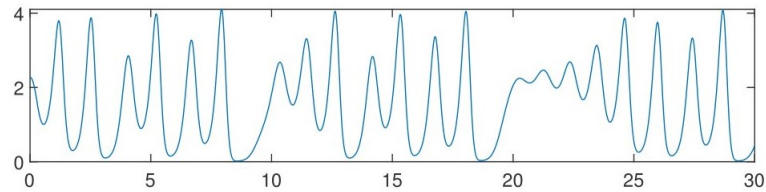


Figure 3. y against t when $c = 18.35978$

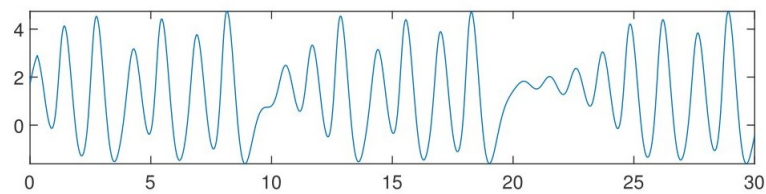


Figure 4. z against t when $c = 18.35978$

$\phi(x(t)) = \begin{pmatrix} 0 \\ -xz \\ xy \end{pmatrix}$ and $\|\phi(x(t))\|^2 = x^2(y^2 + z^2) \leq x^2(x^2 + y^2 + z^2) \leq 5.1529\|x\|^2$, whence $\mu = 5.1529$. This fulfills condition (i) in Theorem 3.1.

Furthermore, take $A = I_3$ (the 3×3 identity matrix) and the set of Impulse Control Matrices (ICM) $\{J_k\}$ such that

$$J_k = \frac{k+1}{k} \begin{pmatrix} -0.2 & 0 & 0 \\ 0 & -0.8 & 0 \\ 0 & 0 & -0.8 \end{pmatrix}.$$

Note that $\frac{k+1}{k} \rightarrow 1$ as $k \rightarrow \infty$ so that $J_k \rightarrow J_\infty$. Hence,

$$J_1 = 2.000 \begin{pmatrix} -0.2 & 0 & 0 \\ 0 & -0.8 & 0 \\ 0 & 0 & -0.8 \end{pmatrix},$$

$$J_2 = 1.500 \begin{pmatrix} -0.2 & 0 & 0 \\ 0 & -0.8 & 0 \\ 0 & 0 & -0.8 \end{pmatrix},$$

$$J_3 = 1.333 \begin{pmatrix} -0.2 & 0 & 0 \\ 0 & -0.8 & 0 \\ 0 & 0 & -0.8 \end{pmatrix},$$

...

$$J_\infty = \begin{pmatrix} -0.2 & 0 & 0 \\ 0 & -0.8 & 0 \\ 0 & 0 & -0.8 \end{pmatrix}.$$

From condition (ii) of Theorem 3.1,

$$\mathcal{V} = \frac{\lambda_x[M^T A + AM] + 2\lambda_x(A^T A)\mu}{\lambda_n(A)} + \frac{2\lambda_x(A^T A)\|N\|^2}{\lambda_x[(I + J_k)^T A(I + J_k)]} > 0$$

$$0 < \delta_k < -\frac{\ln(\lambda_x[(I + J_k)^T A(I + J_k)]/\lambda_n(A))}{\mathcal{V}}$$

For $k = 1$, we have

$$\mathcal{V} = \frac{38.9732 + 2(5.1529)}{1} + \frac{2(8.1225)}{0.36} = 94.404 > 0$$

$$0 < \delta_1 < -\frac{\ln(0.36)}{94.404} = 0.010822$$

Choose $\delta_1 = 0.01$.

For $k = 2$, we have

$$\mathcal{V} = \frac{38.9732 + 2(5.1529)}{1} + \frac{2(8.1225)}{0.49} = 82.43206 > 0$$

$$0 < \delta_2 < -\frac{\ln(0.49)}{82.43206} = 0.008654$$

Choose $\delta_2 = 0.008$.

For $k = 3$, we have

$$\mathcal{V} = \frac{38.9732 + 2(5.1529)}{1} + \frac{2(8.1225)}{0.5379} = 79.47978 > 0$$

$$0 < \delta_3 < -\frac{\ln(0.5379)}{79.47978} = 0.007802$$

Choose $\delta_3 = 0.007$.

For $k = \infty$, we have

$$\mathcal{V} = \frac{38.9732 + 2(5.1529)}{1} + \frac{2(8.1225)}{0.64} = 74.6618 > 0$$

$$0 < \delta_\infty < -\frac{\ln(0.64)}{74.6618} = 0.005977$$

Choose $\delta_\infty = 0.005$.

The choice of the impulsive interval δ_k varies from one control instance to the other as shown in **Table 1**. However, it is such that all the properties in Theorem 3.1 are obtained for all k as $k \rightarrow \infty$.

5. Simulation with Control

Figure 5, **Figure 6**, **Figure 7** show the control effects in x , y , z against t when $c = 18.35978$, respectively.

Table 1 summarizes the data of time of synchronization along axes x , y and z and the instantaneous impulsive intervals.

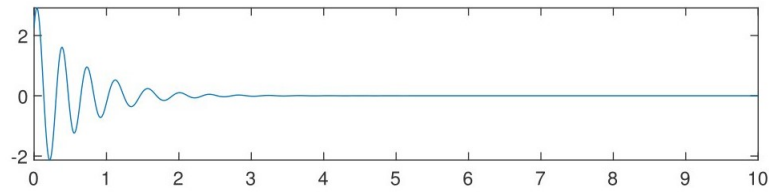


Figure 5. Control in x against t when $c = 18.35978$

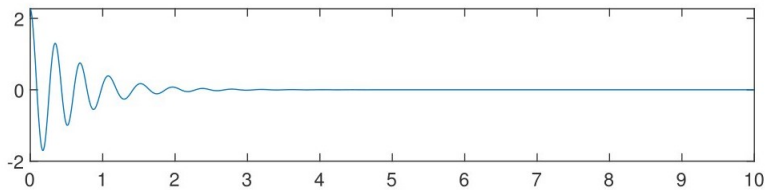


Figure 6. Control in y against t when $c = 18.35978$

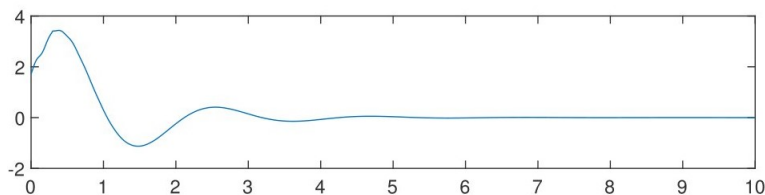


Figure 7. Control in z against t when $c = 18.35978$

Table 1. Synchronization time along axes.

J_k	x	y	z	δ_k
$k = 1$	2.718	2.457	4.911	0.01
$k = 2$	1.344	1.482	4.242	0.008
$k = 3$	1.185	1.224	3.726	0.007
\vdots	\vdots	\vdots	\vdots	\vdots
$k = \infty$	1.173	1.083	3.192	0.005

6. Conclusion

In this paper, a condition which is sufficient to synchronize two hyperchaotic systems via impulsive control is given. The set $\{\delta_k\}$ of the upper bounds of the impulses at different instances which will make the synchronized error system to be uniformly asymptotically stable is also given. This gives infinitely many ways to control the error system.

Finally, the simulations show that this method is effective as the control could be achieved in less time than when it could be achieved in [8]. With the control method proposed in this paper, the control is more efficient, quick and flexible.

Acknowledgments

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