Three Essays in Economics

by

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under the supervision of

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Certificate of Original Authorship

I, Cong Tao, declare that this thesis is submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the Business School at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Abstract

This thesis consists of three chapters. Two of them are a series of theoretical and experimental studies on the unusual auction formats implemented in the perishable goods market. The other is a behavioural model on the toy vending machine mechanism in Gacha games.

Chapter 1 studies a remarkable auction used in several fish markets around the world, notably in Honolulu and Sydney, whereby high-quality fish are sold fast through a hybrid auction that combines the Dutch and the English formats in one auction. Speedy sales are of essence for these perishable goods. Our theoretical model incorporating "time costs" demonstrates that such Honolulu-Sydney auction is preferred by the auctioneer over the Dutch auction when there are few bidders or when bidders have high time costs. Our laboratory experiments confirm that with a small number of bidders, Honolulu-Sydney auctions are significantly faster than Dutch auctions. Bidders overbid in Dutch, benefiting the auctioneer, but bidding approaches risk-neutral predictions as time costs increase. Bidders fare better in the Honolulu-Sydney format compared to Dutch across all treatments. We further observe bidder attempts to tacitly lower prices in Honolulu-Sydney auctions, substantiating existing concerns about pricing in some fish markets.

Chapter 2 explores the Istanbul Flower Auction's unique format and compare it with traditional Dutch and English auctions, focusing on the pressing need to auction large volumes swiftly. In a model with time costs, we study how this auction format, which cleverly combines Dutch and English auction mechanisms, manages time costs by dynamically adapting to initial bidding behaviors. Our numerical analysis, which considers some specific time cost functions, reveals Istanbul Flower Auction's high performance compared to standard auction formats with respect to auctioneer and bidder utilities in the presence of time costs. This work highlights the critical role of auction design in improving social welfare, particularly in scenarios demanding the quick sale of numerous lots.

Chapter 3 explains the mechanics of toy vending machines in mobile games using prospect theory and naivety. It demonstrates that naive players who perceive independent probabilities in a history-dependent way can be induced to play for longer than optimal, thereby increasing the game provider's profits. Free initial rounds can entice naive players to start a game that will ultimately harm them. Bundling two rounds together increases the likelihood that naive players continue to play due to their probability misperception. Numerical examples further support the feasibility and profitability of this mechanism.

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Chapter 1

When Speed is of Essence: Perishable Goods Auctions

Joint work with Isa Hafalir, University of Technology Sydney; Onur Kesten, University of Sydney; Katerina Sherstyuk, University of Hawaii at Manoa.

1.1 Introduction

Many fish and flower auctions around the world are characterized by large volumes of highly perishable and highly variable in quality goods that are auctioned off sequentially, by individual units or lots. Speed of auctions is of essence given the perishable nature and large volumes being traded in a short amount of time; however, competitive bidding on each item is also an essential requirement for price discovery, given the large variability in quality and other characteristics of each item.[1]

We explore a seemingly peculiar and largely under-studied dynamic auction format employed in several perishable goods markets around the world, such as in Honolulu and Sydney fish markets. In this auction, the auctioneer sets a starting price that is neither as low as in an English auction nor as high as in a Dutch, but at a middle ground, allowing bidders to bid at the onset (either by raising their hands during a verbal Honolulu auction or by clicking a button during an automated Sydney clock auction). If at least one bidder bids at the announced starting price, the auction proceeds as an English (ascending price) auction. If there is no initial interest, the price begins to drop, as in a Dutch (descending price) auction. However, once a bidder bids, other bidders are allowed to counter-bid, potentially

[1] According to Hawaii-Seafood.org (2015) description of Honolulu fish auction, "Hundreds of fish are displayed on pallets on the auction floor. The United Fishing Agency auctioneer moves down the rows of fish surrounded by buyers who openly bid against each other for value, the best prices and quality fish. The majority of fish are sold individually. This competition continues until all the fish are sold. Up to 100,000 pounds of fish can be auctioned in a day... The... system allows for the efficient sale of the range of fish species, size and quality to suit each special market niche." See also Peterson (1973) for an early detailed record.

reverting the auction to an English auction. Although this auction format has been documented (Feldman, 2006) and is apparently employed in a number of fish markets in France and Denmark (Guillotreau and Jiménez-Toribio, 2006; Laksá and Marszalec, 2020) as well as in Honolulu and Sydney,[2] little is understood about the reasons for its emergence and its advantages over more traditional descending-only Dutch auction that is also commonly used for perishable goods. To the best of our knowledge, no previous theoretical or experimental investigations have been conducted.

Our goal is to understand the reason for the existence and the patterns of observed behavior exhibited in this distinctive auction format. In both the Honolulu and Sydney fish markets, bidders do not usually wait until the price reaches extremely low or close to zero values during the Dutch stage; on the contrary, bidding typically starts with little delay, and each fish is auctioned off within seconds. This behavior does not align with standard explanations based on risk-neutral, risk-averse, loss-averse, or regret-avoiding bidder preferences, as none of these preferences give an apparent reason to end the Dutch stage prematurely when bidders always have the option to bid later at a lower price.[3]

Extensive consultations and discussions with the experts, administrators and bidders participating in both the Honolulu and Sydney auctions led us to deduce that this auction process is specifically designed to expedite the proceedings, preventing lengthy periods of price increases or decreases. For auctioneers, speed is of essence given the perishable nature of the good and the need to sell thousands of individual fishes or boxes within a 4-5 hour time frame. Similarly, wholesalers and restaurant owners who are repeat buyers encounter time constraints when they aim to purchase significant amounts of highly perishable products.[4]

Motivated by the above, we propose a model that incorporates time costs, where both the auctioneer and bidders are impatient and strictly prefer a fast auction to a slow one, and test this model in a controlled laboratory environment. Taking time costs into account,

[2]Feldman (2006) provides the following description (p. 326, footnote 39): "At the Honolulu Fish Exchange... a modified form of the Dutch auction is used, with the auctioneer starting at a high price and then dropping it until a buyer places the first bid. The auctioneer then calls out a higher price with the hope of getting other buyers to start bidding..." Guillotreau and Jiménez-Toribio (2006) document a similar format used in fish sales in two French ports, and Laksá and Marszalec (2020) provide an identical description of the auction in the Faroe Fish market in Denmark. See Appendix 1.A for more details. Finally, at the Ayazaga flower auction in Turkey, the starting price is determined by the auctioneer, and then may go up or down depending on buyers' bids (however, this flower auction does not allow for the price to go up again, once the price starts going down). This auction was recently analyzed in Hafalir et al. (2024).

[3]Under the standard assumptions, it is a weakly dominant strategy for all bidders to wait until the price drops to zero in the Dutch stage, and then bid in the English (ascending) auction stage. Given this, the auctioneer may prefer the outcome-equivalent but less complex ascending bid English auction.

[4]Cassady (1967, p.60) notes that speed is of essence in many auction markets. In personal communication, an auction expert and experimental economics pioneer Charlie Plott also shared with us his belief that many auction procedures, such as the Honolulu-Sydney auction format, stem from the necessity to quickly conclude the auction.

impatient bidders must balance the advantages of waiting for a lower price with the costs related to the auction's length. As a result, they may opt to bid earlier if they find waiting to be more detrimental. Our findings indicate that both bidders' and auctioneers' time costs can explain the various auction stages observed in the Honolulu-Sydney auction format.

We also examine the Dutch auction alongside the Honolulu-Sydney auction in our theoretical model and in the experiments, as Dutch auctions are also commonly used in perishable goods markets, such as fish, flowers, fresh produce, and meat (Cassady, 1967). In the Sydney Fish Market, for instance, there are two clocks running Dutch auctions, typically for less valuable seafood. The third clock, on the other hand, employs the Honolulu-Sydney auction format and sells high-quality seafood such as sashimi-grade tuna or live lobsters.

In our model with time costs, the well-known Revenue Equivalence result (Myerson, 1981; Riley and Samuelson, 1981) does not apply, meaning that different auction formats yielding the same allocation might produce varying utilities for the auctioneer. Comparing the Honolulu-Sydney to Dutch auctions, we find that utility comparisons are uncertain and depend on the time cost parameters of both the auctioneer and bidders. The blend of nominal revenue, which represents the object's sale price, and time cost effects, which reflect the auctioneer's welfare loss due to the auction's duration, establish the utility comparison between different auction formats.

In a Honolulu-Sydney auction, the auctioneer selects the optimal starting price, whereas the Dutch auction's starting price is typically set at the highest possible value. This "starting price effect" could potentially favor the Honolulu auction as it might result in a shorter auction duration compared to the Dutch, especially when there are few bidders. Intuitively, with small number of bidders, the expected selling price is significantly lower than the highest possible value (which is the Dutch auction's starting price), and this puts the Dutch auction at a disadvantage in terms of time costs. This effect decreases as the number of bidders grows and the expected selling price increases.

It is important to note that while using a reserve price can speed up the sales, it can also lead to inefficiencies in the auction. For perishable goods, promptly selling them is crucial, as their value diminishes over time and they can become unsellable later. This is why reserve prices are generally not used in practice at the Honolulu and Sydney Fish markets. Therefore, in this paper, we do not consider reserve prices.

Our experimental results provide strong evidence of the speed advantage Honolulu-Sydney auctions compared to Dutch with a small number of bidders; as predicted, this advantage decreases with more bidders. However, we also find that Honolulu-Sydney auctions have lower prices than Dutch, as the former are more susceptible to suppressed price competition with a small number of bidders, while the latter exhibit consistent overbidding.

As a result, Honolulu-Sydney auction benefits relative to Dutch are somewhat lower than predicted for the auctioneer, but higher than predicted for the bidders.

The paper proceeds as follows. In Section 1.2, we briefly discuss the relevant literature. The theoretical model of Honolulu-Sydney and Dutch auctions with time costs is presented in Section 1.3. Experimental design is discussed in Section 1.4. Section 1.5 presents experimental results and participant feedback on both auction formats. We conclude in Section 1.6.

1.2 Brief review of the literature

Much of the extant auction literature focuses on the ascending English and the descending Dutch auctions and their sealed-bid analogs in various types of environments (see, for example, Klemperer (1999), Klemperer (2004) and Milgrom (2004) for excellent overviews). We contribute to this literature by studying an original auction format that has already been in use around the world for several decades.

To the best of our knowledge, Katok and Roth (2004) is the only existing analytical study that discusses "alternative" Dutch auctions where the price may go down and then back up. However, they consider auctions of multiple homogeneous goods with divisible lots, and attribute the unusual price dynamics to the presence of synergies between parts of the lot. In Honolulu-Sydney auctions, each fish is sold separately, is indivisible, and units differ in quality, suggesting heterogeneous goods and separable values across units.[5]

The speed of auctions has been recognized as an important consideration for auction design in many contexts, notably the Federal Communication Commission spectrum auctions (Banks et al., 2003; Kwasnica et al., 2005); however, it has not been formally incorporated into the objective function of the auctioneer. For multi-unit auctions with single-unit demand bidders, Andersson and Erlanson (2013) propose a hybrid Vickrey-English-Dutch auction and show numerically that it has a speed advantage over Vickrey-English and Vickrey-Dutch formats. Yet they do not discuss a real-world application of this auction mechanism.[6]

Katok and Kwasnica (2008) study the effect of Dutch auction clock speed on bidding behavior in Dutch auctions. They suggest a theoretical model that explains more overbidding in slower auctions by bidders' intrinsic cost of time, and support their explanation with ex-

[5] "The oral method of the Dutch auction ... is used mainly for the sale of nonstandardized items where quality differences require flexibility..." (Cassady, 1967, p.63). Graddy (2006) further writes that "Fish is more perishable,... and individual fish are more heterogeneous than most agricultural products."

[6] Laboratory experiments also document that people value time along with money. In a recent work, Breaban et al. (2020) conduct an experiment where the subjects must stay in the laboratory for a set time but can bid on the amount of time to leave early.

perimental data. In comparison, in our experiments, we keep the speed of the clock constant, induce time costs through bidder payoffs, and compare bidder behavior in Honolulu-Sydney and Dutch auctions with high and low bidder costs of time. Although we observe that some bidders delay their bids more than is optimal given the induced cost of time, overall, our data provide strong evidence of behavior consistent with positive time costs. In a recent work, Azevedo et al. (2020) propose a "Channel auction" that features an upper-bound price that descends (like a Dutch auction) and a lower-bound price that ascends (like an English auction), creating a narrowing channel of prices. In contrast to our paper, Azevedo et al. (2020) focus on a framework where the main aim is mitigating costly information acquisition (rather than time costs). In another recent work, Komo et al. (2024) show that a hybrid auction that begins with an English auction at the monopoly price, then, should there be no takers, concludes with a Dutch auction, is "weakly shill-proof."

There is vast experimental literature documenting overbidding in first-price sealed bid and Dutch auctions (Cox et al., 1982), often attributing it to bidder risk aversion (Cox et al., 1988) or regret aversion (Filiz-Ozbay and Ozbay, 2007). In line with the previous studies, we observe overbidding in our Dutch experimental auctions. We further use post-auction questionnaires to compare bidder sentiments of satisfaction, regret and sensitivity to time costs between Dutch and Honolulu-Sydney auctions.[7]

1.3 Model and Theoretical Results

We consider a single-item auction with n bidders in which both the auctioneer and the bidders are impatient. Bidders' private values are independently and identically distributed according to a twice differentiable cumulative distribution function F and a corresponding density function f over $[0, 1]$.

If the auction ends when bidder i with value v wins at price p after t units of time has passed, the auctioneer's utility is

$$U_A = p \cdot c_A(t)$$

and the winning bidder's utility is

$$U_B = (v - p) \cdot c_B(t),$$

where the time-adjustment function of the auctioneer $c_A(t)$ and that of a bidder $c_B(t)$ are

[7]We are grateful to Yan Chen for suggesting that the Honolulu-Sydney format allows to alleviate the loser's regret for not bidding early enough to win in the Dutch auction, as it provides a "second chance to bid." We investigate this conjecture by comparing bidder post-auction affective states under the Honolulu-Sydney and Dutch auction formats.

strictly decreasing functions in time t. More specifically, we have $c_A(\cdot), c_B(\cdot) : [0,2] \to [0,1]$[8] and $c'_A(\cdot)$, $c'_B(\cdot) < 0$. The bidders who don't win the item receive zero utility.

In our utility specification, we incorporate time adjustment functions in a multiplicative form within the utility functions. An alternative approach would be to use additive time adjustment functions, as in Katok and Kwasnica (2008). We choose the multiplicative formulation primarily for its analytical simplicity. Additionally, this approach ensures that bidders do not receive negative utility as long as the price paid is lower than their valuations - something that is not always guaranteed with the additive formulation.

1.3.1 Honolulu-Sydney Auction

The Honolulu-Sydney auction proceeds as follows. It begins with an initial price announced by the auctioneer. If no one bids at the initial price, the price starts going down until someone bids (i.e., operates as a Dutch auction). When a bidder bids, if at least one other bidder also bids at the current price, the price starts going up due to the excess demand for the item. The price continues to rise until only one bidder remains (i.e., operates as an English auction). If there is an interest at the starting price, the auction instantly becomes an English auction without any price drop, provided that at least one more bidder shows interest at the starting price. Hence, the Honolulu-Sydney auction can work in a pure Dutch, Dutch-then-English, or a pure English format depending on bidding behavior.

In this setting with impatient bidders, the time cost bidders incur from participating in the auction creates a non-trivial cost-benefit trade-off. On the one hand, an early bid helps save the cost of waiting at the expense of potentially paying a high price. In particular, early bidding is beneficial and helps avoid unnecessary wait times if a bidder anticipates excess demand around the opening price (otherwise, the price will first drop and rise again to the initial price level leading to unnecessary waiting costs). Consequently, each bidder is incentivized to start bidding before the price drops all the way to zero in the opening. On the other hand, considering that the auction may end up working as a pure Dutch auction (e.g., the first bid is above the valuation of the remaining bidders and discourages other bidders from starting an English stage), waiting before bidding first may help reduce the final price provided that no other bidder shows interest.

Next, we formalize this auction and the involved trade-offs. We consider a symmetric perfect Bayesian equilibrium of the Honolulu-Sydney auction where, when the starting price is s, each bidder with value v bids at the price $p(v, s)$ and in the English stage, each bidder will remain in the auction until the price reaches her value.

[8] Note that the maximum amount of time the Honolulu-Sydney auction can take is 2, and the Dutch auction can take is 1.

Consider the auction starting at a price s. The price will decrease until a bidder bids for the first time. Considering symmetric bidding strategies, let us represent this bid price as a function of bidder value given the starting price by $p(v,s) \in [0,s]$ and assume $p(v,s)$ to be strictly increasing in v whenever $0 < p(v,s) < s$ (allowing for "pooling at the starting price and at 0"). After one bidder bids at p, in the ascending auction stage, due to the "$(v-p) \cdot c_B(t)$" formulation of utilities and $c_B(t)$ being non-negative, all bidders with value greater than p will remain in the auction until the price reaches their values. Consider a bidder with value v who bids when the price decreases to p. Her expected utility is given by:

$$EU_B^H(p; v, s) = G(p)(v-p)c_B(s-p) + \int_p^v (v-x)c_B(s+x-2p)dG(x)$$

where $G(x)$ denotes the probability distribution of the highest of $n-1$ random variables independently distributed according to F, i.e., $G(x) = F(x)^{n-1}$. This is because, (i) with probability $G(p)$, no other bidder will increase the price after p, and $s-p$ time has passed since then, and (ii) for a given price $x > p$, with probability $G(x)$, the highest competing bidder has a value less than x and $s+x-2p$ amount of time would pass if the price first drops from s to p and then increases to x. As bidders are utility maximizers, for each bidder value v, and given the starting price s, each bidder will choose the bid price $p(v,s)$ to solve:

$$\max_{p \in [0,s]} EU_B^H(p; v, s)$$

It is easy to see that $p(v,s) < v$, since $p(v,s) \geq v$ clearly results in an expected utility of zero or less.

The auctioneer's expected utility when choosing the starting price s is given by:

$$EU_A^H(s) = \int_0^1 \left(\int_0^{p(v,s)} p(v,s)c_A(s-p(v,s))h(v,x)dx \right. \\ \left. + \int_{p(v,s)}^v xc_A(s+x-2p(v,s))h(v,x)dx \right) dv$$

where $h(v,x)$ is the joint density of the highest (denoted by v) and the second highest (denoted by x) of n random variables identically and independently distributed according to F.[9] This formulation follows because (i) if x is smaller than $p(v,s)$, then the selling price will be $p(v,s)$ and $s-p(v,s)$ time would pass until the auction ends, and (ii) if x is greater than $p(v,s)$, then the selling price will be x and $s+x-2p(v,s)$ time would pass until the auction ends.

[9] We have $h(v,x) = n(n-1)f(v)f(x)F(v)^{n-2}$.

In the Honolulu-Sydney auction, the auctioneer would choose the starting price s to maximize $EU_A^H(s)$.

Note that the equilibrium that we consider for the Honolulu-Sydney auction is efficient. This is because, (i) we assume that the bid functions in the Dutch stage are weakly increasing in values, and (ii) in the English (ascending) stage, each bidders remains in the auction until the price reaches her value.[10] This gives us the following remark.

Remark 1.1 *In equilibrium, Honolulu-Sydney auction results in an efficient allocation, in that the item is allocated to the highest-value bidder.*

In order to get more insights into the equilibrium of the Honolulu-Sydney auction, let us consider an example.

Example 1.1 *Consider 2 bidders,* $F(v) = v$, *and* $c_B(t) = c_A(t) = 1 - \frac{t}{2}$.

Below we solve for the optimal bids for an arbitrary starting price s for this simple setup. The utility of a bidder who has value v and bids at a price $p \in [0, s]$ is given by:

$$\begin{aligned} EU_B^H(p; v, s) &= p(v-p)\left(1 - \frac{s-p}{2}\right) + \int_p^v (v-x)\left(1 - \frac{s+x-2p}{2}\right) dx \\ &= \frac{1}{12}(v-p)\left(6p + 6v - 3ps + 5pv - 3sv + 2p^2 - v^2\right) \end{aligned}$$

We have

$$\frac{\partial}{\partial p} EU_B^H(p; v, s) = 6ps - 12p - 6pv - 6p^2 + 6v^2.$$

First, note that

$$\frac{\partial^2}{\partial p^2} EU_B^H(p; v, s) = 6s - 12 - 6v - 12p$$

which is negative for all p, s, and v. Hence, $EU_B^H(p; v, s)$ is a concave function of p and would be maximized (at an interior solution) when $\frac{\partial}{\partial p} EU_B^H(p; v, s) = 0$.

By solving the first-order condition $\frac{\partial}{\partial p} EU_B^H(p; v, s) = 0$, we get the unique candidate for the maximizer as

$$\frac{1}{2}\left(\sqrt{s^2 - 2sv - 4s + 5v^2 + 4v + 4} - v + s - 2\right)$$

Note that this function is increasing with s and v. Moreover,

$$p(v, s) \leq s$$

[10] This implies that, in equilibrium, the first bidder to bid in the Dutch phase is the ultimate winner of the auction. However, other bidders have nothing to lose and have a potential gain (in case of the off-the-equilibrium behavior of other players) by remaining in the ascending stage of the auction until the price reaches their values.

if and only if

$$s \leq \frac{v^2}{v+2}.$$

The highest value $\frac{v^2}{v+2}$ *can take is* $\frac{1}{3}$*. Therefore, when* $s \geq \frac{1}{3}$*, there is always an interior solution. On the other hand, when* $s \leq \frac{1}{3}$*, we may have* $p(v,s) > s$ *for* $v > \frac{1}{2}s + \frac{1}{2}\sqrt{s(s+8)}$*. Hence, in general, we have*

$$p(v,s) = \min\{s, \frac{1}{2}\left(\sqrt{s^2 - 2sv - 4s + 5v^2 + 4v + 4} - v + s - 2\right)\}. \tag{1.1}$$

Now, let us calculate the expected utility of the auctioneer with arbitrary s:

$$EU_A^H(s) = 2\int_0^1 \left(\int_0^{p(v,s)} p(v,s)\left(1 - \frac{s - p(v,s)}{2}\right)dx \right.$$
$$\left. + \int_{p(v,s)}^{v} x\left(1 - \frac{s + x - 2p(v,s)}{2}\right)dx\right)dv$$

where $p(v,s)$ *is given by Equation 1.1. Suppressing the dependence of* p *on* v *and* s *and with a little algebra, we can write*

$$\begin{aligned} EU_A^H(s) &= 2\int_0^1 \left(p^2\left(1 - \frac{s-p}{2}\right) + \frac{(v-p)}{12}\left(6p + 6v - 3ps + 4pv - 3sv + 4p^2 - 2v^2\right)\right)dv \\ &= 2\int_0^1 \left(\frac{1}{2}pv^2 - \frac{1}{4}p^2 s - \frac{1}{4}sv^2 + \frac{1}{2}p^2 + \frac{1}{6}p^3 + \frac{1}{2}v^2 - \frac{1}{6}v^3\right)dv \end{aligned}$$

Although an analytical solution is difficult to obtain due to the specification of $p(v,s)$*, we can numerically calculate the optimal starting price as* $s \approx 0.28$*.*

Our first result is the following:

Proposition 1.1 *In a Bayesian Nash equilibrium of the Honolulu-Sydney auction, we can observe all three price dynamics:(i) pure Dutch (descending only), (ii) pure English (ascending-only), and (iii) Dutch and then English (descending then ascending) auction price dynamics.*

Proof. Consider Example 1.1. In this example, the starting price will be approximately 0.28. Let v_1 denote bidder 1's value, and v_2 denote bidder 2's value. If $v_1 \geq 0.9$ and $v_2 \geq 0.28$, then we will observe pure English auction dynamics. If $v_1 < 0.9$ and $v_2 < p(v_1, 0.28)$, then we will observe pure Dutch auction dynamics. If $v_1 < 0.9$ and $v_1 > v_2 > p(v_1, 0.28)$, then we will observe first-Dutch-then-English auction price dynamics. ■ □

This result implies that all three observed price dynamics: price only going down, price only going up, and price going down, then up, are theoretically possible.

1.3.2 Dutch Auctions

To compare our predictions for the Honolulu-Sydney auction with its main competitor, we now turn to the analysis of the Dutch auction with impatient bidders.

The equilibrium analysis of the Dutch auction is more straightforward. We examine a standard Dutch auction where the price continuously declines from 1 until a bidder shows interest, i.e., bids on the item, at which point the object is awarded at that price. We concentrate on a symmetric equilibrium in which all bidders bid according to a strictly increasing and differentiable function $\beta(\cdot) \to [0, 1]$. In this equilibrium, a bidder with a value of v bidding as though their value is v' will attain the expected utility of:

$$EU_B^D = (v - \beta(v'))c_B(1 - \beta(v'))G(v')$$

A necessary condition for β to be a symmetric equilibrium strategy is that the first-order derivative of the expression above with respect to v', evaluated at $v' = v$, must be zero. Consequently, we derive the following differential equation for β:

$$-\beta'(v)G(v)\left[c_B(1 - \beta(v)) + (v - \beta(v))c_B'(1 - \beta(v)\right] + (v - \beta(v))c_B(1 - \beta(v))g(v) = 0$$

The auctioneer's expected utility in the Dutch auction is given by

$$EU_A^D = \int_0^1 \beta(x)c_A(1 - \beta(x))dF^n(x).$$

It is worth noting that the Dutch equilibrium we consider here is efficient, given that we assume bid functions are strictly increasing in values.

