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# Enhanced Maximum Torque Per Ampere Control with Predictable Core Loss for the Interior Permanent Magnet Synchronous Motor

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Abstract—Due to the availability of extended applications for the reluctance torque, increasing incorporation of interior permanent magnet synchronous motors (IPMSMs) has been observed in the electric drive system, and to save energy and improve operation efficiency, the maximum torque per ampere (MTPA) control has attracted a lot of academic attention. However, in the traditional MTPA, the optimization objective of tracking the minimum armature current only applies to the situation where the energy consumption in the winding resistance is imposed as the sole constraint. The energy consumption in the core material, i.e. core loss, will grow as motor load and frequency increase, and hence the efficiency optimization control of the IPMSM should also consider the core loss. Firstly, this article proposes a novel mathematical model of the IPMSM which enables the establishment of control algorithms with predictable core loss. Then, a novel MTPA is proposed which can simultaneously optimize the copper loss and core loss to maximize the utilization of phase currents and minimize the electromagnetic losses of the IPMSM. To verify the superiority of the proposed MTPA, the analytical results are compared with the conventional MTPA and *I*<sub>d</sub>=0 control methods.

*Index Terms*—Core loss, copper loss, current control, maximum torque per ampere, interior permanent magnet synchronous motor.

## I. INTRODUCTION

**P**ERMANENT magnet (PM) synchronous motors (PMSMs) continue to be a research focus in motor design and control and have been increasingly extensively applied in industrial drives, such as electric vehicles, rail transport, wind power generation, robotics, *etc.* [1-3]. Since the electric energy cost accounts for more than 90% of all cost, in which the remaining costs include motor price and maintenance cost, PMSM efficiency improvements are consistently gaining significant attention [4].

One of the most significant objectives of PMSM control is to reduce electromagnetic loss and thereby improve operation efficiency; consequently, the maximum torque per ampere (MTPA) control has flourished [5-10]. [5] investigated an online MTPA by injecting a small virtual angle signal into the current angle to track the MTPA points. [6] designed a mathematical-optimization-method-based MTPA control for the double inverter fed wound rotor induction motor, while an MTPA for a mono inverter dual parallel SPMSM was designed in [7]. Look-up tables in MTPA can help to reduce test time and workloads; therefore, [8] established a machine learning regularization theory to generate the look-up tables with good accuracy. The most common practice in the MTPA is to adopt a PI controller to regulate the velocity loop, while [9] proposed a sliding mode controller to replace the PI controller to improve the anti-interference capability of the IPMSM system. In the field of the no-insulation high-temperature superconducting synchronous motor, [10] found that the MTPA method is also applicable to this type of motor to improve the operation efficiency. However, in the above-mentioned research, only the energy consumed in winding resistance, i.e. copper loss, is taken into consideration while the energy consumed in the core material is neglected when developing the MTPA. As the energy consumed in the core material of the PMSM grows significantly when the motor speed and load increase [11, 12], the major motivation of this article is to develop a novel control method of the IPMSM to manage the core loss and further optimize motor efficiency.

This paper proposes a novel MTPA control method for the IPMSM, which is superior in electromagnetic loss management, motor efficiency improvement and simulation of real motor operation conditions. The two main original contributions are listed as follows:

 An IPMSM mathematical model capable of predicting the energy consumed by both winding resistance and core material in the two-phase rotating coordinate system is proposed, which facilitates the construction of the IPMSM control strategies by affording fast and accurate motor performance in practical applications.

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 A novel MTPA control strategy, including energy consumption management of both winding resistance and core material, is introduced, which is conductive to the reduction of electromagnetic losses.

