

# Non-probabilistic Informed Structural Health Assessment with Virtual Modelling Technique



Q. Wang, Y. Feng, D. Wu, and W. Gao

**Abstract** In real-life engineering, non-probabilistic structural information is very common in many and varied disciplines. This class of information is characterized by incompleteness and imprecision, such as interval, fuzzy sets, etc. Non-probabilistic structural information can be reflected in the structural performance and cause it to fluctuate within a specific range, instead of being deterministic. Thus, without appropriate consideration of non-probabilistic information, serious or even disastrous accidents may occur. Therefore, fully estimating the structural health status using non-probabilistic information, especially detecting the lower and upper bounds of the concerned structural response, is extremely significant in uncertainty-sensitive fields. To conquer this challenge, a virtual modeling technique underpinning a structural health assessment framework is introduced. The twin extended support vector regression (T-X-SVR) approach is embedded for virtual model construction. Continuous, differentiable expression of the established virtual model allows the optimal solutions for each interval analysis to be easily achieved. Information update is another inherent feature, which enables structural health assessment to be implemented with updated conditions without rebuilding the virtual model. To demonstrate the applicability of the proposed virtual modeling technique underpinned structural health assessment framework, the non-probabilistic informed elastoplastic nonlocal damage analysis was investigated for engineering structures.

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# 1 Introduction

In the practical engineering field, structural health assessments attract more attention to meet the current higher requirement for smart engineering. Mass information is obtained via various devices (e.g., actuators, monitors, sensors, etc.), but the problem is transferring from acquiring the data to information extraction or mining of the data.

Engineering structures may contain, experience or confront multifarious information from different sources. Non-probabilistic information is one method to represent information with the features of incompleteness and imprecision, such as interval, fuzzy sets, etc. [1, 2]. Moreover, from industry practice, accumulated evidence has repeatedly revealed that stochastic- or probabilistic-based information quantification can have the dilemma that the probability distribution characteristics are challenging to be credibly determined, because of insufficient amount or poor quality of the experimental data. Thus, non-probabilistic structural information has widespread applicability in real-life industries [3].

By considering the non-probabilistic characteristics within the information of system inputs, the structural response correspondingly presents non-probabilistic features (e.g., interval, fuzzy, or imprecise). Without loss of generality, non-probabilistic information is herein considered as a fuzzy parameter. A structural health assessment involving fuzzy information was conducted by seeking the fuzzy-valued bounds or membership functions of the structural response. Through a level-cut strategy, the fuzzy problem was transformed into a series of optimization algorithms on the interval realizations.

However, in practical engineering applications, the relationship between system information and the quantity of interests is normally underpinned, sophisticated, and implicit. Directly implementing the optimization algorithms on this constitutive relationship is extremely challenging. As a single deterministic calculation could already be very time consuming, non-probabilistic uncertainty quantification with large simulations to search for the extremes would become computationally infeasible. Thus, an alternative strategy to tackle the non-probabilistic informed structural health assessment is proposed based on a supervised machine learning technique, namely twin extended support vector regression (T-X-SVR) [4]. Supreme mathematical features of T-X-SVR allow the feasibility of optimal solutions on given intervals being effectively and efficiently obtained in the established virtual model. Furthermore, the virtual model-aided health assessment has an inherent advantage of information updates without the need to reconstruct the model.

## 2 Methods

### 2.1 Non-probabilistic Information

The non-probabilistic information herein is considered as the fuzzy variable. A fuzzy variable  $\xi^F$  is a gradual weighting of a vector space  $\Upsilon$  with the membership functions  $\{\mu_{\xi^F}(x) : \Upsilon \rightarrow [0, 1], \forall x \in \mathfrak{R}\}$ . This membership function can be written as a set of ordered pairs,

$$\{(x, \mu(x)) | x \in \Upsilon \wedge \mu(x) \in [0, 1]\} \tag{1}$$

The set of all fuzzy sets  $\Upsilon$  is denoted by  $\Gamma(\Upsilon)$ . For numerical implementations, it is necessary to introduce  $\alpha$  – levels. For a fuzzy variable  $\xi^F$  and  $\alpha \in [0, 1]$ ,  $\alpha$  – level cut can be written as,

$$\xi_\alpha^F := \{x \in \Upsilon : \mu_{\xi_\alpha^F} \geq \alpha\} \tag{2}$$

It is significant to note that for each given degree of truth (or membership level)  $\alpha$ , the problem converts to an interval form. Each interval problem is conducted by seeking the lower bound (LB) and upper bound (UB) of the concerned structural response, which can be formulated as follows,

$$\begin{aligned} & \underset{\xi_\alpha^F}{optimize} : X(\xi_\alpha^F) \\ & s.t., \xi_\alpha^F \in [\underline{\xi}_\alpha^F, \overline{\xi}_\alpha^F] \end{aligned} \tag{3}$$