Remark 1.2 *In equilibrium, the Dutch auction results in an efficient allocation, in that the item is allocated to the highest-valued bidder.*

As it is analytically challenging to work with general time-adjustment functions, we consider linear time-adjustment functions in Appendix 1.B. More specifically, we consider the case where for each bidder, the payoff shrinks linearly with time by a factor $c_B(t) = 1 - bt$. Likewise, the auctioneer's pay-off shrinks linearly with time by the factor $c_A(t) = 1 - ct$. We use these linear time-adjustment functions to obtain our numerical results.

1.3.3 Comparison of the Two Auction Formats

With the ability to numerically solve for the equilibria for both Dutch and Honolulu-Sydney auctions in the case of linear time-adjustment functions, we can evaluate their performance based on various criteria, such as auction duration, the auctioneer's expected utility, and bidders' ex-ante expected utility in equilibrium.

First, let us examine an extreme scenario with patient bidders and an impatient auctioneer, $b = 0$ and $c > 0$. In this case, it is not difficult to see that the optimal starting price in the Honolulu-Sydney auction would be 0.[11] Consequently, the Honolulu-Sydney auction transforms into an English auction, and in both the Dutch and Honolulu-Sydney auctions, the behavior of the bidders becomes identical to that of the standard risk-neutral case. Under these circumstances, we can determine the exact conditions that make Honolulu-Sydney or Dutch auctions more favorable.

In this scenario, we have the following.

$$
\begin{aligned}
EU_A^H &= \int_0^1 x\,(1-cx)\,dG\,(x) \\
EU_A^D &= \int_0^1 \beta^N\,(x)\left(1-c\left(1-\beta^N\,(x)\right)\right)dF^n\,(x)
\end{aligned}
$$

where β^N represents the standard risk-neutral equilibrium bid function in Dutch auctions and is given by

$$\beta^N(x) = \frac{1}{G(x)}\int_0^x ydG(y)$$

Owing to the revenue equivalence result[12], we know that

$$\int_0^1 xdG\,(x) = \int_0^1 \beta^N\,(x)\,dF^n\,(x)$$

Let us denote this value (which is the expected selling price in the standard auctions with no time cost) by R.

[11] When bidders have no time costs, they will not bid at a positive price, since they have nothing to gain by it. Then, since the auctioneer has a strictly positive time cost, she will start the auction at the price of 0.

[12] The revenue equivalence holds for this case since bidders have no time costs.

Then, we can write

$$\begin{aligned} EU_A^H &= R - c\int_0^1 x^2 dG(x) \\ EU_A^D &= R - c\int_0^1 \beta^N(x)\left(1-\beta^N(x)\right) dF^n(x) \end{aligned}$$

Thus, we can state that:

$$EU_A^H > EU_A^D$$

if and only if

$$\int_0^1 x^2 dG(x) < \int_0^1 \beta^N(x)\left(1-\beta^N(x)\right) dF^n(x),$$

or:

$$\int_0^1 x^2 dG(x) + \int_0^1 \left(\beta^N(x)\right)^2 dF^n(x) < R = \int_0^1 x dG(x).$$

It becomes apparent that as the number of bidders, n, increases, the left side of the inequality will exceed the right side, as both terms on the left and the single term on the right approach 1. Therefore, with a larger number of bidders, the Dutch auction becomes more appealing to the auctioneer. We summarize these observations as follows:

Proposition 1.2 *When the bidders are patient, $b = 0$, and the auctioneer is impatient, $c > 0$, for any given value distribution, there exists n^* such that when the number of bidders is small enough, i.e., $n < n^*$, the auctioneer prefers the Honolulu-Sydney auction over the Dutch auction, and when $n \geq n^*$, the auctioneer prefers the Dutch auction over the Honolulu-Sydney auction.*

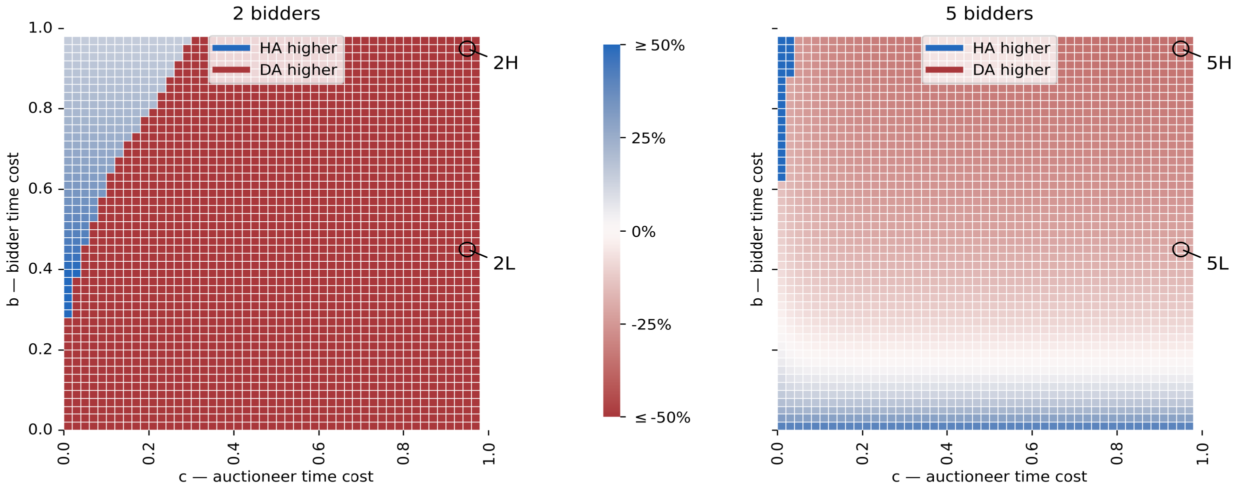
In fact, when F is uniform, some algebraic manipulation allows us to conclude that the Dutch auction yields a higher expected utility than the Honolulu-Sydney when there are at least three bidders.

Corollary 1.1 *When $b = 0$, $c > 0$, and values are uniformly distributed, the Dutch auction yields a higher expected utility than the Honolulu-Sydney auction if and only if $n \geq 3$.*

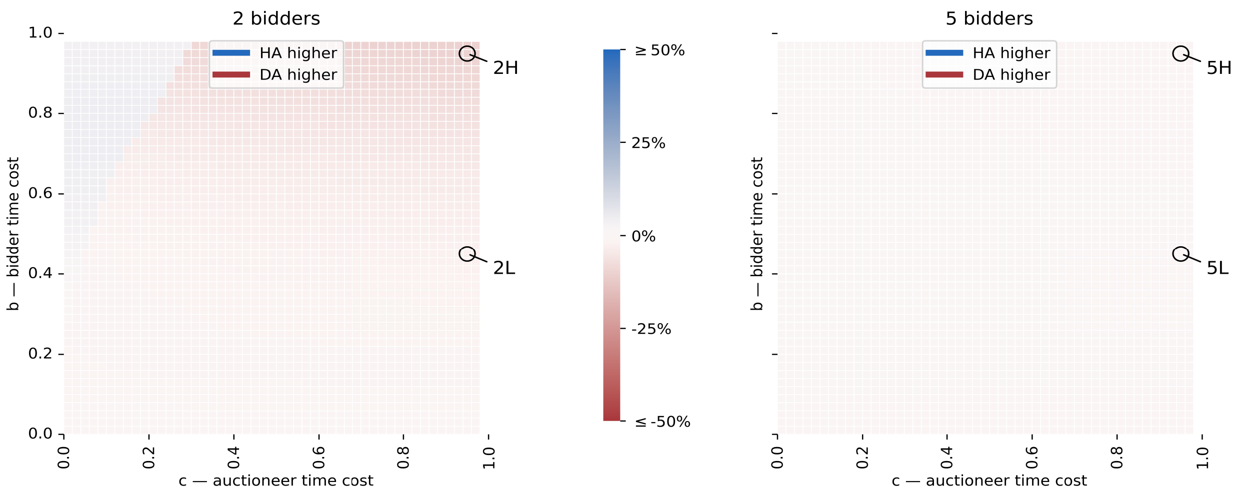
By the continuity of the auctioneer's utility function, this result extends to the case of impatient bidders when $b > 0$ is sufficiently low. Our numerical calculations presented below illustrate this point.

1.3.4 Numerical results

Our numerical algorithm enables us to compute the equilibrium bids, expected utilities, and expected selling prices and durations for both Dutch and Honolulu-Sydney auctions under



(a) Auction duration

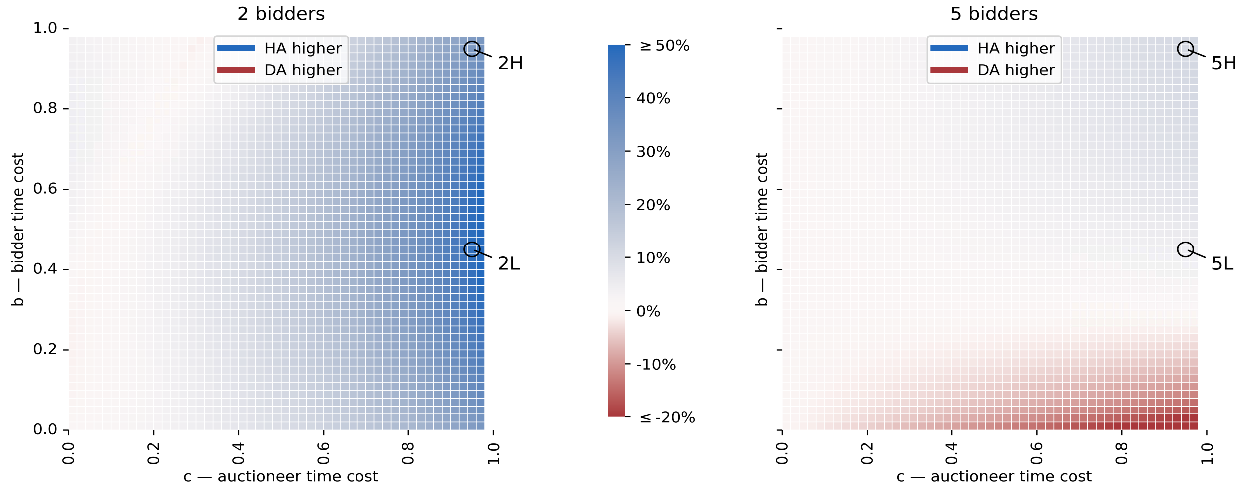


(b) Selling prices

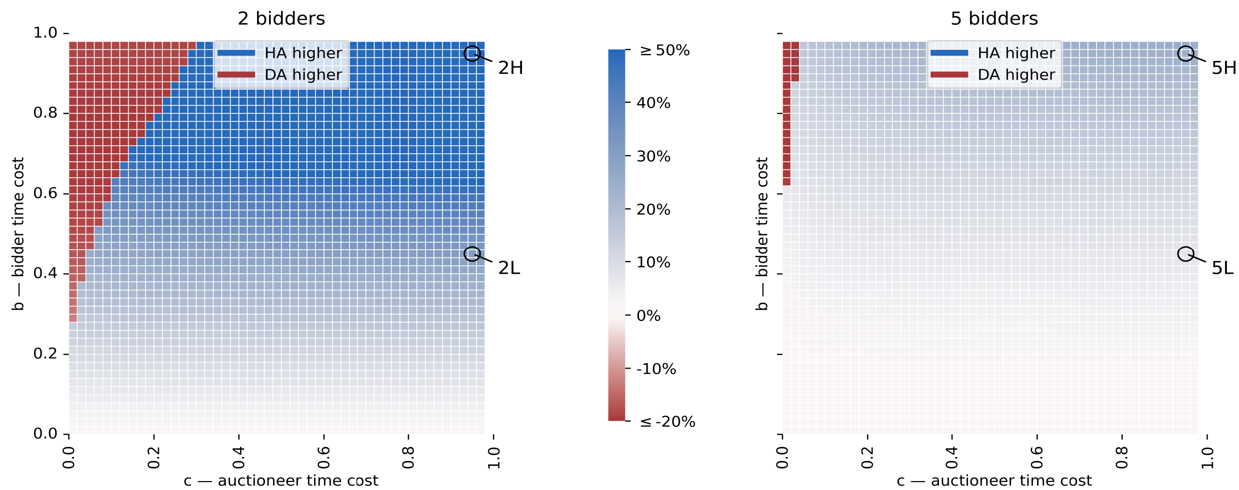
Figure 1.1: Predicted duration and price differences: Honolulu-Sydney vs. Dutch auctions

a wide range of parameter specifications. In our calculations, we assume F to be uniform and allow for variations in the number of bidders (n), the bidders' time cost parameter (b), and the auctioneer's time cost parameter (c). The calculations confirm that, as observed in our extreme example, the Honolulu-Sydney auction's relative performance advantage (in terms of auctioneer utility) against the Dutch auction is less pronounced as the number of bidders grows. This observation aligns with real-world scenarios: Honolulu-Sydney auctions are employed for "premium seafood," which typically involves a smaller number of bidders compared to more conventional fish auctions, where Dutch auctions are commonly utilized.

The numerical estimations allow us to derive the following predictions regarding the relative performance of Dutch and Honolulu-Sydney auctions in terms of duration, selling



(a) Utility for the auctioneer



(b) Utility for the bidder

Figure 1.2: Predicted utility differences: Honolulu-Sydney vs. Dutch auctions

price, and auctioneer and bidder utilities, depending on the number of bidders and the bidder and auctioneer's cost of time, as illustrated in Figures 1.1 – 1.2. The figures plot the differences in the corresponding metrics between the two auction formats relative to Dutch, $\frac{H-D}{D}$.[13]

Predictions 1.1 (Relative performance of Honolulu-Sydney and Dutch auctions) *Assume the distribution of bidder values is uniform, and the time-adjustment function is linear. Then, under a wide range of parameter values,*

1. *(Auction duration) Honolulu-Sydney auctions are faster than Dutch auctions, i.e., their*

[13] Appendix 1.C and Figures 1.3-1.6 therein provide more details.

average duration is shorter. The relative advantage of Honolulu-Sydney auctions over Dutch in terms of duration decreases with the number of bidders.

2. *(Selling prices) The difference in average selling prices between Honolulu-Sydney and Dutch auctions is small; it does not exceed 10 percent.*

3. *(Auctioneer utility) Assume the auctioneer cost of time is relatively high. Then Honolulu-Sydney auctions are always preferred to Dutch in the two-bidder case. For auctions with more than two bidders, Honolulu-Sydney auctions are preferred to Dutch when bidder cost of time is high, and Dutch auctions are preferred to Honolulu-Sydney when bidder cost of time is low.*

4. *(Buyer utility) Buyers prefer Honolulu-Sydney auction to Dutch under a wide range of parameter values. For auctions with a small number of bidders, the advantage of Honolulu-Sydney auctions over Dutch in terms of buyer utility increases with bidder cost of time. The relative advantage of Honolulu-Sydney auctions over Dutch decreases with the number of bidders.*

1.3.5 Summary of Theoretical and Numerical Results

The table below summarizes our theoretical and numerical findings. It further refers to the corresponding findings from the laboratory experiment that we discuss next.

Auction performance	Theory	Experiment
Price dynamics	Proposition 1.1	Result 1.1
Efficiency	Remark 1.1 (Honolulu-Sydney) Remark 1.2 (Dutch)	Result 1.2
Duration	Prediction 1	Result 1.3
Selling prices	Prediction 2	Result 1.4
Auctioneer utility	Proposition 1.2 Prediction 3	Result 1.5
Bidder utility	Prediction 4	Result 1.6

1.4 Experiment Objectives and Design

The experiment is designed to evaluate and compare the performance of Honolulu-Sydney (referred to as "Honolulu" hereafter for brevity) and Dutch auctions in view of the above theoretical analyses. Specifically, we address the following questions. First, do Honolulu auctions manifest the predicted price adjustment flexibility, with prices going up, or down, or down then up, depending on the demand, as predicted? Second, are they as efficient as Dutch auctions? Third, are Honolulu auctions considerably faster than Dutch if the number of bidders is small? Fourth, are the relative advantages of Honolulu auctions over Dutch in terms of speed, auctioneer utility, and bidder payoffs more pronounced when the number of bidders is small and bidder cost of time is high? Further, we explore how the differences in the experimental auction outcomes from the theoretical predictions, if any, can be explained by bidder behavior. Finally, we compare bidder feedback on the two auction institutions.

1.4.1 Auction Design

Values, Auction Duration and Payoffs Our intent is to reproduce, in a laboratory setting, the auction institutions and environments similar to those in the existing fish markets, with speedy sales and noticeable costs of delay for participants.

All experimental participants are assigned the roles of bidders; the auctioneer role is carried out by computer. At the beginning of an auction period, each participant is randomly assigned a private item value drawn from the uniform integer distribution on $[0, 50]$ experimental points, and is randomly matched with $(n-1)$ other participants to compete for a unit of a fictitious good. The participants are explicitly instructed that their earnings from the purchase will depend on the difference between their value for the good and the price they pay, and on how long the auction lasts; see experimental instructions included in the Experimental Materials. The payoff of a bidder with value v who makes a purchase at price p after t units of time is calculated as:

$$U_B = (v-p)\, c_B(t), \tag{1.2}$$

where the "time-adjustment factor" $c(t)$, as it is referred to in the experimental instructions, depends on the common to all bidders time cost parameter b:[14]

$$c_B(t) = \begin{cases} (1-bt) & \text{if } v \geq p; \\ (1+bt) & \text{if } v < p. \end{cases} \tag{1.3}$$

For both Dutch and Honolulu auctions, a virtual clock is used to determine the auction duration. The virtual clock ticks every second. For each tick of the virtual clock, the

[14] Consistent with the model in Section 1.3, buyer positive payoffs (earnings) shrink in proportion of auction duration; yet the negative payoffs (losses) increase in proportion to auction duration. We replaced the time-adjustment function $(1-bt)$ by $(1+bt)$ for the negative payoff range to avoid undesirable effects of bidder losses possibly shrinking with longer auction duration if a bidder stays active in the ascending stage of the auction when the price surpasses their value. Since such behavior never occurs in equilibrium, the model predictions are not affected by this modification.

price changes (decreases or increases, depending on the auction format and its stage) by one point,[15] and the available buyer payoff shrinks according to the time-adjustment factor $c_B(t)$. At each point when the auction is open, each participant is given real-time information on the current price of the item, the auction time elapsed, and their unadjusted $(v - p)$ and time-adjusted $(v - p) \cdot c_B(t)$ payoffs if they were to buy the item at this price and time. Examples of a participant auction screen are given in the Experimental Materials. The bidder who buys the item in the auction receives their time-adjusted payoff at the time and price of sale; all other bidders receive zero payoffs in this auction.

The auctioneer payoff from a sale at price p and time t, used to assess the auction performance, is calculated as

$$U_A = p \cdot c_A(t),$$

where $c_A(t) = (1 - ct)$, and c is the auctioneer's cost of time parameter.

Auction institutions The Dutch auction is implemented in the standard way: the auction opens at the price of 50, and the price decreases by one point with every tick of the virtual clock. The first subject to click the "Bid" button buys the item and receives a payoff equal to her displayed adjusted payoff at the time of her bid.

Under the Honolulu format, the auction opens at the auctioneer's optimal starting price set by the experimenter. Then either a bidder bids at the opening price, or the Dutch stage begins with the price decreasing by one point with every tick of the virtual clock until a bidder bids, becoming a "provisional buyer." Other bidders are then given 10 seconds to challenge this buyer by indicating their willingness to continue bidding. During this "Contest" stage, the virtual clock is stopped, and the price and time do not change. If no one challenges, then the auction ends and the provisional buyer is assigned the object at the price and time of their bid. If anyone challengers by clicking "Bid," the auction proceeds into the ascending price (English) stage, with the virtual clock ticking and the price rising by one point every second, until all but one bidder leave the auction. The remaining bidder is assigned the item at the last dropout price and time, and receives their corresponding time-adjusted payoff, while all others receive zero.

We compare the performance of Dutch and Honolulu auctions using a within-subject design, with a sequence of auctions under one institution followed by a sequence of auctions under the other institution. The sessions are conducted under either *DH* (Dutch-then-Honolulu), or *HD* (Honolulu-then-Dutch) sequence, and are counter-balanced for order.

[15]Therefore, the price changes by 2% of the maximum item value every second, which is a fairly fast clock according to Katok and Kwasnica (2008). Their fast, medium and slow clocks are equivalent to 5%, 0.5% and 0.17% change per second, respectively.

Treatments To explore the effect of the number of bidders and the bidder cost of time on the auction performances, we implement a 2×2 between-subject design, with the number of bidders per auction $n \in \{2, 5\}$, and the bidder cost of time parameter $b \in \{H = 0.95, L = 0.45\}$, corresponding to the experimental buyer payoff shrinking by 1.9% and 0.9% with every tick of the virtual clock in the H (high cost) and L (low cost) environments, respectively. The auctioneer cost of time parameter is fixed at $c = 0.95$, corresponding to a 1.9% auctioneer payoff reduction per tick.[16] The combinations of parameter values n, b and c determine the optimal (for the auctioneer) starting prices s in Honolulu auctions, which are numerically estimated (see Section 1.3) and used in the corresponding treatments. The treatments are labeled $2H, 2L, 5H, 5L$, respectively, and summarized in Table 1.1.

Parameter values are chosen to provide a considerable variation in the relative performances of Honolulu and Dutch auctions across treatments. The time cost parameters are set high enough to have a noticeable effect on participant payoffs. We further accounted for the likely overbidding relative to the risk-neutral prediction in the Dutch auctions (Cox et al., 1982; Katok and Kwasnica, 2008) by choosing parameter values that would, in theory, favor Honolulu auctions over Dutch in all treatments, as we expected the Dutch auction to benefit the auctioneer more than predicted due to higher than predicted prices and shorter than predicted durations.[17] Figures 1.1 - 1.2 illustrate the theoretically predicted differences between the formats in expected auction durations, selling prices, and auctioneer and buyer payoffs under the four different treatments, with treatments parameter locations indicated by circles on each panel; see also Table 1.3 below. Consistent with Predictions 1– 4, Honolulu auctions are expected to be shorter than Dutch and more preferred by the auctioneer and the bidders under all treatments, but the advantages of Honolulu over Dutch in terms of duration and auctioneer utility are predicted to become less pronounced in five-bidder auctions than in two-bidder auctions.

1.4.2 Experimental procedures

The auction experiment is programmed using oTree (Chen et al., 2016), with student participants recruited using ORSEE recruitment system (Greiner, 2015). Eight to 12 participants

[16]Parameter values b and c are given for the unit value scale of the theoretical model of Section 1.3 and then rescaled to the experimental value range of $[0, 50]$.

[17]In calibrating the experimental parameters, we conducted four pilot sessions with 40 participants to inform the experimental design. The pilot sessions had the value range of $[0, 100]$, a slower clock of one price tick per 1.3 seconds, and lower cost of time parameters; these auctions proved to be extremely slow and did not reflect the fast nature of the auctions we seek to model. We also observed that the cost of time was negligible under the original design. Consequently, we re-scaled the value range to $[0, 50]$ interval, increased the clock speed to one price tick per second, and increased the cost of time parameters.

Table 1.1: Experimental design and session summary

Treatment	# of bidders	Bidder cost of time[*]	Starting price	# of auctions per institution	Sequence	# of sessions	# of participants
2H	2	0.019	21	18	Dutch-Honolulu	2	20
					Honolulu-Dutch	2	20
					Total	4	40
2L	2	0.009	12	18	Dutch-Honolulu	3	26
					Honolulu-Dutch	1	12
					Total	4	38
5H	5	0.019	32	25-28[**]	Dutch-Honolulu	2	20
					Honolulu-Dutch	2	25
					Total	4	45
5L	5	0.009	27	28	Dutch-Honolulu	2	20
					Honolulu-Dutch	1	15
					Total	3	35
All treatments						15	158

[*] Bidder cost of time parameters b and the starting prices are re-scaled for the experimental value and price range of $[0, 50]$.
[**] One 5H session had only 18 auctions per institution.

are recruited for each two-bidder auction session, and 10 to 15 participants for each five-bidder auction session. Before proceeding with the auctions, the experimenter reads aloud the experimental instructions and answers any questions, while the participants follow the instructions and complete tests for understanding on the computer screen. Each session includes two practice and 18-28 paid Honolulu auction rounds followed by an identical number of Dutch auction rounds (HD sequence) or vice versa (DH sequence). The session participants are randomly re-matched into groups of two (in 2-bidder treatments) or five (in 5-bidder treatments) for each round. Each auction institution is followed by a brief questionnaire soliciting participant feedback on the institution and participant affective states.[18] At the conclusion of a session, the participants are paid in private their cumulative earnings from the auctions, plus a show-up fee. The total duration of each session is between one and a half and two hours, including instructions.

We conducted a total of 15 independent sessions, with 158 unique student participants at the experimental laboratories of the University of Hawaii and the University of Technology Sydney, with 3-4 sessions per treatment. The sessions are summarized in Table 1.1. The average payments (including the participation fees of $5 USD or $10 AUD) were $28.95 USD and $52.57 AUD, respectively. Experimental instructions, screenshots and the post-auction survey are included in the Experimental Materials supplement.

[18] Qualitative questions measuring regret and affective states are based on Camille et al. (2004). Each experimental session also included short pre- and post-auction surveys that assessed participants cognitive ability, risk, time and competitiveness preferences, and basic demographics. See Cacho (2023) for details.

