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The Ordered Weighted Average Sector Liquid Return Index: A Method for Determining Financial Recovery from Sectoral Debt

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Abstract: The primary aim of this article is to demonstrate that using the average of ratios as a representative value for measuring the health of a sector does not constitute a valid procedure. After mathematically demonstrating this objective, the article will then focus on introducing a new index for estimating the potential debt return value for a sector or group of companies. Next, the article details the start of the process for creating a new index to improve investors' understanding of the risk associated with a sector or a group of companies meeting short-term obligations based on assigned probabilities of future sales. Given that said value is intended to represent an indicator of expected liquid solvency, its construction will take treasury tensions into account. An Ordered Weighted Average type of aggregation function is used to aggregate the magnitudes in this scenario. Consequently, the second objective of the present work is the creation of this index, which provides an initial estimate of how much money can be recovered from a sector's debt.

Keywords: financial mathematics; ratios; acid test; monetary debt; OWA operators; sector; investment risk

MSC: 91G80

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1. Introduction

The use of ratios to analyze financial accounting statements was established at the end of the 19th century [1]. It remains useful at the beginning of the 21st century because, in addition to being an essential tool for financial analysis [2], it is also a source of new research [3–5].

One of the most common ways of applying the ratios is to obtain a classification of a company's state of health according to its numerical value. By way of example, the fixed asset financing ratio, which is calculated by dividing the company's permanent funds by its net fixed assets, provides an idea of its debt strategy. Thus, when the ratio for a given company is greater than or equal to 1, this indicates that we are in a scenario of long-term financial equilibrium. However, if the ratio is less than 1, this indicates negative working capital, warning us that the company may have solvency problems.

Ratios has proven useful when carrying out studies on a company's evolution [6], and a great number of works in the field of accounting mathematics, therefore, list values ratios for all types of companies [7–10]. It is hardly surprising, then, that research with ratios has aroused the interest of researchers attempting to use them to obtain information on a sector or group of companies [11–13]. With regard to this line of research, in recent years, a number of works have warned that the simple use of ratios entails serious problems [14–16] and that false conclusions may be drawn in this regard, especially when using the simple or weighted arithmetic mean of the ratios to extract information from a sector [17,18]. Thus, it is necessary to develop new descriptive statistical tools that allow precise information to be obtained on the state of health of a sector or group of companies.

This work has two primary objectives. The first is to demonstrate that using the average of ratios as a representative value for measuring the health of a sector is not a procedure that can be considered valid. This objective aligns with the research framework introduced by Aitchison [19], known as Compositional Data. In recent years, this theory has sparked a significant debate on the challenges associated with using ratios within areas of accounting at the sector level [20,21]. The present work aims to make a novel contribution to this specific research area. The second aim is to estimate a sector's liquidity analysis by developing an index that incorporates both pessimistic and optimistic projections for future sector payments. This expansion of our analysis aligns with the research direction initiated by Yager [22], which focuses on the application of Ordered Weighted Average aggregation operators [23–25]. An index designed for this specific purpose will be introduced later in this article.

The approach used to address the problem and present potential solutions is structured as follows: Section 2 introduces key terms and concepts related to the acid test ratio used throughout this work, emphasizing the need to move away from using the arithmetic average of ratios as an indicator for comparing sector solvencies. We also introduce a new indicator that makes sectoral value sensitive to real challenges in dealing with the liquidation of short-term liabilities. This includes a restriction that allows the use of individual acid ratios and an aggregation function to create a sectoral index that is responsive to changes in the liquidation of short-term liabilities within a sector. Here, we detail the approach taken to analyze a sector's liquidity and treasury by creating an index that incorporates pessimistic and optimistic projections for future payments within the sector. The index serves as a decision-making tool for assessing investment risk in a given sector. Section 3 includes an illustrative numerical example of the above. And finally, in Section 4 we present our conclusions, study limitations and potential avenues for future research.

2. Materials and Methods

2.1. Financial Ratios

The Royal Spanish Academy defines the term "ratio" as synonymous with "proportion", describing it as the quotient between two numbers or, more generally, two quantities that can be compared. While there are various ways to represent a ratio, the most common method today involves expressing it as a fraction, denoted as A/B.

