International Journal of Game Theory (2025) 54:9
https://doi.org/10.1007/s00182-025-00936-4

ORIGINAL PAPER

Communication frictions and equilibrium pragmatics

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Accepted: 16 February 2025

Abstract

This paper introduces a common-interest communication game that generates pragmatics, where meaning emerges from the use of a preexisting language under equilibrium selection driven solely by efficiency. A key feature is that the sender describes the current state to the receiver by combining preexisting statements. This approach allows us to formalize two communication frictions: (i) longer descriptions incur higher costs, and (ii) with some probability, the receiver interprets only the conventional meaning. The absence of one friction leads to some efficient equilibria exhibiting pragmatics that disregards conventional meaning. However, when communication costs are sufficiently small, given the other friction, any efficient equilibrium exhibits natural pragmatics that refines conventional meaning, reflecting the context provided by the probability distribution of states. The resulting equilibrium pragmatics aligns with major linguistic theories, including Grice's cooperative principle (1975) and Sperber and Wilson's relevance theory (1986).

Keywords Equilibrium pragmatics · Communication costs · Naive receiver · Gricean implicature · Relevance theory

JEL Classification 83

1 Introduction

One of the central questions in pragmatics is how individuals use language in context to convey and infer meaning beyond what is explicitly stated. One way to model this within a game-theoretic framework is to consider a common-interest game where messages have preexisting meanings, and equilibrium is selected based on

I thank the two anonymous referees for their helpful comments and Joel Sobel for a stimulating discussion that motivated me to write this paper, as well as for providing thoughtful feedback.

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Published online: 06 March 2025

“linguistic salience”—the linguistically natural use of language serving as the focal point. While this approach offers valuable insights, it treats linguistically natural pragmatics as a given, leaving open the question of what drives such natural use. To fully understand the mechanics behind the natural use of language, we need an approach where equilibrium selection is driven by factors unrelated to pragmatics. This paper introduces a common-interest communication game that generates linguistically natural pragmatics, with equilibrium selection based solely on efficiency. This framework sheds light on the interplay between efficiency and linguistic behavior, advancing our understanding of pragmatics in a novel way.

The key feature of the current paper is a preexisting language that has a structure for composition and conventional meaning. Specifically, there is a set of preexisting statements called “elementary statements,” and the sender describes her private information by combining them, i.e., “description.” This formalization has two benefits. First, it allows us to distinguish the conventional meaning of a description and the meaning determined by its usage, i.e., “semantics and pragmatics.” This distinction is essential for understanding how players use a preexisting language to communicate. The second benefit is that the explicit model of language guides us to set up the cost of communication that reflects the length of a description. Since different types of languages can have different associations between the length of a description and the precision of its semantic meaning, it is important to specify the structure of the preexisting language players use. In addition to the cost of communication, this paper also takes into account another form of communication friction: the receiver’s bounded rationality. Specifically, with some probability, the receiver can be a “naive type” who can only follow the conventional meaning of a description.

This paper shows that if the model lacks one friction, it can make some efficient equilibria exhibit pragmatics that contradicts conventional meaning. However, if the communication cost is sufficiently small given the probability of having a naive receiver, any efficient equilibrium exhibits natural pragmatics that refines conventional meaning, reflecting the context provided by the probability distribution of states. The current paper also offers a common ground for interpreting linguistic theories. Given the interpretation, this paper shows that if the cost of communication is sufficiently small, the cooperative principle of Grice (1975) leads to efficient equilibria in the current model. By contrast, the relevance theory of Sperber and Wilson (1986) is consistent with a wider set of equilibria.

Section 2 formally introduces the model. There is a sender and a receiver. The set of states is finite, and the prior probability of having each state is assumed to be different for simplicity. The sender describes the current state with a preexisting language. Given a description, the receiver chooses an action. Players share a common objective: both get rewarded if and only if the receiver’s action matches the current state. In this paper, players share a preexisting language that has a compositional structure. Specifically, there is a set of preexisting elementary statements. The sender describes the state by combining some elementary statements and negations. The semantic meaning of a description is then defined as the set of states that are consistent with the description. The set of elementary statements is rich enough to fully characterize every state based on semantics, i.e., each state has a “fully explicit description.” Given the model of language, this paper considers two types of associated communication frictions. First, using a longer description is assumed to be more costly for both players. Second, how the receiver interprets the sender's description depends on whether his type is rational or naive. The former can follow equilibrium inference, whereas the latter's inference is based on the semantic meaning. Since we do not wish to rule out the use of fully explicit descriptions directly by the cost, we focus on the setting where the cost of using a fully explicit description is smaller than the benefit of successful communication. This paper then analyzes the perfect Bayesian equilibria of the game.

Section 3 provides the analysis. As in other communication games, the current game has multiple equilibria. However, since this paper considers a common-interest setting, equilibria that both players most prefer, i.e., efficient equilibria, can be considered the most natural equilibria to study. In addition to efficient equilibria, this paper also considers "fully effective equilibria," where the sender can induce the right action at every state regardless of the receiver's type. Fully effective equilibria are relevant to linguistics since an efficient equilibrium can use an expression that misleads a naive receiver to save the expected communication cost, whereas linguistics often analyzes an expression, assuming a speaker intends to convey information successfully without misleading a listener.

Before the main analysis, this paper clarifies the role of each communication friction in the current model. First, Proposition 1 shows that if the communication cost is zero, there is always an efficient and fully effective equilibrium where the sender describes every state precisely based on the conventional meaning. Thus, if the communication cost is zero, pragmatic meaning is not a general property of either efficient or fully effective equilibria in the current framework. Second, Proposition 2 shows that if the probability of having a naive receiver is zero, no description can mislead the receiver as long as the sender uses a unique description for each state. Thus, if there is a fully effective equilibrium that respects semantics, we can always find another fully effective equilibrium where the sender flips the use of two descriptions whose semantic meanings contradict each other. Similarly, if there is an efficient equilibrium that respects semantics, we can always find another efficient equilibrium where the sender flips the use of two equal-length descriptions whose semantic meanings contradict each other. Hence, in the current framework, the probability of having a naive receiver needs to be strictly positive to obtain pragmatics aligned with semantics as a general property of efficient or fully effective equilibria.

Turning to the main analysis, Lemma 1 establishes an important property of fully effective equilibria: in any fully effective equilibrium, the pragmatic meaning of each equilibrium description is determined by the most likely state, given the semantic meaning of the description. In other words, the pragmatic meaning of each description can be derived from its semantic meaning, reflecting the context provided by the probability distribution of states. Moreover, because the state and message spaces are constructed from elementary statements in the current model, the rational receiver's equilibrium inference embodies typical pragmatic inference based on counterfactual reasoning: "If it were also x, it must have been mentioned. Therefore,.." The next result is Proposition 3. Since this paper considers a common-interest communication game with a preexisting language, an off-path belief that respects the semantic meaning can be considered natural. Proposition 3 shows that there exists a fully effective equilibrium under such an off-path belief and that no fully effective equilibrium is more efficient.

An efficient equilibrium can exhibit counterintuitive pragmatics, such as those that disregard semantic meanings as well as those that misalign with the context provided by the probability distribution of states. Proposition 4, the main result of this paper, provides a condition under which all efficient equilibria exhibit natural pragmatics that refines semantic meaning, reflecting the context provided by the probability distribution of states. Since Lemma 1 shows that such equilibrium pragmatics can be obtained in fully effective equilibria, the main result is established by identifying a condition under which any efficient equilibrium is fully effective, i.e., Lemma 3. Specifically, it is shown that if the communication cost is sufficiently small relative to the probability of having the naive type—satisfying a certain inequality condition—any efficient equilibrium is fully effective. The basic intuition behind this result is as follows. The expected communication cost can be reduced if a shorter description is used for a more frequent state, but this may also mislead a naive receiver. Thus, when communication costs are high and minimizing these costs is the primary concern, an efficient equilibrium may disregard the semantics of descriptions. However, if the communication cost is small enough to satisfy the inequality condition, the benefit of reducing communication costs by disregarding semantics never outweighs the cost of misleading a naive receiver. As a result, finding efficient equilibria reduces to finding the most efficient fully effective equilibria, where no equilibrium description misleads a naive receiver. The condition provided in Proposition 4 is not a necessary condition. However, this paper presents an example where violating the condition causes all efficient equilibria not only to fail in refining semantic meaning in context but even to contradict it.