1.5 Experimental Results

1.5.1 Auction performance

Price dynamics As predicted, we observe all three price dynamics under Honolulu auctions in the experiment: descending price only (Dutch), ascending price only (English), and descending followed by ascending prices (Dutch-then-English) (Table 1.2).

Table 1.2: Price dynamics in Honolulu auctions

		Dutch then English		Dutch only		English only*	
Treatment	Statistics	Predicted	Actual	Predicted	Actual	Predicted	Actual
2H	Count	137	222	156	113	67	25
	Percentage	38.1%	61.7%	43.3%	31.4%	18.6%	6.9%
2L	Count	163	250	65	77	114	15
	Percentage	47.7%	73.1%	19.0%	22.5%	33.3%	4.4%
5H	Count	94	135	41	27	75	48
	Percentage	44.8%	64.3%	19.5%	12.9%	35.7%	22.9%
5L	Count	35	115	35	25	126	56
	Percentage	17.9%	58.7%	17.9%	12.8%	64.3%	28.6%

* To account for possible participant delays in executing their bids, bids within two points (two price ticks) from the starting price are considered as immediate bids, with the corresponding auction dynamics classified as English (ascending price) only.

Yet the Dutch-then-English price dynamics are significantly more frequent, and the English-only dynamics are less frequent than predicted under all treatments, as supported by the Wilcoxon signed-rank tests comparing the actual percentage of these price dynamics to predicted ($p < 0.01$ and $p < 0.05$ for 2-bidder and 5-bidder auctions respectively, and $p < 0.01$ for all treatments pooled; see Table 1.7 in the Appendix 1.E.) This suggests that the participants often allowed prices to drop before bidding even if they would be better off bidding at the opening price. We explore this behavioral pattern in more detail in Section 1.5.2 below.

Result 1.1 *As predicted, experimental Honolulu auctions are characterized by price adjustment flexibility, displaying all three price dynamics: descending only (Dutch), ascending only (English) and descending-then-ascending (Dutch-then-English). Compared to the theoretical predictions, the Dutch-then-English pattern is significantly more frequent, while the English-only pattern is less frequent.*

We next compare the main performance characteristics of Honolulu and Dutch auctions by treatment, as summarized in Table 1.3 and Figures 1.3 and 1.4. Results of hypotheses testing on the comparative performances of the two institutions, benchmarked against the theoretical predictions as given in Remarks 1.1, 1.2 and Predictions 1– 4, are summarized in Table 1.4. All test results reported in Table 1.4 and in the remainder of the paper rely on session-clustered bootstrap estimations, unless noted otherwise. The estimation details can be found in Appendix 1.D.

Table 1.3: Predicted and actual auction characteristics by treatment

	2H		2L		5H		5L	
	Predicted	Actual	Predicted	Actual	Predicted	Actual	Predicted	Actual
Value efficiency								
Dutch	100%	94.8%	100%	95.4%	100%	98.6%	100%	95.5%
Honolulu	100%	96.7%	100%	96.8%	100%	98.4%	100%	98.5%
Auction duration								
Dutch	28.4	29.9	32.1	29.5	15.7	14.3	16.4	13.0
Honolulu	12.5	20.6	14.2	19.6	10.3	18.4	9.0	15.5
H/D, %	44.0%	68.9%	44.2%	66.4%	65.6%	128.7%	54.9%	119.2%
Selling price								
Dutch	21.6	20.6	17.9	21.1	34.3	36.3	33.6	37.6
Honolulu	19.8	17.0	18.1	15.6	33.9	33.4	33.3	32.2
H/D, %	91.7%	82.5%	101.1%	73.9%	98.8%	92.0%	99.1%	85.6%
Auctioneer payoff								
Dutch	11.0	10.8	7.9	11.0	24.9	27.3	23.8	29.3
Honolulu	14.8	9.7	11.9	7.9	27.3	21.7	27.1	22.0
H/D, %	134.5%	90.0%	150.6%	71.8%	109.6%	79.5%	113.9%	75.1%
Buyer payoff								
Dutch	5.9	4.5	11.1	8.1	5.1	3.3	6.7	1.5
Honolulu	11.7	9.7	14.0	14.2	6.1	4.3	8.1	7.8
H/D, %	198.3%	215.6%	126.1%	175.3%	119.6%	130.3%	120.9%	520.0%

Predictions are based on bidder values drawn. Actual values displayed are averages. Value efficiency is in percent, auction duration is in seconds, and prices and auctioneer and buyer payoffs are in experimental points. Buyer payoff is conditional on buying. Bidder ex-ante utility can be obtained by dividing buyer utility by the number of bidders.

Value efficiency Value efficiency[19] is 95% or higher in all treatments under both auction formats. However, Honolulu auctions have about 1.5 percent higher efficiency than Dutch overall ($p < 0.001$ for the pooled data), and higher efficiency in three out of four treatments ($p < 0.1$, one-sided); see Figure 1.3 and Table 1.4. This higher efficiency is likely due to the majority of Honolulu auctions ending with the English stage, which is known to have the highest efficiency among all common auction formats.

[19]The value efficiency is defined in the usual way, as the percent of the buyer item value to the highest value in the market.

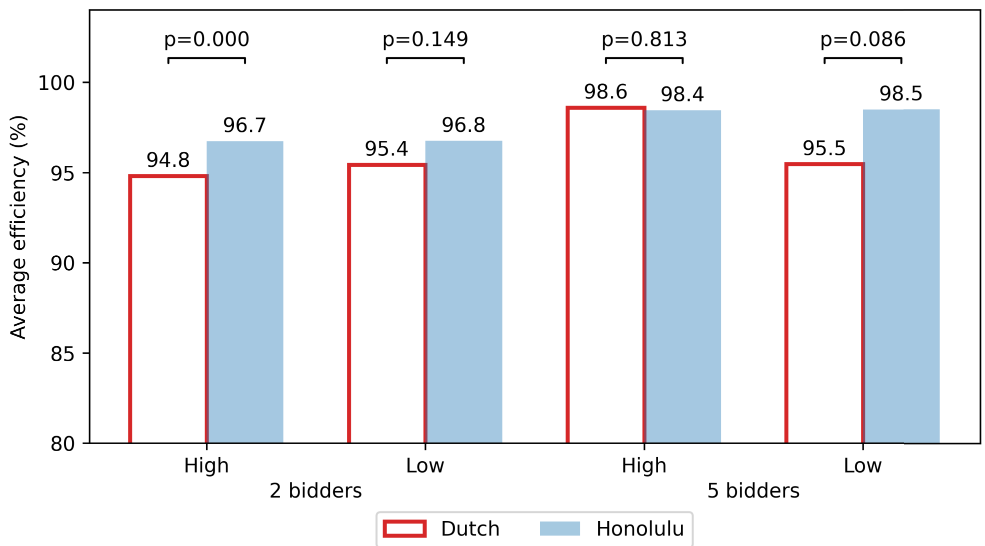


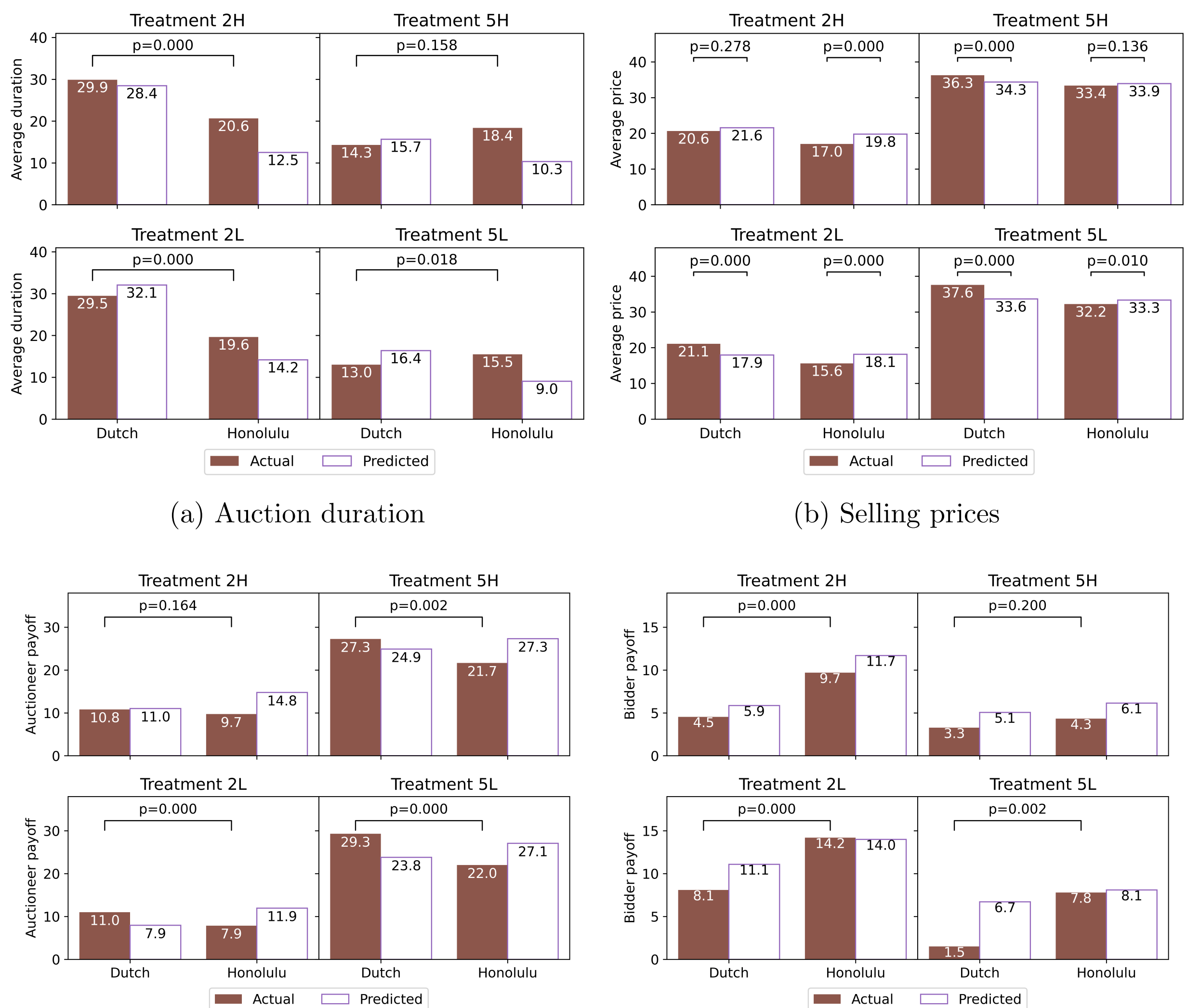
Figure 1.3: Auction efficiency by treatment

Table 1.4: Hypotheses tests of theoretical predictions

Characte-ristic	Prediction-based hypotheses*	bidders/ cost	Observed coeff.	Bootstr. std. err.	p-value	Prediction supported?
Efficiency	$H = D$	pooled	1.54	.45	0.000	no
	$H^h = D^h$	2 bidders	1.92	.48	0.000	no
	$H^l = D^l$	2 bidders	1.32	.91	0.148	yes
	$H^h = D^h$	5 bidders	-.15	.64	0.815	yes
	$H^l = D^l$	5 bidders	3.02	1.81	0.095	no
Duration	$D^2 > H^2$	high cost	9.26	1.70	0.000	yes
	$D^2 > H^2$	low cost	9.85	.93	0.000	yes
	$D^5 > H^5$	high cost	-4.08	2.90	0.921	no
	$D^5 > H^5$	low cost	-2.50	1.05	0.991	no
	$H^5/D^5 > H^2/D^2$	high cost	.60	.22	0.004	yes
	$H^5/D^5 > H^2/D^2$	low cost	.53	.13	0.000	yes
Selling price	$H^2/D^2 > 0.9$	high cost	-.075	.024	0.001	no
	$H^2/D^2 > 0.9$	low cost	-.159	.041	0.000	no
	$H^5/D^5 > 0.9$	high cost	.020	.007	0.998	yes
	$H^5/D^5 > 0.9$	low cost	-.043	.044	0.163	yes
Auctioneer utility	$H^2 > D^2$	high cost	-1.09	.77	0.842	no
	$H^2 > D^2$	low cost	-3.14	.55	1.000	no
	$H^5 > D^5$	high cost	-5.60	1.85	0.997	no
	$H^5 > D^5$	low cost	-7.30	2.205	0.999	no
Buyer utility	$H^2 > D^2$	high cost	5.15	.77	0.000	yes
	$H^2 > D^2$	low cost	6.10	.68	0.000	yes
	$H^5 > D^5$	high cost	1.05	.84	0.106	no
	$H^5 > D^5$	low cost	6.30	1.90	0.001	yes
	$H^h/D^h > H^l/D^l$	2 bidders	.379	.326	0.123	no
	$H^2/D^2 > H^5/D^5$	high cost	.81	.41	0.025	yes
	$H^2/D^2 > H^5/D^5$	low cost	-3.44	19.31	0.429	no

* H – Honolulu, D Dutch; h – high-cost, l – low-cost; 2 – 2-bidders, 5 – 5-bidders. Hypotheses for efficiency are based on Remarks 1.1- 1.2; for duration, selling prices, auctioneer and buyer utilities – on Predictions 1 – 4. Observed coefficients are for the difference between the LHS and the RHS expressions in the corresponding hypothesis; see Online Appendix 1.D for the estimation details.

Result 1.2 *Efficiency is high under both auction formats and in all treatments, as predicted. Overall, Honolulu auctions are more efficient than Dutch.*



(a) Auction duration

(b) Selling prices

(c) Auctioneer payoffs

(d) Bidder payoffs

Figure 1.4: Auction performance by treatment

Auction duration As is evident from Figure 1.4a and Table 1.3, Honolulu auctions are significantly faster than Dutch in 2-bidder case ($p < 0.001$); we observe approximately one-third (32.1%) shorter duration in Honolulu auctions, although, on average, the difference is smaller than predicted. Yet, Honolulu auctions are no faster and actually slower than Dutch in 5-bidder treatments, while they are predicted to be 39% faster. This is explained by longer-than-predicted Honolulu auctions combined with somewhat shorter-than-predicted Dutch auctions. The comparative statics prediction is confirmed: the relative advantage of

Honolulu auctions over Dutch decreases with the number of bidders, $H5/D5 > H2/D2$, for both high and low costs ($p < 0.01$, one-sided, in both cases; Table 1.4). Moreover, the observed auction durations are significantly shorter than those predicted under zero cost of time for all treatments and both auction formats (Figure 1.9 in Appendix 1.F).

Result 1.3 *Honolulu auctions are significantly faster than Dutch with two bidders. Yet Honolulu auctions take longer than predicted and are no faster than Dutch in five-bidder auctions. As predicted, the relative advantage of Honolulu auctions over Dutch, in terms of duration, decreases with the number of bidders.*

Selling prices From Figure 1.4b and Table 1.3, Dutch auctions have higher than predicted ($p < 0.01$) selling prices in all treatments other than 2H. This is not surprising as overbidding is commonly observed in Dutch auctions (Cox et al., 1982; Katok and Kwasnica, 2008). For Honolulu auctions, selling prices are lower than predicted ($p < 0.001$) with 2 bidders, and they are still lower (in 5L) or not significantly different from predicted (in 5H) with 5 bidders (Figure 1.4b). The relative price differences between Honolulu and Dutch auctions significantly exceed the predicted by Proposition 2 ten percent for 2-bidder auctions ($p < 0.01$), but is not significantly different from ten percent for 5-bidder auctions ($p > 0.05$); see Table 1.4.[20]

Result 1.4 *Honolulu auctions have significantly lower prices than Dutch, with the price gap significantly larger than predicted in two-bidder auctions, but within the predicted range in five-bidder auctions.*

Although the prices have a direct effect on the auctioneer's and bidders' utility, the standard revenue comparison is largely irrelevant because of the time costs. Auctioneer utility comparison is more relevant as it incorporates both revenue and time cost considerations.

Auctioneer payoffs As illustrated in Figure 1.4c and Table 1.3, the auctioneer utility is not significantly different between Honolulu and Dutch auctions in 2H treatment ($p = 0.164$, two-sided). In other treatments, Dutch auctions benefit the auctioneer significantly more than Honolulu auctions, even when the opposite is predicted ($p > 0.9$ for the prediction-based hypothesis of $H > D$ for all treatments, Table 1.4). This is explained by auctioneer benefiting less than predicted from Honolulu auctions, while benefiting more than predicted

[20]We further tested the hypotheses on the relative prices in Honolulu and Dutch auctions based on the bidder values drawn, as listed in Table 1.3. The percentage price differences between Honolulu and Dutch auctions are significantly different from those predicted in all treatments; see Table 1.6 in Online Appendix 1.D.

from Dutch auctions. The shortfall of auctioneer utility in Honolulu auctions as compared to the predictions appears greater with two bidders, likely due to lower than predicted prices, as well as longer than predicted duration (Table 1.3); whereas the excess of auctioneer utility over predictions in Dutch auctions is likely due to higher than predicted prices.

Result 1.5 *Auctioneer payoffs under Dutch and Honolulu are not significantly different in 2-bidder, high-cost treatment. In other treatments, Dutch auctions benefit the auctioneer more than Honolulu auctions, even when the opposite is predicted.*

Bidder payoffs Figure 1.4d and Table 1.3 indicate that buyers are significantly better off or no worse off under Honolulu auctions than under Dutch in all but one treatment ($p = 0.106$ for 5H and $p < 0.01$ for all other treatments, for one-sided hypothesis $H > D$, Table 1.4). For 2-bidder auctions, buyer average payoff under Honolulu is twice as high as under Dutch in 2H treatment, and 75 percent higher in 2L treatment, exceeding the predicted differences. The high cost of time leads to a significant reduction of buyer payoffs under both Dutch and Honolulu 2-bidder auctions; yet there is not enough evidence to conclude that Honolulu auctions become more beneficial relative to Dutch as the cost of time increases ($p = 0.123$ for the hypothesis $H^h/D^h > H^l/D^l$, Table 1.4). For 5-bidder auctions with high costs (5H treatment), the advantage of Honolulu relative to Dutch is significantly smaller than in 2-bidder 2H treatment, as predicted ($p = 0.026$); yet it is not smaller in 5L treatment compared to 2L ($p = 0.574$), likely due to significant over-bidding in the 5L Dutch auctions.

Result 1.6 *Bidders are significantly better off or no worse off in Honolulu auctions as compared to Dutch in all treatments. The benefit of Honolulu auctions compared to Dutch with 5 bidders persists more than predicted in some treatments.*

Overall, we obtain strong support for the theoretical predictions on the versatility of price dynamics and efficiency of Honolulu auctions (Proposition 1.1 and Remark 1.1), as well its speed advantages and higher buyer benefits relative to Dutch auctions(Predictions 1, 4). However, compared to the predictions regarding prices and auctioneer utilities (Predictions 2, 3), Honolulu auctions underperform relative to Dutch.[21] To summarize:

[21]We are grateful to Anthony Kwasnica for the following suggestion. In addition to comparing Honolulu-Sydney and Dutch auctions from the auctioneer and the bidder separate standpoints, one could assess the aggregate social welfare, defined as the sum of the auctioneer and bidder utilities. We analyze the social welfare in Appendix 1.C.6. Honolulu-Sydney auctions are predicted to outperform the Dutch along the social welfare metric, and more so in auctions with fewer bidders. Indeed, the social welfare is significantly higher in our Honolulu-Sydney experimental auctions with two bidders, although it falls short of the Dutch auctions social welfare with five bidders.

Conclusion 1.1 *As predicted, Honolulu auctions are highly efficient, and are considerably faster than Dutch auctions with a small number of bidders. While the auctioneer's benefits from Honolulu auctions relative to Dutch are lower than predicted, the bidders' relative benefits often exceed the predictions.*

1.5.2 Behavioral patterns

We next explore individual behavioral patterns that would explain the observed auction performances. Behavior varied widely across individuals, resulting in large differences of bidder payoffs.

To compare bidder payoffs across individuals with different value draws, we normalize the actual payoffs under a given institution as percentages of the theoretically predicted:[22]

$$\%Pay = \frac{Actual\ Total\ Payoff}{Predicted\ Total\ Payoff} \times 100\%.$$

We then categorize participants into Top and Bottom earners, depending on whether their percentage payoff falls above or below the median in their treatment.

Figure 1.5 displays participant median percentage payoff (the cutoff value separating the Top and Bottom earners), and average percentage payoffs for Top and Bottom earners, by treatment. The differences between Top and Bottom earners percentage payoffs are substantial under all treatments for both auction formats, with average Top earners often making higher earnings than predicted, while average Bottom earners making as low as 25 percent of the predicted payoffs in some treatments.

Below we compare the decisions of more successful, Top-earning bidders, with those of less successful, Bottom-earning bidders, to understand which bidding patterns are associated with earning success.

Dominated decisions A strictly dominated decision would involve bidding above value in the Dutch auction, leading to a sure loss. A weakly dominated decision in the Honolulu auction would involve either risking a loss by bidding above value in the Dutch and Contest stages of Honolulu auctions, or staying in bidding above value in the English stage; or "leaving money on the table" by dropping out of bidding in the Contest or English stages

[22] The drawback of this measure are possibly extreme values for bidders who are predicted to earn zero or near zero, due to low value draws. We address this issue by censoring percentage payoff at -10% and 210%; the payoff percentages for only only eight bidders in Dutch auctions and four bidders in Honolulu auctions (out of 158 total bidders) had to be censored. As an alternative, one could consider the average per period point deviations from the theoretical prediction by period; however, such measure would not be comparable across treatments. All qualitative findings are robust to the use of this alternative measure.

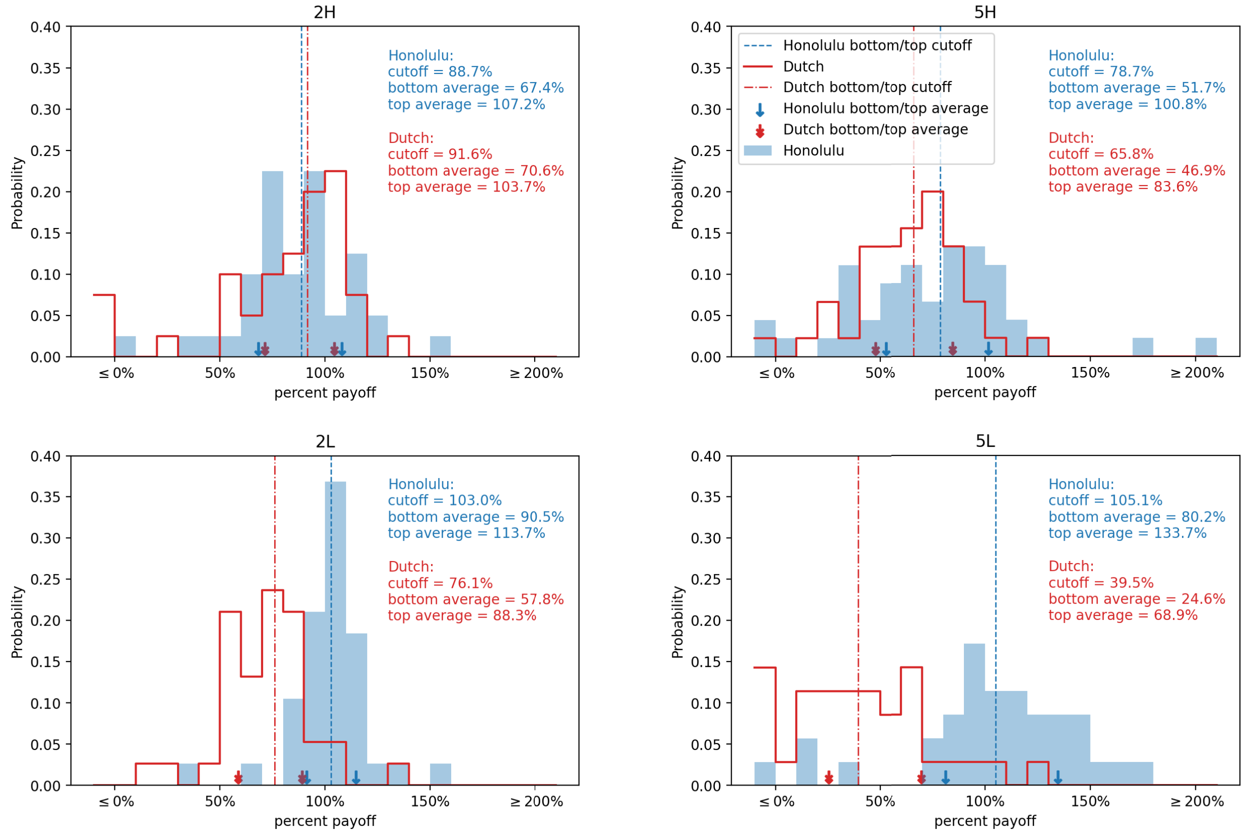


Figure 1.5: Distribution of percent payoffs, by treatment

at a price below one's value. Table 1.5 lists the frequencies of dominated decisions in Dutch and Honolulu auctions (see Table 1.8 in Online Appendix 1.E for dominated actions by auction stage). Almost all bids in Dutch auctions are undominated, as compared to 78 to 95 percent of (weakly) undominated decisions in Honolulu auctions. The share of undominated decisions is no lower or higher for Top earners than for Bottom earners, under all stages of the auctions and under all treatments; the differences between Top and Bottom earners are highly significant for the pooled data, as well as for the most of the treatments and stages ($p < 0.05$, Wicoxon sign-rank test; see Table 1.9 in Online Appendix 1.E.)