Mathematically, a ratio's numerical value represents how many times one number fits into another. This interpretation relies on the concept of equivalence between fractions, allowing us to say that in a ratio with a value of 3/2 (or 1.5/1), quantity A is one and a half times quantity B. Similarly, in a ratio with a value of 1/5 (or 20/100), quantity A is 20% of quantity B.

Financial ratios, which are essentially ratios between economic measures, have gained widespread adoption in accounting practices due to these interpretations. They complement the information provided by financial statements expressed in monetary terms. Moreover, financial ratios facilitate comparisons over time, enabling analysis of a company's evolution, and across different companies within the same industry.

This work begins with the selection of specific ratios to assess the short-term solvency of a sector. These ratios are the current ratio and the acid test ratio.

The current ratio divides current assets (CA) by current liabilities (CL):

$$Current\ ratio = \frac{Current\ Assets}{Current\ Liabilities} \tag{1}$$

Current assets are the assets of any entity or company that can be made liquid, that is, converted into money, in a period of less than 12 months. This is the case, for example, with bank money, financial investments or the stocks of any entity or company.

Current liabilities comprise those liabilities that are made up of any entity or company's short-term obligations. Thus, current liabilities are made up of debts and obligations with a duration of less than 1 year.

If the ratio is greater than 1, this indicates that current capital is positive, which implies the possibility that the company can convert its short-term investments into cash in an amount sufficient to meet the short-term maturities of its debts. One of the main drawbacks of this ratio, which is usually referred to as the "current ratio", is the fact that it includes stock within the set of investments to be liquidated in order to meet short-term payments. This magnitude introduces considerable uncertainties, both in relation to the time required for assets to become liquid and their realizable value, given that inventories are typically accounted for at their cost price within the company. Consequently, we have chosen to center our study on the acid test ratio (AT), often referred to as the "quick ratio", which can be expressed mathematically as follows:

$$Acid\text{-}Test\ (AT) = \frac{Current\ Assets - Stocks}{Current\ Liabilities} = \frac{CA - S}{CL} \quad (2)$$

Calculating the acid test ratio offers an effective means of analyzing a company's ability to absorb short-term debt in times of instability. Although no ideal amount is valid for all situations, it is traditionally estimated that this value can be below the unit. It must be close to 1, however, to ensure that the company has short-term financial stability, given that the remaining part must be contributed by its permanent resources.

2.2. Demonstration of the Need to Abandon the Use of the Arithmetic Mean of Ratios as an Indicator for Comparing Sector Solvencies

Unfortunately, when using an aggregation of various ratios to obtain a reference for the liquid health status in a sector, the fact that the ratio is allowed to be greater than the unit causes a serious distortion problem, as we will see below.

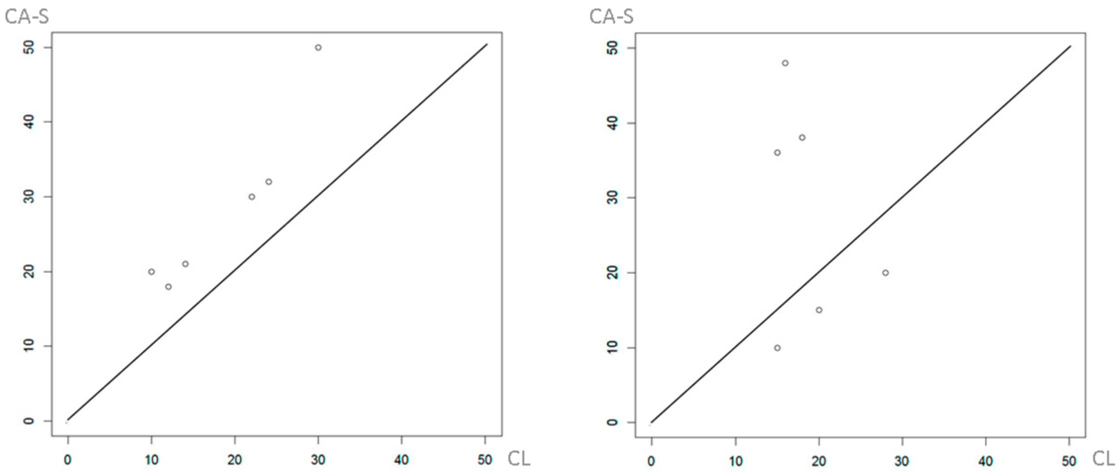
Let us consider two sectors, denoted by {1} and {2}, each comprising six companies, whose volumes of current assets minus stocks are shown in thousands of euros in Table 1.