The current framework also provides a common ground for interpreting linguistic theories. In Sect. 4, we interpret Grice's (1975) cooperative principle and Sperber and Wilson's (1986) relevance theory within the context of the current formal framework and investigate the conditions under which these linguistic theories can be compatible with fully effective or efficient equilibria. Based on this interpretation, we show that the cooperative principle leads to the most efficient fully effective equilibria. Therefore, from the main result in Sect. 3, if the communication cost is sufficiently small, the cooperative principle leads to efficient equilibria in the current model. In contrast, relevance theory is consistent with any fully effective equilibrium in the current model, making it compatible with a wider range of equilibria than the cooperative principle.

Section 5 provides discussions. First, we discuss the robustness of the results in a more general common-interest preference. Second, we explain the role of a naive receiver as a communication friction. Finally, it is illustrated how a myopic best reply process that starts with semantic meanings can reach a most efficient fully effective equilibrium.

The paper is concluded in Sect. 6.

Related literature: The current paper can be categorized into economics literature that studies common-interest communication that incorporates "communication frictions" motivated by the use of a language. There are at least four ways to incorporate such communication frictions. First, a language creates communication friction when it is not rich enough to describe the exact state. Cremer et al. (2007) and Jäger et al. (2011) consider such a setting and study the optimal use of coarse messages. Blume and Board (2013) study a situation where a player can have a poor vocabulary depending on her language type and demonstrate that the possibility of having such a poor language type can make the equilibrium meaning imprecise. Communication frictions can also be various "communication costs." When a message has a preexisting meaning, using a message at a state that is inconsistent with the preexisting meaning can be interpreted as lying.[1] Sobel (2023) provides a model with lying cost where the sender's equilibrium message can be referential or conative depending on the payoff structure. Another type of communication cost is associated with complexity. For example, Sobel (2012) endogenizes the use of coarse messages by taking into account the complexity cost of communication. Suzuki (2020) shows that when there is a message cost, the sender's silence can exhibit the defining property of indexicals in efficient communication. Suzuki (2023) demonstrates that if the cost of using a message is not constant across messages, some messages in the efficient equilibrium can exhibit linguistic ambiguity, shifting the meaning across some contexts. Finally, communication frictions can also be created by behavioral players; an "honest sender" sends a message non-strategically, whereas a "naive sender" takes the conventional meaning seriously.[2]

The current paper incorporates two of the above frictions: the cost of communication and the possible presence of a naive receiver. It is shown that a lack of one friction makes some efficient equilibria exhibit pragmatics that contradicts underlying semantics. Moreover, whether all efficient equilibria can produce natural pragmatics depends on whether one friction, the cost of communication, is sufficiently small given the other friction, the possibility of having a naive receiver.

The current paper is also related to the game-theoretical pragmatics literature in linguistics.[3] Before delving into detailed comparisons, it is important to highlight a key difference in approach. This literature typically employs tailored games to examine specific pragmatic phenomena, such as the scalar implicature "some but not all," by incorporating topic-driven restrictions. In contrast, the current paper develops a general framework for analyzing a wide range of pragmatic phenomena, where the state and message spaces are constructed from elementary statements without imposing any exogenous constraints on message use.

Turning to a specific comparison, the current paper analyzes pragmatics in efficient equilibria, as in Parikh (2001). However, the current paper considers a different game. Specifically, since Parikh (2001) restricts the set of available messages to be consistent with the sender's private information, it is technically a verifiable information game.[4] By contrast, in the current paper, the sender can choose any

[1] Lying cost is originally introduced by Kartik (2009) to study "language inflation" in the cheap talk model.

[2] In cheap talk game literature, a multi-audience cheap talk model was introduced by Farrell and Gibbons (1989), and a naive receiver was introduced by Chen (2011).

[3] For comprehensive surveys, see Jäger (2011), Franke (2013), and Benz and Stevens (2018).

[4] This class of games was introduced by Milgrom (1981) and Grossman (1981) to analyze the strategic disclosure of hard evidence. While economists often hesitate to restrict the set of messages based on the sender's type unless some messages are actually unavailable depending on her type, e.g., pieces of hard

message at any state without any restriction. Another difference is the communication friction of a naive receiver: with some probability, the receiver can be a naive type who only processes the semantic meaning, although the probability can be very small. The current paper shows that any efficient equilibrium respects semantics if the communication cost is sufficiently small given the probability of having a naive receiver. However, without the possibility of having a naive receiver, some efficient equilibria can use messages that contradict their semantic meaning.

In addition to efficient equilibria, the current paper also analyzes fully effective equilibria. As discussed in Sect. 5, the most efficient fully effective equilibria can be obtained by a simple iterated best response process. Thus, the current paper is also related to game theoretical pragmatics literature that utilizes iterated best response (IBR), e.g., Jäger (2007), Franke (2009), and Jäger (2011). The current paper suggests that when the cost of communication is sufficiently small given the probability of having a naive receiver, an efficient equilibrium can be obtained by a simple iterative best response, but this is not always the case unless the cost is sufficiently small.

Finally, the current paper considers the possibility of having a naive receiver. This setting is related to the Rational Speech Act (RSA) framework introduced by Frank and Goodman (2012). The basic RSA model involves a "literal receiver" who infers the state based on the literal meaning of a message, a "pragmatic sender" who follows a stochastic messaging rule that assigns a higher probability to messages yielding higher expected payoffs, given the literal receiver's response, and a "pragmatic receiver" who infers the state based on the pragmatic sender's messaging rule.[5] Since RSA is a stochastic choice model with limited rounds of iterative reasoning, it is not an equilibrium theory. Moreover, because its predictions can depend on parameter values, such as the number of iterations and the noise parameter, a formal comparison with the current model is difficult. However, in the basic RSA model, the pragmatic sender can choose messages in line with the most efficient fully effective equilibrium strategies in the current paper.

2 Model

2.1 Basics

There is a sender (she) and a receiver (he). Let Ω be a finite set of states, and let $\pi(\omega)$ be the probability distribution, which is common knowledge between players. For simplicity, assume that there are no states that are equally likely, i.e., $\pi(\omega') \neq \pi(\omega'')$ if $\omega' \neq \omega''$. The sender observes the current state $\omega \in \Omega$ privately and describes ω to the receiver with a preexisting language, which will be explained in the next

Footnote 4 (continued)

evidence, such a restriction is common in linguistics literature. The simplification is useful for obtaining sharp linguistic insight; for example, Pavan (2013) and Rothschild (2013) explain scalar implicature based on iterative admissibility, exploiting the restriction.

[5] See Degen (2023) for a comprehensive survey of recent developments.

subsection. Both players wish the receiver to choose an action that matches the current state. Specifically, let $A = \Omega$ be the set of the receiver's actions. Then, both players get rewarded by $v > 0$ if the receiver chooses $a = \omega$ at ω, whereas there is no reward for both players if the receiver chooses $a \neq \omega$ at ω. That is, Ω consists entirely of payoff-relevant states.