Result 1.7 *The share of undominated decisions is significantly higher under Dutch than under Honolulu auctions. Top earners make dominated decisions less frequently than Bottom earners under both auction formats under most treatments.*

A higher share of dominated decisions may indicate a higher complexity of Honolulu auction compared to Dutch. Alternatively, leaving below value may be rationalizable in the framework of multi-period supergame, and may be explained by bidder attempts to suppress price competition. We explore these alternative explanations below when analyzing individual behavior in Honolulu auctions.

Table 1.5: Frequencies of dominated decisions, by treatment

	2H		2L		5H		5L	
Decision	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom
DUTCH AUCTION								
Bid above value	0	9	0	1	0	2	0	15
	0.0%	5.5%	0.0%	0.6%	0.0%	2.2%	0.0%	15.3%
Undominated bid	189	156	167	170	117	91	98	83
	100.0%	94.5%	100.0%	99.4%	100.0%	97.8%	100.0%	84.7%
HONOLULU AUCTION								
Leave below value -all	23	53	38	41	20	35	14	38
	6.4%	14.7%	11.1%	12.0%	4.1%	6.2%	2.9%	7.5%
- *foregone purchase*[2]	*23*	*53*	*38*	*41*	*13*	*21*	*7*	*24*
	6.4%	*14.7%*	*11.1%*	*12.0%*	*2.7%*	*3.7%*	*1.5%*	*4.8%*
Bid above value -all	5	25	3	13	7	45	10	25
	1.4%	6.9%	0.9%	3.8%	1.4%	8.0%	2.1%	5.0%
- *actual loss*[2]	*0*	*11*	*1*	*6*	*0*	*6*	*1*	*6*
	0.0%	*3.1%*	*0.3%*	*1.8%*	*0.0%*	*1.1%*	*0.2%*	*1.2%*
Undominated decision	332	282	301	288	461	482	452	441
	92.2%	78.3%	88.0%	84.2%	94.5%	85.8%	95.0%	87.5%

[1] The table lists the number of decisions in each category, and their percentage out of all decisions. Due to possible delays in bid transmission, decisions within 2 points of bidder value are considered to be "at value."

[2] Decisions in the corresponding category where the final auction price was below value (for foregone purchases), or the purchase was made at a price above value (for actual losses).

Bidding patterns in Dutch auctions While there is no analytical solution for the equilibrium bidding strategy in the Dutch auction if the bidder cost of time is positive, $b > 0$ (Section 1.3), the equilibrium bidding strategy can be closely approximated by a linear function of bidder value:

$$DutchBid \approx \alpha_0 + \alpha_1 \times v, \tag{1.4}$$

where v is bidder value, and α_0 and α_1 are the constant and the coefficient on value, respectively, which may both depend on the number n of bidders and bidder cost of time parameter b. By the continuity of the bidding function in the cost of time parameter, and given the analytical solution $DutchBid = \frac{(n-1)}{n} * v$ for the case of no time costs, $b = 0$, we expect the intercept to be close to zero, $\alpha_0 \approx 0$, and the coefficient on value to satisfy $\alpha_1 \in (0, 1)$, for all n and b. Figure 1.6 depicts all bids submitted in the Dutch auctions against bidder values. The numerically estimated prediction line is marked as "theory", with added regression lines of actual bids on values, displayed separately for Top and Bottom earners.

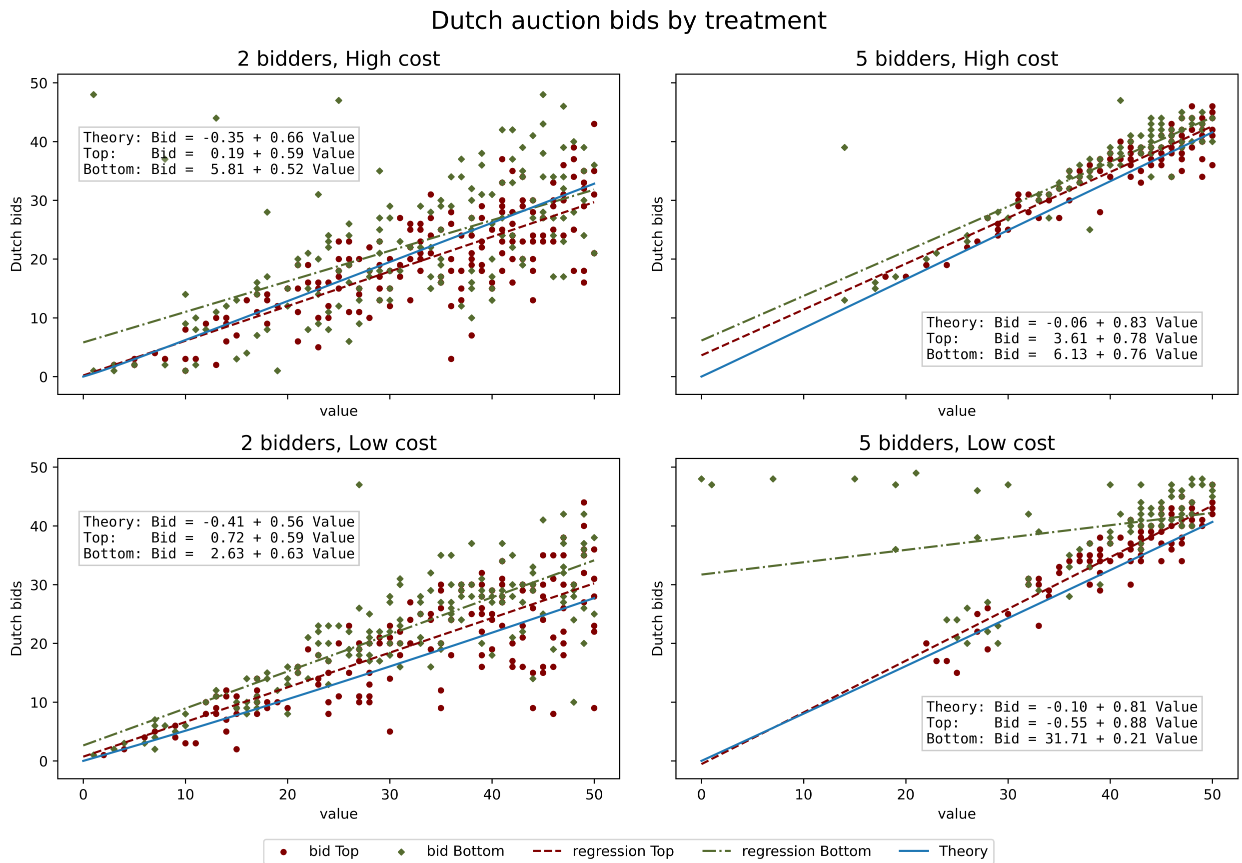


Figure 1.6: Dutch bids by value, by treatment

Overall, the bidding behavior tracks the theoretical predictions rather well for both Top and Bottom earners; the slopes of the bidding functions on value are not significantly different from the theoretically predicted ($p > 0.05$) in all treatments, except for the Bottom earners in the 5L treatment. However, overbidding is common in all but the 2H treatment, and especially for the Bottom earners (see also Table 1.10 in Online Appendix 1.E).

Figure 1.7 (left panel) further illustrates that, indeed, bidders overbid, on average, in all treatments in the Dutch auctions, except for the 2H treatment. Moreover, Top earners bid significantly lower and closer to the theoretical predictions than Bottom earners ($p < 0.05$ for 2-bidder auctions and $p < 0.001$ for 5-bidder auctions).[23] Another interesting observation is that bidding is significantly ($p < 0.01$) closer to theory in the high-cost treatments (with the average over-bidding at 0.75 points only) than in the low-cost treatments (with the average over-bidding at 4.39 points); yet we cannot reject the hypothesis that this is due to higher predicted bids under the high cost of time, rather than differences in bidder behavior. [24]

[23]Reported p-values are obtained by regressing the bid deviations from the predictions on the payoff segment dummies, with bootstrapped standard errors clustered on the session.

[24]Katok and Kwasnica (2008) document more overbidding in slower Dutch clock auctions, attributing

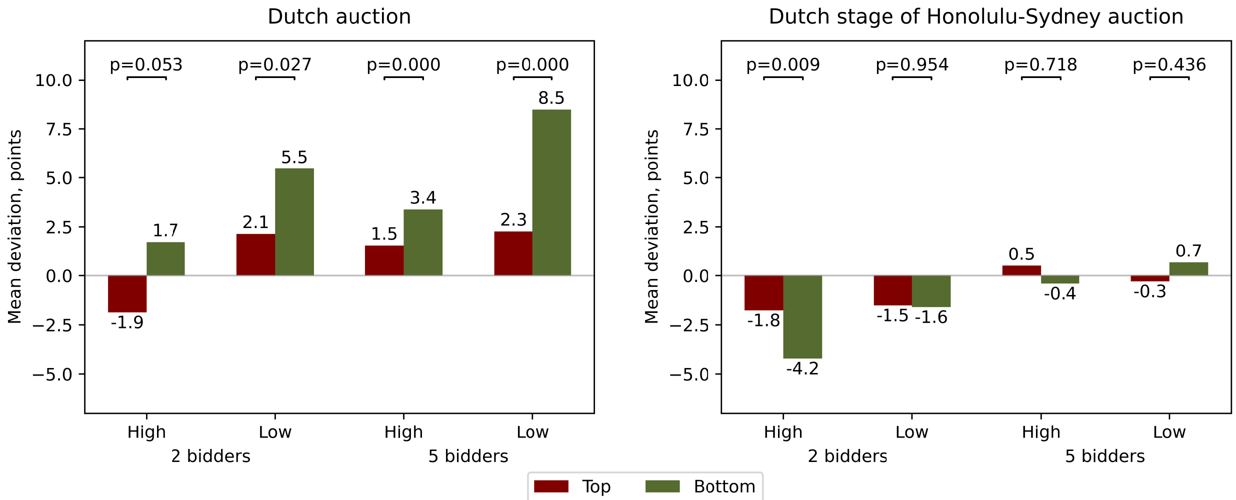


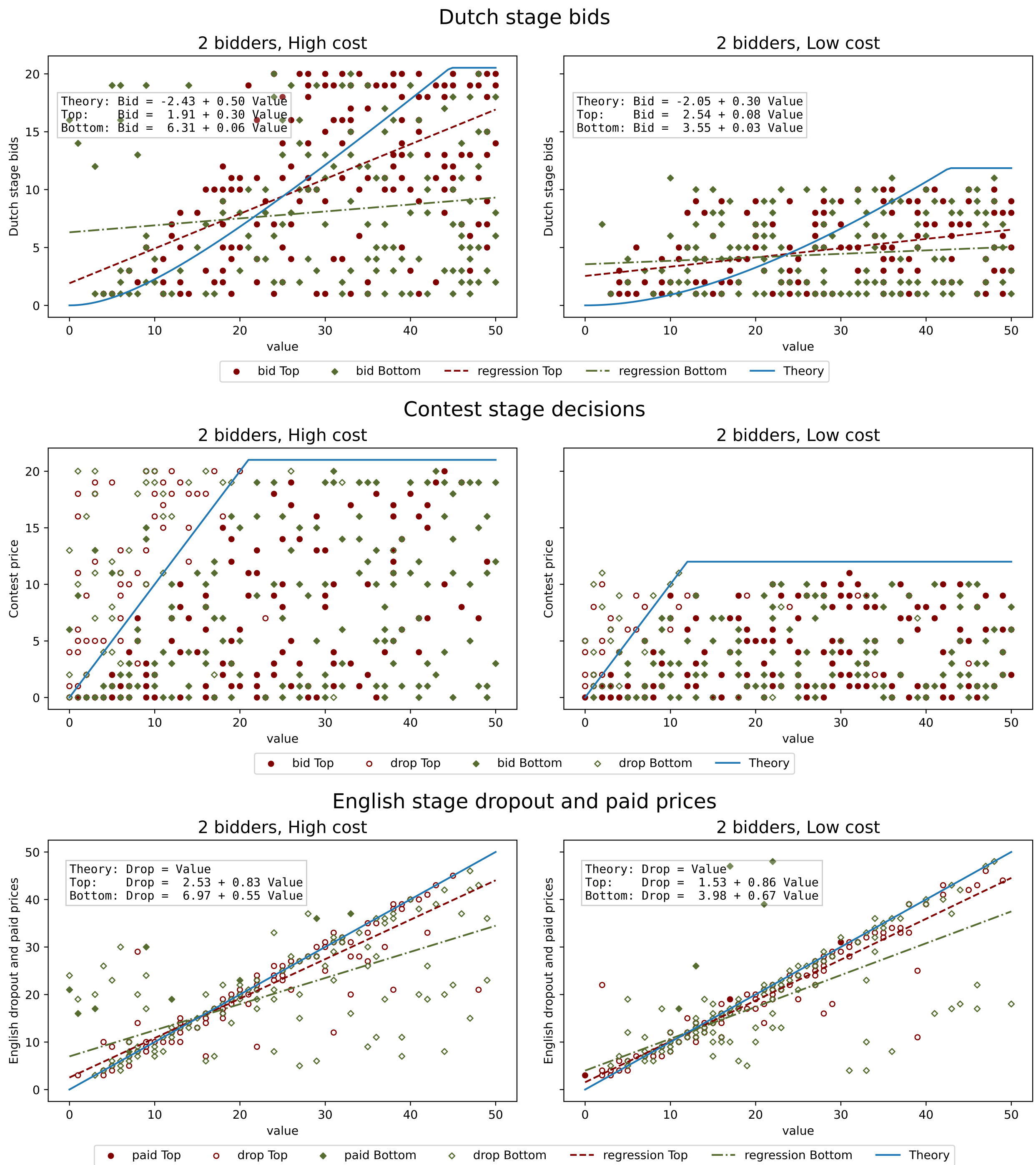
Figure 1.7: Bid deviations from theory in Dutch auctions and at the Dutch stage of Honolulu-Sydney auctions, for Top and Bottom earners

Importantly, the observed bidding in all treatments is consistent with the presence of positive time costs, as no time costs are predicted to result in even lower bids.

Result 1.8 *Bidders overbid in all but one treatment in the Dutch auctions. Bidding closer to the theoretical prediction is associated with significantly higher bidder earnings, whereas overbidding is associated with lower earnings. Bidders overbid significantly less in auctions with the high cost of time.*

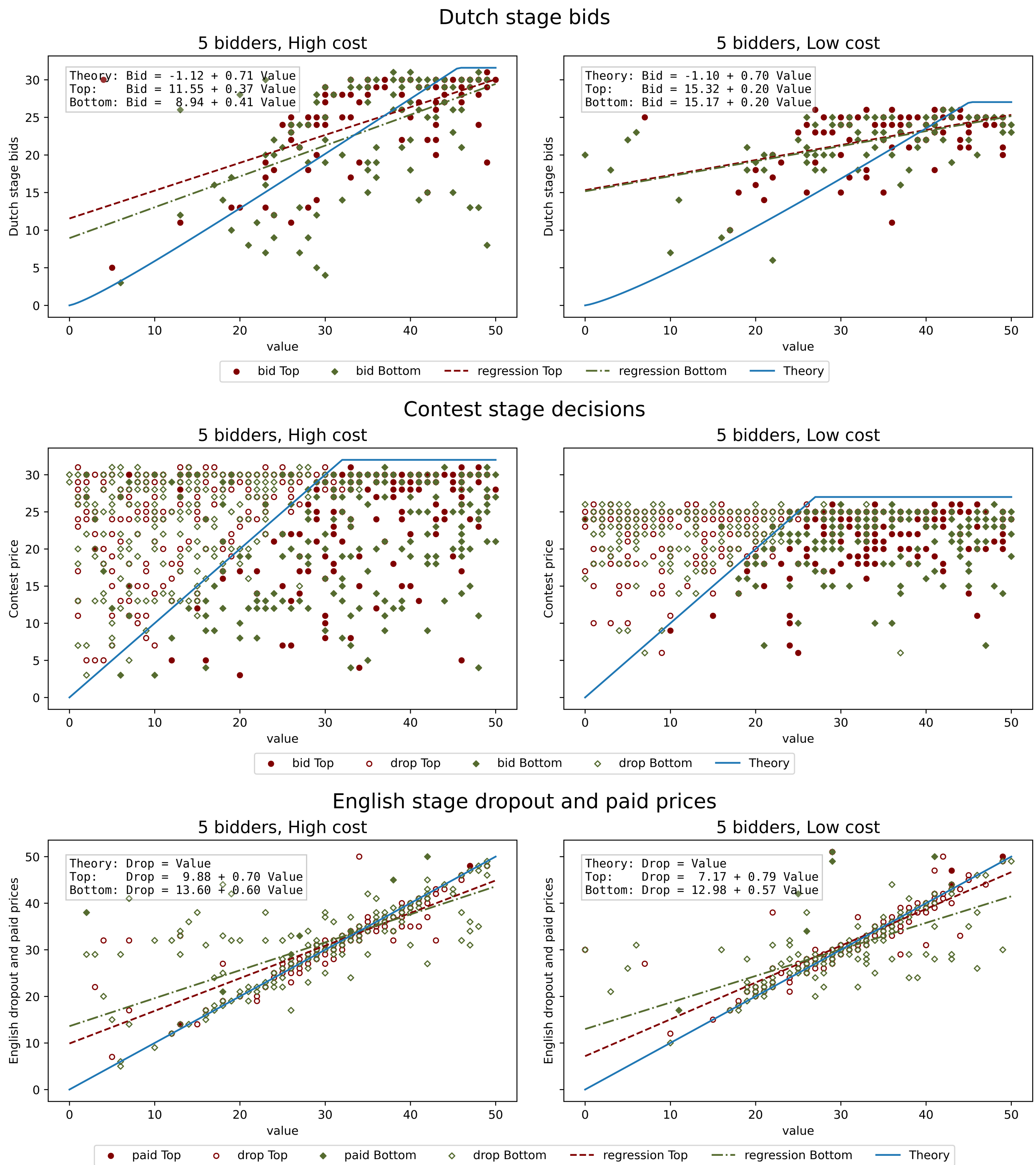
Bidding patterns in Honolulu auctions Figures 1.8 and 1.9 illustrate bidding behavior by stage against the theoretical predictions in 2-bidder and 5-bidder Honolulu auctions, respectively. The figures present evidence of both over-bidding and under-bidding compared to predicted at the Dutch stage, and a sizeable share of early dropouts at the final English stage. In our analysis, we aim to disentangle to what extent such behaviors may be attributed to bounded rationality, and bidder under-valuing time in particular, and to what extent to the bidder attempts to suppress price competition and gain from lower auction prices. Lower than competitive prices that benefit bidders may be supported as equilibria in a multi-period super game even with random rematching (Kandori, 1992).

this phenomenon to bidders' intrinsic impatience. While the speed of the virtual clock is the same in all treatments in our experiment, the higher induced cost of time translates into faster payoff shrinkage and may have a similar effect on behavior as a faster clock, leading to less overbidding. However, our regression estimations of bids on value (Table 1.10 in Online Appendix 1.E) indicate that the bids are not statistically different between high- and low-cost treatments ($p > 0.1$), suggesting that less overbidding compared to predictions under the high cost of time may be due to higher predicted bids rather than changes in bidder behavior.



Note: For the English stage, the only paid prices displayed are those above value.

Figure 1.8: Honolulu auction bidding behavior by stage, 2 bidder treatments



Note: For the English stage, the only paid prices displayed are those above value.

Figure 1.9: Honolulu auction bidding behavior by stage, 5 bidder treatments

First, observe that the overwhelming majority of Dutch-stage bids (72% of bids in 2-bidder auctions and all bids in 5-bidder auctions) are at prices above zero;[25] the share

[25]Bids within 2 points of zero are considered "near zero."

of near-zero bids not consistent with the equilibrium prediction is only 23.6% in 2-bidder auctions. Moreover, out of 96 Dutch- stage bids predicted to be near zero, more than half (49%) were at prices above zero. Recalling that in the absence of positive time costs, it is a weakly dominant strategy for bidders to allow the prices to drop to zero in the Dutch stage, we conclude:

Result 1.9 *The overwhelming majority of decisions at the Dutch stage of Honolulu auctions with two bidders, and all decisions with five bidders, are consistent with the presence of positive time costs.*

Yet, in drastic contrast to bids in Dutch auctions, conditional on bidding, submitted Dutch-stage bids are, on average, at prices no higher or lower than predicted for both Top and Bottom earners in all treatments (Figure 1.7, right panel). Further, bids are flatter in value than predicted, with over-bidding compared to the prediction at low values and under-bidding at high values, as indicated by simple regression estimations included in the top panels of Figures 1.8- 1.9 (see additional Dutch-stage bid estimation results in Table 1.11 in Online Appendix 1.E). The overall effect is dominated by high-value bidders bidding at lower-than-predicted prices, resulting in the Dutch stage lasting, on average, over 4 price ticks longer than predicted in all treatments. In two-thirds (68%) of all auctions with two bidders, and in almost three-quarters (74%) of all auctions with five bidders, the Dutch stage lasted more than two price ticks longer than predicted.

Result 1.10 *Longer-than-predicted duration of Honolulu auction is explained by Dutch-stage bidding delays by high-value bidders.*

On the auction level, Dutch-stage bidding delays are associated with significantly lower purchase prices in both 2-bidder and 5-bidder auctions. On average, one price-tick increase in the Dutch-stage duration above the prediction is associated with 0.48 point reduction in the purchase price from the equilibrium level in 2-bidder auctions, and 0.21 point reduction in 5-bidder auctions ($p < 0.01$ in both cases).[26] This suggests an anti-competitive motive for delayed bidding. Alternatively, bidders may delay bidding simply because they underestimate the cost of time.

To understand the likely motives behind such delays, we link bidder actions at the Dutch stage with their decisions in later stages. A Dutch-stage individual decision is considered a delay if a bidder does not bid at the predicted price, thus allowing the Dutch-stage price to drop lower. A Dutch-stage delay followed by an early below-value dropout in the Contest

[26] Based on regressing the purchase price deviation on bidding delay, with bootstrapped standard errors clustered on session.

or English stage is consistent with non-competitive behavior, whereas a Dutch-stage delay followed by bidding until the price reaches one's value in the English stage is competitive and may be attributed to under-valuing time.[27] Finally, a Dutch-stage delay followed by a purchase below the predicted equilibrium price is consistent with both non-competitive behavior, and competitive behavior combined with under-valuing time.

Table 1.6 displays frequencies of various individual decision-outcome patterns in Honolulu auctions, sorted on the Dutch-stage bidder delays. Overall, 41% of individual Dutch-stage decisions with two bidders are classified as delays, as compared to only 18% of decisions with five bidders. These differences between 2-bidder and 5-bidder auctions are highly significant ($p < 0.01$) based on the regression estimation presented in Table 1.7 below.[28] Among all occurrences of Dutch-stage delays in 2-bidder auction, about a half (23% out of 41% in 2H treatment, and 17% out of 41% in 2L treatment) are followed by low-price drop or buy outcomes and are therefore consistent with non-competitive behavior, and the other half – by competitive decisions at the Contest or English stage. In comparison, three-quarters of Dutch-stage delays in 5-bidder auctions (14% out of 19% under 2H, and 13% out of 17% under 2L treatment) are followed by competitive behaviors at later auction stages (Table 1.6). Keeping in mind that Dutch-stage delays followed by low-price purchases may be attributed to both competitive and non-competitive behavior, we conclude:

Table 1.6: Decision-outcome patterns in Honolulu auctions

	2-bidder auctions		5-bidder auctions		All
Decision-outcome pattern, percent	2H	2L	5H	5L	
Delay, Low Price Buy	17.36	12.13	2.57	2.45	7.54
Delay, Low Price Drop	5.56	4.82	2.38	1.43	3.26
Delay, Competitive Drop or Buy	17.92	24.12	14.00	13.47	16.69
No Delay, Low Price Buy	2.64	3.51	0.95	1.63	2.01
No Delay, Low Price Drop	5.00	6.73	2.86	3.88	4.37
No Delay, Competitive Drop or Buy	47.78	46.35	72.48	73.78	62.46
No Delay, Overbid	3.75	2.34	4.76	3.37	3.67
Total	100.00	100.00	100.00	100.00	100.00

"Delay:" Dutch-stage non-bid at the predicted bid price; "Low Price:" below equilibrium prediction; "Competitive:" at or above equilibrium prediction; "Overbid:" bid above value. Bidding above value following Dutch-stage delays is extremely rare (7 out of 3434 observations) and is lumped with the "Competitive" category. Decisions within 2 points of the corresponding prediction are considered "at prediction."

[27] Our estimate of the share of non-competitive behavior is therefore conservative, as some Dutch-stage delays followed by competitive bidding could indicate failed attempts to lower prices.

[28] The delays are much less frequent at the individual level than at the auction level, as the auction-level Dutch-stage outcomes are driven by the behavior of a subset of bidders who bid first.