Table 1. CA − S and CL of six similar companies in two different sectors. The data for the companies in this table are entirely fictional and have been intentionally selected to produce extreme values for the ratios in alignment with our primary objective, even if this results in potentially surprising significant variations in the data structure. Additionally, to enhance the clarity of comparisons between sectors, we have maintained an equal number of companies and the same total current liabilities.

***CA−S* {1}**	***CL* {1}**	***CA−S* {2}**	***CL* {2}**
18	12	15	20
20	10	10	15
21	14	20	28
32	24	38	18
30	22	36	15
50	30	48	16

In order to visualize the problem, we will represent both volumes in sectors {1} and {2} by means of coordinate axes, with the y-axis representing CA − S and the x-axis representing current liabilities, as shown in Figure 1a,b.

Note that the bisector of the first quadrant angle appears in Figure 1a,b. This line indicates the volumes for which the numerator and denominator of the acid test ratio are the same. Consequently, the value of the ratio is 1. The bisector divides the first quadrant into two complementary regions: the region comprising both the upper part of the bisector and the bisector itself, where current assets minus inventories is greater than or equal to current liabilities and whose acid test must be greater than or equal to one; and the region located below, where the current assets minus inventories are less than its current liabilities and the acid test is consequently less than one.



(**a**) (**b**)

Figure 1. (**a**) Graphic representation of the CA − S and CL variables in sector {1}); (**b**) Graphic representation of the CA − S and CL variables in sector {2}.

The graphs visually represent the fact that in sector {1} we did not find any company in the acid test region with a value less than 1, while in sector {2} three companies appear in this region whose distance from the bisector indicates a potential difficulty in meeting all short-term payments.

This is confirmed numerically by the explicit calculation of the acid test shown in Table 2, company by company, for the two sectors.

Table 2. Calculations of individual acid tests for companies in sectors {1} and {2}.

Acid Test Ratio **{1}**	*Acid Test Ratio* **{2}**
1.50	0.75
2.00	0.67
1.50	0.71
1.33	2.11
1.36	2.40
1.67	3.00

If, by way of example, we were to calculate the value of the acid test ratio for the sector as the simple mean of the individual ratios, which we have denoted as ATs, then we would obtain the following overall values for the sector:

$$ATs\{1\} = \frac{1.50 + 2.00 + 1.50 + 1.33 + 1.36 + 1.67}{6} = 1.56$$

$$ATs\{2\} = \frac{0.75 + 0.67 + 0.72 + 2.11 + 2.40 + 3.00}{6} = 1.61$$

Given the values obtained, it is possible to think that sector {2) is more solvent than sector {1}, although, as we have already seen in the individual study for each company in the sector, unlike sector {1}, sector {2} shows signs of difficulties in dealing with the settlement of its short-term liabilities, and the liquidity of this sector would therefore be partially compromised overall.

2.3. The Sector Liquid Return Index: An Index Used to Assess the Risk Associated with a Specific Sector Facing Difficulties in Meeting Its Short-Term Financial Obligations

In view of the above, the use of this type of indicator to compare solvencies of sectors must be discarded while, at the same time, acknowledging the need to create new indicators that make it possible for sectoral indicator values to be sensitive to real difficulties in dealing with the liquidation of total short-term liabilities.

In this respect, the Sector Liquid Return index [17,26], denoted as SLR, is a clear example of an index that is sensitive to this situation. The index is defined as follows, introducing a restriction based on the idea that no company will return more than 100% of the current liabilities it is holding:

$$SLR = \frac{\sum min(AT_i, 1) \cdot \mathrm{CL}_i}{\sum \mathrm{CL}_i} \quad (3)$$

The numerical value of the index can be interpreted as the unit coefficient for the capacity to repay current liabilities invested in the sector that the group of companies is able to return.