2.2 Language

This paper considers a preexisting language that has a structure for composition and conventional meaning. Specifically, let $S = \{s_1, s_2, .., s_N\}$ be a set of preexisting elementary statements, where each statement s_n is true or false at each ω.[6] Let $x_n(\omega) \in \{0, 1\}$ be a function that assigns 1(0) if statement s_n is true (false) at ω. Suppose that S is rich enough that each ω is fully characterized by a unique binary string $(x_n(\omega))_{n=1}^N$ i.e., the state space is constructed from elementary statements.[7]

Example 1 Consider a situation where the sender wants to describe her friend Mike to the receiver. For simplicity, suppose there are only two available elementary statements: s_1 is "Mike is a game theorist," and s_2 is "Mike is a poet." Then, there are four states as illustrated in Table 1, e.g., ω_2 is the state where Mike is a game theorist but not a poet, i.e., s_1 is true and s_2 is false.

Although $\Omega = \{0, 1\}^N$ in Example 1, it is possible that S contains statements such that if one statement is true, it determines whether some other statements are true or false, i.e., entailment.[8] Thus, $\Omega \subset \{0, 1\}^N$ in general.

The sender describes ω by combining elementary statements and negation. Specifically, a **description** m is a vector $m = (m_1, m_2, .., m_N) \in M = \{0, 1, \emptyset\}^N$, where $m_n = 1$ if m includes statement s_n; $m_n = 0$ if m includes the negation of s_n; $m_n = \emptyset$ if m does not include either s_n or its negation.[9] For instance, in the setting of Example 1, the description $m = (1, 0)$ corresponds to "Mike is a game theorist but not a poet" in English, whereas the description $m = (1, \emptyset)$ corresponds to "Mike is a

[6] This paper uses the term elementary statement in the sense of atomic proposition in propositional logic. For example, "Mike is a game theorist" and "Mike lives in NY" are elementary statements. However, "Mike is a game theorist who lives in NY" is not an elementary statement since, although it is one sentence in English, whether it is true or not depends on whether Mike is a game theorist as well as whether he lives in NY. That is, it is a conjunction statement.

[7] Since Ω consists of payoff-relevant states, this setting implies that S consists only of elementary statements relevant to communication between the sender and the receiver.

[8] For example, if s_1 is "Mike lives in NY," and s_2 is "Mike lives in the US," there is no state characterized by the string (1, 0).

[9] Formally, a description m is a conjunctive formula, $\bigwedge_{n \in \{n' : m_{n'} = 1\}} s_n \wedge \bigwedge_{n \in \{n' : m_{n'} = 0\}} \neg s_n$. We can extend the current setting to accommodate disjunctions as well. However, even if they are available, the sender does not use them in efficient equilibria of the current game. Thus, we focus on conjunctive statements to keep the notation simple.

game theorist."[10] There is no exogenous restriction on the use of m at each ω, i.e., the sender can use any $m \in M$ at every state.

2.3 Communication frictions

This paper incorporates two kinds of communication frictions. One is a communication cost, and another is the receiver's bounded rationality. For the first friction, suppose that both players incur a processing cost when using a description m: speaking, writing, listening, and reading are costly activities. Then, it is assumed that processing a longer description is more costly for both players. Formally, let $k(m)$ be the number of $m_n \neq \varnothing$ in m, i.e., the length of m. For simplicity, assume that the cost of using m for the sender and the receiver are the same. Then, for both players, the cost of using m is $c(m) = \hat{c}(k(m))$, where $\hat{c}(k)$ is strictly increasing in k.[11]

For the receiver's bounded rationality, suppose that how the receiver interprets m depends on his type. Specifically, the receiver can be either the **rational type** $t = \alpha$, who can interpret m according to the equilibrium inference, or the **naive type** $t = \beta$, who can only understand what is explicitly stated in m. Let $\lambda > 0$ be the probability of having the naive type. One possible determinant of the receiver's type is his interpretation cost.[12] If the receiver's cost of interpreting m is negligible, he is the rational type, whereas he is the naive type if the cost is prohibitively high. In this interpretation, λ can depend on various situational factors. For example, if the receiver is busier for other tasks, λ can be higher as multi-tasking increases the cost of interpreting a description. Another factor could be familiarity with the current communication: λ can be close to zero in routine communication where the receiver is already familiar with the interpretations of most descriptions.

To formalize the naive type's inference, let m_n be the n-th element in m. Then, the **semantic meaning** of m is

$$L(m) = \{\omega : \mathrm{x}_n(\omega) = m_n \text{ for all } m_n \neq \varnothing\}.$$

In short, $L(m)$ is the set of every ω that matches the description m. Then, when a naive receiver gets m, he simply infers that the state is in $L(m)$. Specifically, if $L(m) \neq \emptyset$,

[10] In this paper, each description m is a proposition, and it does not reflect how the sender phrases it. For example, there is no distinction between saying, "Mike is a game theorist who lives in NY," and "Mike is a game theorist, and he lives in NY."

[11] If m is the conjunction of s_1 and s_2, i.e., $m = (1, 1)$, the cost of m is $\hat{c}(2)$, although it can be phrased concisely in one English sentence, e.g., "Mike is a game theorist and a poet." This simplification is sensible since the current model intends to capture the fact that the concise expression is still longer than "Mike is a game theorist" or "Mike is a poet."

[12] The interpretation cost is different from the receiver's processing cost $c(m)$: the former is the cost of inferring the equilibrium meaning of m, whereas the latter is the cost of processing what is explicitly mentioned in m.

Table 1 The set of states in Example 1

	$(x_1(\omega), x_2(\omega))$
ω_1	(1, 1)
ω_2	(1, 0)
ω_3	(0, 1)
ω_4	(0, 0)

$$\pi(\omega|L(m)) = \begin{cases} \frac{\pi(\omega)}{\sum_{\omega' \in L(m)} \pi(\omega')} & \text{if}\, \omega \in L(m) \\ 0 & \text{if}\, \omega \notin L(m) \end{cases}.$$

If $L(m) = \emptyset$, m is semantically non-sensical, i.e., m cannot be true at any ω. Then, since the naive type can only learn from the semantic meaning of m, assume $\pi(\omega|L(m)) = \pi(\omega)$.

Consider Example 1, if $m = (1, 0)$ "Mike is a game theorist but not a poet," then $L(m) = \{\omega_2\}$. Thus, a naive receiver infers the state is ω_2. By contrast, if $m = (1, \emptyset)$ "Mike is a game theorist," then $L(m) = \{\omega_1, \omega_2\}$. Hence, the naive receiver infers that Mike is a game theorist but he may or may not be a poet.

It might be worth noting that the semantic accuracy of m, i.e., $|L(m)|$ does not determine $c(m)$. For example, suppose there are only two elementary statements: s_1 is "Mike lives in NY"; s_2 is "Mike lives in the US." Then, there are three possible states: ω_1 at which Mike lives in NY; ω_2 at which Mike lives in the US but not in NY; ω_3 at which Mike does not live in the US. Consider $m' = (1, \emptyset)$ "Mike lives in NY" and $m'' = (\emptyset, 1)$ "Mike lives in the US." Then, $|L(m')| = |\{\omega_1\}| = 1$ whereas $|L(m'')| = |\{\omega_1, \omega_2\}| = 2$, but $c(m') = c(m'')$.

2.4 Strategies and payoff functions

In this paper, we restrict our attention to pure strategies since mixed strategies do not play a substantial role in the current model.

Period 1. The nature selects ω according to $\pi(\omega)$. The sender observes ω and chooses a description $m \in M$. Her communication strategy is $\sigma(\omega)$ where $\sigma : \Omega \to M$.

Period 2. The nature selects the receiver's type t. The receiver then infers ω from m according to his type t. He then chooses an action $a \in A$. His decision-strategy is $f(m, t)$ where $f : M \times T \to A$.

