Decoding the Game: A Quantitative Analysis of Market Manipulation and Machine-Human Interactions

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Statement of Originality

I certify that this dissertation, titled "Decoding the Game: A Quantitative Analysis of Market Manipulation and Machine-Human Interactions', has not previously been submitted for a degree nor has it been submitted as part of requirement for a degree except as fully acknowledged within the text.

I also certify that the dissertation has been written by me. Any help that I have received in my research and the preparation of the dissertation itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the dissertation.

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To my family.

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Abstract

The financial markets have experienced a technology revolution, which has led to various innovations such as the advent of limit order books, the increased use of social media in investment decisions, and the expansion of FinTech services. This dissertation investigates market manipulation and machine-human interactions in the context of the technology revolution in the modern finance industry.

Chapter 2 introduces a novel machine learning technique, Q-learning, as a learning tool to a dynamic limit order market (LOM) to examine how order book information and learning affect the strategic trading behaviour of bounded rational traders. In equilibrium, informed traders favour limit (resp. market) orders when the magnitude of mispricing is small (resp. large). In contrast, uninformed traders tend to "chase the trend", that is, submit market buys (resp. sells) following market buys (resp. sells) from the informed. Interestingly, informed traders can manipulate the market by anticipating a mispricing reversal when there is small overpricing (resp. underpricing) and a high depth imbalance at the best bid (resp. ask). Going against their equilibrium preference of limit buys (resp. sells) due to small mispricing, the informed use market buys (resp. sells) to trigger market buys (resp. sells) from the uninformed to enhance execution probability and profitability of later informed limit sells (resp. buys). Consequently, the profitability of the uninformed trend-chasing market order strategy is reduced. These findings suggest that informed manipulation can be learned as an equilibrium trading strategy in a dynamic LOM.

Chapter 3 analyses how meme investing, a retail buying frenzy coordinated through social media, affects the investment efficiency of firm managers. The buying frenzy is modelled as a cost on (informed and manipulative) short sellers in a model where managers learn about the quality of an investment from stock prices. Small shorting costs improve investment efficiency, intermediate costs may improve or harm, while high costs and short-sale bans harm investment efficiency. This occurs as manipulative short sellers are more sensitive to shorting costs. The buying frenzy is also modelled as asymmetric noise trading where a noise buy is more likely to occur than a noise sell. In the asymmetric noise trading setting, the buying frenzy can improve investment efficiency as it induces extra cost to manipulative short sellers.

Chapter 4 investigates the impact of the interaction between quantitative traders reliant on computer models and discretionary human traders on price efficiency. Quants are modelled as traders with greater information processing capability but weaker flexibility to adapt to market conditions than discretionary traders. The market efficiency depends on how these effects impact the trading aggressiveness of traders. Quants tend to trade more (resp. less) aggressively due to greater information processing (resp. weaker flexibility), and discretionary traders take advantage of the weaker flexibility of quants. Consequently, the price efficiency is non-monotonic with respect to the level of quant trading.

Overall, this dissertation unveils the impact of the technology revolution on the strategic behaviour of financial market and the corresponding market quality consequences. The analysis suggests that technological developments can disrupt the practice of market manipulation that has been in existence for centuries. The analysis of the real economic impact of meme investing suggests that such a retail buying frenzy can be a natural remedy to manipulative short selling and can thus improve the investment efficiency of firm managers. In addition, the investigation of the increased popularity of machines in trading reveals that the strategic interaction between quantitative machine and discretionary human traders and the corresponding impact of growth in quantitative investing can be decomposed into empirically testable components primarily driven by greater information processing and weaker flexibility of machines.

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Chapter 1

Introduction

1.1 Introduction

The advancement of technology in recent decades has brought about numerous innovations in the modern finance industry, including the adoption of limit order books, the increased involvement of social media in personal investment decisions, and the growth of FinTech such as blockchain and robot advisory services. Therefore, the impact of the technology revolution on strategic behaviours of market participants and its effect on market quality in the modern finance industry is a topic that continues to spark intense debate among scholars, market practitioners, and regulators (see, e.g., Dugast and Foucault (2018), Dugast and Foucault (2021), Abis (2022), Malikov and Pasquariello (2022)).

This thesis aims to contribute to this ongoing debate by focusing on two important issues in the field of market microstructure studies: market manipulation and quantitative investing. Market manipulation, defined as the intentional deception of investors by artificially affecting the supply or demand for a security by Allen and Gale (1992), has been a long-standing practice in the financial market. Understanding market manipulation is crucial for promoting fair and transparent markets, protecting investors, and ensuring the stability of the financial system (e.g., Aggarwal and Wu (2006)). This thesis investigates whether technological developments will disrupt the practice of market manipulation that has been in existence for several hundred years. Specifically, the thesis addresses two key questions about market manipulation: will the shift from the traditional auction market to the limit order market influence the form of market manipulation, and can meme investing, a retail buying frenzy coordinated on social media, be a natural remedy to manipulative short selling?

Quantitative investing, which utilises mathematical and AI models to make automated trading decisions, has gained popularity in recent years. The market share of machine-driven quant funds rose from 20% to over 36% between 2014 and 2019 in the US market.[1] This thesis investigates whether technical development, particularly the increased popularity of machines in the finance industry, will lead to novel types of strategic interaction between quantitative machines and discretionary human traders, and influence market price dynamics. The thesis addresses two key questions about quantitative investing: how will quantitative investing, which relies on machine computation, and discretionary investing, which relies on human skills, strategically interact with each other, and what are the market efficiency implications of such strategic interaction?

This chapter provides a concise overview of the main topics covered in each chapter of the thesis, along with an outline of the modelling approaches and key findings. It aims to provide a comprehensive understanding of the thesis as a whole before delving into the specifics of each chapter. Additionally, this chapter depicts the relationship between the thesis and the relevant finance and economics literature, with the detailed comparison of each chapter with existing papers left as a separate section in the corresponding chapter.

1.1.1 Chapter 2: Strategic trading and manipulation: Machine learning in limit order markets

Since the beginning of trading on organised exchanges, speculators have manipulated markets to profit at the expense of others. In quote-driven markets with designated market makers (DMMs), prevailing market microstructure models depict market manipulation through the manipulative informed trader misleading other market participants by trading occasionally in the “wrong” direction (e.g., John and Narayanan (1997)) or introducing noise components to trades (e.g., Huddart, Hughes and Levine (2001)). However, with the growing dominance of limit order markets (LOMs) over quote-driven markets, new forms of market manipulation,

[1] For example, see the Economist’s report on quants, “The stock market is now run by computers, algorithms and passive managers”.

such as spoofing, quote stuffing, and momentum ignition, have surfaced. Consequently, there is a need for academics and regulators to attain a deeper theoretical understanding of market manipulation in LOM. Towards that end, Chapter 2 of the thesis investigates whether the equilibrium in an LOM populated by traders who learn to trade using a novel machine learning technique is learnable, and if market manipulation could be learned as an equilibrium strategy. Furthermore, this chapter aims to examine how the form of manipulation in LOM would differ from that in traditional quote-driven models.

Chapter 2 introduces a novel machine learning technique, Q-learning, to a dynamic LOM. The continuous-time order book is populated by two types of risk-neutral agents, informed traders who know the current fundamental value, and uninformed traders who know the lagged fundamental value. Through trial and error, traders learn to maximise expected cumulative payoffs of current and future periods using Q-learning.

The belief convergence and successful replication of a selection of stylised facts in Chapter 2 suggest that equilibrium is learnable in a dynamic LOM with information asymmetry. Specifically, the stylised facts replicated by the converged model include a hump-shaped order book, absence of autocorrelations of returns, and slow decaying autocorrelations of absolute returns. Chapter 2 also shows that the informed and the uninformed trade strategically in equilibrium and demonstrate predictable trading patterns conditional on order book information, fundamental volatility, and informed trading level. Informed traders prefer limit (resp. market) orders when the magnitude of mispricing is small (resp. large), while uninformed traders tend to "chase the trend", that is, submit market buys (resp. sells) following market buys (resp. sells) from the informed.

Informed manipulation can be learned as an equilibrium trading strategy in the dynamic LOM. Informed traders may deviate from predictable trading patterns to manipulate the market by taking the "wrong" make-take decision. When there is a small-in-size positive (resp. negative) mispricing along with a high depth imbalance at the best bid (resp. ask), informed traders expect a reversal of the mispricing at a later time. Going against the preference for limit buys (resp. sells) due to low mispricing, informed traders opt for market buys (resp. sells) to prompt trend-chasing uninformed traders to also place market buys (resp. sells). Due to the manipulation of the informed, the uninformed experience a profit reduction in trend-chasing market orders.

This study is the first to implement a continuous-time Q-learning algorithm in a simulated artificial stock market, and its findings suggest that Q-learning can be used to model market manipulation in dynamic LOMs. The use of this novel machine learning technique opens up new avenues for further research on market manipulation and its impact on market efficiency.

1.1.2 Chapter 3: Buying frenzies, short selling costs and their impact on investment efficiency

While Chapter 2 shows how the advent of the limit order book influences the form of market manipulation, Chapter 3 shows that meme investing, a retail buying frenzy coordinated through social media, could be a natural solution to manipulative short selling. Trading frenzies in financial markets occur when speculators trade en masse in one direction and result in significant price pressure. A recent example of this phenomenon occurred in January 2021, when multiple stocks with high short interest became popular among retail investors via social media, leading to a significant surge in their prices. These stocks, commonly known as "meme stocks", allowed retail investors to coordinate their purchases through social media, resulting in substantial costs for short sellers and a consequent reduction in short interest. Chapter 3 explores the real economic impact of such a buying frenzy on firms' investment efficiency.

There are two sides to short selling. On the one hand, short sellers keep prices from overinflating, guiding capital to its best uses and improving welfare. On the other hand, some activist short sellers engage in manipulative tactics that create panic and destroy firm fundamentals. The dichotomy of short sellers creates a challenge for policymakers investigating the welfare implications of short selling. Chapter 3 extends Goldstein and Guembel (2008) to allow costly short-selling and investigates the real investment effects of retail buying frenzy as a friction on short sales.

The model incorporates a feedback mechanism whereby the financial market outcomes influence the real investment value of a firm, allowing the firm manager to glean information about the quality of potential investments from observed stock prices. The model consists of a risk-neutral speculator (who can be positively informed, negatively informed, or uninformed), a noise trader, a risk-neutral market maker, and the firm manager. After observing stock price realisations in the financial market, the firm manager makes a decision regarding a real investment opportunity. In the benchmark equilibrium without frictions, negatively informed and uninformed speculators always sell in the two trading periods. While the sell – sell strategy employed by the negatively informed speculator helps the firm manager correctly reject a poor investment, the same strategy from the uninformed speculators is manipulative because it leads the firm manager to incorrectly reject a good investment based on non-information-driven selling pressure.

In order to investigate the real investment effects of the retail buying frenzy, Chapter 3 extends the benchmark model in three ways. First, an outright short-sale ban is modelled. Second, a cost on short selling is introduced to reflect the fact that a retail buying frenzy increases the stock price and poses an additional cost to short sellers. Third, we model asymmetric noise traders, who are more likely to buy than sell.

A short-sale ban always harms both price efficiency and real efficiency relative to the benchmark equilibrium. A short-sale ban eliminates informed (resp. manipulative) short selling, decreasing (resp. increasing) the stock price efficiency. Additionally, a short-sale ban decreases the information content of positive order flows. This is because positive orders may come from all types of speculators, unlike the benchmark equilibrium where they only originate from the positively informed speculator. Consequently, a short-sale ban brings a net price efficiency reduction, which is then translated into an inferior investment decision by the firm manager.

With costly short selling, we show that, as the short selling cost increases, the uninformed short sellers are driven out of the market quicker than the manipulative short sellers. Small shorting costs do not alter the short-selling intensity of speculators, resulting in no change in price efficiency and investment efficiency compared to the benchmark model. Intermediate shorting costs only deter manipulative short selling and unambiguously improve the stock price efficiency and the real investment efficiency. Relatively high shorting costs eliminate manipulative short selling, but also lead the negatively informed short seller to change from selling in both trading periods to no trade in $t = 1$ and sell in $t = 2$. Relatively high shorting costs thus may improve or harm price efficiency and real efficiency.

Extremely high shorting costs eliminate all short selling and are effectively the same as a short-sale ban.

Another way to incorporate the retail buying frenzy into the benchmark model is to model asymmetric noise trading such that they buy more often than they sell. In this model, noise buy is more likely than noise sell in $t = 2$. In this chapter, we show that such coordinated noise buys cause a cost that is specific to manipulative short sellers and can improve price efficiency and investment efficiency. To be more precise, increased noise buys push up the overall prices and lead the firm manager to increase his propensity to invest. When the short selling is manipulative, a correction of under-investment and an improvement in fundamental value occur, resulting in a higher cost to cover the manipulative short. When the short selling is informative, an over-investment and a deterioration in fundamental value occur, resulting in a lower cost to cover the informed short. This mechanism imposes a cost specific to manipulative short sellers.

In summary, Chapter 3 suggests that meme investing reduces manipulative short selling and does not harm informed short selling for certain types of firms and market conditions, since manipulative short sellers are more sensitive to the same short selling cost increase and may experience extra cost increase during the retail buying frenzy when compared with informed short sellers.

1.1.3 Chapter 4: Interaction of quantitative and discretionary investing

Chapters 2 and 3 explore the potential impact of the technical revolution in the finance industry, including the emergence of limit order book and the increased use of social media in investment decisions, on the practice of market manipulation. Chapter 4 builds on this argument and suggests that such technical revolution can also generate new types of strategic interaction, particularly the interaction between quantitative machines and discretionary human traders.

The use of advanced mathematics and technology has led to a rise in machine-based quantitative investing, with quant funds' market share in US increasing from 20% to over 36% between 2014 and 2019. As quantitative investing relies on machines and rule-based criteria, whereas traditional discretionary investing relies on human skills, it is important to examine the strategic interaction between these two approaches to avoid market quality deterioration brought on by quants, such as the Quant Meltdown of 2007.

Focusing on the fundamental trade-off between capacity and flexibility when machines replace humans in the investment industry, Chapter 4 addresses two fundamental questions related to the impact of the rise of quants on market dynamics. First, what strategic interaction between discretionary and quantitative investing will arise? Second, how does the growth in machine-based quantitative investing impact market efficiency when such interaction exists?

Chapter 4 introduces a Kyle-type model that considers the interaction between two investment firms, namely a fully discretionary firm (firm A) and a firm consisting of both a quant department Q and a discretionary department D (firm B). The model also features liquidity traders and competitive market makers as in the standard Kyle (1985) model. The prior probability of the quant department's existence in firm B measures the quantitative investing level in the economy.

In the model, there are two key differentiating features of the quant and discretionary trading. First, the quant department has greater information processing capacity compared to the discretionary department. There are several reasons why quants may be more effective than human traders in accessing information, such as analysing data more quickly with the help of powerful computers, accessing a wide range of data sources from news, social media, and financial statements, and monitoring the market 24/7. The mathematical models and algorithms that quants use are designed to analyse vast amount of data to identify patterns, trends, and opportunities than human traders may miss. As a result, quants often have better access to information and are able to process and interpret it more quickly than human traders.

Despite the advantage of better information processing capacity, there are also some limitations to quant trading. One of the main limitations is that quants are less flexible than human traders. The fact that quants rely on algorithms and mathematical models means they may miss out on opportunities that require intuition and creative thinking. Additionally, quants may struggle to adapt to sudden and unexpected changes in market conditions. That is, quants are strategically less flexible compared to human traders. The quant department's greater processing capacity (resp. weaker strategic flexibility) than discretionary participants is modelled as access to an additional piece of information (resp. incorrectly believing that the opponent firm A does not react to firm B's demand).

The increase in quantitative investing level has three impacts on the strategic interaction between the two firms. First, it renders firm B more capable of extracting fundamental information, increasing its trading aggressiveness, which is called the "*capacity enhancing effect*". Second, the higher strategic weakness implies firm B is more inclined to overestimate the opponent's trading aggressiveness, reducing firm B's trading aggressiveness, which is called the "*strategy oblivion effect*". Finally, being fully discretionary, firm A tends to trade more aggressively to profit from firm B's increased strategic weakness, which is called the "*internalising effect*". Given such strategic interaction, growth in quantitative investing reduces (resp. increases) overall trading aggressiveness, and, thus, price efficiency when the negative strategy oblivion effect dominates (resp. is dominated by) the other two effects. A growth in quantitative investing can also have a non-monotonic impact on the overall trading aggressiveness and price efficiency. If the quality of the additional piece of information of quants is moderate, the sum of the capacity enhancing and internalising effects start off smaller than the strategy oblivion effect, but then dominate as the quantitative investing level increases.

Chapter 4 extends the model to include the setting where each of the two firms consists of a quant department and a discretionary department and the setting where Q incorrectly believes that the fully discretionary firm A would not strategically react to B's demand with a probability of h, but has a correct belief about the fully discretionary firm with a probability of $1 - h$. The comparison between the extended and the baseline models shows that the impacts of a ceteris paribus increase in firm B's quantitative investing level on the strategic interaction and on price efficiency stay robust to alternative more general model settings.

In addition, when firms A and B each have a quant department, aside from the three effects described above, an increase in firm B's quantitative investing level causes firm A's quant department to have a deteriorated information advantage given more fierce competition from firm B. This effect is called the "*competition effect*". Furthermore, growth in firm B's quantitative investing level is more likely to decrease overall trading aggressiveness and price efficiency when firm A's quantitative investing level is higher. This is due to the weakened internalising effect:

firm A's discretionary department puts less effort into exploiting firm B's trading aggressiveness reduction caused by increased strategic weakness.

Overall, Chapter 4 provides a conceptual framework that explicitly illustrates the fundamental trade-off between capacity and flexibility in the context of machines replacing humans in trading. It also offers novel insights into the consequential implications for market quality by analytically decomposing the impact of machine-based quantitative investing into several quantifiable components that are empirically verifiable.

1.2 Literature review

This section reviews the related literature on market manipulation and on machine-human interaction in financial markets to contextualise the topics explored in the thesis. In addition, it examines the literature on order choice problems, the application of machine learning in economics and finance, managerial learning from stock prices, and relevant extensions of the Kyle model to help understand the methodologies employed in the thesis. The detailed comparisons of each chapter with relevant papers are left as a separate section in the corresponding chapter.

1.2.1 Market manipulation

The manipulation of financial markets has been extensively studied in the literature, which categorises manipulation techniques into three forms: (i) action-based manipulation, referring to intentionally taking actions that change the value (or the perceived value) of the asset (e.g., Vila (1989)); (ii) information-based manipulation, referring to the act of spreading false or deceptive information or rumours (e.g., Benabou and Laroque (1992), Van Bommel (2003), Eren and Ozsoylev (2006)); and (iii) trade-based manipulation, referring to the act of manipulating the stock price through trading activities (e.g., John and Narayanan (1997), Brunnermeier (2000), Huddart et al. (2001), Takayama (2021)).

Chapter 2 of this thesis analyses the third category of manipulation conducted by informed traders. This chapter contributes to the literature by demonstrating that informed traders in the LOM can learn to manipulate the market using make-take decisions, rather than buy/sell and amount decisions in the existing studies of quote-driven markets. Chapter 3 is associated with the third category of manipulation conducted by uninformed traders and contributes to this literature by investigating the real investment effects of a retail buying frenzy in the presence of both informed and manipulative short sellers. For a comprehensive overview of the literature on market manipulation, see Vives (2010), Putnins (2012), and Putnins (2020).

1.2.2 Machine-human interaction in financial markets

The thesis is closely related to the literature on the machine-human interaction in financial markets (see, e.g., Barbopoulos, Dai, Putnins and Saunders (2021), Wang, Cao, Yang and Jiang (2021), Abis (2022), Coleman, Merkley and Pacelli (2022), Malikov and Pasquariello (2022)). For instance, Abis (2022) shows that the trade-off of capacity and inflexibility results in different investment styles (e.g., different macroeconomic timing and stock picking abilities) of quantitative and discretionary mutual funds. Meanwhile, Malikov and Pasquariello (2022) evaluate trading strategies and market quality outcomes, such as efficiency, liquidity, volatility and volume in response to the entry of new quant funds or the switch by incumbent discretionary funds to quantitative investing. Wang et al. (2021) empirically document that the price forecasts generated by a hybrid approach combining machine and human analysts outperform the forecasts of the human-only and machine-only analysts. Coleman et al. (2022) empirically show that machine analysts are less inclined to recommend glamour stocks and firms with potential investment banking needs than their human counterparts.

Chapter 4 of this thesis contributes to this literature by analytically decomposing the impact of growth in machine-based quantitative investing on strategic interaction among traders and on market efficiency into empirical testable components primarily driven by the capacity enhancing effect (i.e., greater information processing capacity) and the strategy oblivion effect (i.e., weaker strategic flexibility).

1.2.3 Endogenous order choices

The literature on order choice problems in limit order markets with information asymmetry has been criticised for oversimplifying the order choice problems of uninformed traders (e.g., Goettler, Parlour and Rajan (2009)). For instance, in a static model, Chakravarty and Holden (1995) draw the uninformed's order from random distributions. The uninformed order choice problems in dynamic LOM models show dependence on exogenous parameters such as private value (e.g., Goettler et al. (2009), Rosu (2020)). To address this issue, Chapter 2 follows Chiarella, He and Wei (2015) and He and Lin (2022) by building a dynamic LOM model with endogenous order choices, thereby providing a more accurate representation of uninformed traders' order choices.

1.2.4 Application of machine learning in economics and finance

The application of machine learning in economics and finance is an emerging field, and most studies have focused on empirical analyses. These studies generally use machine learning to extract unstructured information (e.g., Renault (2017)) or apply machine learning to test theories that imply stock return predictability (e.g., Easley, López de Prado, O'Hara and Zhang (2021)). Chapter 2 adds to this literature by using a novel machine-learning technique, Q-learning, in the context of the dynamic LOM model. This approach demonstrates the potential of machine learning in theoretical frameworks, specifically by integrating reinforcement learning with the market microstructure theory. For comprehensive surveys of this literature, see Varian (2014), Athey (2018), and Athey and Imbens (2019).

1.2.5 Managerial learning from stock prices

The thesis is also closely related to the literature on managerial learning from stock prices. Managerial learning occurs because stock prices aggregate information of many different market participants who do not have direct channels for communication with the firm outside the trading process and are able to complement the information of managers to make corporate investment decisions. For theoretical literature, see, for example, Dow and Gorton (1997) and Subrahmanyam and Titman (1999) and for empirical literature, see, for example, Chen, Goldstein and Jiang (2007), Foucault and Fresard (2012), Aliyev, Allahverdiyeva and Putnins (2023)). Chapter 3 theoretically examines managerial learning in the presence of market manipulation and short-selling frictions, motivated by the retail buying frenzy of meme stocks.

1.2.6 Extensions of the Kyle model

The Kyle (1985) model is one of the workhorse models of market microstructure theory that explores the impact of asymmetric information on the price formation process. Subsequent extensions of the Kyle model include risk averse informed traders and market makers (Subrahmanyam (1991)), discretionary liquidity traders (Admati and Pfleiderer (1988)), multiple informed traders with correlated information (Foster and Viswanathan (1993)), discrete actions of traders (Goldstein and Guembel (2008)), and the impact of feedback from the financial market on the real investment value of a firm (Goldstein and Guembel (2008)) to name a few. Building on Goldstein and Guembel (2008), Chapter 3 extends the Kyle model by introducing a retail buying frenzy as short sale friction. Chapter 4, on the other hand, incorporates quantitative traders into the Kyle model and models machine-based quants' greater information processing capacity as access to an additional piece of information and quants' weaker flexibility as ignorance of the strategic impact of their own trades.

1.3 Thesis outline

The remaining sections of the thesis are divided into three primary chapters, each focusing on a separate study exploring the following topics:

- Chapter 2: Strategic trading and manipulation in an artificial limit order market where agents learn via reinforcement learning.
- Chapter 3: The impact of retail buying frenzies on real investment efficiency.
- Chapter 4: Implications of the interaction between quantitative investing and discretionary investing for market quality.

Chapter 5 encompasses a summary of the thesis's findings, presents reflections on the discoveries made, and proposes potential directions for future research.

Chapter 2

Strategic trading and manipulation: Machine learning in limit order markets

2.1 Introduction

The ignoble history of market manipulation dates back to the foundation of the world's first stock market, the Amsterdam Stock Exchange, over three hundred years ago. Existing theoretical market microstructure models tend to capture market manipulation in a quote-driven market, where designated market makers (DMMs) take the other side of traders' market orders. In these models, the manipulative informed trader misleads other participants by taking the wrong action when facing the buy/sell or amount decisions, e.g., occasionally trading in the opposite direction of his information (John and Narayanan (1997)) or adding noise components to trades (Huddart et al. (2001)).

Given that limit order markets (LOM) have replaced quote-driven markets and become the dominant form of financial market organisation, in which numerous new types of manipulation appear, such as spoofing, quote stuffing and momentum ignition, more theoretical insights on the market manipulation in the LOM is of interest to academics and regulators. Towards that end, this paper introduces Q-learning, a novel machine learning technique, to a dynamic limit order market, and addresses the following questions: Is the equilibrium learnable in an LOM populated by bounded rational Q-learning traders? Would market manipulation be learned as an equilibrium strategy? And if so, would our setting generate a different form of informed manipulation compared to the traditional DMM setting?

The advantages of applying Q-learning in our model are threefold. First, it demonstrates the potential of integrating reinforcement learning techniques (RL) into the market microstructure theoretical framework, as an alternative belief updating rule. Traditional Bayesian updating assumes agents have adequate knowledge of model priors (e.g., the joint distribution for fundamental value and order flow). Reinforcement learning techniques do not make such an assumption and represent one step towards realism. Second, with Q-learning traders who maximise expected cumulative payoffs of current and future periods (i.e., Q-values), our model is better able to reflect the dynamic nature of order choice problems than existing LOM models (see, e.g., Foucault (1999), Goettler et al. (2009), Rosu (2020)) – a trader not only factors in the impact of future traders on his current order's payoff as in existing dynamic models, but also considers the impact of his current order on future market conditions and further on all his future orders' payoffs. By considering the two impacts simultaneously, the Q-learning traders maximise their expected lifetime utility. Last, Q-learning allows us to fully endogenise the order choice problems of traders, without introducing exogenous parameters such as private value (Goettler et al. (2009)) or time preference (Rosu (2020)).

In our model, a continuous-time order book is populated by two types of risk-neutral agents, informed traders who know the current fundamental value, and uninformed traders who know the lagged fundamental value. A trader submits a limit order, or a market order, or chooses not to trade upon arrival. We show that equilibrium is learnable in a dynamic LOM with information asymmetry. Measured by Q-value criteria and average reward criteria, belief convergence of traders is achieved. The converged model is able to replicate stylised facts documented in the empirical LOM literature, including a hump-shaped order book (Bouchaud, Mézard and Potters (2002)), absence of autocorrelations of returns (Cont (2001)), and slowly decaying autocorrelations of absolute returns (Cont (2001), Schnaubelt, Rende and Krauss (2019)).

We also show that, in equilibrium, the informed and the uninformed trade strategically and demonstrate predictable trading patterns conditional on order book information, fundamental volatility, and informed trading level. Informed traders submit more market (limit) orders when the mispricing is large (small), reflecting increased sensitivity to execution risk. Uninformed traders "chase the trend" and are more prone to place market buy orders following a market buy order. Increased informed trading leads informed traders to undercut each other using more aggressive limit but not more market orders, increases (reduces) aggressive limit orders for the informed (uninformed), and improves efficiency and liquidity simultaneously.

Most importantly, market manipulation can be learned as an equilibrium strategy by bounded rational Q-learning traders in our model. Informed traders may strategically violate their predictable trading patterns when facing make-take decisions, in order to mislead uninformed traders. Informed manipulation emerges as a result of the strategic interaction between traders. Informed traders anticipate a later mispricing reversal when a small-in-size positive (negative) mispricing is accompanied by high depth imbalance at the best bid (ask): given uninformed traders' preference for market buy (sell) following a market buy (sell), informed traders go against their preference for limit buys (sells), and react strategically by using market buys (sells) to trigger uninformed market buys (sells) and enhance the execution probability and profitability of later informed traders' limit sells (buys). This strategy of the informed is both manipulative and collusive, because informed traders confuse uninformed traders by taking the "wrong" action when facing make-take decisions, submit market buys (sells) when they should have submitted limit buys (sells), and sacrifice their own profit difference between limit and market buys (sells) in exchange for profit increases of later arriving informed traders. We thus add to the market manipulation literature by showing that informed manipulation in the LOM can go beyond the buy/sell or amount dimension as in existing studies within the DMM setting (see, e.g., John and Narayanan (1997), Brunnermeier (2000), Huddart et al. (2001), Takayama (2021)).

Our paper also innovates the literature of machine learning applications in finance and economics. To the best of our knowledge, this paper is the first to implement a continuous-time Q-learning algorithm in a simulated artificial stock market. Recent studies within this strand of literature have mostly been empirical and focus on employing machine learning techniques to construct novel variables such as investor sentiment (e.g., Renault (2017), Easley et al. (2021), Bryzgalova, Pelger and Zhu (2020), Gu, Kelly and Xiu (2020)). In this line of research, our work is perhaps most relevant to Philip (2021), which applies discrete-time Q-learning to real-world limit order book data to determine the value of the option to cancel a limit order. Nevertheless, given the empirical nature of Philip (2021), our work is fundamentally different from his, and shows that reinforcement learning can be a promising and powerful tool for theorists.

The rest of the paper proceeds as follows. Section 2.2 reviews the related literature. Section 2.3 introduces Q-learning and the order book information classifier system into a dynamic limit order market. Section 2.4 defines the concepts of the numerical equilibrium and evaluates the model against a selection of stylised facts. Section 2.5 investigates the role of order book information in order choices and strategic trading. Section 2.6 analyses the informed traders' and uninformed traders' trading behaviours under different volatility regimes and different informed trading levels. Section 2.7 illustrates informed traders' manipulative behaviours, i.e., strategic and deliberate violation of their predictable trading patterns, and uninformed traders' reactions. Section 2.8 concludes.

2.2 Related literature

This paper is related to the literature of order choice problems in static and dynamic limit order markets with information asymmetry (see Parlour and Seppi (2008) and Rosu et al. (2012) for excellent surveys). Existing static and dynamic LOM models with information asymmetry are prone to oversimplify the order choice problems for uninformed traders for model tractability (Goettler et al. (2009)). For static models, Chakravarty and Holden (1995) posit that the profit-maximising informed trader can choose the order size, order type and the limit order price, but draw the uninformed's order from random distributions. For dynamic models, Goettler et al. (2009) numerically solve a continuous-time game featured by the endogenous cancellation and endogenous information acquisition, in which agents are endowed with positive, negative or zero private values. The order choice of the uninformed relies on the private value: an uninformed trader with a large positive (large negative) private value is more inclined to submit a market buy (market sell). In Rosu (2020), equilibrium strategies of the uninformed are deterministically determined by exogenously given waiting costs. In these papers, the order choice problem of uninformed traders is either not explicitly modelled or highly dependent on exogenous parameters (private value, patience). Relaxing these possibly unrealistic assumptions, we contribute to this line of research by establishing causality from information asymmetry to differences between the informed trader's and the uninformed trader's order choices, and also by generating testable predictions for the empirical literature on order aggressiveness by different trader types (see Duong, Kalev and Krishnamurti (2009) and Chiu, Chung and Wang (2014) for institutional and individual traders; see Beber and Caglio (2005) for high PIN and low PIN trading periods).

This paper is also related to the theoretical and empirical literature on informed trading's impact on liquidity in limit order market – especially on resiliency. For empirical studies, Kempf, Mayston and Yadav (2009) examine the electronic limit order market XETRA using market order imbalance to represent the unobservable information variable, and conclude that the impacts of informed trading on both spread resiliency and depth resiliency are strongly negative. Menkhoff, Osler and Schmeling (2010) study the interdealer forex market for Russian rubles, and show that the limit order submission rate of the informed is much more positively responsive to an increase in spread, a drop in depth or a drop in cumulative depth than that of the uninformed. They argue that informed trading is thus resiliency improving. The two empirical studies are constrained by the extent to which their proxies represent the information. Our work is different from them, because our simulated data has exact identification of the informed traders and the uninformed traders.

For theoretical studies, Rosu (2020) shows that an increase in the fraction of informed traders always improves spread, has no effect on price impact, and improves resiliency. Nevertheless, he provides no further explanations on possible underlying channels that drive the improvement in resiliency. Our work is thus complementary to Rosu (2020) in two ways – first, by showing that the resiliency improving effect is caused by the differences between equilibrium liquidity provision strategies of the informed trader and the uninformed trader, and more fundamentally, driven by information asymmetry; and second, by showing that more intensive intertemporal competition between informed traders can be liquidity improving (in terms of spread and depth), rather than liquidity deteriorating as his work.

This paper is most relevant to the literature on market manipulation (surveyed in Vives (2010), Putnins (2012), Putnins (2020)). The growing literature categorizes manipulation techniques into three forms: (i) action-based manipulation, which involves taking actions that change the value (or the perceived value) of the asset (Vila (1989)), e.g., a company manager can divest a factory to depress the stock value; (ii) information-based manipulation, which involves spreading misleading information or rumours (Benabou and Laroque (1992), Van Bommel (2003), Eren and Ozsoylev (2006)); and (iii) trade-based manipulation, which involves influencing stock price purely through trading (John and Narayanan (1997), Brunnermeier (2000), Huddart et al. (2001), Takayama (2021)). Our work analyses the third category of manipulation conducted by the informed. In existing REE literature on informed trade-based manipulation, a trader with long-lived information manipulates to sabotage other participants' technical analysis if faced with ex-post trade disclosure requirements (John and Narayanan (1997), Huddart et al. (2001)), or future public announcements about the fundamental (Brunnermeier (2000)). In these papers, a manipulative informed trader hides or enhances his own information advantage, either by occasionally trading in the opposite direction of his information (John and Narayanan (1997), Takayama (2021)), or by adding noise components to trades (Huddart et al. (2001)), i.e., choosing the "wrong" action when facing buy/sell or amount decisions. In contrast, in our setting, an informed trader acts collusively and manipulates to enhance the profit of other later arriving informed traders, by choosing the "wrong" action when facing market/limit order decisions.

Lastly, our work contributes to the emerging literature of applying machine learning in economics and finance (see Varian (2014), Athey and Imbens (2019) and Athey (2018) for excellent surveys). Recent studies are mainly empirical, and use machine learning to extract unstructured information and construct novel variables such as investor sentiment (Renault (2017)) or apply machine learning to test theories that imply stock return predictability (Easley et al. (2021), Bryzgalova et al. (2020), Gu et al. (2020)). Our work unveils the promising future of applying machine learning to theory, especially the possibility of integrating reinforcement learning with market microstructure theory framework. Using reinforcement learning to update the agent's belief in our dynamic LOM rather than the traditional Bayesian updating rule enables us to relax a set of strict assumptions like the agent's perfect knowledge of the model's probability structure and avoid using time preference and private value parameters. Fewer parametrisations can help reveal the importance and consequence of information itself, which matters particularly in the context of market microstructure studies.

2.3 The model

We consider a dynamic limit order market model of trading a single risky asset, which is motivated by Goettler et al. (2009), Chiarella et al. (2015), and He and Lin (2022). Let time to be denoted as t, where $t \in \{0, \infty\}$. The innovations to the fundamental value of the asset v_t follow a Poisson process with parameter θ. If an innovation takes place, the fundamental value increases or decreases by Δ ticks with equal probability. There are N risk-neutral traders who enter the market randomly following a Poisson process at rate λ. Among them, N_I are informed, and N_U are uninformed. When entering the market, an informed trader knows the current fundamental value v_t, while an uninformed trader only knows the lagged fundamental value $v_{t-\tau}$, where τ is a positive integer. The only difference between informed and uninformed traders is their knowledge of the fundamental value. Different from Goettler et al. (2009) where the trader only trades one share in his lifetime, repeatedly visits the market if no execution, and leaves the market forever after an execution, our Q-learning traders can repeatedly visit the market despite all his prior executions and no executions.

2.3.1 Traders' order choices

The set of available actions for any trader is related to his choices about trading or no trading, market or limit order, order direction (buy or sell), and limit order aggressiveness. When re-entering the market, the trader cancels his last limit order if unexecuted, and optimally chooses an action that maximises his expected cumulative payoffs given the trading history H_t, the current state of the limit order book, and his type.

Formally, the types of buy orders submitted can be defined as follows. A market buy order (mb) is a request to complete the transaction immediately at the best ask. An extremely aggressive limit buy order ($ealb$) lies within the spread and is posted at $a_t - 1$, a price that is one tick below the best ask a_t. A moderately aggressive limit buy order (alb) lies within the spread and is posted at $b_t + 1$, a price that is one tick above the best bid b_t. An ordinary limit buy order (lb) is at the best bid b_t. An unaggressive limit buy order (ulb) is at $b_t - 1$, one tick below the best bid. The market sell order (ms), extremely aggressive limit sell order ($eals$), moderately aggressive limit sell order (als), and ordinary limit sell order (ls) can be defined analogously. A trader can also choose not to trade (nt). In summary, the trader's action space contains 11 actions: let $\mathcal{A}_b = \{mb, ealb, alb, lb, ulb\}$, and $\mathcal{A}_s = \{ms, eals, als, ls, uls\}$, then the action space is $\mathcal{A} = \mathcal{A}_b \cup \mathcal{A}_s \cup \{nt\}$.

2.3.2 Order book information state space

A limit order book $L_t = \{l_t^i\}_{i=1}^{\infty}$ contains the history of order book information at time t, l_t^i, consisting of a backlog of unexecuted limit orders at each discrete price level, p^i, with the standard price-time priorities for limit order execution. The continuous information state space of the limit order book is discretised as a finite set of states denoted as $\mathcal{C} = \{(s_t, E(v_t) - p_t^m, R_t, b_t, a_t, d_t^a - d^b, D_t^a - D_t^b, LT_t)^j\}_{j=1}^J$. State variables are the spread condition s_t, the expected fundamental minus midprice $E(v_t) - p_t^m$, Rosu's signal $R_t = |\bar{v}_t - p_t^m|/s_t$, the current best bid b_t, the current best ask a_t, the depth imbalance measured at the best ask and bid $d_t^a - d_t^b$, the cumulative depth imbalance measured at the whole sell side and buy side $D_t^a - D_t^b$, the last trade direction LT_t. The expected fundamental value equals v_t for the informed and equals $v_{t-\tau}$ for the uninformed. Rosu's signal is the level of mispricing observed, in which $\bar{v}_t = v_t$ for the informed and $\bar{v}_t = (1/N)\sum_{t'=t-L+1}^{t} p_{t'}$, a moving average price of the past L periods, for the uninformed.

For computational tractability, state variables are further discretised using the classifier system developed in Chiarella et al. (2015) and He and Lin (2022). Appendix 2.1 reports the classified rules (CRs) of the classifier system. A feasible state a trader may encounter is a vector of the values of state variables. For instance, a possible value of s_t is "30", where "3" denotes that the current spread is larger than 2, and "0" denotes that the order book is not empty; a possible value of $E(v_t) - p_t^m$ is "1", which means the expected fundamental is higher than the midprice; a possible value of Rosu's signal $R_t = |\bar{v}_t - p_t^m|/s_t$ is "1", which means the mispricing measure is between 0 and 0.5; a possible value of b_t (or a_t) is "0", which means the current bid (or ask) equals the last bid (or ask); a possible value of $d_t^a - d_t^b$ (or $D_t^a - D_t^b$) is "1", which means the depth at the best ask (or at the whole sell side) is larger than the best bid (or at the whole buy side); a possible value of LT_t is "1", which means the last trade is buyer-initiated; the resulting state vector is thus "301100111". After applying the classifier rules, the final discretised state space is composed of 14580 feasible states.

For dynamical strategic trading in LOM with such a large number of state variables, it becomes extremely challenging to have an analytic solution. In this paper, we solve the trading game numerically by formalising the limit order book's evolution through time as a Markov Decision Process (MDP) and applying the Q-learning algorithm (Watkins (1989)), a reinforcement learning technique, to obtain the traders' equilibrium beliefs about the expected cumulative payoffs of possible state-action combinations.

2.3.3 Trader objective and Q-learning

The Markov Decision Process of the limit order book is characterised by a 4-tuple $(\mathcal{C}, \mathcal{A}, \mathbb{P}(c'|c,a), \rho(c,a))$. As defined before, $\mathcal{C}$ is the set of feasible states of the order book, and $\mathcal{A}$ is a trader's action space. $\mathbb{P}(c'|c,a)$ is an unobserved Markov probability transition matrix, which determines new order book state $c' \in \mathcal{C}$ at the trader's next entry given his current action $a \in \mathcal{A}$ and the current order book state $c \in \mathcal{C}$. $\rho(c,a)$ is an underlying reward rate that is constant for each state-action combination. $\rho(c,a)$ and the time it takes to transfer from c to c' (i.e., the time elapsed since the trader's current entry and until his next entry) jointly determine the reward r observed by the trader when he re-enters the market.

By viewing the order book's evolution through time as an MDP, we can formulate the trader's dynamic optimization problem. Each trader has a type (I, β), where I represents his information type (informed or not). As in Goettler et al. (2009), β is a discount rate that reflects the cost of market monitoring until a limit order's execution and the opportunity costs like delaying trades in other assets if the trader is implementing a portfolio strategy. The reward r for a trader whose buy order was executed at time t before his re-entry is the difference between the fundamental value v_t and the price he paid, or the order profit. The reward r for an executed sell order can be defined analogously. The reward associated with no execution or no order placement is 0. Formally, when re-entering the market, conditional on his last order placement decision, a trader receives:

$$r = \begin{cases} v_t - p_t & \text{if he executes a buy order at } t \\ p_t - v_t & \text{if he executes a sell order at } t \\ 0 & \text{if he has no execution or submission} \end{cases} \tag{2.1}$$

Let $Q_\pi(c,a)$ represent the discounted cumulative rewards that can be expected (i.e., expected cumulative payoffs) if action a is taken in state c, and a given strategy π is followed thereafter:

$$\begin{aligned}Q_\pi(c,a) = \sum_{c'\in\mathcal{C}} \Pr(c'|c,a) \int_0^\infty \int_0^{\Delta t} e^{-\beta s}\rho(c,a)dsdF(\Delta t)\\ + \sum_{c'\in\mathcal{C}} \Pr(c'|c,a) \int_0^\infty e^{-\beta\Delta t} Q_\pi(c',a')dF(\Delta t),\end{aligned} \tag{2.2}$$

where $\Pr(c'|c,a)$ is an element of the transition matrix $\mathbb{P}$, and Δt is the random time between the trader's two consecutive entries, of which the CDF is denoted as $F(\Delta t)$. Since both the informed and uninformed enter the market randomly following a Poisson process at a rate λ, Δt follows an exponential distribution with mean $1/\lambda$. For the optimal strategy π^*, we have:

$$\begin{aligned}\pi^*(c,a) = \max_\pi \Bigg(\sum_{c'\in\mathcal{C}} \Pr(c'|c,a) \int_0^\infty \int_0^{\Delta t} e^{-\beta s}\rho(c,a)dsdF(\Delta t)\\ + \sum_{c'\in\mathcal{C}} \Pr(c'|c,a) \int_0^\infty e^{-\beta\Delta t} Q_\pi(c',a')dF(\Delta t)\Bigg)\end{aligned} \tag{2.3}$$

$$\begin{aligned}Q^*(c,a) =& \sum_{c'\in\mathcal{C}} \Pr(c'|c,a) \int_0^\infty \int_0^{\Delta t} e^{-\beta s}\rho(c,a)dsdF(\Delta t)\\ &+ \sum_{c'\in\mathcal{C}} \Pr(c'|c,a) \int_0^\infty e^{-\beta\Delta t} \max_{a'\in\mathcal{A}} Q^*(c',a')dF(\Delta t)\\ =& \mathbb{E}\Bigg(\int_0^\infty \int_0^{\Delta t} e^{-\beta s}\rho(c,a)dsdF(\Delta t)\\ &+ \int_0^\infty e^{-\beta\Delta t} \max_{a'\in\mathcal{A}} Q^*(c',a')dF(\Delta t)\Bigg| C_t = c, A_t = a\Bigg).\end{aligned} \tag{2.4}$$

Note that $Q^*(c,a) \equiv Q_{\pi^*}(c,a)$. The last two lines of Eq.(2.4) are two forms of Bellman optimality equations for the trader's continuous time dynamic optimisation problem. If Markov probability transition matrix $\mathbb{P}$ and the underlying reward rate $\rho(c,a)$ were known, the value function $Q^*(c,a)$ can be explicitly solved from the Bellman optimality equations for any state-action combination (c,a), as there are the same numbers of equations and unknowns. Without such perfect knowledge, both informed and uninformed traders are thus assumed to learn their equilibrium beliefs about expected cumulative payoffs and optimal strategies conditional on their information sets using Q-learning, a model-free reinforcement learning algorithm.

Based on Eq.(2.4), the Q-learning iteration procedure includes the following four steps. (i) Arbitrarily initialize the value function Q^* for every state, every action. (ii) After observing the current state c, a trader chooses the best action a given current belief with probability $1-\varepsilon$, and chooses an inferior action with probability $\varepsilon/10$ (there are 11 feasible actions). He trembles to avoid local optima. (iii) Upon next entry, the trader observes reward r and the new state $c' \in \mathcal{C}$, and updates his belief for the value function following the rule depicted by Eq.(2.5),

$$Q^{(k+1)}(c,a) = Q^{(k)}(c,a) + \alpha\left(\frac{1-e^{-\beta\Delta t}}{\beta}\rho(c,a) + e^{-\beta\Delta t}\max_{a'\in\mathcal{A}} Q^{(k)}\left(c',a'\right) - Q^{(k)}(c,a)\right), \quad (2.5)$$

where Δt is the realised value of time between two entries. $1/\beta\cdot(1-e^{-\beta\Delta t})\rho(c,a)$ is the observed reward r. k is the number of times action a has been chosen in state c by traders of the same type, and α is the learning rate. (iv) Repeat the previous two steps until traders' beliefs converge.

2.4 Equilibrium

This section illustrates the equilibrium concept in our trading game, characterised by Q-value convergence and average reward convergence criteria, using a benchmark parametrisation. The benchmark equilibrium results allow us to investigate informed and uninformed traders' order choices conditional on various order book states in later sections. In equilibrium, informed traders trade on mispricing signals. The converged model demonstrates statistical properties consistent with empirical data including hump-shaped mean depth profiles, absence of autocorrelations of returns, and slowly decaying autocorrelations of absolute returns.

2.4.1 A benchmark

For illustrative purposes, we regard each trading period as 1 minute and 360 trading periods as a 6-hour trading day, and set benchmark parameter values as follows. We choose the total number of traders populated in the market to be $N = 1000$, where $N_I = 150$ are informed and $N_U = 850$ are uninformed. Informed and uninformed traders do not differ in trading speed, with the same returning rate of 1/60 and make an order choice 6 times every trading day. The uninformed's information lag $\tau = 180$, which corresponds to half a trading day. We set the tick size to 1, and the initial fundamental value to $v_0 = 5000$ ticks (say, i.e., \$50). The expected time between innovations about fundamental value is set to 1 minute, i.e., $\theta = 1$. After an innovation occurs, the fundamental value will either go up or down by $\Delta = 4$ ticks with equal probability.

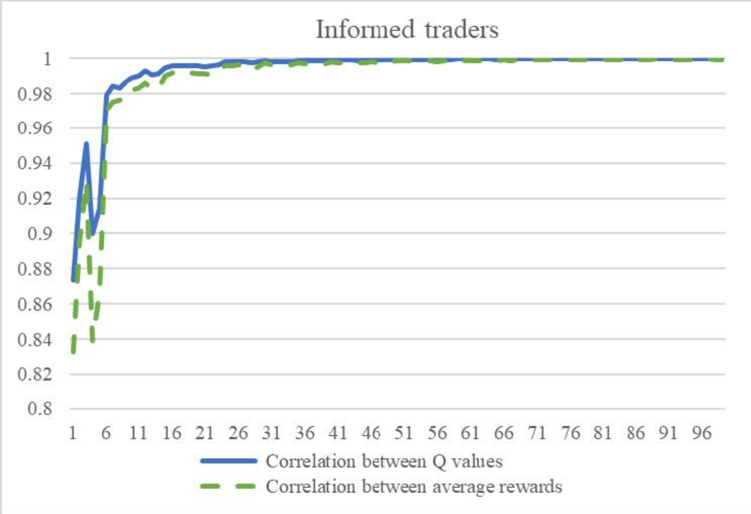
As for Q-learning belief updating, the learning rate is set to $\max(0.0003, 1/(n+1))$ for all traders, where n represents the number of trading rounds (the "trading round" concept here is equivalent to the "training episode" concept in machine learning literature, and each round consists of 360,000 trading periods). All traders, despite their information types, have the same continuous discount rate of $\beta = 0.05$ and tremble rate of $\epsilon = 0.01$.

2.4.2 Convergence criteria

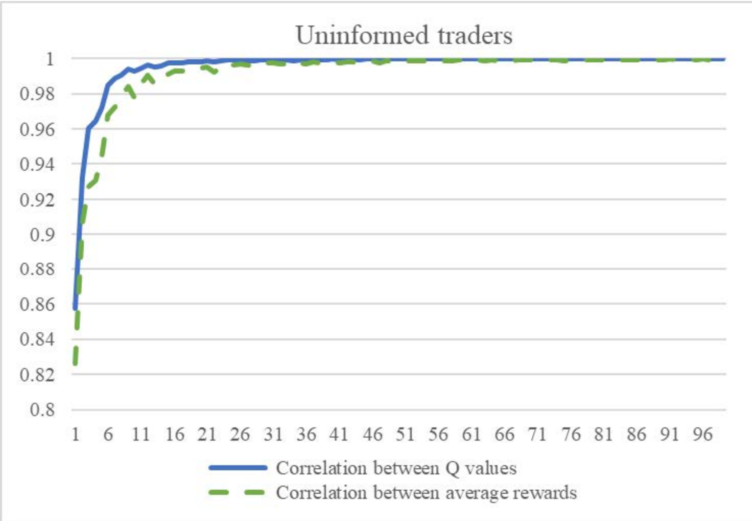
We consider that a numerical equilibrium is reached if both the convergence of Q-values and the convergence of traders' average rewards (order profits) are satisfied. Intuitively, the convergence of traders' estimates of expected cumulative payoffs (Q-values) mirrors the fixed point problem in REE models. Moreover, the convergence of traders' average rewards is a more stringent criterion than the convergence of Q-values given the dynamic feedback mechanism between trading behaviours and limit order book: it requires that both the informed's and uninformed's strategies stabilise, and it also requires that the order book settles into equilibrium states, such that the traders can obtain equilibrium rewards from each interaction with the order book. Formally, the equilibrium concept and the two corresponding in-sample convergence criteria are defined as follows.

Definition 2.1. *A numerical equilibrium of the limit order market under Q-learning, defined by informed and uninformed traders' beliefs about their own cumulative payoffs and corresponding optimal strategies, is considered to be reached after the n-th trading round for sufficiently large n when the following two criteria are satisfied:*

- *Q-value criterion: The correlation of Q-values of active trading strategies between the n-th and the (n+1)-th rounds reaches 0.999;*
- *Average reward criterion: The correlation of traders' average rewards between the n-th and the (n+1)-th rounds reaches 0.999.*



(A) Informed traders



(B) Uninformed traders

Figure 2.1: Convergence of learning in the benchmark model.
The figure shows the convergence results of the benchmark model. Panels (A) and (B) show the convergence of the informed's and the uninformed's learning, respectively. The horizontal axis corresponds to the number of trading rounds, and each round consists of 360,000 trading periods. For the Q-value convergence criterion, uninformed traders' learning converges faster than informed traders' learning (uninformed convergence: round 24; informed convergence: round 40); For the average reward convergence criterion, the two types of traders' learning converges simultaneously at round 59.

We simulate the model under the benchmark parametrisation for 100 rounds and check for convergence along with the training. The convergence results are reported in Figure 2.1. The horizontal axis is trading round n, and the vertical axis is the correlation coefficient of Q-values (blue solid line) and the correlation coefficient of average rewards (green dashed line) over two adjacent rounds.