Result 1.11 *Dutch-stage bidding delays are significantly more frequent in auctions with two bidders, with up to a half of the delays attributable to non-competitive behavior. In 5-bidder auctions, the delays occur less frequently, and at most a quarter of these delays are attributable to non-competitive behavior. Overall, between 17 and 24 percent of all decisions in Honolulu auctions are consistent with competitive behavior with under-valuing time.*

Table 1.6 provides further evidence for weaker competition in 2-bidder auctions as compared to 5-bidder auctions, documenting an additional sizeable share of low-price dropouts following no delays at the Dutch stage; see also Table 1.5 for overall frequencies of "Leave below value" decisions. The share of bidder decisions resulting in low price dropouts or purchases is significantly higher in 2-bidder auctions than in 5-bidder auctions ($p < 0.01$[29]). And yet, almost a half of individual decisions in auctions with two bidders and almost three-quarters of decisions in auctions with five bidders are consistent with the equilibrium behavior with no Dutch-stage delays (Table 1.6, "No Delay, Competitive Drop or Buy" category).[30]

Regression estimations of behavior presented in Table 1.7 confirm that under-bidding at the Dutch stage is more prevalent for bidders with high-value draws, but is significantly less common in 5-bidder auctions ($p < 0.01$ in both cases). Dutch-stage bidding delays persist in later rounds ($p < 0.05$), suggesting again an anti-competitive motive behind many of the delays. Bidders are significantly more likely to stay above value in 5-bidder auctions than in 2-bidder auctions ($p < 0.01$), possibly due to a lower risk of buying above value in auctions with more than two active bidders. The precedence of both leaving the auction at a price below value and bidding above value at the English stage is reduced significantly in later rounds ($p < 0.05$), indicating participant learning. Consistent with the theoretical prediction, the cost of time parameter does not have a significant effect on bidding at the English stage.

Finally, the table documents significant differences in behavior between Top and Bottom earners. Compared to Bottom earners, Top earners are significantly less likely to delay their bids at the Dutch stage ($p < 0.05$), and both to drop out early and over-bid at the English stage ($p < 0.01$ in both cases). An English-stage dropout price regression estimation (Table 1.12 in Appendix 1.E) confirms that Top earners' dropout prices are closer to the value than those of the Bottom earners. We further address heterogeneity in bidder behavior using multi-dimensional clustering based on similarity method. The analysis provides an

[29]Wilcoxon Mann-Whitney test using sessions averages as units of observation.

[30]Moreover, the modal behavior is consistent with equilibrium, i.e., it falls under either "No Delay, Competitive Drop or Buy" or "No Delay, Low Price Buy" categories, for three-quarters (73 percent) of bidders in 2-bidder auctions, and the overwhelming majority (96 percent) of bidders in 5-bidder auctions. We therefore do not sort individuals into behavioral types, but instead focus on behavioral differences between Top and Bottom earners.

Table 1.7: Regression estimation of individual decisions in Honolulu auctions, by stage

	(1) Dutch stage		(2) Contest stage		(3) English stage		(4) Final outcome	
Bid/leave below equilibrium								
item value	0.13***	(0.01)	0.01	(0.01)	0.02***	(0.00)	0.06***	(0.00)
5 bidders	-1.89***	(0.28)	-0.97	(0.63)	-0.34	(0.38)	-1.22***	(0.37)
high cost	0.02	(0.29)	-0.23	(0.43)	0.10	(0.46)	0.21	(0.36)
5 bidders high cost	0.18	(0.50)	0.26	(0.85)	-0.20	(0.57)	-0.35	(0.53)
Dutch auction first	0.13	(0.32)	0.25	(0.43)	-0.14	(0.34)	-0.09	(0.29)
round	0.03**	(0.01)	-0.03	(0.02)	-0.04***	(0.01)	-0.04***	(0.01)
Top earner	-0.37**	(0.16)	-0.36	(0.23)	-0.67***	(0.17)	-0.29***	(0.09)
Constant	-4.12***	(0.38)	-2.55***	(0.54)	-1.76***	(0.45)	-2.17***	(0.34)
Competitive bid/leave: base outcome								
Bid above value								
item value	-0.13	(0.10)	-0.10**	(0.05)	-0.14***	(0.01)	-0.04***	(0.01)
5 bidders	0.97	(9.95)	11.95***	(3.43)	2.50***	(0.88)	0.14	(0.76)
high cost	1.80	(9.00)	13.47***	(2.21)	0.53	(0.86)	0.47	(0.72)
5 bidders high cost	-2.08	(10.98)	-10.74***	(4.05)	-0.47	(1.03)	-0.15	(0.93)
Dutch auction first	-2.36	(6.09)	-2.21	(2.21)	-0.90	(0.56)	-1.02**	(0.50)
round	-0.01	(0.07)	-0.07	(0.07)	-0.03**	(0.01)	-0.02*	(0.01)
Top earner	-2.21	(4.32)	-1.85	(2.59)	-1.31***	(0.39)	-1.49***	(0.41)
Constant	-2.77	(8.71)	-14.36***	(2.12)	0.45	(0.97)	-1.39*	(0.83)
Observations	3434		2381		2064		3434	
Pseudo R^2	0.360		0.137		0.171		0.169	

Multinomial logit estimation. Bootstrapped standard errors clustered on session in parentheses.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

additional support that the behavior of high earners is qualitatively different and closer to the equilibrium bidding strategy, especially at the Contest and English stages, than that of low earners who tend to either over-bid or under-bid (see Appendix 1.G for details.)

Two interesting observations are due in relation to the apparent bidder attempts to lower prices through bidding delays and early dropouts. First, these attempts do not pay off equally for all bidders; while Top earners earn, on average, above the prediction in Honolulu auctions, the Bottom earners earn below the prediction (Figure 1.5); overall, the bidders are no better off than they would be under the predicted non-cooperative equilibrium behavior in any of the treatments (Table 1.3). Second, these deviations from the predicted behavior do not distort the allocative efficiency of Honolulu auctions (Result 1.2).

Result 1.12 *Almost a half of all individual decisions in Honolulu auctions with two bidders, and an overwhelming majority of decisions with five bidders, are consistent with the theoretical predictions, with the frequency of such decisions increasing in later rounds. Compared to Bottom earners, Top earners bid closer to the equilibrium predictions at all stages.*

We summarize the bidding behavior and its link to the auction performances as follows:

Conclusion 1.2 *Overall, competitive bidding consistent with the theoretical predictions is the modal behavior in Honolulu auctions. Observed under-performance of Honolulu auctions relative to Dutch for the auctioneer is explained, on one hand, by consistent overbidding in the Dutch auctions, and, on the other hand, by bidding delays and non-competitive dropouts in a sizeable share of instances in Honolulu auctions.*

1.5.3 Participant feedback

To evaluate what mattered to participants in their bidding decisions, and to assess their post-auction affective states, we conducted short surveys soliciting participant feedback immediately following each auction institution.[31] Consistent with the observed behavior, participants reported that they cared more about buying fast in Dutch than in Honolulu auctions, and cared more about getting a lower price in Honolulu than in Dutch. Further, they reported experiencing significantly less winner and loser regret in Honolulu than in Dutch auctions (Figures 1.10- 1.11). Finally, participants felt about equally happy when they made a purchase in Honolulu and in Dutch, but felt significantly less unhappy when they did not buy in Honolulu as compared to Dutch; see Table 1.13 in Online Appendix 1.E. These responses suggest that, in addition to having a speed advantage over Dutch auctions when the number of bidders is small, Honolulu auctions reduce bidder regret and make the participants happier.

1.6 Conclusions

In this paper, we study a distinctive auction mechanism used in several fish markets around the world, including Honolulu and Sydney fish markets. This auction facilitates the rapid sale of premium quality fish—a necessity given the perishable nature of the goods in question. This hybrid auction format, blending the traits of both Dutch and English auctions, has shown its efficacy not only in real-world markets but also in our theoretical model and in the laboratory.

Our theoretical framework highlights the pivotal role of "time costs." Our results show that the Honolulu-Sydney auction format is more favorable for the auctioneer (as compared to the standard Dutch auction format) in scenarios where there's a limited number of bidders or when these bidders bear high time costs. This is substantiated by our experimental data, revealing that Honolulu-Sydney auctions conclude significantly faster than their Dutch counterparts when there is a low number of bidders. The individual bidder behavior is

[31] See Cacho (2023) for an extended analysis of participants behavioral features.

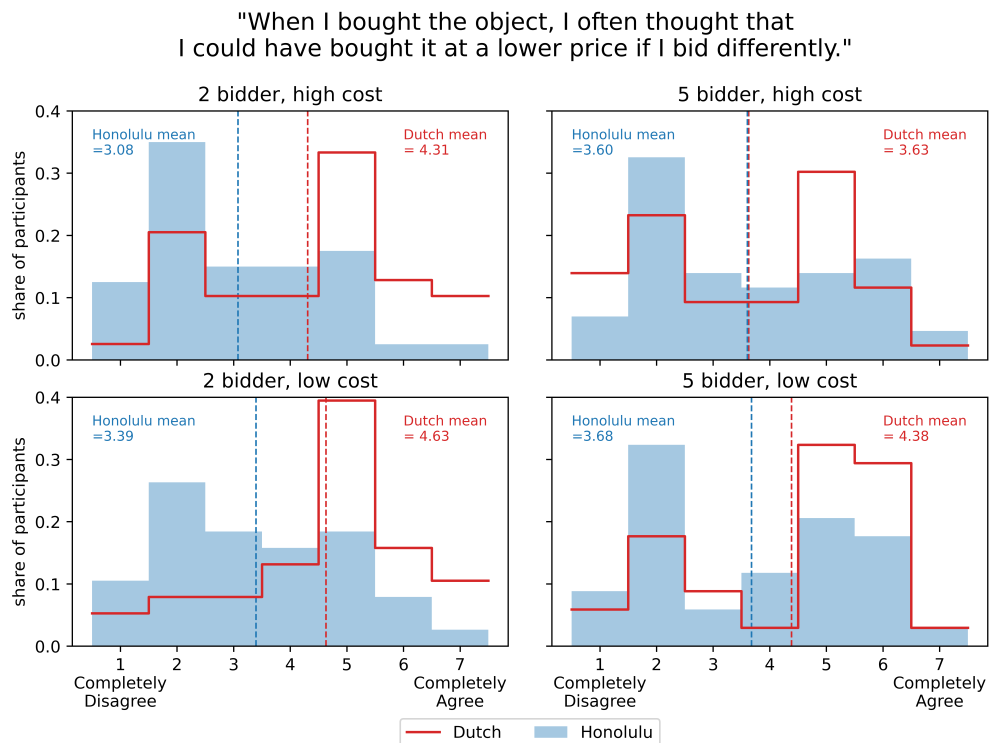


Figure 1.10: Post-auction questionnaire, Dutch vs Honolulu: Winner Regret

overwhelmingly consistent with the presence of time costs.

In our experimental results, we observe overbidding in Dutch auctions, a trend advantageous for the auctioneer. However, as time costs increase, bidding patterns in the Dutch format shift closer to risk-neutral predictions. Moreover, the Honolulu-Sydney auction consistently outperforms the Dutch in terms of bidder payoffs, and appears to reduce the feeling of both winner and loser regret. This suggests that not only does the hybrid format expedite the auction process, but it also offers better outcomes for bidders.

An unanticipated albeit not surprising insight provided by our experiments is the evidence of bidder attempts to suppress price competition in Honolulu-Sydney auctions, especially when the number of bidders is small. Bidder collusion has been studied in the context of procurement auctions (Hendricks and Porter, 1989); school milk contracts (Pesendorfer, 2000), cattle (Phillips et al., 2003) and spectrum (Kwasnica and Sherstyuk, 2007) auctions, among others. Concerns about collusion in fish markets have also been raised (Graddy, 2006; Fluvia et al., 2012). Our laboratory experiment confirms that attempts to tacitly lower prices may be present in such markets, especially given that many buyers are professionals who participate in these auctions on a day-to-day basis.

In our laboratory experiment under the hybrid Honolulu-Sydney auction format, the apparent attempts to lower prices manifest themselves through delayed bidding at the

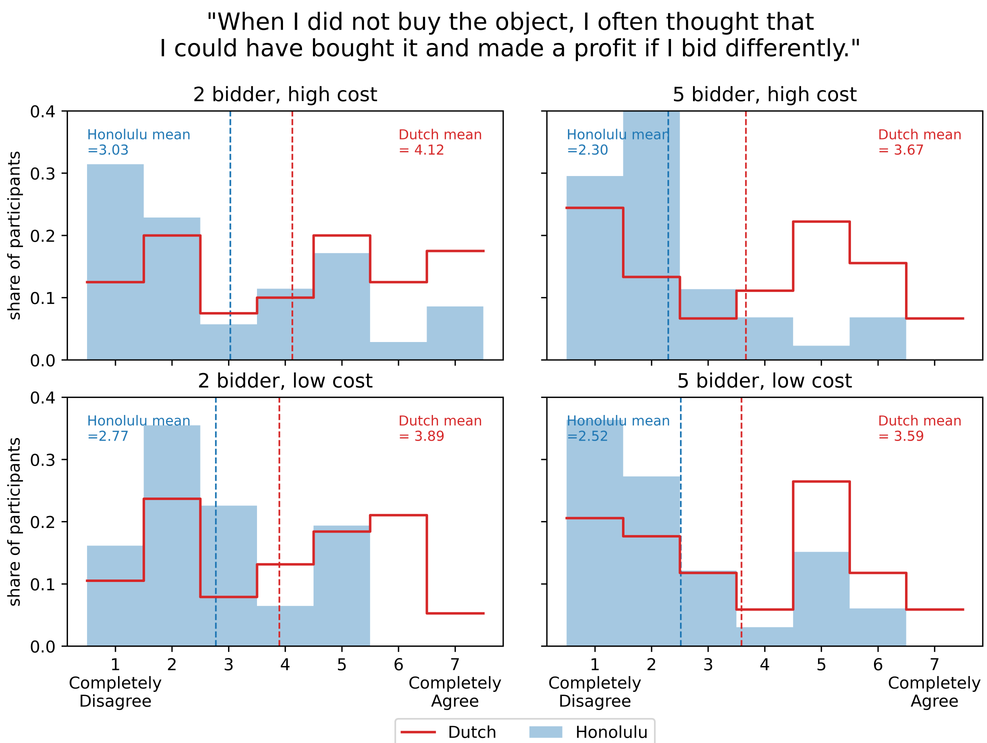


Figure 1.11: Post-auction questionnaire, Dutch vs Honolulu: Loser Regret

descending-price stage, and dropping out of the auction early at the ascending-price stage; such attempts are especially noticeable in small two-bidder auctions. However, in our experiments, these departures from competitive bidding do not increase the overall welfare of the bidders. While bidders who frequently delay their bids and drop out of the competition early suffer from lower earnings, the benefits of their actions are acquired by those bidders who behave more competitively and bid closer to the equilibrium prediction, often collecting higher than predicted earnings. It is an open question whether professional repeat buyers in fish markets could be able to better tacitly coordinate on a bid rotation scheme and distribute the benefits among themselves more evenly.

We further obtain evidence that the hybrid Honolulu-Sydney auctions result in superior allocative efficiency compared to the Dutch auctions, in spite of the observed departures from equilibrium behavior.

In sum, the Honolulu-Sydney auction represents a creative solution for markets where time is of the essence and when the auctioneer also cares about bidder satisfaction in addition to her own payoff.

Appendix

1.A Fish auctions around the world

Many fish and flower auctions around the world are characterized by large volumes of highly perishable and highly variable in quality goods that are auctioned off sequentially, by individual units or lots. Speed of sales is of the essence given large volumes; however, competitive bidding on each item is also an essential requirement for price discovery, given the large variability in quality and other characteristics of each item. Graddy (2006) specifically attributes the need for a centralized market to heterogeneity of fish; she further notes that "many... major fish markets, including Tsukiji, Sydney, Portland and Boston are auction markets" (p. 219). Because of the variability, the buyers are typically given a chance to examine each unit of the good for sale (fish or flowers) before or at the time of bidding, with the bidding process then taking only a few seconds per unit.

These auctions are organized under a variety of formats, including purely ascending, purely descending or a combination of both, with each of these arguably providing for competitive bidding and speed.[32] For example, the Tokyo (formerly Tsukiji) fish market, arguably one of the most famous fish auctions around the world, uses an ascending price auction format, where buyers, after having inspected fish before the auction, use sign language to signal their bids to the auctioneer; the auctioneer calls out the highest bid, and bidders then may sign higher bids. Cassady (1967) notes the advantage of allowing many bidders to submit bids at the same time: "The auction system is potentially very fast...It is amazing to realize how quickly the auctioneer can interpret the bids and decide which is the highest... Because speed is essential in the sale of fish, the auctioneer must knock each lot down as quickly as possible" (p. 66).

Many other auctions where large amounts of goods are auctioned have elements of descending, or Dutch, format. Aside from the famous Dutch flower auction, Cassady (1967) gives examples of descending oral auction method used "... in some other Continental, and even British, communities, in certain Middle eastern countries [such as Israel], and elsewhere" (p. 60). He further writes (p. 63): "The oral method of the Dutch auction (but not the clock method) is used mainly for the sale of nonstandardized items where quality differences require flexibility..." It is instructive that Cassady (1967) explicitly refers to the trade-off between the starting price and the speed of auction: "The auctioneer must start

[32]Plott (2017) emphasizes the role of speed and competition: "...When items are perishable (fish and flowers) reserve prices play no role... Many of auction procedures are related to the need to have the auction over quickly... Efficiency both in terms of gains from trade and in terms of "market making" time is a big driver of auction rules and procedures."

the quotation at a sufficiently high level to permit the bidder with the highest demand price to register his maximal bid. On the other hand, *he should not start too high, particularly in market situations where speed is essential* (p. 60; the emphasis is ours).

Honolulu fish auction, run by the United Fishing Agency since 1952, is claimed to be based on the Japanese fish market auction;[33] however, it is really a combination of descending (Dutch) and ascending (English) auctions. Feldman (2006) provides the following description (p. 326, footnote 39): "At the Honolulu Fish Exchange... a modified form of the Dutch auction is used, with the auctioneer starting at a high price and then dropping it until a buyer places the first bid. The auctioneer then calls out a higher price with the hope of getting other buyers to start bidding..."

Such a hybrid descending-ascending auction format is not unique to the fish auction in Honolulu, with variations of it encountered in different parts of the world. Guillotreau and Jiménez-Toribio (2006) document a similar format used in fish sales in two French ports: " In the ... port [of Lorient], offshore boats have been selling fish in a trading room with descending-ascending auctions... An opening price is proposed by the auctioneer before going down around the clock. When a buyer makes a bid the clock stops... During this signalled delay, other bidders may intervene with a higher bid, until a single buyer remains in the auction.... The second example is given by seven ports of south Brittany... One of these ports – Saint Gue´nole´ – implemented a Dutch system... Six neighboring shout auction systems – including that of Le Guilvinec – were equipped...in April 2002 by mobile ECAS [Electronic clock auction systems], with a descending-ascending bidding process, similar to the offshore fish market in Lorient..." (pp.525-6). Finally, Laksá and Marszalec (2020) provide a description of a very similar descending-then-ascending price clock auction mechanism used at the Faroe Fish market in Denmark.

The above indicates that fish and flower auctions have evolved to meet the essential criteria of combining speed and price efficiency, and that the descending-ascending auction format is a viable institution under these criteria.

1.B The Case of Linear Time-Adjustment Functions

The analysis for the general time-adjustment functions are too complex to allow for closed-form solutions; they are also not suitable for numerical solutions with the generality of the time-adjustment function. In this section, we focus on a specific type of time-adjustment

[33] "The Honolulu Fish Auction is based on the famous Tokyo auction, where large fish are sold individually rather than by the boatload to a wholesaler. The auction provides a marketing service for fisherman and helps them with quality improvement to get the best prices for their catch" (Hawaii-Seafood.org, 2015).

function for both the bidders and the auctioneer. Assume that for each bidder, the payoff shrinks linearly with time by a factor $1 - bt$, where t represents the total duration of the auction and $b \leq 1/2$ is the bidder unit time cost. Likewise, suppose that for the auctioneer, the payoff shrinks linearly with time by the factor $1 - ct$, with t denoting the total auction duration and $c \leq 1/2$ being the auctioneer unit time cost.[34]

We begin by examining the Honolulu-Sydney auction.

1.B.1 Honolulu-Sydney Auction

The auctioneer starts the auction at price $s \in [0, 1]$. Consider a bidder with value v who bids at the time when the price decreases to p, his/her expected utility can now be written as

$$EU_B^H(p; v, s) = G(p)(v - p)(1 - bs + bp) + \int_p^v (v - x)(1 - bs - bx + 2bp)\, dG(x)$$

For each v, let us denote the solution to the maximization problem

$$\max_{p \in [0,s]} EU_B^H(p; v, s)$$

by $p(v, s)$. We can easily argue that $p(v, s) \leq v$ and also by definition $p(v, s) \in [0, s]$.

The first-order condition for the maximization problem is to have the partial derivative of $EU_B^H(p; v, s)$ with respect to p is equal to 0 at $p = p(v, s)$. We have:

$$\begin{aligned}
\frac{\partial}{\partial p} EU_B^H(p; v, s) &= g(p)(v - p)(1 - bs + bp) \\
&\quad - G(p)(1 - bs + bp) \\
&\quad + G(p)(v - p)\, b \\
&\quad - (v - p)(1 - bs - bp + 2bp)\, g(p) \\
&\quad + 2b \int_p^v (v - x)\, dG(x)
\end{aligned}$$

Since the first and fourth lines cancel each other and by integration by parts in the fifth line,

[34] These constraints are not essential but they guarantee that the cost of time component of the utility remains positive for all possible durations up to the highest $t \leq 2$.

we can rewrite this as

$$\begin{aligned}\frac{\partial}{\partial p}EU_B^H\left(p;v,s\right) &= G\left(p\right)\left(vb-2bp-1+bs\right)+2b\left(\int_p^v G\left(x\right)dx-\left(v-p\right)G\left(p\right)\right)\\ &= 2b\int_p^v G\left(x\right)dx-G\left(p\right)\left(1-bs+bv\right)\end{aligned}$$

Given that $b \leq 1/2$, we can first argue that the second-order derivative is always negative. This is because

$$\frac{\partial}{\partial p}\left(\frac{\partial}{\partial p}EU_B^H\left(p;v,s\right)\right)=-2bG\left(p\right)-g\left(p\right)\left(1-bs+bv\right)$$

and $G\left(p\right),\ g\left(p\right),\ 1-bs+bv$ are all positive.

Moreover, let us denote the first-order derivative $\frac{\partial}{\partial p}EU_B^H\left(p;v,s\right)$ by

$$K\left(p;v,s\right)\equiv 2b\int_p^v G\left(x\right)dx-G\left(p\right)\left(1-bs+bv\right) \tag{1.5}$$

Then $K\left(p;v,s\right)$ is strictly decreasing in p (since second order derivative is negative) and there is a unique solution to $K\left(p;v,s\right)=0$ in $p\in\left[0,v\right]$. This is because: (i) at $p=0$, $K\left(p;v,s\right)=2b\int_0^v G\left(x\right)dx>0$, and (ii) at $p=v$, $K\left(p;v,s\right)=-G\left(v\right)\left(1-bs+bv\right)<0$.

Let us denote this unique solution p that equates $K(p;v,s)$ to 0 by $k\left(v,s\right)$. If $k\left(v,s\right)\leq s$, then it would be the price that a bidder with value v would bid. Otherwise, the bidder would bid s. Therefore, we can write the equilibrium bid of a bidder with value v as

$$p\left(v,s\right)=\min\{s,k\left(v,s\right)\},$$

which can be solved numerically.

Given that we can obtain a numerical solution for equilibrium bid functions, we can write the expected utility of the auctioneer as follows:

$$\begin{aligned}EU_A^H(s)=&\int_0^1\int_0^{p(v,s)}p(v,s)(1-c(s-p(v,s)))h(v,x)dxdv\\ &+\int_0^1\int_{p(v,s)}^{v}x(1-c(s+x-2p(v,s)))h(v,x)dxdv,\end{aligned}$$

The starting price s that maximizes the above equation will be chosen by the auctioneer and this optimal s again can be computed numerically.

Given the optimal starting price, the expected duration of the auction and the expected selling price (the auction revenue) are as follows (where the maximizer is used to perform the calculations):

$$ED^H = \int_0^1 \left(\int_0^{p(v,s)} (s - p(v,s))h(v,x)dx + \int_{p(v,s)}^{v} (s + x - 2p(v,s))h(v,x)dx \right) dv$$

$$ER_A^H = \int_0^1 \left(\int_0^{p(v,s)} p(v,s)h(v,x)dx + \int_{p(v,s)}^{v} xh(v,x)dx \right) dv$$

Lastly, interim and ex-ante expected utilities of the bidders are given by

$$EU_B^H(v) = G(p)(v - p(v,s))\left(1 - bs + bp(v,s)\right) + \int_{p(v,s)}^{v} (v - x)\left(1 - bs - bx + 2bp(v,s)\right) dG(x)$$

and

$$EU_B^H = \int_0^1 EU_B^H(v)dF(v)$$

1.B.2 Dutch Auction

With regards to the Dutch auction, with linear time-adjustment functions, we can rewrite the necessary condition for the equilibrium bid function β as follows:

$$g(v)(v - \beta(v))(1 - b(1 - \beta(v))) = \beta'(v)G(v)(1 + 2b\beta(v) - b - bv)$$

This is an ordinary differential equation for β with boundary condition $\beta(0) = 0$.