In order to clarify the terms used, Tables 3 and 4 provide a detailed model for calculating the index based on data from the initial example:

Table 3. Intermediate calculations of the Sector Liquid Return index for sector {1}.

Name	$CA_i - S_i$	CL_i	AT_i	$\mathbf{min(AT_i,1)}$	$\mathbf{min(AT_i,1) \cdot CL_i}$
Firm 1	18	12	1.50	1	12
Firm 2	20	10	2.00	1	10
Firm 3	21	14	1.50	1	14
Firm 4	32	24	1.33	1	24
Firm 5	30	22	1.36	1	22
Firm 6	50	30	1.67	1	30

In this case:

$$SLR\{1\} = \frac{12 + 10 + 14 + 24 + 22 + 30}{12 + 10 + 14 + 24 + 22 + 30} = \frac{112}{112} = 1$$

We note that, in sector {1}, since all of the acid test ratios are greater than 1, all of the companies have the capacity to return 100%, progressively liquidating their short-term investments. In line with this, an extreme value is obtained for the index, indicating that the sector has the capacity to return 100% of the total investment in liabilities made within it.

Table 4. Intermediate calculations of the Sector Liquid Return index for sector {2}.

Name	$CA_i - S_i$	CL_i	AT_i	$\mathbf{min(AT_i,1)}$	$\mathbf{min(AT_i,1) \cdot CL_i}$
Firm 1	15	20	0.75	0.75	15
Firm 2	10	15	0.67	0.67	10
Firm 3	20	28	0.71	0.71	20
Firm 4	38	18	2.11	1	18
Firm 5	36	15	2.40	1	15
Firm 6	48	16	3.00	1	16

In this case:

$$SLR\{2\} = \frac{15 + 10 + 20 + 18 + 15 + 16}{20 + 15 + 28 + 18 + 15 + 16} = \frac{94}{112} = 0.84$$

A value of 0.84 is obtained for sector {2}, which cannot handle all the debt of its current liabilities. This indicates that, according to the data, at the time in question there is no safety margin for the repayment of all short-term indebtedness since this sector only has the capacity to repay 84% of total indebtedness.

Note that the information offered by the proposed index reflects the situation of sector {2} having less capacity to repay its current liabilities.

2.4. The OWA Sector Liquid Return Index: A Sectoral Indicator Value That Reflects Sensitivity to Challenges in Relation to Debt Liquidation

The aim of this section is to create an index to be used by businesses to determine the best option by which to sell their products. This section is based on the idea that the Sector Liquid Return index presented in the previous section is not necessarily the one with the greatest likelihood of representing the liquid returns of a specific company. This recognition arises from the understanding that estimation, which involves making educated guesses, is a crucial aspect that must be considered in all real-world business applications. In this respect, there are many open lines of research that assess future estimation using mathematical models. In order to process the estimation of a future value in the return on the debt for a group of companies in a sector, this article adopts OWA-type (Ordered Weighted Average) aggregation operators [22].

An OWA operator is defined as a function f of $\mathbb{R}^n \to \mathbb{R}$, where n is the amount of data to be aggregated, having an associated weight vector W of length n, denoted by $W = (w_1, w_2, \ldots, w_n)$, where:

$w_{\rm i} \in [0, 1]\ \forall i$
$\sum_{\rm i=1}^{\rm n} w_{\rm i} = 1$,
$\mathrm{f}(a_1, a_2, \ldots, a_n) = \sum_{\rm i=1}^{\rm n} w_{\rm i} a_{<i>}$
and where $a_{<i>}$ is the i-th largest value of the sequence, ordered from largest to smallest, of the values $a_1, a_2, \ldots, a_n$.

The intrinsic idea behind OWA-type operators is the reordering of elements, causing there to be an association between that ordering and the weights w_j.