Panel (A) shows the convergence of the informed's learning. Panel (B) shows the convergence of the uninformed's learning. On the Q-value criterion, the uninformed converge to their equilibrium beliefs at round 25 with a correlation coefficient of 99.92%, while the informed converge to their equilibrium beliefs at round 41 with a correlation coefficient of 99.91%. Uninformed traders learn faster because there are more of them in the trading crowd (850 out of 1000). As expected, the average reward criterion is satisfied at round 60, which is 19 rounds later than when all traders have formed their optimal strategies.

2.4.3 Order book statics and stylised facts

After the benchmark model reaches the equilibrium, we fix traders' Q values, disallow the tremble, and simulate for another 1000 trading days (360,000 trading periods). We report overall frequencies of all order book state variables in Table 2.1, except for $E(v_t) - p_t^m$ and Rosu's signal. As shown in the table, the buy side and the sell side of the order book are symmetric and on average quite balanced over the whole 1000 trading-day simulation.

Since informed and uninformed traders mainly differ in observations of $E(v_t) - p_t^m$ and Rosu's signal due to information asymmetry in terms of fundamental value, we report frequencies of the two state variables by trader types separately in Table 2.2. The informed's observations on the difference between expected fundamental value and midprice justify the usage of midprice as a proxy for contemporal fundamental value in the existing empirical literature (see, e.g., Beltran, Grammig and Menkveld (2005), Chacko, Jurek and Stafford (2008)). For him, $E(v_t)$ is equal to v_t, and he is observing $v_t - p_t^m > 0$ and $v_t - p_t^m < 0$ of roughly the same probability, i.e., 49.57% and 49.66%, meaning that the current midprice is very close to the current fundamental value. On the other hand, for the uninformed, $E(v_t)$ is equal to v_{t-180}, and he is observing $v_{t-180} - p_t^m > 0$ and $v_{t-180} - p_t^m < 0$ of relatively different probability, i.e., 48.96% and 49.79%, meaning that the current midprice is relatively far from the lagged fundamental value.

The relatively large differences between the uninformed trader's and the informed trader's observation frequencies of Rosu's signal indicate that Rosu's signal might be an inaccurate mispricing signal for the uninformed trader (since he is estimating $\bar{v}_t$, using moving average past prices, while for the informed $\bar{v}_t = v_t$).

Table 2.1: Order book state frequencies for all traders in the benchmark model.
The table shows the frequencies (in percentage) of spread s_t, bid trend $b_t - b_{t-1}$, ask trend $a_t - a_{t-1}$, depth imbalance $d^a_t - d^b_t$, and cumulative depth imbalance $D^a_t - D^b_t$ for all traders based on the converged benchmark model. The frequencies of bid trend, ask trend, depth imbalance, cumulative depth imbalance, and last trade direction reflect the symmetry of buy and sell sides.

Spread: s_t				Bid trend: $b_t - b_{t-1}$			Ask trend: $a_t - a_{t-1}$		
$= 1$	$= 2$	> 2	emp.	> 0	< 0	$= 0$	> 0	< 0	$= 0$
53.01	17.17	27.94	1.87	4.05	4.09	91.86	4.39	4.39	91.24

Depth imbalance: $d^a_t - d^b_t$			Cumulative depth imbalance: $D^a_t - D^b_t$			Last trade direction: LT_t	
> 0	< 0	$= 0$	> 0	< 0	$= 0$	> 0	< 0
37.71	38.81	23.48	49.05	49.82	1.13	49.44	50.56

Table 2.2: Frequencies of $E(v_t) - p^m_t$ and $|\bar{v}_t - p^m_t|/s_t$ for informed and uninformed traders in the benchmark model.
The table shows the frequencies (in percentage) of expected fundamental minus midprice $E(v_t) - p^m_t$ and Rosu's signal $|\bar{v}_t - p^m_t|/s_t$ for informed and uninformed traders, respectively, in the converged benchmark model. The large difference between uninformed and informed traders' observation frequencies of Rosu's signal indicates that it is an inaccurate mispricing signal for uninformed traders.

Expected fundamental value: $E(v_t)$			Rosu's signal: $|\bar{v}_t - p^m_t|/s_t$			
$> p^m_t$	$< p^m_t$	$= p^m_t$	$\leqslant 0.5$	$\leqslant 1.5$	$\leqslant 2.5$	$\leqslant 3.5$
49.57	49.66	0.77	5.59	12.40	8.17	6.26
48.96	49.79	1.25	33.02	29.47	11.90	6.41

We now evaluate our model based on stylized facts documented in the empirical LOM literature (Cont (2001), Bouchaud et al. (2002), Schnaubelt et al. (2019)), including (i) a hump-shaped order book, (ii) absence of autocorrelations of returns, and (iii) slowly decaying autocorrelations of absolute returns.

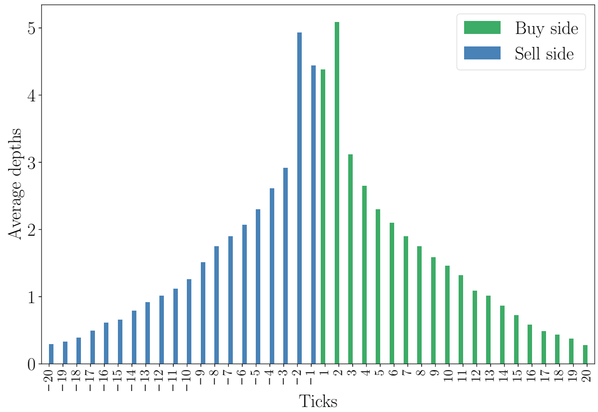
During 360,000 trading periods simulated using the converged benchmark model, we record the order book every 100 trading periods. The average order book shape is then calculated using the mean depths of the 3600 snapshots. We record the price series period by period and calculate the log returns accordingly. The resulting average order book shape, the autocorrelations of returns, and the autocorrelations of absolute returns are depicted in Figure 2.2.

As shown in Panel (A) of Figure 2.2, our order book has a "hump" located at one tick away from the best bid (ask), with average depth increases from the best bid (ask) to the second-best bid (ask), then subsequently decreases. In particular, the average depth at the best bid (ask) is 4.38 (4.44), and at the second-best bid (ask) is 5.09 (4.93). Panel (B) suggests that autocorrelations (ACs) of returns are only negative for the initial 3 lags, reflecting the bid-ask bounce, and then quickly approach 0. Panel (C) indicates that the ACs of absolute returns are slowly decaying from 0.26 to around 0.05 even with 30 lags, reflecting long memory. All three stylised effects are reproduced in the paper.

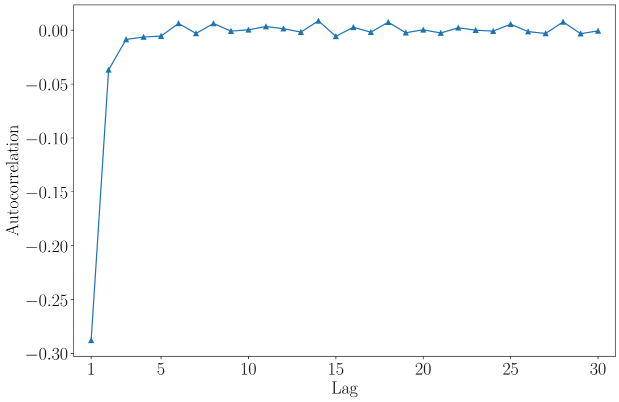
2.5 Order choices and order book states

In this section, we first characterise the equilibrium liquidity provision and consumption roles of traders using unconditional probabilities for each order type, and then show how order book information impact uninformed and informed traders' trading behaviours, and hence the consequent implication on market quality. In our model, informed (uninformed) traders place more (less) aggressive quotes out of their liquidity supply when depth at the same-side best quote is large, consistent with empirical findings of Aitken, Almeida, Harris and McInish (2007) and Chiu et al. (2014) about institutional and individual traders' order aggressiveness strategies, and which, to date, have not been depicted by theoretical studies.

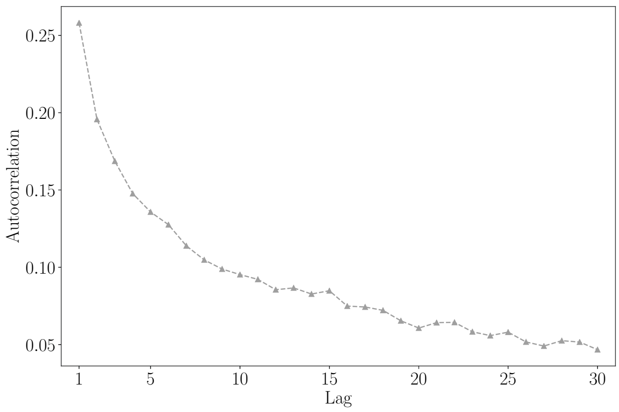
Panels (A) and (B) in Table 2.3 report the unconditional and conditional probabilities of trader's order choices based on order book states. The probabilities of *nt* are scaled by two for ease of comparison. Since the buy side and the sell side are quite symmetric, we only report on the buy side.



(A) Hump-shaped order book



(B) Absence of ACs of returns



(C) Slowly decaying ACs of absolute returns

Figure 2.2: Stylised facts in the simulated order book.
The figure shows the stylized facts in the simulated order book. Panel (A) shows the order book's mean depth profile generated by the converged benchmark model. Green bars represent the buy side, and blue bars represent the sell side. The depth of each price level is recorded every 100 trading periods for the 20 best quotes on both sides. Panel (B) shows the absence of autocorrelations (ACs) of price returns in the converged model. Panel (C) shows the slowly decaying ACs of absolute returns in the converged model.

Table 2.3: The relationship between order choices and order book states in the benchmark model.
The table shows the unconditional and conditional probabilities of order choices based on feasible values of all state variables. Panels (A) and (B) show order choices of informed and uninformed traders, respectively. Given the symmetry, only buy side probabilities are reported. The probabilities of not trading are scaled by two for ease of comparison.

Panel A: Informed traders

Conditional probability (%)

	Spread: s_t				Expected fundamental value: $E(v_t)$			Rosu's signal: $\lvert\bar{v}_t - p_t^m\rvert/s_t$					Bid trend: $b_t - b_{t-1}$		
Order type	$=1$	$=2$	>2	emp.	$>p_t^m$	$<p_t^m$	$=p_t^m$	$\leqslant 0.5$	$\leqslant 1.5$	$\leqslant 2.5$	$\leqslant 3.5$	>3.5	>0	<0	$=0$
mb	32.32	21.91	12.23	4.84	49.04	0.11	0.28	0.13	4.27	14.90	16.35	32.01	27.21	6.57	25.08
ealb	-	9.89	6.38	11.50	7.40	0.05	0.17	0.12	1.22	5.67	5.51	4.04	1.75	11.86	3.42
alb	-	-	4.27	8.39	2.56	0.03	0.57	1.38	4.07	2.52	1.87	0.57	0.53	3.18	1.24
lb	4.49	1.71	2.01	5.07	6.56	0.13	0.74	0.62	1.89	1.26	2.30	4.16	5.81	3.45	3.21
ulb	1.67	1.17	0.85	2.79	2.65	0.13	0.83	0.25	0.55	1.04	0.98	1.71	2.12	1.21	1.36
nt	10.61	14.85	22.4	14.89	15.65	13.27	46.60	47.21	35.99	24.58	22.07	6.24	12.62	15.85	14.75

	Ask trend: $a_t - a_{t-1}$			Depth imbalance: $d_t^a - d_t^b$			Cumulative depth imbalance: $D_t^a - D_t^b$			Last trade direction: LT_t	
Order type	>0	<0	$=0$	>0	<0	$=0$	>0	<0	$=0$	Buy	Sell
mb	14.87	23.10	24.88	31.17	18.11	23.96	22.94	26.21	9.01	33.50	15.53
ealb	10.28	1.47	3.51	6.10	2.18	2.34	3.83	3.65	0.50	3.59	3.80
alb	2.88	1.57	1.21	1.20	1.13	1.70	1.16	1.43	1.22	0.91	1.66
lb	3.44	3.55	3.32	5.40	1.30	3.35	3.16	3.46	4.61	3.79	2.88
ulb	1.38	2.50	1.34	2.14	0.65	1.40	1.33	1.45	0.98	1.68	1.10
nt	17.72	12.20	14.68	14.52	14.14	15.95	14.69	14.40	28.89	15.46	13.97

Unconditional probability (%)

mb: 24.41	*elb*: 3.70	*alb*: 1.29	*lb*: 3.33	*ulb*: 1.39	*nt*: 14.71

Panel B: Uninformed traders

Conditional probability (%)

Order type	Spread: s_t				Expected fundamental value: $E(v_t)$			Rosu's signal: $\lvert\bar{v}_t - p_t^m\rvert/s_t$					Bid trend: $b_t - b_{t-1}$		
	$= 1$	$= 2$	> 2	emp.	$> p_t^m$	$< p_t^m$	$= p_t^m$	$\leqslant 0.5$	$\leqslant 1.5$	$\leqslant 2.5$	$\leqslant 3.5$	> 3.5	> 0	< 0	$= 0$
mb	9.61	8.59	4.29	3.85	9.21	6.60	1.47	6.69	8.27	9.45	10.96	7.11	11.49	6.31	7.74
ealb	-	7.68	4.18	3.80	3.37	1.78	0.45	3.43	2.79	2.35	1.37	1.23	1.84	5.03	2.48
alb	-	-	4.46	3.80	1.99	0.66	0.06	2.25	1.29	0.65	0.42	0.47	1.50	2.29	1.27
lb	7.07	4.39	3.69	3.35	9.08	2.18	0.09	4.77	5.84	6.02	6.28	6.14	8.13	4.13	5.55
ulb	5.96	4.23	3.52	3.45	9.15	0.76	0.03	4.38	4.92	5.21	5.36	5.57	5.16	3.88	4.97
nt	27.65	25.22	30.55	33.05	28.19	27.60	47.92	28.81	27.59	26.38	25.84	29.73	23.47	23.87	28.54

Order type	Ask trend: $a_t - a_{t-1}$			Depth imbalance: $d_t^a - d_t^b$			Cumulative depth imbalance: $D_t^a - D_t^b$			Last trade direction: LT_t	
	> 0	< 0	$= 0$	> 0	< 0	$= 0$	> 0	< 0	$= 0$	Buy	Sell
mb	14.18	10.21	7.46	8.30	7.04	8.40	7.54	7.92	16.84	10.15	5.58
ealb	8.77	1.40	2.34	2.69	2.43	2.57	2.54	2.61	1.37	3.30	1.84
alb	3.83	1.26	1.21	1.15	1.30	1.60	1.39	1.25	1.03	1.50	1.14
lb	4.28	5.90	5.64	5.14	6.25	5.24	5.51	5.66	5.81	6.16	5.04
ulb	3.94	5.32	4.96	3.79	6.16	4.73	4.89	4.98	4.79	4.95	4.91
nt	21.89	21.51	28.70	28.01	28.47	27.84	27.98	28.50	19.88	27.81	28.48

Unconditional probability (%)

mb: 7.84 *elb*: 2.56 *alb*: 1.32 *lb*: 5.59 *ulb*: 4.93 *nt*: 28.14

2.5.1 Liquidity consumption and provision

On liquidity provision and consumption, based on unconditional probabilities reported in the last rows of the two panels in Table 2.3, we can see that informed traders use more market orders (24.41%) than limit orders (9.71%), and uninformed traders use less market orders (7.84%) than limit orders (14.40%).

Also, the fraction of market (limit) orders informed traders contribute to all orders submitted by all trader types is 15.24% (6.06%), and the fraction of market (limit) orders uninformed traders contribute to all orders submitted by all trader types is 27.74% (50.95%). On average, informed traders mainly consume liquidity, while uninformed traders mainly provide liquidity. Nevertheless, informed traders can switch to endogenous liquidity provision when the spread is large or the mispricing signal is large.

2.5.2 Trade/no-trade decision

The scaled unconditional probability of nt of informed traders is 14.71%, and that of uninformed traders is 28.14%, meaning that uninformed traders choose to not trade for more than half of the time due to their information disadvantage. Our results indicate that $E(v_t)-p_t^m$, $|\bar{v}_t - p_t^m|/s_t$ are more important for informed traders to decide between trading and not trading than for uninformed traders, while uninformed traders are more reliant on bid trend and ask trend than informed traders are. Additionally, the impacts of $d_t^a - d_t^b$ on informed traders' and uninformed traders' trading interests are of opposite directions. Non-zero depth imbalance at the best quote motivates informed traders but discourages uninformed traders to trade.

We now elaborate on informed and uninformed traders' differences in trade/no-trade decisions conditional on order book information using Table 2.3. When $E(v_t) - p_t^m$ changes from 0 to greater than (smaller than) 0, the probability of nt of informed traders drops from 46.60% to 15.65% (13.27%), while the probability of nt of uninformed traders does not decrease as much (from 47.92% to 28.19% or 27.60%).

When $|\bar{v}_t - p_t^m|/s_t$ increases from (0, 0.5] to (3.5,+∞), the probability of nt for the informed monotonically decreases from 47.21% to 6.24%, while the probability of

nt for uninformed traders first decreases from 28.81% to 25.84% then increases back to 29.73%. Due to inaccurate observations, uninformed traders barely profit from observed fundamental directional movements or enlarged mispricing compared to informed traders.

When $d_t^a - d_t^b$ changes from 0 to greater than (smaller than) 0, the probability of nt of informed traders drops from 15.95% to 14.52% (14.14%), while the probability of nt of uninformed traders increases from 27.84% to 28.01% (28.47%). A possible explanation is that the informed have a better ability than the uninformed to profit from picking off stale quotes during sudden directional changes of fundamental value. More specifically, $d_t^a - d_t^b > 0$ is very likely to be observed when the fundamental value's long-run decreasing trend ceases and suddenly starts to rise, during which times, an informed traders' most likely action type is a market buy (his probability of placing market buy if $d_t^a - d_t^b > 0$ is 31.17%), while the uninformed traders are most likely to sell.

As for bid trend and ask trend, uninformed is more reliant on these to profit from trade since the lack of correct directional information about the fundamental value, their trading interests unambiguously increase with any directional movements in bid trend or ask trend, while the same is not true for the informed traders.

2.5.3 Buy/sell decision

Our results indicate that $E(v_t) - p_t^m$ is most important for informed traders' buy/sell decisions, while uninformed traders' buy/sell decisions are more reliant on bid trend and ask trend than those of informed traders. Additionally, the impacts of $d_t^a - d_t^b$ on informed traders' and uninformed traders' buy/sell decisions are of opposite directions. To elaborate on informed and uninformed traders' differences in buy/sell decisions conditional on order book information, we calculate conditional probabilities of traders' buy orders out of all orders under feasible states, and results are reported in Table 2.4.

Table 2.4 indicates that informed traders learn almost perfectly to exploit their private information advantage, and buy with a probability of 99.28% when the asset is undervalued (when $E(v_t) - p_t^m > 0$).

Table 2.4: Percentage of buy orders under different states.
The table shows conditional probabilities of traders' buy orders out of all orders based on feasible values of expected fundamental minus midprice, bid trend, ask trend, depth imbalance, cumulative depth imbalance and last trade direction. Informed traders learn to exploit their information advantage, and uninformed traders chase the trend.

	Expected fundamental value: $E(v_t)$			Bid trend: $b_t - b_{t-1}$			Ask trend: $a_t - a_{t-1}$		
	$> p_t^m$	$< p_t^m$	$= p_t^m$	> 0	< 0	$= 0$	> 0	< 0	$= 0$
Informed	99.28	0.61	38.22	50.05	38.46	48.68	50.89	42.59	48.49
Uninformed	75.21	26.77	50.26	53.01	41.38	51.27	62.23	42.25	50.72
	Depth imbalance: $d_t^a - d_t^b$			Cumulative depth imbalance: $D_t^a - D_t^b$			Last trade direction: LT_t		
	> 0	< 0	$= 0$	> 0	< 0	$= 0$	Buy		Sell
Informed	64.84	32.59	48.09	45.90	50.84	38.67	62.93		34.65
Uninformed	47.92	53.82	50.83	49.67	52.12	49.54	58.68		42.99

The uninformed traders learn to chase the trend due to lacking private information. They prefer buy orders, and especially market buy orders, following a market buy. They are more reliant on the directional movements in bid trend and ask trend than the informed do: the informed traders have almost equal probabilities of buying and selling when the order book is moving up (when bid trend > 0, P(buy|informed and trade) = 50.05%; and when ask trend > 0, P(buy|informed and trade) = 50.89%), while the uninformed traders are inclined to buy if the order book moves up, sell if the order book moves down. Moreover, uninformed traders' buying probability is 53.82% when the depth at the best bid is large.

Interestingly, informed traders' probability of buying equals 64.84% when the depth at the best ask is large. The prior discussion of Table 2.3 shows that they are buying using market orders under such market conditions. We argue that they are able to infer from the imbalance at the inside bid/ask about when there are stale quotes standing at the best bid/ask. This, again, is because they have the correct directional information about the fundamental value. Uninformed traders are not able to infer the possible existence of stale quotes from the imbalance at the inside bid/ask, and in fact, have to infer directional information about the fundamental value from $d_t^a - d_t^b$ due to the lack of private information advantage.

2.5.4 Market/limit order decision

Our results indicate that market/limit order decisions of the informed and the uninformed have different responses to illiquidity and mispricing. A possible underlying mechanism may be information asymmetry.

Table 2.5: Percentage of limit orders under different states.
The table shows conditional probabilities of traders' limit orders out of all orders based on feasible values of state variables spread, Rosu's signal, depth imbalance, and cumulative depth imbalance. Informed traders increase their market order usage when mispricing is large, same-side depth at the best quote level, and the cumulative level is large. Uninformed traders increase their market order usage when same-side depth at the cumulative level is large. Both informed and uninformed traders respond to spread widening by submitting more limit orders, but informed traders are more responsive than uninformed traders.

Panel A: Informed traders

	Spread: s_t				Rosu's signal: $\lvert\bar{v}_t - p_t^m\rvert/s_t$					Depth imbalance: $d_t^a - d_t^b$			Cumulative depth imbalance: $D_t^a - D_t^b$		
Limit/All orders	$=1$	$=2$	>2	emp.	$\leqslant 0.5$	$\leqslant 1.5$	$\leqslant 2.5$	$\leqslant 3.5$	>3.5	>0	<0	$=0$	>0	<0	$=0$
Buyers	16.04	34.92	52.50	83.66	94.78	64.38	41.29	39.48	24.69	32.26	22.50	26.84	29.23	27.60	44.79
Sellers	15.95	36.82	53.16	86.27	94.90	61.93	41.28	35.88	25.22	22.09	33.04	27.66	27.57	30.17	49.67
Both sides	15.99	35.86	52.86	85.16	94.85	62.98	41.29	37.62	24.96	28.69	29.61	27.27	28.33	28.86	47.78

Panel B: Uninformed traders

	Spread: s_t				Rosu's signal: $\lvert\bar{v}_t - p_t^m\rvert/s_t$					Depth imbalance: $d_t^a - d_t^b$			Cumulative depth imbalance: $D_t^a - D_t^b$		
Limit/All orders	$=1$	$=2$	>2	emp.	$\leqslant 0.5$	$\leqslant 1.5$	$\leqslant 2.5$	$\leqslant 3.5$	>3.5	>0	<0	$=0$	>0	<0	$=0$
Buyers	57.55	65.49	78.70	78.93	68.93	64.21	60.10	55.09	65.34	60.61	69.63	62.70	65.51	64.66	43.59
Sellers	57.93	64.97	79.34	80.31	69.43	64.70	59.06	58.17	63.82	68.99	61.22	63.37	63.89	66.29	52.36
Both sides	57.74	65.23	79.01	79.57	69.18	64.45	59.58	56.62	64.59	64.97	65.75	63.03	64.70	65.44	48.02

To elaborate the role of information asymmetry on market/limit order decisions, we calculate conditional probabilities of limit orders out of all orders under the four discussed states, and report the results in Table 2.5.

In equilibrium, both informed and uninformed traders consume liquidity when it is ample and supply liquidity when it is scarce. When spread increases from $= 1$ to > 2, the informed trader's limit order submission rate monotonically increases from 16.04% to 52.50%. In other words, endogenous informed liquidity provision emerges when spread surpasses 2. As spread increases from $= 1$ to > 2, the uninformed trader's limit order submission rate monotonically increases from 57.55% to 65.49%. Both informed and uninformed traders are more willing to submit limit orders because the price of immediacy is high, but the response of uninformed traders is not as strong, since for them there is another force of opposite direction at work – the spread widening could reflect the increase in adverse selection risk, or the intensification of information disadvantage.

In terms of Rosu's signal, the Q-learning informed trader learns to play a threshold strategy as in the REE model of Rosu (2020). His limit (market) order submission rate monotonically decreases (increases) from 94.85% (5.15%) to 24.96% (75.01%) when observed mispricing increases, reflecting increased sensitivity to execution risk. In particular, in Rosu (2020) the informed trader would deterministically and optimally submit a buy market order if he observes mispricing above a threshold, and deterministically and optimally submit a buy limit order when he observes mispricing below the threshold (but positive). In our model, we also detect a "threshold": when $|\bar{v}_t - p_t^m|/s_t$ moves below the (1.5, 2.5] region, the informed trader's limit order submission rate surpasses 50% and reaches 62.98%, and endogenous informed liquidity provision emerges. On the other hand, when $|\bar{v}_t - p_t^m|/s_t$ moves beyond the (1.5, 2.5] region, the informed trader's market order submission rate surpasses 50% and reaches 58.71%, and he favors market order. The uninformed trader fails to learn such a threshold strategy. When the mispricing signal increases from (0, 0.5] to (3.5, $+\infty$), the limit order submission rate decreases at first from 69.18% to 56.62% then increases back to 64.59%. Due to his information disadvantage, his observed mispricing is inaccurate, and he will not treat it as an equivalent to increases in the implicit cost of non-execution.

Though informed traders submit more (less) market orders when same-side depths at inside quote level and the cumulative level are large (small), uninformed traders submit more (less) market orders only when same-side depth at the cumulative

Table 2.6: Percentage of ALO under different depth imbalance levels.
The table shows conditional probabilities of traders' aggressive limit orders out of all limit orders based on feasible values of the state variable depth imbalance $d_t^a - d_t^b$. Informed (uninformed) traders increase (reduce) their ALO/LO ratio when same-side depth at the best quote level is large.

	Depth imbalance: $d_t^a - d_t^b$		
ALO/Limit orders	> 0	< 0	$= 0$
Informed buyers	49.21	63.01	45.94
Uninformed buyers	30.08	23.11	29.49
Informed sellers	58.91	49.25	55.67
Uninformed sellers	22.41	29.08	28.21

level is large (small). The informed buyer's limit order submission rate when $d_t^a - d_t^b < 0$ is 22.50%, lower than his limit order submission rates when the same state variable equals 0 or is greater than 0, i.e., 32.26% and 26.84%. The informed buyer's limit order submission rate when $D_t^a - D_t^b < 0$ is 27.60%, again lower than his limit order submission rates when the same state variable equals 0 and when the same variable is greater than 0, i.e., 29.23% and 44.79%. As for the uninformed buyer, his limit order submission rate with buy-side depth imbalance at the cumulative level is ($D_t^a - D_t^b < 0$) is 64.66%, which is lower than his limit order submission rate of 65.51% with no depth imbalance at the cumulative level, but is higher than his limit order submission rate of 43.59% with sell-side depth imbalance at the cumulative level. Note that the scenario of no depth imbalance at the cumulative level has a low occurrence frequency of 1.13% and can be negligible. We argue that the two reasons why informed and uninformed traders' market/limit order decisions and limit order aggressiveness have different responses to depth at the inside level are: (i) private information moves the informed trader's trade-off between price risk, execution risk, and adverse selection risk more towards the execution risk side than the uninformed trader since he has a higher implicit cost of non-execution; and (ii) the informed trader has correct directional information about the fundamental value, so he can learn better about execution probability from the order book information $d_t^a - d_t^b$ than the uninformed trader can.

Further, as shown in Table 2.6, when same-side depth at inside quote level is large, informed traders increase their ALO usage (both *ealb* and *alb*) out of all limit orders, while uninformed traders decrease their ALO usage out of all limit orders. This is consistent with Aitken et al.(2007)'s and Chiu et al. (2016)'s empirical findings of institutional (possibly informed) and individual traders' (possibly uninformed) order aggressiveness strategies.

2.5.5 Liquidity provision and information asymmetry

We now further validate the different responses of informed traders' and uninformed traders' market/limit order decisions react to order book information differently using logistic regression, demeaned logistic regression with intercept, and demeaned OLS regression with intercept. The results are consistent with the last subsection and suggest that informed traders are resiliency improving. The regressions are inspired by Menkhoff et al. (2010), and are mainly different from them in the sense that our simulated data have exact identification of informed traders and uninformed traders, while Menkhoff et al. (2010) use dealer trading activity and dealer trading location as proxies for information.

The regressions are conducted for the informed traders and uninformed traders, respectively, using the 360,000 trading period simulated data generated by the converged benchmark model. The dependent variable is a dummy variable that takes 1 when the order is a limit order and takes 0 when the order is a market order. The independent variables are the 8 discretised order book state variables. The demeaning is carried out on the state variables by subtracting their means. Similar to Menkhoff et al. (2010), we make directional adjustments on the depth imbalance variable such that it takes 1 when $d_t^b - d_t^a > 0$ (< 0) is faced by buyer (seller), it takes 0 when $d_t^b - d_t^a = 0$ is faced by the buyer (seller), and it takes -1 when $d_t^b - d_t^a < 0$ (> 0) is faced by the buyer (seller). In other words, increases of the adjusted depth imbalance variable reflect increases in the relative magnitudes of the depth of a trader's own side compared to the other side. The cumulative depth imbalance variable is adjusted similarly.

The variable $E(v_t) - p_t^m$ (expected fundamental minus midprice) is adjusted such that the adjusted variable takes 1 when $E(v_t) - p_t^m = \pm 1$, and takes 0 when $E(v_t) - p_t^m = 0$. In other words, the expected fundamental minus midprice variable is adjusted such that it reflects whether there is any directional movement in fundamental value: 0 indicates no movement and 1 indicates the existence of movement. The direction of the last trade direction variable is adjusted such that it takes 1 when a buyer (seller) observes a market buy (sell), and it takes -1 when a buyer (seller) observes a market sell (buy). The regression results are reported in Table 2.7.

Table 2.7: Impact of private information on liquidity provision.
The table shows regression results of market/limit order decisions on order book information. Directional adjustments are made to expected fundamental minus midprice, bid trend, ask trend, depth imbalance, cumulative depth imbalance, and last trade direction. Informed traders are resiliency improving according to the coefficients of spread, depth imbalance, and cumulative depth imbalance: when spread enlarges or same-side cumulative depth decreases, both informed and uninformed increase their limit order usage, but informed traders demonstrate much stronger responses. Additionally, when the same-side depth at best quote decreases, the informed increase limit order usage, but the uninformed increase market order usage.

	Logistic		Demeaned logistic		Demeaned OLS	
	Informed	Uninformed	Informed	Uninformed	Informed	Uninformed
Spread	0.5509	0.4087	0.6823	0.3834	0.1836	0.1056
	(220.170)	(341.984)	(222.370)	(205.649)	(290.210)	(262.746)
Expected fundamental minus midprice	−1.0587	0.5092	−1.8776	0.5241	−0.3710	0.1305
	(−6.257)	(273.793)	(−11.206)	(275.618)	(−22.245)	(321.787)
Rosu's signal	−0.3886	0.0035	−0.1106	−0.0085	−0.0176	0.0015
	(−358.913)	(5.051)	(−28.671)	(−8.801)	(−27.541)	(6.413)
Bid trend	−0.1329	−0.0242	−0.1336	−0.0228	−0.0258	−0.0231
	(−15.242)	(−4.466)	(−15.396)	(−4.212)	(−10.733)	(−21.965)
Ask trend	−0.1311	−0.0277	−0.1301	−0.0295	−0.0296	−0.0198
	(−15.372)	(−5.994)	(−15.330)	(−6.387)	(−18.080)	(−19.210)
Depth imbalance	−0.2861	0.1221	−0.2931	0.1218	−0.0578	0.0338
	(−95.196)	(71.465)	(−97.162)	(71.255)	(−106.464)	(88.980)
Cumulative depth imbalance	−0.0456	−0.0315	−0.0498	−0.0330	−0.0085	−0.0031
	(−18.096)	(−21.037)	(−19.698)	(−22.000)	(−18.567)	(−9.613)
Last trade direction	−0.1994	−0.1311	−0.1996	−0.1337	−0.0460	−0.0355
	(−75.075)	(−83.864)	(−74.839)	(−85.120)	(−94.281)	(−104.195)
Intercept	None	None	−0.8344	0.6678	0.3172	0.6439
			(−128.393)	(414.384)	(293.569)	(811.870)
No. of observations	847326	2098132	847326	2098132	847326	2098132
Pseudo/Adjusted R^2	0.1165	0.03694	0.1305	0.03769	0.348	0.140

The first two columns are results from the logistic regression, the third and the fourth columns are results from demeaned logistic regression with intercept, and the last two columns are results from the demeaned OLS regression with intercept. We focus on the logistic regressions without intercept, since the demeaned regressions deliver quite consistent results.

The coefficient of spread for the informed (uninformed) is 0.5509 (0.4087), reflecting that both the informed traders and uninformed traders will shift to limit orders in response to higher costs of immediacy. The smaller magnitude of the spread variable's coefficient for the informed is driven by the fact that the lack of private information moves the uninformed trader's trade-off between price risk, execution risk, and adverse selection risk more towards the adverse selection risk side: they would have a tendency to interpret rises in spreads as intensifications of the adverse selection risk.

The coefficient of expected fundamental minus midprice for the informed is -1.0587, and the coefficient of expected fundamental minus midprice for the uninformed is 0.5092. These two coefficients reflect that the informed (uninformed) are inclined to profit by using market orders (unaggressive limit orders) when they observe any directional movements in the fundamental value (lagged fundamental value).

The bid trend variable's coefficient for the informed (uninformed) is -0.1329 (-0.0242), and the ask trend variable's coefficient for the informed (uninformed) is -0.1311 (-0.0277). When informed and uninformed buyers observe that an order book is moving up, they tend to use market orders to improve their execution probability. The depth imbalance variable's coefficient for the informed (uninformed) is -0.2861 (0.1221), and the cumulative depth imbalance variable's coefficient for the informed (uninformed) is -0.0456 (-0.0315). For all these four variables, the informed trader's responses are more negative than the uninformed because the information advantage renders the informed traders more sensitive to execution risk than the uninformed traders.

The last trade direction variable's coefficient for the informed (uninformed) is -0.1994 (-0.1311), reflecting the "diagonal effect", a well-documented stylised fact whereby a market buy (sell) is more likely to be followed by a market buy (sell). For the last trade direction variable, the uninformed trader's response is less negative because when an uninformed seller observes a market buy, he has a tendency to interpret it as a rise in the fundamental value only known to the informed, i.e., an intensification of adverse selection risk, which discourages his usage of limit sell.

Most importantly, the informed traders are resiliency improving according to the coefficients of spread, depth imbalance, and cumulative depth imbalance: when spread enlarges or same-side cumulative depth decreases, both informed and uninformed increase their limit order usage, but informed traders demonstrate much stronger responses. Additionally, when the same-side depth at best quote decreases, the informed increase limit order usage, but the uninformed increase market order usage.

2.6 Volatility and informed trading

We now show how the informed traders and the uninformed traders differ in their limit order submission strategies when responding to the changes in fundamental volatility and the proportion of informed traders, and the ensuing impact on market quality.

2.6.1 Limit order submission and volatility

We leave all the parameters in the benchmark parametrisation unchanged and only alter the volatility level. The volatility regimes are $\delta = \{2, 4, 6, 8\}$. We apply the same convergence criteria as defined in Section 2.4.2, and then fix Q-values, disallow the tremble, and simulate for another 360,000 trading periods. For illustrative purposes, we divide the orders submitted by traders into three broad categories, i.e., MO, ALO and NLO. MO are market orders; ALO consists of *ealb*, *alb*, *eals*, and *als*; and NLO consists of *lb*, *ulb*, *ls* and *uls*.

Table 2.8 reports the limit order submission strategies conditional on volatility changes. As discussed by Copeland and Galai (1983), limit order traders give market order traders a timing option. When the volatility increases, the value of the option increases, and spread, in general, should enlarge, and the cost of immediacy increases. Consequently, the uninformed trader submits less MO (because it is more expensive) and submits more ALO: when the volatility of fundamental value increases from 2 to 8, the uninformed trader's usage of MO monotonically decreases from 34.41% to 32.65%, so he is increasing his limit order submission rate in general. In particular, his usage of ALO monotonically increases from 18.64%

Table 2.8: Order aggressiveness conditional on volatility levels.
The table shows the order aggressiveness strategies of informed and uninformed traders conditional on volatility changes. When fundamental volatility increases, uninformed traders decrease MO and increase ALO, while informed traders increase MO and decrease ALO.

Volatility of fundamental value	2	4	6	8
P(Spread $\leqslant$ 2) %	71.58	57.42	52.93	52.61
Informed order choices				
Market order %	73.81	74.79	76.00	76.11
Aggressive limit order %	15.68	15.39	14.13	12.99
Nonaggressive limit order %	10.51	9.82	9.86	10.90
Uninformed order choices				
Market order %	34.41	33.91	33.15	32.65
Aggressive limit order %	18.64	22.71	23.27	23.33
Nonaggressive limit order %	46.95	43.39	43.58	44.02

to 23.33%. Though the increased adverse selection imposed by informed traders should induce the uninformed trader to reduce the usage of ALO, the adverse selection effect is outweighed by the cost of immediacy effect.

Now, as volatility increases, the private information the informed trader possesses again moves his trade-off between price risk, execution risk, and adverse selection more toward the execution risk side than he used to be when the volatility is at its lowest level. His information is more valuable, and his cost of non-execution is higher. Consequently, the informed trader increases his usage of MO due to increased sensitivity to execution risk, reduces his usage of ALO (both due to increases in MO and to avoid future adverse selection), and does not vary his usage of NLO much.

As the volatility monotonically increases from 2 to 8, P(spread $>$ 2) increases from 28.42% to 47.39%. The informed should be more responsible for the spread widening since they increase MO submission and reduce ALO submission. The uninformed, compared to the informed, are liquidity improving because of their increased ALO submission. The finding of volatility increases leading to informed's (uninformed's) increased (reduced) usage of MO and reduced (increased) usage of ALO is different from He and Lin (2022), in which both informed and uninformed use more ALO, and is also different from Goettler et al. (2009), in which both informed and uninformed traders submit more conservative limit orders.

2.6.2 Limit order submission and informed trading

We leave all the parameters in the benchmark parametrisation unchanged and only alter the fractions of informed and uninformed traders. The fractions we use are $N_I{:}N_U = \{0.100{:}0.900, 0.125{:}0.875, 0.150{:}0.850, 0.200{:}0.800\}$. We apply the same convergence criteria as defined in Section 2.4.2, fix Q-values, disallow the tremble, and simulate for another 360,000 trading periods. We focus on the case when mispricing is low, i.e., $|\bar{v}_t - p_t^m|/s_t \in (0, 0.5]$.

Table 2.9 reports the limit order submission strategies conditional on informed trading level changes. In the $|\bar{v}_t - p_t^m|/s_t \in (0, 0.5]$ region, since the informed trader observes the accurate and rather small mispricing information, his sensitivity to execution risk becomes rather low. Therefore, he drastically decreases his market order usage compared to other mispricing scenarios and becomes a "de facto" market maker. Taking the benchmark case as an example, the informed trader's unconditional limit order submission rate (out of all his orders) is 28.74%, while at the low mispricing level scenario his limit order submission rate becomes 94.84%. When informed trading levels increase from 10% to 20% at the low mispricing time, the price discovery improves and decreases from 0.58 to 0.29, mainly because the informed undercut each other and compete by using more ALO (increasing its usage from 38.00% to 59.97%) but not more MO, i.e., they compete in liquidity provision. For informed traders, there is also a weakening information effect, caused by more informed trading and more efficient prices, which reduces market order profit and augments the ALO usage increase brought by the competition effect. The uninformed traders respond to the informed share increases by submitting less ALO (decreasing its usage from 31.14% to 16.08%) to avoid being picked off due to intensified information disadvantage. The spread decreases from 1.47 to 1.36, and depth at the best quote increases from 3.16 to 5.21 because the effect of increased intertemporal competition between informed liquidity providers outweighs the increased adverse selection risk imposed on uninformed traders. The standard intuition that liquidity deteriorates given more adverse selection is violated. In this case, the informed trader is liquidity improving due to their increased usage of ALO.

As for welfare consequences, using the mean Q-value as the welfare measure, increases in informed trader proportion decrease their welfare from 2.10 to 1.78,

Table 2.9: Order aggressiveness conditional on informed trading levels. The table shows the order aggressiveness strategies of informed and uninformed traders conditional on volatility changes. When fundamental volatility increases, uninformed traders decrease MO and increase ALO, informed traders increase MO and decrease ALO.

Proportion of informed traders	10%	12.5%	15%	17.5%	20%
Market quality					
Quoted spread	1.47	1.44	1.41	1.40	1.36
Depth at best bid	3.16	3.51	3.65	4.83	5.21
Price discovery $\lvert p_t - v_t \rvert / v_t$ %	0.58	0.48	0.37	0.33	0.29
Welfare					
Informed trader	2.10	2.03	1.94	1.93	1.78
Uninformed trader	−0.21	−0.15	−0.09	−0.07	−0.06
Informed order choices					
Market order %	5.89	5.19	5.15	4.66	4.20
Aggressive limit order %	38.00	49.37	52.32	53.78	59.97
Nonaggressive limit order %	56.11	45.44	42.52	41.56	35.84
Uninformed order choices					
Market order %	31.22	31.56	30.82	28.34	27.40
Aggressive limit order %	31.14	28.11	26.01	19.47	16.08
Nonaggressive limit order %	37.64	40.33	43.17	52.19	56.52

because they can extract less rent from their information advantage. The uninformed trader's welfare becomes less negative and increases from −0.21 to −0.06, because he free rides the price efficiency improvement brought by the intensified informed trader competition.

In summary, in this subsection, we show that increases in informed trading intensify competition among the informed and adverse selection for the uninformed, reducing market orders and increasing (reducing) aggressive limit orders for the informed (uninformed), reducing (improving) welfare for the informed (uninformed), and improving price efficiency and market liquidity. Different from Rosu (2020), in which the competition effect leads to larger information decay and larger slippage component of the spread, and hence deteriorating liquidity, we show that intertemporal competition between the informed traders can be liquidity improving if informed traders compete by submitting more aggressive limit orders.

2.7 Manipulative behaviours of informed traders

As previously shown in Table 2.5, informed traders unambiguously favour limit orders (market orders) when mispricing is low (high). It is thus intriguing to investigate when and why would informed traders, faced with low mispricing, go against their natural tendency of using limit orders and employ market orders instead.

Our analysis suggests that the seemingly irrational market order usage turns out to be partially attributable to informed traders' collusive manipulation. The intuition can be expressed as follows. Taking buy side as an example, when small positive mispricing is accompanied by high buy-side depth imbalance ($d_t^a - d_t^b < 0$) rather than zero depth imbalance ($d_t^a - d_t^b = 0$), informed traders could anticipate a mispricing reversal in the near future (mispricing direction changes from $v_t > p_t^m$ to $v_t < p_t^m$), which can possibly be caused by upward price changes following buying pressure. Given that the probability of uninformed traders placing market buy following a market buy is 10.15% (shown in Panel (B) of Table 2.3), the highest among all types of uninformed orders, informed traders might react by strategically using market buys to trigger uninformed market buys, sacrifice current profit difference between limit and market buys in exchange for enhanced execution probability and profitability of later informed traders' limit sells, and collusively manipulate for the good of informed traders as a group.

To justify this intuition, when the actual mispricing varies from small positive ($v_t > p_t^m$ and $|v_t - p_t^m|/s_t \leqslant 1.5$) to large positive ($v_t > p_t^m$ and $|v_t - p_t^m|/s_t > 3.5$), we compare and contrast the following statistics of high buy-side depth imbalance condition with those of zero depth condition, for the informed and uninformed s respectively: (i) the probability of market buy placement at current depth, i.e., $\mathrm{P}^{\text{placement}}$(MB|at current depth); (ii) the probability of limit sell placement in the 20 orders interval after the same-group market buy at current depth , i.e., $\mathrm{P}^{\text{placement}}$(LS|after MB at current depth); (iii) the probability of limit sell execution in the 20 orders interval after observing current depth, i.e., $\mathrm{P}^{\text{execution}}$(LS|after current depth); (iv) the profit per trade (PPT) of LS in the 20 orders interval after observing current depth; (v) the profit per order (PPO) of LS in the 20 orders interval after the same-group market buy at current depth; (vi) the PPO of uninformed MB in the 20 orders interval after observing informed MB; and (vii) the probability of uninformed MB placement in the 20 orders interval after observing current depth, i.e., $\mathrm{P}^{\text{placement}}$(MB| after current depth). For the uninformed, statistics (vi) and (vii) are unique to them, and their statistics (i)-(v) are presented and discussed in Appendix 2.2.

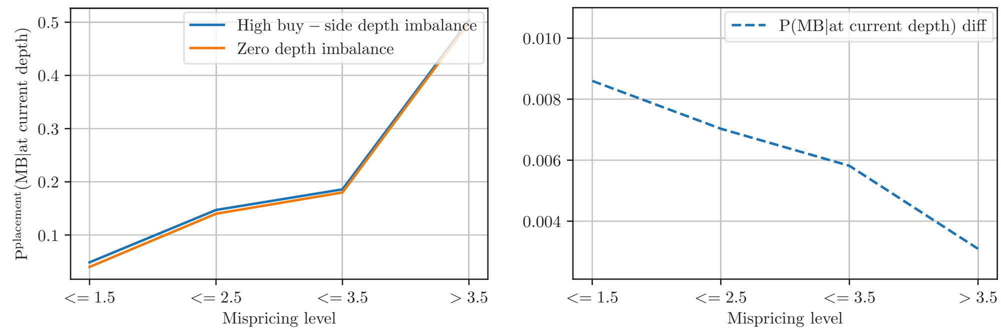
Blue (yellow) solid lines on the left-hand side of Figure 2.3 Panel (A) report statistics (i)-(v) of informed traders under high buy-side depth imbalance condition (zero depth imbalance condition). Blue (yellow) solid lines on the left-hand side of Figure 2.3 Panel (B) report statistics (vi)-(vii) of uninformed traders under high buy-side depth imbalance condition (zero depth imbalance condition). Blue (yellow) dotted lines on the right-hand side of Figure 2.3 report the difference between these statistics of high buy-side depth imbalance condition and zero depth imbalance condition.

We first examine informed trader behaviours shown in Panel (A). For $P^{\text{placement}}$(MB| at current depth), statistic (i), of informed traders, both blue and yellow solid lines monotonically rise as mispricing enlarges, reflecting the ceteris paribus effect in Section 2.5 that greater mispricing increases informed traders' implicit cost of non-execution; the blue solid line lies on top of the yellow solid line, somehow reflecting the ceteris paribus effect in Section 2.5 that deeper depth imbalance at the best bid/ask encourages same-side informed traders' to jump the queue. The difference between statistic (i) under high buy-side depth imbalance and zero depth imbalance conditions monotonically decreases with mispricing from 0.86% to 0.31%. Current informed traders increase MB usage if small positive mispricing is accompanied by high buy-side depth imbalance rather than zero depth imbalance.

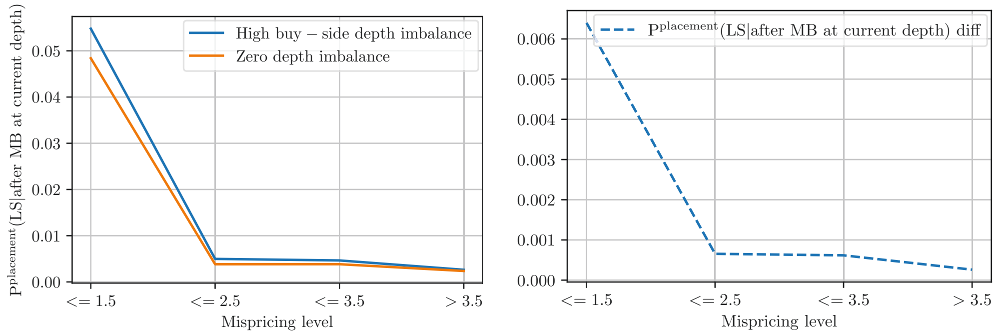
For $P^{\text{placement}}$(LS|after MB at current depth) and the corresponding PPO, statistics (ii) and (v), of informed traders, both blue and yellow solid lines monotonically decrease as mispricing enlarges. Intuitively, the larger the current positive mispricing is, the more likely it is that limit sells placed after the current period are going to lose. Yellow solid lines are significantly higher than (rather close to) corresponding blue solid lines when mispricing is no larger than 3.5 (greater than 3.5), somehow reflecting the ceteris paribus effect in section 2.5 that deeper depth imbalance at the best bid/ask generally tilts informed traders more towards limit rather than market orders. The difference between statistic (ii) under high buy-side depth imbalance and zero depth imbalance monotonically decreases with mispricing from 0.64% to 0.03%. Later informed traders increase LS usage following the current informed MB if small positive mispricing is accompanied by high buy-side depth imbalance rather than zero depth imbalance.

Further, when Rosu's mispricing signal is no larger than 1.5, the difference between statistic(v) (statistic(iv)) under high buy-side depth imbalance and zero

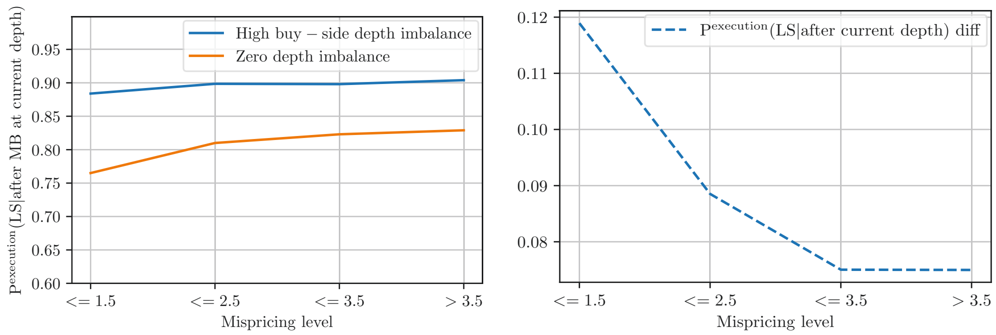
(i) $\mathrm{P}^{\text{placement}}(\text{MB}|\text{at current depth})$



(ii) $\mathrm{P}^{\text{placement}}(\text{LS}|\text{after MB at current depth})$



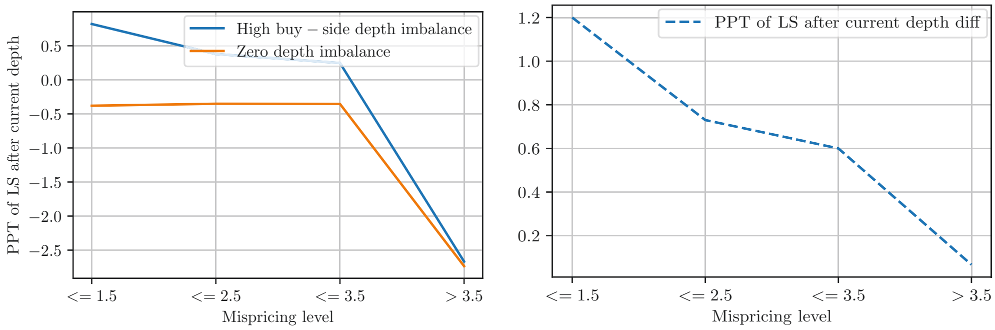
(iii) $\mathrm{P}^{\text{execution}}(\text{LS}|\text{after current depth})$



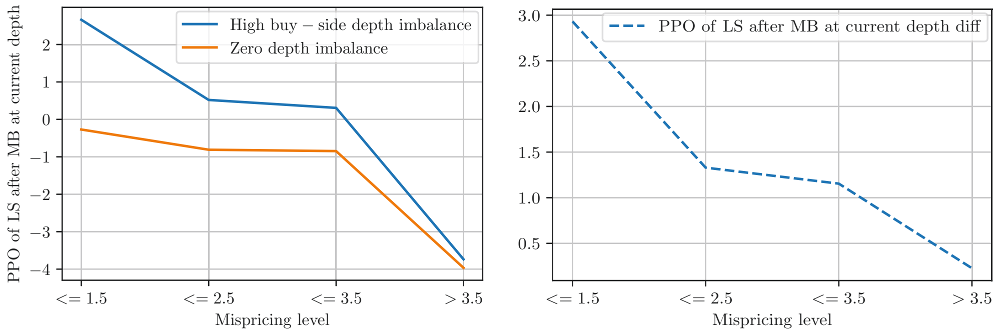
(A) Informed traders

Figure 2.3: Informed manipulation.