## II. MATHEMATICAL MODEL OF THE IPMSM WITH PREDICTABLE CORE LOSS

## A. Mathematical Model

Benefiting from Park-Clark orthogonal transformation, the mathematical model of the IPMSM in the stationary coordinate system can be transformed into a model in the d-q axes coordinate frame attached to the motor rotor, which is extremely favorable for the decoupling control of the magnetic flux and torque. Furtherly, the proposed models in the d-q axes coordinate frame considering the core loss are shown as follows:

$$\begin{cases} V_d = R_s I_d + L_d \cdot dI_d^m / dt - \omega_e L_q I_q & (1) \\ V_q = R_s I_q + L_q \cdot dI_q^m / dt + \omega_e L_d I_d + \omega_e \varphi_f & \\ \begin{cases} I_d = I_d^m + I_d^c & (2) \\ I_q = I_q^m + I_q^c = I_{PM1} + I_{PM2} + I_{PM3} + I_{oq} & \end{cases} \end{cases}$$

where  $V_d$ ,  $I_d$ , and  $L_d$  are the *d*-axis voltage, current, and inductance, respectively.  $V_q$ ,  $I_q$ , and  $L_q$  are the *q*-axis voltage, current, and inductance, respectively.  $R_s$  is the resistance of per phase winding,  $\varphi_f$  the PM flux linkage, and  $\omega_e$  the electrical angular frequency in rad/s. Due to the inclusion of the core loss, the number of current branches in the proposed model has increased. Correspondingly, the *d*- and *q*-axes currents are divided into the magnetization current components ( $I_d^m$  and  $I_q^m$ ) and the core loss current components ( $I_d^c$  and  $I_q^c$ ).  $I_{PMI}$ ,  $I_{PM2}$ , and  $I_{PM3}$  are currents that flow through the equivalent resistances due to the PMs magnetic flux, while  $I_{oq}$  is the current that flows through electromotive force  $\omega_e \varphi_f$ . The proposed mathematical model is a generalized model for the PMSM, where the inductances of the dq axes are approximately equal in the surface-mount PMSM.

Furtherly, the current branches can be respectively calculated as  $I_d^c = -\omega_e L_q I_q / R_{lc}$ ,  $I_q^c = \omega_e L_d I_d / R_{lc}$ ,  $I_{PM1} = \omega_e \varphi_f / R_{PM1}$ ,  $I_{PM2} = \omega_e \varphi_f / R_{PM2}$ ,  $I_{PM3} = \omega_e \varphi_f / R_{PM3}$ , where  $R_{lc}$  is the equivalent core loss resistance due to the motor load.  $R_{PM1}$ ,  $R_{PM2}$ , and  $R_{PM3}$  are equivalent resistances due to the PMs magnetic flux, and indicate anomalous loss, eddy current loss, and hysteresis loss, respectively. These four equivalent core loss resistances can depict the core loss of the IPMSM over the entire operating range with highly accurate and detailed information.

To profoundly reveal the mechanism of each core loss component with the IPMSM's electric angular frequency and phase current, the equivalent resistances can be modelled as follows  $R_{PM1} = k_3 \sqrt{\omega_e}$ ,  $R_{PM2} = k_2$ ,  $R_{PM3} = k_1 \omega_e$ ,  $R_{lc} = f(I_p, \omega_e)$ , where  $I_p$  is the phase current, and  $R_{lc}$  is a function of phase current and rotor speed, while  $k_1$ ,  $k_2$ , and  $k_3$ are constant coefficients. These constant coefficients and the function which determines  $R_{lc}$  are obtained by curve-fitting the core loss data of the IPMSM under various motor speeds or excitation frequencies and load currents. Further, they depend

### B. Parameters in the Mathematical Model of the IPMSM

 $k_2=14.74$ , and  $k_3=0.02486$ , respectively.

The structure of the IPMSM for electric vehicle propulsion is illustrated in Fig.1, and the speed range is 0 to 5500 r/min, while the electromagnetic torque range is 0 to 40 kW. The core material of the IPMSM prototype is the non-oriented silicon steel sheet 35WW270, with a thickness of 0.35 mm and a mass density of 7650 kg/m<sup>3</sup>.



Fig. 1. IPMSM structure: (a) stator, and (b) rotor.

The parameters in the mathematical model of the IPMSM are calculated via ANSYS software, and at a specified motor speed, the values of the equivalent core loss resistances are deterministic and presented in Table I.