Example: If we have $W = (0.5, 0.3, 0.1, 0.1)$, then $f(6,8,4,7) = 0.5 \cdot 8 + 0.3 \cdot 7 + 0.1 \cdot 6 + 0.1 \cdot 4 = 7.2$

Some of the most important properties of OWA operators are as follows:

(i) Commutativity. If $(a_1, a_2, \ldots, a_{\rm n})$ are the arguments to be aggregated and $(p_1, p_2, \ldots, p_{\rm n})$ is a permutation of them, then: $f(a_1, a_2, \ldots, a_{\rm n}) = f(p_1, p_2, \ldots, p_{\rm n})$ For example, if we continue with $W = [0.5, 0.3, 0.1, 0.1]$, $f(4,6,8,7) = 7.2$
(ii) Monotony. $f(a_1, a_2, \ldots, a_{\rm n}) \le f(c_1, c_2, \ldots, c_{\rm n})$ if $a_{\rm i} \le c_{\rm i}\ \forall i$
(iii) Boundary. $\mathrm{Min}(a_i) \le f(a_1, a_2, \ldots, a_{\rm n}) \le \mathrm{Max}(a_i)$
(iv) Idempotence. $f(a, a, \ldots, a) = a$

There are three relevant cases according to the weighting function [22]:

1. $\mathrm{OWA}^* = \mathrm{Max}(a_i)$ if $W = (1, 0, \ldots, 0)$
2. $\mathrm{OWA}^* = \mathrm{Min}(a_i)$ if $W = (0, 0, \ldots, 1)$
3. $\mathrm{OWA}_{\rm ave} = \overline{a}$ if $W = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})$ where $\overline{a}$ = average $(a_1, a_2, \ldots, a_n) = \frac{1}{n}\sum a_i$

The aim of our proposal is to establish a sectoral indicator that is sensitive to difficulties in dealing with the liquidation of a debt contracted by a group of companies. It should incorporate the observer's subjective estimations of the companies' acid tests.

Let $E = \{E_1, E_2, E_3, \ldots, E_n\}$ be the set of companies in a given sector, whose total amounts of current assets minus inventories, current liabilities and acid tests are represented by $\{CA_1 - S_1, CA_2 - S_2, \ldots, CA_n - S_n\}$, $\{CL_1, CL_2, \ldots, CL_n\}$ and $\{AT_1, AT_2, AT_3, \ldots, AT_n\}$. Let $W = (w_1, w_2, \ldots, w_{\rm n})$ also be a weight vector where the components can be associated with an estimate of the likelihood of selling to companies in the sector, ordered from most to least solvent. For example, a weight vector $W = (0.5, 0.3, 0.2, 0, \ldots, 0)$ would identify an estimate that it is expected to sell 50% of production to the most solvent company, 30% to the second most solvent and the rest to the third most solvent.

Based on the data considered, we define the specific OWA Sector Liquid Return index as follows:

$$SLR_{OWA} = \sum_{i=1}^{n} w_i \cdot \min(AT_{<i>}, 1) \quad (4)$$

We note from the definition of the index that the following properties will always be fulfilled:

1. $SLR_{OWA} \geq 0$
This property is simply a product of the index definition itself given that $\omega_i \geq 0$ and $\min\left(ra_{<i>}, 1\right) \geq 0$ whatever the value of i.
2. $SLR_{OWA} \leq 1$
This property is also a product of the index definition itself given that

$$SLR_{OWA} = \sum_{i=1}^{n} \omega_i \cdot \min(AT_{<i>}, 1) \leq \sum_{i=1}^{n} \omega_i \cdot 1 = \sum_{i=1}^{n} \omega_i = 1$$

3. For two sequences of ratios (a_i), (b_i) that satisfy $a_i \leq b_i \; \forall i$ and given a vector of weights $W = (w_1, w_2, \ldots, w_n)$, the following will always hold true:

$$SLR_{OWA}(a_i) \leq SLR_{OWA}(b_i)$$

given that

$$\min(AT_{<i>}, 1) \leq \min(AT_{<i>}, 1) \; \forall i$$

and therefore,

$$SLR_{OWA}(a_i) = \sum_{i=1}^{n} \omega_i \cdot \min(AT_{<i>}, 1) \leq \sum_{i=1}^{n} \omega_i \cdot \min(AT_{<i>}, 1) = SLR_{OWA}(b_i)$$

4. If $W = (w_1, w_2, \ldots, w_n) = \left(\frac{CL_{<1>}}{\sum CL_i}, \frac{CL_{<2>}}{\sum CL_i}, \ldots, \frac{CL_{<n>}}{\sum CL_i}\right)$, with $CL_{<j>}$ being the current liability of the company that has as an acid test the value $AT_{<j>}$, then in this particular case: $slr_{OWA} = slr$

In other words, the Sector Liquid Return index is a specific case of the OWA Sector Liquid Return index.