Turning to the payoff functions, both players only care about whether the receiver's action matches the state and the communication cost. Specifically, let

$$u(a, \omega) = \begin{cases} v & \text{if } a = \omega \\ 0 & \text{if}\, a \neq \omega \end{cases}.$$

where $v > 0$. Then, if the sender uses m at ω, and the receiver chooses a, both players get the payoff of $u(a, \omega) - c(m)$. This paper focuses on a situation where the cost of communication is small. Specifically, assume that $v/2 > c(m)$ for all $m \in M$.[13]

2.5 Equilibrium concept

This paper analyzes the *perfect Bayesian equilibria* of the game. Let $M_\sigma = \{m : \exists \omega, \sigma(\omega) = m\}$ be the set of on-path m given σ. A tuple (σ, f, μ) is a perfect Bayesian equilibrium if

(i) the receiver with $t = \alpha$ follows Bayes' rule whenever possible, i.e., for all $m \in M_\sigma$,

$$\mu_\alpha(\omega|m) = \frac{\pi(\omega)}{\sum_{\tilde{\omega} \in \{\omega' : \sigma(\omega') = m\}} \pi(\tilde{\omega})};$$

(ii) the receiver with $t = \beta$ updates his belief based on $L(m)$, i.e., for all m,

$$\mu_\beta(\omega|m) = \pi(\omega|L(m));$$

(iii) given m, the receiver with t chooses a to maximize his expected payoff under μ_t, i.e., for all m and t,

$$f(m, t) \in \arg\max_{a \in A} \sum_\omega [u(a, \omega) - c(m)]\mu_t(\omega|m);$$

(iii) given ω, the sender chooses m to maximize her expected payoff, i.e., for all ω,

$$\sigma(\omega) \in \arg\max_{m \in M} [\lambda u(f(m, \beta), \omega) + (1 - \lambda)u(f(m, \alpha), \omega) - c(m)].$$

3 Equilibrium analysis

3.1 Efficient and fully effective equilibria

The current game has many equilibria as in other communication games. Since this is a common-interest game, an equilibrium is considered more efficient if a player gets a higher expected payoff. We call an equilibrium **efficient** if there is no equilibrium that is more efficient. In common-interest communication games, the sender can credibly commit to an efficient equilibrium strategy. Thus, it is common to analyze efficient equilibria to obtain the main results in the literature.[14]

[13] Since this paper obtains the main results under small communication costs, we can keep the same result even if we allow higher communication costs. However, focusing on this set of costs simplifies the exposition of the paper.

[14] For example, Blume and Board (2013) and Suzuki (2023) focus on efficient equilibria to obtain the main results. Blume et al. (1993) show that when there are small message costs, an equilibrium that the sender prefers is evolutionary stable, which corresponds to an efficient equilibrium in the current model.

Another class of equilibria we study is one in which communication is always successful at every state. We call an equilibrium **fully effective** if the receiver chooses $a = \omega$ at every ω in the equilibrium. Note that an efficient equilibrium is not necessarily fully effective since there can be a situation where an efficient equilibrium uses a description that misleads the receiver at some state in order to save the expected communication cost. Fully effective equilibria can be considered more relevant to linguistics; linguists usually analyze the pragmatics of an expression, presuming that the utterer intends to be understood by the listener successfully. This paper analyzes each class of equilibria, including their relationship.

3.2 Preliminary observations

In this subsection, we clarify the role of each communication friction. In this paper, we define the pragmatic meaning of m as the equilibrium meaning of m, i.e., "meaning by use." That is, the pragmatic meaning of m is determined by the set of states at which m is used in a communication strategy σ. Formally, the **pragmatic meaning** of m in σ is

$$P_\sigma(m) = \{\omega : \sigma(\omega) = m\}.$$

For instance, in Example 1, the semantic meaning of "Mike is a game theorist" is $\{\omega_1, \omega_2\}$. If σ uses the description only at ω_2, the pragmatic meaning of "Mike is a game theorist" is $\{\omega_2\}$, i.e., "Mike is a game theorist but not a poet." We can consider that an equilibrium exhibits pragmatics if the semantic meaning of a description is not exactly the same as the equilibrium meaning of the description, i.e., $L(\sigma(\omega)) \neq P_\sigma(\sigma(\omega))$ for some ω.

Our first result suggests that if the cost of using any description is zero, we cannot obtain pragmatic meaning as a general property of efficient or fully effective equilibria in the current model.

Proposition 1 *If the cost of using m is zero for all m, then there exists an equilibrium that is both efficient and fully effective, and $L(\sigma(\omega)) = P_\sigma(\sigma(\omega))$ for all ω.*

Proof Suppose the sender uses σ where every ω is fully described. Then, both the pragmatic meaning and the semantic meaning of $\sigma(\omega)$ are $\{\omega\}$ at every ω. Since the receiver with any type infers the state is ω when he receives $\sigma(\omega)$, he chooses $a = \omega$ at every ω regardless of his type. Moreover, since there is no communication cost, σ is an equilibrium strategy under any off-path belief. Clearly, this equilibrium is both fully effective and efficient. □

Turning to the role of a naive receiver, note that players only use conjunctive statements in the current model, i.e., listing whether the current state does or does not satisfy some properties. Thus, it would be unnatural if the pragmatic meaning of a description contradicts what it explicitly states. For example, it would be strange if "Mike is a poet" could mean Mike is not a poet but a game theorist,[15]

The next result suggests that if the receiver is always rational, both efficient and fully effective equilibria can exhibit pragmatics that contradicts underlying semantics.

Proposition 2 *Suppose that the probability of having a naive receiver is zero. There exists a fully effective equilibrium where $P_\sigma(\sigma(\omega)) \cap L(\sigma(\omega)) = \emptyset$ for some ω. Moreover, there also exists an efficient equilibrium where $P_\sigma(\sigma(\omega)) \cap L(\sigma(\omega)) = \emptyset$ for some ω.*

Proof Note that since $v > c(m)$ for all m, if an equilibrium is efficient or fully effective under $\lambda = 0$, the equilibrium strategy must be separating, i.e., $P_\sigma(\sigma(\omega)) = \{\omega\}$. Thus, if $P_\sigma(\sigma(\omega)) \cap L(\sigma(\omega)) = \emptyset$, then $\omega \notin L(\sigma(\omega))$, i.e., the semantic meaning of $\sigma(\omega)$ contradicts ω.

For the first part, if there is no naive type, any separating equilibrium is fully effective. Thus, we can always construct a fully effective equilibrium with a separating strategy where the semantic meaning of a description contradicts the pragmatic meaning.

For the second part, suppose σ is an efficient equilibrium where the pragmatic meaning of $\sigma(\omega)$ never contradicts the semantics at any ω, i.e., $\omega \in L(\sigma(\omega))$ for all ω. Consider m' that only states s_1 is true, and m'' that only states s_1 is false. If m' and m'' are used in the efficient equilibrium, construct σ' from σ by flipping the use of m' and m'', disregarding semantically contradictory uses. Then, σ' is still separating and has the same expected message cost as σ. But then, σ' is also an efficient equilibrium strategy, a contradiction. If m' is not used in σ, choose any ω' at which s_1 is false and $\sigma(\omega') \neq (\emptyset, \emptyset, .., \emptyset)$, and construct σ' by simply replacing $\sigma(\omega')$ with m'. Then, σ' uses m' at the state where $s_1$is false. Since $c(m') \leq c(\sigma(\omega'))$ and σ' is separating, σ' is also an efficient equilibrium strategy, a contradiction. □

3.3 Main analysis

We start with the analysis of fully effective equilibria. We say the state ω is the **most likely state (MLS)** given $L(m)$ if

$$\{\omega\} = \arg\max_{\omega'} \pi(\omega' | L(m)).$$

[15] The most relevant pragmatics in the current setting is *conversational implicature* introduced by Grice (1975). One of the essential properties of conversational implicature is calculability; a receiver can "calculate" what a speaker implicates based on the conventional meaning of what is stated with the help of a tacit communication rule called "the cooperative principle" and the context.