Given that we have a solution for equilibrium bids, we can write the expected utility of the auctioneer as follows:

$$EU_A^D = \int_0^1 \beta(x)(1 - c(1 - \beta(x)))dF^n(x).$$

The expected duration of the auction and the expected selling price are as follows:

$$ED^D = \int_0^1 (1 - c(1 - \beta(x)))dF^n(x)$$

$$ER_A^D = \int_0^1 \beta(x)dF^n(x)$$

Lastly, interim and ex-ante expected utilities of the bidders are given by

$$EU_B^D(v) = (v - \beta(v))(1 - b(1 - \beta(v))G(v)$$

and

$$EU_B^D = \int_0^1 EU_B^D(v)dF(v)$$

1.C Numerical results and prediction details

Note that the starting price is chosen by the auctioneer to maximize her utility, so for the expected utility for the auctioneer EU_A, the differences between Dutch and Honolulu auctions are always changing smoothly in the 2D parameter space (b, c). In fact, for given (b, c), $EU_A(s)$ is a function of the starting price $s \in [0, 1]$ such that it is first increasing, then possibly decreasing, and finally may be increasing again. So, the optimal s can either be some interior value at the peak of the increasing then decreasing inverse U-shaped curve, or simply be at the boundary when the functional value finally increases and exceeds the previous peak (see Figure 1.2 for a visual demonstration). Such a jump in the optimal starting price results in discontinuity in the relative performance of the Dutch and Honolulu-Sydney auctions in terms of expected duration, selling prices and bidder expected utility. This is the reason for a sharp change in the relative performances displayed in the top left corner of these plots.

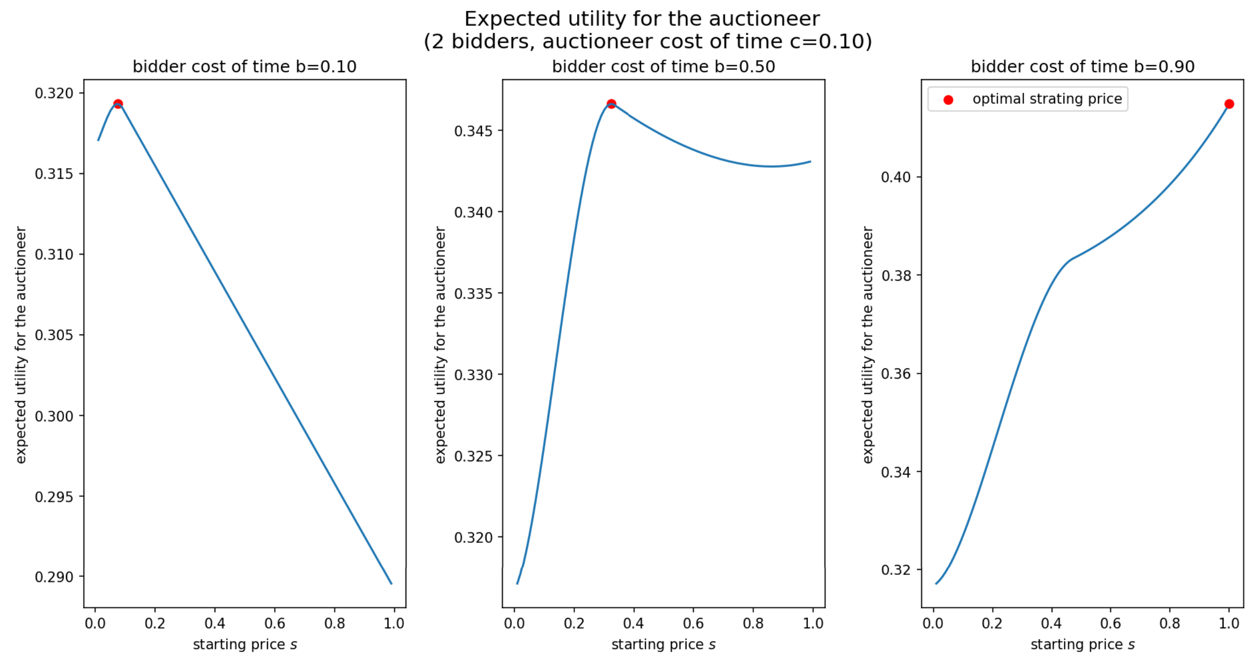


Figure 1.2: Auctioneer expected utility as a function of starting price

The performance difference between Dutch and Honolulu-Sydney auctions can take drastic changes when the optimal starting price jumps from an interior point to 1. For example (Figure 1.3), Honolulu-Sydney auctions are always shorter than Dutch under 2-bidder auctions when the optimal starting price is in the interior, while this pattern is completely reversed to Dutch auctions are always shorter when the optimal starting price jumps to 1. We mainly focus on the vast right-hand side area where the optimal starting price is an interior solution. The meaning of the words “under a wide range of parameter values” in the premise of Predictions 1– 4 is to exclude such cases which relate to the aforementioned top-left corner of those plots.

1.C.1 Theoretical performance comparison

The table below is the theoretical comparison between the two auction formats along the key auction metrics by treatment.

Table 1.2: Theoretical performance comparison re-scaled for experimental setting

Treatment	2H	2L	5H	5L
n – Number of bidders	2	2	5	5
b – Bidder unit cost of time	0.95	0.45	0.95	0.45
c – Auctioneer unit cost of time	0.95	0.95	0.95	0.95
s^* – Optimal Honolulu-Sydney auction starting price	0.4104	0.2370	0.6317	0.5405
Expected auction duration				
Dutch	0.5654	0.6382	0.3074	0.3232
Honolulu	0.2456	0.2729	0.1991	0.2582
Honolulu/Dutch ratio	43.4%	42.8%	64.8%	79.9%
Expected selling prices				
Dutch	0.4346	0.3618	0.6926	0.6768
Honolulu	0.3922	0.3526	0.6899	0.6773
Honolulu/Dutch ratio	90.2%	97.5%	99.6%	100.1%
Expected utility for the auctioneer				
Dutch	0.2245	0.1593	0.5034	0.4816
Honolulu	0.2964	0.2342	0.5581	0.5039
Honolulu/Dutch ratio	132.0%	147.0%	110.9%	104.6%
Expected utility for the bidder				
Dutch	0.0596	0.1117	0.0205	0.0270
Honolulu	0.1176	0.1452	0.0253	0.0289
Honolulu/Dutch ratio	197.3%	130.0%	123.4%	107.0%

Note: The calculations in this table are based on the value range of $[0, 1]$, as in the theoretical model setting.

1.C.2 Auction duration

The observations below are made for the wide range of parameters resulting in an interior solution of the optimal starting price, i.e., the top-left corner of the plots is excluded.

The general observation is that Honolulu-Sydney auctions are faster than Dutch auctions in most cases, which is evident from the overwhelming red color on the graphs. In all four treatments we use in the experiment, Honolulu-Sydney auctions are at least 20% faster than Dutch.

Focusing on the same configuration of parameters (b, c) in each panel of the graph for different numbers of bidders, we find that the relative advantage of Honolulu-Sydney auctions over Dutch decreases as the number of bidders increases. The dark red color on the 2-bidder graph fades, and even turns blue on 4- and 5-bidder graphs for very low bidder cost of time. Specifically, Dutch auctions are faster when the time cost of bidders $b < 0.1$ in the 4-bidder case and $b < 0.2$ in the 5-bidder case. This comparison relates to $2H$ versus $5H$ and $2L$ versus $5L$ in Table 1.2, where Honolulu-Sydney auctions are about 50% shorter than Dutch for 2 bidders while they shrink to roughly 30% shorter when the number of bidders increases

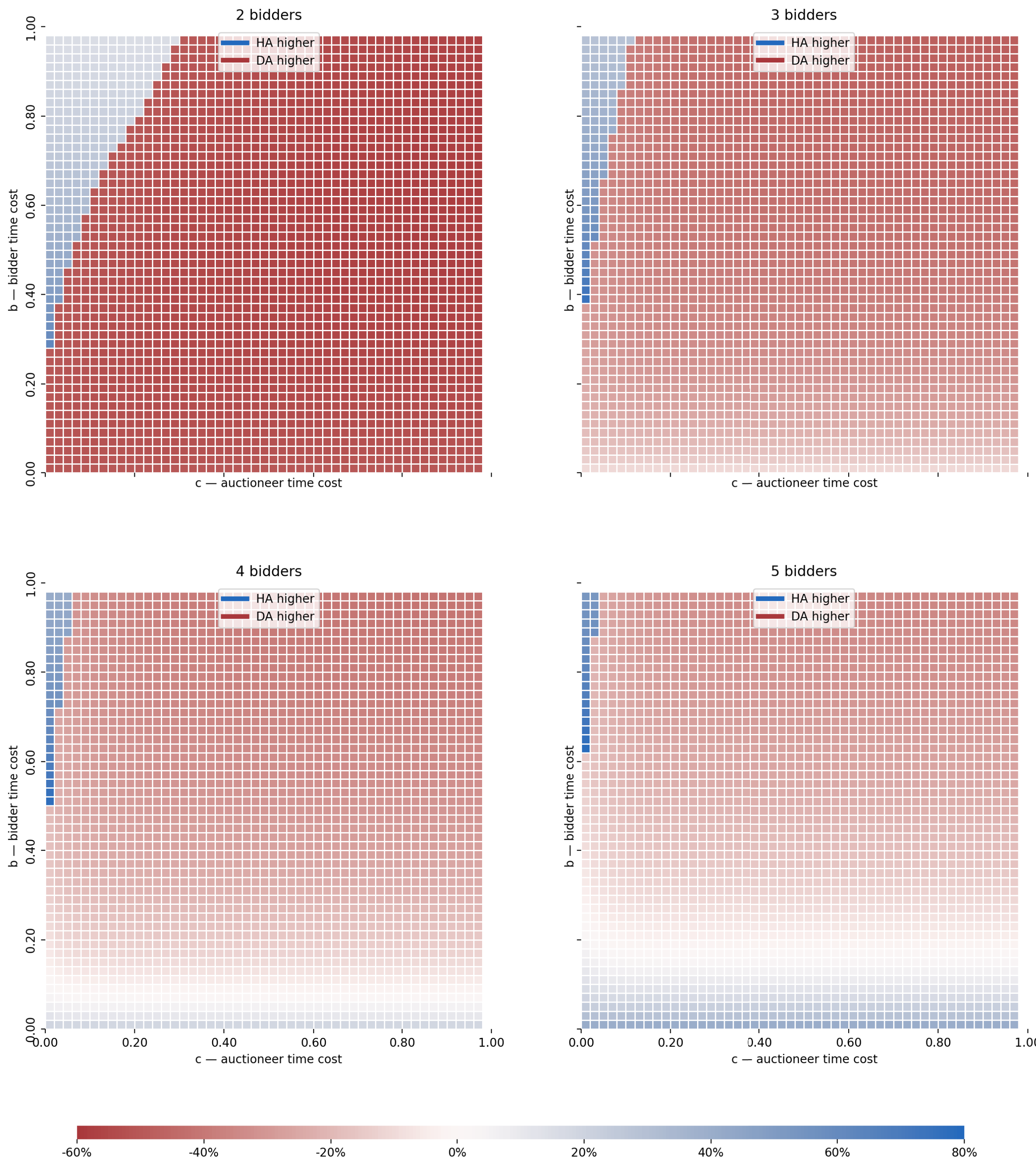


Figure 1.3: Auction duration differences, $\frac{H-D}{D}$

to 5.

For a given bidder cost of time, which is a fixed horizontal slice of the graph, the relative advantage of Honolulu-Sydney auctions over Dutch slightly increases with higher auctioneer cost of time. The greatest increment when comparing the lowest and highest auctioneer cost of time on the graph does not exceed 10% in all cases.

For a given auctioneer cost of time, which is a fixed vertical slice of the graph, the relative advantage of Honolulu-Sydney auctions over Dutch increases with higher bidder cost of time under 4- and 5-bidder cases — Honolulu-Sydney auctions are 35.2% faster than Dutch under $5H$ but only 20.1% faster under $5L$, while it first increases and then decreases under 2- and 3-bidder cases. Furthermore, this possible decrease is always small — the greatest decrement does not exceed 5% and 1% in 2- and 3-bidder auctions, respectively. Our treatment $2H$ is on the decreasing interval, and the relative time advantage of Honolulu-Sydney auctions over Dutch shrinks very little by 0.6% compared to $2L$.

1.C.3 Selling prices

The observations below are made for the wide range of parameters resulting in an interior solution of the optimal starting price, i.e., the top-left corner of the plots is excluded.

Since we assume the presence of positive time costs for the bidder and the auctioneer in the model, the revenue equivalence theorem no longer holds. However, the expected unadjusted revenue for the auctioneer, which is equivalent to the expected selling price, differs only slightly between the Dutch and Honolulu-Sydney auction formats.

This difference never exceeds 10% in either positive or negative directions, and is usually shrinks with the number of bidders. This is because the auction becomes faster as competition increases, which offsets the role of time cost.

After excluding the top-left corner of the plots, Dutch selling prices are always higher than Honolulu-Sydney for 2-bidder auctions. This advantage vanishes as the number of bidders increases. For auctions with more than 3 bidders, a light blue area where Honolulu-Sydney selling prices are higher than Dutch emerges in the left middle of the plot and is expanding as the number of bidders increases — our $5L$ treatment lies in this area, where Honolulu-Sydney expected selling prices are 0.1% higher than Dutch.

1.C.4 Utility for the auctioneer

The utility for the auctioneer is the only auction performance indicator that changes smoothly in the whole domain of parameters. Thus, we consider the entire graph for our analysis.

We find that the comparison between Dutch and Honolulu-Sydney auctions is overall indistinguishable when the time cost of the auctioneer is relatively low (less than 0.3) — it never exceeds 6%, as represented by the very light red or blue area on the left of each graph.

The remaining observations are made for the auctioneer time cost greater than 0.3.

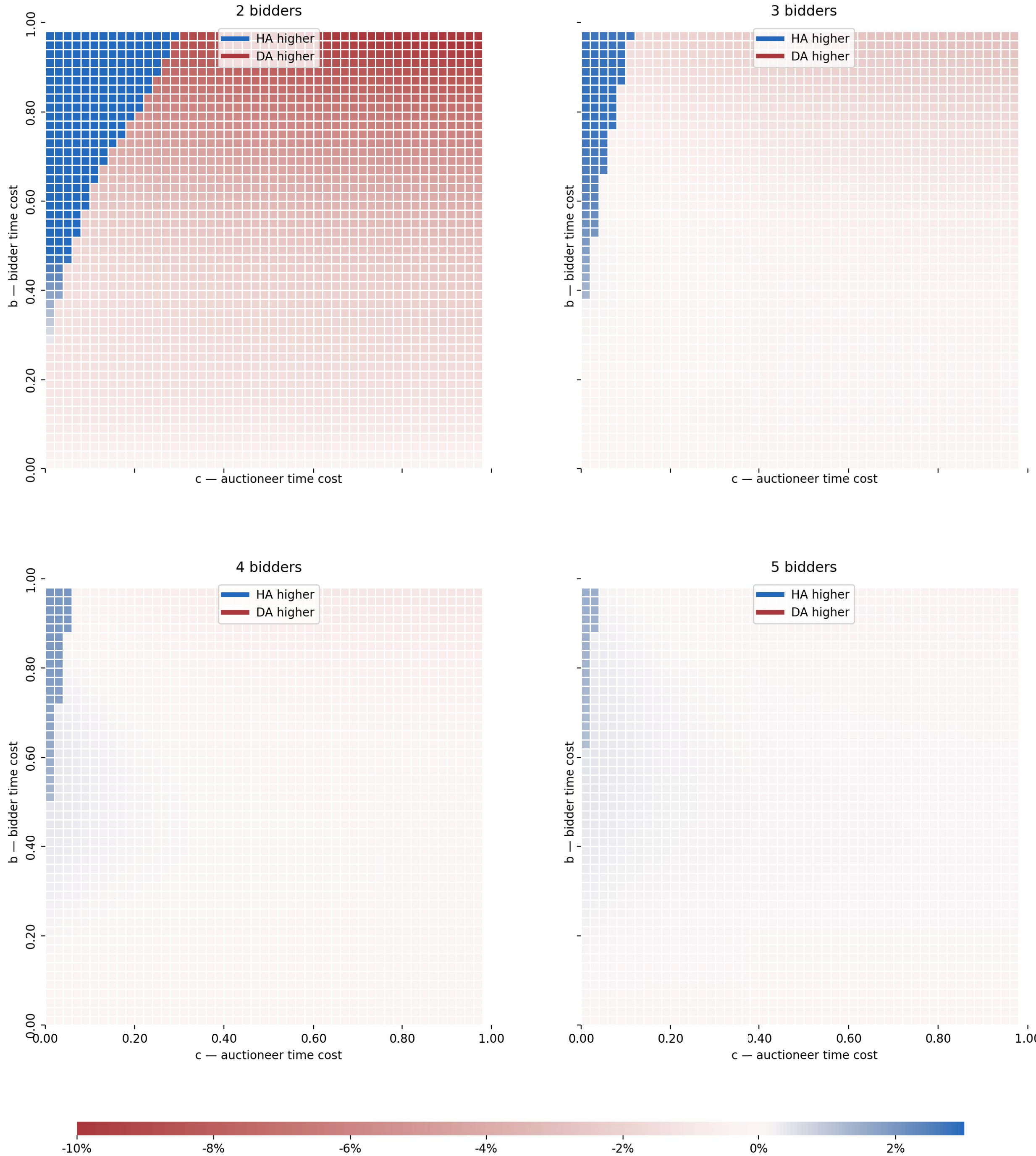


Figure 1.4: Selling price differences, $\frac{H-D}{D}$

Honolulu-Sydney auctions are always preferred by the auctioneer in the 2-bidder case, while Dutch auctions become more preferred with low bidder cost of time and when the number of bidders increases. Specifically, Dutch auctions are preferred when the bidder time cost is smaller than 0.1, 0.2 and 0.24 in 3-, 4- and 5-bidder auctions, respectively.

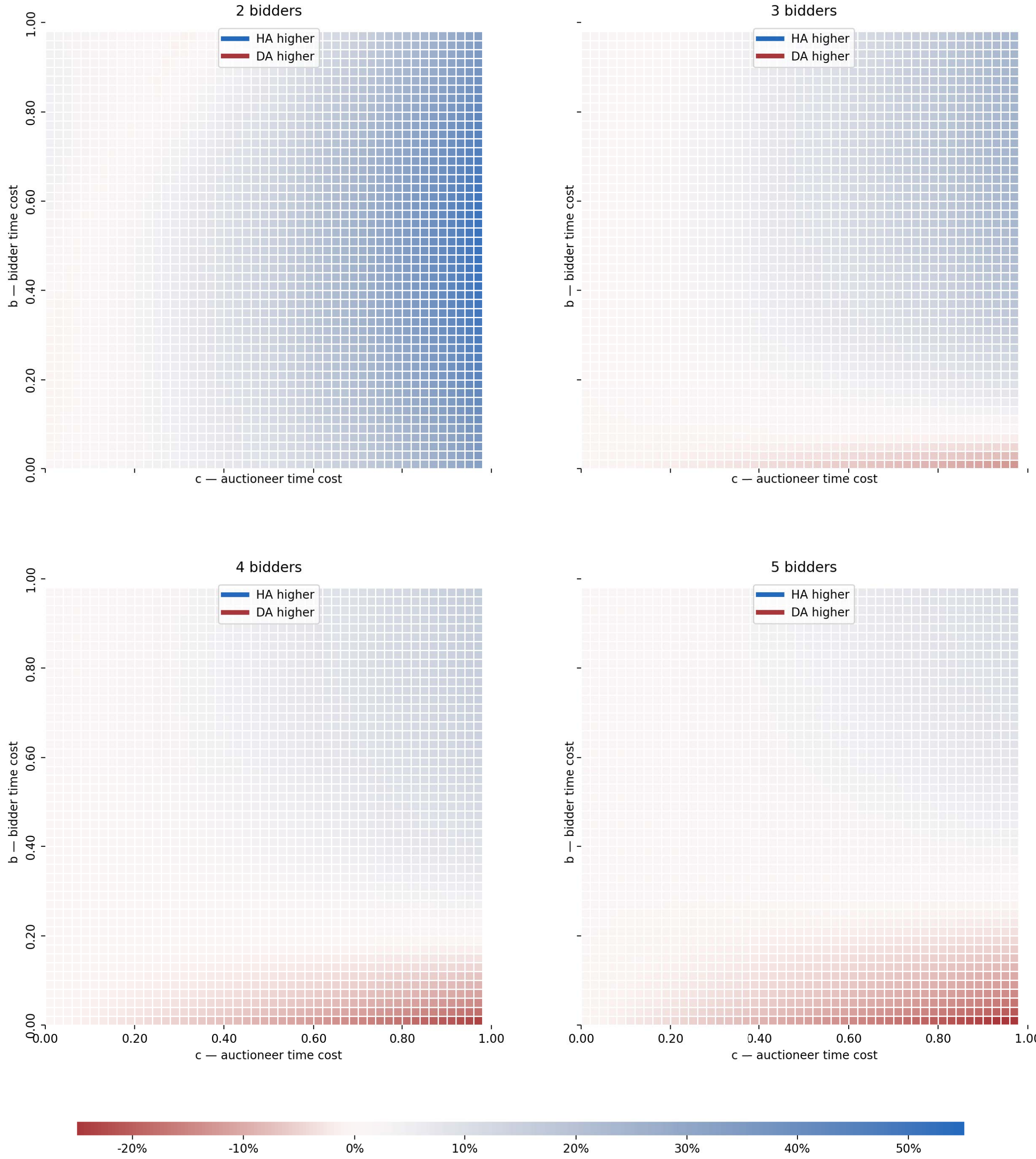


Figure 1.5: Auctioneer utility differences, $\frac{H-D}{D}$

Focusing on the same configuration of parameters (b, c) in each panel of the graph for different numbers of bidders, the relative advantage of Honolulu-Sydney auctions over Dutch usually decreases as the number of bidders increases. Furthermore, related to our experimental parameter choices (Table 1.2), this advantage is always decreasing when the auctioneer's time cost $c > 0.6$. Thus, we have this relative advantage that the Honolulu-Sydney auctions drop from more than 30% under 2-bidder treatments to around 10% under 5-bidder treatments.

For a given auctioneer cost of time, which is a fixed vertical slice of the graph, the relative advantage of Honolulu-Sydney auctions over Dutch increases with higher bidder cost of time, except for very impatient bidders whose time cost exceeds a threshold ranges from 0.5 to 0.9 depending on the number of bidders. This threshold is increasing with more bidders. Therefore, we have treatment $2L$ close to the end of the increasing interval at 0.5 where Honolulu-Sydney auctions bring 47% higher expected utility for the auctioneer than Dutch, while this advantage shrinks to 32% after moving along the decreasing interval to treatment $2H$. In contrast, treatment $5H$ is close to the threshold around 0.9, such a minor decrease makes the Honolulu-Sydney advantage of 10.9% still higher than that of 4.6% under treatment $5L$.

For a given bidder cost of time, which is a fixed horizontal slice of the graph, as the auctioneer cost of time goes up, the differences between Dutch and Honolulu-Sydney auctions become more pronounced and could go in either direction: Honolulu auctions become relatively more advantageous for sufficiently high bidder cost of time, but Dutch auctions can become relatively more advantageous for auctions with more than two bidders and low bidder cost of time (Figure 1.5) [35]

1.C.5 Utility for the bidder

The observations below are made for the wide range of parameters that result in an interior solution of the optimal starting price, which means the top left corner of the plots is excluded.

Buyers prefer Honolulu-Sydney auctions to Dutch auctions under a wide range of parameter values. Specifically, after excluding the top-left corner, Honolulu-Sydney auctions are always preferred when there are no more than 4 bidders, and are preferred in the 5-bidder case when the bidder cost of time is not too small (greater than 0.02). In our experimental settings (Table 1.2), the expected utility for the bidder is up to 100% higher in Honolulu-Sydney auctions than in Dutch auctions under different treatments.

For a given auctioneer cost of time, which is a fixed vertical slice of the graph, the darker blue color from bottom to top indicates that the advantage of Honolulu-Sydney auctions over Dutch in terms of buyer utility increases with bidder cost of time, especially for small number of bidders. The expected relative benefit for buyers in Honolulu-Sydney auctions increases from about 30% under treatment $2L$ to almost 100% under treatment $2H$.

[35] Thus we chose a high value of auctioneer cost of time for experimental design to obtain a greater expected difference between Dutch and Honolulu auctions.