TABLE I Parameters in the Mathematical Model

| TARAMETERS IN THE MATHEMATICAL MODEL          |                             |
|---|-----------------------------|
| Parameter                                     | Value                       |
| Permanent magnet flux linkage ( $\varphi_f$ ) | 0.04789 Wb                  |
| Stator resistance $(R_s)$                     | 0.0655 Ω                    |
| <i>d</i> -axis inductance $(L_d)$             | 83.955 μH                   |
| q-axis inductance $(L_q)$                     | 328.365 µH                  |
| Equivalent hysteresis loss resistance         | 12.5 Ω                      |
| ( <i>R</i> <sub><i>PM3</i></sub> )@1200 r/min |                             |
| Equivalent eddy current loss                  | $14.74 \Omega$              |
| resistance $(R_{PM2})$                        |                             |
| Equivalent anomalous loss                     | 295 Ω                       |
| resistance (R <sub>PMI</sub> ) @1200 r/min    |                             |
| Equivalent load core loss resistance          | 7.1786e <sup>0.0088Ip</sup> |
| $(R_{lc})$ @1200 r/min                        |                             |

#### III. PROPOSED MTPA CURRENT CONTROL FOR THE IPMSM

The control objective of the MTPA is to identify the minimum stator current for a demanded torque. Therefore, the *d*-axis current  $I_d$  and *q*-axis current  $I_q$  should satisfy the constraint condition of  $I_s = \min \sqrt{I_d^2 + I_q^2}$ , where  $I_s$  is the stator current. When considering the power energy dissipation due to the core loss of the IPMSM, the torque calculation formula is modified as follows:

$$T_{e} = \frac{3}{2} P[\varphi_{f}I_{q} + L_{d}I_{d}I_{q} - L_{q}I_{q}I_{d} - \frac{\omega_{e}(L_{d}^{2}I_{d}^{2} + L_{q}^{2}I_{q}^{2})}{R_{lc}} - (\frac{\omega_{e}\varphi_{f}^{2}}{R_{PM1}} + \frac{\omega_{e}\varphi_{f}^{2}}{R_{PM2}} + \frac{\omega_{e}\varphi_{f}^{2}}{R_{PM3}})]$$
(3)

where  $T_e$  is the electromagnetic torque of the IPMSM, and P is the number of IPMSM pole pairs.

Introduce the Lagrange coefficient  $\lambda$  and establish an auxiliary function as follows:

$$F = I_{d}^{2} + I_{q}^{2} + \lambda \begin{cases} \frac{3}{2} P[\varphi_{f}I_{q} + L_{d}I_{d}I_{q} - L_{q}I_{q}I_{d} - \frac{\omega_{e}(L_{d}^{2}I_{d}^{2} + L_{q}^{2}I_{q}^{2})}{R_{lc}} \\ -(\frac{\omega_{e}\varphi_{f}^{2}}{R_{PM1}} + \frac{\omega_{e}\varphi_{f}^{2}}{R_{PM2}} + \frac{\omega_{e}\varphi_{f}^{2}}{R_{PM3}})] - T_{e} \end{cases}$$

$$(4)$$

After calculating the partial differential equations of (4) to  $I_d$ ,  $I_q$ , as well as  $\lambda$  and then making them equal to zero, the following equations can be derived:

$$\frac{\partial F}{\partial I_d} = 2I_d + \frac{3}{2}\lambda P[L_d I_q - L_q I_q - \frac{2\omega_e L_d^2}{R_{lc}}I_d] = 0$$
(5)

$$\frac{\partial F}{\partial I_q} = 2I_q + \frac{3}{2}\lambda P[\varphi_f + L_d I_d - L_q I_d - \frac{2\omega_e L_q^2}{R_{lc}}I_q] = 0$$
(6)

$$\frac{\partial F}{\partial \lambda} = \frac{3}{2} P[\varphi_f I_q + L_d I_d I_q - L_q I_q I_d - \frac{\omega_e (L_d^2 I_d^2 + L_q^2 I_q^2)}{R_{lc}} - (\frac{\omega_e \varphi_f^2}{R_{PM1}} + \frac{\omega_e \varphi_f^2}{R_{PM2}} + \frac{\omega_e \varphi_f^2}{R_{PM3}})] - T_e = 0$$
(7)