Since there are no equally likely states in the current setting, the MLS given $L(m)$ is a single state. Note that any ω is the MLS given the fully explicit description of ω. Thus, any ω has at least one description m such that ω is the MLS given $L(m)$. Let

$$\hat{M}(\omega) = \{m : \{\omega\} = \arg\max_{\omega'} \pi(\omega' | L(m))\}.$$

That is, this is the set of m such that ω is the MLS given $L(m)$.

Example 2 Consider the setting of Example 1. Since game theorists and poets are rather uncommon, it is reasonable to assume that (i) regardless of whether Mike is a game theorist or not, the probability that Mike is a poet is lower than the probability that he is not, i.e., $\pi(\omega_1 | L((1, \emptyset))) < \pi(\omega_2 | L((1, \emptyset)))$, and $\pi(\omega_3 | L((0, \emptyset))) < \pi(\omega_4 | L((0, \emptyset)))$; (ii) regardless of whether Mike is a poet or not, the probability that Mike is a game theorist is lower than the probability that he is not, i.e., $\pi(\omega_1 | L((\emptyset, 1))) < \pi(\omega_3 | L((\emptyset, 1)))$ and $\pi(\omega_2 | L((\emptyset, 0))) < \pi(\omega_4 | L((\emptyset, 0)))$. If $m = (1, \emptyset)$, i.e., "Mike is a game theorist," the MLS given $L(m)$ is ω_2 where Mike is a game theorist but not a poet; if $m = (0, \emptyset)$, i.e., "Mike is not a game theorist," the MLS given $L(m)$ is ω_4; if $m = (\emptyset, 1)$, i.e., "Mike is a poet," the MLS given $L(m)$ is ω_3; if $m = (\emptyset, 0)$, i.e., "Mike is not a poet," the MLS given $L(m)$ is ω_4. (Table 2).

Lemma 1 *In any fully effective equilibrium,* $\sigma(\omega) \in \hat{M}(\omega)$ *for all* ω.

Proof Note that if the receiver is naive and gets m, he chooses the action that matches the MLS given $L(m)$. Thus, if σ is a fully effective strategy, then the MLS given $L(\sigma(\omega))$ must be ω for all ω. That is, $\sigma(\omega) \in \hat{M}(\omega)$ for all ω. □

Lemma 1 states that each equilibrium m refers to the MLS given $L(m)$ in any fully effective equilibrium. Thus, the pragmatic meaning of m can be calculated from its semantic meaning and the probability distribution of ω. This suggests that in any fully effective equilibrium, the pragmatic meaning of m reflects the context provided by the probability distribution of states.

It is important to note that although the use of m is computed from its semantic meaning and the probability distribution of ω, the pragmatic inference is captured by the rational receiver's equilibrium inference. Specifically, since the message and state spaces are constructed from elementary statements, the profile of m in the strategy ensures coherent counterfactual reasoning, which is commonly found in pragmatic inference. For example, in the setting of Example 2, consider any fully effective equilibrium where "Mike is a poet" refers to the state where he is a poet, not a game theorist. In linguistics, the pragmatic reasoning behind this inference would be that if he were also a game theorist, the sender would have mentioned it; therefore, he must be just a poet. In the current paper, this is captured by the rational receiver's equilibrium inference, since, according to Lemma 1, "Mike is a poet and a game theorist" is used at a different state in any fully effective equilibrium.

Proposition 3 *There exists a fully effective equilibrium under the off-path belief system that respects the semantic meaning in a fully effective equilibrium i.e.,* $\mu(\omega|m) = \pi(\omega|L(m))$. *Moreover, there is no fully effective equilibrium that is more efficient.*

Proof Let $\sigma(\omega)$ be a strategy such that $\sigma(\omega) \in \arg\min_{m \in \hat{M}(\omega)} c(m)$ for all ω. If $m \in \hat{M}(\omega')$, then $m \notin \hat{M}(\omega)$ for all $\omega \neq \omega$'. Thus, σ is a separating strategy. Then, since $\sigma(\omega) \in \hat{M}(\omega)$, the strategy induces $a = \omega$ at every ω regardless of the receiver's type. By the construction of σ, the sender has no incentive to deviate if the off-path belief is $\mu(\omega|m) = \pi(\omega|L(m))$. For the second part, suppose σ' is a fully effective equilibrium strategy. From Lemma 1, $\sigma'(\omega) \in \hat{M}(\omega)$ for all ω. By construction, $c(\sigma(\omega)) \leq c(\sigma'(\omega))$ for all ω. Thus, σ' cannot be more efficient than σ. □

Example 3 The strategy in Table 3 illustrates the most efficient fully effective equilibrium strategy in the setting of Example 2. In this strategy, the sender makes a statement explicit only if it matches the state and it is less common than its negation; for example, if Mike is a game theorist (or poet), it is mentioned, but if he is not, it is left unmentioned.

Turning to the analysis of efficient equilibria, note that if there is no naive type, i.e., $\lambda = 0$, it is obvious that any efficient equilibrium needs to be separating. However, since the rational and the naive types can choose a different action given m, there are non-separating strategies with which the receiver can choose $a = \omega$ with positive probability at every ω. Since such a strategy can save communication costs, it is not obvious whether every efficient equilibrium uses a separating strategy if $\lambda > 0$.

Lemma 2 *Any efficient equilibrium strategy is separating.*

Proof See appendix. □

The basic idea of Lemma 2 is that if there is an efficient equilibrium where the sender's strategy σ is not separating, the sender can improve the expected payoff by using a fully explicit description. More specifically, suppose there are states ω' and ω'' where the sender uses the same description m', and the rational type chooses the right action only at ω', whereas the naive type chooses the right action only at ω''. First, the fully explicit descriptions of ω' and ω'' should be off-path descriptions in σ: since they cannot save communication costs, if they are used in an efficient equilibrium, they should be used only at the right state. If σ is an efficient equilibrium strategy, the benefit of using the fully explicit description of ω'' at ω'' should be smaller than the increased communication cost. Then, since the use of the fully explicit description at ω'' makes the rational type choose the right action at ω'', the probability of having the rational type needs to be sufficiently low. Now, suppose the sender uses the fully explicit description of ω' at ω' so that the naive type can choose the right action at ω'. We can show that the gain from the use of the fully explicit description at ω' exceeds the increased communication cost whenever the probability of having the naive type is high enough to make the use of the fully explicit description at ω'' not profitable.

Although it does not play an important role in the current paper, there is another property of efficient equilibria. In the current setting, any efficient equilibrium uses "silence" $(\emptyset, \emptyset, .., \emptyset)$ at the ex-ante most likely state. This captures the empirical regularity that if nothing is mentioned, people can interpret it as an indication that the current situation is "normal" or "the same as usual."[16] However, in a more general setting, the use of silence in efficient equilibria can be more subtle. For instance, if the receiver has the option to choose " inaction," which guarantees a constant payoff across states, efficient equilibria do not use silence unless the ex-ante most likely state has a sufficiently high probability.

As mentioned earlier, an efficient equilibrium may or may not be fully effective in general. The next lemma provides a sufficient condition for any efficient equilibrium to be fully effective.

Lemma 3 *If* $\hat{c}(N) - \hat{c}(1) < \lambda v$, *any efficient equilibrium is fully effective.*

Proof Suppose there exists an efficient equilibrium that is not fully effective when $\hat{c}(N) - \hat{c}(1) < \lambda v$. Then, Lemma 1 implies that there must exist ω' at which $\sigma(\omega') \notin \hat{M}(\omega')$. Note that the fully explicit description of ω', denoted by $m^{\omega'}$, cannot reduce communication costs. Thus, if $m^{\omega'}$ is not used at ω' in an efficient equilibrium strategy, it must be an off-path description in σ.