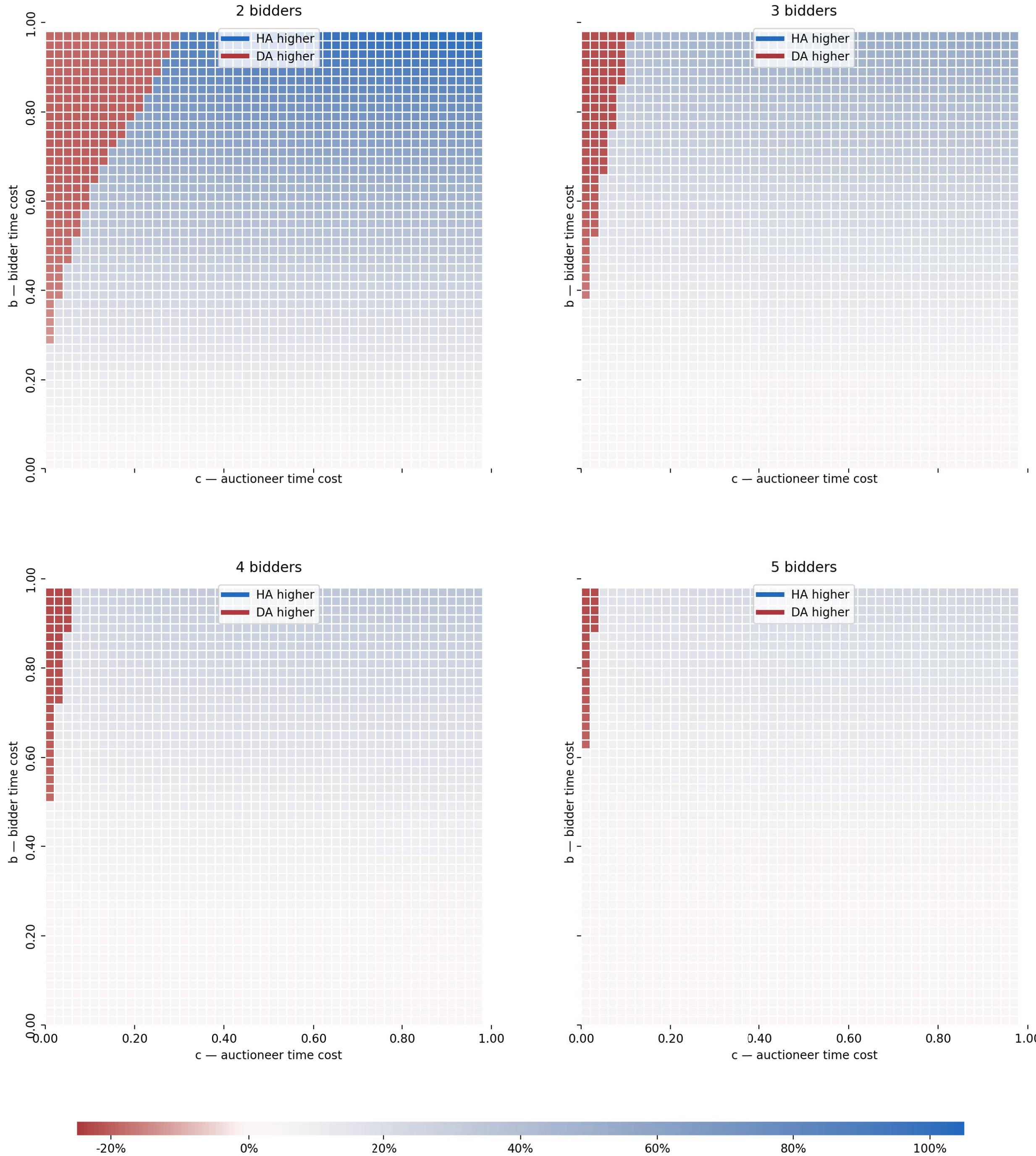


Figure 1.6: Bidder utility differences, $\frac{H-D}{D}$

Using the same configuration of the parameters (b, c) in each panel of the graph, the blue color fades as the number of bidders increases, which means that the relative advantage of Honolulu-Sydney auctions over Dutch decreases with the number of bidders. This relative advantage experiences a rapid drop from almost 100% to around 20% when increasing the number of bidders from 2 to 5 in high-cost treatments, but only a slight drop of no more than 20% in low-cost treatments.

1.C.6 Social welfare

The social welfare of an auction U_{AB} is defined as the sum of auctioneer and bidders utilities, $U_{AB} = U_A + \sum_i U_{B_i}$, and the expected social welfare EU_{AB} is given by $EU_{AB} = EU_A + \sum EU_{B_i}$. Equivalently, we may use the buyer utility to replace the sum of all bidder utilities. The numerical predictions based on our theoretical model are displayed in Figure 1.7 below, where we plot the relative difference between the two auction formats $\frac{EU^H_{AB} - EU^D_{AB}}{EU^D_{AB}}$.

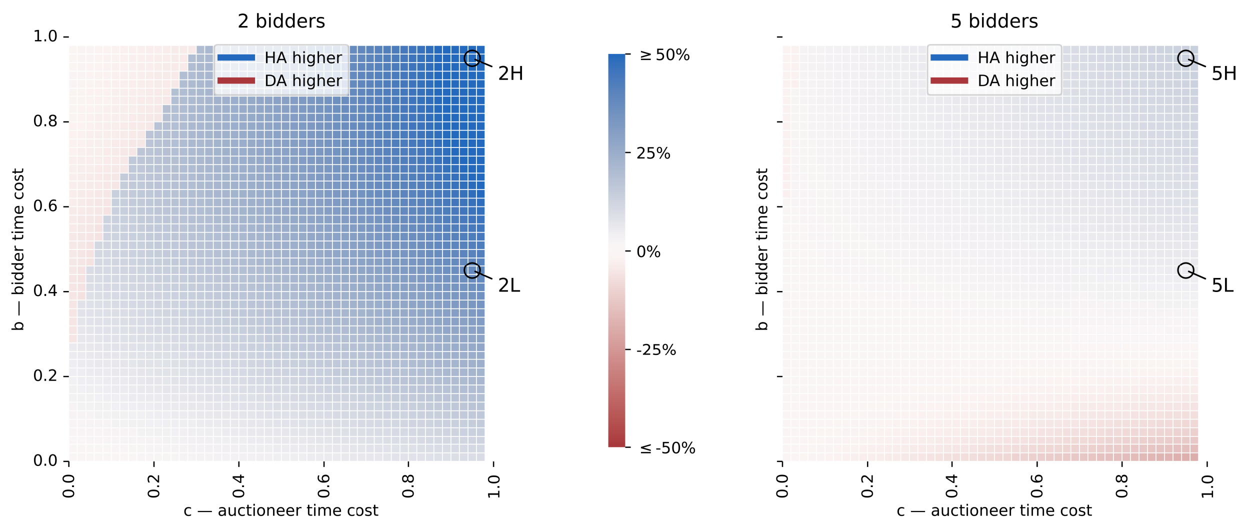


Figure 1.7: Social welfare differences, $\frac{H-D}{D}$

Predicted (correspondingly, actual) social welfare for the experimental auctions are calculated based on the actual values drawn, as the sum of predicted (correspondingly, actual) auctioneer and buyer utilities. 12 out of 2216 experimental auctions that ended without a winner are assessed zero welfare. These measures are displayed in Figure 1.8 and Table 1.3.

Table 1.3: Predicted and actual social welfare by treatment

	2H		2L		5H		5L	
	Predicted	Actual	Predicted	Actual	Predicted	Actual	Predicted	Actual
Social welfare								
Dutch	16.9	15.3	19.0	19.0	30.0	30.5	30.5	30.8
Honolulu	26.5	19.4	25.9	22.0	33.5	26.0	35.2	29.8
H/D, %	156.8%	126.8%	136.3%	115.8%	111.7%	85.2%	115.4%	96.8%

Both predicted and actual measures are calculated based on bidders values drawn.

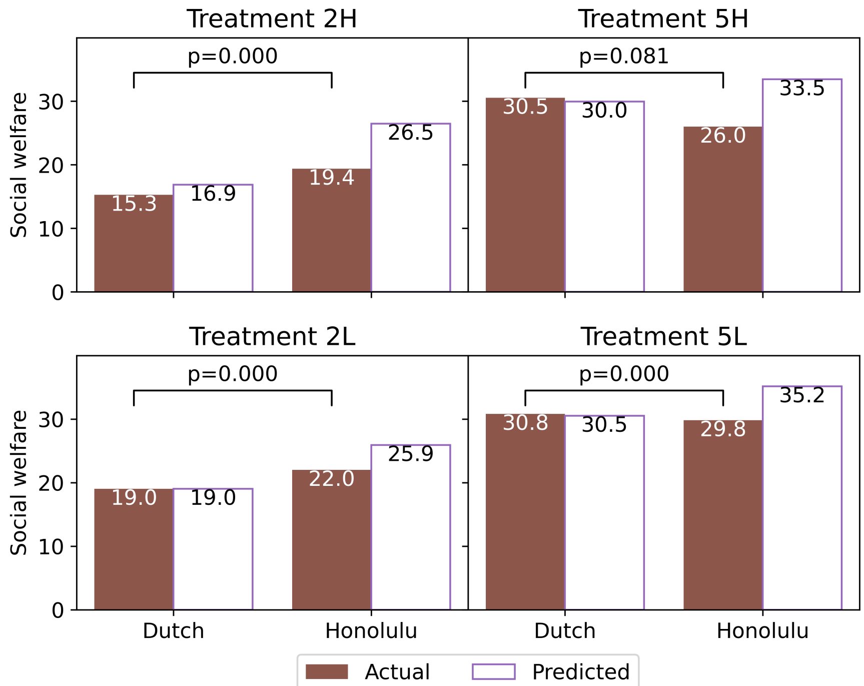


Figure 1.8: Social welfare in the experiment

The social welfare is predicted to be higher in Honolulu auctions as compared to Dutch in all experimental treatments. In the actual experiment, the welfare is significantly different (higher) in Honolulu auctions compared to Dutch in two-bidder auctions, but is not significantly different, or lower, in Honolulu as compared to Dutch in five-bidder auctions.

1.D Hypotheses tests of experimental auction comparative performances

This section provides details of hypotheses testing on comparative performances of Honolulu and Dutch auctions. The hypotheses as based on the theoretical predictions given in Remarks 1.1 - 1.2 and Prediction 1.1. These prediction and the corresponding hypotheses are listed below, with D and H denoting the corresponding performance characteristics under Dutch and Honolulu auctions, respectively:

1. (Efficiency: Remarks 1.1 - 1.2) Both auctions are efficient.

   Corresponding hypotheses

   (a) $D = H$ for 2-bidder and 5-bidder auctions, high and low cost

2. (Auction duration: Prediction 1) Honolulu-Sydney auctions are faster than Dutch auctions, i.e., their average duration is shorter. The relative advantage of Honolulu-Sydney auctions over Dutch in terms of duration decreases with the number of bidders.

   Corresponding hypotheses

   (a) $D > H$ for 2-bidder and 5-bidder auctions, high and low cost
   (b) $H^5/D^5 > H^2/D^2$, high cost
   (c) $H^5/D^5 > H^2/D^2$, low cost

3. (Selling prices: Prediction 2) The difference in average selling prices between Honolulu and Dutch auctions is small; it does not exceed 5-10 percent.

   Corresponding hypotheses

   (a) $0.9 <= H^2/D^2 <= 1.1$, for 2-bidder and 5-bidder auctions, high and low cost

   Hypotheses based on the actual value draws as given in Table 1.3:

   (a) $H^2/D^2 = 0.917$, high cost
   (b) $H^2/D^2 = 1.011$, low cost
   (c) $H^5/D^5 = 0.988$, high cost
   (d) $H^5/D^5 = 0.991$, low cost

4. (Auctioneer utility: Prediction 3 Assume the auctioneer cost of time is relatively high. Then Honolulu-Sydney auctions are always preferred to Dutch in the two-bidder case. For auctions with more than two bidders, Honolulu-Sydney auctions are preferred to Dutch when bidder cost of time is high, and Dutch auctions are preferred to Honolulu-Sydney when bidder cost of time is low.

   Corresponding hypotheses:

   (a) $H^2 > D^2$, high cost
   (b) $H^2 > D^2$, low cost
   (c) $H^5 > D^5$, high cost
   (d) $H^5 > D^5$, low cost[36]

[36] The bidder cost of time in the "low"-cost treatment is high enough so that the prediction is still $H^5 > D^5$. However, given the predicted difference is small, and based on earlier experimental evidence of over-bidding relative to the risk-neutral predictions in Dutch auctions, we may expect to observe $H^5 < D^5$ under the low cost.

5. (Buyer utility: Prediction 4) Buyers prefer Honolulu-Sydney auction to Dutch under a wide range of parameter values. For auctions with a small number of bidders, the advantage of Honolulu-Sydney auctions over Dutch in terms of buyer utility increases with bidder cost of time. The relative advantage of Honolulu-Sydney auctions over Dutch decreases with the number of bidders.

   Corresponding hypotheses:

   (a) $H > D$, high and low cost, 2 and 5 bidders

   (b) $H^h/D^h > H^l/D^l$, 2 bidders

   (c) $H^2/D^2 > H^5/D^5$, high cost

   (d) $H^2/D^2 > H^5/D^5$, low cost

To test the above hypotheses, we regressed auction efficiency, duration, selling prices, and auctioneer and buyer utilities on auction type and treatment dummies, as displayed in Tables 1.4 - 1.5 below. We then used the regression coefficients to estimate the test statistic equal to the difference between LHS and the RHS expressions in the corresponding hypotheses, $coeff \equiv (LHS - RHS)$ (e.g., $coeff \equiv H^5/D^5 - H^2/D^2$ for high cost for hypotheses 2(b) above), and bootstrapped standard errors on the statistic to test each hypotheses. The hypotheses tests results are given in Table 1.4 in the main text. Additionally, Table 1.6 in this Appendix shows the hypotheses test results for selling prices based on the actual value draws as given in Table 1.3.

Table 1.4: Baseline regression estimation for hypotheses testing, by the number of bidders

	Efficiency			Auctioneer payoff		Buyer payoff	
	pooled	2 bidders	5 bidders	2 bidders	5 bidders	2 bidders	5 bidders
REGRESSION							
HNL auction	1.54**	1.32	3.02	-3.14***	-7.30**	6.10***	6.30**
	(0.52)	(0.97)	(1.89)	(0.54)	(2.22)	(0.68)	(1.95)
high cost		-0.62	3.12*	-0.19	-2.04	-3.55***	1.78
		(1.90)	(1.60)	(1.02)	(2.58)	(0.69)	(1.45)
high cost HNL		0.60	-3.17	2.05*	1.70	-0.95	-5.25**
		(1.09)	(2.01)	(0.99)	(2.95)	(1.05)	(2.14)
Constant	95.85***	95.44***	95.48***	11.01***	29.30***	8.10***	1.50
	(0.72)	(1.31)	(1.57)	(0.31)	(2.52)	(0.43)	(1.40)
Observations	2216	1404	812	1404	812	1392	812
R^2	0.003	0.003	0.020	0.029	0.141	0.144	0.106
PREDICTION							
H^h – HNL high cost		96.74	98.45	9.73	21.66	9.69	4.33
H^l – HNL low cost		96.76	98.49	7.86	22.01	14.20	7.80
D^h – Dutch high cost		94.82	98.60	10.82	27.27	4.54	3.28
D^l – Dutch low cost		95.44	95.48	11.01	29.30	8.10	1.50

OLS estimation. Standard errors clustered by session in parentheses.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 1.5: Baseline regression estimation for hypotheses testing, by bidder cost of time

	Auction duration		Selling price		Buyer payoff	
	high cost	low cost	high cost	low cost	high cost	low cost
REGRESSION						
HNL auction	-9.26***	-9.85***	-3.61***	-5.47***	5.15***	6.10***
	(1.81)	(0.99)	(0.71)	(1.06)	(0.80)	(0.69)
5 bidders	-15.60***	-16.48***	15.62***	16.49***	-1.26*	-6.59***
	(1.09)	(1.84)	(1.11)	(1.83)	(0.65)	(1.47)
5 bidders HNL	13.34***	12.35***	0.73	0.10	-4.10**	0.20
	(3.62)	(1.45)	(0.78)	(2.34)	(1.19)	(2.07)
Constant	29.90***	29.49***	20.63***	21.07***	4.54***	8.10***
	(1.01)	(0.50)	(1.03)	(0.51)	(0.54)	(0.43)
Observations	1140	1076	1140	1076	1133	1071
R^2	0.239	0.310	0.421	0.444	0.133	0.199
PREDICTION						
H^5 – HNL 5 bidders	18.38	15.50	33.37	32.30	4.33	7.80
H^2 – HNL 2 bidders	20.64	19.63	17.02	15.61	9.69	14.20
D^5 – Dutch 5 bidders	14.30	13.00	36.26	37.57	3.28	1.50
D^2 – Dutch 2 bidders	29.90	29.49	20.63	21.07	4.54	8.10

OLS estimation. Standard errors clustered by session in parentheses.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 1.6: Hypotheses tests of predictions on relative prices in Honolulu and Dutch auctions based on actual values drawn

Characteristic	Prediction-based hypotheses*	cost	Observed coeff.	Bootstr. std. err.	p-value	Prediction supported?
Selling price	$H^2/D^2 = 0.917$	high cost	-.092	.024	0.000	no
	$H^2/D^2 = 1.011$	low cost	-.260	.040	0.000	no
	$H^5/D^5 = 0.988$	high cost	-.068	.007	0.000	no
	$H^5/D^5 = 0.991$	low cost	-.134	.045	0.003	no

* H – Honolulu, D Dutch; 2 – 2-bidders, 5 – 5-bidders. Hypotheses for the selling price ratios are based on Table 1.3 predictions. Observed coefficients are for the difference between the LHS and the RHS expressions in the corresponding hypotheses.

1.E Additional tables

Table 1.7: Wilcoxon signed-rank tests on price dynamics in Honolulu auctions

Treatment	Alternative hypothesis	Price dynamics	Statistic	p-value	# of Obs*
All treatments	Actual > Predicted	Dutch then English	119.0	0.000	15
	Actual < Predicted	Dutch only	22.5	0.030	
		English only	0.0	0.000	
	Actual ≠ Predicted	Dutch then English	1.0	0.000	
		Dutch only	12.5	0.059	
		English only	0.0	0.001	
2-bidder	Actual > Predicted	Dutch then English	36.0	0.004	8
	Actual < Predicted	Dutch only	11.0	0.191	
		English only	0.0	0.004	
	Actual ≠ Predicted	Dutch then English	0.0	0.008	
		Dutch only	11.0	0.383	
		English only	0.0	0.008	
2H	Actual > Predicted	Dutch then English	10.0	0.063	4
	Actual < Predicted	Dutch only	0.0	0.063	
		English only	0.0	0.063	
	Actual ≠ Predicted	Either	0.0	0.125	
2L	Actual > Predicted	Dutch then English	10.0	0.063	4
		Dutch only	8.5	0.188	
	Actual < Predicted	English only	0.0	0.063	
	Actual ≠ Predicted	Dutch then English	0.0	0.125	
		Dutch only	1.5	0.375	
		English only	0.0	0.125	
5-bidder	Actual > Predicted	Dutch then English	27.0	0.016	7
	Actual < Predicted	Dutch only	2.5	0.046	
		English only	0.0	0.014	
	Actual ≠ Predicted	Dutch then English	1.0	0.031	
		Dutch only	2.5	0.093	
		English only	0.0	0.028	
5H	Actual > Predicted	Dutch then English	9.0	0.125	4
	Actual < Predicted	Dutch only	1.0	0.143	
		English only	0.0	0.054	
	Actual ≠ Predicted	Dutch then English	1.0	0.250	
		Dutch only	1.0	0.285	
		English only	0.0	0.109	
5L	Actual > Predicted	Dutch then English	6.0	0.125	3
	Actual < Predicted	Dutch only	0.0	0.125	
		English only	0.0	0.125	
	Actual ≠ Predicted	Either	0.0	0.250	

*The tests compare actual and predicted frequencies, with session averages as units of observation.

Table 1.8: Frequencies of strictly and weakly dominated decisions, by treatment

		2H		2L		5H		5L	
Stage	Decision	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom
DUTCH AUCTION									
	Bid above value	0	9	0	1	0	2	0	15
		0.0%	5.5%	0.0%	0.6%	0.0%	2.2%	0.0%	15.3%
	Undominated bid	189	156	167	170	117	91	98	83
		100.0%	94.5%	100.0%	99.4%	100.0%	97.8%	100.0%	84.7%
HONOLULU AUCTION									
Dutch stage	Bid above value	0	9	0	1	1	3	1	5
		0.0%	5.9%	0.0%	0.6%	1.0%	2.7%	1.0%	5.2%
	Undominated bid	181	144	147	165	96	110	98	92
		100.0%	94.1%	100.0%	99.4%	99.0%	97.3%	99.0%	94.8%
Contest stage	Leave below value	7	12	12	12	7	12	6	11
		10.8%	24.5%	29.3%	32.4%	3.0%	4.8%	2.7%	4.8%
	Undominated leave	58	37	29	25	229	239	214	220
		89.2%	75.5%	70.7%	67.6%	97.0%	95.2%	97.3%	95.2%
	Bid above value[2]	0	6	0	0	5	24	1	1
		0.0%	3.8%	0.0%	0.0%	3.2%	12.1%	0.6%	0.6%
	Undominated bid	114	152	154	139	150	174	156	173
		100.0%	96.2%	100.0%	100.0%	96.8%	87.9%	99.4%	99.4%
English stage	Leave below value	16	41	26	29	13	23	8	27
		7.1%	15.2%	9.8%	10.9%	5.5%	7.7%	3.3%	10.4%
	Stay above value	5	22	3	12	7	45	10	25
		2.2%	8.2%	1.1%	4.5%	2.9%	15.1%	4.1%	9.6%
	Undominated decision	204	206	235	225	218	230	226	208
		90.7%	75.5%	89.0%	84.6%	91.6%	77.2%	92.6%	80.0%
Final outcome	Leave below value -all	23	53	38	41	20	35	14	38
		6.4%	14.7%	11.1%	12.0%	4.1%	6.2%	2.9%	7.5%
	- foregone purchase[3]	*23*	*53*	*38*	*41*	*13*	*21*	*7*	*24*
		6.4%	*14.7%*	*11.1%*	*12.0%*	*2.7%*	*3.7%*	*1.5%*	*4.8%*
	Bid above value -all	5	25	3	13	7	45	10	25
		1.4%	6.9%	0.9%	3.8%	1.4%	8.0%	2.1%	5.0%
	- actual loss[3]	*0*	*11*	*1*	*6*	*0*	*6*	*1*	*6*
		0.0%	*3.1%*	*0.3%*	*1.8%*	*0.0%*	*1.1%*	*0.2%*	*1.2%*
	Undominated decision	332	282	301	288	461	482	452	441
		92.2%	78.3%	88.0%	84.2%	94.5%	85.8%	95.0%	87.5%

[1] The table lists the number of decisions in each category, and their percentage out of all decisions at the corresponding auction stage. Decisions within 2 points of bidder value are considered to be "at value."

[2] Dutch stage provisional winners are excluded from the Contest stage decisions.

[3] Decisions in the corresponding category where the final auction price was below value (for foregone purchases), or the purchase was made at a price above value (for actual losses).

Table 1.9: Wilcoxon signed-rank tests on the share of undominated decisions for Top and Bottom earners

Treatment	Hypothesis	Type of decision	Statistic	p-value	# of Obs*
All treatments	Top > Bottom	Dutch auction bid	45.0	0.004	15
		Dutch stage bid	19.0	0.037	
		Contest stage leave	89.0	0.054	
		Contest stage bid	36.0	0.055	
		English stage leave	110.0	0.001	
		Honolulu auction final outcome	113.0	0.001	
2-bidder	Top > Bottom	Dutch auction bid	10.0	0.034	8
		Dutch stage bid	6.0	0.054	
		Contest stage leave	27.0	0.125	
		Contest stage bid	6.0	0.054	
		English stage leave	32.0	0.027	
		Honolulu auction final outcome	33.0	0.020	
2H	Top > Bottom	Dutch auction bid	6.0	0.054	4
		Dutch stage bid	3.0	0.090	
		Contest stage leave	8.0	0.188	
		Contest stage bid	6.0	0.054	
		English stage leave	10.0	0.063	
		Honolulu auction final outcome	10.0	0.063	
2L	Top > Bottom	Dutch auction bid	1.0	0.159	4
		Dutch stage bid	1.0	0.159	
		Contest stage leave	7.0	0.322	
		Contest stage bid	Not applicable**		
		English stage leave	6.0	0.438	
		Honolulu auction final outcome	7.0	0.313	
5-bidder	Top > Bottom	Dutch auction bid	10.0	0.021	7
		Dutch stage bid	5.0	0.143	
		Contest stage leave	20.0	0.188	
		Contest stage bid	14.0	0.232	
		English stage leave	26.0	0.023	
		Honolulu auction final outcome	27.0	0.016	
5H	Top > Bottom	Dutch auction bid	3.0	0.079	4
		Dutch stage bid	2.0	0.327	
		Contest stage leave	6.0	0.438	
		Contest stage bid	8.0	0.188	
		English stage leave	10.0	0.063	
		Honolulu auction final outcome	10.0	0.063	
5L	Top > Bottom	Dutch auction bid	6.0	0.125	3
		Dutch stage bid	1.0	0.159	
		Contest stage leave	6.0	0.125	
		Contest stage bid	0.0	0.910	
		English stage leave	4.0	0.375	
		Honolulu auction final outcome	5.0	0.250	

*The tests compare the share of undominated decisions for top and bottom earners in each session.

**For the contest stage bid decisions under treatment $2L$, all decisions are undominated, so the test is not applicable.

Table 1.10: Dutch auction bids regression estimation

	2-bidder auctions			5-bidder auctions		
Earning Category	Theory	Bottom	Top	Theory	Bottom	Top
item value	0.56***	0.63***	0.59***	0.81***	0.21***	0.88***
	(0.00)	(0.05)	(0.05)	(0.00)	(0.08)	(0.06)
item value high cost	0.11***	-0.11	0.00	0.02***	0.55***	-0.10
	(0.00)	(0.09)	(0.07)	(0.00)	(0.09)	(0.07)
high cost	0.06***	3.18	-0.53	0.04***	-25.58***	4.16
	(0.01)	(3.88)	(1.86)	(0.00)	(4.55)	(2.67)
Constant	-0.41***	2.63***	0.72	-0.10***	31.71***	-0.55
	(0.01)	(0.86)	(0.69)	(0.00)	(4.12)	(2.46)
Observations	1404	336	356	2030	191	215
Adjusted R^2	1.000	0.484	0.614	1.000	0.436	0.845

OLS estimation. Bootstrapped standard errors clustered by session in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The joint hypothesis that the coefficients on "item value high cost" and "high cost" are both equal to zero cannot be rejected for either Bottom or Top earners ($p > 0.1$) in both 2-bidder and 5-bidder auctions, except for 5L bottom earners. Hence, we cannot reject the hypothesis that bids as a function of value are the same in high-cost and low-cost cases, for both 2-bidder and 5-bidder auctions.