(iv) PPT of LS after current depth

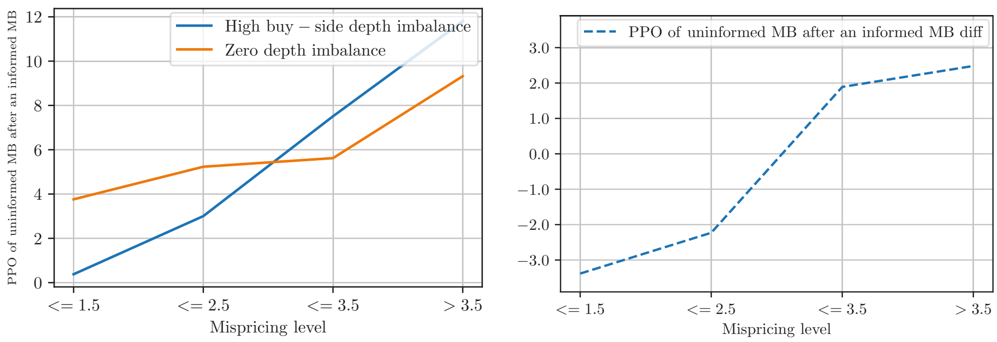


(v) PPO of LS after MB at current depth

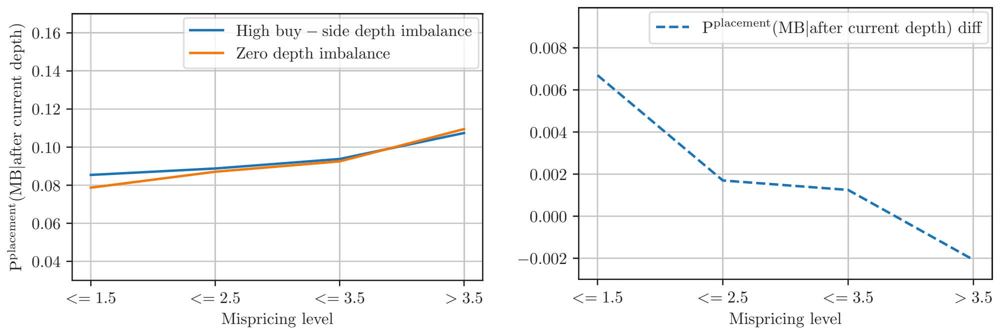


(A) Informed traders

(vi) PPO of uninformed MB after an informed MB



(vii) $P^{\text{placement}}$(MB|after current depth)



(B) Uninformed traders

Figure 2.3: Informed manipulation (continued).

depth imbalance reaches its peak at 2.93 ticks (1.20 ticks), and the difference between $P^{\text{execution}}$(LS|after current depth), statistic (iii), under high buy-side depth imbalance and zero depth imbalance conditions reaches its peak at 11.89%. According to Panel (A), informed traders, faced with low mispricing, despite their natural tendency to use limit orders under low mispricing, indeed appear to use market buys to try to trigger uninformed market buys after observing high buy-side depth imbalance because they anticipate a later mispricing reversal, and thereby increase the execution probability and profitability of later informed traders' limit sells. Though a low mispricing level combined with high buy-side depth imbalance are informative about future mispricing reversal for the informed, according to statistics (i)-(v) of the uninformed discussed in Appendix 2.1, the uninformed do not observe the actual mispricing level and are not able to extract such mispricing reversal related information.

We now examine how uninformed traders respond to informed manipulation by investigating Panel (B). In terms of the PPO of uninformed MB after an informed MB-statistic (vi), the yellow solid line is above (below) the blue solid line for relatively low (high) actual mispricing, and both of two lines monotonically increase. The high buy-side depth imbalance condition, compared to the zero depth imbalance condition, reinforces uninformed traders' tendency to interpret an informed MB as positive mispricing that will proceed into the future: strong buying pressure means current price is relatively low. An (informed) market buy observing uninformed trader would thus use more market buy orders at high buy-side depth imbalance than zero depth imbalance, a *chasing* effect. When a low mispricing level presents and a mispricing reversal is more likely, informed traders have a stronger motive to use market buys to mislead uninformed traders, the *chasing* effect thereby results in a higher chance to get fooled and a lower statistic (vi) for high buy-side depth imbalance observing uninformed traders than zero depth imbalance observing peers. When a high mispricing level prevails, since the misleading motive weakens and informed market buys are more "genuine", the *chasing* effect results in a higher chance to trade in the right direction and a higher statistic (vi) for high buy-side depth imbalance observing uninformed traders than zero depth imbalance observing peers. As for the two blue dotted lines, when mispricing increases, the difference between uninformed MB placement probability after high buy-side depth imbalance and zero depth imbalance monotonically decreases from 0.67% to −0.21%, and the difference between PPO of uninformed MB after an informed MB at high buy-side depth and zero depth imbalance monotonically increases from -3.38 ticks to 2.48 ticks. This indicates that high depth imbalance and low mispricing observing informed traders successfully trick uninformed traders into using more market buys than they should have. Further, informed traders' limit sell profit rise is at least partially achieved via reducing trend-chasing uninformed traders' market buy profit.

So far, what we have discussed is the scenario where the mispricing direction does change from $v_t > p_t^m$ to $v_t < p_t^m$. However, it is worth pointing out that even if small positive mispricing does not result in a reversal, i.e., $v_t > p_t^m$ persists, informed traders, when faced with the *high buy-side depth imbalance condition* rather than the *zero-depth imbalance condition*, might still have a stronger incentive to deviate from LB to MB at the current period, since the previously discussed chasing effect increases trend-chasing MB usage of the uninformed and pushes up the price, leading to a reduction in uninformed trend-chasing MB profits.

Trade-based manipulation has been widely studied in the rational expectations literature (Huddart et al. (2001), John and Narayanan (1997), Takayama (2021)). In these models, an informed trader, who wants to preserve his own information advantage longer, tricks uninformed traders by choosing the "wrong" action when facing buy/sell or amount decisions. Different from them, we contribute to this theoretical literature by presenting a novel form of informed manipulation originating from the inherent differences in informed and uninformed make-take strategies, where an informed trader, who acts collusively and is willing to sacrifice his current profit in exchange for profit increases of later arriving informed traders, tricks uninformed traders by choosing the "wrong" action when facing market/limit decisions.

2.8 Conclusion

We develop a dynamic limit order book populated by informed and uninformed traders who learn to trade via Q-learning. Q-learning enables us to fully endogenise traders' order choice problems. In general, it is promising to integrate reinforcement learning with the market microstructure theory framework and use RL as an alternative belief updating rule, because RL enables us to relax a set of strict assumptions, such as the agent's perfect knowledge of model priors.

Trial-and-error learning of bounded rational agents from order book information gives rise to strategic trading, of which a key component is predictable trading behaviours. With information advantage, the informed are most reliant on the fundamental information (other than other order book information) to determine their order choices. The informed are resiliency improving and unambiguously favour limit (market) orders when the magnitude of mispricing is small (large). Due to information disadvantage and learning, the uninformed “chase the trend” and are more prone to place market buy orders following a market buy.

Most importantly, informed manipulation can be learned as an equilibrium trading strategy in our dynamic LOM, where informed deviants deliberately “deviate” from their own predictable trading behaviours and exploit uninformed traders’ predictable trading behaviours. Given uninformed traders’ trend chasing tendency, informed traders, who anticipate a mispricing reversal when observing both small-in-size positive (negative) mispricing and high depth imbalance at the best bid (ask), react by strategically going against their own preference for limit buys (sells) and using market buys (sells) to trigger uninformed market buys (sells), sacrifice current profit difference between limit and market buys (sells) in exchange for enhanced execution probability and profitability of later informed traders’ limit sells (buys), and collusively manipulate the market for the good of informed traders as a group. The novelty of the form of market manipulation emerging in our dynamic LOM is that informed traders take the “wrong” action to mislead uninformed traders when faced with make-take decisions, instead of when faced with buy/sell or amount decisions as in existing market manipulation studies.

Our findings that machines can learn to manipulate LOM have practical implications for investors and regulators. From the regulatory perspective, current market manipulation legislation in most jurisdictions primarily focuses on the “intent” — the state of mind or purpose behind the actions of suspected individuals or entities. However, this focus is more applicable to humans than to machines. Given our analysis and the fact that machines can also learn to manipulate markets, the current legislation needs to be amended to also include market manipulation by machines. In addition, the surge of innovative AI and its prevalent use in market manipulation increases the risk for long-term investors who are primarily interested in resource allocation. To ensure market integrity, our analyses suggest that regulators should adhere to the overarching rule that market abuse is market abuse, regardless of whether it is from a human or a machine.

Appendix 2.1.

Table A.2.1. classified rules (CRs) for state variable discretisation.

This table presents eight classification rules (CRs) in the classifier system based on the spread, the expected fundamental value, the mispricing signal, order book movements, depth imbalances, and the last trade direction. Using the classifier system, the continuous state space is transformed into a discrete one that contains 14,580 possible states.

Classified rules	Possible values
Spread s_t	Current spread is equal to 1 Current spread is equal to 2 Current spread is higher than 2 Empty on buy side (emp^{+}) Empty on sell side (emp_{-}) Empty on both sides (emp_{-}^{+})
Expected fundamental minus midprice $E(v_t) - p_t^m$	Expected fundamental is higher than p_t^m Expected fundamental is lower than p_t^m Expected fundamental equals p_t^m
Rosu's signal $\frac{\lvert\overline{v}_t - p_t^m\rvert}{s_t}$	Mispricing signal is in range [0, 0.5] Mispricing signal is in range (0.5, 1.5] Mispricing signal is in range (1.5, 2.5] Mispricing signal is in range (2.5, 3.5] Mispricing signal is in range (3.5, $+\infty$)
Bid trend $b_t - b_{t-1}$	Current bid is higher than last bid Current bid is lower than last bid Current bid equals last bid
Ask trend $a_t - a_{t-1}$	Current ask is higher than last ask Current ask is lower than last ask Current ask equals last ask
Depth imbalance $d_t^a - d_t^b$	Depth at the best ask is higher Depth at the best ask is lower Depth at the best ask and best bid are equal
Cumulative depth imbalance $d_t^a - d_t^{\mathrm{b}}$	Depth at the sell side is higher Depth at the sell side is lower Depth at the sell and buy sides are equal
Last trade direction LT_t	Last market order is a buy order Last market order is a sell order

Appendix 2.2. Uninformed traders' inability to infer mispricing reversal

The uninformed cannot extract mispricing reversal related information and do not demonstrate manipulative behaviours resemble those of informed traders'. To illustrate this, we look at Figure A2.1. For $\mathrm{P}^{\mathrm{placement}}(\mathrm{MB}|\text{at current depth})$, statistic (i), of uninformed traders, the blue and yellow solid lines do not display monotonicity, an artefact driven by the assumption that uninformed traders have no access to actual mispricing. The blue solid line is below the yellow solid line, consistent with the ceteris paribus effect in Section 2.5 that deeper depth imbalance at the best bid/ask decreases same-side uninformed traders' tendency to submit market orders.

More importantly, the difference between $\mathrm{P}^{\mathrm{placement}}(\mathrm{MB}|\text{at current depth})$ of high buy-side depth imbalance and zero depth imbalance fluctuates around -0.70%, reflecting that uninformed traders do not discriminate between various depth imbalance and mispricing combinations when it comes to market buy placement probability at current period.

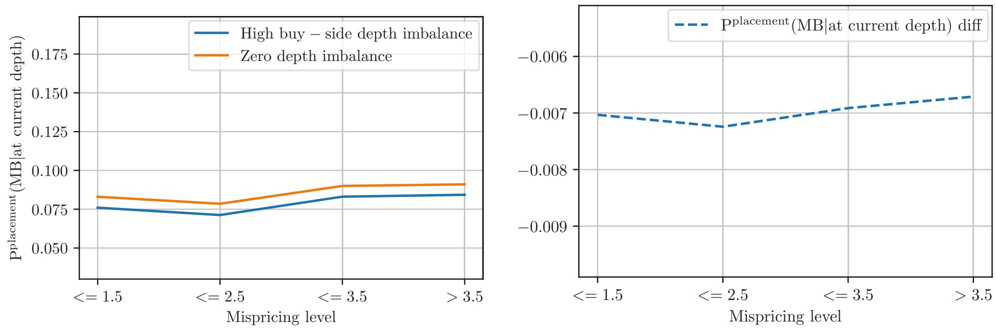
For $\mathrm{P}^{\mathrm{placement}}(\mathrm{LS}|\text{after MB at current depth})$, statistic (ii), of uninformed traders, the blue and yellow solid lines again have no monotonical patterns because uninformed traders do not observe actual mispricing. The blue solid line is below the yellow solid line, consistent with the ceteris paribus effect in Section 2.5 that deeper depth imbalance at the best bid/ask decreases other-side uninformed traders' tendency to submit limit orders. This ceteris paribus effect can be justified by the corresponding PPO of $\mathrm{P}^{\mathrm{placement}}(\mathrm{LS}|\text{after MB at current depth})$. As seen from statistic (v), when the actual mispricing is greater than 3.5, accounting for more than 60% of the simulated sample, the PPO of high buy-side imbalance is lower than that of zero depth imbalance, and the difference has a nonnegligible magnitude of -1.79 ticks.

More importantly, the difference between $\mathrm{P}^{\mathrm{placement}}(\mathrm{LS}|\text{after MB at current depth})$ of high buy-side depth imbalance and zero depth imbalance fluctuates around -0.60%, reflecting that uninformed traders do not discriminate between various depth imbalance and mispricing combinations when it comes to limit sell placement probability after the current uninformed MB.

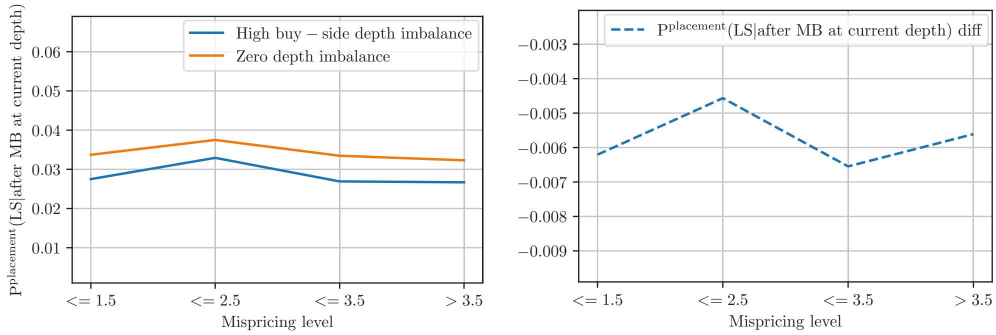
Furthermore, when Rosu's mispricing signal is no larger than 1.5, the difference between $P^{\text{execution}}(LS|\text{after current depth})$, statistic (iii), of uninformed traders under high buy-side depth imbalance and zero depth imbalance is not at its highest value. The same applies for statistic (iv) – the PPT of uninformed LS after current depth, and statistic (v) – the PPO of uninformed LS after uninformed MB at current depth.

To sum up, uninformed traders cannot extract mispricing reversal related information, and do not vary current MB usage and future LS usage across different depth imbalance and mispricing combinations.

(i) $P^{\text{placement}}(\text{MB}|\text{at current depth})$



(ii) $P^{\text{placement}}(\text{LS}|\text{after MB at current depth})$



(iii) $P^{\text{execution}}(\text{LS}|\text{after current depth})$

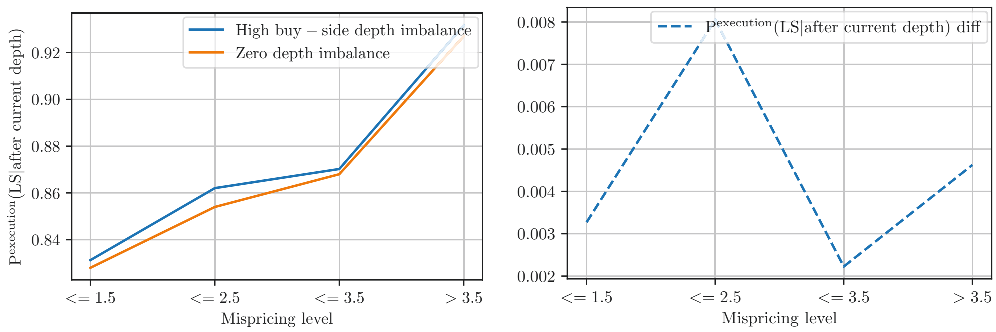
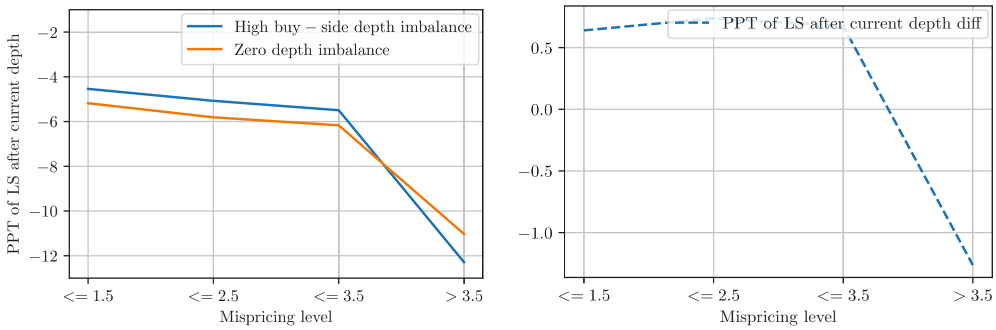


Figure A2.1: The uninformed's inability to infer mispricing reversal.

(iv) PPT of LS after current depth



(v) PPO of LS after MB at current depth

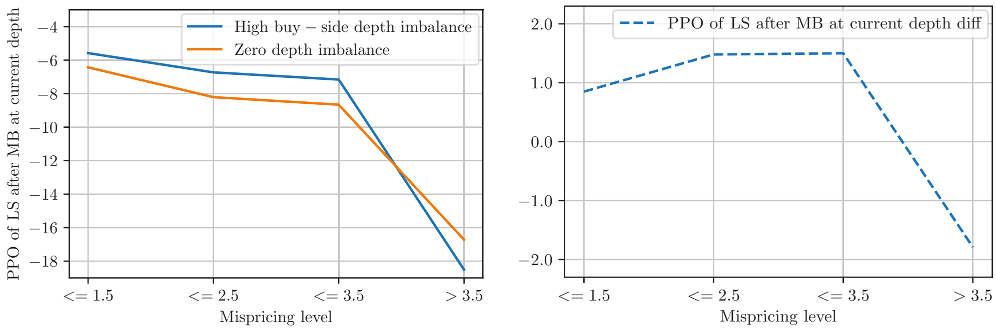


Figure A2.1: The uninformed's inability to infer mispricing reversal (continued).

Chapter 3

Buying frenzies, short selling costs and their impact on investment efficiency

3.1 Introduction

Trading frenzies in financial markets occur when many speculators trade in the same direction, leading to significant price pressure. For example, multiple stocks with an excessive short interest experienced a dramatic price increase in January 2021 when retail investors coordinated to purchase the stocks using social media. As their share prices skyrocketed to new highs, a flood of media coverage accompanied them, and their shares became known as "meme stocks".[2] The ability of individual retail investors to coordinate through social media to purchase the "meme stocks" imposed costs on short sellers, resulting in a substantial loss to short sellers and, consequently, a significant reduction in the amount of short interest in these stocks.[3] In this paper, we are interested in how such a buying frenzy impacts firms' investment efficiency.

[2] Among the so-called "meme stocks", an American video game company GameStop Corp ("GameStop") was a particularly extreme case. The level of short interest (the ratio of borrowed shares to total outstanding shares) in the GameStop share was 122.97% at its peak, as reported by SEC (2021). A short-interest ratio can exceed 100% when successive purchasers lend the same shares multiple times.

[3] For example, investors who sold GameStop short have lost around $25 billion (see O'Hara (2021)). The amount of short interest in the GameStop share reduced to around 10% as of 15 October 2021 (see, for example, https://www.ortex.com/symbol/NYSE/GME).

There are generally two sides to short selling. On the one hand, short sellers play a crucial role in financial markets, keeping prices from overinflating or entering bubbles, guiding capital to its best uses and improving welfare. On the other hand, some activist short sellers massively bet stock prices to fall and create panic in the market by exaggerating their statements. Such short selling is often perceived as market manipulation. Manipulative short sellers inhibit the beneficial role of the financial market in terms of resource allocation and destroy firm fundamentals in a way that might benefit their trading position. This dichotomy of short sellers makes it difficult for policymakers to investigate the welfare implications of short selling.

In this paper, we investigate the real investment effects of retail buying frenzy as a friction on short sales. We first develop a benchmark model in the Goldstein and Guembel (2008) framework that extends Kyle (1985) to allow a feedback effect. The feedback effect allows the firm manager to learn about the quality of an investment opportunity from the stock price. To examine the real effects, we introduce a model with four types of participants: a risk-neutral speculator (who can be positively informed, negatively informed, or uninformed), a noise trader, a risk-neutral market maker, and the firm manager in a four-date economy. The speculator observes a private signal about the true state of the economy in $t = 0$; the speculator and the noise trader trade with the market maker in $t = 1$ and $t = 2$. Based on the stock price realisations in the financial market, the firm manager makes the investment decision in $t = 3$.

The benchmark equilibrium as in Goldstein and Guembel (2008) features a sell – sell strategy by the negatively informed speculator as well as the uninformed speculator. [4] While a sell – sell strategy from the negatively informed speculator helps the firm manager to avoid a bad investment opportunity by correctly rejecting the investment, a sell – sell strategy from the uninformed speculator leads the firm manager to miss a good investment opportunity by incorrectly rejecting the investment. Such a manipulative strategy is profitable as the uninformed speculator utilises the feedback mechanism (i.e., managerial learning) from market prices to the manager's investment. He sets up a manipulative and non-information driven short position in $t = 1$ since he knows that his own selling pressure in $t = 2$ would

[4] When the speculator implements a sell – sell strategy, he sells in $t = 1$, and sells again in $t = 2$ if his type is not fully revealed.

lead to a rejection of a good investment opportunity, improving the profitability of the first short position.

To investigate the real investment effects of a retail buying frenzy, we then extend the benchmark model in three aspects. First, we introduce an outright short-sale ban to the benchmark model, meaning that a sell order is only permissible if the speculator has already bought the share in $t = 1$. The risk that retail investors can coordinate through social media to purchase stocks leads to price distortions that are not corrected by short sellers. Second, instead of imposing an outright short-sale ban, we introduce a cost of short selling to reflect the fact that a retail buying frenzy increases the stock price and introduces an additional cost on short sellers. We then investigate the real investment effects of different levels of cost to short selling. Third, we model asymmetric noise traders who are more likely to buy than sell.

We first show that a short-sale ban always harms both stock price efficiency and real investment efficiency relative to the benchmark equilibrium. On the one hand, a short-sale ban eliminates manipulative short selling, increasing the information content of negative order flows. On the other hand, a short-sale ban eliminates informed short selling, decreasing the stock price efficiency. The third effect of a short-sale ban is that it decreases the information content of positive order flows. This is because, in the benchmark equilibrium, positive orders can only come from the positively informed speculator, whereas in the equilibrium with a short-sale ban, positive orders can come from other types of speculators. The second and third effects, added together, dominate the first effect, resulting in a net reduction in the price efficiency. The reduction in the price efficiency brought by a short-sale ban then translates into an inferior investment decision by the firm manager.

In an extended model, we relax the strict ban on short selling and permit traders to short the stock while incurring a cost of c. The model with costly short selling generalises both the benchmark model and the model with a short-sale ban – a small cost on short selling ($c < c_1$) is equivalent to the benchmark model, and an extremely high short selling cost ($c > c_5$) is equivalent to the model with a short-sale ban. We investigate two other interesting short-sale cost regions: (i) an intermediate level of short-sale cost ($c_2 < c < c_3$) and (ii) a relatively high short-sale cost ($c_4 < c < c_5$).

An intermediate level of cost on short selling unambiguously improves the stock price efficiency and the real investment efficiency relative to the benchmark equilibrium. This occurs because an intermediate level of cost only deters manipulative short selling given that informed short sellers anticipate greater profits from their short selling activities compared to manipulative short sellers. The informed short seller is less sensitive to a short-sale cost due to his information advantage. The deterrence of manipulative short selling but not the informed improves the information content of negative order flows, leading to a more efficient stock price and investment decision.

A relatively high cost on short selling eliminates manipulative short selling, but also leads the negatively informed short seller to change from a sell – sell strategy to a no trade – sell strategy.[5] A relatively high level of short-sale cost thus reduces the information content of positive order flow in $t = 1$ (because no trade by the negatively informed speculator added together with a noise buy could also generate a positive order flow), but improves the information content of negative order flow in $t = 2$ (because the manipulative short selling is eliminated). The outcome of the opposing changes in the informativeness of different orders is that the real investment efficiency may improve or deteriorate depending which effect dominates. We derive the necessary and sufficient condition under which the investment efficiency improves with a relatively high cost on short selling.

We then develop a third model without an explicit cost on short sellers, where noise traders trade asymmetrically so that noise buys are more likely than noise sells in $t = 2$. This means that a short-sale position in $t = 1$ bears a risk of coordinated noise buys in $t = 2$. The coordinated noise buys present a trade-off in terms of stock price efficiency and investment efficiency. On the one hand, the increase in noise buys disguises the negatively informed speculator's sell orders, reducing the information content of no-trade event – we refer to this effect as the "*order flow disguising effect*".

On the other hand, the increase in noise buys pushes up the overall prices and leads to an increase in the manager's propensity to invest. When the short selling is manipulative, it means a correction of underinvestment and an improvement in fundamental value, resulting in a higher cost to cover the manipulative short. When the short selling is informative, however, it means an overinvestment and a

[5]When the speculator implements a no trade – sell strategy, he does not trade in $t = 1$, and sells in $t = 2$ if his type is not fully revealed.

deterioration in fundamental value, resulting in a lower cost to cover the informed short. Such a cost increase to cover the short sale is specific to the uninformed speculator – we thus refer to this effect as the "*uninformed-specific short-sale cost*".

While the order flow disguising effect harms the stock price and investment efficiency (because no trade has less information content), the uninformed-specific short-sale cost improves the stock price and investment efficiency (because it only applies to manipulative short sellers). We derive the necessary and sufficient condition under which the uninformed-specific short-sale cost dominates the order flow disguising effect, so that the investment efficiency is improved. We show that in the presence of coordinated noise buys, an investment decision is more likely to be efficient when (i) the fraction of uninformed speculator is large, (ii) the project has a large ex-ante NPV, and (iii) the uncertainty about the profitability of the investment is small. The rest of the paper is organised as follows. In Section 3.2, we discuss the related literature. In Section 3.3, we develop and solve the equilibrium in the benchmark model. In Section 3.4, we extend the benchmark model in three different ways and present our main results. Section 3.5 concludes.

3.2 Related literature

The paper contributes to the literature that analyses how stock prices affect corporate investment (e.g., Barro (1990), Morck, Shleifer, Vishny, Shapiro and Poterba (1990)) and the managerial learning from stock prices (e.g., Dow and Gorton (1997), Subrahmanyam and Titman (1999), Foucault and Gehrig (2008), Edmans, Jayaraman and Schneemeier (2017)). Stock prices aggregate information of many different market participants who do not have direct channels for communication with the firm outside the trading process (e.g., Dow and Gorton (1997), Subrahmanyam and Titman (1999)). The idea that stock prices are a useful source of information dates back to Hayek (1945). Thus, stock prices complement the information of managers and guide them in making corporate investment decisions (referred to as the managerial learning channel from stock prices in the extant literature).

Empirically, it has been shown that the managerial learning from stock prices is driven by the amount of private information in stocks (e.g., Chen et al. (2007)), cross-listing in multiple exchanges (e.g., Foucault and Fresard (2012)), peers' stock prices (e.g., Foucault and Fresard (2014), Dessaint et al. (2019)), and firms' capital constraints (e.g., Baker, Stein, and Wurgler (2003)). Closely related to our paper, Aliyev, Aly and Putnins (2021) show that market manipulation also distorts the managerial learning process from stock prices by reducing the sensitivity of corporate investment to stock prices and harms firms' future operating performance.

Consistently, we model the managerial learning process in the presence of both manipulative and informed short sellers as in Goldstein and Guembel (2008). In the presence of a feedback effect from the financial market to the firm's real investment, manipulative short selling is a profitable strategy because an initial short position leads to the rejection of a good investment opportunity (i.e., underinvestment), resulting in a reduction in the stock price and thereby improving the profitability of the short position. The paper is therefore also related to the theoretical market manipulation literature (e.g., Allen and Gorton (1992), Vila (1989), Benabou and Laroque (1992), Chakraborty and Yilmaz (2004), Takayama (2021)).

We contribute to this literature by investigating the real investment effects of a retail buying frenzy in the presence of both informed and manipulative short sellers. We model the retail buying frenzy as (i) imposing a short-sale ban to an otherwise Goldstein and Guembel (2008) model, (ii) introducing an additional cost to short selling, and (iii) asymmetric noise trading, in which noise buy is more likely to occur than noise sell. All three analyses are motivated by the frenzied retail traders piling into a blistering rally in "meme stocks" during the coronavirus pandemic.

Our model is different from the literature that investigates the impact of short-sale constraints without the presence of manipulative short selling. The literature shows that short-sale constraints play a trivial role – market efficiency is always reduced when short selling constraints bind, and the market becomes overvalued because the informed short sellers cannot perform an important role of driving the price back to the fundamental value (Bai, Chang and Wang (2006) and Cao, Zhang and Zhou (2007)).

We contribute to this literature by showing that different levels of the short-sale cost impact the price and real efficiency differently as both informed and manipulative short sellers are affected by such a cost. While the reduction in informed short selling due to short-sale constraints harms price and real efficiency, the reduction in manipulative short selling improves the price and real efficiency. We show that an outright short-sale ban unambiguously harms both price and real investment efficiency. However, an intermediate level of short-sale cost eliminates manipulative short selling but not informed short selling, resulting in improvement in the stock price and real investment efficiency. To our knowledge, this paper is the first to investigate the real investment effects of a retail buying frenzy as an additional cost on short sellers.

We also model a retail buying frenzy as asymmetric noise trading where a noise buy is more likely to occur than a noise sell. A large theoretical literature in market microstructure following Glosten and Milgrom (1985) and Kyle (1985) treat noise buys and sells as symmetric. In such a model, sellers and buyers are equally likely to be informed, leading to a symmetric price impact. This assumption is questionable because of factors such as short-sale constraints which make it easier to exploit good news than bad news. Allen and Gorton (1992) show that price manipulation is possible when noise traders trade asymmetrically. To capture the recent retail buying frenzy, we also introduce asymmetry into noise trading as in Allen and Gorton (1992). We show that a higher probability of noise buys gives rise to two opposing effects (the order flow disguising effect and uninformed-specific short-sale cost) that ultimately drive the real investment efficiency.

More broadly, our paper is related to the role of finance in terms of the broader real economy (see, for example, Levine (2005) and Bond, Edmans and Goldstein (2012) for an extensive survey). Research that clarifies our understanding of the role of finance in economic growth has policy implications and shapes future policy-oriented research (Levine (2005)). We contribute to the finance-real economy nexus by showing how informed short selling helps firm managers to learn from stock prices, how manipulative short selling distorts the managerial learning process and the harms and benefits of the retail buying frenzy on firm managers' investment decisions and the real economy. Our paper implies that natural market forces such as the retail buying frenzy for manipulative short selling may improve the real investment efficiency by driving out manipulative short selling.

3.3 The benchmark model

We develop the benchmark model in the Goldstein and Guembel (2008) framework to allow the firm manager to learn about the quality of the investment opportunity from the stock price. The benchmark model enables us to characterise manipulative short selling due to the existence of the feedback effect. The benchmark model also provides a contrast to subsequent models that capture the impacts of the retail buying frenzy coordinated through social media.

3.3.1 The benchmark model setup

The model consists of four dates, denoted as $t \in \{0, 1, 2, 3\}$, and a firm of which shares are traded in unit supply. There are four types of participants: a risk-neutral speculator, a noise trader, a risk-neutral market maker, and the firm manager. The firm manager needs to make an investment decision. However, the manager faces uncertainty over the quality of the investment opportunity and uses stock price realisations to learn about the quality of the investment project. There are two potential states of the firm's investment profitability $s \in \{l, h\}$ that each occurs with a probability of 0.5.

In $t = 0$, the speculator observes a private signal s about the true state of the economy. With probability α, the signal is perfectly informative, denoted as $s \in \{l, h\}$, and with probability $1-\alpha$, the signal is uninformative, denoted as $s = \emptyset$. Following Goldstein and Guembel (2008), we use the terms "positively informed speculator" when $s = h$, "negatively informed speculator" when $s = l$, and "uninformed speculator" when $s = \emptyset$. Conditional on receiving a perfectly informative signal, the speculator is equally likely to be positively informed or negatively informed.

Trading takes place in $t = 1$ and $t = 2$. When making trading decisions, the speculator rationally maximises his expected profits by choosing to sell ($u_t = -1$), not trade ($u_t = 0$), or buy ($u_t = 1$), generating an order flow $u_t \in \{-1, 0, 1\}$. The noise trader, who is not strategic, randomly draws an action from the three actions, each with an equal probability. We denote the noise trader's order flow at time t as n_t. We also assume that the noise trader's orders are serially uncorrelated, meaning n_1 and n_2 are independent.

The market maker, as in Kyle (1985), quotes prices after observing the total order flow $Q_t = n_t + u_t \in \{-2, -1, 0, 1, 2\}$. The $t = 1$ price is the expected firm value conditional on Q_1, $P_1 (Q_1) = E [V|Q_1]$ and similarly, the $t = 2$ price is the expected firm value conditional on Q_1 and Q_2, $P_2(Q_1, Q_2) = E[V|Q_1, Q_2]$. In $t = 3$, the manager makes the investment decision on a project based on price/order flow realisations in the financial market. The firm value is $V^+ > 0$ when the manager encountered the high state ($s = h$) invests, and is $V^- < 0$ when the manager encountered the low state ($s = l$) invests. The firm value is 0 with no investment. The manager does not have private information about the project profitability and, analogous to Goldstein and Guembel (2008), we assume that the project has positive ex-ante NPV, i.e., $V^+ \geqslant -3V^-$. Absent further information, the firm manager will opt to invest. Additionally, a crucial condition for the model is that information conveyed by a sell order is sufficiently strong, i.e.,

$$\frac{\alpha}{2}V^- + (1-\alpha)\bar{V} < 0. \tag{3.1}$$

Otherwise, the firm manager does not call off the investment after sell orders, and the uninformed speculator does not impact the real investment and firm value, resulting in no feedback between the financial market and real investment. Figure 3.1 illustrates the model timeline and the set of actions at each time t.

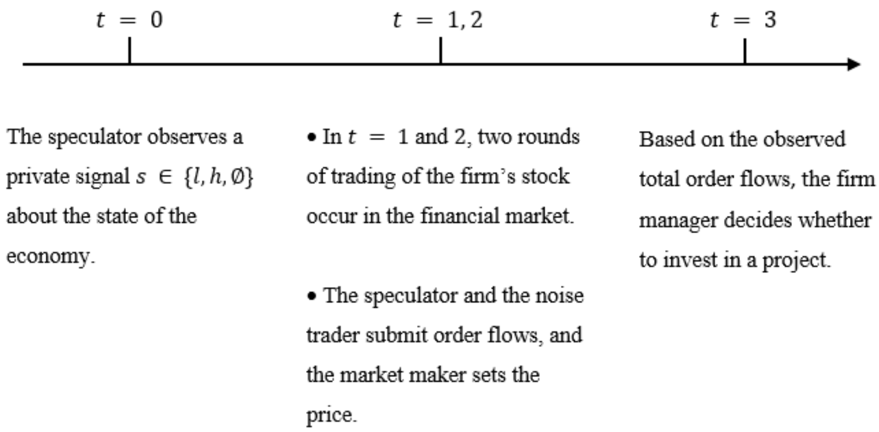


Figure 3.1: Model timeline.
This figure illustrates the timeline in the baseline model. In $t = 0$, the speculator observes a private signal. In $t = 1$ and $t = 2$, trading occurs in the financial market. In $t = 3$, the firm manager decides whether or not to invest in a project using the information from the financial market.

3.3.2 The benchmark equilibrium

The equilibrium concept throughout the chapter is perfect Bayesian Nash equilibrium. It is defined as follows: (i) the speculator chooses a trading strategy

$\{u_1(s), u_2(s, Q_1, u_1)\}$ that maximises his expected final payoff, (ii) the firm manager chooses an investment strategy that maximises the expected value of the firm, (iii) the market maker chooses a price-setting strategy $\{p_1(Q_1), p_2(Q_1, Q_2)\}$ that breaks even in expectations, (iv) the firm manager and the market maker update beliefs using Bayes' theorem, and (v) all agents have rational expectations (i.e., each player's belief about the other players' strategies is correct in equilibrium).

We now characterise the equilibrium strategies of the speculator in the benchmark model. Lemma 3.1 shows the best response of the negatively informed speculator ($s = l$) along with that of the uninformed speculator ($s = \emptyset$). To derive the best responses of negatively informed and uninformed speculators, we fix the positively informed speculator's strategy profile as to buy in $t = 1$ and buy again in $t = 2$ if his type is not revealed. Throughout the paper, we follow the same reasoning to characterise the best responses of negatively informed and uninformed speculators.

Lemma 3.1. *Suppose that the positively informed speculator's strategy profile is to buy in $t = 1$ and buy again in $t = 2$ if his type is not revealed, the following holds in $t = 2$:*

(i) When Q_1 perfectly reveals a speculator's information or reveals that he is not informed of the high state, then he is indifferent between buy, sell, and no trade.

(ii) When Q_1 reveals that the speculator is not informed of the low state, then the best response in $t = 2$ of the uninformed speculator is to sell.

(iii) When Q_1 reveals that the speculator is not uninformed, then the best response in $t = 2$ of negatively informed speculator who buys or does not trade in $t = 1$ is to sell.

(iv) When Q_1 does not reveal the speculator's type, then the best response in $t = 2$ of the uninformed speculator who does not trade in $t = 1$ is to not trade, the best response in $t = 2$ of uninformed speculator who sells in $t = 1$ is to sell, and the best response in $t = 2$ of the negatively informed speculator is to sell.

The best responses in $t = 2$ characterised in Lemma 3.1 pin down four candidate equilibriums. In each candidate equilibrium, we calculate the market prices as the expected firm values conditional on order flows given the hypothesised speculator strategies. We then calculate each type of speculator's total profit in these candidate equilibriums and show that the benchmark equilibrium with manipulative short selling (formally defined in Proposition 3.2) is the one with no profitable deviation. Table 3.1 shows the equilibrium strategies of the speculator and the equilibrium prices of the market maker.

In the benchmark equilibrium, in $t = 3$, the firm manager chooses not to invest if the order flow suggests that the speculator can not be positively informed, and invests otherwise. This means that, given the equilibrium best responses in Lemma 3.1, the firm chooses not to invest for any Q_2 following $Q_1 \in \{-2, -1\}$ and for $Q_2 \in \{-2, -1\}$ following $Q_1 \in \{0, 1\}$. The firm manager chooses to invest otherwise. The equilibrium prices in $t = 2$ are expected investment profit, conditional on $t = 1$ and $t = 2$ order flows, and equilibrium prices in $t = 1$ is the expected $t = 2$ price, conditional on $t = 1$ order flow.

We now formally characterise the benchmark equilibrium in Proposition 3.2. The benchmark equilibrium features both a manipulative and informed short selling in $t = 1$. Such an equilibrium allows us to determine the impact of retail buying pressure (in the form of an additional cost on short sellers or a short-sale ban in the extreme and coordinated noise trading) on the informed and manipulative short sellers, and consequently on market prices and firm value in subsequent models.

Proposition 3.2. *An equilibrium where the positively informed speculator buys in both periods if his type is not revealed exists only if the negatively informed speculator and the uninformed speculator sells in $t = 1$ and sells (resp. is indifferent between buy/sell/no trade) in $t = 2$ if Q_1 is non-negative (resp. negative).*

Note that manipulative short selling occurs only if the uninformed speculator implements a sell – sell strategy. With the feedback effect, the uninformed speculator sets up a short position in $t = 1$ since he knows that his own selling pressure in $t = 2$ would lead to the rejection of an investment opportunity, improving the profitability of the first short position. The manipulative short selling in $t = 1$ impacts the investment's ex-ante efficiency and hence the expected firm value. Corollary 3.3 establishes the expected firm value in the benchmark equilibrium.

Corollary 3.3. *In the benchmark equilibrium, the real efficiency (i.e., the expected value of the firm) is given by*

$$RE_{benchmark} = \frac{(1-\alpha)\bar{V}}{9} + \frac{\alpha V^-}{18} + \frac{\alpha V^+}{2}, \tag{3.2}$$

where $\bar{V} = \frac{1}{2}V^+ + \frac{1}{2}V^- > 0$ is the ex-ante NPV of the investment opportunity.

Table 3.1: Equilibrium strategies and prices in the benchmark setting.
This table reports the equilibrium strategies of the speculator and the equilibrium prices of the market maker. Panel (A) shows the equilibrium strategies of the speculator in $t = 1$ and 2. Panel (B) shows the equilibrium prices of the market maker in $t = 1$ and 2.

Panel (A): Benchmark equilibrium strategies

	Equilibrium strategy in $t = 1$
$u_1(s = h)$	1
$u_1(s = l)$	-1
$u_1(s = \emptyset)$	-1

	Equilibrium strategy in $t = 2$				
	$Q_1 = -2$	$Q_1 = -1$	$Q_1 = 0$	$Q_1 = 1$	$Q_1 = 2$
$u_2(s = h)$	Not applicable	Not applicable	1	$\{-1, 0, 1\}$	$\{-1, 0, 1\}$
$u_2(s = l)$	$\{-1, 0, 1\}$	$\{-1, 0, 1\}$	-1	Not applicable	Not applicable
$u_2(s = \emptyset)$	$\{-1, 0, 1\}$	$\{-1, 0, 1\}$	-1	Not applicable	Not applicable

Panel (B): Benchmark equilibrium prices

Round 1 of trading	Round 2 of trading
$P_1(0) = \frac{\bar{V}}{3} + \frac{\alpha}{3} V^+$	$P_2(0, 0) = \bar{V}$
$P_1(1) = P_1(2) = V^+$	$P_2(0, 1) = P_2(0, 2) = V^+$
$P_1(-1) = P_1(-2) = 0$	$P_2(0, -1) = P_2(0, -2) = 0$
	$P_2(1, \cdot) = P_2(2, \cdot) = V^+$
	$P_2(-1, \cdot) = P_2(-2, \cdot) = 0$

3.4 The real effects of retail buying frenzy

We now modify the benchmark model in three different ways. First, we introduce an outright short-sale ban to the benchmark model. This allows us to investigate the real effects of a short-sale ban in a model in which both informed and manipulative short sellers are present. This is different from the models with only informed short sellers that show that the market efficiency is always reduced when short selling constraints bind, and the market becomes overvalued because the informed short sellers cannot perform an important role of driving the price back to the fundamental value (e.g., Bai et al. (2006) and Cao et al. (2007)).

Second, instead of imposing an outright short-sale ban, we introduce a monetary cost c to short selling, to reflect the fact that the retail buying pressure increases the stock price and introduces an additional cost on short sellers, and investigate the real effects of different levels of cost to short selling. The model with a short selling cost generalises both the benchmark model and the model with a short-sale ban – in this setting, $c = 0$ is equivalent to the benchmark model, and an extremely high short selling cost c is equivalent to the model with a short-sale ban. Third, we explicitly increase the probability of noise buys in the benchmark model to capture the real investment effects of coordinated noise buys. We assume the noise trader buys with a probability of $(1+\delta)/3$, sells with a probability of $(1-\delta)/3$, or does not trade with a probability of $1/3$. The magnitude of the coordinated noise buys is determined by the coordination coefficient $0 \leqslant \delta \leqslant 1$. Glosten and Milgrom (1985), Kyle (1985) and the large subsequent literature treat noise traders as symmetric, that is, equally likely to be buyers and sellers. By introducing an asymmetry into noise trading in Glosten and Milgrom (1985), Allen and Gorton (1992) demonstrate an example of price manipulation. The manipulative short selling in our setting occurs because of the feedback loop between the financial market and firm value. The coordinated buying pressure in our model, however, pushes up the market prices compared to the benchmark model and ultimately act as a cost on short sellers. All three models aim to capture different dimensions of the transitory retail buying frenzy.

3.4.1 A short-sale ban

In this setting, we describe an equilibrium derivation in an economy where a short-sale is prohibited. The only difference of this setting from the benchmark is that a sell order is only permissible if the speculator has already bought the share in $t = 1$. This means that the total potential order flow that the market observes in $t = 1$ is $Q_1 \in \{-1, 0, 1, 2\}$ and in $t = 2$ is $Q_2 \in \{-2, -1, 0, 1, 2\}$.

Similar to the benchmark model, we first characterise the best responses of the negatively informed and uninformed speculators when the positively informed speculator's strategy profile is to buy in $t = 1$ and buy again in $t = 2$ if his type is not fully revealed.[6] We replace the subgame with the corresponding best responses of the negatively informed and uninformed speculators, leading to several candidate equilibriums. In the candidate equilibriums, the negatively informed speculator selects between a buy – sell strategy and a no trade – no trade strategy, and the uninformed speculator selects between a strategy that buys in $t = 1$ and a no trade – no trade strategy.[7] We search for the short-sale ban equilibrium by checking these candidate equilibriums and excluding the ones that have a profitable deviation. Lemma 3.4 shows the best responses of the negatively informed and uninformed speculators with a short-sale prohibition.

Lemma 3.4. *In the presence of a short-sale ban, suppose that positively informed speculator implements a buy – buy strategy, the following holds in $t = 2$:*

(i) When Q_1 perfectly reveals a speculator's information or reveals that he is not informed of the high state, then the speculator is indifferent between buy, sell (if he has an initial position), and no trade.

(ii) When Q_1 reveals that the speculator is not informed of the low state, then the best response $t = 2$ of the uninformed speculator is to sell.

(iii) When Q_1 reveals that the speculator is not uninformed, then the best response in $t = 2$ of the negatively informed speculator is to sell.

(iv) When Q_1 does not reveal anything, the negatively informed speculator sells if he has an initial position, or does not trade if he does not trade in $t = 1$, and the uninformed speculator does not trade in $t = 2$ if he does not trade in $t = 1$.

[6] This strategy profile is hereinafter reffered to as a buy – buy strategy.

[7] When the speculator adopts a buy – sell strategy, he buys in $t = 1$, and sells in $t = 2$. When the speculator adopts a no trade – no trade strategy, he does not trade in $t = 1$, and does not trade in $t = 2$ if his type is not fully revealed.

In the presence of a short-sale prohibition, the uninformed speculators are not allowed to choose a manipulative sell – sell strategy as in the benchmark model. Instead, they can choose a buy – sell strategy, but they refrain from that strategy since no trade – no trade is more profitable. Based on the above best responses, Proposition 3.5 characterises the short-sale ban equilibrium in which the negatively informed and uninformed speculators chooses to not trade in $t = 1$ and $t = 2$.

Proposition 3.5. *In the presence of a short-sale ban, an equilibrium where the positively informed speculator buys in both periods if his type is not revealed exists only if the negatively informed or the uninformed speculator does not trade in* $t = 1$ *and does not trade (resp. indifferent between buy and no trade) in* $t = 2$ *if* $Q_1 \neq -1$ *(resp.* $Q_1 = -1$*).*

In our model, the short-sale ban alters the equilibrium behaviours of the speculators compared to the benchmark equilibrium. Table 3.2 reports the equilibrium strategies and corresponding stock prices, where the equilibrium prices are the expected firm value given the observed order flow as before.

The changes in the equilibrium strategies of the speculators change the information content of order flows, thereby impacting the stock price efficiency. A short-sale ban eliminates both the informed and manipulative short selling. On the one hand, the elimination of manipulative short selling leads to an increase in the price efficiency. On the other hand, a short-sale ban eliminates the negative informed speculator's short selling and decreases the price efficiency by deterring the negative information from being impounded in prices. An additional effect that the short-sale ban has is that it decreases the information content of the positive order flow. In the benchmark equilibrium, positive order flows can only come from the positively informed ($s = h$) speculator, and are fully revealing, whereas in the short-sale ban equilibrium positive order flows can come from other types of speculators, reducing the informativeness of the positive order flows.

We formalise the above intuition about the informational effect of a short-sale ban using conditional probabilities. We define the information content of negative order flow in $t = 1$ as $\Pr(s = l|Q_1 < 0)$ and in $t = 2$ as $\Pr(s = l|Q_1 = 0, Q_2 < 0)$. The information content of no-trade in $t = 1$ is $1-\Pr(s = l|Q_1 = 0)$ and in $t = 2$ is $1 - \Pr(s = l|Q_1 = Q_2 = 0)$. Similarly, the information content of a positive order flow in $t = 1$ is $1 - \Pr(s = l|Q_1 > 0)$ and in $t = 2$ is $1 - \Pr(s = l|Q_1 = 0, Q_2 > 0)$.

Table 3.2: Equilibrium strategies and prices in the short-sale ban setting.
This table reports the equilibrium strategies of the speculator and equilibrium prices of the market maker in the setting where short selling is prohibited. Panel (A) shows the equilibrium strategies of the speculator in $t = 1$ and 2. Panel (B) shows the equilibrium prices of the market maker in $t = 1$ and 2.

Panel (A): Short-sale ban equilibrium strategies

	Equilibrium strategy in $t = 1$			
$u_1(s = h)$	1			
$u_1(s = l)$	0			
$u_1(s = \emptyset)$	0			
	Equilibrium strategy in $t = 2$			
	$Q_1 = -1$	$Q_1 = 0$	$Q_1 = 1$	$Q_1 = 2$
$u_2(s = h)$	Not applicable	1	1	$\{-1, 0, 1\}$
$u_2(s = l)$	$\{0, 1\}$	0	0	Not applicable
$u_2(s = \emptyset)$	$\{0, 1\}$	0	0	Not applicable

Panel (B): Short-sale ban equilibrium prices

Round 1 of trading	Round 2 of trading
$P_1(0) = P_1(1) = \frac{2\bar{V}}{3} + \frac{\alpha}{6}V^+$	$P_2(0,0) = P_2(0,1) = P_2(1,0) = P_2(1,1) = \bar{V}$
$P_1(2) = V^+$	$P_2(0,2) = P_2(1,2) = V^+$
$P_1(-1) = 0$	$P_2(0,-1) = P_2(1,-1) = 0$
	$P_2(2,\cdot) = V^+$
	$P_2(-1,\cdot) = 0$

Table 3.3: Information content of order flows in the short-sale ban setting.
The table shows the information content of order flows in the short-sale ban equilibrium and compares it with that of the benchmark equilibrium in $t = 1$ and 2. The information content of positive order flow is given by $1 - \Pr(s = l|Q_1 > 0)$, the information content of no trade is given by $1 - \Pr(s = l|Q_1 = 0)$, and the information content of negative order flow is given by $\Pr(s = l|Q_1 < 0)$. The information content of the no-trade and negative order flows in the short-sale ban setting is the same as that of the benchmark model. A short-sale ban impedes the positive information from getting impounded into the price and reduces the information content of the positive order flow relative to the benchmark equilibrium.