Now, consider an alternative strategy that replaces $\sigma(\omega')$ with $m^{\omega'}$. From Lemma 2, σ is a separating strategy. Thus, $\sigma(\omega')$ ensures that the rational receiver chooses the correct action at ω', whereas the naive receiver selects an incorrect action at ω'. Since the alternative strategy makes both types choose the correct action at ω', the expected gain from this strategy is λv. Since $\sigma(\omega') \notin \hat{M}(\omega')$ and σ is an efficient equilibrium strategy, we have $\sigma(\omega') \neq (\emptyset, \emptyset, .., \emptyset)$. Hence, $c(\sigma(\omega')) \geq \hat{c}(1)$. The increased communication cost at ω' is at most $\hat{c}(N) - \hat{c}(1)$. Therefore, if $\hat{c}(N) - \hat{c}(1) < \lambda v$, the alternative strategy is an equilibrium strategy that strictly increases the expected payoff, a contradiction. $\square$

To see the basic idea behind Lemma 3, note that the sender uses a description that misleads a naive receiver in an efficient equilibrium only if it saves the communication cost at the state. Thus, if the communication cost $\hat{c}(N)$ is sufficiently small given the probability of having the naive type, the benefit of saving communication costs cannot exceed the cost of misleading a naive receiver. As a result, the sender never uses a description that misleads a naive receiver in efficient equilibria if the communication cost is small enough to satisfy the condition in Lemma 3.

[16] Suzuki (2020) shows that silence can convey meaning in a manner analogous to indexicals, such as "it" and "now."

Table 2 $\hat{M}(\omega)$ in Example 2

	$(x_1(\omega), x_2(\omega))$	$\hat{M}(\omega)$
ω_1	(1, 1)	$\{(1,1)\}$
ω_2	(1, 0)	$\{(1,\varnothing),(1,0)\}$
ω_3	(0, 1)	$\{(\varnothing,1),(0,1)\}$
ω_4	(0, 0)	$\{(\varnothing,\varnothing),(0,\varnothing),(\varnothing,0),(0,0)\}$

The main result of this paper is as follows.

Proposition 4 *If $\hat{c}(N) - \hat{c}(1) < \lambda v$, the pragmatic meaning of $\sigma(\omega)$ in any efficient equilibrium is the MLS given $L(\sigma(\omega))$.*

Proof The result follows from Lemma 1 and Lemma 3. □

This result suggests that if the communication cost is sufficiently small, the equilibrium pragmatics refines semantic meaning, reflecting the context provided by the probability distribution of states.

The condition in Proposition 4 is not a necessary condition. However, if $N = 2$ and $\Omega = \{0, 1\}^2$, the condition is necessary and sufficient. The next example illustrates that when the communication cost is not small enough to satisfy the condition, all efficient equilibria not only fail to provide pragmatics that refines semantic meaning in context but may even contradict it.

Example 4 Consider the setting in Example 2, where $N = 2$ and $\Omega = \{0, 1\}^2$. Let σ' be the strategy that is constructed from the strategy in Table 3 by replacing (1, 1) with $(0, \varnothing)$ at ω_1. Clearly, σ' disregards the semantic meaning of $(0, \varnothing)$ at ω_1. However, if $\hat{c}(N) - \hat{c}(1) \geq \lambda v$, σ' is an efficient equilibrium strategy. Moreover, there is another efficient equilibrium, which uses $(\varnothing, 0)$ at ω_1, replacing $(0, \varnothing)$ in σ'. Clearly, this also disregards the semantic meaning of $(\varnothing, 0)$ at ω_1. By contrast, if $\hat{c}(N) - \hat{c}(1) < \lambda v$, the strategy in Table 3 is the unique efficient equilibrium strategy (Table 4).

If $\hat{c}(N) - \hat{c}(1) \geq \lambda v$, the cost of communication can be large enough to ignore the semantics of m at some states. Specifically, suppose ω' is the most likely state given $L(m)$ and $L(m')$. If efficient equilibria use m at ω' as m is cheaper than m',

Table 3 The strategy in Example 3

	$(x_1(\omega), x_2(\omega))$	σ^*
ω_1	(1, 1)	(1, 1)
ω_2	(1, 0)	$(1,\varnothing)$
ω_3	(0, 1)	$(\varnothing,1)$
ω_4	(0, 0)	$(\varnothing,\varnothing)$

but m' is also fairly cheap, m' can be used in a state that is inconsistent with its semantic meaning to reduce communication costs.

4 Relationship with linguistic theories

The explicit model of a preexisting language in this paper allows us to interpret linguistic concepts in terms of mathematical elements within the framework.[17] With this interpretation, we can analyze the conditions under which efficient or fully effective equilibria can be compatible with specific linguistic theories.

4.1 Gricean implicature

Grice (1975) argues that a speaker can convey her intended meaning to a listener beyond what is explicitly stated since they share a tacit rule of communication known as the "cooperative principle." The listener can then infer the speaker's intended meaning, assuming the speaker adheres to this principle. The cooperative principle consists of four maxims: Quality, Quantity, Relation, and Manner.

The quality maxim demands the utterer "try to make your contribution one that is true." In the current model, we can consider that using a description m at ω satisfies this maxim if m is true at ω, i.e., $m \in L(\sigma(\omega))$. In any fully effective equilibrium, the quality maxim is always satisfied at every ω. By contrast, the maxim can be violated in efficient equilibria at some ω. However, from Lemma 3, the maxim is satisfied at every ω in any efficient equilibrium if the cost of communication is sufficiently small.

The quantity maxim relates to "the quantity of information to be provided." The maxim consists of two parts: (i) "Make your contribution as informative as is required" and (ii) "Do not make your contribution more informative than is required." Since the sender's purpose of communication is to make the receiver choose $a = \omega$ at ω in the current setting, we can consider a description m "as informative as is required" in the current framework if it can make the receiver always choose $a = \omega$. Thus, in this paper, if $m \in \hat{M}(\omega)$, we can consider (i) to be satisfied. Note that since m is a conjunction, if a description m is not the shortest in $\hat{M}(\omega)$, it provides more information about ω than the shortest description in $\hat{M}(\omega)$. Thus, if m does not make "the contribution more informative than is required," satisfying (ii), it must be the shortest one in $\hat{M}(\omega)$, i.e., $\sigma(\omega) \in \arg\min_{m \in \hat{M}(\omega)} c(m)$, in the current model.[18] The strategy that satisfies (i) and (ii) at every ω is a most efficient fully effective equilibrium strategy. It follows that we can consider that the quantity maxim is satisfied at every ω in any most efficient fully effective equilibrium. Then,

[17] Needless to say, the current model, which focuses on simple referential communication, is not well-suited for defining linguistic concepts in greater generality. This section aims to interpret linguistic concepts within the current context, though the approach could also be extended to more general settings.

[18] Being the shortest itself is not essential to satisfy the quantity maxim in general, but it is an implication of the maxim in the current model, where m is a conjunction.

Table 4 The strategy σ' in Example 4

	$(x_1(\omega), x_2(\omega))$	σ'
ω_1	(1, 1)	$(0, \emptyset)$
ω_2	(1, 0)	$(1, \emptyset)$
ω_3	(0, 1)	$(\emptyset, 1)$
ω_4	(0, 0)	$(\emptyset, \emptyset)$

from Lemma 3, if the communication cost is sufficiently small, the quantity maxim is satisfied at every ω in any efficient effective equilibrium.