This property is proven as follows :

$$SLR_{OWA} = \sum_{i=1}^{n} \omega_i \cdot \min(AT_{<i>}, 1) = \sum_{i=1}^{n} \frac{CL_{<i>}}{\sum CL_i} \cdot \min(AT_{<i>}, 1)$$
$$= \frac{\sum_{i=1}^{n} CL_{<i>} \cdot \min(AT_{<i>}, 1)}{\sum CL_i} = \frac{\sum_{i=1}^{n} CL_i \cdot \min(AT_i, 1)}{\sum CL_i} = SLR$$

Once the properties have been verified, we need to obtain a financial meaning for the proposed formula. Given that the sectoral liquidity index allows an investor to have an idea of the risk that a given sector offers if it is unable to meet its short-term obligations, we can interpret the OWA Sector Liquid Return index as one that allows an entrepreneur to estimate the return on investment of a given sector.

Let us now consider several numerical examples to help us better understand the idea. Table 5 presents the specific case of six companies for a given period.

Table 5. Sample data.

Name	CA_i-S_i	CL_i	AT_i	$\min(AT_i,1)$
Firm 1	14	20	0.700	0.7
Firm 2	15	25	0.600	0.6
Firm 3	18	24	0.750	0.75
Firm 4	30	10	3.000	1
Firm 5	35	12	2.917	1
Firm 6	52	15	3.467	1

We know that Company 1 has the ability to return 70% of its investment. Company 2 can return 60%, Company 3 can return 75% and the rest 100%.

Let us suppose that a company suspects that it can manage to sell its merchandise proportionally to the three best companies in that sector. Its weight vector would be $W = (0.\hat{3}, 0.\hat{3}, 0.\hat{3}, 0, 0, 0)$ and, in this case, it would obtain the value:

$$SLR_{OWA} = \sum_{i=1}^{n} w_i \cdot \min(AT_{<i>}, 1) = 0.\hat{3} \cdot 1 + 0.\hat{3} \cdot 1 + 0.\hat{3} \cdot 1 + 0 + 0 + 0 = 1$$

This, in theory, would indicate that the estimate of payments for the sector is 100% of the debt contracted with the company.

Let us now suppose that the company suspects it can manage to sell its production proportionally to the three worst companies in that sector. Its weight vector would be W = (0, 0, 0, 0.$\hat{3}$, 0.$\hat{3}$, 0.$\hat{3}$) and, in this case, the following value would be obtained as the OWA Sector Liquid Return index:

$$SLR_{OWA} = \sum_{i=1}^{n} w_i \cdot \min(AT_{<i>}, 1) = \\ 0 + 0 + 0 + 0.\hat{3} \cdot 0.75 + 0.\hat{3} \cdot 0.7 + 0.\hat{3} \cdot 0.6 = 0.68\hat{3}$$

This indicates the estimated initial recovery of the debt that the sector has contracted with the company as 68.$\hat{3}$%.

Let us suppose that the company's sales are expected to be distributed according to the following weight vector: W = (0, 0, 0.2, 0.5, 0.3, 0). In other words, based on experience, the entrepreneur believes that nothing is going to be sold to the two companies with the best return capacity, or those with the worst, and that the estimate is for 20% of production to be sold to the third best company, 50% to the fourth best company and the remaining 30% of products to the fifth best company.