	Order flow informativeness in $t = 1$		
	Positive order flow $1 - \Pr(s = l\|Q_1 > 0)$	No-trade $1 - \Pr(s = l\|Q_1 = 0)$	Negative order flow $\Pr(s = l\|Q_1 < 0)$
Benchmark equilibrium	1	$1 - \frac{\alpha}{2}$	$\frac{\alpha}{2-\alpha}$
Short-sale ban equilibrium	$\frac{2}{2+\alpha}$	$1 - \frac{\alpha}{2}$	$\frac{\alpha}{2-\alpha}$
	Order flow informativeness in $t = 2$		
	Positive order flow $1 - \Pr(s = l\|Q_1 = 0, Q_2 > 0)$	No-trade $1 - \Pr(s = l\|Q_1 = 0, Q_2 = 0)$	Negative order flow $\Pr(s = l\|Q_1 = 0, Q_2 < 0)$
Benchmark equilibrium	1	$1 - \frac{\alpha}{2}$	$\frac{\alpha}{2-\alpha}$
Short-sale ban equilibrium	$\frac{2}{2+\alpha}$	$1 - \frac{\alpha}{2}$	$\frac{\alpha}{2-\alpha}$

Table 3.3 contrasts the informativeness of the order flows in the benchmark and short-sale ban equilibrium. The table shows that positive order flows in the short-sale ban equilibrium reveal less information than they do in the benchmark equilibrium, reducing from 1 to $2/(2+\alpha)$. Interestingly, no trade and negative order flows in the benchmark and the short-sale ban equilibrium have the same informativeness. The intuition for the unchanged informativeness of no trade and negative order flows is that the harmful effect of the eliminated informed short selling and the beneficial effect of the eliminated manipulative short selling cancel each other out for these two types of order flows.

In our model, the firm manager's ability to learn from the stock price (i.e., the managerial learning) about the quality of the investment opportunity changes the firm value. The changes in the informativeness of the order flows and the resulting changes in the stock price efficiency should then translate into the changes in the real investment efficiency of the firm due to the feedback effect. Corollary 3.6 shows the expected firm value given all trading game participants' strategies in the presence of a short-sale ban and how it compares to the benchmark model.

Corollary 3.6. *(i) In the presence of a short-sale ban, the real efficiency (i.e., the expected value of the firm) is given by*

$$RE_{ssban} = \frac{\alpha V^+}{2} + \frac{2\alpha V^-}{9} + \frac{4(1-\alpha)\bar{V}}{9}. \tag{3.3}$$

(ii) A short-sale ban always harms the real efficiency.

The corollary quantifies the expected firm value in the presence of a short-sale ban. Comparing Eq.(3.3) to Eq.(3.2) reveals that an outright short-sale ban always harms the firm value due to the unambiguous drop in the price efficiency relative to the benchmark model. This means that an extremely large implicit cost imposed on (both informed and manipulative) short sellers due to the extreme transitory retail buying pressure results in the price and real investment inefficiency. To investigate the real investment effects of varying levels of cost on short sellers instead of an outright short-sale ban, we next explicitly introduce a linear cost on short sellers.

3.4.2 Costly short selling

To relax the outright short-sale ban, we now introduce a cost $c > 0$ per unit of shorting position. This is to reflect that retail buying pressure in fact pushes the market prices up and imposes an additional cost on short selling. In practice, many short-sale constraints impose an additional cost on short sellers but are less restrictive than an outright ban. When the short selling cost is extremely high, the costly short selling model resembles the model with a short-sale ban, and when the short selling cost is extremely low, the model appears like the benchmark model. Thus, the costly short selling model enables us to compare the benchmark equilibrium, the short-sale ban equilibrium, and the equilibrium with a costly short selling. Lemma 3.7 characterises the best responses of the speculators in the costly short selling model.

Lemma 3.7. *Let a_1, a_2, a_3, and a_4 be given respectively by Eqs.(A3.1.8), (A3.1.5), (A3.1.7), and (A3.1.6) in the Appendix. In the presence of a short selling cost $c > 0$, suppose that the positively informed speculator implements a buy – buy strategy, the following holds in $t = 2$:*

(i) When Q_1 perfectly reveals a speculator's information or reveals that he is not informed of the high state, then the speculator is indifferent between buy and no trade.

(ii) When Q_1 reveals that the speculator is not informed of the low state, then the uninformed speculator sells (resp. does not trade) in $t = 2$ if $c < a_2$ (resp. $c > a_2$).

(iii) When Q_1 reveals that the speculator is not informed of the high state, then the negatively informed speculator as well as the uninformed speculator are indifferent between buy and no trade.

(iv) When Q_1 reveals that the speculator is not uninformed, then the negatively informed speculator buying in $t = 1$ sells (resp. does not trade) in $t = 2$ if $c < a_4$ (resp. $c > a_4$) and the negatively informed speculator not trading in $t = 1$ sells (resp. does not trade) in $t = 2$ if $c < a_3$ (resp. $c > a_3$).

(v) When Q_1 does not reveal the speculator's type, the uninformed speculator not trading in $t = 1$ also does not trade in $t = 2$, and the uninformed speculator selling in $t = 1$ sells again (resp. does not trade) in $t = 2$ if $c < a_1$ (resp. $c > a_1$); the negatively informed speculator buying in $t = 1$ sells (resp. does

not trade) in $t = 2$ if $c < a_4$ (resp. $c > a_4$), the negatively informed speculator not trading in $t = 1$ sells (resp. does not trade) in $t = 2$ if $c < a_3$ (resp. $c > a_3$), and the negatively informed speculator selling in $t = 1$ sells again (resp. does not trade) in $t = 2$ if $c < a_1$ (resp. $c > a_1$).

The equilibrium is obtained through the following procedure. We first dissect the range of short selling cost c into four regions: (i) $(0, a_1)$, (ii) (a_1, a_2), (iii) (a_2, a_3), and (iv) $(a_3, +\infty)$. Then, in each region, we replace the subgame in $t = 2$ with corresponding best responses. Finally, we determine the range of c for each type of equilibrium by imposing that there is no profitable deviation for the speculator. When the positively informed speculator implements a buy – buy strategy, four equilibrium outcomes can arise:

- Benchmark equilibrium: both the negatively informed and uninformed speculator sell in both trading rounds.
- No manipulative shorting *(NMS)* equilibrium: the speculator sells in both trading rounds when he receives $s = l$, and the speculator does not trade in both trading rounds if $Q_1 = 0$ (does not trade in $t = 1$ and sells in $t = 2$ if $Q_1 = 1$) when he receives $s = \emptyset$.
- Reduced informed and no manipulative shorting ($RINMS$) equilibrium: the speculator does not trade in $t = 1$ and sells in $t = 2$ when he receives $s = l$, and does not trade in both trading rounds when he receives $s = \emptyset$.
- Short-sale ban (SSB) equilibrium: both negatively informed and uninformed speculators choose to not trade in $t = 1$ and $t = 2$.

Proposition 3.8 shows how the equilibrium outcomes vary with the different levels of short selling cost. Threshold values of short selling cost in the proposition are:

$$c_1 = \frac{\alpha}{9}V^+ - \frac{V^+ - V^-}{18}, \tag{3.4}$$

$$c_2 = \frac{\alpha}{9}V^+ - \frac{\alpha}{9(2-\alpha)}\left(V^+ - \bar{V}\right), \tag{3.5}$$

$$c_3 = \frac{\alpha}{6}V^+ - \frac{\alpha}{6(2-\alpha)}\left(V^+ - \bar{V}\right), \tag{3.6}$$

$$c_4 = \frac{\alpha\left(V^+ - V^-\right)}{6}, \tag{3.7}$$

$$c_5 = \frac{V^+ - V^-}{6}. \tag{3.8}$$

Proposition 3.8. *Let c_1, c_2, c_3, c_4 and c_5 be given respectively by Eqs.(3.4), (3.5), (3.6), (3.7) and (3.8). In the presence of a short selling cost $c > 0$, the trading game has the following equilibria in which the positively informed speculator implements a buy – buy strategy:*

(i) When $c < c_1$, the only equilibrium is the benchmark equilibrium.

(ii) When $c_2 < c < c_3$, the only equilibrium is the NMS equilibrium.

(iii) When $c_4 < c < c_5$, the only equilibrium is the RINMS equilibrium.

(iv) When $c > c_5$, the only equilibrium is the SSB equilibrium.

Proposition 3.8 (i) suggests that with a small cost on short selling ($c < c_1$), the equilibrium features both informed and manipulative short selling as in the benchmark model. This is intuitive because with a small cost on short selling, it is still profitable for the uninformed speculator to engage in the manipulative short selling. However, the proposition also shows that the uninformed speculator is more sensitive to short selling cost compared to the negatively informed speculator. An intermediate level of cost on short selling ($c_2 < c < c_3$) eliminates the manipulative short seller but does not impact the informed short seller.[8] A relatively high cost on short selling ($c_4 < c < c_5$) also impacts the informed short seller, leading him to change from the sell – sell strategy in the *NMS* equilibrium to the no trade – sell strategy in the *RINMS* equilibrium. Finally, an extremely high cost ($c > c_5$) eliminates both informed and manipulative short selling, leading to the equilibrium in the presence of a short-sale ban. Table 3.4 shows the equilibrium strategies and the corresponding prices for each type of equilibrium.

To further explain the changes in the equilibrium strategies of the speculators, let's consider the extreme case in which the speculators who are not positively informed choose between sell – sell and no trade – no trade strategies. By selling in $t = 1$ and selling again in $t = 2$ if not revealed, the uninformed short seller's expected profit is

[8]Note that in the *NMS* equilibrium, the fact that uninformed speculator does not trade in $t = 1$ and sells in $t = 2$ if $Q_1 = 1$ does not imply a manipulative short selling. When $Q_1 = 1$, the uninformed speculator simply takes the benefit of his information advantage compared to the market maker as he knows $Q_1 = 1$ is due to the noise trader (since he has not traded) but the market maker was not aware of that. Manipulative short selling occurs when the uninformed speculator implements a sell – sell strategy. In fact, no trade – sell strategy by the uninformed speculator occurs without the feedback effect.

Table 3.4: Equilibrium strategies and prices in the costly short selling setting.
This table reports the equilibrium strategies of the speculator and equilibrium prices of the market maker in the costly short selling setting. Panels (A) and (C) show the strategies of the speculator in $t = 1$ and 2 in the "No manipulative short" (*NMS*) equilibrium and "Reduced informed and no manipulative short" (*RINMS*) equilibrium, respectively. Panels (B) and (D) show the prices quoted by the market maker in $t = 1$ and 2 in the *NMS* equilibrium and *RINMS* equilibrium. The "Short-sale ban" (*SSB*) equilibrium has been discussed before and is thus neglected here.

Panel (A): Strategies in the *NMS* equilibrium

			Equilibrium strategy in $t = 1$		
$u_1(s = h)$			1		
$u_1(s = l)$			-1		
$u_1(s = \emptyset)$			0		
			Equilibrium strategy in $t = 2$		
	$Q_1 = -2$	$Q_1 = -1$	$Q_1 = 0$	$Q_1 = 1$	$Q_1 = 2$
$u_2(s = h)$	Not applicable	Not applicable	1	1	$\{-1, 0, 1\}$
$u_2(s = l)$	$\{0, 1\}$	$\{0, 1\}$	-1	Not applicable	Not applicable
$u_2(s = \emptyset)$	Not applicable	$\{0, 1\}$	0	-1	Not applicable

Panel (B): Prices in the *NMS* equilibrium

Round 1 of trading	Round 2 of trading
$P_1(0) = \frac{\bar{V}}{3} + \frac{\alpha}{6}V^+$	$P_2(0, 0) = \bar{V}$
$P_1(1) = \frac{\alpha}{2-\alpha}V^+ + \frac{2(1-\alpha)}{2-\alpha}\bar{V}$	$P_2(0, 1) = P_2(0, 2) = V^+, P_2(0, -1) = P_2(0, -2) = 0$
$P_1(2) = V^+$	$P_2(1, 0) = \frac{\alpha}{2-\alpha}V^+ + \frac{2(1-\alpha)}{2-\alpha}\bar{V}$
$P_1(-2) = P_1(-1) = 0$	$P_2(1, 1) = P_2(1, 2) = V^+, P_2(1, -1) = P_2(1, -2) = \bar{V}$
	$P_2(2, \cdot) = V^+$
	$P_2(-2, \cdot) = P_2(-1, \cdot) = 0$

Table 3.4: Equilibrium strategies and prices in the costly short selling setting (continued).

Panel (C): Strategies in the *RINMS* equilibrium

	Equilibrium strategy in $t = 1$
$u_1(s = h)$	1
$u_1(s = l)$	0
$u_1(s = \emptyset)$	0

	Equilibrium strategy in $t = 2$			
	$Q_1 = -1$	$Q_1 = 0$	$Q_1 = 1$	$Q_1 = 2$
$u_2(s = h)$	Not applicable	1	1	$\{-1, 0, 1\}$
$u_2(s = l)$	$\{0, 1\}$	-1	-1	Not applicable
$u_2(s = \emptyset)$	$\{0, 1\}$	0	0	Not applicable

Panel (D): Prices in the *RINMS* equilibrium

Round 1 of trading	Round 2 of trading
$P_1(0) = P_1(1) = \frac{2-2\alpha}{3}\bar{V} + \frac{\alpha}{3}V^+$	$P_2(0,0) = P_2(1,0) = \bar{V}$
$P_1(2) = V^+$	$P_2(0,1) = P_2(1,1) = \frac{\alpha}{2-\alpha}V^+ + \frac{2(1-\alpha)}{2-\alpha}\bar{V}$
$P_1(-1) = 0$	$P_2(0,2) = P_2(1,2) = V^+$
	$P_2(0,-1) = P_2(0,-2) = P_2(1,-1) = P_2(1,-2) = 0$
	$P_2(2,\cdot) = V^+$
	$P_2(-1,\cdot) = 0$

$$P_{s=\emptyset} = \underbrace{\frac{1}{3}P_1(0) + \frac{1}{9}P_2(0,0)}_{\textit{Shorting proceeds}} \; - \; \underbrace{\frac{2}{9}\bar{V}}_{\textit{Disadvantage}} \; - \; \frac{4}{3}c, \tag{3.9}$$

whereas the negatively informed short seller's expected profit is

$$P_{s=l} = \underbrace{\frac{1}{3}P_1(0) + \frac{1}{9}P_2(0,0)}_{\textit{Shorting proceeds}} \; \underbrace{-\frac{2}{9}V^-}_{\textit{Advantage}} - \frac{4}{3}c. \tag{3.10}$$

The shorting proceeds terms in Eqs.(3.9) and (3.10) represent the revenue arising from setting up short positions. The second terms are the expected covering costs of the short position in the benchmark economy, i.e., the products of the probability of accepting the investment opportunity and the expected firm value if the manager invests. While the second term decreases the uninformed speculator's expected trading profit, it increases the informed speculator's expected trading profit. This is because the informed has an information advantage over the uninformed speculator. Given this information advantage, the informed short sellers are driven out of the market slower than the manipulative short sellers.

We now examine the effects of short selling cost on the information content of order flows. Table 3.5 compares the probabilities of a given signal conditional on the order flow in the costly short selling model with those of the benchmark. Not surprisingly, the informativeness of the no trade ($Q_1 = 0, Q_2 = 0$) is the same as the benchmark, since the market maker and the firm manager do not learn anything with zero order imbalance. When the short-sale cost is intermediate and *NMS* equilibrium holds, we observe an improvement in the information content of negative order flow in the costly short selling setting (α) compared to the benchmark ($\frac{\alpha}{2-\alpha}$). This occurs because *NMS* equilibrium eliminates the manipulative short selling, but does not impact the informed short selling.

When the short-sale cost is relatively high and *RINMS* equilibrium holds, the price efficiency is deteriorated in $t = 1$ and improved in $t = 2$. It deteriorates in $t = 1$ because the informativeness of the positive order flow is reduced (due to the positive possibility that the negatively informed speculator can generate $Q_1 = 1$),

Table 3.5: Information content of order flows in the costly short selling setting.
This table shows the information content of order flows in the costly short selling equilibriums: "No manipulative short" (*NMS*) equilibrium and "Reduced informed and no manipulative short" (*RINMS*) equilibrium, and compares them with that of the benchmark equilibrium. The "Short-sale ban" (*SSB*) equilibrium has been discussed before and is thus neglected here. The information content of positive order flow is given by $1 - \Pr(s = l|Q_1 > 0)$, the information content of no trade is given by $1 - \Pr(s = l|Q_1 = 0)$, and the information content of negative order flow is given by $\Pr(s = l|Q_1 < 0)$. No-trade events have the same information content in all four equilibriums. The *NMS* equilibrium (resp. the *SSB* equilibrium) has more informative (resp. less informative) order flows than the benchmark equilibrium. The *RINMS* equilibrium is associated with an reduction in informativeness in $t = 1$, and an improvement in informativeness in $t = 2$.

	Order flow informativeness in $t = 1$		
	Positive order flow $1 - \Pr(s = l\|Q_1 > 0)$	No-trade $1 - \Pr(s = l\|Q_1 = 0)$	Negative order flow $\Pr(s = l\|Q_1 < 0)$
Benchmark equilibrium	1	$1 - \frac{\alpha}{2}$	$\frac{\alpha}{2-\alpha}$
NMS equilibrium	1	$1 - \frac{\alpha}{2}$	α
RINMS equilibrium	$\frac{2}{2+\alpha}$	$1 - \frac{\alpha}{2}$	$\frac{\alpha}{2-\alpha}$
	Order flow informativeness in $t = 2$		
	Positive order flow $1 - \Pr(s = l\|Q_1 = 0, Q_2 > 0)$	No-trade $1 - \Pr(s = l\|Q_1 = 0, Q_2 = 0)$	Negative order flow $\Pr(s = l\|Q_1 = 0, Q_2 < 0)$
Benchmark equilibrium	1	$1 - \frac{\alpha}{2}$	$\frac{\alpha}{2-\alpha}$
NMS equilibrium	1	$1 - \frac{\alpha}{2}$	α
RINMS equilibrium	1	$1 - \frac{\alpha}{2}$	α

whereas the informativeness of the no trade and negative order flows remain the same as the benchmark. It improves in $t = 2$ because the informativeness of the negative order flow increases due to eliminated manipulative shorting, whereas the informativeness of the no trade and positive order flows remain the same as the benchmark. When the short-sale cost is extremely high, the *SSB* equilibrium is associated with an unambiguous drop in the price efficiency as in Subsection 3.4.1.

The changes in the information content of order flows and the stock price efficiency translate into changes in the real investment efficiency of the firm due to the managerial learning from the stock price. Corollary 3.9 shows the expected firm value given all trading game participants' strategies in the presence of a linear short selling cost and how it compares to the benchmark model.

Corollary 3.9. *(i) In the NMS equilibrium, the real efficiency (i.e., the expected value of the firm) is given by*

$$RE_{NMS} = \frac{\alpha V^+}{2} + \frac{\alpha V^-}{18} + \frac{5(1-\alpha)\bar{V}}{9}. \tag{3.11}$$

(ii) The NMS equilibrium always improves the real efficiency.
(iii) In the RINMS equilibrium, the real efficiency is given by

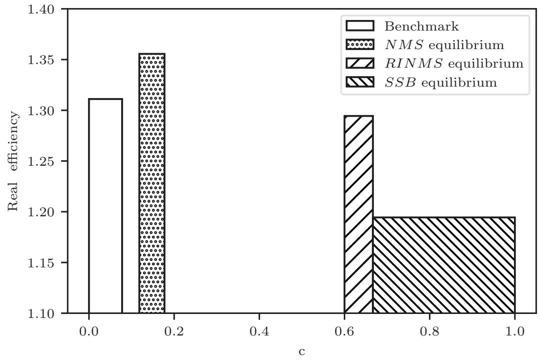
$$RE_{RINMS} = \frac{\alpha V^+}{2} + \frac{\alpha V^-}{9} + \frac{4(1-\alpha)\bar{V}}{9}. \tag{3.12}$$

(iv) The RINMS equilibrium improves the real efficiency if and only if the condition $\frac{\alpha V^-}{18} + \frac{3(1-\alpha)\bar{V}}{9} > 0$ *holds.*

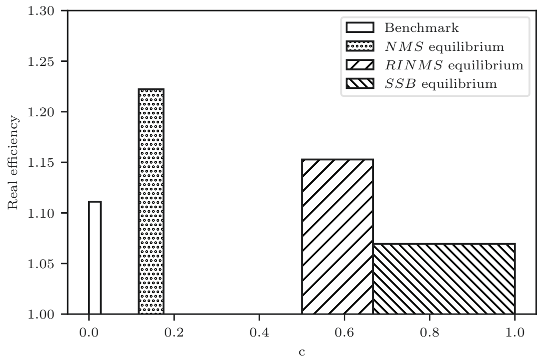
Corollary 3.9 suggests that the *NMS* equilibrium always improves the real efficiency because the price efficiency always improves compared to the benchmark due to the elimination of manipulative short selling. Thus, the manager learns more from the stock price, improving the quality of the investment decision. The corollary also suggests that the *RINMS* equilibrium may improve or deteriorate the real efficiency depending on whether the reduced information content of positive Q_1 or improved information content of negative Q_2 has a dominating impact on the investment decision.

Figure 3.2 provides a numerical example of the real efficiency in different equilibriums. Panel (A) illustrates four equilibriums when $\alpha = 0.9$, $V^+ = 3$, and $V^- = -1$, whereas Panel (B) illustrates the same when $\alpha = 0.75$, $V^+ = 3$, and

$V^- = -1$. In both panels, the *NMS* equilibrium has a higher real investment efficiency and the *SSB* equilibrium has a lower real investment efficiency compared to the benchmark. Panel (A) (resp. Panel (B)) illustrates the case in which the *RINMS* equilibrium improves (resp. deteriorates) the real investment efficiency.



(A)



(B)

Figure 3.2: Real efficiency in the costly short selling setting.
This figure illustrates numerical examples of the real efficiency in the costly short selling equilibriums: (i) Benchmark equilibrium, (ii) No manipulative shorting (*NMS*) equilibrium, (iii) Reduced informed and no manipulative shorting (*RINMS*) equilibrium, and (iv) Short-sale ban (*SSB*) equilibrium. Panel (A) illustrates the real efficiency when $\alpha = 0.9$, $V^+ = 3$, and $V^- = -1$, and Panel (B) illustrates the same when $\alpha = 0.75$, $V^+ = 3$, and $V^- = -1$.

3.4.3 Coordinated noise buys

We now consider the model in which the noise trader submits $\{1, 0, -1\}$ with the probability of $\{1/3, 1/3, 1/3\}$ in $t = 1$ as in the benchmark model and submits the same orders with the probability of $\{(1+\delta)/3, 1/3, (1-\delta)/3\}$ in $t = 2$, where $0 \leqslant \delta \leqslant 1$. Now, the noise trader in $t = 2$ is more likely to be a buyer than a seller, and δ captures the intensity of coordinated noise buy. This means that a short position in $t = 1$ bears an additional risk of coordinated noise buy in $t = 2$.

We first get an intuitive understanding of how an increased probability of noise buys influences the speculators' expected profits. Intuitively, increased buying pressure should push up the overall prices, and decrease the managers' ex-ante probability of observing negative order flows, leading to an increase in the manager's propensity to invest in the project. If the speculator is a manipulative short seller, it means a correction of underinvestment and an improvement in fundamentals, resulting in a higher cost to cover the manipulative short. If the speculator is an informed short seller, it means an overinvestment and a deterioration in fundamentals, resulting in a lower cost to cover the informed short. That is to say, an increased probability of noise buys reduces the profitability of the manipulative shorts but increases that of the informed shorts. More formally, Lemma 3.10 characterises the best responses of speculators.

Lemma 3.10. *In the presence of coordinated noise buys, suppose that positively informed speculator implements a buy – buy strategy, the following holds in t = 2:*

(i) When Q_1 perfectly reveals a speculator's information or reveals that he is not informed of the high state, then the speculator is indifferent between buy, sell, and no trade.

(ii) When Q_1 reveals that the speculator is not informed of the low state, then the uninformed speculator sells in $t = 2$.

(iii) When Q_1 reveals that the speculator is not uninformed, then the negatively informed speculator has a best response in $t = 2$ of selling.

(iv) When Q_1 does not reveal the speculator's type, the negatively informed speculator has a best response in $t = 2$ of selling; the uninformed speculator does not buy in $t = 2$, and does not trade in $t = 2$ if he does not trade in $t = 1$.

Given the best responses characterised in Lemma 3.10, we obtain the equilibrium as follows. First, we replace the subgame in $t = 2$ with the corresponding best responses and pin down several candidate equilibriums. Second, we exclude the candidate equilibriums that have a profitable deviation for the speculator regardless of the value of δ. Finally, we calculate the range of δ for the remaining candidate equilibriums by imposing that the given equilibrium strategy is more profitable than any possible alternative strategy. When the positively informed speculator implements a buy – buy strategy and $0 \leqslant \delta \leqslant 1$, three equilibrium outcomes can arise:

- Benchmark equilibrium: the negatively informed speculator as well as the uninformed speculator all sell in both trading rounds.
- Reduced informativeness and manipulative shorting (*RIMS*) equilibrium: the negatively informed speculator as well as the uninformed speculator all sell in both trading rounds, but the information content of a no-trade event is lower than the benchmark equilibrium.
- No manipulative shorting (*NMS*) equilibrium: the speculator sells in both trading rounds when he receives $s = l$ and the speculator does not trade in both trading rounds if $Q_1 = 0$ (does not trade in $t = 1$ and sells in $t = 2$ if $Q_1 = 1$) when he receives $s = \emptyset$.

Proposition 3.11 shows how equilibrium outcomes and speculator behaviours vary with noise trader buying intensity δ. Threshold values of noise trader buying intensity in the proposition are:

$$\delta_1 = \frac{2\alpha V^+ - V^+ + V^-}{V^+ + \alpha V^+ - V^-}, \tag{3.13}$$

$$\delta_2 = -\frac{\alpha - \alpha\sqrt{G} + \alpha\eta + 2\alpha^2\eta + 2\alpha^2 + 4}{2\left(3\alpha + 3\alpha\eta - \alpha^2\eta - \alpha^2 + 2\right)}, \tag{3.14}$$

$$\delta_3 = \frac{3\alpha\eta - 2\eta - 2\alpha + 2\alpha^2\eta + 5\alpha^2 + \sqrt{E}}{6\alpha^2\eta - 2\alpha\eta + 6\alpha^2}, \tag{3.15}$$

$$\delta_4 \equiv \frac{\sqrt{\alpha\left(17\alpha + 8\eta + 18\alpha\eta + 5\alpha\eta^2 + 4\eta^2\right)} - \alpha - \alpha\eta}{4\alpha + 2\alpha\eta}. \tag{3.16}$$

where η is a function of V^+ and V^-, and G and E are functions of η and informed fraction α, and are expressed as follows

$$V^+ \equiv -(1+\eta)V^-, \tag{3.17}$$

$$G = (1+\eta)\left(25\eta - 4\alpha - 4\alpha\eta + 4\alpha^2\eta + 4\alpha^2 + 49\right), \tag{3.18}$$

$$\begin{aligned} E = {} & 16\alpha^4\eta^2 + 56\alpha^4\eta + 49\alpha^4 + 8\alpha^3\eta^2 - 10\alpha^3\eta \\ & - 44\alpha^3 - 23\alpha^2\eta^2 + 48\alpha^2\eta + 4\alpha^2 - 4\alpha^2\eta + 8\alpha\eta + 4. \end{aligned} \tag{3.19}$$

Proposition 3.11. *Let δ_1, δ_2, δ_3, and δ_4 be given respectively by Eqs.(3.13), (3.14), (3.15), and (3.16). The trading game has the following equilibria in which the positively informed speculator buys in $t = 1$ and buys again in $t = 2$ if his type is not revealed:*

(i) *When $\delta = 0$, the only equilibrium is the benchmark equilibrium.*

(ii) *When $0 < \delta < \min\{\delta_1, \delta_2\}$, the only equilibrium is the RIMS equilibrium.*

(iii) *When $\delta > \max\{\delta_3, \delta_4\}$, the only equilibrium is the NMS equilibrium.*

Proposition 3.11 shows that when the intensity of coordinated noise buy is sufficiently low, $0 < \delta < \min\{\delta_1, \delta_2\}$, the *RIMS* equilibrium that we obtain is similar to the benchmark equilibrium, in which both the negatively informed speculator who knows $s = l$ and the uninformed speculator who knows $s = \emptyset$ sell in $t = 1$ and $t = 2$ if his type is not revealed. The only difference between the *RIMS* and the benchmark equilibrium is that the information content of a no-trade event in the *RIMS* is lower than the benchmark equilibrium.[9] This occurs because the increase in noise buys disguises the negatively informed speculator's sell orders – we refer to this effect as the the "*order flow disguising effect*". In the benchmark equilibrium, after observing no trade, $\{Q_1 = 0,\ Q_2 = 0\}$, the probability of $s = l$ is given by $\alpha/2$, whereas in the *RIMS* equilibrium it is given by $\alpha k/2$, where

$$k = \frac{1+\delta}{1+\delta(1-\alpha)} > 1. \tag{3.20}$$

Therefore, the informativeness of no trade, $\{Q_1 = 0, Q_2 = 0\}$, is reduced from $1-\alpha/2$ in the benchmark equilibrium to $1-\alpha k/2$ in the *RIMS* equilibrium. Since $\partial k/\partial\delta > 0$, the larger the intensity of coordinated noise buy δ is, the stronger the order flow disguising effect is in the *RIMS* equilibrium.

Proposition 3.11 also shows that when the intensity of coordinated noise buy is sufficiently high, $\delta > \max\{\delta_3, \delta_4\}$, the *NMS* equilibrium applies. In the *NMS* equilibrium, the negatively informed speculator still submits sell – sell orders, whereas the strategy of not trading in $t = 1$, and selling (resp. not trading) in $t = 2$ if $Q_1 = 1$ (resp. $Q_1 = 0$) is observed for the uninformed speculator. It is not surprising that increased intensity of coordinated noise buy eliminates only manipulative shorts, but not the informed shorts. This is because the expected profits of a sell –

[9] The information content of order flows other than $\{Q_1 = 0, Q_2 = 0\}$ in the *RIMS* equilibrium remains the same as the benchmark equilibrium, and there are no changes in the speculator's equilibrium trading strategy and price efficiency.

sell strategy by the uninformed and negatively informed speculator is, respectively, given by

$$P_{s=\emptyset} = \underbrace{\frac{1}{3}P_1(0) + \frac{1}{9}P_2(0,0)}_{\textit{Shorting proceeds}} - \underbrace{\frac{2}{9}\bar{V}}_{\textit{Disadvantage}} - \underbrace{\frac{2\delta}{3}\bar{V}}_{\substack{\textit{Covering cost}\\ \textit{increase}}}, \tag{3.21}$$

$$P_{s=l} = \underbrace{\frac{1}{3}P_1(0) + \frac{1}{9}P_2(0,0)}_{\textit{Shorting proceeds}} - \underbrace{\frac{2}{9}V^-}_{\textit{Advantage}} - \underbrace{\frac{2\delta}{3}V^-}_{\substack{\textit{Covering cost}\\ \textit{reduction}}}. \tag{3.22}$$

The shorting proceeds term is computed using the equilibrium prices reported in Table 3.6. Both the shorting proceeds and the information advantage (resp. disadvantage) term for the informed (resp. uninformed) short sellers are as defined in Subsection 3.4.2. The third term in Eqs.(3.21)-(3.22) represents the amount the short seller pays to close out his existing short positions due to the increase in δ. Since $\partial Shorting\ proceeds/\partial\delta < 0$ and covering cost for the uninformed increases in δ, $\partial P_{s=\emptyset}/\partial\delta < 0$. That means an increase in the intensity of coordinated noise buys always reduces the profitability of manipulative short selling. However, depending on the relative magnitudes of $\partial Shorting\ proceeds/\partial\delta$ and the covering cost reduction, the expected profit of the negatively informed speculator can be either increasing or decreasing in δ.

Table 3.7 compares the information content of order flows in different equilibriums. The informativeness of no-trade in the *NMS* equilibrium, $1 - \alpha(1+\delta)/2$, is still lower than that of the benchmark, $1 - \alpha/2$. The order flow disguising effect in the *NMS* equilibrium is still increasing in δ as in the *RIMS* equilibrium. Different from the *RIMS* equilibrium, however, the *NMS* equilibrium features an increased informativeness of the negative order flows. This is because of a feedback loop between the financial market and the manager's decision to invest or not. When the speculator is uninformed of the economic state, a relatively high noise buying intensity δ leads to a feedback from the market to the manager to correct under-investment due to the fewer occurrences of negative order flows and the feedback from the manager to the market to increase the cost to cover the manipulative shorts due to improved fundamentals and, thus, a reduction in manipulative shorts

Table 3.6: Equilibrium strategies and prices in the coordinated noise buys setting.
This table reports the equilibrium strategies of the speculator and equilibrium prices of the market maker in the coordinated noise buys setting. Panel (A) and (C) show the strategies of the speculator in $t = 1$ and 2 in the "Reduced informed and no manipulative short" (*RIMS*) equilibrium and "No manipulative short" (*NMS*) equilibrium, respectively. Panel (B) and (D) show the prices quoted by the market maker in $t = 1$ and 2 in the *RIMS* equilibrium and *NMS* equilibrium, respectively.

Panel (A): Strategies in the *RIMS* equilibrium

			Equilibrium strategy in $t=1$		
$u_1(s=h)$			1		
$u_1(s=l)$			-1		
$u_1(s=\emptyset)$			-1		
			Equilibrium strategy in $t=2$		
	$Q_1=-2$	$Q_1=-1$	$Q_1=0$	$Q_1=1$	$Q_1=2$
$u_2(s=h)$	Not applicable	Not applicable	1	$\{-1,0,1\}$	$\{-1,0,1\}$
$u_2(s=l)$	$\{-1,0,1\}$	$\{-1,0,1\}$	-1	Not applicable	Not applicable
$u_2(s=\emptyset)$	$\{-1,0,1\}$	$\{-1,0,1\}$	-1	Not applicable	Not applicable

Panel (B): Prices in the *RIMS* equilibrium

Round 1 of trading	Round 2 of trading
$P_1(0) = \frac{\alpha}{2}V^+ + \frac{(1+\delta)(1-\alpha)}{3}\bar{V} + \frac{(1+\delta)\alpha}{6}V^-$	$P_2(0,0) = \frac{(1-\delta)\alpha V^+ + (1+\delta)\alpha V^- + 2(1+\delta)(1-\alpha)\bar{V}}{1+\alpha\delta-\delta}$
$P_1(1) = P_1(2) = V^+$	$P_2(0,1) = P_2(0,2) = V^+$
$P_1(-1) = P_1(-2) = 0$	$P_2(0,-1) = P_2(0,-2) = 0$
	$P_2(1,\cdot) = P_2(2,\cdot) = V^+$
	$P_2(-1,\cdot) = P_2(-2,\cdot) = 0$

Table 3.6: Equilibrium strategies and prices in the coordinated noise buys setting (continued).

Panel (C): Strategies in the *NMS* equilibrium

	Equilibrium strategy in $t=1$				
$u_1(s=h)$			1		
$u_1(s=l)$			-1		
$u_1(s=\emptyset)$			0		
	Equilibrium strategy in $t=2$				
	$Q_1=-2$	$Q_1=-1$	$Q_1=0$	$Q_1=1$	$Q_1=2$
$u_2(s=h)$	Not applicable	Not applicable	1	1	$\{-1,0,1\}$
$u_2(s=l)$	$\{0,1\}$	$\{0,1\}$	-1	Not applicable	Not applicable
$u_2(s=\emptyset)$	Not applicable	$\{0,1\}$	0	-1	Not applicable

Panel (D): Prices in the *NMS* equilibrium

Round 1 of trading	Round 2 of trading
$P_1(0)=\frac{\alpha}{2}V^+ + \frac{(2+\delta)(1-\alpha)}{3}\bar{V} + \frac{(1+\delta)\alpha}{6}V^-$	$P_2(0,0)=\frac{(1-\delta)\alpha}{2}V^+ + \frac{(1+\delta)\alpha}{2}V^- + (1-\alpha)\bar{V}$
$P_1(1)=\frac{\alpha}{2-\alpha}V^+ + \frac{2(1-\alpha)}{2-\alpha}\bar{V}$	$P_2(0,1)=\frac{\alpha V^+ + 2(1-\alpha)(1+\delta)\bar{V}}{2-\alpha+2\delta-2\alpha\delta}$
$P_1(2)=V^+$	$P_2(0,2)=V^+, P_2(0,-1)=P_2(0,-2)=0$
$P_1(-2)=P_1(-1)=0$	$P_2(1,0)=\frac{\alpha(1-\delta)V^+ + 2(1-\alpha)(1+\delta)\bar{V}}{2-\alpha+2\delta-3\alpha\delta}$
	$P_2(1,1)=P_2(1,2)=V^+, P_2(1,-1)=P_2(1,-2)=\bar{V}$
	$P_2(2,\cdot)=V^+$
	$P_2(-2,\cdot)=P_2(-1,\cdot)=0$

Table 3.7: Information content of order flows in the coordinated noise buys setting.
This table shows the information content of order flows in the coordinated noise buys equilibriums: "Reduced informativeness and no manipulative short" (*RIMS*) equilibrium and "No manipulative short" (*NMS*) equilibrium, and compares them with that of the benchmark equilibrium. The information content of positive order flow is given by $1 - \Pr(s = l|Q_1 > 0)$, the information content of no trade is given by $1 - \Pr(s = l|Q_1 = 0)$, and the information content of negative order flow is given by $\Pr(s = l|Q_1 < 0)$. Positive order flows have the same information content in all three equilibriums. The *RIMS* equilibrium results in less informative order flows than the benchmark. The *NMS* equilibrium is associated with more (resp. less) informative negative order flows (resp. no-trade events).

	Order flow informativeness in $t = 1$		
	Positive order flow $1 - \Pr(s = l\|Q_1 > 0)$	No-trade $1 - \Pr(s = l\|Q_1 = 0)$	Negative order flow $\Pr(s = l\|Q_1 < 0)$
Benchmark equilibrium	1	$1 - \frac{\alpha}{2}$	$\frac{\alpha}{2-\alpha}$
RIMS equilibrium	1	$1 - \frac{\alpha}{2}$	$\frac{\alpha}{2-\alpha}$
NMS equilibrium	1	$1 - \frac{\alpha}{2}$	α
	Order flow informativeness in $t = 2$		
	Positive order flow $1 - \Pr(s = l\|Q_1 = 0, Q_2 > 0)$	No-trade $1 - \Pr(s = l\|Q_1 = 0, Q_2 = 0)$	Negative order flow $\Pr(s = l\|Q_1 = 0, Q_2 < 0)$
Benchmark equilibrium	1	$1 - \frac{\alpha}{2}$	$\frac{\alpha}{2-\alpha}$
RIMS equilibrium	1	$1 - \frac{\alpha(1+\delta)}{2(1+\delta-\delta\alpha)}$	$\frac{\alpha}{2-\alpha}$
NMS equilibrium	1	$1 - \frac{\alpha(1+\delta)}{2}$	$\frac{2\alpha-\alpha\delta}{2+\alpha\delta-2\delta}$

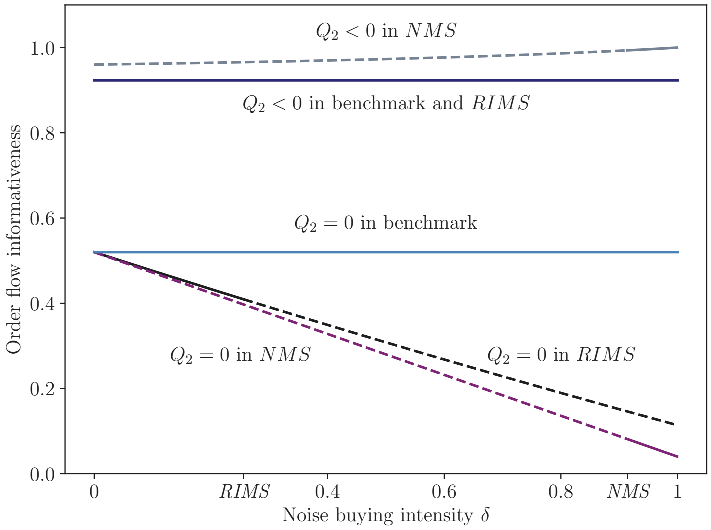


Figure 3.3: Informational content of no trade and negative order flows with respect to the intensity of noise buys.
This figure provides a numerical example of order flow informativeness in $t = 2$ when $\alpha = 0.96$ and $V^+ = -3V^-$. The light blue line is the informativeness of no-trade ($Q_2 = 0$) in the benchmark equilibrium. The dashed black line is the informativeness of no-trade ($Q_2 = 0$) in the "Reduced informed and manipulative short" equilibrium. The dashed purple line is the informativeness of no-trade ($Q_2 = 0$) in the "No manipulative short" equilibrium. The dark blue line represents the informativeness of negative order flow ($Q_2 < 0$) in the benchmark and the *RIMS* equilibriums. The dashed grey line is the informativeness of negative order flow ($Q_2 < 0$) in the *NMS* equilibrium.

and a rise in the information content of negative order flows. Such a cost increase to cover the short sale is specific to the uninformed speculator – we therefore refer to this effect as the "*uninformed-specific short-sale cost*". Figure 3.3 illustrates a numerical example of the information content of order flows in $t = 2$ when $\alpha = 0.96$ and $V^+ = -3V^-$.

The discussion about the information content of order flows and consequently the manager's ability to learn from prices have implications for the real investment efficiency. For the *RIMS* equilibrium, the informational efficiency is always worse than the benchmark equilibrium. Accordingly, one may reasonably expect a decrease in the quality of investment decision and the real investment efficiency in the *RIMS* equilibrium. For the *NMS* equilibrium, however, the real investment efficiency can increase or decrease relative to the benchmark equilibrium because the informativeness of negative order flows increases (i.e., uninformed-specific short-sale cost), whereas the informativeness of a no-trade event decreases (i.e., order flow disguising effect). Corollary 3.12 formally shows the real investment efficiency in different equilibriums.

Corollary 3.12. *(i) In the RIMS equilibrium, the real investment efficiency (i.e, the expected firm value) is given by*

$$RE_{RIMS} = \frac{\alpha V^+}{2} + \frac{\alpha(1+\delta)V^-}{18} + \frac{(1-\alpha)(1+\delta)\bar{V}}{9}. \tag{3.23}$$

(ii) The RIMS equilibrium always harms the real efficiency.
(iii) In the NMS equilibrium, the real investment efficiency is given by

$$RE_{NMS} = \frac{\alpha V^+}{2} + \frac{\alpha(1+\delta)V^-}{18} + \frac{(1-\alpha)(5+\delta)\bar{V}}{9}. \tag{3.24}$$

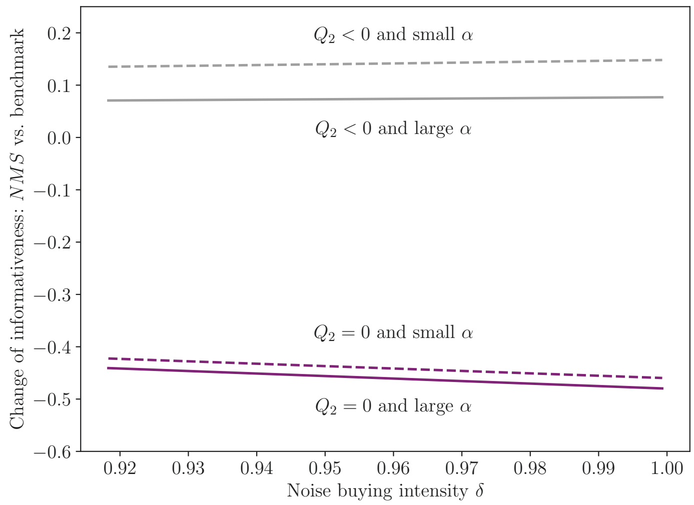
(iv) The NMS equilibrium improves the real efficiency if and only if

$$\frac{(1-\alpha)(4+\delta)\bar{V}}{9} + \frac{\alpha\delta(\bar{V}-U)}{18} > 0, \tag{3.25}$$

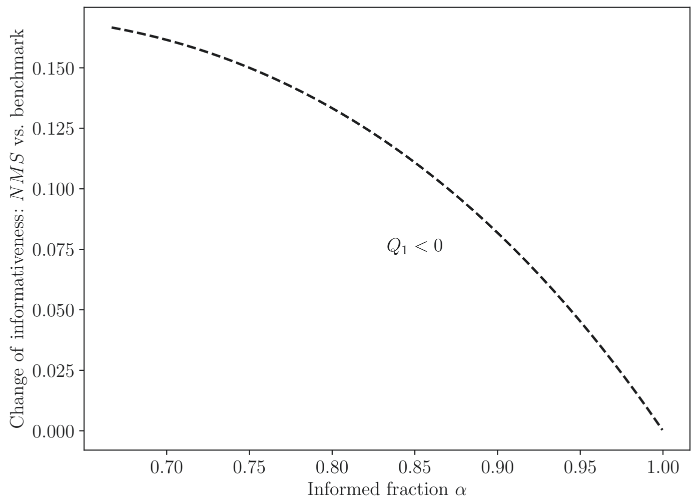
where $U = V^+ - \bar{V}$ is the measure of uncertainty.
(v) The real efficiency change in the NMS equilibrium relative to the benchmark equilibrium (i.e., $RE_{NMS} - RE_{benchmark}$) decreases in the fraction α of informed speculator, increases in the ex-ante NPV of the project ($\bar{V}$), and decreases in the uncertainty about the profitability of the investment (U).

Corollary 3.12 shows that the *RIMS* equilibrium ($\delta < \min\{\delta_1, \delta_2\}$) always harms real efficiency. This occurs as the *RIMS* equilibrium is associated with a decrease in the price informativeness due to the order flow disguising effect and the resulting overinvestment after no-trade events. The corollary derives the condition under which the uninformed-specific short-sale cost dominates the order flow disguising effect in the *NMS* equilibrium ($\delta > \max\{\delta_3, \delta_4\}$), resulting in the improvementin the real efficiency. The corollary additionally shows for what types of conditions, the uninformed-specific short-sale cost dominates the order flow disguising effect in the equilibrium. The uninformed-specific short-sale cost is more likely to dominate the order flow disguising effect when (i) the fraction of uninformed speculator ($1-\alpha$) is large, (ii) the project has a large ex-ante NPV ($\bar{V}$), and (iii) the uncertainty about the profitability of the investment (U) is small.

First, when the fraction of the uninformed speculator is large (i.e., α is small), the benchmark equilibrium is associated with more manipulative shorting and the improvement in the information content of negative order flows due to the uninformed-specific short-sale cost is thus stronger than the deterioration in the



(A)



(B)

Figure 3.4: The impact of α on the information content of orders in the *NMS* equilibrium.
Panel (A) shows the change in the information content of no trade and negative order flows in the "No manipulative short" equilibrium relative to the benchmark equilibrium in $t = 2$. The parameter setting of the solid lines are $\alpha = 0.96$ and $V^+ = -3V^-$; the parameter setting of the dashed lines are $\alpha = 0.94$ and $V^+ = -3V^-$. Panel (B) shows the change in the information content of the negative order flow in the *NMS* equilibrium relative to the benchmark in $t = 1$ when $V^+ = -3V^-$.

information content of no trade due to the order flow disguising effect (see Figure 3.4). The grey dashed line in the figure is higher than the grey solid line, implying that a smaller α enlarges the improvement in the information content of $Q_2 < 0$ in the *NMS* equilibrium relative to the benchmark equilibrium. The purple dashed line is above the purple solid line, implying that a smaller α weakens the decrease in the information content of $Q_2 = 0$ in the *NMS* equilibrium relative to the benchmark equilibrium. Also, the black dashed line in Panel (B) is downward-sloping, implying that enlarges the improvement in the information content of $Q_1 < 0$ in the *NMS* equilibrium relative to the benchmark equilibrium. Therefore, firms with a high uninformed trading fraction or a small informed trading fraction tend to improve the quality of the manager's investment decision following the transitory retail buying frenzy.

Second, the investment project having a higher ex-ante NPV means a higher loss in the expected firm value associated with successful manipulative short selling. Thus, firms with high growth opportunities also tend to improve the quality of investment decisions following a retail buying frenzy. Finally, managers tend to learn more from the financial markets and improve real investment efficiency during the normal conditions (i.e., when the uncertainty about the profitability of the investment is small).

3.5 Conclusion

In this paper, we develop a model in the presence of both manipulative and informed short sellers to explain how a transitory retail buying frenzy (as short-sale friction) impacts the market and real investment efficiency. We use three different extensions of an otherwise Goldstein and Guembel (2008) model to conduct our analysis. We first introduce a short-sale ban to the model and show that a short-sale ban always harms the real investment efficiency. We then introduce a short-sale cost and show that an intermediate level of short-sale cost improves the investment efficiency, whereas a relatively high cost on short selling may improve or deteriorate the investment efficiency. An extremely large short-sale cost always jams the managers' learning from the stock price and harms the quality of his investment decisions as in the short-sale ban.

We also model a retail buying frenzy as asymmetric noise trading such that noise buys are more likely than noise sells. We show that asymmetric noise trading gives rise to two opposing effects on the informativeness of order flows: the order flow disguising effect and uninformed-specific short-sale cost. The order flow disguising effect harms the investment efficiency, whereas the uninformed-specific short-sale cost improves it. We show that uninformed-specific short-sale cost is more likely to dominate the order flow disguising effect so that the investment efficiency is improved when (i) the fraction of the uninformed speculator is large, (ii) the project has a large ex-ante NPV, and (iii) the uncertainty about the profitability of the investment is small.

Our analysis offers insights into the regulation of short selling as well as meme investing. Regulation of short-selling is a delicate balancing activity for regulators since an extremely high short-sale cost, driving out manipulative and informed shorting altogether, deteriorates real efficiency. Different levels of short-sale cost can also be thought of as different intensities of price run-ups driven by the retail buying frenzies. An intermediate level of short-sale cost, corresponding to an intermediate increase in the stock price, improves the investment efficiency because it only deters manipulative shorting. These efficiency improvements in the short-sale cost model and the asymmetric noise trading model suggest that regulations investigating retail buying frenzies in meme stocks for potential involvement in pump and dump schemes and seeking to prevent such activities need to be carefully curated.

Appendix 3.1. Proofs

In this Appendix, we prove our main results. The proofs of Lemma 3.1 and Proposition 3.2 are omitted for brevity as they respectively follow from the proofs of Lemma 3.7 and Proposition 3.8 when the cost of short sales is zero. Throughout the derivations, the possible information sets after $t = 1$ order flow Q_1 are:

(i). Q_1 perfectly reveals the speculator's type;

(ii). Q_1 reveals that he is not negatively informed;

(iii). Q_1 reveals that he is not positively informed;

(iv). Q_1 reveals that he is not uninformed;

(v). Q_1 does not reveal any new information.

Proof of Corollary 3.3. The expected firm value follows from the expected firm value of each type of speculator in equilibrium. The benchmark equilibrium strategies of speculators who are not positively informed are sell – sell strategies, and the benchmark equilibrium strategy of the positively informed speculator is a buy – buy strategy. For the negatively informed speculator implementing the sell – sell strategy, the expected firm value (before trading occurs) is $\frac{V^-}{9}$. For the uninformed speculator who has a sell – sell strategy, the expected firm value is $\frac{\bar{V}}{9}$. For the positively informed speculator who has a buy – buy strategy, the expected firm value is V^+. Given the probability of the positively informed speculator and the probability of the negatively informed speculator are both $\frac{\alpha}{2}$, and that of the uninformed speculator is $1 - \alpha$, the expected firm value (or the real efficiency) in the benchmark equilibrium follows as

$$RE_{benchmark} = \frac{\alpha V^+}{2} + \frac{\alpha V^-}{18} + \frac{(1-\alpha)\bar{V}}{9}. \tag{A3.1.1}$$

■

Proof of Lemma 3.4. We investigate the best responses of the negatively informed and uninformed speculators for the possible information sets after the order flow in $t = 1$.