Although the derivation of the most efficient fully effective equilibrium strategies is based on the naive receiver's responses, the rational receiver's equilibrium inference captures standard pragmatic reasoning based on the quantity maxim. To illustrate this, consider the setting in Example 3. The standard pragmatic inference from "Mike is a poet" under the quantity maxim is that if he were also a game theorist, the sender, who aims to be "as informative as required," would have mentioned it. This inference is captured by the rational receiver's equilibrium inference, as the description "Mike is a game theorist and a poet" is indeed used in a different state under the strategy. Thus, while a strategy satisfying the quantity maxim can be derived using the naive receiver, pragmatic reasoning is ultimately captured by the rational receiver's equilibrium inference. To understand the connection, note that while message choice at each state is determined by the naive receiver's response, pragmatic reasoning is shaped by the profile of those choices across states. The message and state spaces, constructed from elementary statements, ensure coherent counterfactual reasoning, which plays a key role in pragmatic inference for implicatures.

The relation maxim demands that an utterance "be relevant." As mentioned in Sect. 2, the state space in this paper is payoff-relevant and constructed from elementary statements. Thus, every $m \in M$, consisting of relevant elementary statements, is relevant to communication. Hence, under this conservative interpretation of the relation maxim, any equilibrium can be considered to satisfy the maxim in this framework.

In the pragmatics literature, implicatures based on the relation maxim often take into account the degree of relevance—some expressions are more relevant to a given reference than others, and satisfying the relation maxim entails being the most relevant. In the current framework, the degree of relevance of m to ω can be defined by the probability of ω given the semantic meaning of m, i.e., $\pi(\omega|L(m))$. Under this more refined interpretation, the maxim is satisfied only by the most efficient fully effective equilibria, where every equilibrium description refers to the MLS given its semantic meaning. Then, from Lemma 3, if the communication cost is sufficiently small, only efficient equilibria satisfy the relation maxim.

It might be useful to provide a linguistic example to illustrate how a most efficient fully effective equilibrium captures the implicature based on the relation maxim. Suppose we wish to derive the implicature of "I'm busy," uttered in response to another person's suggestion to go out. The standard pragmatic reasoning is that if the sender adheres to the relation maxim, his answer—whether he can go—should be the one most relevant to "I'm busy"; that is, he cannot go. To see how the current model derives this implicature, consider the set of relevant elementary statements: "I'm busy" and "I can go," which generate four possible states. If $\pi(\omega)$ reflects the empirical regularity that a person is more likely to go out when he is not busy than when he is busy, the MLS given the semantic meaning of "I'm busy" corresponds to the state where he is busy and cannot go out.[19]

Turning to the manner maxim, Grice (1975) describes it as "relating not to what is said but, rather, to HOW what is said is to be said." Specifically, it demands an utterance "be perspicuous." He then provides four sub-maxims: "avoid obscurity," "avoid ambiguity," "be brief," and "be orderly." For example, "It is not the case that Mike is not a poet" violates the maxim since one could make it brief: "Mike is a poet." In the current model, the set of available descriptions is restricted to the set of simple conjunctive statements, excluding unnecessary logical operations, such as double negation. Thus, every description in M has a unique semantic meaning in the current setting. Then, since there is no room for choosing "how what is said is to be said," we can consider that the sender's description always satisfies the manner maxim.

According to the above analysis, the equilibrium that satisfies the cooperative principle in the current framework aligns with the most efficient fully effective equilibria. Lemma 3 then implies that the cooperative principle aligns with any efficient equilibrium if the cost of communication is sufficiently small, given the probability of having a naive receiver.

If the cost of communication is not sufficiently small, efficient equilibria can fail to produce pragmatics consistent with the cooperative principle, as it can violate both the quality and quantity maxims, as illustrated in Example 4. Finally, fully effective equilibria, excluding the most efficient ones, violate the quantity maxim by using a description that is more informative than necessary to induce the correct action.

4.2 Relevance theory

Relevance theory, introduced by Sperber and Wilson (1986), provides an alternative formulation of Gricean implicature, replacing the four maxims with the following two principles. The first one is the cognitive principle: human cognition is geared to the maximization of relevance. The second one is the communicative principle: utterances create expectations of optimal relevance. They argue that pragmatic inference can be understood with those two principles without the four maxims in Grice (1975).

According to Sperber and Wilson (1986), relevance is determined by two factors: (i) cognitive effect, i.e., the outcome of communication, and (ii) processing effort, i.e., the cognitive effort required to interpret an expression. In the current setting, (ii)

[19] Although all of the most efficient fully effective equilibria exhibit some form of implicature based on the relation maxim, "I'm busy" can be used in some of them. However, if we introduce a small cost for using negation, it can be used in all of the most efficient fully effective equilibria.

is irrelevant because there is no cost of pragmatic inference for the rational receiver, just as there is no cost of semantic inference for the naive receiver. Since the outcome of communication that both players want at ω is $a = \omega$, the relevance of m is maximized at ω if m always induces $a = \omega$ at ω. Consequently, the communicative principle demands that the receiver expect the sender to use m that always induces $a = \omega$ at ω. In the current model, any fully effective equilibrium is consistent with such communication. Moreover, from Lemma 3, if the communication cost is sufficiently small, such communication is also consistent with any efficient equilibrium.

A common criticism of relevance theory is that the notion of "the maximization of relevance" is ambiguous. The current framework offers an objective definition of "maximum relevance" in referential communication. As discussed earlier, the use of m that maximizes relevance is consistent with fully effective equilibria. From Lemma 1, any equilibrium m is MLS given $L(m)$ in any fully effective equilibrium. Thus, if we consider $\pi(\omega|L(m))$ measures the relevance of ω to m, the interpretation of m based on the maximization of relevance is equivalent to the maximization of $\pi(\omega|L(m))$ with respect to ω. That is, the maximization of relevance in the current setting can be defined as the maximization of "statistical relevance."[20]

In contrast to the cooperative principle, which aligns with the most efficient fully effective equilibria, the pragmatics of relevance theory is consistent with any fully effective equilibrium. This indeterminacy arises because maximizing relevance does not favor particular descriptions when they produce the same effect and there is no cost to interpreting them.[21]

5 Discussions

5.1 More general common-interest preference

The current paper considers a simple common-interest setting in which players are rewarded only if the receiver's action correctly matches the state. This setting is appropriate for the current paper because it aims to derive pragmatics that is comparable with pragmatics literature; in linguistics, it is common to study the meaning of an expression without detailed background information, such as payoff functions, assuming that the speaker simply intends to be understood correctly. However, it is worth clarifying how the results of this paper could be affected if a more general common-interest preference is considered in the current setting.

First, in the current model, if the communication cost is sufficiently small, any efficient equilibrium is fully effective. When the communication cost is sufficiently small, ensuring that both types of receivers choose the state-optimal action becomes

[20] This definition is specific to the current setting, where there is no cost to interpreting m. If such a cost exists, the definition must be modified to account for it.

[21] If we consider $c(m)$ as part of the cost of interpreting m, maximizing relevance favors the most efficient fully effective equilibria. However, it would be unnatural to qualify a shorter, less precise description as "more relevant" than a longer, more precise one.

the most critical factor for an equilibrium to be efficient. Thus, as long as players share the same preference order over A given each state in the current setting, the result remains unchanged.

Second, in the current model, any description in a fully effective equilibrium refers to the most likely state given its semantic meaning. This result holds as long as the payoff function does not include a "safer action"—an action that is optimal in a particular state but also performs fairly well across some suboptimal states, whereas the performance of other actions is more sensitive to the state.

When the payoff function includes such a "safer action," a description in a fully effective equilibrium does not necessarily refer to the most likely state given its literal meaning. The fact that pragmatics in efficient equilibria can depend on the payoff function does not diminish the value of the current model. On the contrary, it highlights a key feature of pragmatic inference: the meaning of an expression evolves as additional contextual information about the communication is introduced.