In this case, the following value would be obtained as the OWA Sector Liquid Return index:

$$SLR_{OWA} = \sum_{i=1}^{n} w_i \cdot \min(AT_{<i>}, 1) = 0 + 0 + 0.2 \cdot 1 + 0.5 \cdot 0.75 + \\ 0.3 \cdot 0.7 = 0.785$$

Finally, we observe that under conditions of total uncertainty in which we do not know proportions of the distribution company's sales, the weight vector would be W = (1/6, 1/6, 1/6, 1/6, 1/6, 1/6) and, in this case, we would obtain the value:

$$SLR_{OWA} = \sum_{i=1}^{n} w_i \cdot \min(AT_{<i>}, 1) = 1/6 \cdot 0.7 + 1/6 \cdot 0.6 + 1/6 \cdot 0.75 + \\ 1/6 \cdot 0.7 + 1/6 \cdot 1 + 1/6 \cdot 1 = 0.84$$

This would, in theory, indicate that the level of estimated payments for the sector is 84% of the debt contracted with the company. Note how high uncertainty can lead us to a different forecast.

3. Results

In this section, we will present the analysis of a practical case in order to visualize how the index is used. To this end, we will use data from real companies in three different sectors.

It is important to note that both the year and the sector have been randomly selected. This work represents a purely theoretical construction, aimed at developing new mathematical tools that may find applications in future accounting objectives.

Let us consider a company that can manufacture consumable products for the sectors classified as "Wholesale fruit marketing", "Other processing and preservation of fruits and vegetables" and "Production of fruit and vegetable juices". To better illustrate the situation, let us consider that the company will sell similar volumes at the same prices to the three sectors and that there is no difference in producing for the different sectors. To determine the real situation of each sector, the company consults the data for the companies associated with these sectors in the SABI (Iberian Balance sheet Analysis System) database, developed by Informa D&B in collaboration with Bureau Van Dijk, which contains the financial statements of over two million Spanish and more than 500,000 Portuguese companies, using the search criteria outlined below. The year 2019 was selected arbitrarily from the most recent accounting years for which all data are available. Box 1 shows the search data.

Box 1. Search data.

Company status: Active
Region/Country: Catalonia (SPAIN)
Availability of data for 2019
And one of the following three CNAE codes with activity restrictions:

(1) CNAE 2009(Only primary codes): 4631-Sale of fruits and vegetables.
(2) CNAE 2009(Only primary codes): 1039-Other processing and preservation of fruits and vegetables.
(3) CNAE 2009(Only primary codes): 103 -Production of fruit and vegetable juices.

Using the search data with the first CNAE code, of the thirty-three companies obtained, fifteen would be excluded for having a value of 0 in current liabilities, which is a value necessary to calculate the index. Consequently, the number of usable companies would be eighteen. Table 6 shows the results obtained for this search.

Table 6. Data from first search.

Current Assets (Thousands EUR)	Inventories (Thousands EUR)	Current Liabilities (Thousands EUR)
29.243	4.577	28.467
21.780	2.754	12.445
20.481	479	11.259
7.396	210	6.508
8.239	1.758	5.160
1.403	40	827
1.916	31	697
2.015	5	1.195
643	14	292
1.363	83	1.344
1.002	46	973
480	83	337
152	20	148
327	31	206
120	35	101
48	2	17
113	37	40
9	3	3

The search using the second CNAE code offered an initial result of sixteen companies, one of which would be excluded as it has 0 for the values necessary to calculate the ratios or coordinates. Consequently, the number of usable companies in this case would be fifteen. Table 7 shows the results obtained for this second search.

Table 7. Data from second search.

Current Assets (Thousands EUR)	Inventories (Thousands EUR)	Current Liabilities (Thousands EUR)
45.210	6.718	31.824
6.155	4.219	1.321
1.996	453	1.405
5.504	904	1.015
2.561	1.176	787
1.324	311	1.107
2.749	667	2.278
1.905	66	638
1.566	279	572
1.195	100	244
258	74	155
586	48	262
103	37	73
285	65	61
36	17	10

Finally, the search using the third CNAE code offered an initial result of six companies, one of which would be excluded as it has 0 in the values necessary to calculate the index. Consequently, the number of usable companies in this case would be five. Table 8 shows the results obtained for this last search.

Table 8. Data from third search.