(i). When Q_1 perfectly reveals the speculator's type, the price equals the speculator's expected firm value, and all actions generate zero profit. Hence, the speculator shows indifference between buying, selling (if he has an initial position), and not trading.

(ii). When Q_1 reveals that the speculator is not negatively informed, the firm always invests, and $t = 2$ strategy is independent of $t = 1$ order flow. Consequently, we only need to check profit in $t = 2$. Suppose that the uninformed speculator's strategy for this information set is to sell in $t = 2$. Then, his $t = 2$ trading profit is $\frac{1}{3}\left(P_2(\cdot,0) - \bar{V}\right) + \frac{1}{3}\left(P_2(\cdot,-1) - \bar{V}\right) + \frac{1}{3}\left(P_2(\cdot,-2) - \bar{V}\right)$. Substituting the prices $P_2(\cdot,0)$, $P_2(\cdot,-1)$, and $P_2(\cdot,-2)$ leads the uninformed speculator's profit in $t = 2$ to $\frac{\alpha}{2-\alpha}\frac{V^{+} - V^{-}}{6} > 0$. If he deviates from selling to not trading, his profit in $t = 2$ is 0. If he deviates from selling to buying, his profit in $t = 2$ is $\bar{V} - \left(\frac{1}{3}P_2(\cdot,0) + \frac{1}{3}P_2(\cdot,1) + \frac{1}{3}P_2(\cdot,2)\right) = \bar{V} - \frac{2}{3}V^{+} - \frac{1}{3}\frac{\alpha}{2-\alpha}V^{+} - \frac{1}{3}\frac{2(1-\alpha)}{2-\alpha}\bar{V} < 0$. Hence, he does not have an incentive to deviate.

(iii). When Q_1 reveals that the speculator is not positively informed, the firm's manager cancels the investment due to the expected NPV loss suggested by Eq.(3.1). Since the expected firm value and prices are 0 after any order flow realisation, the negatively informed and uninformed speculators are indifferent between all three actions.

(iv). When Q_1 reveals that the speculator is not uninformed, action in $t = 2$ will affect profit in $t = 1$ due to possible divestment. We therefore calculate the total profit rather than the profit in $t = 2$.

Let's assume that, within this information set, the strategy adopted by the uninformed speculator is to sell in $t = 2$. The negatively (resp. positively) informed speculator generates the order flow of $Q_2 = \{-2,-1,0\}$ (resp. $Q_2 = \{0,1,2\}$), and the market maker sets $P_2(\cdot,0) = \bar{V}$, $P_2(\cdot,1) = P_2(\cdot,2) = V^{+}$ and $P_2(\cdot,-1) = P_2(\cdot,-2) = 0$. For the negatively informed speculator, selling in $t = 2$ is associated with a total profit of $\frac{1}{3}P_2(\cdot,0) - P_1(\cdot) = \frac{1}{3}\bar{V} - P_1(\cdot)$. If he deviates to buying, the total profit is $2V^{-} - P_1(\cdot) - \frac{1}{3}P_2(\cdot,0) - \frac{1}{3}P_2(\cdot,-1) - \frac{1}{3}P_2(\cdot,-2) = 2V^{-} - P_1(\cdot) - \frac{1}{3}\bar{V}$. If he deviates to not trading, the total profit is $\frac{2}{3}V^{-} - P_1(\cdot)$. No deviation is superior to selling in $t = 2$.

Let's assume that, within this information set, the strategy adopted by the uninformed speculator is to not trade in $t = 2$ instead. The positively (resp.

negatively) informed speculator generates the order flow of $Q_2 = \{0, 1, 2\}$ (resp. $Q_2 = \{-1, 0, 1\}$), and the market maker sets $P_2(\cdot, 0) = P_2(\cdot, 1) = \bar{V}$, $P_2(\cdot, 1) = V^-$, and $P_2(\cdot, 2) = V^+$. The negatively informed speculator's total profit when not trading in $t = 2$ is $\frac{2}{3}V^- - P_1(\cdot)$. The deviation to selling gives rise to a total profit of $\frac{1}{3}P_2(\cdot, 0) - P_1(\cdot)$. $Q_2 = \{-1, -2\}$ is off-equilibrium and can lead to divestment. The deviation to buying gives rise to a total profit of $2V^- - \frac{1}{3}P_2(\cdot, 0) - \frac{1}{3}P_2(\cdot, 1) - \frac{1}{3}P_2(\cdot, 2) - P_1(\cdot) = \frac{5}{3}V^- - \frac{2}{3}V^+ - P_1(\cdot)$. Since $\frac{1}{3}\bar{V} - P_1(\cdot) > \frac{2}{3}V^- - P_1(\cdot) > \frac{5}{3}V^- - \frac{2}{3}\bar{V} - P_1(\cdot)$, it is profitable for the negatively informed speculator to deviate from not trading to selling in $t = 2$.

Finally, let's assume that the negatively informed speculator's strategy when reaching the information set is to buy in $t = 2$. Because period-2 order flow does not seperate the negatively informed speculator who knows $s = l$ from the positively informed speculator who knows $s = h$, $P_2(\cdot, 0) = P_2(\cdot, 1) = P_2(\cdot, 2) = \bar{V}$. The negatively informed speculator, by buying in $t = 2$, earns a total profit of $2V^- - \frac{1}{3}P_2(\cdot, 0) - \frac{1}{3}P_2(\cdot, 1) - \frac{1}{3}P_2(\cdot, 2) - P_1(\cdot) = \frac{3}{2}V^- - \frac{1}{2}V^+ - P_1(\cdot)$. If he deviates to selling, the total profit is $\frac{1}{3}P_2(\cdot, 0) - P_1(\cdot) = \frac{1}{3}\bar{V} - P_1(\cdot)$. If he deviates to not trading, the total profit is $V^- - P_1(\cdot)$. Since $\frac{1}{3}\bar{V} - P_1(\cdot) > V^- - P_1(\cdot) > \frac{3}{2}V^- - \frac{1}{2}V^+ - P_1(\cdot)$, both selling and not trading are profitable deviations for the negatively informed speculator who buys a unit of stock in $t = 2$. Consequently, at the information set of scenario (iv), the negatively informed speculator's best response in $t = 2$ is to sell.

(v). When Q_1 does not reveal any new information about the speculator's type, we split the analysis of the negatively informed speculator's best responses into two cases: (a) he buys in $t = 1$; (b) he does not trade in $t = 1$. In (a), following a similar argument as in scenario (iv), one can easily show the negatively informed speculator in $t = 2$ has a best response of selling. In (b), assume that instead of not trading, the negatively informed speculator's strategy is to buy in $t = 2$. He can't sell at this node because of no initial position. He then earns a total profit of $2V^- - \frac{1}{3}P_2(\cdot, 0) - \frac{1}{3}P_2(\cdot, 1) - \frac{1}{3}P_2(\cdot, 2) - P_1(\cdot) < 0$. By deviating and not trading in $t = 2$, he secures a total profit of 0. When the order flow Q_1 does not reveal the trader's type, a negatively informed speculator sells if he places a buy order in $t = 1$, or does not trade if he does not trade in $t = 1$. For the uninformed speculator, we show it is optimal for him to not trade in $t = 2$ if he does not trade in $t = 1$. Suppose instead, he buys in $t = 2$, his total profit is $\bar{V} - \frac{1}{3}P_2(\cdot, 0) - \frac{1}{3}P_2(\cdot, 1) - \frac{1}{3}P_2(\cdot, 2) =$

$\bar{V} - \frac{1}{3}\bar{V} - \frac{1}{3}\left(\frac{\alpha}{2-\alpha}V^{+} + \frac{2(1-\alpha)}{2-\alpha}\bar{V}\right) - \frac{1}{3}P_2(\cdot,1) < 0$, where $P_2(\cdot,1)$ equals (resp. is greater than) $\bar{V}$ if negatively informed speculator chooses to not trade (resp. sell) in $t=2$.

■

Proof of Proposition 3.5. Best responses from Lemma 3.1 pin down four candidate equilibriums in Table A3.1. Column 1 of the tables show speculator's types, and columns 2-3 show order flow in $t=1$ and $t=2$. We show that only candidate equilibriums 3 has no profitable deviations.

(a). In candidate equilibrium 1, we have the following prices $P_2(0,0) = P_2(1,0) = \bar{V}$, $P_2(0,-1) = P_2(1,-1) = 0$, $P_2(0,-2) = P_2(1,-2) = 0$, $P_2(0,1) = P_2(1,1) = \frac{\alpha}{2-\alpha}V^{+} + \frac{2-2\alpha}{2-\alpha}\bar{V}$ and $P_2(0,2) = P_2(1,2) = V^{+}$. Period-1 price $P_1(0)$ is expected period-2 prices:

$$\begin{aligned} P_1(0) &= \sum_{i=-2}^{2} \Pr(Q_2 = i \mid Q_1 = 0)\, P_2(0,i) \\ &= \frac{\alpha}{6}V^{+} + \frac{1}{3}\bar{V} + \left(\frac{1}{3} - \frac{\alpha}{6}\right)\left(\frac{\alpha}{2-\alpha}V^{+} + \frac{2-2\alpha}{2-\alpha}\bar{V}\right). \end{aligned} \tag{A3.1.2}$$

$P_1(1)$ equals $P_1(0)$, whereas $P_2(2,0) = \frac{\alpha}{2-\alpha}V^{+} + \frac{2(1-\alpha)}{2-\alpha}V^{-}$, $P_2(2,1) = P_2(2,2) = V^{+}$, and $P_2(2,-1) = P_2(2,-2) = 0$. Period 1 price for the order flow of $Q_1 = 2$ is $P_1(2) = \frac{1}{3}P_2(2,0) + \frac{2}{3}\frac{\alpha}{2-\alpha}V^{+}$. The buy – sell strategy profit of the speculator who is negatively informed can be calculated as follows: When $Q_1 = 0$, the negatively informed speculator who buys in $t=1$ and sells in $t=2$ has a total profit of $\frac{1}{3}P_2(0,0) + \frac{1}{3}P_2(0,-1) + \frac{1}{3}P_2(0,-2) - P_1(0) = -\frac{\alpha}{6}V^{+} - \left(\frac{1}{3} - \frac{\alpha}{6}\right)\left(\frac{\alpha}{2-\alpha}V^{+} + \frac{2-2\alpha}{2-\alpha}\bar{V}\right) < 0$. When $Q_1 = 1$, he has a total profit of $\frac{1}{3}P_2(1,0) + \frac{1}{3}P_2(1,-1) + \frac{1}{3}P_2(1,-2) - P_1(1) = -\frac{\alpha}{6}V^{+} - \left(\frac{1}{3} - \frac{\alpha}{6}\right)\left(\frac{\alpha}{2-\alpha}V^{+} + \frac{2-2\alpha}{2-\alpha}\bar{V}\right) < 0$. When $Q_1 = 2$, he has a total profit of $\frac{1}{3}P_2(2,0) + \frac{1}{3}P_2(2,-1) + \frac{1}{3}P_2(2,-2) - P_2(1) = \frac{1}{3}P_2(2,0) - \frac{1}{3}P_2(2,0) - \frac{2}{3}\frac{\alpha}{2-\alpha}V^{+} = -\frac{2}{3}\frac{\alpha}{2-\alpha}V^{+} < 0$. Therefore, the negatively informed speculator is willing to deviate to the action of not trading in both periods to secure a profit of 0, meaning that candidate equilibrium 1 cannot sustain.

(b). In candidate equilibrium 2, we have $P_2(1,0) = P_2(1,1) = \bar{V}$, $P_2(1,2) = V^{+}$, and $P_2(1,-1) = P_2(1,-2) = 0$. Period 1 price $P_1(1) = \frac{\alpha}{6}V^{+} + \frac{1}{3}\bar{V} + \frac{\alpha}{3}\bar{V}$. For $Q_1 = 0$, we have $P_2(0,0) = P_2(0,1) = \bar{V}$, $P_2(0,2) = V^{+}$, $P_2(0,-1) = 0$, and $P_2(0,-2) = \bar{V}$. Period 1 price $P_1(0) = \frac{2}{3}\bar{V} + \frac{\alpha}{6}V^+$. For $Q_1 = 2$, $P_2(2,0) = \frac{\alpha}{2-\alpha}V^+ + \frac{2(1-\alpha)}{2-\alpha}\bar{V}$, $P_2(2,1) = P_2(2,2) = V^+$, and $P_2(2,-1) = P_2(2,-2) = \bar{V}$. Period 1 price $P_1(2) = \frac{\alpha}{2-\alpha}V^+ + \frac{2(1-\alpha)}{2-\alpha}\bar{V}$. The buy – sell strategy profit of the uninformed speculator is as follows: When $Q_1 = 0$, the uninformed speculator who buys in $t = 1$ and sells in $t = 2$ has a total profit of $\frac{1}{3}P_2(0,-2) + \frac{1}{3}P_2(0,-1) + \frac{1}{3}P_2(0,0) - P_1(0) = \frac{1}{3}\bar{V} + 0 + \frac{1}{3}\bar{V} - \frac{2}{3}\bar{V} - \frac{\alpha}{6}V^+ = -\frac{\alpha}{6}V^+ < 0$. When $Q_1 = 1$, he has a total profit of $\frac{1}{3}P_2(1,-2) + \frac{1}{3}P_2(1,-1) + \frac{1}{3}P_2(1,0) - P_1(1)$. When $Q_1 = 2$, he has a total profit of $\frac{1}{3}P_2(2,-2) + \frac{1}{3}P_2(2,-1) + \frac{1}{3}P_2(2,0) - P_1(2) = -\frac{\alpha}{3}\bar{V} - \frac{\alpha}{6}V^+$ $= \frac{2}{3}\bar{V} - \frac{2}{3}\left(\frac{\alpha}{2-\alpha}V^+ + \frac{2(1-\alpha)}{2-\alpha}\bar{V}\right) < 0$. Therefore, the uninformed speculator has an incentive to not trading in both periods to secure a profit of 0, meaning that candidate equilibrium 2 cannot sustain.

(c). In candidate equilibrium 4, if the uninformed speculator implements a buy – buy strategy, he has a profit of $2\bar{V} - \frac{1}{2}P_2(0,0) - \frac{1}{3}P_2(0,1) - \frac{1}{3}P_2(0,2) - p_1(\cdot) = \frac{5}{3}\bar{V} - \frac{4-2\alpha}{3}\left(\frac{\alpha}{2-\alpha}V^+ + \frac{2-2\alpha}{2-\alpha}\bar{V}\right) - \frac{1}{3}\left(\alpha V^+ + (2-2\alpha)\bar{V}\right)$. If the uninformed speculator implements a buy – sell strategy, he obtains a profit of $0 - P_1(\cdot) < 0$. If the uninformed speculator implements a buy – no trade strategy, he obtains a profit of $\frac{2}{3}\bar{V} - P_1(\cdot) = \frac{\bar{V}}{2} - \left(\frac{\alpha}{2-\alpha}V^+ + \frac{2-2\alpha}{2-\alpha}\bar{V}\right)\left(\frac{1}{3} - \frac{\alpha}{6}\right) - \frac{\alpha}{6}V^+ < 0$. All traders buying in $t = 1$ cannot be an equilibrium, because the uninformed speculator is willing to to deviate to no trade in both periods to secure a profit of 0. Candidate equilibrium 3 is the only Nash equilibrium in which nobody has an incentive to deviate.

■

Proof of Corrollary 3.6. The equilibrium strategies in the short-sale ban setting of the speculators who are not positively informed are no trade – no trade strategies, and the positively informed speculator is a buy – buy strategy. For the negatively informed speculator that implements a no trade – no trade strategy, the condition expectation of the firm value is $\frac{4V^-}{9}$. For the uninformed speculator that implements a no trade – no trade strategy, the expected firm value is $\frac{4\bar{V}}{9}$. For the positively informed speculator that implements a buy – buy strategy, the conditional expectation of the firm value is V^+. Given that the probability of the positively informed speculator together with that of the negatively informed speculator both equal $\frac{\alpha}{2}$, and that of the uninformed speculator is $1-\alpha$, the expected firm value (or the real efficiency) in the short-sale ban equilibrium follows as

$$RE_{ssban} = \frac{\alpha V^+}{2} + \frac{2\alpha V^-}{9} + \frac{4(1-\alpha)\bar{V}}{9}. \qquad \text{(A3.1.3)}$$

Table A3.1. Candidate equilibriums in the short-sale ban setting

This table reports the speculators' strategies in the candidate equilibriums in the short-sale ban setting. Column 1 is the speculator's type, and columns 2-3 are the orders in $t = 1$ and $t = 2$. In the table, NT stands for not trade.

Candidate equilibrium 1

Speculator	$t = 1$ order	$t = 2$ order
$s = h$	Buys	Buys
$s = l$	Buys	Sells
$s = \varnothing$	NT	NT if not fully revealing

Candidate equilibrium 2

Speculator	$t = 1$ order	$t = 2$ order
$s = h$	Buys	Buys
$s = l$	NT	NT if not fully revealing
$s = \varnothing$	Buys	Sells

Candidate equilibrium 3

Speculator	$t = 1$ order	$t = 2$ order
$s = h$	Buys	Buys
$s = l$	NT	NT if Q_1 does not reveal that
$s = \varnothing$	NT	he is not $s = h$ speculator

Candidate equilibrium 4

Speculator	$t = 1$ order	$t = 2$ order
$s = h$	Buys	Buys
$s = l$	Buys	Sells
$s = \varnothing$	Buys	Buys, sells, or NT

It then follows from the difference of Eqs. (A3.1.3) and (A3.1.1) that the short-sale ban always harms real efficiency, since

$$RE_{ssban} - RE_{benchmark} = \frac{3(1-\alpha)\bar{V}}{9} + \frac{3\alpha V^-}{18} < 0. \tag{A3.1.4}$$

■

Proof of Lemma 3.7. Similar to the proof of Lemma 3.4, we show the best responses of the negatively informed and uninformed speculators for the possible information sets after the order flow in $t = 1$.

(i). When Q_1 perfectly reveals the speculator's type, the price equals the speculator's expected firm value, and all actions generate zero profit. Hence, the speculator shows indifference between buying and not trading.

(ii). When Q_1 reveals that the speculator is not negatively informed, the firm always invests, and $t = 2$ strategy is independent of $t = 1$. Consequently, we only need to check the profit in $t = 2$. Suppose that the uninformed speculator's strategy for this information set is to sell in $t = 2$. Trading profit earned in $t = 2$ by him is then given by $\frac{1}{3}\left(P_2(\cdot, 0) - \bar{V}\right) + \frac{1}{3}\left(P_2(\cdot, -1) - \bar{V}\right) + \frac{1}{3}\left(P_2(\cdot, -2) - \bar{V}\right) - c = \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{6} - c$. Deviation to not trading yields a profit of 0. Deviation to buying yields $\bar{V} - \left(\frac{1}{3}P_2(\cdot, 0) + \frac{1}{3}P_2(\cdot, 1) + \frac{1}{3}P_2(\cdot, 2)\right) = \bar{V} - \frac{1}{3}\left(\frac{\alpha}{2-\alpha}V^+ + \frac{2(1-\alpha)}{2-\alpha}\bar{V}\right) - \frac{2}{3}V^+ < 0$. The uninformed speculator does not deviate from selling as long as $c < a_2$, where

$$a_2 = \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{6}. \tag{A3.1.5}$$

Suppose that the uninformed speculator's strategy for this information set is to buy in $t = 2$. His profit in $t = 2$ is $\bar{V} - \left(\frac{1}{3}P_2(0, 0) + \frac{1}{3}P_2(0, 1) + \frac{1}{3}P_2(0, 2)\right) = \bar{V} - \frac{\alpha}{2-\alpha}V^+ - \frac{2(1-\alpha)}{2-\alpha}\bar{V}$. The uninformed speculator never buys under this scenario. Hence, when Q_1 reveals that the speculator is not informed of the low state, the uninformed speculator sells (resp. not trade) in $t = 2$ if $c < a_2$ (resp. $c > a_2$).

(iii). When Q_1 reveals that the speculator is not positively informed, both the actions of buying and not trading can generate a profit of 0 due to the cancellation of the investment. Negatively informed and uninformed speculators are indifferent between buy and no trade.

(iv). When Q_1 reveals that the speculator is not uninformed, we first consider the case that the negatively informed speculator buys in $t = 1$. Let's assume the negatively informed speculator's strategy for this information set is to sell in $t = 2$. He earns a gross trading profit of: $\frac{1}{3}P_2(\cdot, 0) - P_1(\cdot) - c = \frac{1}{3}\bar{V} - P_1(\cdot) - c$. Deviation to not trading leads to a gross trading profit of $\frac{2}{3}V^- - P_1(\cdot)$. Deviation to buying gives rise to a gross trading profit of $2V^- - P_1(\cdot) - \frac{1}{3}P_2(\cdot, 0) - \frac{1}{3}P_2(\cdot, 1) - \frac{1}{3}P_2(\cdot, 2) = \frac{11}{6}V^- - \frac{5}{6}V^+ - P_1(\cdot)$. He does not deviate from selling as long as $\frac{1}{3}\bar{V} - c > \frac{2}{3}V^-$, which is equivalent to $c < a_4$, where

$$a_4 = \frac{V^+}{6} - \frac{V^-}{2}. \tag{A3.1.6}$$

Let's assume the negatively informed speculator's strategy when reaching this information set is to buy in $t = 2$. His total trading profit is then $2V^- - P_1(\cdot) - \frac{1}{3}P_2(\cdot, 0) - \frac{1}{3}P_2(\cdot, 1) - \frac{1}{3}P_2(\cdot, 2) = \frac{3}{2}V^- - \frac{1}{2}V^+ - P_1(\cdot)$. Deviating to not trading gives rise to a profit of $\frac{2}{3}V^- - P_1(\cdot)$, which is always superior than buying in $t = 2$. Buying in $t = 2$ cannot be the best response for the negatively informed speculator who buys in $t = 1$. Let's assume his strategy when reaching this information set is to not trade in $t = 2$. His total trading profit is then given by $\frac{2}{3}V^- - P_1(\cdot)$. Deviation to selling results in a profit of $\frac{1}{3}\bar{V} - P_1(\cdot) - c$. Deviation to buying results in a profit of $2V^- - P_1(\cdot) - \frac{1}{3}P_2(\cdot, 0) - \frac{1}{3}P_2(\cdot, 1) - \frac{1}{3}P_2(\cdot, 2) = \frac{5}{3}V^- - \frac{2}{3}V^+ - P_1(\cdot)$. He does not deviate from not trading as long as $\frac{2}{3}V^- > \frac{1}{3}\bar{V} - c$, which is equivalent to $c > a_4$. Hence, when Q_1 reveals that the speculator is not uninformed, the negatively informed speculator who buys in $t = 1$ sells (resp. does not trade) in $t = 2$ if $c < a_4$ (resp. $c > a_4$).

Now, consider the case he does not trade in $t = 1$. The negatively informed speculator is comparing between buying in $t = 2$ with a profit of $V^- - \frac{1}{3}P_2(\cdot, 0) - \frac{1}{3}P_2(\cdot, 1) - \frac{1}{3}P_2(\cdot, 2) < 0$, selling in $t = 2$ with a profit of $\frac{1}{3}P_2(\cdot, 0) - \frac{1}{3}V^- - c$ and not trading with a trading profit of 0. Hence, when Q_1 reveals that the speculator is not uninformed, the negatively informed speculator who buys in $t = 1$ sells (resp. does not trade) in $t = 2$ if $\frac{1}{3}\bar{V} - \frac{1}{3}V^- - c < 0$ $\Leftrightarrow c < a_3$ (resp. $c > a_3$), where

$$a_3 = \frac{V^+}{6} - \frac{V^-}{6}. \tag{A3.1.7}$$

(v). When Q_1 does not reveal any new information about the speculator's type, the best responses of the negatively informed speculator who buys one unit or chooses to not trade in $t = 1$ follow from the derivation in (iv). For the negatively informed speculator who sells in $t = 1$, the profit of buying in $t = 2$ is $P_1(\cdot) - c - \frac{1}{3}P_2(\cdot, 0) - \frac{1}{3}P_2(\cdot, 1) - \frac{1}{3}P_2(\cdot, 2)$, the profit of not trading in $t = 2$ is $P_1(\cdot) - \frac{2}{3}V^- - c$, and the profit of selling in $t = 2$ is $P_1(\cdot) + \frac{1}{3}P_2(\cdot, 0) - \frac{2}{3}V^- - 2c$. Buying in $t = 2$ is strictly dominated by not trading in $t = 2$. Hence, when Q_1 does not reveal the speculator's type, the negatively informed speculator who sells in $t = 1$ sells (resp. does not trade) in $t = 2$ if $\frac{1}{3}\bar{V} - \frac{2}{3}V^- - 2c > -\frac{2}{3}V^- - c$, which is equivalent to $c < a_1$ (resp. $c > a_1$), where

$$a_1 = \frac{1}{3}\bar{V}. \tag{A3.1.8}$$

The derivation of uninformed best responses under this scenario is as follows. First, consider the case that the negatively informed speculator chooses to sell in $t = 2$, and the uninformed speculator chooses to sell in $t = 1$. If the uninformed speculator's best response in $t = 2$ is to buy, the corresponding trading profit is $P_1(\cdot) - c - \frac{1}{3}P_2(\cdot, 0) - \frac{1}{3}P_2(\cdot, 1) - \frac{1}{3}P_2(\cdot, 2)$, with $P_2(\cdot, 0) = \bar{V}$ and $P_2(\cdot, 1) = P_2(\cdot, 2) = \frac{\alpha}{2-\alpha}V^+ + \frac{2(1-\alpha)}{2-\alpha}\bar{V}$. Deviation to selling in $t = 2$ is profitable if and only if $P_1(\cdot) + \frac{1}{3}P_2(\cdot, 0) - \frac{2}{3}\bar{V} - 2c > P_1(\cdot) - c - \frac{1}{3}P_2(\cdot, 0) - \frac{1}{3}P_2(\cdot, 1) - \frac{1}{3}P_2(\cdot, 2) \Leftrightarrow c < \frac{2}{3}\left(\frac{\alpha}{2-\alpha}V^+ + \frac{1-\alpha}{2-\alpha}V^-\right)$. Deviation to not trading is associated with the trading profit of $P_1(\cdot) - 2c - \frac{1}{3}\bar{V}$ which strictly dominates buying. If his strategy in $t = 2$ is to not trade, the trading profit is $P_1(\cdot) - c - \frac{2}{3}\bar{V}$. Deviation to sell gives $P_1(\cdot) - 2c - \frac{1}{3}\bar{V}$. The deviation to buying yields $P_1(\cdot) - c - \frac{1}{3}P_2(\cdot, 0) - \frac{1}{3}P_2(\cdot, 1) - \frac{1}{3}P_2(\cdot, 2) = P_1(\cdot) - c - \frac{1}{3}\bar{V} - \frac{1}{3}V^+ - \frac{1}{3}\left(\frac{\alpha}{2-\alpha}V^+ + \frac{2(1-\alpha)}{2-\alpha}\bar{V}\right)$ which is strictly worse than not trading. The uninformed speculator doesn't deviate from not trading when $-c - \frac{2}{3}\bar{V} > -2c - \frac{1}{3}\bar{V} \Leftrightarrow c > a_1$. If his strategy in $t = 2$ is to sell, the trading profit is $P_1(\cdot) - 2c - \frac{1}{3}\bar{V}$. Buying is a profitable deviation if and only if $P_1(\cdot) - c - \frac{2}{3}V^+ - \frac{1}{3}\bar{V} > P_1(\cdot) - 2c - \frac{1}{3}\bar{V} \Leftrightarrow c < \frac{2}{3}V^+$. The uninformed speculator has no incentive to deviate from selling to not trading if and only if $2c + \frac{1}{3}\bar{V} < c + \frac{2}{3}\bar{V} \Leftrightarrow c < a_1$. When the negatively informed speculator does not trade in $t = 2$, the derivation of the $t = 2$ best response of the uninformed speculator who sells in $t = 1$ follows from the above procedure, with slightly different prices, and draws the same conclusion about the cutoff of c, surpassing which leads to a preference of not trading over submitting sell orders in $t = 2$. Hence, the uninformed speculator who sells in $t = 1$ sells again (resp. not trade) in $t = 2$ if $c < a_1$ (resp. $c > a_1$). Finally, we prove that the uninformed speculator who does not trade in $t = 1$ also does not trade in $t = 2$. The profit of not trading in both periods is 0. The profit of chooses to not trade in $t = 1$ then buy in $t = 2$ is $\bar{V} - \frac{1}{3}P_2(\cdot, 0) - \frac{1}{3}P_2(\cdot, 1) - \frac{1}{3}P_2(\cdot, 2)$. The former equation never surpasses 0: $Q_2 = 2$ implies the non-existence of negatively informed speculator and $P_2(\cdot, 2) > \bar{V}$; furthermore, $P_2(\cdot, 1) > \bar{V}$ ($P_2(\cdot, 1) = \bar{V}$) as long as the negatively informed speculator sells (does not trade) in $t = 2$. The profit of not trading in $t = 1$ and then selling in $t = 2$ is $\frac{1}{3}\left(P_2(\cdot, 0) - \bar{V}\right) - c < 0$.

■

Proof of Proposition 3.8. To start with, we show that neither the negatively informed nor the uninformed speculator buys in $t = 1$. Conditional on reaching information sets (ii) or (v) and the speculator is uninformed and buys in $t = 1$, selling in $t = 2$ leads to a profit of $\frac{1}{3}P_2(\cdot, 0) - P_1(\cdot) - c < 0$; not trading in $t = 2$ leads to a profit of $\frac{2}{3}\bar{V} - P_1(\cdot) < 0$. To show that $P_1(\cdot) > \frac{2}{3}\bar{V}$, we know that $P_1(\cdot) = Pr(Q_2 = 0|Q_1 = \cdot)P_2(\cdot, 0) + Pr(Q_2 = 1|Q_1 = \cdot)P_2(\cdot, 1) + Pr(Q_2 = 2|Q_1 = \cdot)P_2(\cdot, 2) > \frac{1}{3}\bar{V} + \frac{\alpha}{6}\bar{V} + \frac{\alpha}{6}V^+ > \frac{2}{3}\bar{V}$. Thus, an uninformed speculator never buys in $t = 1$ due to the negative profit entailed. Conditional on reaching information sets (iv) or (v) and the speculator knows $s = l$ (i.e., is negatively informed) and buys in $t = 1$, selling in $t = 2$ results in a profit of $\frac{1}{3}P_2(\cdot, 0) - P_1(\cdot) - c < 0$, not trading results in a profit of $\frac{2}{3}V^- - P_1(\cdot) < 0$. Thus, a speculator informed about the low state never buys in $t = 1$ due to the negative profit entailed.

(a). When $c \in \left[0, \frac{\bar{V}}{3}\right)$, candidate equilibrium 1.1 has the following strategy profile: the positively informed speculator buys in $t = 1$ and buys in $t = 2$ if his type is not fully revealed (otherwise, indifferent between buying and not trading in $t = 2$); the negatively informed speculator does not trade in $t = 1$, and sells (is indifferent between buying and not trading) in $t = 2$ if $Q_1 \neq -1$(if $Q_1 = -1$); the uninformed speculator sells in $t = 1$, and sells (is indifferent between buying and not trading) in $t = 2$ if $Q_1 = 0$ (if $Q_1 \neq 0$).The corresponding prices are $P_2(1, 0) = P_2(0, 0) = \bar{V}$, $P_2(0, 1) = P_2(0, 2) = V^+$, and $P_1(0) = \frac{1}{3}\bar{V} + \frac{\alpha}{3}V^+$. The corresponding trading profit of the negatively informed speculator who no trade – sell is $\frac{1}{3}\left(\frac{1}{3}P_2(0, 0) - \frac{V^-}{3} - c\right) + \frac{1}{3}\left(\frac{1}{3}P_2(1, 0) - \frac{V^-}{3} - c\right)$. The best deviation for him is sell – sell, which yields $\frac{1}{3}\left(\frac{1}{3}P_2(0, 0) + P_1(0) - \frac{2}{3}V^-\right) - \frac{4}{3}c$.

The negatively informed speculator thus does not deviate if and only if $\frac{1}{9}P_2(1,0) > \frac{1}{3}P_1(0) - \frac{2}{3}c \Leftrightarrow c > \frac{\alpha}{6}V^+$. However, $\frac{\alpha}{6}V^+ > \frac{1}{3}\bar{V}$, this candidate equilibrium is invalid.

Candidate equilibrium 1.2 in this region has the following strategy profile: the positively informed speculator implements a buy – buy strategy; the negatively informed speculator does not trade in $t = 1$, and sells (is indifferent between buying and not trading) in $t = 2$ if $Q_1 \neq -1$(if $Q_1 = -1$); the uninformed speculator does not trade in $t = 1$ and does not trade (is indifferent between buying and not trading) in $t = 2$ if $Q_1 \neq -1$ (if $Q_1 = -1$). Following a similar vein as in candidate equilibrium 1.1, one can show that candidate equilibrium 1.2 is invalid.

Candidate equilibrium 1.3 in this region is the benchmark equilibrium, with the following prices: $P_2(0,0) = \bar{V}$, $P_2(0,1) = P_2(0,2) = V^+$, $P_1(0) = \frac{\bar{V}}{3} + \frac{\alpha}{3}V^+$, $P_1(1) = P_1(2) = V^+$, and $P_1(-1) = P_1(-2) = 0$. The profit of negatively informed speculator who sell – sell is $\frac{1}{3}\left(\frac{1}{3}P_2(0,0) + P_1(0) - \frac{2}{3}V^-\right) - \frac{4}{3}c$. The best deviation for him is no trade – sell, which yields the trading profit of $\frac{1}{3}\left(\frac{1}{3}P_2(0,0) - \frac{V^-}{3} - c\right) + \frac{1}{3}\left(\frac{1}{3}P_2(1,0) - \frac{V^-}{3} - c\right)$. The negatively informed speculator does not deviate if and only if $\frac{1}{9}P_2(1,0) < \frac{1}{3}P_1(0) - \frac{2}{3}c$, equivalent to $c < \frac{\alpha}{6}V^+ - \frac{V^+ - V^-}{12}$. The equilibrium profit of uninformed speculator who sell – sell is $\frac{1}{3}\left(\frac{1}{3}P_2(0,0) + P_1(0) - \frac{2}{3}\bar{V}\right) - \frac{4}{3}c$. The best deviation for him is "no trade in $t = 1$, and sell (no trade) in $t = 2$ if the order flow suggests that not negatively informed (does not reveal or exclude any type)", which generates $\frac{1}{3}\left(\frac{1}{3}P_2(1,0) - \frac{\bar{V}}{3} - c\right)$. The uninformed speculator does not have a profitable deviation from the benchmark equilibrium if and only if $\frac{1}{3}P_2(1,0) - c < P_1(0) - 4c$, equivalent to $c < c_1$. c_1 is defined in Eq.(A3.1.9). We know that $c_1 < \frac{\bar{V}}{3}$ and $V^+ \geqslant -3V^-$ is a sufficient condition for $c_1 > 0$ to hold. Combining Eqs.(A3.1.8) and (A3.1.9), the unique equilibrium when $c < c_1$ is the benchmark equilibrium.

$$c_1 = \frac{\alpha}{9}V^+ - \frac{V^+ - V^-}{18}. \tag{A3.1.9}$$

Candidate equilibrium 1.4 in this region is *NMS* equilibrium, with following prices: $P_2(0,0) = \bar{V}$, $P_2(1,0) = P_2(0,1) = \frac{\alpha}{2-\alpha}V^+ + \frac{2(1-\alpha)}{2-\alpha}\bar{V}$, $P_2(1,1) = P_2(1,2) = P_2(0,2) = V^+$, $P_2(-1,\cdot) = P_2(-2,\cdot) = 0$, and $P_1(0) = \frac{\alpha}{3}V^+ + \frac{2}{3}\bar{V} - \frac{\alpha\bar{V}}{3}$. The equilibrium profit of negatively informed speculator who sell – sell is $\frac{1}{3}\left(\frac{1}{3}P_2(0,0)+P_1(0)-\frac{2}{3}V^-\right)$, and his alternative strategy of no trade – sell generates $\frac{1}{3}\left(\frac{1}{3}P_2(0,0)-\frac{V^-}{3}-c\right)+\frac{1}{3}\left(\frac{1}{3}P_2(1,0)-\frac{V^-}{3}-c\right)$. The negatively informed speculator does not have an incentive to deviate if and only if $\frac{1}{9}P_2(1,0)<\frac{1}{3}P_1(0)-\frac{2}{3}c$, equivalent to $c<c_3$, where

$$c_3=\frac{\alpha}{6}V^+-\frac{\alpha}{6(2-\alpha)}\left(V^+-\bar{V}\right). \tag{A3.1.10}$$

The profit of the uninformed speculator who "no trade in $t=1$, and sell (no trade) in $t=2$ if Q_1 suggests that not negatively informed (does not reveal anything)" is $\frac{1}{3}\left(\frac{1}{3}P_2(1,0)-\frac{\bar{V}}{3}-c\right)$. His alternative strategy of sell – sell gives rise to $\frac{1}{3}\left(\frac{1}{3}P_2(0,0)+P_1(0)-\frac{2}{3}\bar{V}\right)-\frac{4}{3}c$. The uninformed speculator does not has an incentive to deviate if and only if $\frac{1}{3}P_2(1,0)-c>P_1(0)-4c$, which is equivalent to $c>c_2$. c_2 is defined by Eq.(A3.1.11). Given that $c_2>\frac{\alpha}{9}V^+-\frac{V^+-V^-}{18}$ and $c_3<\frac{\bar{V}}{3}$, combining Eq.(A3.1.10) and Eq.(A3.1.11), the unique equilibrium when $c_2<c<c_3$ is *NMS*.

$$c_2=\frac{\alpha}{9}V^+-\frac{\alpha}{9(2-\alpha)}\left(V^+-\bar{V}\right). \tag{A3.1.11}$$

(b). When $c\in\left(\frac{\bar{V}}{3},\frac{\alpha}{2-\alpha}\frac{V^+-V^-}{6}\right)$, candidate equilibrium 2.1 has the following strategy profile: the positively informed speculator implements a buy – buy strategy, the negatively informed speculator implements a sell – no trade strategy, whereas the uninformed speculator does not trade in $t=1$, and sells (does not trade) in $t=2$ if $Q_1=1$ ($Q_1=0$). The corresponding prices are $P_2(0,0)=P_2(0,1)=\bar{V}$, $P_2(0,2)=V^+$, $P_1(0)=\frac{2}{3}\bar{V}+\frac{\alpha}{6}V^+$, and $P_2(1,0)=\frac{\alpha}{2-\alpha}V^++\frac{2(1-\alpha)}{2-\alpha}\bar{V}$. The negatively informed speculator's profit from sell – no trade is $\frac{1}{3}\left(P_1(0)-\frac{2}{3}V^-\right)-c$. The deviation to strategy no trade – sell leads to $\frac{1}{3}\left(\frac{1}{3}P_2(1,0)-\frac{1}{3}V^--c\right)+\frac{1}{3}\left(\frac{1}{3}P_2(0,0)-\frac{1}{3}V^--c\right)$. There's no deviation for the negatively informed speculator if and only if $\frac{1}{3}\left(P_1(0)-\frac{2}{3}V^-\right)-c>\frac{1}{3}\left(\frac{1}{3}P_2(1,0)-\frac{1}{3}V^--c\right)+\frac{1}{3}\left(\frac{1}{3}P_2(0,0)-\frac{1}{3}V^--c\right)\Leftrightarrow c<\frac{\bar{V}}{3}+\frac{\alpha V^+}{6}-\frac{1}{3}\left(\frac{\alpha}{2-\alpha}V^++\frac{2(1-\alpha)}{2-\alpha}\bar{V}\right)$. As the term on the right hand side of this inequality is smaller than $\frac{\bar{V}}{3}$, candidate equilibrium 2.1 is invalid.

Candidate equilibrium 2.2 in this region has the following strategy profile: the positively informed speculator implements a buy – buy strategy, the negatively informed and the uninformed speculator implements a sell – no trade strategy. The corresponding prices are $P_2(0,0)=P_2(0,1)=\bar{V}$, $P_2(0,2)=P_2(1,0)=V^+$, and $P(0)=\frac{2}{3}\bar{V}+\frac{\alpha}{6}V^+$. To avoid a profitable deviation of the negatively informed speculator from sell – no trade to no trade – sell, we have $\frac{1}{3}\left(P_1(0)-\frac{2}{3}V^-\right)-c > \frac{1}{3}\left(\frac{1}{3}P_2(1,0)-\frac{1}{3}V^- - c\right)+\frac{1}{3}\left(\frac{1}{3}P_2(0,0)-\frac{1}{3}V^- - c\right) \Leftrightarrow c < \frac{1}{3}\bar{V}+\frac{\alpha}{6}V^+ - \frac{V^+}{3}$. As the term on the right hand side is smaller than $\frac{\bar{V}}{3}$, candidate equilibrium 2.2 is invalid.

Candidate equilibrium 2.3 in this region has the following strategy profile: the positively informed speculator implements a buy – buy strategy, the negatively informed speculator implements a no trade – sell strategy, and the uninformed speculator implements a sell – no trade strategy. The corresponding prices are $P_2(0,0)=P_2(1,0)=\bar{V}$, $P_2(0,1)=\frac{\alpha}{2-\alpha}V^+ + \frac{2(1-\alpha)}{2-\alpha}\bar{V}$, $P_2(1,1)=V^+$ and $P_1(0)=\frac{2-\alpha}{3}\bar{V}+\frac{\alpha}{3}V^+$. The negatively informed speculator does not deviate from no trade – sell if and only if $\frac{1}{3}\left(\frac{1}{3}P_2(1,0)-\frac{1}{3}V^- - c\right)+\frac{1}{3}\left(\frac{1}{3}P_2(0,0)-\frac{1}{3}V^- - c\right) > \frac{1}{3}\left(P_1(0)-\frac{2}{3}V^-\right) - c \Leftrightarrow c > \frac{\alpha\left(V^+-V^-\right)}{6}$. As $\frac{\alpha\left(V^+-V^-\right)}{6}$ is greater than $\frac{\alpha}{2-\alpha}\frac{V^+-V^-}{6}$, candidate equilibrium 2.3 is invalid.

Candidate equilibrium 2.4 in this region has the following strategy profile: the positively informed speculator implements a buy – buy strategy, the negatively informed speculator implements a no trade – sell strategy, and the uninformed speculator does not trade in $t=1$, and sells (does not trade) in $t=2$ if $Q_1=1$ ($Q_1=0$). The corresponding prices are $P_2(0,0)=P_2(1,0)=\bar{V}$, $P_2(1,1)=P_2(0,1)=\frac{\alpha}{2-\alpha}V^+ + \frac{2(1-\alpha)}{2-\alpha}\bar{V}$, $P_2(1,2)=P_2(0,2)=V^+$, and $P_1(0)=\frac{2-\alpha}{3}\bar{V}+\frac{\alpha}{3}V^+$. Following a similar procedure as candidate equilibrium 2.3, one can show that the negatively informed speculator is not able to profitably deviate from no trade – sell to sell – no trade if and only if $c > \frac{\alpha\left(V^+-V^-\right)}{6}$. As $\frac{\alpha\left(V^+-V^-\right)}{6}$ is greater than $\frac{\alpha}{2-\alpha}\frac{V^+-V^-}{6}$, candidate equilibrium 2.4 is invalid.

(c). When $c \in \left(\frac{\alpha}{2-\alpha}\frac{V^+-V^-}{6}, \frac{V^+-V^-}{6}\right)$, given $t=2$ best responses defined in Lemma 3.7, the negatively informed speculator chooses between sell – no trade and no trade – sell, and the uninformed speculator chooses between sell – no trade and no trade – no trade. Therefore, the strategy profile of candidate equilibrium 3.1 is : the positively informed speculator implements a buy – buy strategy, the negatively informed speculator implements a sell – no trade strategy, and the uninformed speculator implements a no trade – no trade strategy. The strategy profile of candidate equilibrium 3.2 is: the positively informed speculator implements a buy – buy strategy, the negatively informed speculator implements a sell – no trade strategy, and the uninformed speculator implements a sell – no trade strategy. The strategy profile of candidate equilibrium 3.3 is: the positively informed speculator implements a buy – buy strategy, the negatively informed speculator implements a no trade – sell strategy, and the uninformed speculator implements a sell – no trade strategy. None of these three candidate equilibriums are valid in the given region of c.

Candidate equilibrium 3.4 is the *RINMS* equilibrium. The corresponding prices are listed in Table 3.4. The negatively informed speculator compares the equilibrium profit of no trade – sell, $\frac{1}{3}\left(\frac{1}{3}P_2(1,0)-\frac{1}{3}V^- - c\right)+\frac{1}{3}\left(\frac{1}{3}P_2(0,0)-\frac{1}{3}V^- - c\right)$, with the profit of sell – no trade, $\frac{1}{3}\left(P_1(0)-\frac{2}{3}V^-\right)-c$. The negatively informed speculator does not profitably deviate if and only if $\left(\frac{1}{3}P_2(1,0)-\frac{1}{3}V^-\right)+\left(\frac{1}{3}P_2(0,0)-\frac{1}{3}V^-\right)>\left(P_1(0)-\frac{2}{3}V^-\right)-c$, equivalent to $c>c_4$, where

$$c_4=\frac{\alpha\left(V^+-V^-\right)}{6}. \tag{A3.1.12}$$

The uninformed speculator compares the profit of no trade – no trade, with the profit of his alternative strategy sell – no trade, $\frac{1}{3}\left(P_1(0)-\frac{2}{3}\bar{V}\right)-c$. The uninformed speculator does not deviate if and only if $\frac{1}{3}\left(P_1(0)-\frac{2}{3}\bar{V}\right)-c>0$, equivalent to

$$c>\frac{\alpha\left(V^+-V^-\right)}{18}. \tag{A3.1.13}$$

Combining Eqs.(A3.1.12) and (A3.1.13), as other candidate equilibriums in $c\in\left(\frac{\alpha}{2-\alpha}\frac{V^+-V^-}{6},\frac{V^+-V^-}{6}\right)$ are invalid, it is proved the only Nash equilibrium is the *RINMS* equilibrium, when

$$c_4<c<\frac{V^+-V^-}{6}\equiv c_5. \tag{A3.1.14}$$

(d). When $c\in\left(\frac{V^+-V^-}{6},+\infty\right)$, given $t=2$ best responses defined in Lemma 3.7, both the negatively informed and the uninformed speculator chooses between sell – no trade and no trade – no trade. Candidate equilibrium 4.1 in this region of c is: the positively informed speculator employs a buy – buy strategy, the negatively informed speculator employs a sell – no trade strategy, and the uninformed speculator no trade – no trade. The corresponding prices are $P_2(0,0)=P_2(0,1)=\bar{V}$, $P_2(0,2)=V^+$, $P_2(1,0)=P_2(1,1)=$

$\frac{\alpha}{2-\alpha}V^+ + \frac{2(1-\alpha)}{2-\alpha}\bar{V}$, $P_2(1,1) = V^+$, and $P_1(0) = \frac{2}{3}\bar{V} + \frac{\alpha}{6}V^+$. The negatively informed speculator possess no profitable deviation from sell – no trade to no trade – no trade if and only if $\frac{1}{3}\left(P_1(0) - \frac{2}{3}V^-\right) - c < 0 \Leftrightarrow c < \frac{\alpha}{18}V^+ + \frac{V^+-V^-}{9}$. As the term on the right hand side is smaller than $\frac{V^+-V^-}{6}$, candidate equilibrium 4.1 is invalid.

Candidate equilibrium 4.2 in this region of c is: the positively informed speculator opts to buy – buy, the negatively informed speculator opts to no trade – no trade, and the uninformed speculator opts to sell – no trade strategy. Candidate equilibrium 4.3 in this region of c is: the positively informed speculator opts to buy – buy strategy, the negatively informed speculator and the uninformed speculator both opt to sell – no trade. For these two candidate equilibriums, $P_1(0) = \frac{2}{3}\bar{V} + \frac{\alpha}{6}V^+$, and the uninformed speculator does not deviate from sell – no trade to no trade – no trade if and only if $\frac{1}{3}\left(P_1(0) - \frac{2}{3}\bar{V}\right) - c > 0 \Leftrightarrow c < \frac{\alpha}{18}V^+ < \frac{V^+-V^-}{6}$. Candidate equilibrium 4.2 and candidate equilibrium 4.3 are invalid.

Candidate equilibrium 4.4 is the short sale ban equilibrium, in which $P_1(0) = \frac{2}{3}\bar{V} + \frac{\alpha}{6}V^+$. The negatively informed speculator's equilibrium trading profit of no trade – no trade is 0. His best deviation is sell – no trade, which generates $\frac{1}{3}\left(P_1(0) - \frac{2}{3}V^-\right) - c$.

$$c > \frac{\alpha}{18}V^+ + \frac{V^+ - V^-}{9}, \tag{A3.1.15}$$

$$c > \frac{\alpha}{18}V^+. \tag{A3.1.16}$$

He does not deviate if and only if $\frac{1}{3}\left(P_1(0) - \frac{2}{3}V^-\right) - c < 0$, equivalent to Eq.(A3.1.15). The uninformed speculator does not deviate from no trade – no trade to sell – no trade if and only if $\frac{1}{3}\left(P_1(0) - \frac{2}{3}\bar{V}\right) - c < 0$, equivalent to Eq.(A3.1.16). Combining the two equations, given that all other candidate equilibriums associated with $c \in \left(\frac{V^+-V^-}{6}, +\infty\right)$ are invalid, we proved: when $c > \frac{V^+-V^-}{6} \equiv c_5$, the only equilibrium is the *SSB* equilibrium.

■

Proof of Corollary 3.9. (i) In the *NMS* equilibrium, the uninformed speculator does not trade in $t = 1$, does not trade in $t = 2$ if $Q_1 = 0$, and sells in $t = 2$ if $Q_1 = 1$. The negatively informed speculator has a sell – sell strategy, and the positively informed speculator has a buy – buy strategy. The expected firm value conditional on the speculator being uninformed is $\frac{5\bar{V}}{9}$: $Q_2 < 0$ following $Q_1 = 1$ does not lead the manager to cancel the investment as it fully reveals the uninformed speculator's type. The expected firm value conditional on the speculator being negatively informed is $\frac{V^-}{9}$ and being positively informed is V^+. The expected firm value in the *NMS* equilibrium then follows as

$$RE_{NMS} = \frac{\alpha V^+}{2} + \frac{\alpha V^-}{18} + \frac{5(1-\alpha)\bar{V}}{9}. \tag{A3.1.17}$$

(ii) The *NMS* equilibrium always improves the real investment efficiency since

$$RE_{NMS} - RE_{benchmark} = \frac{4(1-\alpha)\bar{V}}{9} > 0. \tag{A3.1.18}$$

(iii) When the *RINMS* equilibrium sustains, the uninformed speculator employs a no trade – no trade strategy, the negatively informed speculator employs a no trade – sell strategy, and the positively informed speculator has a buy – buy strategy. The expected firm value conditional on the speculator being uninformed is $\frac{4\bar{V}}{9}$. The expected firm value conditional on the speculator being negatively informed is $\frac{2V^-}{9}$. The expected firm value conditional on the speculator being positively informed is V^+. The expected firm value in the *RINMS* equilibrium is

$$RE_{RINMS} = \frac{\alpha V^+}{2} + \frac{\alpha V^-}{9} + \frac{4(1-\alpha)\bar{V}}{9}. \tag{A3.1.19}$$

(iv) The *RINMS* equilibrium raises the real investment efficiency if and only if

$$RE_{RINMS} - RE_{benchmark} = \frac{\alpha V^-}{18} + \frac{3(1-\alpha)\bar{V}}{9} > 0. \tag{A3.1.20}$$

■

Proof of Lemma 3.10. We show the best responses of the negatively informed and uninformed speculators at the possible information sets after the order flow in $t = 1$ in the presence of asymmetric noise buys.