By solving (5) - (7), the MTPA controller output currents should be:

$$I_{d} = \frac{-[\varphi_{f} + \frac{2\omega_{e}}{R_{lc}}(L_{d}^{2} - L_{q}^{2})I_{q}] + \sqrt{[\varphi_{f} + \frac{2\omega_{e}}{R_{lc}}(L_{d}^{2} - L_{q}^{2})I_{q}]^{2} + 4(L_{d} - L_{q})^{2}I_{q}^{2}}{2(L_{d} - L_{q})}$$
(8)

$$I_{q} = \frac{\left[\frac{2\omega_{e}}{R_{lc}}(L_{d}^{2} - L_{q}^{2})I_{d}\right] - \sqrt{\left[\frac{2\omega_{e}}{R_{lc}}(L_{d}^{2} - L_{q}^{2})I_{d}\right]^{2} + 4(L_{d} - L_{q})[\varphi_{f}I_{d} + (L_{d} - L_{q})I_{q}^{2}]}{2(L_{d} - L_{q})}$$
(9)

## IV. COMPARISON OF THE PROPOSED MTPA WITH TRADITIONAL CONTROL METHODS

The schematic diagram of the proposed IPMSM control system with the novel MTPA is presented in Fig. 2.



**Fig. 2.** Schematic overview of the proposed IPMSM control system with the novel MTPA.

Simulations of the suggested novel MTPA, traditional MTPA without core loss, and  $I_d=0$  control are established in MATLAB/Simulink to compare control performances. Fig. 3 indicates the *d*- and *q*-axes current responses when reference torque is 20 Nm and motor speed is 1200 r/min. More specially,

when the IPMSM reaches a steady state, the average values of the *d*- and *q*-axes currents are -23.04 A and 69.13 A in the proposed novel MTPA, and -22.23 A and 70.38 A in the traditional MTPA and -0.043 A and 77.85 A in the  $I_d=0$  control, respectively. The stator currents are calculated as 72.87 A, 73.81 A and 77.85 A in the novel MTPA, traditional MTPA and  $I_d=0$ , respectively. It is seen that for the same torque command of 20 Nm, the stator current in the proposed novel MTPA is the smallest, followed by traditional MTPA, and the maximum current is the  $I_d=0$  control.



**Fig. 3.** *d*- and *q*-axes current responses at the speed of 1200 r/min: (a) novel MTPA; (b) tradition MTPA; and (c)  $I_d=0$ .

The copper loss  $P_{cu}$ , core loss  $P_c$ , and major electromagnetic losses (the summation)  $P_{loss}$  can be calculated as follows:

$$P_{cu} = \frac{3}{2} (R_s I_d^2 + R_s I_q^2)$$
(10)

$$P_{c} = \frac{3}{2} \left[ (\omega_{e} \varphi_{f})^{2} (\frac{1}{R_{h}} + \frac{1}{R_{e}} + \frac{1}{R_{an}}) + \frac{(\omega_{e} L_{q} I_{q})^{2} + (\omega_{e} L_{d} I_{d})^{2}}{R_{i}} \right]$$
(11)  
$$P_{loss} = P_{cu} + P_{c}$$
(12)

Therefore, the major electromagnetic losses of the IPMSM under the operating condition described above in the proposed novel MTPA, traditional MTPA, and  $I_d$ =0 are 666.56 W, 680.6 W and 743.95 W, respectively. It can be seen that the proposed novel MTPA has the optimum performance in reducing the electromagnetic losses.

Fig. 4 presents the torque responses in three control methods, in terms of the proposed novel MTPA, traditional MTPA, and  $I_d$ =0, when the torque changes from 20 Nm to 10 Nm at 3 s while the motor speed maintains at 1200 r/min. The starting torque of the traditional MTPA is the largest, followed by the proposed novel MTPA, and the  $I_d$ =0 control is the lowest. Moreover, all these three control methods can effectively follow the torque mutation command well.



**Fig. 4.** Torque responses of three methods from 20 Nm to 10 Nm at 3 s.

#### V. CONCLUSION

This article proposes a novel mathematical model of the IPMSM which can predetermine the energy consumed in core material under the two-phase rotating coordinate system. This model not only improves the motor characteristic analysis solutions but also broadens the motor control strategy when considering the core loss. In addition, this article develops a novel MTPA which can manage both the core loss and copper loss. Compared with the traditional MTPA and  $I_d=0$  control methods, the proposed novel MTPA can obtain the minimum stator current and electromagnetic losses when the motor torques are equal. Simultaneously, the proposed MTPA has good performance in following sudden torque changes.

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