### 2.2 Virtual Model Construction

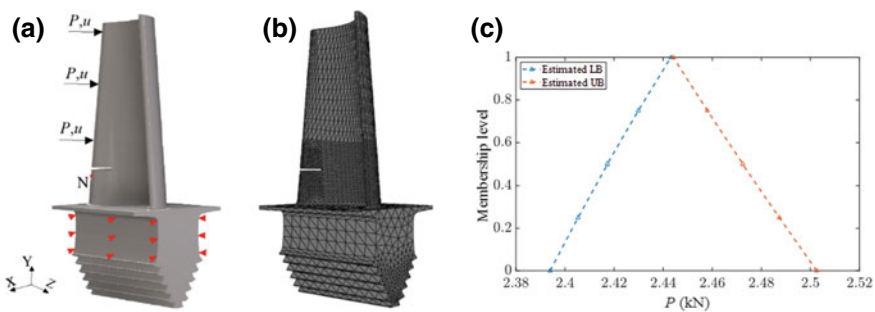
To provide a more robust effective and efficient manner to tackle this engineering-simulated problem, a supervised machine learning technique, namely T-X-SVR [4] was used for the virtual model construction. The established virtual model alternatively depicts the implicit constitutive relationship between the system inputs and the concerned structural responses. T-X-SVR aims to minimize the gap between two bounds of the virtual model and the datasets from two different directions. The optimal solution for weights and bias can be effectively obtained by solving two quadratic programming problems (QPPs).

### 2.3 Virtual Model Aided Non-probabilistic Uncertainty Quantification

After the virtual model construction, a series of optimization programming problems for fuzzy informed engineering problems can be conducted on the virtual model instead of the original constitutive relationship. There are two main benefits in utilizing the virtual model as an alternative: (1) A mathematical function for each model possesses cheap computational costs; (2) On any specific interval of the inputs, the optimal solutions for the estimation of extremes can be effectively obtained through the derivation methods.

### 2.4 Numerical Investigation

A jet engine was investigated by considering fuzzy information within the material properties [5]. According to that convergence study, the virtual model was constructed by learning from 160 training samples.  $1e3$  Monte Carlo simulation (MCS) results were considered as the benchmark. The established virtual model had relatively high accuracy, with R-square nearly 1, and root mean squared error (RMSE) about  $8e - 4$ . The Poisson's ratio  $\nu_C$  and density  $\rho_C$  of the ceramic are considered as fuzzy parameters.  $\nu_C$  was assumed to follow the triangular membership function, with the support of  $[0.33, 0.37]$ , and top of 0.35;  $\rho_C$  followed the trapezoid membership function, with the support of  $[3.168, 3.232]$   $g/cm^3$ , and top of  $[3.198, 3.202]$   $g/cm^3$ . The corresponding fuzzy-valued LB and UB of the concerned structural response (i.e., critical load  $P$  (kN) of the damage analysis) was estimated through the proposed strategy, as shown in Fig. 1c.



**Fig. 1** **a** Numerical model, **b** adopted FEM mesh of notched blade, and **c** estimated fuzzy-valued lower bound and upper bound of the concerned structural response

### 3 Discussion

This extended abstract introduced a virtual model-aided non-probabilistic structural health assessment framework for engineering structures. The non-probabilistic information was considered as a fuzzy parameter. Through a level-cut strategy, the problem was formulated as a series of non-linear programming problems in the virtual model. For the virtual model construction, a supervised machine learning technique, namely T-X-SVR, has been adopted. The implicit, computational expansive constitutive relationship can be alternatively depicted as a mathematical equation, with continuous and differentiable features. Thus, the optimal solution for fuzzy-valued bounds can be effectively obtained in the virtual model. Moreover, information update can be fulfilled by importing updated information into the virtual model, instead of reconstructing the virtual model. The introduced virtual model-aided non-probabilistic informed structural health assessment has great potential applicability in real-life engineering.

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