(i). When Q_1 perfectly reveals the speculator's type, then the speculator is indifferent between buy, sell, and no trade.

(ii). When Q_1 reveals that the speculator is not negatively informed, we only consider the profit in $t = 2$ since the manager always invests when reaching this information set. Suppose that the uninformed speculator's strategy when reaching this information set is to sell in $t = 2$. His $t = 2$ trading profit is then calculated as below

$$\frac{1-\delta}{3}\left(P_2(\cdot,0)-\bar{V}\right)+\frac{1}{3}\left(P_2(\cdot,-1)-\bar{V}\right)+\frac{1+\delta}{3}\left(P_2(\cdot,-2)-\bar{V}\right)=\frac{\alpha(1-\delta)V^{+}+2(1-\alpha)(1+\delta)\bar{V}}{3\alpha(1-\delta)+6(1-\alpha)(1+\delta)} -\frac{1}{3}\bar{V}>0. \tag{A3.1.21}$$

Deviation to not trading gives rise to a zero date-2 profit. Deviation to buying leads to

$$\bar{V}-\left(\frac{1-\delta}{3}P_2(\cdot,0)+\frac{1}{3}P_2(\cdot,1)+\frac{1+\delta}{3}P_2(\cdot,2)\right)=\bar{V}-\frac{1-\delta}{3}\left(\frac{\alpha(1-\delta)V^{+}+2(1-\alpha)(1+\delta)\bar{V}}{\alpha(1-\delta)+2(1-\alpha)(1+\delta)}\right) -\frac{2+\delta}{3}V^{+}<0. \tag{A3.1.22}$$

The uninformed speculator therefore does not deviate from selling in $t = 2$. Now suppose that the uninformed speculator's best response for this information set is to buy. We then have

$$P_2(\cdot,0)=P_2(\cdot,1)=P_2(\cdot,2)=\frac{\alpha}{2-\alpha}V^{+}+\frac{2(1-\alpha)}{2-\alpha}\bar{V}. \tag{A3.1.23}$$

The profit in $t = 2$ is $\bar{V}-\left(\frac{1}{3}P_2(\cdot,0)+\frac{1}{3}P_2(\cdot,1)+\frac{1}{3}P_2(\cdot,2)\right)=\bar{V}-P_2(\cdot,1)<0$, so that the uninformed speculator does not buy in $t = 2$. Finally, suppose that the uninformed speculator's strategy for this information set is to not trade. We then have $P_2(\cdot,0)=\frac{\alpha(1-\delta)V^{+}+2(1-\alpha)\bar{V}}{\alpha(1-\delta)+2(1-\alpha)}$. The profit from not trading in $t = 2$ is 0. By deviating to selling, the uninformed speculator earns a profit of

$$\frac{1+\delta}{3}\left(P_2(\cdot,0)-\bar{V}\right)=\frac{1+\delta}{3}\left(\frac{\alpha(1-\delta)V^{+}+2(1-\alpha)\bar{V}}{\alpha(1-\delta)+2(1-\alpha)}-\bar{V}\right)>0. \tag{A3.1.24}$$

Hence, when Q_1 reveals that the speculator is not informed of the low state, the uninformed speculator sells in $t = 2$.

(iii). When Q_1 reveals that the speculator is not positively informed, buying, selling and not trading all generate zero profit given the investment cancellation. Negatively informed and uninformed speculators are indifferent between all three actions. We combine (i) and (iii) in the main body of the paper.

(iv). When Q_1 reveals that the speculator is not uninformed, we first consider the case that the negatively informed speculator buys in $t = 1$. By selling in $t = 2$, his total profit is $\frac{1+\delta}{3}P_2(\cdot,0)-P_1(\cdot)$. By not trading in $t = 2$, his total profit is $\frac{2+\delta}{3}V^{-}-P_1(\cdot)$. By buying in $t = 2$, his total profit is $2V^{-}-P_1(\cdot)-\frac{1-\delta}{3}P_2(\cdot,0)-\frac{1}{3}P_2(\cdot,1)-\frac{1+\delta}{3}P_2(\cdot,2)$. Selling strictly dominates not trading and buying since $\frac{1+\delta}{3}P_2(\cdot,0) > 0 > \frac{2+\delta}{3}V^- > 2V^- - \frac{1-\delta}{3}P_2(\cdot,0) - \frac{1}{3}P_2(\cdot,1) - \frac{1+\delta}{3}P_2(\cdot,2)$. Now consider the case that the negatively informed speculator does not trade in $t = 1$. By selling in $t = 2$, his total profit is $\frac{1+\delta}{3}\left(P_2(1,0) - V^-\right)$. By not trading in $t = 2$, his total profit is 0. By buying in $t = 2$, his total profit is $V^- - \frac{1-\delta}{3}P_2(\cdot,0) - \frac{1}{3}P_2(\cdot,1) - \frac{1+\delta}{3}P_2(\cdot,2)$. Selling strictly dominates not trading and buying since $\frac{1+\delta}{3}\left[P_2(1,0) - V^-\right] > 0 > V^- - \frac{1-\delta}{3}P_2(\cdot,0) - \frac{1}{3}P_2(\cdot,1) - \frac{1+\delta}{3}P_2(\cdot,2)$. Hence, when Q_1 reveals that the speculator is not uninformed, the negatively informed speculator's best response in $t = 2$ is to sell.

(v). When Q_1 does not reveal any new information, the derivation of $t = 2$ best responses for the negatively informed who buys or does not trade in $t = 1$ directly follow from the derivation in scenario (iv). For the negatively informed speculator who sells in $t = 1$, if he buys in $t = 2$, his total profit is $P_1(\cdot) - \frac{1+\delta}{3}P_2(\cdot,2) - \frac{1}{3}P_2(\cdot,1) - \frac{1-\delta}{3}P_2(\cdot,0)$. If he does not trade in $t = 2$, his total profit is $P_1(\cdot) - \frac{2+\delta}{3}V^-$. If he sells in $t = 2$, his total profit is $P_1(\cdot) + \frac{1+\delta}{3}P_2(\cdot,0) - \frac{2+2\delta}{3}V^-$. Selling strictly dominates buying and not trading as $\frac{1+\delta}{3}P_2(\cdot,0) - \frac{2+2\delta}{3}V^- > -\frac{2+2\delta}{3}V^- > 0 > -\frac{1+\delta}{3}P_2(\cdot,2) - \frac{1}{3}P_2(\cdot,1) - \frac{1-\delta}{3}P_2(\cdot,0)$. Hence, when Q_1 does not reveal the speculator's type, the negatively informed speculator's best response in $t = 2$ is to sell.

We then derive uninformed speculator behaviors at the information set (v). For the uninformed speculator who buys in $t = 1$, if he buys again in $t = 2$, his total profit is $2\bar{V} - P_1(\cdot) - \frac{1-\delta}{3}P_2(\cdot,0) - \frac{1}{3}P_2(\cdot,1) - \frac{1+\delta}{3}P_2(\cdot,2)$. If he sells in $t = 2$, his total trading profit in the two trading periods is $\frac{1+\delta}{3}P_2(\cdot,0) - P_1(\cdot)$. If he chooses to not trade in $t = 2$, his trading profit in the two trading periods is $\frac{2+\delta}{3}\bar{V} - P_1(\cdot)$. Not trading strictly dominates selling as $P_2(\cdot,0) < \bar{V}$.

To show the negatively informed speculator who buys in $t = 1$ does not trade in $t = 2$, suppose that by way of contradiction that he buys in $t = 2$. Because the positively informed speculator buys and the negatively informed speculator sells in $t = 2$, we have $P_2(\cdot,0) = \frac{(1-\delta)\alpha V^+ + (1+\delta)\alpha V^- + 2(1-\delta)(1-\alpha)\bar{V}}{2\alpha + 2(1-\delta)(1-\alpha)}$, and $P_2(\cdot,1) = P_2(\cdot,2) = \frac{\alpha}{2-\alpha}V^+ + \frac{2(1-\alpha)}{2-\alpha}\bar{V}$. Substituting $P_2(\cdot,0)$, $P_2(\cdot,1)$ and $P_2(\cdot,2)$ into the profit difference between buying and not trading obtains

$$2\bar{V} - \frac{1-\delta}{3}P_2(\cdot,0) - \frac{1}{3}P_2(\cdot,1) - \frac{1+\delta}{3}P_2(\cdot,2) - P_1(\cdot) - \left(\frac{2+\delta}{3}\bar{V} - P_1(1)\right) =$$
$$\left(\frac{(\delta-1)^2}{3\alpha + 6\delta - 6\alpha\delta - 6} + \frac{1}{3\alpha - 6} - \frac{\delta}{2} + \frac{1}{3}\right)V^+ + \left(\frac{1 - 2\delta + \delta^2 + 2\alpha\delta - 2\alpha\delta^2}{3\alpha + 6\delta - 6\alpha\delta - 6} - \frac{\alpha - 1}{3\alpha - 6} - \frac{\delta}{6} + \frac{2}{3}\right)V^-, \quad \text{(A3.1.25)}$$

where

$$\frac{(\delta-1)^2}{3\alpha+6\delta-6\alpha\delta-6}+\frac{1}{3\alpha-6}-\frac{\delta}{2}+\frac{1}{3}<\frac{(\delta-1)^2}{3\alpha-6}-\frac{\delta}{2}+\frac{1}{3}+\frac{1}{3\alpha-6}<-\frac{1-2\delta+\delta^2}{6}-\frac{1}{6}-\frac{\delta}{2}+\frac{1}{3}<0, \quad \text{(A3.1.26)}$$

and

$$\frac{1-2\delta+\delta^2+2\alpha\delta-2\alpha\delta^2}{3\alpha+6\delta-6\alpha\delta-6}-\frac{\alpha-1}{3\alpha-6}-\frac{\delta}{6}+\frac{2}{3}>-\frac{1}{3}-\frac{1}{6}-\frac{\delta}{6}+\frac{2}{3}>0. \quad \text{(A3.1.27)}$$

Therefore, the negatively informed speculator who buys in $t=1$ does not buy in $t=2$ and in fact does not trade in $t=2$.

For the uninformed speculator who sells in $t=1$, buying in $t=2$ gives rise to $P_1(1)-\frac{1+\delta}{3}P_2(\cdot,2)-\frac{1}{3}P_2(\cdot,1)-\frac{1-\delta}{3}P_2(\cdot,0)<P_1(\cdot)-\frac{1+\delta}{3}\bar{V}-\frac{1}{3}\bar{V}-0=P_1(\cdot)-\frac{2+\delta}{3}\bar{V}$, not trading in $t=2$ gives rise to $P_1(\cdot)-\frac{2+\delta}{3}\bar{V}$, and selling in $t=2$ gives rise to $P_1(1)+\frac{1+\delta}{3}P_2(0,0)-\frac{2+2\delta}{3}\bar{V}$. While not trading in $t=2$ is strictly dominated by not trading, the uninformed speculator who sells in $t=1$ may either prefer not trading or selling in $t=2$, depending on the parameter ranges. For the uninformed speculator who does not trade in $t=1$, not trading in $t=2$ yields a total profit of 0. Buying in $t=2$ yields a total profit of $\bar{V}-\frac{1-\delta}{3}P_2(\cdot,0)-\frac{1}{3}P_2(\cdot,1)-\frac{1+\delta}{3}P_2(\cdot,2)<0$, where $P_2(\cdot,2)=V^+$ and $P_2(\cdot,1)>\bar{V}$. Selling in $t=2$ yields a total profit of $\frac{1+\delta}{3}\left(P_2(\cdot,0)-\bar{V}\right)$, where $P_2(\cdot,0)<\bar{V}$. Hence, when Q_1 does not reveal a speculator's type, the uninformed speculator does not buy in $t=2$, and does not trade in $t=2$ if he does not trade in $t=1$.

■

Proof of Proposition 3.11. To start with, we demonstrate that neither the negatively informed speculator nor the uninformed speculator buys in $t=1$. For the negatively informed speculator reaching information set (iv), his best response is to sell in $t=2$, and the profit from buy – sell is $\frac{1+\delta}{3}P_2(\cdot,0)-P_1(\cdot)=\frac{1+\delta}{3}P_2(\cdot,0)-\frac{1}{3}P_2(\cdot,0)-\frac{1}{3}\frac{1}{2}P_2(\cdot,1)-\frac{1+\delta}{3}\frac{1}{2}P_2(\cdot,2)=\frac{\delta}{3}P_2(\cdot,0)-\frac{1}{6}V^+-\frac{1+\delta}{6}V^+<0$. For the negatively informed speculator reaching information set (v), his best response is to sell in $t=2$, and his profit from buy – sell given that the uninformed speculator chooses to not trade in $t=2$ is $\frac{1+\delta}{3}P_2(\cdot,0)-P_1(\cdot)=\frac{1+\delta}{3}P_2(\cdot,0)-\frac{1}{3}P_2(\cdot,0)-\left(\frac{1}{3}\frac{\alpha}{2}+\frac{1+\delta}{3}(1-\alpha)\right)P_2(\cdot,1)-\frac{1+\delta}{3}\frac{\alpha}{2}P_2(\cdot,2)=\frac{\delta}{3}P_2(\cdot,0)-\left(\frac{\alpha}{6}+\frac{(1+\delta)(1-\alpha)}{3}\right)P_2(\cdot,1)-\frac{(1+\delta)\alpha}{6}P_2(\cdot,2)<0$, where $P_2(\cdot,0)<\bar{V}$, $P_2(\cdot,1)>\bar{V}$ and $P_2(\cdot,2)=V^+$. His profit from buy – sell if the uninformed speculator sells in $t=2$ is $\frac{1+\delta}{3}P_2(\cdot,0)-$

$\frac{1+\delta-\alpha\delta}{3}P_2(\cdot,0)-\frac{\alpha}{6}P_2(\cdot,1)-\frac{(1+\delta)\alpha}{6}P_2(\cdot,2)=\frac{\alpha\delta}{3}P_2(\cdot,0)-\frac{2\alpha+\alpha\delta}{6}V^+<0$. While the negatively informed speculator receives a negative profit from buying in $t=1$, no trade – no trade guarantees a profit of 0, he thus does not buy in $t=1$.

For the uninformed speculator reaching information set (ii), his best response is to sell in $t=2$, and his earns profit, from buy – sell, of $\frac{1+\delta}{3}P_2(\cdot,0)+\frac{1}{3}P_2(\cdot,-1)+\frac{1-\delta}{3}P_2(\cdot,-2)-P_1(\cdot)=\frac{1+\delta}{3}P_2(\cdot,0)+\frac{2-\delta}{3}\bar{V}-\left(\frac{\alpha}{2-\alpha}V^++\frac{2(1-\alpha)}{2-\alpha}\bar{V}\right)<0$, where $P_2(\cdot,0)<\frac{\alpha}{2-\alpha}V^++\frac{2(1-\alpha)}{2-\alpha}\bar{V}$. For the uninformed speculator reaching information set (v), his best response is to not trade in $t=2$, and his profit from buy – no trade is $\frac{2+\delta}{3}\bar{V}-P_1(\cdot)<0$, where $P_1(\cdot)=\frac{\alpha}{2}V^++\frac{(2+\delta)(1-\alpha)}{3}\bar{V}+\frac{(1+\delta)\alpha}{6}V^-$. While the uninformed speculator receives a negative profit if he buys in $t=1$, no trade – no trade guarantees a profit of 0, he thus does not buy in $t=1$. Recall that in Lemma 4, the uninformed speculator who sells in $t=1$ may either sell in $t=2$ or not trade in $t=2$ if Q_1 does not reveal that he's not positively informed, depending on parameter values. However, a sell – no trade strategy by the uninformed speculator does not cause the manipulative short selling equilibrium. Hereafter, to remain comparability with prior models, we consider the case where the uninformed speculator who sells in $t=1$ sells again in $t=2$ if Q_1 does not reveal that he's not positively informed.

In the increased noise buying setting, there are four candidate equilibriums. Candidate equilibrium 1 is: the positively informed speculator employs a buy – buy strategy, the negatively informed speculator employs a no trade – sell strategy, and the uninformed speculator employs a sell – sell strategy. Candidate equilibrium 2 is: the positively informed speculator employs a buy – buy strategy, the negatively informed speculator employs a no trade – sell strategy, and the uninformed speculator employs a no trade – no trade strategy. Candidate equilibrium 3 is the *RIMS* equilibrium. Candidate equilibrium 4 is the *NMS* equilibrium.

(*a*). In candidate equilibrium 1, $P_2(1,0)=\bar{V}-\frac{\delta V^+}{2}+\frac{\delta V^-}{2}$ and $P_1(0)=\frac{1+\delta}{3}\bar{V}+\frac{\delta}{3}V^+-\frac{\alpha\delta}{6}V^+$. By not trading in $t=1$ and selling (resp. showing indifference between all three actions) in $t=2$ if $Q_1=0,1$ (resp. if $Q_1=-1$), the negatively informed speculator receives $\frac{1}{3}\left(\frac{1+\delta}{3}\left(P_2(1,0)-V^-\right)\right)+\frac{1}{3}\left(\frac{1+\delta}{3}\left(P_2(0,0)-V^-\right)\right)$. His best deviation is sell – sell, which yields a profit of $\frac{1}{3}\left(P_1(0)+\frac{1+\delta}{3}P_2(0,0)-\frac{2+2\delta}{3}V^-\right)$. The negatively informed speculator does not deviate from no trade – sell if and only if $\frac{1+\delta}{3}P_2(1,0)>P_1(0)\Leftrightarrow\delta(1+\delta)\left(V^+-V^-\right)<(\alpha\delta-2\alpha)V^+$, which never holds for any

$\delta \in [0,1]$. Candidate equilibrium 1 is not sustainable in the increased noise buying setting.

(b). In candidate equilibrium 2, $P_2(1,0) = (1-\delta)\frac{\alpha}{2}V^+ + (1+\delta)\frac{\alpha}{2}V^- + (1-\alpha)\bar{V}$, $P_1(0) = \frac{\alpha}{2}V^+ + \frac{(2+\delta)(1-\alpha)}{3}\bar{V} + \frac{(1+\delta)\alpha}{6}V^-$. The negatively informed speculator again does not deviate from no trade – sell to sell – sell if and only if $\frac{1+\delta}{3}P_2(1,0) > P_1(0) \Leftrightarrow \frac{\delta}{3}\left((1-\delta)\frac{\alpha}{2}V^+ + (1+\delta)\frac{\alpha}{2}V^- + (1-\alpha)\bar{V}\right) > \frac{\alpha}{6}V^+ + \frac{1+\delta}{3}(1-\alpha)\bar{V} + \frac{1+\delta}{3}\frac{\alpha}{2}V^+$, which never holds for any $\delta \in [0,1]$. Candidate equilibrium 2 is not sustainable in the increased noise buying setting.

(c). In the *RIMS* equilibrium, the prices are $P_2(1,0) = P_2(0,2) = P_2(0,1) = V^+$, $P_2(0,0) = \frac{(1-\delta)\alpha V^+ + (1+\delta)\alpha V^- + 2(1+\delta)(1-\alpha)\bar{V}}{2\alpha + 2(1+\delta)(1-\alpha)}$, and $P_1(0) = \frac{\alpha V^+}{2} + \frac{1+\delta}{3}(1-\alpha)\bar{V} + \frac{(1+\delta)\alpha}{6}V^-$. The profit of the negatively informed speculator who sell – sell is $\frac{1}{3}\left(P_1(0) + \frac{1+\delta}{3}P_2(0,0) - \frac{2+2\delta}{3}V^-\right)$. His alternative strategy of no trade – sell yields $\frac{1}{3}\left(\frac{1+\delta}{3}\left(P_2(1,0) - V^-\right)\right) + \frac{1}{3}\left(\frac{1+\delta}{3}\left(P_2(0,0) - V^-\right)\right)$. He does not deviate if and only if $P_1(0) + \frac{1+\delta}{3}P_2(0,0) - \frac{2+2\delta}{3}V^- > \frac{1+\delta}{3}\left(P_2(1,0) - V^-\right) + \frac{1+\delta}{3}\left(P_2(0,0) - V^-\right)$. Rearranging terms, we have $\delta < \frac{3P_1(0)}{P_2(1,0)} - 1$, which is equivalent to

$$\delta < \delta_1 \equiv \frac{2\alpha V^+ - (V^+ - V^-)}{(V^+ - V^-) + \alpha V^+}. \tag{A3.1.28}$$

When $V^+ \geq -3V^-$ and $\alpha > \frac{2}{3}$, $2\alpha V^+ - (V^+ - V^-) = (2\alpha - 1)V^+ - V^- > 0$, meaning that $\delta_1 > 0$; $2\left(2\alpha V^+ - (V^+ - V^-)\right) - (V^+ - V^-) - \alpha V^+ = 3\alpha V^+ - 3V^+ + 3V^- < 0$, meaning that $\delta_1 < \frac{1}{2}$.

On the other hand, the equilibrium profit of the uninformed speculator who has a sell – sell strategy is $\frac{1}{3}\left(P_1(0) + \frac{1+\delta}{3}P_2(0,0) - \frac{2+2\delta}{3}\bar{V}\right)$. His alternative strategy is "no trade in $t=1$, and sell (no trade) in $t=2$ if the order flow suggests that he is not negatively informed (does not reveal or exclude any type)", leading to a profit of $\frac{1}{3}\frac{1+\delta}{3}\left(P_2(1,0) - \bar{V}\right)$. He has no incentive to deviate if $\frac{1}{3}\left(P_1(0) + \frac{1+\delta}{3}P_2(0,0) - \frac{2+2\delta}{3}\bar{V}\right) > \frac{1}{3}\frac{1+\delta}{3}\left(P_2(1,0) - \bar{V}\right)$. Rearranging terms, we have

$$P_1(0) + \frac{1+\delta}{3}P_2(0,0) - \frac{1+\delta}{3}P_2(1,0) - \frac{1+\delta}{3}\bar{V} > 0 \Leftrightarrow$$

$$\frac{\left(-\alpha^2 V^+ + 3\alpha V^+ - 2V^-\right)\delta^2 + \left(2\alpha^2 V^+ + \alpha V^+ - 4V^-\right)\delta - 2V^- - 2\alpha V^+}{(6\alpha - 6)\delta - 6} > 0 \Leftrightarrow$$

$$\delta < \delta_2 \equiv -\frac{\alpha - \alpha\sqrt{(\eta+1)\left(25\eta - 4\alpha - 4\alpha\eta + 4\alpha^2\eta + 4\alpha^2 + 49\right)} + \alpha\eta + 2\alpha^2\eta + 2\alpha^2 + 4}{2\left(3\alpha + 3\alpha\eta - \alpha^2\eta - \alpha^2 + 2\right)}. \tag{A3.1.29}$$

where $\eta \in R^+$ is defined by $V^+ = -(1+\eta)V^-$. Taking first order derivative in respect to η, we have $\frac{\partial \delta_2}{\partial \eta} > 0$, indicating $\lim_{\eta \to 2} \delta_2 < \delta_2 < \lim_{\eta \to +\infty} \delta_2$. We also have $\lim_{\eta \to 2} \delta_2 = \frac{\alpha\sqrt{36\alpha^2 - 36\alpha + 297} - 3\alpha - 6\alpha^2 - 4}{-6\alpha^2 + 18\alpha + 4} > \frac{1}{5}$ and $\lim_{\eta \to +\infty} \delta_2 = \frac{2\alpha + 1 - \sqrt{(4\alpha^2 - 4\alpha + 25)}}{2(\alpha - 3)} < \frac{\sqrt{217} - 7}{14}$ for $\alpha > \frac{2}{3}$.

(d). In the *NMS* equilibrium, the prices ar $P_2(1,0) = \frac{\alpha(1-\delta)V^+ + 2(1-\alpha)(1+\delta)\bar{V}}{\alpha(1-\delta) + 2(1-\alpha)(1+\delta)}$, $P_2(0,0) = \frac{(1-\delta)\alpha}{2} V^+ + \frac{(1+\delta)\alpha}{2} V^- + (1-\alpha)\bar{V}$, and $P_1(0) = \frac{\alpha V^+}{2} + \frac{(2+\delta)(1-\alpha)}{3}\bar{V} + \frac{(1+\delta)\alpha}{6} V^-$. The negatively informed speculator shows no deviation from his equilibrium strategy of sell – sell to the alternative strategy of no trade – sell if $P_1(0) + \frac{1+\delta}{3} P_2(0,0) - \frac{2+2\delta}{3} V^- > \frac{1+\delta}{3}\left(P_2(1,0) - V^-\right) + \frac{1+\delta}{3}\left(P_2(0,0) - V^-\right)$. Rearranging terms, we obtain

$$P_1(0) > \frac{1+\delta}{3} P_2(1,0) \Leftrightarrow$$

$$\frac{(A\delta^2 + B\delta + C)}{D} > 0 \Leftrightarrow$$

$$\delta > \delta_3 \equiv \frac{3\alpha\eta - 2\eta - 2\alpha + 2\alpha^2\eta + 5\alpha^2 + \sqrt{E}}{6\alpha^2\eta - 2\alpha\eta + 6\alpha^2}, \tag{A3.1.30}$$

where $A = \alpha V^+ + \alpha V^- - 3\alpha^2 V^+$, $B = 3\alpha V^+ - 2V^- - 2V^+ + 5\alpha V^- + 2\alpha^2 V^+ - 3\alpha^2 V^-$, $C = 2\alpha V^- - 2V^- - 2V^+ + \alpha^2 V^+ - \alpha^2 V^-$, $D = (18\alpha - 12)\delta + 6\alpha - 12$, and $E = 16\alpha^4\eta^2 + 56\alpha^4\eta + 49\alpha^4 + 8\alpha^3\eta^2 - 10\alpha^3\eta - 44\alpha^3 - 23\alpha^2\eta^2 + 48\alpha^2\eta + 4\alpha^2 - 4\alpha^2\eta + 8\alpha\eta + 4$. As $\frac{\partial \frac{3\alpha\eta - 2\eta - 2\alpha + 2\alpha^2\eta + 5\alpha^2}{6\alpha^2\eta - 2\alpha\eta + 6\alpha^2}}{\partial \eta} > 0$, $\delta_3 > \lim_{\eta \to 2} \frac{3\alpha\eta - 2\eta + 2\alpha + 2\alpha^2\eta + 5\alpha^2}{6\alpha^2\eta - 2\alpha\eta + 6\alpha^2} = \frac{9\alpha^2 + 4\alpha - 4}{18\alpha^2 - 4\alpha} > \frac{1}{2}$ for any $\frac{2}{3} < \alpha \leqslant 1$. The uninformed speculator does not deviate from his equilibrium strategy of "no trade in $t = 1$, and sell (no trade) in $t = 2$ if the order flow suggests that he is not negatively informed (does not reveal or exclude any type)" to no trade – sell if $\frac{1}{3}\frac{1+\delta}{3}\left(P_2(1,0) - \bar{V}\right) > \frac{1}{3}\left(P_1(0) + \frac{1+\delta}{3} P_2(0,0) - \frac{2+2\delta}{3}\bar{V}\right)$, which is equivalent to

$$\begin{aligned}&\left(\frac{\alpha V^-}{6} - \frac{\alpha V^+}{6}\right)\delta^2 + \left(\frac{\alpha V^+}{6} - \frac{V^+}{3} - \frac{V^-}{3} + \frac{\alpha V^-}{3} + \frac{2(1-\alpha)\bar{V}}{3}\right)\delta \\ &- \frac{V^+}{6} - \frac{V^-}{6} + \frac{\alpha V^+}{2} + \frac{\alpha V^-}{6} + \frac{2(1-\alpha)\bar{V}}{3} < 0 \Leftrightarrow\end{aligned}$$

$$\delta > \delta_4 \equiv \frac{\sqrt{\alpha\left(17\alpha + 8\eta + 18\alpha\eta + 5\alpha\eta^2 + 4\eta^2\right)} - \alpha - \alpha\eta}{4\alpha + 2\alpha\eta} > \frac{1}{2}. \tag{A3.1.31}$$

■

Proof of Corollary 3.12. (i) In the *RIMS* equilibrium, the uninformed speculator and the negatively informed speculator have a sell – sell strategy, and the positively informed speculator has a buy – buy strategy. The expected firm value for the uninformed speculator is $\frac{(1+\delta)\bar{V}}{9}$. The expected firm value for the negatively informed speculator is $\frac{(1+\delta)V^-}{9}$. The expected firm value for the positively informed speculator is V^+. The real investment efficiency in the *RIMS* equilibrium then follows as

$$RE_{RIMS} = \frac{\alpha V^+}{2} + \frac{\alpha(1+\delta)V^-}{18} + \frac{(1-\alpha)(1+\delta)\bar{V}}{9}. \tag{A3.1.32}$$

(ii) The *RIMS* equilibrium always harms the real investment efficiency since

$$RE_{RIMS} - RE_{benchmark} = \frac{(1-\alpha)\delta\bar{V}}{9} + \frac{\alpha\delta V^-}{18} < 0. \tag{A3.1.33}$$

(iii) In the *NMS* equilibrium, the uninformed speculator does not trade in $t = 1$, does not trade in $t = 2$ if $Q_1 = 0$, sells in $t = 2$ if $Q_1 = 1$. The negatively informed speculator has a sell – sell strategy and the positively informed speculator has a buy – buy strategy. The expected firm value for the uninformed speculator is $\frac{(5+\delta)\bar{V}}{9}$. The expected firm value for the negatively informed speculator is $\frac{(1+\delta)V^-}{9}$. The expected firm value for the positively informed speculator is V^+. The real investment efficiency in the *NMS* equilibrium then follows as

$$RE_{NMS} = \frac{\alpha V^+}{2} + \frac{\alpha(1+\delta)V^-}{18} + \frac{(1-\alpha)(5+\delta)\bar{V}}{9}. \tag{A3.1.34}$$

(iv) The *NMS* equilibrium improves the real efficiency if and only if

$$RE_{NMS} - RE_{benchmark} = \frac{(1-\alpha)(4+\delta)\bar{V}}{9} + \frac{\alpha\delta(\bar{V} - U)}{18} > 0. \tag{A3.1.35}$$

■

Chapter 4

Quantitative vs discretionary investing: Implications for market efficiency

4.1 Introduction

Advances in mathematics and technology, from quantum physics to machine learning algorithms such as deep neural networks, have led to a breed of tech-savvy investors known as "quants" taking over the global financial markets in recent years.[10] Quantitative investing relies on machines and makes trading decisions using rule-based criteria, whereas traditional discretionary investing relies on human skills. Despite the rising regulatory concern over machine-based quantitative investing, academic research about the impact of quants on market quality is scant.[11] This paper thus aims to address the following two questions: Given the differences in the decision-making process of machines and humans, what strategic interaction between discretionary and quantitative investing will arise? And, with

[10] Quantitative hedge funds now account for 27% of U.S. stock trades (see, for example, https://www.wsj.com/articles/the-quants-run-wall-street-now-1495389108). Quantitative investment products also become increasingly accessible to retail investors via the popularity of quantitative mutual funds and online backtesting platforms such as Quantopian (Beggs, Brogaard and Hill-Kleespie (2021)).

[11] In 2021, The Securities and Exchange Commission announced charges against Sergei Polevikov, a former quantitative analyst, for perpetrating a front-running scheme that generated illicit profits of over 8.5 million USD, see https://www.sec.gov/news/press-release/2021-186.

the presence of such strategic interaction, how does the growth in machine-based quantitative investing impact market efficiency?

Machines are less susceptible to incentive and sentiment bias and possess a greater capacity to process information compared to humans due to their fast computing speed. But machines are not impeccable and may involve the risks of overfitting and crowding, as well as weaker flexibility to adapt to ever-changing market conditions than humans given their inherent reliance on fixed rules. Motivated by Abis (2022), we focus on the fundamental trade-off between capacity and flexibility when machines replace humans in the investment industry.

This paper builds a Kyle-type model populated by a fully discretionary investment firm, an investment firm composed of a quant department and a discretionary department, and liquidity traders, as well as competitive market makers. The two firms are referred to as firms A and B. Before trading takes place, nature determines if the quant department Q or discretionary department D of firm B is present. The prior probability of the quant department's existence measures the quantitative investing level in the economy. The quant department's greater information processing capacity (resp. weaker strategic flexibility) than discretionary participants is modelled as access to an additional piece of information (resp. ignorance of the strategic impact of its own trades). Discretionary participants observe the component v_1 of security's liquidation value $v = v_1 + v_2$, whereas the quant department additionally observes a noisy signal θ_2 of the component v_2. Reflecting the strategic weakness of the quant, Q incorrectly believes that the opponent firm would not strategically react to the demand of firm B. Our modelling of strategic inflexibility of the quant is in the spirit of Malikov and Pasquariello (2022) but technically different from them. In Malikov and Pasquariello (2022), the inflexibility of the quant is modelled as adherence to a backtested trading strategy with constant parameters. Instead, we model the strategic inflexibility of Q as an incorrect belief about the opponent firm's equilibrium strategy and allow Q to choose a strategy that optimises its expected trading profit with such misbelief.

We find that the strategic interaction between the two investment firms is impacted in three ways as the quantitative investing level increases. First, this increase renders firm B more capable of information processing and extracting the noisy signal θ_2. Firm B thereby trades more aggressively on θ_2 given its enhanced information advantage in inferring v_2, which is the "*capacity enhancing effect*". Second, being more likely to ignore the fully discretionary firm's strategic response, firm B is more inclined to overestimate the opponent's trading aggressiveness on v_1. Firm B thereby trades less aggressively with respect to v_1 given its increased strategic weakness, which is the "*stratey oblivion effect*". Finally, being fully discretionary, firm A trades more aggressively on v_1 in order to profit from firm B's increased strategic weakness. This change in firm A's trading behaviour effectively internalises firm B's trading aggressiveness reduction on v_1, and is referred to as the "*internalising effect*".

Given such strategic interaction, growth in quantitative investing reduces overall trading aggressiveness and thus price efficiency for low-quality signal θ_2 since the negative strategy oblivion effect dominates, whereas it increases trading aggressiveness and price efficiency for high-quality signal θ_2 since the negative strategy oblivion effect is dominated. Interestingly, for moderate-quality signal θ_2, growth in quantitative investing has a non-monotonic impact on overall trading aggressiveness and price efficiency.

We extend our analysis to include the setting where each of the two firms consists of a quant department and a discretionary department (see Section 4.4) and the setting where the quant department Q incorrectly believes that the fully discretionary firm A would not strategically react to B's demand with a probability of h, but has correct belief about the fully discretionary firm with a probability of $1-h$ (see Appendix 4.2). The comparison between the extended and the baseline models shows that the impacts of a ceteris paribus increase in firm B's quantitative investing level on the strategic interaction and on price efficiency are robust to different model settings. In the extended model where firms A and B each have a quant department, aside from the capacity enhancing effect, the strategy oblivion effect, and the internalising effect, a ceteris paribus increase in firm B's quantitative investing level additionally causes firm A to have deteriorated information advantage with regard to θ_2 given increased competition from firm B. The resulting trading aggressiveness reduction on θ_2 by firm A is referred to as the "*competition effect*".

Moreover, growth in quantitative investing level by firm B is more prone to decrease the overall trading aggressiveness and harm price efficiency when the opponent firm's quantitative investing level is higher due to the weakened internalising effect: firm A's discretionary department would put less effort into exploiting firm B's trading aggressiveness reduction on v_1.

Overall, our theory explicitly models the fundamental trade-off between capacity and flexibility when machines replace humans and sheds new light on the related market quality consequences. First, we analytically decompose the impact of growth in machine-based quantitative investing on overall trading aggressiveness and market efficiency into components driven by the capacity enhancing effect, the strategy oblivion effect, the internalising effect, and the competition effect. Given such decomposition, it is possible to test the four channels empirically.[12] Our theory additionally provides an alternative explanation to Farboodi, Matray, Veldkamp and Venkateswaran (2022)'s empirical observation that big data growth corrodes price informativeness for the universe of publicly traded U.S. stocks but enhances price informativeness for S&P 500 stocks. While Farboodi et al. (2022) attribute the price informativeness divergence to different average size growth rates of S&P 500 and non-S&P 500 stocks, our model suggests that different relative magnitudes of the capacity enhancing effect to the strategy oblivion effect of S&P 500 and non-S&P 500 stocks could be another driver for this divergence. Second, our model implies that regulators and practitioners should pay close attention to the potentially harmful growth in quantitative investing if there is a myriad of Artificial Intelligence ETFs and other quantitative products.

The rest of the paper is organised as follows. In Section 4.2, we discuss the related literature. In Section 4.3, we solve for the equilibrium in the benchmark model. In Section 4.4, we extend the benchmark model to the setting where each investment firm has a quant department. Section 4.5 concludes.

4.2 Related literature

Our paper contributes to several strands of literature. First, our finding contributes to the emerging literature on the machine-human interaction in financial markets (see, e.g., Barbopoulos et al. (2021), Wang et al. (2021), Abis (2022), Coleman et al. (2022), Malikov and Pasquariello (2022)). Abis (2022) shows that the trade-off of capacity and inflexibility results in different investment styles of

[12] The data on quantitative and discretionary investors are not readily available, making the manual collection of such data necessary (e.g., Beggs et al. (2021)). There are also no straightforward proxies for the strategic inflexibility of quants, though Abis (2022) demonstrate that the inflexibility of quants can result in a weak ability to time macroeconomic shocks. In this chapter, we focus on the theory of interaction between quants and discretionary investors and its implications for market efficiency and leave the empirical analysis for future work.

quantitative and discretionary mutual funds. In the model of Abis (2022), compared with quant funds, discretionary funds possess a higher macroeconomic timing ability and a lower stock picking ability and hold less diversified portfolios. Malikov and Pasquariello (2022) analyse the trading strategies and market quality outcome including efficiency, liquidity, volatility, and volume both when new quant funds enter and when incumbent discretionary funds switch to quantitative investing. They model the incumbent's transition to quantitative investing as increased adherence to the backtested strategy. Wang et al. (2021) empirically train a machine-only AI analyst and a man + machine analyst, and show that the price forecasts of the man + machine analyst outperform the forecasts of the human-only and machine-only analysts, since combining AI's computational power and the human art of interpreting soft information can generate synergy. Coleman et al. (2022) find empirical evidence that machine analysts are less inclined to recommend glamour stocks and firms with potential investment banking needs than human analysts are. Our work innovates this line of research by explicitly modelling the fundamental trade-off between capacity and flexibility when machines replace humans, and analytically decomposing the impact of growth in machine-based quantitative investing on strategic interaction and market efficiency into empirically testable components primarily driven by the capacity enhancing effect and the strategy oblivion effect.

Our paper is also broadly related to theoretical studies about the influence of irrational traders on price evolution (see, e.g., De Long, Shleifer, Summers and Waldmann (1990), Gervais and Odean (2001), Hirshleifer, Subrahmanyam and Titman (2006), Kogan, Ross, Wang and Westerfield (2006), Pouget, Sauvagnat and Villeneuve (2017)). According to De Long et al. (1990), bullish noise traders bear more systematic risk than rational investors do, and thus earn higher expected returns and survive in the long run as the market rewards risk taking. Hirshleifer et al. (2006) show that irrational traders can influence underlying cash flows due to the feedback effect and thus earn higher profits than rational informed traders do. Kogan et al. (2006) suggest that irrational traders (pessimistic traders and strongly optimistic traders), who take bets on extremely unlikely states, can give rise to significant price impact even when their wealth is negligible. Gervais and Odean (2001) propose a model with traders becoming overconfident via learning from the successes and failures of their past price predictions. They show that both volume and volatility increase with a trader's overconfidence. Pouget et al. (2017) build a model in which some traders are prone to confirmatory bias and ignore information inconsistent with their prior views, and find that such bias yields excess volume, excess volatility, as well as momentum. We add to this line of research by modelling the quant's strategic inflexibility, a form of irrationality, as an incorrect belief about the opponent firm's equilibrium strategy.

We also contribute to the literature on how FinTech adoption affects market efficiency. In the big data literature, increased computing power may only benefit the information corporation of large firms (Farboodi et al. (2022)), and may cause imprecise signals to crowd out precise signals and harm efficiency (Dugast and Foucault (2018)). Moreover, high dimensional learning problems, coupled with data abundance, can distort conventional efficiency measures (Martin and Nagel (2022)). In the algorithmic trading (AT) literature, AT benefits efficiency if its liquidity provision raises informed profits (Hendershott, Jones and Menkveld (2011)), and harms efficiency if back-running erodes information rents (Weller (2018)). We contribute to this literature by analysing the price efficiency impacts of quantitative investing, a relatively underexplored subset of algorithmic trading.[13]

4.3 The model

We consider a Kyle-type model where quantitative and discretionary traders coexist. The model enables us to study the interaction between quantitative and discretionary traders and its impact on price efficiency.

4.3.1 Model setup

Three groups of participants trade risky security: (i) market makers, (ii) noise traders, and (iii) two investment firms (firms A and B). The liquidation value v of the security is given by $v = v_1 + v_2$, where v_1 and v_2 are independent and identically distributed normal variables with mean zero and variance Σ_0. Investment firms are composed of a discretionary department D and a quant department Q. The discretionary department observes v_1 and trades akin to the rational Kyle (1985)

[13] According to Hasbrouck and Saar (2013), AT can be divided into two categories, proprietary, i.e., quantitative, and agency algorithms. Agency algorithms are used by buy-side institutions to minimise the price impact of executing trades when implementing changes in their investment portfolios. Agency algorithms are non-fundamental oriented, e.g., the VWAP execution algorithm. Proprietary algorithms can be either non-fundamental-oriented (e.g., liquidity provision and back-running) or fundamental-oriented.

insider. The quant department additionally observes a noisy signal $\theta_2 = v_2 + \epsilon$ with $\epsilon \sim N(0, \sigma_\epsilon^2)$ about the second component of the liquidation value.

We first investigate the case where firm A is entirely discretionary, and firm B has a discretionary department D and a quant department Q. The next section solves the model for the general case where both firms have both departments. The model has three dates, $t \in \{0, 1, 2\}$, as illustrated in Figure 4.1. At $t = 0$, nature determines which department of firm B is in the market. The probability of the quant department's presence is γ, whereas the probability of the discretionary department's presence is $1 - \gamma$, where $0 \leqslant \gamma \leqslant 1$ measures the level of quantitative investing. The realisation of department Q's or department D's presence is only observed by firm B.

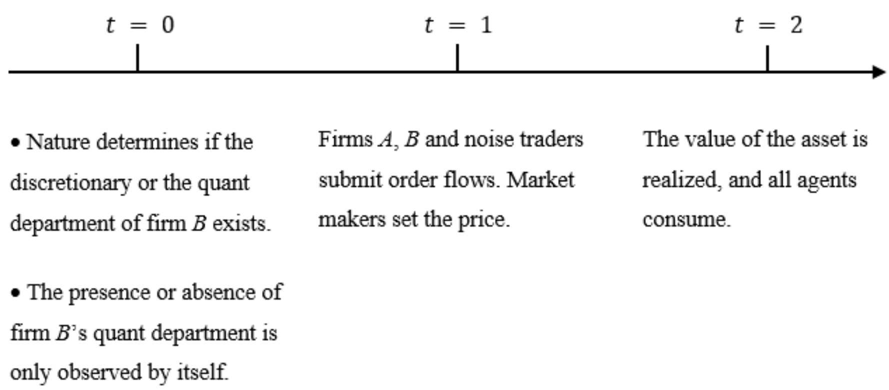


Figure 4.1: Model timeline.
This figure illustrates the timeline in the baseline model. In $t = 0$, nature determines which department of firm B presents. In $t = 1$, trading occurs for firms A, B and noise traders. In $t = 2$, the security is liquidated.

The trading date is $t = 1$. Let us denote the price and aggregate order flow at $t = 1$ as p and w, respectively. We first assume that traders rationally conjecture the price function as $p = \lambda \cdot \omega$. Given the conjectures about the pricing schedule $p = \lambda \cdot \omega$ and the opponent's demand of x_j shares, trader $i \in \{A, D, Q\}$ endowed with the information set ϕ_i submits x_i shares of market orders to maximise the expected trading profits, i.e.,

$$E\left[(v - p(\phi_i, x_i, x_j))x_i | \phi_i, x_i\right]. \tag{4.1}$$

For the discretionary departments, $\phi_A = \phi_D = v_1$. For the quant department, $\phi_Q = \{v_1, \theta_2\}$. The additional signal θ_2 captures the greater information processing capacity of the quant department. The quality of the additional signal is $q =$

$\Sigma_0/(\Sigma_0 + \sigma_\epsilon^2)$. Although the quant department has greater information processing capacity, it is strategically weaker than the discretionary department. To capture the strategic weakness of the quant department, we assume that it ignores the competition from itself can reduce the trading aggressiveness of firm A. While the actual demand of firm A is $x_A = \frac{E(v|\phi_A)}{2\lambda} - \frac{E(x_B|\phi_A)}{2}$, the quant department perceives the demand of firm A as $x_A = \frac{E(v|\phi_A)}{2\lambda}$. This means that the strategically weaker quant department ignores its strategic impact on the opponent.

Noise traders trade according to their liquidity or hedging needs exogenous to the model. We assume they demand $u \sim N(0, \Sigma_0)$ units for simplicity. Market makers observe the aggregate order flow (sum of all traders' demands) as $\omega = x_A + \gamma \cdot x_Q + (1-\gamma) \cdot x_D + u$. They are risk neutral and competitive. Therefore, their equilibrium pricing rule satisfies weak market efficiency and allows them to break even in expectation, i.e., $p = E[v|\omega]$.

4.3.2 Equilibrium

The equilibrium concept throughout the chapter is perfect Bayesian Nash equilibrium.[14] It is defined as follows: (i) the trader i chooses a trading strategy $x_i(\phi_i, E(x_j))$ that maximises his expected trading profit based on the expectation of opponent's demand $E(x_j)$, (ii) the market maker chooses a pricing strategy $p(\omega)$ that breaks even in expectation, (iii) the traders and market makers use Bayes' theorem to form beliefs.

To derive the equilibrium, we start with optimization problems defined in Eq.(4.1) and derive the first-order conditions. Solving the maximisation problem of firm A yields its best response $x_A = \alpha_A v_1$, with

$$\alpha_A = \frac{1}{2\lambda} - \frac{\gamma\alpha_Q}{2} - \frac{(1-\gamma)\alpha_D}{2}. \tag{4.2}$$

Solving the maximisation problem faced by the quant department of firm B yields its best response $x_Q = \alpha_Q v_1 + \beta_Q \theta_2$, with

$$\alpha_Q = \frac{1}{4\lambda},\ \beta_Q = \frac{q}{2\lambda}. \tag{4.3}$$

[14] The linear equilibrium in our model exists uniquely because we adopt the Kyle (1985) framework, and particularly because we assume normality of the exogenous random variables.

Solving the maximisation problem faced by the discretionary department of firm B yields its best response $x_D = \alpha_D v_1$, with

$$\alpha_D = \frac{1}{2\lambda} - \frac{\alpha_A}{2}. \tag{4.4}$$

Combining Eqs.(4.2)-(4.4) with $\lambda = cov(v,\omega)/var(\omega)$ gives a system of five equations and five unknowns $(\lambda, \alpha_A, \alpha_Q, \beta_Q, \alpha_D)$. The following proposition gives the solution to the system, which summarises the equilibrium outcomes in the economy with a discretionary and a quantitative firm.

Proposition 4.1. *There is a unique linear equilibrium if an investment firm is fully discretionary, and the other firm has a quantitative investing intensity of γ. The equilibrium pricing rule of market makers is characterised by $p = \lambda\omega$, and the equilibrium trading strategies of traders A, Q, and D are given by $x_A = \alpha_A v_1$, $x_Q = \alpha_Q v_1 + \beta_Q \theta_2$, and $x_D = \alpha_D v_1$, with*

$$\lambda = \sqrt{\frac{(\gamma+4)(\gamma+2)}{4(\gamma+3)^2} + \frac{q\gamma(2-\gamma)}{4}}, \tag{4.5}$$

$$\alpha_Q = \frac{1}{4\lambda},\ \alpha_D = \frac{\gamma+4}{4\lambda(\gamma+3)}, \tag{4.6}$$

$$\beta_Q = \frac{q}{2\lambda},\ \alpha_A = \frac{\gamma+2}{2\lambda(\gamma+4)}. \tag{4.7}$$

4.3.3 Quantitative investing and price efficiency

We now analyse how the growth in quantitative investing affects the trading aggressiveness of investment firms and consequently, affects informational efficiency of the price. We normalise the trading aggressiveness of each firm on each signal by dividing $1/3\lambda$, which is the trading aggressiveness of the firms if both were fully discretionary, leading to

$$\widetilde{\alpha}_A = \frac{3\gamma+6}{2\gamma+6},\ \widetilde{\alpha}_B = \frac{3}{\gamma+3},\ \widetilde{\beta}_B = \frac{3q\gamma}{2}. \tag{4.8}$$

Define the overall trading aggressiveness as $\tau \equiv \widetilde{\alpha}_A + \widetilde{\alpha}_B + \widetilde{\beta}_B$. The following proposition identifies the effects of growth in quantitative investing on overall trading aggressiveness.

Proposition 4.2. *When quantitative investing level γ increases, the impact on overall trading aggressiveness τ hinges on the trade-off among:*

(i) The positive capacity enhancing effect by the quantitative firm B, implying B trades more aggressively on θ_2 due to increased capacity, and is given by

$$\frac{\partial \widetilde{\beta}_B}{\partial \gamma} = \frac{3q}{2}. \tag{4.9}$$

(ii) The negative strategy oblivion effect by the quantitative firm B, implying B trades less aggressively on v_1 due to increased inflexibility, and is given by

$$\frac{\partial \widetilde{\alpha}_B}{\partial \gamma} = -\frac{3}{(\gamma+3)^2}. \tag{4.10}$$

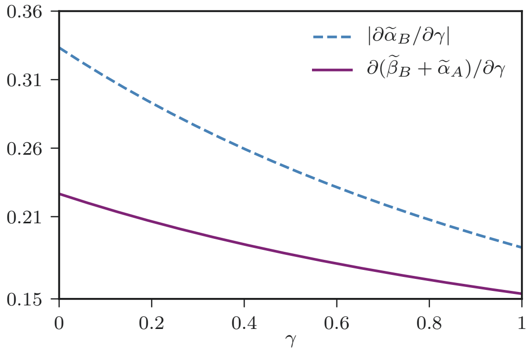
(iii) The positive internalising effect by the fully discretionary firm A, implying A trades more aggressively on v_1 to exploit increased inflexibility of B, and is given by

$$\frac{\partial \widetilde{\alpha}_A}{\partial \gamma} = \frac{3}{2(\gamma+3)^2}. \tag{4.11}$$

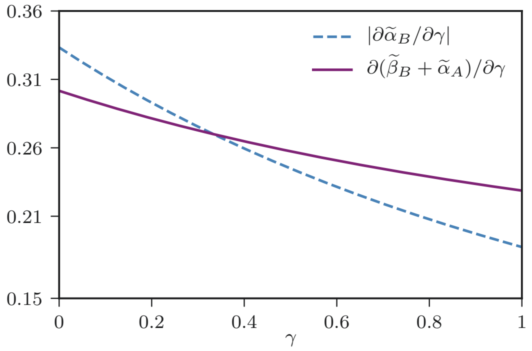
An increase in the quantitative investing level γ increases both information processing capacity and strategic inflexibility. On the one hand, higher information processing capacity permits firm B a higher access to θ_2 and a higher information advantage in inferring v_2. We refer to the resulting increase in firm B's trading aggressiveness on θ_2 as the *capacity enhancing effect*. On the other hand, higher strategic inflexibility makes firm B become more likely to overestimate the trading aggressiveness of firm A. We refer to the resulting decrease in firm B's trading aggressiveness on v_1 as the *strategy oblivion effect*. Additionally, the fully discretionary firm A tries to exploit firm B's increased strategy oblivion by increasing its trading aggressiveness on v_1, leading to the *internalising effect*.

Corollary 4.3 analytically describes how these three effects vary with q and γ. Figure 4.2 graphically illustrates the three effects for different values of q. Panels (A)-(C) show these effects for $q = 0.04, 0.09$ and 0.12, respectively. We combine Figure 4.2 with Corollary 4.3 to analyse the net impact of growth in γ on overall trading aggressiveness under different parameter settings.