5.2 The role of the naive receiver

This paper considers the possibility of having a naive receiver as one of the two communication frictions. If one just aims to obtain pragmatics that respects semantic meaning in efficient equilibria, there are other approaches that do not involve a naive receiver. For example, suppose there is a psychological cost associated with sending a semantically inconsistent message. Then, if such a cost is high enough, all efficient equilibria exhibit pragmatics aligned with semantic meaning. However, since such an approach only regulates semantic consistency, some efficient equilibria may still exhibit unnatural pragmatics; for instance, in the environment of Example 2, the implicature of "Mike is a poet" could be that he is not only a poet but also a game theorist. Thus, in general, such an approach requires equilibrium selection augmented with linguistic salience.

We can consider that the main role of the naive receiver is to regulate the use of language to ensure not only consistency with semantic meaning but also responsiveness to the context created by the probability distribution of states. Needless to say, this paper does not claim that the current setting is the only way to obtain linguistically natural pragmatics in all efficient equilibria. Any friction that regulates the use of language sufficiently well should yield linguistically natural pragmatics in any efficient equilibrium, although a viable alternative is not readily apparent.

5.3 Bounded rationality

Since it is difficult for ordinary people to perform complex computations during conversations, sensible pragmatics must be easily derivable from semantic meaning and basic contextual information. In this paper, the pragmatics of fully effective equilibria has this property: any m refers to the MLS given its semantic meaning. If the communication cost is sufficiently small, any efficient equilibrium is fully effective. Thus, in such cases, the pragmatics of efficient equilibria is also easily derivable.

If the communication cost is not sufficiently small, the pragmatics of efficient equilibria is not easily derived from semantics in general, as the pragmatics of some descriptions may ignore their semantic meanings. A learning process could converge to the pragmatics of such an efficient equilibrium, starting from the use of descriptions based on semantics. However, since such a process relies on "stochastic mutation," it may take an impractically long time to reach an efficient equilibrium when the state space is large. This contrasts with the case of the most efficient fully effective equilibria, which can be easily induced by the following simple iterative best-response process.

Step 1: Both types of the receiver choose the initial response function $b_1^R(m, t)$, where

$$b_1^R(m, t) \in \arg\max_{a \in A} E[u(a, \omega)|L(m)]$$

for both t. That is, b_1^R maximizes the receiver's payoff under the inference of ω based on the semantic meaning of m.

Step n (even): Given $b_{n-1}^R(m, t)$, the sender chooses $b_n^S(\omega)$ so that, for any ω,

$$b_n^S(\omega) \in \arg\max_{m \in M}[\lambda u(b_{n-1}^R(m, \beta), \omega) + (1 - \lambda)u(b_{n-1}^R(m, \alpha), \omega) - c(m)].$$

Step $n > 1$ (odd): Given $b_{n-1}^S(\omega)$, the rational receiver chooses $b_n^R(m, \alpha)$ so that, for any $m \in b_{n-1}^S(\Omega)$,

$$b_n^R(m, \alpha) \in \arg\max_{a \in A} E[u(a, \omega)|m = b_{n-1}^S(\omega)]$$

and $b_n^R(m, \alpha) = b_{n-2}^R(m, \alpha)$ for any $m \notin b_{n-1}^S(\Omega)$. The naive type receiver keeps using $b_1^R(m, \beta)$ for all n. Moreover, assume that $b_{n+1}^S(\omega) = b_n^S(\omega)$ if there is no m that yields a strictly higher expected payoff than $b_n^S(\omega)$.[22]

Note that since $b_2^S(\omega) \in \arg\min_{m \in \hat{M}(\omega)} c(m)$, it follows that $b_2^S(\omega) = b_n^S(\omega)$ for all $n > 2$. Thus, the adjustment process leads to a most efficient fully effective equilibrium.

6 Concluding remarks

This paper introduces a common-interest communication game that can produce linguistically natural pragmatics under equilibrium selection based solely on efficiency. The key feature of the model is a preexisting language with a structure for composition and conventional meaning, which allows us to formalize two communication frictions: communication costs and the possibility of having a naive receiver. It is shown that if the communication cost is sufficiently small, given the probability of having a naive receiver, any efficient equilibrium exhibits natural pragmatics that

[22] This assumption simply prevents the sender from switching between indifferent messages, which could create a non-convergent cycle.

refines conventional meaning, reflecting the context provided by the probability distribution of states. This paper also shows that the pragmatics in any efficient equilibrium aligns with major linguistic theories, such as Grice (1975) and Sperber and Wilson (1986). Since the result holds under any small probability of having a naive receiver, as long as the communication cost is sufficiently small, one way to interpret this result is that communication frictions can serve as perturbations that make efficient equilibria exhibit linguistically natural pragmatics.

Since this paper focuses on explaining the basic idea, we did not explore how well the current framework accounts for various pragmatic phenomena. First, the current framework can derive various implicatures that obey Gricean maxims, such as the scalar implicature "some, not all." By contrast, it requires a more substantial extension to derive implicatures for expressions that flout Gricean maxims in efficient equilibria. This is because, in the current model, it would never be efficient to use an indirect expression that is longer than necessary when a shorter, direct expression is available. One way to extend the model to account for such implicatures is by incorporating sociolinguistic factors, such as politeness and playfulness, into the communication cost. For example, if communicating with an indirect expression is perceived as socially desirable, while using a direct expression is viewed as less desirable, the cost of uttering the indirect expression could be offset by the utility of being perceived as more desirable. Conversely, the cost of using a direct expression could increase due to the disutility of being perceived as less desirable. As a result, using an indirect expression might become more efficient than using a direct one.

Finally, although this paper presents one approach to deriving linguistically natural pragmatics under equilibrium selection based solely on efficiency, it does not argue that other approaches based on equilibrium selection augmented with linguistic salience are less desirable. The choice of "the right approach" should depend on the type of insights one seeks to obtain from the model.

7 Appendix

7.1 Proof of Lemma 2

Suppose there is an efficient equilibrium where the sender's strategy σ is not separating. Then, there are states ω' and ω'' where the sender uses the same description m', and the rational type chooses the right action only at ω' whereas the naive type chooses the right action only at ω''. Note that if m' is also used at another state, the right action is never induced at the state. Thus, if σ is an efficient equilibrium strategy, m' should be used only at ω' and ω''.

Note that fully explicit descriptions are the most expensive descriptions. Then, since the sender cannot save the communication cost with a fully explicit description, whenever an efficient equilibrium strategy uses it, it must be used only at the state that matches the description. Thus, m', which is used at more than one state, cannot be a fully explicit description, and the fully explicit descriptions of ω' and ω'' should be off-path descriptions in σ.

Suppose the sender uses the fully explicit description of ω'' at ω'', while m' is still used at ω'. Then, the rational type chooses the right action at ω''. Since σ is an efficient equilibrium strategy, this gain should be smaller than the increased communication cost at ω''. That is, $(1-\lambda)v < \hat{c}(N) - c(m')$. Then, $\lambda > \frac{1}{2}$ since $v/2 > \hat{c}(N)$.

Now, suppose the sender uses the fully explicit description of ω' at ω', while m' is still used at ω''. Then, the naive type chooses the right action at ω', whereas the rational type chooses the right action at ω''. Thus, the expected gain from this change is $\lambda v\pi(\omega') + (1-\lambda)v\pi(\omega'')$. Since σ is an efficient equilibrium strategy, we must have $\lambda v\pi(\omega') + (1-\lambda)v\pi(\omega'') \le \pi(\omega'')(\hat{c}(N) - c(m'))$. However, since $\lambda > \frac{1}{2}$ and $v/2 > \hat{c}(N)$, the inequality cannot be satisfied, a contradiction.

Funding Open Access funding enabled and organized by CAUL and its Member Institutions.

Data availability This is a theoretial paper and uses no data.

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