Current Assets (Thousands EUR)	Inventories (Thousands EUR)	Current Liabilities (Thousands EUR)
53.024	33.843	50.956
5.729	3.355	1.017
4.053	766	1.740
1.808	47	1.798
185	112	15

If we assume that the company does not know the distribution of the possible sales of its products among the different companies in each possible sector, that is, the vector

$$\left[\frac{\mathrm{CL}_{<1>}}{\sum \mathrm{CL}_i}, \frac{\mathrm{CL}_{<2>}}{\sum \mathrm{CL}_i}, \ldots, \frac{\mathrm{CL}_{<n>}}{\sum \mathrm{CL}_i}\right]$$

can also be taken as the W value, where n is the number of companies in each sector, then the sectoral OWA indices are as follows:

$$SLR_{OWA}(sector1) = 94.40$$

$$SLR_{OWA}(sector2) = 99.29$$

$$SLR_{OWA}(sector3) = 42.70$$

We observe that in planning to supply one of the three possible sectors, the company must focus its attention on adapting its product to the companies in sector 2, since its sale of products in that sector has the highest estimate of being paid for in full, according to the calculated method.

4. Discussion

This research mathematically defines financial ratios and provides an example to demonstrate that using arithmetic means is an invalid procedure for comparing sector solvencies. Accordingly, the first objective of this work has been fulfilled.

Next, an index was developed to estimate future payment collections by a company. Our detailed analysis of how the index was constructed and full example of its application allow us to compare the progress of this research with respect to the "Sector Liquid Return index".

Thus, with the creation of the "OWA Sectoral Liquid Return Index", the second objective of this work has also been achieved.

Consequently, this work expands the line of research related to the use of ratios for obtaining information from a sector [12,13,17,26]. Specifically, it presents the process of creating an index based on the use of OWA operators to estimate the best investment option when it comes to producing merchandise for a company that supplies various sectors or groups of companies. It must be taken into account that since a company's payment capacity depends on current cash flow and future sales, it is evident that the proposal presented is only a first approach to the type of problems posed by the solvency of a particular sector. The expanded study of the subject matter addressed in this work opens a possible new line of research based on the idea of introducing into the study the variables client collection period and payment to suppliers. Also, it is possible to develop further extensions of said index by using other techniques to include uncertainty, such as fuzzy numbers and linguistic variables [27–30], and more complex structures, such as compositional data analysis [19,31,32]. Additionally, a new research direction is emerging, emphasizing the development of more robust indices and their comparative analyses. The proposed index is not robust at all, as small variations in input data can lead to significant fluctuations in its value. This constitutes the first limitation of this study, which can be addressed through further research. Likewise, the index can also be improved if the individual value of the AT is not fully known, or it can be improved to include the measurement of default possibilities. An example of the latter is that two sectors could have average ATs of 5 and 1, respectively, although both have an SLR equal to 1. While the former would be expected to display great resilience, the latter would have a high possibility of falling below 1 at the minimum downturn of events.

A further limitation is working with ratios, which poses the problem of their non-existence when the denominator of the ratio is 0. This can be solved in the present article simply by taking the value 1 for the value $\min(AT_{<i>}, 1)$, since it can be assumed that the company is in a position to return 100% of what it owes and, in such a case, this value is 0.

A third limitation of this work is that it does not present return on investment, in the sense that it does not take into account costs associated with shipments according to the distances between companies, for example. This opens up room for another line of research based on the study of performance using the expression $\frac{\text{sales}-\text{operating expenses}}{\text{operating expenses}} \times 100$.

Finally, it is important to note that the Modigliani-Miller theorem suggests that, under certain assumptions, the method a company uses to finance its growth is irrelevant. This position is only defensible in perfect markets. In reality, however, the value of the company is affected by variables such as interest rates on debt and tax rates. Therefore, it is necessary to pay special attention to the risk of bankruptcy resulting from the inability to collect payments for sales, which would prevent the generation of future cash flows, thereby delimiting the value of the company's solvency. The present work is framed within this context and offers a monitoring tool for evaluating the risk of bankruptcy resulting from the impossibility of collecting payments for sales made.

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