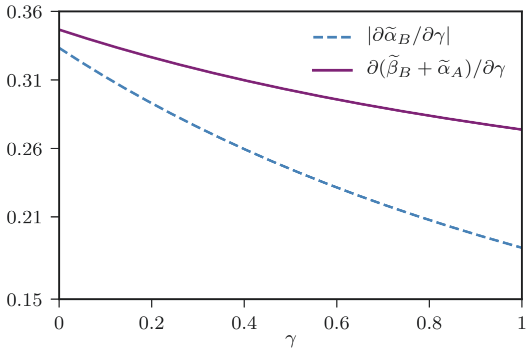
Corollary 4.3. *The magnitude of the capacity enhancing effect increases in signal quality q. The magnitude of the strategy oblivion effect decreases in quantitative investing level γ at a faster rate than the magnitude of the internalising effect does.*



(A)



(B)



(C)

Figure 4.2: Trading aggressiveness when the opponent firm is fully discretionary.
This figure illustrates the derivatives of trading aggressiveness measures with respect to quantitative investing level γ in the setting of fully discretionary firm R when: (i) q is low; (ii) q is moderate; and (iii) q is high. Panel (A) illustrates the derivatives when $q = 0.04$, Panel (B) illustrates the derivatives when $q = 0.09$, and Panel (C) illustrates the same when $q = 0.12$. The other parameter value is $\Sigma_0 = 1$. Among all panels, the blue dashed line is the absolute value of the strategy oblivion effect, and the purple line is the sum of the capacity enhancing effect and the internalising effect.

According to the above corollary, the positive capacity enhancing effect is strengthened when q increases. Consistently, for low quality θ_2 as in Panel (A) of Figure 4.2, the sum of the positive effects is surpassed by the absolute value of the negative strategy oblivion effect, implying that an increase in quantitative investing level decreases the overall trading aggressiveness for any γ. For high quality θ_2 as in Panel (C) of Figure 4.2, the sum of the positive effects exceeds the absolute value of the negative strategy oblivion effect, implying that an increase in quantitative investing level increases the overall trading aggressiveness for any γ.

Corollary 4.3 also states that an increase in γ decreases the magnitude of the strategy oblivion effect. This result is intuitive. Consider two extreme cases of $\gamma = 0$ and $\gamma \to 1$: the same amount of increase in γ should lead to a stronger marginal effect on firm B's trading aggressiveness on v_1 when $\gamma = 0$ than when $\gamma \to 1$, since the increase at $\gamma = 0$ represents an initial deviation from full rationality which fundamentally alters firm B's trading behaviour. Also, an increase in γ dampens the strategy oblivion effect more than it dampens the internalising effect. As a consequence, for moderate quality θ_2 as shown in Panel (B) of Figure 4.2, the sum of the two positive effects starts off smaller than the absolute value of the strategy oblivion effect if γ is small but would dominate if γ increases further, which means the overall trading aggressiveness first decreases then increases when γ increases.

Additionally, an increase in q does not affect the curve representing the strategy oblivion effect, and shifts up the curve representing the sum of the capacity enhancing effect and the internalising effect without changing its shape. This is because the capacity enhancing effect is linear in q, while the other two effects are independent of q.

As trading is the sole channel that impounds information into prices in the model, changes in trading aggressiveness should naturally translate into changes in price efficiency. Following Kyle (1985) and its extant extensions (e.g., Subrahmanyam (1991), Holden and Subrahmanyam (1992), and Yang and Zhu (2020)), the price efficiency measure we use throughout the paper is $var(v|p)^{-1}$. In the baseline equilibrium, the price efficiency is thus given by the following equation:

$$PE_{baseline} = \frac{2\gamma + 6}{(3\gamma - 3q\gamma - q\gamma^2 + 8)\Sigma_0}. \tag{4.12}$$

Proposition 4.4 shows the price efficiency impact of growth in quantitative trading when a firm is fully discretionary and the other has a quant department.

Proposition 4.4. *When quantitative investing level γ increases, the effect on price efficiency depends on the quality of the additional signal obtained by the quant department:*

(i) When $0 < q \leq q_1$, price efficiency decreases with quantitative investing level.

(ii) When $q_1 < q < q_2$, price efficiency decreases if quantitative investing level is below the threshold value $\bar{\gamma}$, and increases otherwise.

(iii) When $q \geq q_2$, price efficiency increases with quantitative investing level.

The sign of the price efficiency impact rests with the value of signal quality q. When signal θ_2 is of low quality ($q \leq q_1$), growth in quantitative investing always harms price efficiency due to the unambiguous drop in overall aggressiveness for any γ. When signal θ_2 is of high quality ($q > q_2$), growth in quantitative investing always improves price efficiency. When signal θ_2 is of moderate quality, growth in quantitative investing initially harms price efficiency, then improves price efficiency if γ increases beyond the threshold $\bar{\gamma}$ and the sum of capacity enhancing, strategy oblivion and internalising effects turns from smaller than to greater than zero. Figure 4.3 provides numerical examples of the proposition.

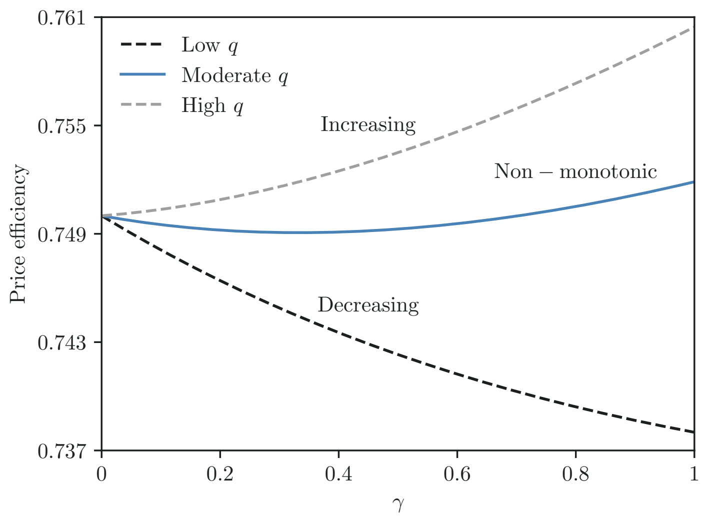


Figure 4.3: Price efficiency with a fully discretionary opponent firm. This figure illustrates the price efficiency impact of growth in quantitative investing level γ in the setting of fully discretionary firm A when: (i) q is low; (ii) q is moderate; and (iii) q is high. The black dashed line, the blue line, and the grey dashed line plot the price efficiency as a function of γ for $q = 0.04$, $q = 0.09$, and $q = 0.12$, respectively. The other parameter value is $\Sigma_0 = 1$.

4.4 Opponent firm with quant department

Thus far, firm A has been assumed to be fully discretionary and there is a monopoly for quantitative technology. In reality, quantitative funds are on the rise and face fierce competition among each other (Abis (2022), Farboodi et al. (2022)). To relax the rather unrealistic assumption about a fully discretionary opponent firm, we analyse a symmetric model in which each firm has a quant department in this section.

Consider now that firm A is also composed of a quant department Q_1 and a discretionary department D_1, different from being fully discretionary in the baseline model. Let firm B's quant department be denoted as Q_2, and its discretionary department be denoted as D_2. At $t = 0$, the nature determines which department presents for each firm. The probability of the presence of firm A's quant department is γ_1, whereas the probability of the presence of firm B's quant department is γ_2, with $0 \leq \gamma_1 \leq 1$ and $0 \leq \gamma_2 \leq 1$.

All traders know that the pricing schedule at $t = 1$ takes the form of $p = \lambda'\omega$. The demands of D_1, Q_1, D_2 and Q_2 are x_{D1}, x_{Q1}, x_{D2}, and x_{Q2}, respectively. Since the opponent firm A is no longer fully discretionary, both firm A's and firm B's ex-ante demand is a weighted sum of demands from their own quant and discretionary departments:

$$x_A = \gamma_1 x_{Q1} + (1 - \gamma_1)x_{D1}, \tag{4.13}$$

$$x_B = \gamma_2 x_{Q2} + (1 - \gamma_2)x_{D2}. \tag{4.14}$$

If present in the market, trader D_2 endowed with v_1 optimally chooses x_{D2} to maximise the expected trading profit defined by Eq.(4.15), whereas trader Q_2 endowed with v_1 and θ_2 optimally chooses x_{Q2} to maximise the expected trading profit defined by Eq.(4.16):

$$E\left[(v - p(v_1, x_{D2}, x_R))x_{D2}|v_1\right], \tag{4.15}$$

$$E\left[(v - p(v_1, x_{Q2}, x_M))x_{Q2}|v_1, \theta_2\right]. \tag{4.16}$$

The optimisation problem of D_1 is symmetric with respect to the optimisation problem of D_2, and the optimisation problem of Q_1 is symmetric with respect to the optimisation problem of Q_2. While the actual demand of D_1 is $x_{D1} = \frac{E(v|v_1)}{2\lambda} - \frac{E(x_B|v_1)}{2}$, the strategically inflexible quant department Q_2 incorrectly thinks x_{D1} takes the following form:

$$x_{D1} = \frac{E(v|v_1)}{2\lambda}. \tag{4.17}$$

Similarly, while the actual demand of Q_1 is $x_{Q1} = \frac{E(v|v_1,\theta_2)}{2\lambda} - \frac{E(x_M|v_1,\theta_2)}{2}$, the strategically inflexible quant department Q_2 incorrectly thinks x_{Q1} takes the following form:

$$x_{Q1} = \frac{E(v|v_1,\theta_2)}{2\lambda}. \tag{4.18}$$

As usual, market makers consider the aggregate order flow ω as the sum of the order flows submitted by firm A, firm B, and a group of noise traders:

$$\omega = \gamma_1 x_{Q1} + (1-\gamma_1)x_{D1} + \gamma_2 x_{Q2} + (1-\gamma_2)x_{D2} + u. \tag{4.19}$$

Market makers again follow the weak market efficiency rule defined by $p = E[v|\omega]$ and set a price that allows them to break even in expectation. The following proposition summarises equilibrium outcomes in the extended economy in which each firm has a quant department.

Proposition 4.5. *Consider an economy in which the two firms each have a quant department. Equilibrium pricing rule of market makers is characterised by $p = \lambda'\omega$, and equilibrium trading strategies of traders Q_1, D_1, Q_2, and D_2 are given by $x_{Q1} = \alpha_{Q1}v_1 + \beta_{Q1}\theta_2$, $x_{D1} = \alpha_{D1}v_1$, $x_{Q2} = \alpha_{Q2}v_1 + \beta_{Q2}\theta_2$, and $x_{D2} = \alpha_{D2}v_2$ with*

$$\lambda' = \sqrt{\frac{(\gamma_1+\gamma_2-2\gamma_1\gamma_2+4)(\gamma_1+\gamma_2+2)}{4(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)^2} + q\eta(1-\eta)}, \tag{4.20}$$

$$\alpha_{Q1} = \alpha_{Q2} = \frac{1}{4\lambda'}, \quad \alpha_{D1} = \frac{\gamma_1+2\gamma_2-\gamma_1\gamma_2+4}{4\lambda'(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)}, \tag{4.21}$$

$$\beta_{Q1} = \frac{2q-q\gamma_2}{4\lambda'}, \quad \alpha_{D2} = \frac{\gamma_2+2\gamma_1-\gamma_1\gamma_2+4}{4\lambda'(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)}, \tag{4.22}$$

$$\beta_{Q2} = \frac{2q-q\gamma_1}{4\lambda'}, \quad \eta \equiv \frac{\gamma_1+\gamma_2-\gamma_1\gamma_2}{2}. \tag{4.23}$$

Note that, when γ_1 or γ_2 approaches zero, the extended economy would degenerate to the baseline economy. Based on the equilibrium trading strategies defined by Proposition 4.5, normalised trading aggressiveness measures of firm A on signals v_1 and θ_2 and of firm B on the similar two signals can be derived as:

$$\widetilde{\alpha}'_A = \frac{3\gamma_2 - 3\gamma_1\gamma_2 + 6}{2(\gamma_1 + \gamma_2 - \gamma_1\gamma_2 + 3)}, \; \widetilde{\beta}'_A = \frac{3q\gamma_1(2-\gamma_2)}{4}, \tag{4.24}$$

$$\widetilde{\alpha}'_B = \frac{3\gamma_1 - 3\gamma_1\gamma_2 + 6}{2(\gamma_1 + \gamma_2 - \gamma_1\gamma_2 + 3)}, \; \widetilde{\beta}'_B = \frac{3q\gamma_2(2-\gamma_1)}{4}. \tag{4.25}$$

4.4.1 Growth in quantitative investing by a single firm

We now turn to analysing the ceteris paribus impacts of growth in quantitative investing by firm B. As before, we define the overall trading aggressiveness τ' as the sum of the four normalised trading aggressiveness measures. The following proposition demonstrates the implications of growth in quantitative investing on overall trading aggressiveness in the extended economy where firm A also has a quant department.

Proposition 4.6. *Consider an economy in which the two firms each have a quant department. When quantitative investing level γ_2 of firm B increases, the impact on overall trading aggressiveness τ' hinges on the trade-off among:*

(i) The positive capacity enhancing effect by investment firm B, which is given by

$$\frac{\partial \widetilde{\beta}'_B}{\partial \gamma_2} = \frac{3q(2-\gamma_1)}{4}. \tag{4.26}$$

(ii) The negative strategy oblivion effect by investment firm B, which is given by

$$\frac{\partial \widetilde{\alpha}'_B}{\partial \gamma_2} = -\frac{3\gamma_1 + 3}{(\gamma_1 + \gamma_2 - \gamma_1\gamma_2 + 3)^2}. \tag{4.27}$$

(iii) The positive internalising effect by the opponent firm A, which implies D_1 trades more aggressively on v_1 to exploit increased inflexibility of B, and is given by

$$\frac{\partial \widetilde{\alpha}'_A}{\partial \gamma_2} = \frac{3 - 3\gamma_1^2}{2(\gamma_1 + \gamma_2 - \gamma_1\gamma_2 + 3)^2}. \tag{4.28}$$

(iv) *The negative competition effect by the opponent firm A, which implies Q_1 trades less aggressively on θ_2 due to increased competition, and is given by*

$$\frac{\partial \widetilde{\beta}'_A}{\partial \gamma_2} = -\frac{3q\gamma_1}{4}. \tag{4.29}$$

Proposition 4.6 suggests the key result in the baseline model and in the extended model featured by partial strategic inflexibility (see Appendix 4.2) continues to hold if both firms have a quant department.

An increase in quantitative investing level γ_2 again causes (i) firm B trade more aggressively on θ_2 since B benefits from exploiting the higher capacity of processing θ_2, (ii) firm B to trade less aggressively on v_1 since B is more strategically inflexible and more likely to overestimate the trading aggressiveness of the opponent firm, and (iii) firm A to trade more aggressively on v_1 as its discretionary department internalises the reduced aggressiveness on v_1 by B. Aside from the capacity enhancing effect, the strategy oblivion effect, and the internalising effect discussed in Section 4.3 and Appendix 4.2, an increase in γ_2 also makes firm A's information advantage with regard to θ_2 become less valuable due to increased competition from firm B, and thus renders firm A to trade less aggressively on θ_2. We refer to this trading aggressiveness reduction by firm A as the *competition effect*. Corollary 4.7 analytically depicts how these four effects on trading aggressiveness vary with q and γ_2.

Corollary 4.7. *Consider an economy in which two firms each have a quant department. The magnitude of the capacity enhancing effect increases in signal quality q at a faster rate than the magnitude of the competition effect. The magnitude of the strategy oblivion effect decreases in quantitative investing intensity γ_2 at a faster rate than the magnitude of the internalising effect does.*

The above results again mirror the findings in Corollary 4.3. The positive capacity enhancing effect is strengthened more by an increment in q than the negative competition effect is, whereas the other two effects are independent of q. Therefore, for high q, the positive capacity enhancing effect (resp. the negative strategy oblivion effect) is more likely to dominate, and the net impact of an increase in quantitative investing level γ_2 on overall trading aggressiveness is positive (resp. negative). The negative strategy oblivion effect is weakened more by an increment in γ_2 than the positive internalising effect is. Therefore, for moderate q, the negative strategy oblivion effect may dominate with low γ_2 but no longer dominates when drastically weakened by the further increase in γ_2, and the overall trading aggressiveness is hump-shaped in γ_2. The graphical comparison between the sum of the positive effects and the sum of the absolute values of the negative effects when q are at low, moderate and high levels would demonstrate similar patterns as Figure 4.2. Thus, for brevity, we do not report the graphical comparison here.

Driven by the above changes in trading aggressiveness, the price efficiency implications of increases in γ_2 in this extended economy resemble the price efficiency implications γ in the baseline economy, though the threshold values determining if q is low, or moderate, or high are different between the two economies. The equilibrium price efficiency measure in the economy where each firm has a quant department is specified by the following equation:

$$PE_{both\ quants} = \frac{4\eta + 6}{(3\gamma_1 + 3\gamma_2 - 2\gamma_1\gamma_2 + 8 - 4q\eta^2 - 6q\eta)\,\Sigma_0}. \tag{4.30}$$

The following proposition formally summarises the price efficiency impact of growth in quantitative investing by firm B when both firms may contain quant departments, and Figure 4.4 graphically illustrates the proposition.

Proposition 4.8. *Consider an economy in which the two firms each have a quant department. When quantitative investing level γ_2 increases, the effect on price efficiency depends on the magnitude of signal quality q:*

(i) When $0 < q \le q_3$, price efficiency decreases with quantitative investing level.

(ii) When $q_3 < q < q_4$, price efficiency decreases if quantitative investing level is below the threshold value $\bar{\gamma}'$, and increases if otherwise.

(iii) When $q \ge q_4$, price efficiency increases with quantitative investing level.

Recall that, when $\gamma = 0$ in the baseline model (see Figure 4.3), the price efficiency stays the same despite the variations in the signal quality n. Nevertheless, when $\gamma_2 = 0$ and $\gamma_1 > 0$ in the current model (see Figure 4.4), the price efficiency would vary if the signal quality γ varies. The two different observations can be explained by the differences in trading behaviours when firm A is fully discretionary and when firm A has a quant department.

More specifically, in models with a fully discretionary opponent firm A, zero quantitative investing level γ by firm B effectively means no active trading on signal θ_2. The signal quality thus becomes irrelevant in terms of market outcomes in such

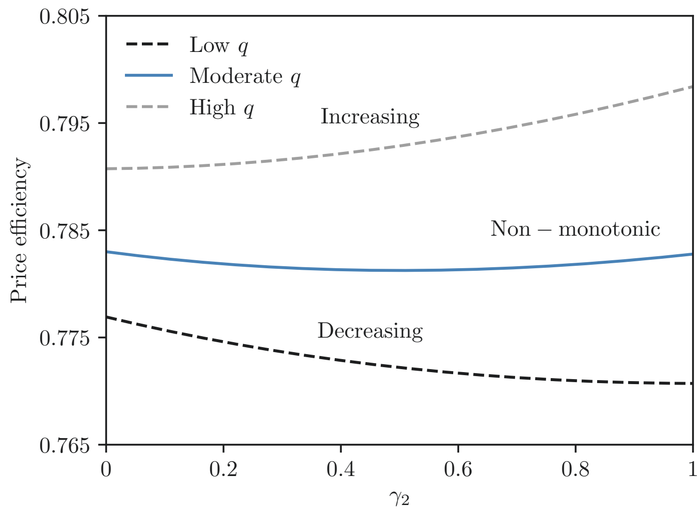


Figure 4.4: Price efficiency when A has a quant department and quantitative investing level γ_1 of A is fixed.
This figure illustrates the price efficiency impact of growth in quantitative investing level γ_2 when γ_1 is fixed in the setting that firm A also has a quant department. The figure plots the price efficiency as a function of γ_2. The black dashed line corresponds to $q = 0.28$, that is, low q. The blue line corresponds to $q = 0.32$, that is, moderate q. The grey dashed line corresponds to $q = 0.37$, that is, high q. The other parameter values are $\Sigma_0 = 1$ and $\gamma_1 = 0.5$, meaning that the opponent firm A's quant department presents in the market with a probability of 0.5.

a setting. If firm A is no longer fully discretionary, despite the zero quantitative investing level by firm B, the quant department of firm A is naturally more willing to trade on signal θ_2 when the signal is more precise (i.e., larger q), which gives rise to higher price efficiency.

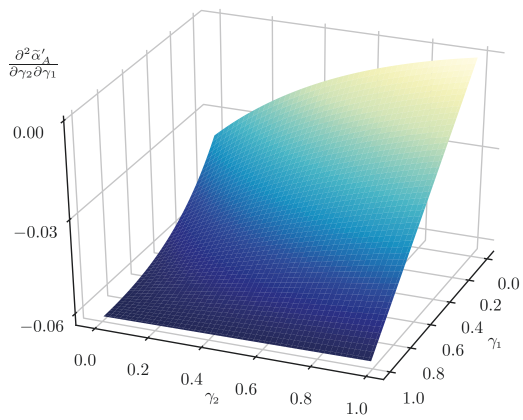
4.4.2 Growth in quantitative investing by both firms

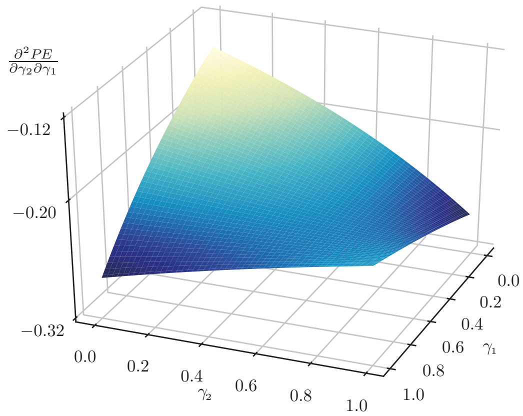
In the last subsection, we analysed the trading aggressiveness and price efficiency implications of an increase in the quantitative investing level of firm B, while keeping the quantitative investing level of firm A fixed. What is the impact on price efficiency in the extended economy when both firms experience a rise in quantitative investing levels? To answer this question, we analyse the second-order cross partial derivatives of $\widetilde{\alpha}'_A$ and the price efficiency measure with respect to γ_1 and γ_2, and summarise the results in the following corollary.

Corollary 4.9. *(i) The internalising effect brought by an increase in the quantitative investing level γ_2 of firm B is decreasing in γ_1.*

(ii) The derivative of price efficiency with respect to the quantitative investing level γ_2 of firm B is decreasing in γ_1.

An increase in the quantitative investing level of firm B is more likely to decrease the overall trading aggressiveness and harm price efficiency with an increase in the quantitative investing level of firm A than without. Recall from the last subsection that the discretionary department of firm A internalises the reduction in firm B's trading aggressiveness on v_1 given growth in firm B's quantitative investing level, that is, $\partial\,\widetilde{\alpha}'_A/\partial\gamma_2 > 0$. According to part (i) of Corollary 4.9, if firm A has a higher quantitative investing level, its discretionary department would put less effort into



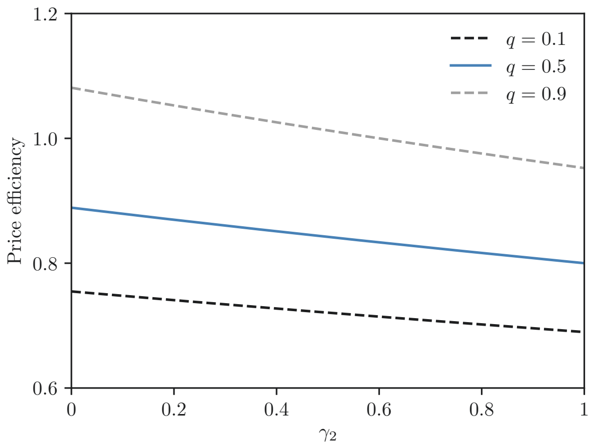


(B)

Figure 4.5: Price efficiency when A has a quant department and γ_1 and γ_2 increase.
The figure illustrates how the relationship between price efficiency and γ_2 varies if γ_1 is increased instead of fixed. Panel (A) plots the second-order partial derivatives of $\widetilde{\alpha}'_A$ on γ_2 and γ_1. Panel (B) plots the second-order partial derivatives of price efficiency with respect to γ_2 and γ_1 when $q = 0.5$ and $\Sigma_0 = 1$.

internalising such trading aggressiveness reduction brought by the growth in firm B's quantitative investing level, that is, $\partial^2\widetilde{\alpha}'_B/\partial\gamma_2\,\partial\gamma_1 < 0$. According to part (ii) of Corollary 4.9, the price efficiency impact of the growth in firm B's quantitative investing level is more likely to be negative when firm A's quantitative investing level is high than when it is low.

Figure 4.5 graphically illustrates the corollary for any combinations of γ_1 and γ_2. To further depict the possible detrimental effect of growth in firm B's quatitative investing level γ_2 on price efficiency when firm A's quantitative investing level is high, we plot price efficiency with respect to γ_2 when $\gamma_1 = 1$ for three different levels of q, i.e., $q = 0.1$, $q = 0.5$ and $q = 0.9$, in Figure 4.6. The downward sloping curves in the figure, together with Eq.(A4.1.29), suggest that price efficiency is always harmed by increases in quantitative level investing level γ_2 when the opponent firm has a quant department but no discretionary department.



(A)

Figure 4.6: Price efficiency when the opponent firm has a quant department but no discretionary department.
This figure illustrates the price efficiency impact of growth in quantitative investing level γ_2 when firm A has a quant department but no discretionary department. The figure plots the price efficiency as a function of γ_2. The black dashed line corresponds to $q = 0.1$, the blue line corresponds to $q = 0.5$, and the grey dashed line corresponds to $q = 0.9$. The other parameter value is $\Sigma_0 = 1$.

One may interpret the quantitative investing level of firm A as the market quantitative investing level excluding firm B. When the market quantitative investing level is already high, regulators and practitioners should pay close attention to increased reliance on quantitative investing by individual firms as such reliance might jeopardise market quality. The findings from both the analytic results and the numerical examples in this subsection indicate that implementing the policy intervention of reducing quantitative investing activities should benefit price efficiency in markets where such activities are extremely prevalent.

4.5 Conclusion

With advancements in technology and access to large amounts of data, quantitative investing has become increasingly popular in recent years. We build a concise model that allows for different quantitative investing levels of investment firms in financial markets. We show that a firm's quantitative investing level increase may result in four strategic effects. First, the capacity enhancing effect means the firm has greater computing power and becomes better at extracting information, increasing the firm's trading aggressiveness. Second, the strategy oblivion effect means the firm has weaker strategic flexibility and is more likely to over-estimate the trading aggressiveness of other traders, decreasing the firm's trading aggressiveness. Third, the internalising effect suggests that discretionary opponents of the firm exploit the weaker strategic flexibility and trade more aggressively. Finally, the competition effect refers to the trading aggressiveness reduction of the opponent firm due to more fierce competition on quantitative technology. Consequently, price efficiency can be non-monotonic in the quantitative investing level.

In our model, growth in quantitative investing level by individual firms is more likely to harm price efficiency when the market quantitative investing level is already high, as discretionary traders are less likely to be present in the market to internalise trading aggressiveness reduction brought by increased strategic inflexibility.

By decomposing the impact of growth in machine-based quantitative investing on overall trading aggressiveness and the market efficiency into empirical testable components primarily driven by the capacity enhancing effect and the strategy oblivion effect, our theory deepens the understanding of the channels through which machine-human interaction may drive the financial market outcome.

Appendix 4.1. Proofs

In this Appendix, we prove our main results. The proofs of Propositions 4.1, 4.2, 4.4 and Corollary 4.3 are omitted for brevity as they respectively follow from the proofs of Propositions 4.5, 4.6, 4.8 and Corollary 4.7 when the quantitative investing level of firm A is zero.

Proof of Proposition 4.5. The quant department of firm B observes $\{v_1, \theta_2\}$ and chooses x_{Q2} to maximise the expected profit $E\left[(v_1 + v_2 - \lambda' x_{Q2} - \lambda' x_A)\, x_{Q2} | v_1, \theta_2\right]$. Using the quant department Q_2's incorrect conjectures about firm A's trading strategies as defined in Eqs.(4.10) and (4.11), we can compute the first-order condition (FOC), which delivers

$$\begin{aligned} x_{Q2} &= \frac{v_1 + E\left(v_2|\theta_2\right)}{2\lambda'} - \frac{E\left(\gamma_1 x_{Q1}|v_1, \theta_2\right)}{2} - \frac{E((1-\gamma_1)x_{D1}|v_1, \theta_2)}{2} \\ &= \frac{v_1}{4\lambda'} + \frac{(2-\gamma_1)q\theta_2}{4\lambda'}. \end{aligned} \tag{A4.1.1}$$

Therefore, the quant department Q_2 has a linear equilibrium trading strategy $x_{Q2} = \alpha_{Q2} v_1 + \beta_{Q2}\theta_2$, with the values of α_{Q2} and β_{Q2} satisfy the following equation:

$$\alpha_{Q2} = \frac{1}{4\lambda'} \text{ and } \beta_{Q2} = \frac{2q - q\gamma_1}{4\lambda'}. \tag{A4.1.2}$$

Since the optimisation problems of the quant departments of firm A and firm B are symmetric, equilibrium trading strategies of the two quant departments are also symmetric. The quant department Q_1 has a linear equilibrium trading strategy $x_{Q1} = \alpha_{Q1} v_1 + \beta_{Q1}\theta_2$. Substituting γ_1 on the right hand side of Eq.(A4.1.2) with γ_2, we know that the values of α_{Q1} and β_{Q1} satisfying the following equation:

$$\alpha_{Q1} = \frac{1}{4\lambda'} \text{ and } \beta_{Q1} = \frac{2q - q\gamma_2}{4\lambda'}. \tag{A4.1.3}$$

The discretionary department of firm B observes only v_1 and chooses x_{D2} to maximise the $E\left[(v_1 + v_2 - \lambda' x_{D2} - \lambda' x_A)\, x_{D2} | v_1\right]$. Taking the FOC of D_2's profit with respect to x_{D2} and rearranging terms yields

$$\begin{aligned} x_{D2} &= \frac{v_1}{2\lambda'} - \frac{E\left(\gamma_1 x_{Q1}|v_1, \theta_2\right)}{2} - \frac{E((1-\gamma_1)x_{D1}|v_1, \theta_2)}{2} \\ &= \frac{v_1}{2\lambda'} - \frac{\gamma_1 v_1}{8\lambda'} - \frac{(1-\gamma_1)x_{D1}}{2}. \end{aligned} \tag{A4.1.4}$$

Since the optimisation problems of the discretionary departments of firm A and firm B are symmetric, equilibrium trading strategies of the two discretionary departments are also symmetric. The strategy of D_1 should satisfy

$$x_{D1} = \frac{v_1}{2\lambda'} - \frac{\gamma_2 v_1}{8\lambda'} - \frac{(1-\gamma_2)x_{D2}}{2}. \tag{A4.1.5}$$

Combining Eqs.(A4.1.4) and (A4.1.5), the discretionary departments D_2 and D_1 have linear equilibrium trading strategies $x_{D2} = \alpha_{D2}v_1$ and $x_{D1} = \alpha_{D1}v_1$, with:

$$\alpha_{D2} = \frac{(4+\gamma_2+2\gamma_1-\gamma_1\gamma_2)}{4\lambda'(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)}, \tag{A4.1.6}$$

$$\alpha_{D1} = \frac{(4+\gamma_1+2\gamma_2-\gamma_2\gamma_1)}{4\lambda'(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)}. \tag{A4.1.7}$$

We now turn to the market maker's decision. The market maker considers the aggregate order flow ω takes the form of $\omega = \gamma_1 x_{Q1} + (1-\gamma_1)x_{D1} + \gamma_2 x_{Q2} + (1-\gamma_2)x_{D2} + u$ and sets $p = E(v|\omega) = \lambda'\omega$. Given that $\lambda' = cov(v,\omega)/var(\omega)$, we have:

$$\lambda' = \frac{(\gamma_1\alpha_{Q1} + (1-\gamma_1)\alpha_{D1} + \gamma_2\alpha_{Q2} + (1-\gamma_2)\alpha_{D2})\Sigma_0 + q(\gamma_1\beta_{Q1}+\gamma_2\beta_{Q2})\Sigma_0}{(\gamma_1\alpha_{Q1} + (1-\gamma_1)\alpha_{D1} + \gamma_2\alpha_{Q2} + (1-\gamma_2)\alpha_{D2})^2\Sigma_0 + q(\gamma_1\beta_{Q1}+\gamma_2\beta_{Q2})^2\Sigma_0 + \Sigma_0}. \tag{A4.1.8}$$

Multiplying both sides of Eq.(A4.1.8) by λ' and rearranging terms, we know that the above equation is equivalent to

$$\begin{aligned} &\lambda'^2(\gamma_1\alpha_{Q1} + (1-\gamma_1)\alpha_{D1} + \gamma_2\alpha_{Q2} + (1-\gamma_2)\alpha_{D2})^2 + q\lambda'^2(\gamma_1\beta_{Q1}+\gamma_2\beta_{Q2})^2 \\ &+ \lambda'^2 = \lambda'(\gamma_1\alpha_{Q1} + (1-\gamma_1)\alpha_{D1} + \gamma_2\alpha_{Q2} + (1-\gamma_2)\alpha_{D2}) + q\lambda'(\gamma_1\beta_{Q1}+\gamma_2\beta_{Q2}). \end{aligned} \tag{A4.1.9}$$

Substituting Eqs.(A4.1.2), (A4.1.3), (A4.1.6), and (A4.1.7) into Eq.(A4.1.9), we can explicitly solve for the value of λ', which is

$$\lambda' = \sqrt{\frac{(\gamma_1+\gamma_2-2\gamma_1\gamma_2+4)(\gamma_1+\gamma_2+2)}{4(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)^2} + q\eta(1-\eta)}, \tag{A4.1.10}$$

where η is defined as a function of γ_1 and γ_2:

$$\eta \equiv (\gamma_1+\gamma_2-\gamma_1\gamma_2)/2. \tag{A4.1.11}$$

The equilibrium values of (λ', α_{Q2}, β_{Q2}, α_{Q1}, β_{Q1}, α_{D2}, α_{D1}) are thus given by Eqs.(A4.1.2), (A4.1.3), (A4.1.6), (A4.1.7) and (A4.1.10).

■

Proof of Proposition 4.6. Let $\psi \equiv 1/3\lambda'$, the normalised trading aggressiveness measure of firm B on signal θ_2 can be calculated as $\widetilde{\beta}'_B = \gamma_2\beta_{Q2}/\psi = 3\lambda'\gamma_2\beta_{Q2}$, which leads to

$$\begin{aligned} \widetilde{\beta}'_B &= 3\lambda'\gamma_2 \cdot \frac{2q - q\gamma_1}{4\lambda'} \\ &= \frac{3q\gamma_2(2-\gamma_1)}{4}. \end{aligned} \tag{A4.1.12}$$

The first-order derivative of $\widetilde{\beta}'_B$ with respect to γ_2 is thus

$$\begin{aligned} \frac{\partial \widetilde{\beta}'_B}{\partial \gamma_2} &= \frac{3q(2-\gamma_1)}{4} \cdot \frac{\partial \gamma_2}{\partial \gamma_2} \\ &= \frac{3q(2-\gamma_1)}{4}. \end{aligned} \tag{A4.1.13}$$

The normalised trading aggressiveness measure of firm B on signal v_1 can be calculated as $\widetilde{\alpha}'_B = (\gamma_2\alpha_{Q2} + (1-\gamma_2)\alpha_{D2})/\psi = 3\lambda'\gamma_2\alpha_{Q2} + 3\lambda'(1-\gamma_2)\alpha_{D2}$, which leads to

$$\begin{aligned} \widetilde{\alpha}'_B &= 3\lambda'\gamma_2 \cdot \frac{1}{4\lambda'} + 3\lambda'(1-\gamma_2) \cdot \frac{\gamma_2 + 2\gamma_1 - \gamma_1\gamma_2 + 4}{4\lambda'(\gamma_1 + \gamma_2 - \gamma_1\gamma_2 + 3)} \\ &= \frac{3\gamma_1 - 3\gamma_1\gamma_2 + 6}{2(\gamma_1 + \gamma_2 - \gamma_1\gamma_2 + 3)}. \end{aligned} \tag{A4.1.14}$$

The first-order derivative of $\widetilde{\alpha}'_B$ with respect to γ_2 is thus

$$\begin{aligned} \frac{\partial \widetilde{\alpha}'_B}{\partial \gamma_2} &= \frac{3((\gamma_1 + \gamma_2 - \gamma_1\gamma_2 + 3)(-\gamma_1) - (\gamma_1 - \gamma_1\gamma_2 + 2)(1-\gamma_1))}{2(\gamma_1 + \gamma_2 - \gamma_1\gamma_2 + 3)^2} \\ &= -\frac{3\gamma_1 + 3}{(\gamma_1 + \gamma_2 - \gamma_1\gamma_2 + 3)^2}. \end{aligned} \tag{A4.1.15}$$

The normalised trading aggressiveness measure of firm A on signal θ_2 can be calculated as $\widetilde{\beta}'_A = \gamma_1\beta_{Q1}/\psi = 3\lambda'\gamma_1\beta_{Q1}$, which leads to

$$\begin{aligned} \widetilde{\beta}'_A &= 3\lambda'\gamma_1 \cdot \frac{2q - q\gamma_2}{4\lambda'} \\ &= \frac{3q\gamma_1(2-\gamma_2)}{4}. \end{aligned} \tag{A4.1.16}$$

The first-order derivative of $\widetilde{\beta}'_A$ with respect to γ_2 is thus

$$\begin{aligned} \frac{\partial \widetilde{\beta}'_A}{\partial \gamma_2} &= \frac{3q\gamma_1}{4} \cdot \frac{\partial(2-\gamma_2)}{\partial \gamma_2} \\ &= -\frac{3q\gamma_1}{4}. \end{aligned} \tag{A4.1.17}$$

The normalised trading aggressiveness measure of firm A on signal v_1 can be calculated as $\widetilde{\alpha}'_A = (\gamma_1\alpha_{Q1} + (1-\gamma_1)\alpha_{D1})/\psi = 3\lambda'\gamma_1\alpha_{Q1} + 3\lambda'(1-\gamma_1)\alpha_{D1}$, which leads to

$$\begin{aligned}\widetilde{\alpha}'_A &= 3\lambda'\gamma_1 \cdot \frac{1}{4\lambda'} + 3\lambda'(1-\gamma_1)\cdot\frac{\gamma_1+2\gamma_2-\gamma_1\gamma_2+4}{4\lambda'(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)}\\ &= \frac{3\gamma_2-3\gamma_1\gamma_2+6}{2(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)}.\end{aligned} \tag{A4.1.18}$$

The first-order derivative of $\widetilde{\alpha}'_A$ with respect to γ_2 is thus

$$\begin{aligned}\frac{\partial\widetilde{\alpha}'_A}{\partial\gamma_2} &= \frac{3((\gamma_1+\gamma_2-\gamma_1\gamma_2+3)(1-\gamma_1)-(\gamma_2-\gamma_1\gamma_2+2)(1-\gamma_1))}{2(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)^2}\\ &= \frac{3-3\gamma_1^2}{2(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)^2}.\end{aligned} \tag{A4.1.19}$$

Therefore, for $\gamma_1 \in [0,1]$ and $\gamma_2 \in [0,1]$, we have $\partial\widetilde{\beta}'_B/\partial\gamma_2 > 0$, $\partial\widetilde{\alpha}'_B/\partial\gamma_2 < 0$, $\partial\widetilde{\beta}'_A/\partial\gamma_2 \leq 0$ (the equal sign holds only when $\gamma_1 = 0$), $\partial\widetilde{\alpha}'_A/\partial\gamma_2 \geq 0$ (the equal sign holds only when $\gamma_1 = 1$).

■

Proof of Corollary 4.7. Using Eq.(A4.1.13) and taking the derivative of the capacity enhancing effect with respect to the signal quality q, we obtain

$$\begin{aligned}\frac{\partial^2\widetilde{\beta}'_B}{\partial\gamma_2\,\partial q} &= \frac{3(2-\gamma_1)}{4}\cdot\frac{\partial q}{\partial q}\\ &= \frac{3(2-\gamma_1)}{4}.\end{aligned} \tag{A4.1.20}$$

We thus have $\partial^2\widetilde{\beta}'_B/\partial\gamma_2\,\partial q > 0$, which, combined with $\partial\widetilde{\beta}'_B/\partial\gamma_2 > 0$, means that the magnitude of the capacity enhancing effect is intensified by increased signal quality q.

Using Eq.(A4.1.17) and taking the derivative of the competition effect with respect to the signal quality q, we obtain

$$\begin{aligned}\frac{\partial^2\widetilde{\beta}'_A}{\partial\gamma_2\,\partial q} &= -\frac{3\gamma_1}{4}\cdot\frac{\partial q}{\partial q}\\ &= -\frac{3\gamma_1}{4}.\end{aligned} \tag{A4.1.21}$$

We thus have $\partial^2\widetilde{\beta}'_A/\partial\gamma_2\,\partial q \leq 0$ (the equal sign holds only when $\gamma_2 = 0$), which, combined with $\partial\widetilde{\beta}'_A/\partial\gamma_2 < 0$, means that the magnitude of the competition effect is intensified by increased signal quality q. Furthermore, we know that

$$\frac{\partial^2\widetilde{\beta}'_B}{\partial\gamma_2\,\partial q} - \left|\frac{\partial^2\widetilde{\beta}'_A}{\partial\gamma_2\,\partial q}\right| = \frac{3}{2}. \tag{A4.1.22}$$

By Eq.(A4.1.22), the magnitude of the capacity enhancing effect increases in signal quality q at a faster rate than the magnitude of the competition effect does.

Using Eq.(A4.1.15) and taking the derivative of the strategy oblivion effect with respect to the quantitative investing level γ_2 yields

$$\begin{aligned}\frac{\partial^2\widetilde{\alpha}'_B}{\partial\gamma_2\,\partial\gamma_2} &= -(3+3\gamma_1)\cdot\Big(-\frac{2}{(3+\gamma_1+\gamma_2-\gamma_1\gamma_2)^3}\Big)\cdot\frac{\partial(3+\gamma_1+\gamma_2-\gamma_1\gamma_2)}{\partial\gamma_2}\\ &= \frac{6(1+\gamma_1)(1-\gamma_1)}{(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)^3}.\end{aligned} \tag{A4.1.23}$$

Since $\partial^2\widetilde{\alpha}'_B/\partial\gamma_2\,\partial\gamma_2 \geq 0$ (the equal sign holds only when $\gamma_1 = 1$) and $\partial\widetilde{\alpha}'_B/\partial\gamma_2 < 0$, the magnitude of the strategy oblivion effect is weakened by the increased quantitative investing level γ_2.

Using Eq.(A4.1.19) and taking the derivative of the internalising effect with respect to the quantitative investing level γ_2 yields

$$\begin{aligned}\frac{\partial^2\widetilde{\alpha}'_A}{\partial\gamma_2\,\partial\gamma_2} &= \frac{(3-3\gamma_1^2)}{2}\cdot\frac{-2(1-\gamma_1)}{(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)^2}\\ &= -\frac{3(1-\gamma_1)^2(1+\gamma_1)}{(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)^2}.\end{aligned} \tag{A4.1.24}$$

Since $\partial^2\widetilde{\alpha}'_A/\partial\gamma_2\,\partial\gamma_2 \leq 0$ and $\partial^2\widetilde{\alpha}'_A/\partial\gamma_2 \geq 0$ (the equal signs hold only when $\gamma_2 = 1$), the magnitude of the internalising effect is weakened by the increased quantitative investing level γ_2. Furthermore, combining Eqs.(A4.1.22) and (A4.1.23), we know that

$$\frac{\partial^2\widetilde{\alpha}'_B}{\partial\gamma_2\,\partial\gamma_2} - \left|\frac{\partial^2\widetilde{\alpha}'_A}{\partial\gamma_2\,\partial\gamma_2}\right| = \frac{3(1+\gamma_1)^2(1-\gamma_1)}{(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)^3} > 0. \tag{A4.1.25}$$

By Eq.(A4.1.25), the magnitude of the strategy oblivion effect decreases in quantitative investing level γ_2 at a faster rate than the magnitude of the internalising effect does.

■

Proof of Proposition 4.8. The covariance between the fundamental value v and the price p can be calculated as following

$$\begin{aligned} cov(v,p) &= cov(v_1+v_2, \lambda'\gamma_1\alpha_{Q1}v_1 + \lambda'(1-\gamma_1)\alpha_{D1}v_1 + \lambda'\gamma_2\alpha_{Q2}v_1 + \\ &\quad \lambda'\gamma_1\beta_{Q1}\theta_2 + \lambda'\gamma_2\beta_{Q2}\theta_2) \\ &= \frac{\gamma_1+\gamma_2-2\gamma_1\gamma_2+4}{2(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)}\Sigma_0 + \frac{q(\gamma_1+\gamma_2-\gamma_1\gamma_2)}{2}\Sigma_0. \end{aligned} \tag{A4.1.26}$$

Because $\lambda = cov(v,\omega)/var(\omega)$, we get $cov(v,p) = var(p)$. Using the projection theorem, the conditional variance $var(v|p)$ is

$$\begin{aligned} var(v|p) &= var(v) - \frac{cov(v,p)^2}{var(p)} \\ &= 2\Sigma_0 - \frac{\gamma_1+\gamma_2-2\gamma_1\gamma_2+4}{2(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)}\Sigma_0 - \frac{q(\gamma_1+\gamma_2-\gamma_1\gamma_2)}{2}\Sigma_0. \end{aligned} \tag{A4.1.27}$$

The first-order derivative of the price discovery measure $var(v|p)^{-1}$ with respect to the quantitative investing level γ_2 satisfies

$$\begin{aligned} \frac{\partial PE_{both\ quants}}{\partial\gamma_2} &= -\frac{1}{\mathrm{var}(v_1+v_2|p)^2}\cdot\frac{\partial var(v|p)}{\partial\gamma_2} \\ &= -\frac{1}{\mathrm{var}(v_1+v_2|p)^2}\left(\frac{(1+\gamma_1)^2}{2(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)^2} + q(\frac{\gamma_1}{2}-\frac{1}{2})\right)\Sigma_0^{-1} \\ &\propto -\left(\frac{(1+\gamma_1)^2}{2(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)^2} + q(\frac{\gamma_1}{2}-\frac{1}{2})\right). \end{aligned} \tag{A4.1.28}$$

To determine the sign of the derivative of the price efficiency with respect to γ_2, we discuss the following two scenarios, i.e., $\gamma_1 = 1$ and $0 \le \gamma_1 < 1$.

(i) If $\gamma_1 = 1$ and firm A is fully quantitative, we have the following relationships:

$$\frac{(1+\gamma_1)^2}{2(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)^2} > 0 \text{ and } \frac{\gamma_1}{2}-\frac{1}{2} = 0. \tag{A4.1.29}$$

Substituting Eq.(A4.1.29) into Eq.(A4.1.28) yields $\partial PE_{both\ quants}/\partial\gamma_2 < 0$ for any γ_2. That is, price efficiency always decreases with the quantitative investing level γ_2 when $\gamma_1 = 1$.

(ii) If $0 \le \gamma_1 < 1$ and firm A has a discretionary department, $\partial PE_{both\ quants}/\partial\gamma_2 >$ 0 holds if and only if

$$
\begin{aligned}
&-\left(\frac{(1+\gamma_1)^2}{2(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)^2}+q(\frac{\gamma_1}{2}-\frac{1}{2})\right)>0 \\
&\Longleftrightarrow \gamma_2 > \frac{1+\gamma_1}{(1-\gamma_1)\sqrt{q\,(1-\gamma_1)}}-\frac{3+\gamma_1}{1-\gamma_1}\equiv\bar{\gamma}'.
\end{aligned}
\tag{A4.1.30}
$$

Under similar parameter settings, $\partial PE_{both\ quants}/\partial\gamma_2 < 0$ holds if and only if

$$
\begin{aligned}
&-\left(\frac{(1+\gamma_1)^2}{2(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)^2}+q(\frac{\gamma_1}{2}-\frac{1}{2})\right)<0 \\
&\Longleftrightarrow \gamma_2 < \frac{1+\gamma_1}{(1-\gamma_1)\sqrt{q\,(1-\gamma_1)}}-\frac{3+\gamma_1}{1-\gamma_1}\equiv\bar{\gamma}'.
\end{aligned}
\tag{A4.1.31}
$$

Note that for $\bar{\gamma}'$ to lie inside of (0,1), the value of q must satisfy the following inequality:

$$
\begin{aligned}
&0<\frac{1+\gamma_1}{(1-\gamma_1)\sqrt{q\,(1-\gamma_1)}}-\frac{3+\gamma_1}{1-\gamma_1}<1 \\
&\Longleftrightarrow q>\frac{(1+\gamma_1)^2}{16(1-\gamma_1)}\equiv q_3 \ \text{ and } \ q<\frac{(1+\gamma_1)^2}{(3+\gamma_1)^2(1-\gamma_1)}\equiv q_4.
\end{aligned}
\tag{A4.1.32}
$$

Therefore, when $q_3 < q < q_4$, price efficiency decreases (resp. increases) with the quantitative investing level γ_2 if γ_2 is below (resp. above) the threshold $\bar{\gamma}'$. When $q < q_3$, we have $\bar{\gamma}' > 1$, which means $\partial PE_{both\ quants}/\partial\gamma_2 < 0$ for any γ_2. When $q = q_3$, we have $\bar{\gamma}' = 1$ and thus $\gamma_2 \leq \bar{\gamma}'$ and $\partial PE_{both\ quants}/\partial\gamma_2 \leq 0$, of which the equal signs hold for $\gamma_2 = 1$. Consequently, when $0 < q \leq q_3$, price efficiency decreases with the quantitative investing level γ_2 of firm B.

When $q > q_4$, we have $\bar{\gamma}' < 0$, which means $\partial PE_{both\ quants}/\partial\gamma_2 > 0$ for any γ_2. When $q = q_4$, we have $\bar{\gamma}' = 0$ and thus $\gamma_2 \leq \bar{\gamma}'$ and $\partial PE_{both\ quants}/\partial\gamma_2 \leq 0$, of which the equal signs hold for $\gamma_2 = 0$. Consequently, when $q \geq q_4$, price efficiency increases with the quantitative investing level γ_2 of firm B.

Substituting $\gamma_1 = 0$ into Eq.(A4.1.32), we obtain the values of q_1 and q_2 in Proposition (4.4) as shown in Eq.(A4.1.33):

$$
q_1 \equiv \frac{1}{16} \ \text{ and } \ q_2 \equiv \frac{1}{9}. \tag{A4.1.33}
$$

■

Proof of Corollary 4.9. We determine the sign of the second-order cross partial derivative of $\widetilde{\alpha}'_A$ with respect to γ_1 and γ_2 as follows:

$$\begin{aligned}\frac{\partial^2 \widetilde{\alpha}'_A}{\partial\gamma_2\,\partial\gamma_1} &= \frac{3}{2}(1-\gamma_1^2)\cdot\left(-\frac{2(1-\gamma_1)}{(3+\gamma_1+\gamma_2-\gamma_1\gamma_2)^3}\right)\\ &= -\frac{3(1-\gamma_1)(1-\gamma_1^2)}{(3+\gamma_1+\gamma_2-\gamma_1\gamma_2)^3}<0.\end{aligned} \tag{A4.1.34}$$

The second-order cross partial derivative of the price efficiency measure satisfies the following:

$$\begin{aligned}\frac{\partial PE_{both\ quants}}{\partial\gamma_2\,\partial\gamma_1} &= \frac{2}{var(v|p)^3}\cdot\frac{\partial var(v|p)}{\partial\gamma_1}\cdot\frac{\partial var(v|p)}{\partial\gamma_2}-\frac{1}{var(v|p)^2}\cdot\frac{\partial var(v|p)}{\partial\gamma_2\partial\gamma_1}\\ &\propto \frac{2}{var(v|p)}\cdot\frac{\partial var(v|p)}{\partial\gamma_1}\cdot\frac{\partial var(v|p)}{\partial\gamma_2}-\frac{\partial var(v|p)}{\partial\gamma_2\partial\gamma_1}.\end{aligned} \tag{A4.1.35}$$

We can derive that $\frac{2}{3}\Sigma_0^{-1} \leqslant var(v|p)^{-1} \leqslant \frac{8}{7}\Sigma_0^{-1}$. Taking first-order derivatives of conditional variance $var(v|p)$ with respect to quantitative investing levels γ_1 and γ_2, we have

$$\frac{\partial var(v|p)}{\partial\gamma_1} = \left(\frac{(1+\gamma_2)^2}{2\left(\gamma_1+\gamma_2-\gamma_1\gamma_2+3\right)^2}+q\left(\frac{\gamma_2}{2}-\frac{1}{2}\right)\right)\Sigma_0, \tag{A4.1.36}$$

$$\frac{\partial var(v|p)}{\partial\gamma_2} = \left(\frac{(1+\gamma_1)^2}{2\left(\gamma_1+\gamma_2-\gamma_1\gamma_2+3\right)^2}+q\left(\frac{\gamma_1}{2}-\frac{1}{2}\right)\right)\Sigma_0. \tag{A4.1.37}$$

From Eqs.(A4.1.36) and (A4.1.37), we can determine the value range of $\partial var(v|p)/\partial\gamma_1$ and $\partial var(v|p)/\partial\gamma_2$ as $-\frac{15}{32}\Sigma_0 \leqslant \partial var(v|p)/\partial\gamma_1 \leqslant \frac{1}{8}\Sigma_0$ and $-\frac{15}{32}\Sigma_0 \leqslant \partial var(v|p)/\partial\gamma_2 \leqslant \frac{1}{8}\Sigma_0$. The second-order cross partial derivative of $var(v|p)$ is

$$\frac{\partial var(v|p)}{\partial\gamma_2\partial\gamma_1} = \left(\frac{q}{2}+\frac{2\left(1+\gamma_1\right)\left(1+\gamma_2\right)}{\left(\gamma_1+\gamma_2-\gamma_1\gamma_2+3\right)^3}\right)\Sigma_0. \tag{A4.1.38}$$

To determine the sign of the second-order cross partial derivative of the price efficiency measure, consider the following five scenarios: (i) $\partial var(v|p)/\partial\gamma_1 < 0$ and $\partial var(v|p)/\partial\gamma_2 > 0$, (ii) $\partial var(v|p)/\partial\gamma_1 < 0$ and $\partial var(v|p)/\partial\gamma_2 < 0$, (iii) $\partial var(v|p)$
$/\partial\gamma_1 > 0$ and $\partial var(v|p)/\partial\gamma_2 < 0$, (iv) $\partial var(v|p)/\partial\gamma_1 > 0$ and $\partial var(v|p)/\partial\gamma_2 > 0$, and (v) $\partial var(v|p)/\partial\gamma_1$ or $\partial var(v|p)/\partial\gamma_2 = 0$.

(i) If $\partial var(v|p)/\partial\gamma_1 < 0$ and $\partial var(v|p)/\partial\gamma_2 > 0$, as $2(var(v|p))^{-1} > 0$, $-\partial var(v|p)$ $/\partial\gamma_1\partial\gamma_2 < 0$, and $(\partial var(v|p)/\partial\gamma_1)(\partial var(v|p)/\partial\gamma_2) < 0$, we can easily obtain that $\partial PE_{both\ quants}/\partial\gamma_2\gamma_1$ is negative.

(ii) If $\partial var(v|p)/\partial\gamma_1 < 0$ and $\partial var(v|p)/\partial\gamma_2 < 0$, since $\frac{2}{3}\Sigma_0^{-1} \leqslant var(v|p)^{-1} \leqslant \frac{8}{7}\Sigma_0^{-1}$ and $0 < -2\partial var(v|p)/\partial\gamma_1 \leqslant \frac{15}{16}\Sigma_0$, we have the following relationship:

$$\begin{aligned}
&\frac{2}{var(v|p)} \cdot \frac{\partial var(v|p)}{\partial\gamma_1} \cdot \frac{\partial var(v|p)}{\partial\gamma_2} - \frac{\partial var(v|p)}{\partial\gamma_2\partial\gamma_1} \\
&\leqslant -\frac{15}{14}\frac{\partial var(v|p)}{\partial\gamma_2} - \frac{\partial var(v|p)}{\partial\gamma_1\partial\gamma_2} \\
&= -\frac{15}{14}\left(\frac{(1+\gamma_1)^2}{2\left(\gamma_1+\gamma_2-\gamma_1\gamma_2+3\right)^2} + \frac{q\gamma_1 - q}{2}\right)\Sigma_0 \\
&- \left(\frac{q}{2} + \frac{2\gamma_1+2\gamma_2+2\gamma_1\gamma_2+2}{\left(\gamma_1+\gamma_2-\gamma_1\gamma_2+3\right)^3}\right)\Sigma_0 \\
&\equiv s.
\end{aligned} \tag{A4.1.39}$$

From Eq.(A4.1.39), $\partial s/\partial q = (1-15\gamma_1)\Sigma_0/28$. If $\gamma_1 \geqslant 1/15$, we have $\partial s/\partial q \leqslant 0$, and thus

$$\begin{aligned}
s \leqslant s|_{q=0} &= -\frac{(1+\gamma_1)(60\gamma_1+71\gamma_2-15\gamma_1^2\gamma_2+15\gamma_1^2+101)}{28(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)^3}\Sigma_0 \\
&< -\frac{\Sigma_0}{28} < 0.
\end{aligned} \tag{A4.1.40}$$

If $\gamma_1 < 1/15$, we have $\partial s/\partial q > 0$ and thus the following inequality:

$$\begin{aligned}
s \leqslant s|_{q=1} &= -\frac{(1+\gamma_1)(60\gamma_1+71\gamma_2-15\gamma_1^2\gamma_2+15\gamma_1^2+101)}{28(\gamma_1+\gamma_2-\gamma_1\gamma_2+3)^3}\Sigma_0 + \left(\frac{1}{28} - \frac{15\gamma_1}{28}\right)\Sigma_0 \\
&< -\frac{1}{28}\Sigma_0 + \frac{1}{28}\Sigma_0 - \frac{15\gamma_1}{28}\Sigma_0 < 0.
\end{aligned} \tag{A4.1.41}$$

According to Eq.(A4.1.40) and Eq.(A4.1.41), we know that $\partial PE_{both\ quants}/\partial\gamma_2\gamma_1$ is negative when $\partial var(v|p)/\partial\gamma_1 < 0$ and $\partial var(v|p)/\partial\gamma_2 < 0$.

(iii) If $\partial var(v|p)/\partial\gamma_1 > 0$ and $\partial var(v|p)/\partial\gamma_2 < 0$, as $2(var(v|p))^{-1} > 0$, $-\partial var(v|p)/\partial\gamma_1\partial\gamma_2 < 0$, and $(\partial var(v|p)/\partial\gamma_1)(\partial var(v|p)/\partial\gamma_2) < 0$, we can easily obtain that $\partial PE_{both\ quants}/\partial\gamma_2\gamma_1$ is negative.

(iv) If $\partial var(v|p)/\partial\gamma_1 > 0$ and $\partial var(v|p)/\partial\gamma_2 > 0$, as $\frac{4}{3}\Sigma_0^{-1} \leqslant 2(var(v|p))^{-1} \leqslant \frac{16}{7}\Sigma_0^{-1}$ and $0 < (\partial var(v|p)/\partial\gamma_1)(\partial var(v|p)/\partial\gamma_2) \leqslant \frac{1}{64}\Sigma_0^2$, we have the following relationship:

$$\frac{2}{var(v|p)} \cdot \frac{\partial var(v|p)}{\partial \gamma_1} \cdot \frac{\partial var(v|p)}{\partial \gamma_2} - \frac{\partial var(v|p)}{\partial \gamma_2 \partial \gamma_1}$$
$$\leqslant \frac{1}{28}\Sigma_0 - \left(\frac{q}{2} + \frac{2\gamma_1 + 2\gamma_2 + 2\gamma_1\gamma_2 + 2}{(\gamma_1 + \gamma_2 - \gamma_1\gamma_2 + 3)^3}\right)\Sigma_0 \tag{A4.1.42}$$
$$< \frac{1}{28}\Sigma_0 - \frac{1}{16}\Sigma_0 < 0.$$

(v) If $\partial var(v|p)/\partial \gamma_1 = 0$ or $\partial var(v|p)/\partial \gamma_2 = 0$, we have $\partial PE_{both\ quants}/\partial \gamma_2\, \partial \gamma_1 = -(var(v|p))^{-2} \partial var(v|p)/\partial \gamma_2 \partial \gamma_1 < 0$.

Based on the discussion of the above five scenarios, it is proved that $\partial PE_{both\ quants}$ $/\partial \gamma_2\, \partial \gamma_1 < 0$ for all combinations of γ_1, γ_2, and q.

■

Appendix 4.2. Partial strategic inflexibility

In the baseline model, we assume a fully discretionary firm and a firm with a quant department. The quant department is assumed to have complete strategic inflexibility, i.e., it completely ignores that its price impact can reduce the trading aggressiveness of the fully discretionary firm. In this Appendix, we relax the assumption of complete strategic inflexibility and extend to the more general case of partial strategic inflexibility. For proportion h of the quant department Q's orders, Q incorrectly ignores that the fully discretionary firm A reacts to the corresponding price impact by reducing trading aggressiveness, where $0 < h \leq 1$. Denote the market maker's pricing function as $p = \lambda^* \omega$, Q now thinks the demand of firm A takes the following form:

$$x_A = \frac{E(v|\phi_A)}{2\lambda^*} - \frac{(1-h)E(x_B|\phi_A)}{2}. \tag{A4.2.1}$$

All other setups remain the same as in the baseline model. The following proposition summarises the equilibrium outcomes in the economy with partial strategic inflexibility.

Proposition 4.10. *Consider an economy with partial strategic inflexibility. Equilibrium pricing rule of market makers is characterised by $p = \lambda^* \omega$, and equilibrium trading strategies of traders A, Q, and D are given by $x_A = \alpha_A^* v_1$, $x_Q = \alpha_Q^* v_1 + \beta_Q^* \theta_2$, and $x_D = \alpha_D^* v_1$, with*

$$\lambda^* = \sqrt{\frac{(\gamma h + 4)(\gamma h + 2)}{4(\gamma h + 3)^2} + \frac{n\gamma(2-\gamma)}{4}}, \tag{A4.2.2}$$

$$\alpha_Q^* = \frac{\gamma h - h + 4}{4\lambda^*(\gamma h + 3)}, \quad \alpha_D^* = \frac{\gamma h + 4}{4\lambda^*(\gamma h + 3)}, \tag{A4.2.3}$$

$$\beta_Q^* = \frac{q}{2\lambda^*}, \quad \alpha_A^* = \frac{\gamma h + 2}{2\lambda^*(\gamma h + 3)}. \tag{A4.2.4}$$

Proof. Using the quant department Q's incorrect conjectures about firm A's trading strategies as defined in Eq.(A4.2.1), we can compute the FOC of Q's maximisation problem about the expected trading profit $E\left[(v_1 + v_2 - \lambda^* x_Q - \lambda^* x_A)\, x_Q | v_1, \theta_2\right]$, which gives

$$
\begin{aligned}
x_Q &= \frac{v_1 + E\left(v_2|\theta_2\right)}{2\lambda^*} - \frac{E\left(x_A|v_1,\theta_2\right)}{2} \\
&= \frac{v_1 + q\theta_2}{2\lambda^*} - \frac{E(v|\phi_A)}{4\lambda^*} + \frac{(1-h)E(x_B|\phi_A)}{4} \\
&= \frac{v_1 + q\theta_2}{2\lambda^*} - \frac{hv_1}{4\lambda^*} - \frac{(1-h)x_A}{2}.
\end{aligned} \tag{A4.2.5}
$$

The FOC of D's maximisation problem about its expected trading profit $E\big(v_1 + v_2 - \lambda^* x_D - \lambda^* x_A)\, x_D|v_1\big)$ leads to

$$
x_D = \frac{v_1}{2\lambda^*} - \frac{x_A}{2}. \tag{A4.2.6}
$$

The FOC of firm A's maximisation problem about its expected trading profit $E(v_1 + v_2 - \lambda^* x_B - \lambda^* x_A)\, x_A|v_1)$ gives rise to

$$
\begin{aligned}
x_A &= \frac{v_1}{2\lambda} - \frac{E(\gamma x_Q|v_1)}{2} - \frac{E((1-\gamma)x_D|v_1)}{2} \\
&= \frac{v_1}{2\lambda^*} - \frac{\gamma\alpha_Q^* v_1}{2} - \frac{(1-\gamma)x_D}{2}.
\end{aligned} \tag{A4.2.7}
$$

Combining Eqs. (A4.2.5), (A4.2.6), and (A4.2.7), we know that

$$
\begin{cases}
\alpha_Q^* = \dfrac{1}{2\lambda^*} - \dfrac{h}{4\lambda^*} - \dfrac{(1-h)\alpha_A^*}{2} \\
\alpha_D^* = \dfrac{1}{2\lambda^*} - \dfrac{\alpha_A^*}{2} \\
\alpha_A^* = \dfrac{1}{2\lambda^*} - \dfrac{\gamma\alpha_Q^*}{2} - \dfrac{(1-\gamma)\alpha_D^*}{2}.
\end{cases} \tag{A4.2.8}
$$

Based on the above equation system, we can express α_A, α_D, and α_Q as functions of λ^*:

$$
\alpha_Q^* = \frac{\gamma h - h + 4}{4\lambda^*(\gamma h + 3)}, \tag{A4.2.9}
$$

$$
\alpha_D^* = \frac{\gamma h + 4}{4\lambda^*(\gamma h + 3)}, \tag{A4.2.10}
$$

$$
\alpha_A^* = \frac{\gamma h + 2}{2\lambda^*(\gamma h + 3)}. \tag{A4.2.11}
$$

From Eq.(A4.2.5), the aggressiveness of the quant department on θ_2 subjects to

$$
\beta_Q^* = \frac{q}{2\lambda^*}. \tag{A4.2.12}
$$

The market maker considers the aggregate order flow ω as $\omega = x_A + \gamma x_Q + (1-\gamma)x_D + u$. The price $p = E(v|\omega) = \lambda^* \omega$ is characterised by

$$\lambda^* = \frac{(\alpha_A^* + \gamma\alpha_Q^* + (1-\gamma)\alpha_D^*)\Sigma_0 + q\gamma\beta_Q^*\Sigma_0}{(\alpha_A^* + \gamma\alpha_Q^* + (1-\gamma)\alpha_D^*)^2\Sigma_0 + q\gamma{\beta_Q^*}^2\Sigma_0 + \Sigma_0}. \tag{A4.2.13}$$

Multiplying both sides of Eq.(A4.2.13) by λ^* and rearranging terms, we know that the above equation is equivalent to

$$\begin{aligned} &{\lambda^*}^2(\alpha_A^* + \gamma\alpha_Q^* + (1-\gamma)\alpha_D^*)^2 + n{\lambda^*}^2(\gamma\beta_Q^*)^2 \\ &+ {\lambda^*}^2 = \lambda^*(\alpha_A^* + \gamma\alpha_Q^* + (1-\gamma)\alpha_D^*) + n\lambda^*\gamma\beta_Q^*. \end{aligned} \tag{A4.2.14}$$

Substituting Eqs.(A4.2.9)-(A4.2.12) into Eq.(A4.2.14), we can explicitly solve for the value of λ^*, which is

$$\lambda^* = \sqrt{\frac{(\gamma h + 4)(\gamma h + 2)}{4(\gamma h + 3)^2} + \frac{q\gamma(2-\gamma)}{4}}. \tag{A4.2.15}$$

The equilibrium values of $(\lambda^*,\ \alpha_Q^*,\ \beta_Q^*,\ \alpha_D^*,\ \alpha_A^*)$ are thus given by Eqs.(A4.2.9)-(A4.2.12) and (A4.2.15).

■

The normalised trading aggressiveness measures of firm A on signal v_1, and of firm B on signals v_1 and θ_2 are derived as:

$$\widetilde{\alpha}_A^* = \frac{3\gamma h + 6}{2\gamma h + 6},\ \widetilde{\alpha}_B^* = \frac{3}{\gamma h + 3},\ \widetilde{\beta}_B^* = \frac{3q\gamma}{2}. \tag{A4.2.16}$$

Define the overall trading aggressiveness as $\tau^* \equiv \widetilde{\alpha}_A^* + \widetilde{\alpha}_B^* + \widetilde{\beta}_B^*$, the following proposition identifies the effects of growth in quantitative investing on overall trading aggressiveness with partial strategic inflexibility.

Proposition 4.11. *Consider an economy with partial strategic inflexibility. When quantitative investing level γ increases, the impact on overall trading aggressiveness τ^* again hinges on the trade-off among:*

(i) *The positive capacity enhancing effect by the quantitative firm B, which is given by*

$$\frac{\partial \widetilde{\beta}_B^*}{\partial \gamma} = \frac{3q}{2}. \tag{A4.2.17}$$

(ii) *The negative strategy oblivion effect by the quantitative firm B, which is given by*

$$\frac{\partial \widetilde{\alpha}_B^*}{\partial \gamma} = -\frac{3h}{(\gamma h + 3)^2}. \tag{A4.2.18}$$

(iii) *The positive internalising effect by the fully-discretionary firm A, which is given by*

$$\frac{\partial \widetilde{\alpha}_A^*}{\partial \gamma} = \frac{3h}{2(\gamma h + 3)^2}. \tag{A4.2.19}$$

Proof. Define $\psi_2 \equiv 1/3\lambda^*$, the normalised trading aggressiveness measure of firm B on signal θ_2 is $\widetilde{\beta}_B^* = \gamma\beta_Q/\psi_2 = 3\lambda^*\gamma\beta_Q$, which yields

$$\widetilde{\beta}_B^* = 3\lambda^*\gamma \cdot \frac{q}{2\lambda^*} = \frac{3q\gamma}{2}. \tag{A4.2.20}$$

The first-order derivative of $\widetilde{\beta}_B^*$ with respect to γ is thus

$$\frac{\partial \widetilde{\beta}_B^*}{\partial \gamma} = \frac{3q}{2}. \tag{A4.2.21}$$

The normalised trading aggressiveness measure of firm B on signal v_1 is $\widetilde{\alpha}_B^* = (\gamma\alpha_Q + (1-\gamma)\alpha_D)/\psi_2$, which can be calculated as

$$\widetilde{\alpha}_B^* = 3\lambda^*\gamma \cdot \frac{\gamma h - h + 4}{4\lambda^*(\gamma h + 3)} + 3\lambda^*(1-\gamma) \cdot \frac{\gamma h + 4}{4\lambda^*(\gamma h + 3)} = \frac{3}{\gamma h + 3}. \tag{A4.2.22}$$

The first-order derivative of $\widetilde{\alpha}_B^*$ with respect to γ is thus calculated as follows

$$\begin{aligned} \frac{\partial \widetilde{\alpha}_B^*}{\partial \gamma} &= -\frac{3}{(\gamma h + 3)^2} \cdot \frac{\partial(\gamma h + 3)}{\partial \gamma} \\ &= -\frac{3h}{(\gamma h + 3)^2}. \end{aligned} \tag{A4.2.23}$$

The normalised trading aggressiveness measure of the fully discretionary firm A on signal v_1 is $\widetilde{\alpha}_A^* = \alpha_A/\psi_2$, which can be calculated as

$$\begin{aligned} \widetilde{\alpha}_A^* &= 3\lambda^* \cdot \frac{\gamma h + 2}{2\lambda^*(\gamma h + 3)} \\ &= \frac{3\gamma h + 6}{2\gamma h + 6}. \end{aligned} \tag{A4.2.24}$$

The first-order derivative of $\widetilde{\alpha}_A^*$ with respect to γ is thus

$$\begin{aligned}\frac{\partial \widetilde{\alpha}_A^*}{\partial \gamma} &= \frac{3}{2} \cdot \frac{(\gamma h + 3)h - (\gamma h + 2)h}{(\gamma h + 3)^2} \\ &= \frac{3h}{2(\gamma h + 3)^2}.\end{aligned} \tag{A4.2.25}$$

Therefore, for $\gamma \in [0, 1]$, we have $\partial \widetilde{\beta}_B^*/\partial \gamma > 0$, $\partial \widetilde{\alpha}_B^*/\partial \gamma < 0$, and $\partial \widetilde{\alpha}_A^*/\partial \gamma > 0$. With partial strategic inflexibility and as quantitative investing level γ increases, firm B trades more aggressively on signal θ_2 and less aggressively on signal v_1, while firm A trades more aggressively on signal v_1.

■

Proposition 4.11 suggests the key result in the baseline model remains valid with partial strategic inflexibility. That is, the growth in quantitative trading still gives rise to the positive capacity enhancing effect, the negative strategy oblivion effect and the positive internalising effect. Note that, the strategy oblivion effect $\partial \widetilde{\alpha}_B^*/\partial \gamma$ and the internalising effect $\partial \widetilde{\alpha}_A^*/\partial \gamma$ are independent of signal quality q and approach zero when inflexibility degree h approaches zero, while the capacity enhancing effect $\partial \widetilde{\beta}_B^*/\partial \gamma$ is independent of h.

Corollary 4.12. *Consider an economy with partial strategic inflexibility. The magnitude of the capacity enhancing effect increases in signal quality q. The magnitude of the strategy oblivion effect increases in inflexibility degree h at a faster rate than the magnitude of the internalising effect does. The magnitude of the strategy oblivion effect decrease in quantitative investing intensity γ at a faster rate than the magnitude of the internalising effect does.*

Proof. Using Eq.(A4.2.17) and taking the derivative of the capacity enhancing effect with respect to the signal quality q, we have

$$\frac{\partial^2 \widetilde{\beta}_B^*}{\partial \gamma\, \partial q} = \frac{3}{2}. \tag{A4.2.26}$$

The second-order derivative $\partial^2 \widetilde{\beta}_B^*/\partial \gamma\, \partial q > 0$ again suggests that the positive capacity enhancing effect is intensified by increased signal quality q.

Using Eq.(A4.2.23) and taking the derivative of the strategy oblivion effect with respect to the inflexibility degree h, we have

$$\begin{aligned}\frac{\partial^2 \widetilde{\alpha}_B^*}{\partial\gamma\,\partial h} &= -3\left(\frac{1}{(\gamma h+3)^2} - \frac{2\gamma h}{(\gamma h+3)^3}\right)\\ &= -\frac{3(3-\gamma h)}{(\gamma h+3)^3}.\end{aligned} \tag{A4.2.27}$$

Using Eq.(A4.2.25) and taking the derivative of the internalising effect with respect to the inflexibility degree h, we have

$$\begin{aligned}\frac{\partial^2 \widetilde{\alpha}_A^*}{\partial\gamma\,\partial h} &= \frac{3}{2}\left(\frac{1}{(\gamma h+3)^2} - \frac{2\gamma h}{(\gamma h+3)^3}\right)\\ &= \frac{3(3-\gamma h)}{2(\gamma h+3)^3}.\end{aligned} \tag{A4.2.28}$$

The second-order derivative $\partial^2 \widetilde{\alpha}_A^*/\partial\gamma\,\partial h > 0$ implies that the positive internalising effect is intensified by increased inflexibility degree h. Based on Eqs.(A4.2.27) and (A4.2.28), we know that

$$\left|\frac{\partial^2 \widetilde{\alpha}_B^*}{\partial\gamma\,\partial h}\right| - \frac{\partial^2 \widetilde{\alpha}_A^*}{\partial\gamma\,\partial h} = \frac{3(3-\gamma h)}{2(\gamma h+3)^3} > 0. \tag{A4.2.29}$$

By Eq.(A4.2.29), the magnitude of the strategy oblivion effect increases in inflexibility degree h at a faster rate than the magnitude of the internalising effect does. Using Eq.(A4.2.23) and taking the derivative of the strategy oblivion effect with respect to the quantitative investing level γ gives

$$\begin{aligned}\frac{\partial^2 \widetilde{\alpha}_B^*}{\partial\gamma\,\partial\gamma} &= -3h\left(-\frac{2h}{(\gamma h+3)^3}\right)\\ &= \frac{6h^2}{(\gamma h+3)^3}.\end{aligned} \tag{A4.2.30}$$

Since $\partial^2 \widetilde{\alpha}_B^*/\partial\gamma\,\partial\gamma > 0$, the negative strategy oblivion effect is weakened by increased quantitative investing level γ. Using Eq.(A4.2.25) and taking the derivative of the internalising effect with respect to the quantitative investing level γ gives

$$\begin{aligned}\frac{\partial^2 \widetilde{\alpha}_A^*}{\partial\gamma\,\partial\gamma} &= \frac{3h}{2}\left(-\frac{2h}{(\gamma h+3)^3}\right)\\ &= -\frac{3h^2}{(\gamma h+3)^3}.\end{aligned} \tag{A4.2.31}$$

Since $\partial^2\widetilde{\alpha}_A^*/\partial\gamma\,\partial\gamma < 0$, the negative strategy oblivion effect is weakened by increased quantitative investing level γ. Based on Eqs.(A4.2.30) and (A4.2.31), we know that

$$\frac{\partial^2\widetilde{\alpha}_B^*}{\partial\gamma\,\partial\gamma} - \left|\frac{\partial^2\widetilde{\alpha}_A^*}{\partial\gamma\,\partial\gamma}\right| = \frac{3h^2}{(\gamma h+3)^3} > 0. \tag{A4.2.32}$$

By Eq.(A4.2.32), the magnitude of the strategy oblivion effect decreases in quantitative investing level γ at a faster rate than the magnitude of the internalising effect does.

■

Corollary 4.12 describes in detail how these three effects vary with q,γ, and h. Two notable observations emerge from Corollary 4.12. First, the monotonicity of the three effects with regard to q and γ remain qualitatively unchanged compared with Corollary 4.3. In particular, an increase in q intensifies the capacity enhancing effect, i.e., $\partial^2\widetilde{\beta}_B^*/\partial\gamma\,\partial q > 0$. Second, a rise in h intensifies the strategy oblivion effect more than it intensifies the internalising effect, i.e., $\partial^2\widetilde{\alpha}_B^*/\partial\gamma\,\partial h < 0$, $\partial^2\widetilde{\alpha}_A^*/\partial\gamma\,\partial h > 0$ and $-\partial^2\widetilde{\alpha}_B^*/\partial\gamma\,\partial h > \partial^2\widetilde{\alpha}_A^*/\partial\gamma\,\partial h$. The rationale behind the intensification of the strategy oblivion effect brought by a rise in h can be stated as follows. When the technology implemented by the quant department becomes more reliant on fixed rules, we can reasonably expect that a certain increment of $\Delta\gamma$ at any level of γ brings a larger increment in total strategic inflexibility and thus an intensified strategy oblivion effect $\partial\widetilde{\alpha}_B^*/\partial\gamma$.

Based on the above discussion, we conjecture the net impact of growth in quantitative investing on overall trading aggressiveness is more likely to be beneficial when q is high or h is low due to the capacity enhancing effect's dominance, and is more likely to be harmful when q is low or h is high due to the strategy oblivion effect's dominance. Figure A4.1 graphically illustrates the three effects for different combinations of q and h, which helps us validate the conjecture. Panels (A)-(C) illustrate the case of varying q while keeping h fixed, and demonstrate similar patterns to Figure 4.2. Panels (D)-(F) illustrate the case of varying h while keeping n fixed.

In Panel (D) with low h, the absolute value of the negative strategy oblivion effect lies below the sum of the positive effects, and the overall trading aggressiveness is improved by growth in quantitative investing level. In Panel (E) with moderate h, the sum of the positive effects and the absolute value of the negative strategy oblivion effect are both shifted upward compared with their values when h is low, since $\partial^2\widetilde{\alpha}_B^*/\partial\gamma\,\partial h < 0$ and $\partial^2\widetilde{\alpha}_A^*/\partial\gamma\,\partial h > 0$. The former upward shift is smaller than

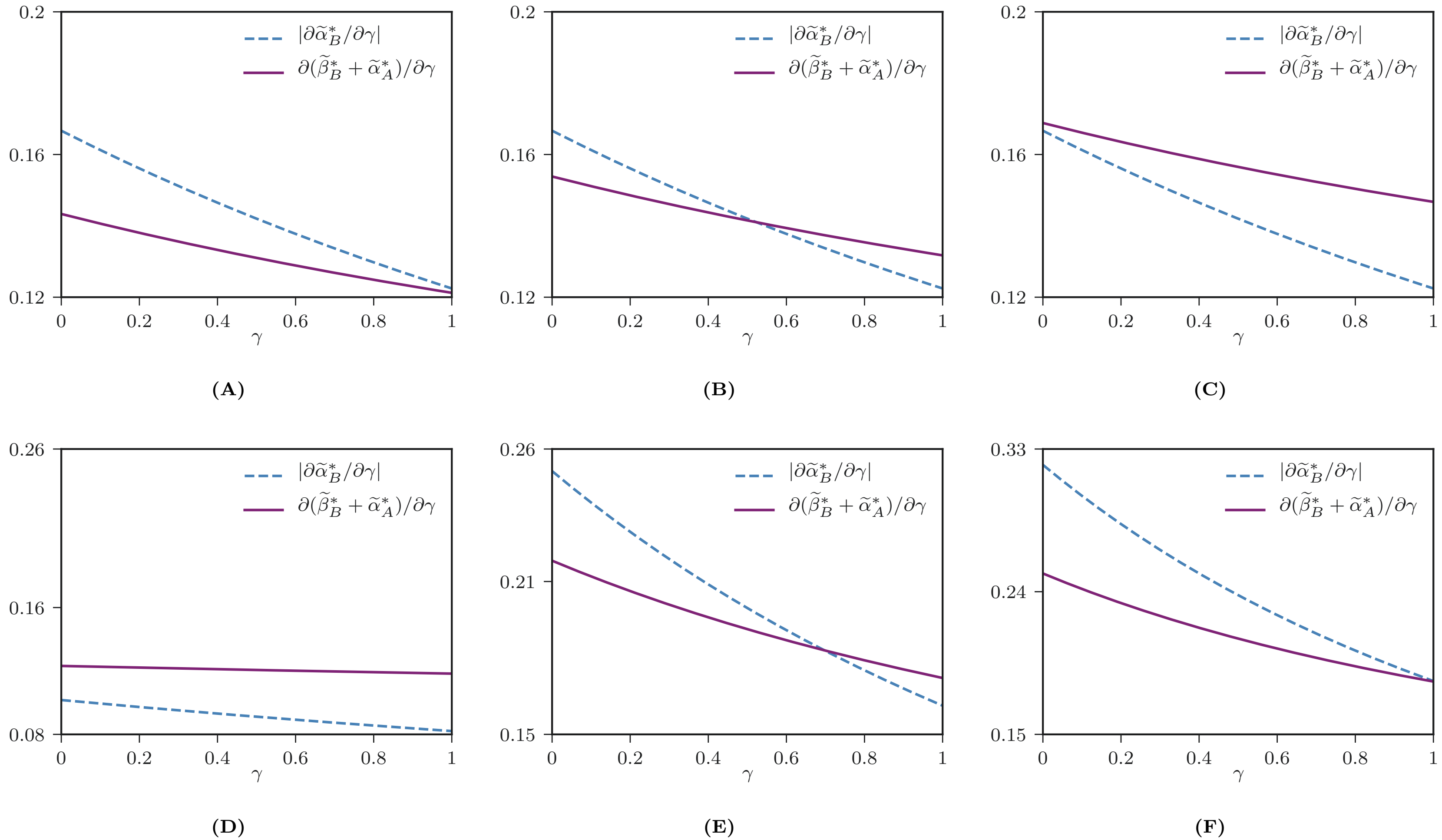


Figure A4.1: Trading aggressiveness with partial strategic inflexibility.
This figure illustrates the derivatives of trading aggressiveness measures with respect to quantitative investing level γ in the setting with partial strategic inflexibility. Panels (A)-(C) illustrate the case of varying q while keeping $h = 0.5$ fixed. We assume $q = 0.04$ in Panel (A), $q = 0.047$ in Panel (B), and $q = 0.057$ in Panel (C). Panels (D)-(F) illustrate the case of varying h while keeping $q = 0.0611$ fixed. We assume $h = 0.32$ in Panel(D), $h = 0.76$ in Panel (E), and $h = 0.96$ in Panel (F). The other parameter value is $\Sigma_0 = 1$. Among all panels, the blue dashed line is the absolute value of the strategy oblivion effect, and the purple line is the sum of the capacity enhancing effect and the internalising effect.

the later upward shift because $-\partial^2\widetilde{\alpha}_B^*/\partial\gamma\,\partial h > \partial^2\widetilde{\alpha}_A^*/\partial\gamma\,\partial h$. Therefore, the absolute value of the strategy oblivion effect is closer to the sum of the positive effects than when h is low, and intersects with the sum of the positive effects. As $-\partial^2\widetilde{\alpha}_B^*/\partial\gamma\,\partial\gamma < \partial^2\widetilde{\alpha}_A^*/\partial\gamma\,\partial\gamma < 0$, we observe that the sum of the positive effects is initially smaller but then greater than the absolute value of the negative strategy oblivion effect. For moderate h, the overall trading aggressiveness decreases with the quantitative investing level if γ is small, and increases with the quantitative investing level if γ is large. In Panel (F) with high h, given $\partial^2\widetilde{\alpha}_B^*/\partial\gamma\,\partial h < 0$ and $\partial^2\widetilde{\alpha}_A^*/\partial\gamma\,\partial h > 0$, the absolute value of the strategy oblivion effect moves up further compared with when h is moderate, and lies above the sum of the positive effects. Interestingly, unlike changes in q, changes in h do alter the shapes of the curve representing the absolute value of the strategy oblivion effect and the curve representing the sum of the two positive effects. This is because neither the strategy oblivion effect nor the internalising effect is linear in h.

The discussion about trading aggressiveness has implications on price efficiency. The equilibrium price efficiency measure in the economy with partially strategic inflexibility is specified by the following equation:

$$PE_{partial\ inflexibility} = \frac{2\gamma h + 6}{(3\gamma h - 3q\gamma - q\gamma^2 h + 8)\Sigma_0}. \tag{A4.2.33}$$

Proposition 4.13 shows the price efficiency impact of growth in quantitative trading when a firm is fully discretionary and another firm's quant department is partially inflexible.

Proposition 4.13. *Consider an economy with partial strategic inflexibility. When quantitative investing level γ increases, the effect on price efficiency depends on the relative magnitude of signal quality q and inflexibility degree h:*

(i) When $0 < q/h \le a_1$, price efficiency decreases in quantitative investing level.

(ii) When $a_1 < q/h < a_2$, price efficiency decreases if quantitative investing level is below the threshold value $\bar{\gamma}^*$, and increases if otherwise.

(iii) When $q/h \ge a_2$, price efficiency increases in quantitative investing level.

Proof. The covariance between the fundamental value v and the price p in the setting of partial strategic inflexibility can be calculated as follows:

$$\begin{aligned} cov(v,p) &= cov(v_1+v_2, \lambda^*\alpha_A v_1 + \lambda^*\gamma\alpha_Q v_1 + \lambda^*(1-\gamma)\alpha_D v_1 + \lambda^*\gamma\beta_Q\theta_2) \\ &= \frac{\gamma h+4}{2\gamma h+6}\Sigma_0 + \frac{q\gamma}{2}\Sigma_0. \end{aligned} \tag{A4.2.34}$$

Because $cov(v,p) = var(p)$, the conditional variance $var(v|p)$ can be calculated via the projection theorem as follows:

$$\begin{aligned} var(v|p) &= var(v) - \frac{cov(v,p)^2}{var(p)} \\ &= 2\Sigma_0 - \frac{\gamma h+4}{2\gamma h+6}\Sigma_0 - \frac{q\gamma}{2}\Sigma_0. \end{aligned} \tag{A4.2.35}$$

According to the chain rule, the first-order derivative of the price discovery measure $var(v|p)^{-1}$ with respect to the quantitative investing level γ satisfies

$$\begin{aligned} \frac{\partial PE_{partial\ inflexibility}}{\partial\gamma} &= -\frac{1}{\mathrm{var}(v_1+v_2|p)^2}\cdot\frac{\partial var(v|p)}{\partial\gamma} \\ &= -\frac{1}{\mathrm{var}(v_1+v_2|p)^2}\left(\frac{h}{2(\gamma h+3)^2} - \frac{q}{2}\right)\Sigma_0^{-1} \\ &\propto -\left(\frac{h}{2(\gamma h+3)^2} - \frac{q}{2}\right). \end{aligned} \tag{A4.2.36}$$

Based on Eq.(A4.2.29), the derivative $\partial PE_{partial\ inflexibility}/\partial\gamma > 0$ holds if and only if

$$-\left(\frac{h}{2(\gamma h+3)^2} - \frac{q}{2})\right) > 0 \Longleftrightarrow \gamma > \frac{1}{\sqrt{qh}} - \frac{3}{h} \equiv \bar{\gamma}^*. \tag{A4.2.37}$$

Similarly, the derivative $\partial PE_{partial\ inflexibility}/\partial\gamma < 0$ holds if and only if

$$-\left(\frac{h}{2(\gamma h+3)^2} - \frac{q}{2})\right) < 0 \Longleftrightarrow \gamma < \frac{1}{\sqrt{qh}} - \frac{3}{h} \equiv \bar{\gamma}^*. \tag{A4.2.38}$$

The value range of n such that $\bar{\gamma}^*$ lie inside of (0,1) can be derived as follows:

$$0 < \frac{1}{\sqrt{qh}} - \frac{3}{h} < 1 \Longleftrightarrow \frac{q}{h} > \frac{1}{(3+h)^2} \equiv a_1 \text{ and } \frac{q}{h} < \frac{1}{9} \equiv a_2. \tag{A4.2.39}$$

Consequently, when $a_1 < q/h < a_2$, the price efficiency decreases (resp. increases) with the quantitative investing level γ if γ is below (resp. above) the threshold value of $\bar{\gamma}^*$.

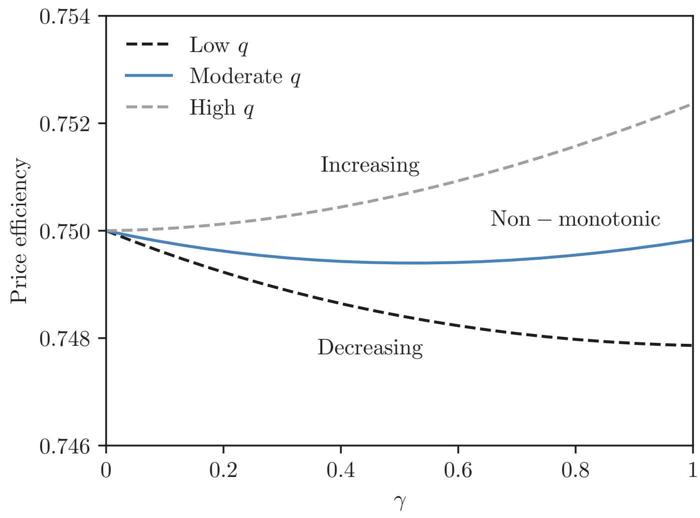
When $q/h < a_1$, we obtain $\bar{\gamma}^* > 1$, which means that $\partial PE_{partial\ inflexibility}/\partial\gamma < 0$. When $q/h = a_1$ and $\bar{\gamma}^* = 1$, we have $\gamma \leq \bar{\gamma}^*$ and $\partial PE_{partial\ inflexibility}/\partial\gamma \leq 0$, of which the equal signs stand for $\gamma = 1$. As a result, when $0 < q/h \leq a_1$, the price efficiency decreases with the quantitative investing level γ.

When $q/h > a_2$, we obtain $\bar{\gamma}^* < 0$, which means that $\partial PE_{partial\ inflexibility}/\partial\gamma > 0$. When $q/h = a_2$ and $\bar{\gamma}^* = 0$, we have $\gamma \geq \bar{\gamma}^*$ and $\partial PE_{partial\ inflexibility}/\partial\gamma \geq 0$, of which the equal signs stand for $\gamma = 1$. As a result, when $q/h \geq a_2$, the price efficiency decreases with the quantitative investing level γ.

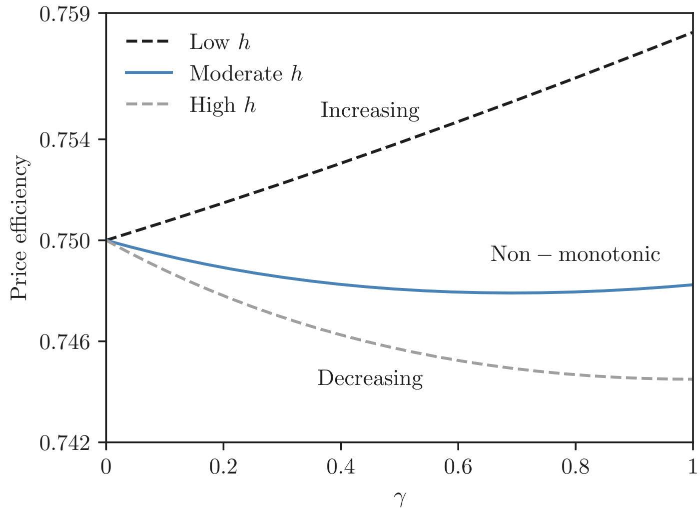
■

If the ratio of q/h is low ($0 < q/h \leq a_1$), indicating low q or high h, the price efficiency is harmed by growth in quantitative investing, because the strategy oblivion effect dominates, and the aggregate trading aggressiveness deteriorates. If the ratio of q/h is high ($q/h \geq a_2$), indicating high q or low h, the price efficiency is improved by growth in quantitative investing, because the capacity enhancing effect dominates, and the aggregate trading aggressiveness is enhanced. If the ratio of q/h is high ($q/h \geq a_2$), indicating high q or low h, the price efficiency is improved by growth in quantitative investing, because the capacity enhancing effect dominates, and the aggregate trading aggressiveness is enhanced.

If the ratio of q/h is moderate ($a_1 < q/h < a_2$), the hump shape of price efficiency in γ is consistent with the trading aggressiveness changes implied by Panels (B) and (E) of Figure A4.1. The price efficiency is initially amplified by growth in quantitative investing when γ is lower than the threshold value $\bar{\gamma}$ but then diminished by growth in quantitative investing if γ increases beyond $\bar{\gamma}$. Figure A4.2 provides numerical examples of the proposition under the same parameter settings as Figure A4.1.



(A)



(B)

Figure A4.2: Price efficiency with partial strategic inflexibility. This figure illustrates the price efficiency impact of growth in quantitative investing level γ in the setting of partial strategic inflexibility. Panel (A) illustrates the case of varying q while keeping $h = 0.5$ fixed. We let the grey dashed line correspond to $q = 0.04$, the blue line correspond to $q = 0.047$, and the black dashed line correspond to $q = 0.057$ in Panel (A). Panel (B) illustrates the case of varying h while keeping $q = 0.0611$ fixed. We let the grey dashed line correspond to $q = 0.32$, the blue line correspond to $q = 0.76$, and the black dashed line correspond to $q = 0.96$ in Panel (B). The other parameter value is $\Sigma_0 = 1$.

Chapter 5

Conclusion and future research

The technology revolution in finance has given rise to numerous innovations, including the introduction of limit order books, the increased involvement of social media in personal investment decisions, and the expansion of FinTech services. This thesis analyses the impact of the technology revolution on strategic behaviours of market participants and its effect on market quality in the modern finance industry, focusing on two important issues in the field of market microstructure studies: market manipulation and quantitative investing.

The thesis addresses two questions about market manipulation: Could the transition from the traditional auction market to the limit order market alter the form of market manipulation, and could meme investing, a retail buying frenzy coordinated on social media, reduce manipulative short selling? The thesis also addresses two questions about quantitative investing: How will quantitative investing, driven by machine computation, and discretionary investing, driven by human skills, strategically interact with each other, and what are the relevant market efficiency implications?

5.1 Learning to strategic trade and manipulate in limit order markets

Chapter 2 introduces Q-learning, a novel machine learning technique, as a learning tool to a dynamic limit order market (LOM) to investigate how order book information and learning impact the strategic trading behaviours of bounded rational traders. Q-learning fully endogenises traders' order choice problems. Overall, this chapter shows the great potential of integrating reinforcement learning (RL) with the market microstructure theory framework and adopting RL as an alternative belief updating rule, since the usage of RL relaxes a set of strict assumptions like the agent's perfect knowledge of model priors.

5.1.1 Order choices in limit order markets

Trial-and-error learning based on order book information of bounded rational agents leads to strategic trading, which is featured by predictable trading behaviours. Informed traders primarily depend on fundamental information (as opposed to other order book information) to determine their order choices. Informed traders improve market resiliency and prefer limit (or market) orders when mispricing is small (or large). Uninformed traders, owing to their information disadvantage, learn to "chase the trend" and have a greater tendency to place market buy orders after observing a previous market buy.

5.1.2 Informed manipulation in limit order markets

The analysis in Chapter 2 shows that informed manipulation can be learned as an equilibrium trading strategy in the dynamic LOM, where informed traders deliberately stray from their usual predictable trading behaviours to exploit uninformed traders' predictable trading behaviours. Anticipating a mispricing reversal when small-in-size positive (negative) mispricing is accompanied by high depth imbalance at the best bid (ask), manipulative informed traders strategically act against their own preference for limit buys (sells) and use market buys (sells) to trigger trend-chasing uninformed market buys (sells), enhancing execution probability and profitability of later informed traders' limit sells (buys). A novel form of market manipulation is presented in our dynamic LOM, in which informed traders take the "wrong" action facing make-take decisions to mislead uninformed traders, as opposed to taking the "wrong" action facing buy/sell or amount decisions as in traditional quote-driven models.

5.2 The influence of meme investing on manipulative short selling

A Kyle-type model with both manipulative and informed short sellers is developed in Chapter 3 to explain how meme investing, i.e., a retail buying frenzy (as short-sale friction), impacts the market and real investment efficiency. To model the retail buying frenzy, an otherwise Goldstein and Guembel (2008) model is extended to include the settings of explicit short-sale constraints and asymmetric noise trading. Manipulative short selling is profitable as it drives down the firm value via the feedback effect from the financial market to real investment. Chapter 3 shows that meme investing can be a natural remedy to manipulative short selling and does not harm informed short selling for certain types of firms and market conditions.

5.2.1 Meme investing as explicit short-sale constraints

Explicit short sale constraints settings in Chapter 3 include costly short selling and a short-sale ban. An intermediate level of short-sale cost enhances investment efficiency, whereas a relatively high cost has mixed effects on investment efficiency. An extremely large short-sale cost can impede managers' ability to learn from stock prices and compromise the quality of their investment decisions, similar to what happens when a short-sale ban is implemented.

5.2.2 Meme investing as asymmetric noise trading

Chapter 3 also models a retail buying frenzy as asymmetric noise trading such that noise buys are more likely than noise sells. Asymmetric noise trading results in two opposing effects on the informativeness of order flows, namely the order flow disguising effect and the uninformed-specific short-sale cost effect. The former effect harms investment efficiency, whereas the latter effect improves it. The latter effect arises since the increase in noise buys pushes up overall prices and increases the manager's investment propensity and imposes a covering cost that is caused by the correction of underinvestment and is specific to the uninformed trader. The uninformed-specific short-sale cost effect is more likely to prevail over the order flow disguising effect, thereby improving investment efficiency when (i) the fraction of uninformed speculator is large, (ii) the ex-ante NPV of the project is large, and (iii) the uncertainty about the profitability of the investment is small.

5.3 The interaction of quantitative and discretionary investing

The strategic interaction of quantitative and discretionary investing is formalised in Chapter 4 through a Kyle-type model populated by a fully discretionary investment firm, an investment firm composed of a quant and a discretionary department, liquidity traders, as well as competitive market makers. The analysis is further extended to the setting in which two firms each consist of a quant and a discretionary department and the setting in which partial strategic inflexibility is present. In essence, Chapter 4 deepens the understanding of the channels through which human-machine interaction may influence financial market outcomes by breaking down the impact of growth in machine-based quantitative investing into empirically testable components, including the capacity enhancing effect and the strategy oblivion effect. Finally, an additional competition effect arises in the setting where both firms have quant departments, referring to the trading aggressiveness reduction of the opponent firm due to more fierce competition on quantitative technology.

5.3.1 Growth in quantitative investing level by individual firms when other things equal

A ceteris paribus increase in a firm's quantitative investing level could lead to various strategic effects in the three aforementioned model settings. First, the capacity enhancing effect implies increased computing power and improved information extraction capabilities of the firm, leading to a rise in the firm's trading aggressiveness. Second, the strategy oblivion effect renders the firm more strategically inflexible and more inclined to overestimate the trading aggressiveness of other traders, leading to a decrease in the firm's trading aggressiveness. Third, the internalising effect implies the firm's discretionary opponents take advantage of the weaker strategic flexibility by trading more aggressively. Consequently, increased quantitative investing level can have a non-monotonic effect on price efficiency.

5.3.2 Growth in quantitative investing level by individual firms with a high market quantitative investing level

Accompanied by a high market quantitative investing level (i.e., a high quantitative investing level of the opponent firm), an increase in a firm's quantitative investing level is more likely to harm price efficiency. This is due to the diminished internalising effect, with which discretionary traders are less likely to be present in the market to internalise trading aggressiveness reduction brought by increased strategic inflexibility.

5.4 Final remarks

To analyse the implications of technological developments in financial markets, the methodologies used in this thesis involve applying reinforcement learning to market microstructure theory and modelling feedback effect from financial markets to real economy in the presence of costly short selling, and modelling the interaction of quantitative and discretionary investing.

Future research may focus on specific tactics of market manipulation such as spoofing, a more realistic form of market manipulation in limit order markets, using reinforcement learning agents. Spoofing is a manipulative practice that involves placing one or more limit orders on a particular side of the limit order book, with the intention of revoking these orders before they are executed. The primary objective of this tactic is to deceive the market by creating a false impression of buying or selling interest, with the aim of influencing other traders' order submission decisions. Researchers can utilize the reinforcement learning agents to comprehend when and how spoofing emerges as an optimal trading strategy, as well as the impact spoofing has on other trading strategies, and more importantly, what market designs are more susceptible to spoofing.

Future research may undertake empirical analysis to explore the impact of machine-based quants replacing human discretionary traders on price efficiency. Since the thesis provides an analytical decomposition of the impact of growth in machine-based quantitative investing on the strategic interaction of humans and machines and on market efficiency into empirically testable components primarily driven by capacity and flexibility channels, our theoretical framework of quantitative investing growth can provide a foundation for such empirical research to examine these channels.

Future research can also shed light on the topics of welfare and market quality consequences of alternative data, i.e., data used by investors to evaluate a company that is not within their traditional data sources. Well-known examples of alternative data include unstructured text and imagery from news feeds, social media, online communities, communications metadata, satellite imagery, and geospatial information. Since alternative data has only recently been applied to investment analysis, traders are likely to be unsure about how reliable such data is upon obtaining it. One possible direction to formalise the topic is to extend standard market microstructure models to allow the speculators to learn about data reliability over time through trading history.

While there are various avenues for future research to expand upon the theoretical groundwork laid out in the thesis, it's important to acknowledge the limitations of the current research. One constraint of Chapter 2 pertains to the realism of the trader objective, particularly in relation to the trading profit of reinforcement learning traders. Currently, this profit is defined as the difference between the fundamental value and the executed price for a buyer, and the difference between the executed price and the fundamental value for a seller. A more realistic approach to defining trading profit would involve considering the price differential between when the trader opens and closes positions. Chapter 3 is constrained by the possibility that there might be additional factors at play beyond the observed heightened short-sale frictions associated with retail investor coordination via social media. Chapter 4 is similarly constrained by the relatively idealized assumptions of the model. While we simulate a Kyle-type financial market involving two investment firms, real-world financial markets encompass a multitude of traders and are characterized by greater competitiveness. Additionally, quantitative and discretionary traders may exhibit varying abilities to diversify across different securities, a nuance not captured in our single-asset model.

The financial market microstructure is constantly evolving as technology developments never stop. This thesis provides only a brief overview of recent developments and is inevitably limited in scope, and additional research is necessary to gain a deeper understanding of future challenges and issues in financial markets.

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