Table 1.11: Honolulu auctions Dutch stage bids regression estimation

Treatment	2L		2H		5L		5H	
	Theory	Actual	Theory	Actual	Theory	Actual	Theory	Actual
item value	0.30***	0.03	0.50***	0.06*	0.70***	0.20***	0.71***	0.41***
	(0.00)	(0.03)	(0.00)	(0.04)	(0.00)	(0.03)	(0.00)	(0.04)
item value Top earner		0.04*		0.24**		0.00		-0.03
		(0.03)		(0.10)		(0.02)		(0.08)
Top earner		-0.94		-4.41		0.15		2.53
		(1.60)		(3.28)		(1.31)		(4.43)
Constant	-2.05***	3.55***	-2.43***	6.31***	-1.10***	15.17***	-1.12***	8.99***
	(0.06)	(1.27)	(0.05)	(2.08)	(0.02)	(1.60)	(0.01)	(2.78)
Observations	684	313	720	334	980	196	1050	210
Pseudo R^2	0.643	0.014	0.723	0.038	1.004	0.064	1.040	0.055

Tobit estimation, with bid upper bound for each treatment censored at the corresponding starting price.
Bootstrapped standard errors clustered by session in parentheses.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 1.12: English stage dropout price regression estimation

	2-bidder auctions			5-bidder auctions		
Earning Category	All	Bottom	Top	All	Bottom	Top
item value	0.76***	0.67***	0.86***	0.67***	0.57***	0.79***
	(0.04)	(0.07)	(0.04)	(0.06)	(0.12)	(0.03)
item value high cost	-0.12	-0.13	-0.03	-0.04	0.03	-0.08
	(0.12)	(0.13)	(0.11)	(0.12)	(0.19)	(0.10)
high cost	2.75	3.00	1.00	2.09	0.63	2.71
	(2.03)	(2.48)	(2.05)	(4.52)	(6.91)	(3.14)
Constant	2.78***	3.98***	1.53*	10.42***	12.98***	7.17***
	(0.48)	(1.07)	(0.87)	(2.19)	(3.86)	(0.91)
Observations	512	270	242	689	380	309
R^2	0.656	0.522	0.856	0.610	0.539	0.739

Linear regression. Bootstrapped standard errors clustered by session in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 1.13: Post-auction survey responses summary

Survey Question	Honolulu mean score μ_H	Dutch mean score μ_D	p-value* for $H_0 : \mu_H = \mu_D$
1. "When I bid, all I cared about was buying as fast as possible"	2.86	3.27	0.025
2. "When I bid, all I cared about was paying the lowest price possible"	4.85	4.07	0.000
3. "When I bought the object, I often thought that I could have bought it at a lower price if I bid differently"	3.43	4.21	0.002
4. "When I did not buy the object, I often thought that I could have bought it and made a profit if I bid differently"	2.63	3.82	0.000
5. "Indicate how you felt when you bought an object in the auction"	5.57	5.51	0.115
6. "Indicate how you felt when you did not buy an object in the auction"	3.88	3.59	0.001

For Questions 1 – 4, the scales are from 1 (Completely Disagree) to 7 (Completely Agree).
For Questions 5 – 6, the scales are from 1 (Extremely Sad) to 7 (Extremely Happy).
* Based on bootstrapped standard errors clustered on session.

1.F Auction duration with zero-cost prediction

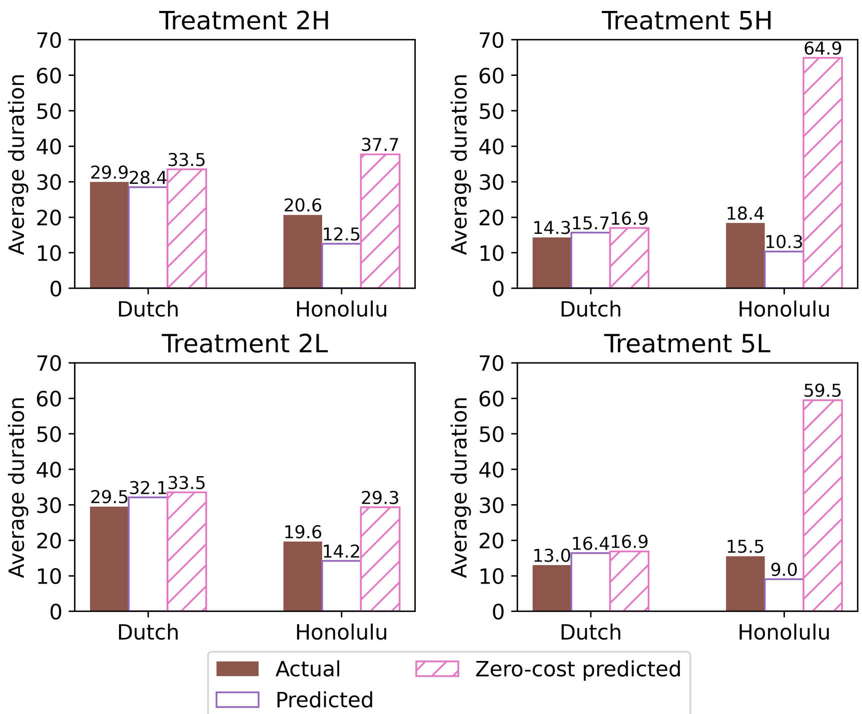


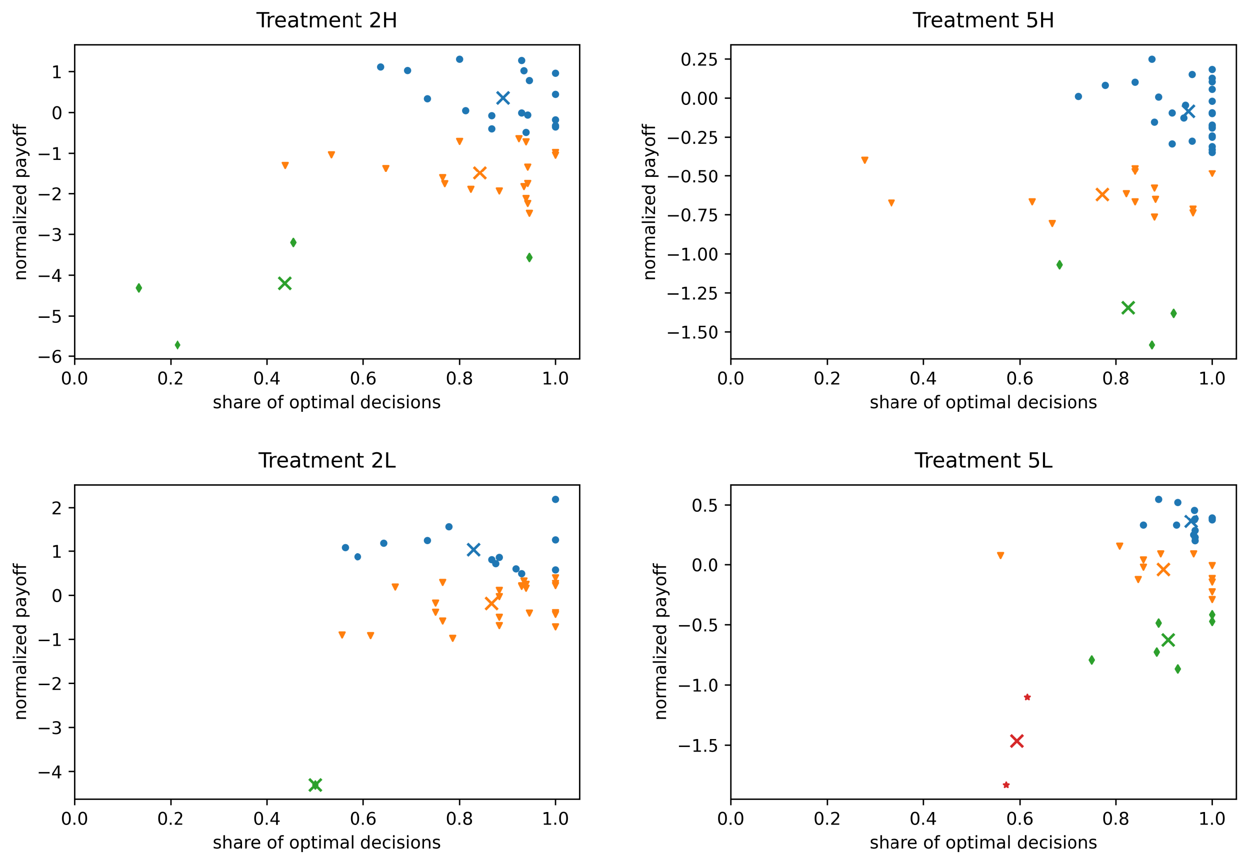
Figure 1.9: Auction duration by treatment

1.G Bidder heterogeneity: clustering based on similarity

We apply a commonly used clustering based on similarity method called K-means algorithm (Lloyd, 1982) to study bidder heterogeneity in Honolulu-Sydney auctions. We characterize each bidder on two aspects, her normalized payoff, and the optimality of her strategy.

For the normalized payoff, we use the alternative measure mentioned in footnote 23 in Section 5.2, which is the average per period point deviations from the theoretical prediction by period. We avoid censoring of extreme values by using this alternative measure, at the cost of losing cross-treatment comparability. We argue that the benefit of this alternate measure outweighs the cost, because the exact position of each data point affects the global clustering result in K-means algorithm; censoring extreme values would distort the optimally chosen clusters.

For the optimality of bidder strategy, we focus on Contest or English stage decisions, and exclude the individual observations of the Dutch-stage winners who are uncontested and become the auction winner without the follow-up English stage.[37] A bidder's joint decision at the Contest and/or English stage is considered optimal if one of the following three conditions is satisfied: (1) She drops out on the Contest stage when the contest price is higher than her item value; (2) She enters the English stage but drops out at the price equal to her item value; (3) She enters the English stage and wins the item at a price equal to or below her item value. (Decisions within two unit from the value are considered at value.) We measure the optimality of a bidder's strategy by the share of optimal decisions in all of her decisions.



Note: Each dot represents one individual. The dots in the same shape belong to the same cluster, and the clustering center is marked with a cross.

Figure 1.10: Clustering based on similarities in bidder characteristics in Honolulu auctions: normalized payoffs and individual decisions at the Contest and English stages

Figure 1.10 shows the clustering results by treatment. In all treatments except 2-bidder

[37] Applying K-means clustering method to individual decisions for each stage revealed that the optimality of Dutch-stage decisions has a smaller association with individual payoffs than their decisions at later stages. We therefore exclude the cases when the auction ended with an uncontested Dutch-stage bid. In total, 6.4% (219 out of 3434 observations) of all decisions are excluded.

low-cost (2L), the cluster with the highest average normalized payoff has also the highest optimal decision rate. The average share of optimal decisions in the top-earning cluster exceeds 95% in 5-bidder auctions, and it exceeds 90% in 2-bidder high-cost (2H) auctions. The share of optimal decisions in these three treatments generally decreases when moving from the cluster with the highest average normalized payoff to the one with the lowest payoff. There is a 18% − 40% gap in the optimal decision rate between the top-earning cluster and bottom-earning cluster. Thus, the clustering result provides evidence for bidder heterogeneity: Top earners employ closer to theoretical optimal strategies, except for the treatment 2L. For the 2-bidder low-cost treatment, clustering based on similarity does not provide much information about the relationship between earnings and optimality of bidding strategies. One possible explanation is the presence of suppressed price competition.

EXPERIMENTAL MATERIALS

1.A Experimental Instructions

[The sample instructions are for 2H treatment. For 5-bidder ("5") or low-cost ("L") treatments, the number of other bidders is changed from **1** to **4**, and the payoff percent adjustment per tick of the virtual clock is changed from "**1.9**" to "**0.9**" (as indicated in the parentheses). The text in the square brackets is not given to participants.]

Introduction

In this part you are going to participate in a sequence of independent auction periods in which you will be buying a fictitious good. At the beginning of each auction, you will be assigned to a market with **1** (**4**) other participants. You will not be told which of the other participants are in your market. *What happens in your market has no effect on the participants in other markets and vice versa.*

In each auction period you will be able to place bids in an auction to purchase a single unit of a good.

Values, Auction Duration and Earnings

Values and Earnings During each market period you are free to purchase a unit of a good in an auction. You will be assigned a number which describes the value of the good for you. These values may differ among bidders and market periods. Each bidder will receive a value between **0** and **50**, with each number being equally likely. *You are not to reveal your value to anyone. It is your own private information.*

Your earnings from the purchase, which are yours to keep, will depend on the difference between your value for the good and the price you pay, and on how long the auction lasts.

Your raw (unadjusted) earnings or loss from the purchase is equal to the difference between your value and the purchase price: That is:

YOUR UNADJUSTED EARNINGS = YOUR VALUE − PURCHASE PRICE.

Example 1 (All values are hypothetical). If you buy a good for 15 points and your value is 32, then your unadjusted earnings are:

$$\text{UNADJUSTED EARNINGS} = 32 - 15 = 17 \text{ points.}$$

Example 2 If you buy the good for 50 points and your value is 32, then your unadjusted earnings are:

$$\text{UNADJUSTED EARNINGS} = 32 - 50 = -18 \text{ points.}$$

ARE THERE ANY QUESTIONS?

Auction Duration and Earnings *Your earnings will further decrease in proportion to how long the auction lasts.* The auction duration will be measured by the virtual clock, which will tick every **1** seconds while the auction is open. Your unadjusted earnings, if positive, will shrink by **1.9** (**0.9**) percent with every tick of the virtual clock; your loss, if negative, will increase by **1.9** (**0.9**) percent with every tick of the virtual clock. That is, your time-adjusted earnings will be equal to

$$\text{YOUR EARNINGS} = (\text{YOUR VALUE} - \text{PURCHASE PRICE}) \times (\text{TIME-ADJUSTMENT FACTOR}),$$

where the time-adjustment factor decreases with every tick of the virtual clock for positive earnings, and increases with every tick of the virtual clock for negative earnings. The table below illustrates how your earnings will be adjusted depending on the auction duration:[38]

Elapsed time, in ticks of the virtual clock	Time-adjustment factor for positive earnings	Time-adjustment factor for negative earnings
0	1	1
1	0.981	1.019
2	0.962	1.038
5	0.905	1.095
10	0.81	1.19
20	0.62	1.38
50	0.05	1.95

Example 1 continued (All values are hypothetical). Suppose that your value for a good is 32, and you buy the good for 15. Suppose the time-adjustment factor is as given above. If the auction ends after 6 ticks of the virtual clock, then your positive earnings will shrink to 88.6 percent, and your time-adjusted earnings will be:

$$\text{TIME-ADJUSTED EARNINGS} = (32 - 15) \times 0.886 = 15 \text{ points.}$$

If, on the other hand, the auction ends after 34 ticks, your earnings will shrink to 35.4 percent by the end of the auction, and your time-adjusted earnings will be:

$$\text{TIME-ADJUSTED EARNINGS} = (32 - 15) \times 0.354 = 6 \text{ points.}$$

Example 2 continued Suppose, as in Example 2 above, that your value is 32, you buy the good for 50, and the time-adjustment factor is as given. If the auction ends after 6 ticks of the virtual clock, then your loss from the purchase will grow in magnitude by 11.4 percent, and you adjusted earnings (loss) will be:

$$\text{TIME-ADJUSTED EARNINGS} = (32 - 50) \times 1.114 = -20 \text{ points.}$$

If, on the other hand, your value and the purchase price are the same as above, but the auction ends after 34 ticks, then your loss from the purchase will increase by 64.6 percent.

[38] These are the table and examples appearing in the instructions for the high-cost treatments "H." For the low-cost treatments "L," the numbers in the table and the numerical examples are adjusted accordingly.

Your time-adjusted negative earnings will be:

$$\text{TIME-ADJUSTED EARNINGS} = (32 - 50) \times 1.646 = -29 \text{ points.}$$

If you do not buy the good in a given period, your earnings will be zero in this period irrespective of how long the auction lasts.

At each point of time during the auction, your computer screen will display your value of the good, the current price at which you may buy, the time adjustment factor, and your unadjusted and time-adjusted payoffs (earnings or losses) if you were to buy the good at the current price and time.

ARE THERE ANY QUESTIONS?

Summary of Values, Duration and Earnings

1. You will participate in an auction to buy a fictitious good. If you buy the good, your earnings from the purchase will depend on the difference between the value of the good and the price you pay, and on how long the auction lasts.

2. If you buy the good at a price above your value, your earnings will be negative (you will lose money).

3. If you buy the good at a price below your value, your earnings will be positive (you will make money).

4. The longer the auction takes, the lower will be your earnings from the purchase, other things being equal.

5. If you do not buy, your earnings will be zero.

Test for Understanding This is a test for understanding about the values, auction duration and earnings. **Please answer all questions carefully.** You will not be able to start the auction until you answer all questions correctly.

1. Suppose your value for the good is 41, and the auction ends after several ticks of the virtual clock at the price of 20. The time-adjustment factor is 0.85 for positive earnings, and 1.15 for negative earnings. If you buy the good at this price and time, what is your earning (rounded to the closest integer)?

   [**Answer: 18**]

2. Suppose your value for the good is 31, and the auction ends after several ticks of the virtual clock at the price of 49. The time-adjustment factor is 0.6 for positive earnings, and 1.4 for negative earnings. If you buy the good at this price and time, what is your earning (rounded to the closest integer)?

   [**Answer: -25**]

Auction Organization [Dutch Auction]

The auction is organized as follows. In each auction you and **1** (**4**) other participants will compete to purchase a fictitious good. The price of the good will start at **50** points and will decrease by **1** point with every tick. Any of the bidders can stop the auction and purchase the good at the price displayed on the screen by clicking the **Bid** button. The first person to click the button buys the good and pays the price displayed on the screen, and the other bidders earn 0 for that auction.

After the good is sold, the auction closes for this period. Your results screen will display whether you bought the good or not; your value for the good, sale price, and your unadjusted and time-adjusted payoffs.

If you do not buy, your results screen will report your actual payoffs as zero, and will also display your payoffs if you were to buy the good at the price and time when it was sold.

Your time-adjusted earnings from all previous auctions will be displayed in the history box at the bottom of each auction's results screen.

ARE THERE ANY QUESTIONS?

Summary of Auction Organization

1. When the auction opens, the price will start to go down. The first person to click the **Bid** button will buy the good at the time and price of their bid.
2. If you buy at a price above your value, your earnings will be negative (you will lose money).
3. If you buy at a price below your value, your earnings will be positive (you will make money).
4. The longer the auction takes, the lower will be your earnings from the purchase, other things being equal.
5. If you do not buy, your earnings will be zero.

Test for Understanding

1. Suppose the price of the good is going down, and someone else, not you, is the first person to bid. Check which statement is correct:
   (a) The auctions will end immediately, and the person who bid first will buy the good at the time and price at which they bid.
   (b) The person who bid will be provisionally assigned the good, but you will have another chance to bid and resume the auction.

   [**Answer: (a)**]

2. Suppose the price is going down, and you are the first person to bid at the price of 33. Suppose your value for the good is 25. Check which statement is correct:

(a) Even though the price of 33 is above my value of value 25, I will not lose money if I bid at this price because another person may also bid and buy the good.

(b) Because the price of 33 is above my value of 25, I will lose money if I bid at this price.

[**Answer: (b)**]

Auction Organization [Honolulu-Sydney Auction]

The auction is organized as follows. In each auction you and **1** (**4**) other participants will compete to purchase a fictitious good.

Price is going down The price of the good will start at some value between **0** and **50** points, set by the experimenter, and will go down by **1** point with every tick of the virtual clock. Any of the bidders can stop the auction and provisionally purchase the good at the price displayed on the screen by clicking the **Bid** button. The first person to click the **Bid** button is provisionally assigned the good at the given price and time.

Another chance to bid When a bidder bids and becomes a provisional buyer, other bidders have **10** seconds to challenge the provisional buyer by clicking the **Bid** button. The virtual clock will stop, and bidder payoffs will not change, during this time.

If a bidder decides not to bid at this stage, they may leave the auction by either clicking **Leave** button, or by taking no action. This bidder then becomes inactive and cannot bid any more.

If no bidder challenges the provisionally assigned buyer by clicking the **Bid** button after **10** seconds, then the auction ends. The provisional buyer buys the good at the price and time of their bid. All other bidders earn 0 for that auction.

Price is going up If one or more other bidders challenge the provisional buyer by clicking the **Bid** button, then the auction and the virtual clock will resume, and the price will go up from the previously bid price by **1** point with every tick of the virtual clock. Only the provisional buyer and those bidders who challenged this buyer by clicking **Bid** will be active in the auction at this stage. (The bidders who left the auction will be inactive and will view the auction without taking any action.)

Any active bidder can then leave the auction at any time by clicking the **Leave** button. The auction will continue, with the price and time increasing, as long as there are two or more active bidders in the auction. The auction ends when the next to the last active bidder clicks **Leave** button. The last bidder to stay active in the auction buys the good at the price and time when the next to last bidder left. All other bidders earn 0 for that auction. If two last active bidders drop out at the same time and price, then the object will be allocated randomly to one of these bidders at the price and time when they left.

After the good is sold, the auction closes for this period. Your results screen will display whether you bought the good or not; your value for the good, sale price, and your unadjusted and time-adjusted payoffs.

If you do not buy, your results screen will report your actual payoffs as zero, and will also display your hypothetical payoff if you were to buy the good at the price and time when it was sold.

Your time-adjusted earnings from all previous auctions will be displayed in the history box at the bottom of each auction's results screen.

ARE THERE ANY QUESTIONS?

Summary of Auction Organization

1. When the auction opens, the price will start to go down. The first person to click the **Bid** button will be provisionally assigned the good at the time and price of their bid. After that, all other bidders will be given another chance to bid.

2. Only the bidders who click **Bid**, either when the price is going down, or at "Another Chance to Bid" stage, will have a chance to buy. Bidders who do not bid will not be able to buy the good in this auction and will earn zero.

3. If more than one bidder clicks the **Bid** button, the price will start going up, and all bidders who are active (i.e., they pressed **Bid** earlier) will be considered willing to buy at the current price. You have to press the **Leave** button to exit bidding. The last bidder to stay active in the auction will buy the good at the price and time when the next to last bidder left.

4. If the price is above your value, and you bid (if the price is going down or at "Another Chance to Bid" stage) or you do not leave the auction (if the price is going up), you may end up buying the good at a price above your value, and lose money.

5. If the price is below your value and you do not bid (at "Another Chance to Bid" stage), or you leave the auction (when the price is going up), you may forgo a chance to buy at a price below your value and earn a positive profit.

Test for Understanding This is another test for understanding about the auction organization. **Please answer all questions carefully.** You will not be able to start the auction until you answer all questions correctly.

1. Suppose the price is going down, and you are the first person to bid at the price of 24. Suppose your value for the good is 17. Check which statement is correct:

   (a) Even though the price of 24 is above your value of value 17, I will not lose money if I bid at this price because another person may also bid and buy the good.

   (b) Because the price of 24 is above my value of 17, I may lose money if I bid at this price.

   [**Answer: (b)**]

2. Suppose the price was going down, but then it stopped at 35, and you are given "Another Chance to Bid". Suppose your value for the good is 41. Check which statement is correct:

(a) If I click **Bid**, I will continue bidding and may have a chance to buy and make a positive earning. If I do not click **Bid** or click the **Leave** button, I will forgo a chance to buy and earn money.

(b) I should not bid. Instead, I should leave the auction because the price is too high.

[**Answer: (a)**]

3. Suppose again that the price was going down, but then it stopped at 18, and you are given "Another Chance to Bid". Suppose now your value for the good is 11. Check which statement is correct:

   (a) If I click **Bid** I will continue bidding and may have a chance to buy and make a positive earning. If I do not click **Bid** or click the **Leave** button, I will forgo a chance to buy and earn money.

   (b) Because the price is above my value, I should not bid if I want to avoid losses. If I click **Bid**, I may buy at the current price or higher, and I will lose money if I buy.

   [**Answer: (b)**]

4. Suppose the price of the good is going up and is currently at 14. Suppose you are an active bidder, and your value of the good is 28. Check which statement is correct:

   (a) I have to click **Bid** with every price increase to remain active in the auction and have a chance to buy the good.

   (b) If I click the **Leave** button at this price, I will forgo my chance to buy at a price below my value and earn a positive payoff.

   (c) Even if I click the **Leave** button at this price, I will still have a chance to bid at a later point and buy in this auction.

   [**Answer: (b)**]

5. Suppose the price of the good is going up and is currently at 14. Suppose you are an active bidder, and your value of the good is 9. Check which statement is correct:

   (a) If I do not leave, I will continue bidding, and may buy the good at a price below my value and earn money. If I click the **Leave** button at this price, I will forgo my chance to buy at a price below value and earn a positive payoff.

   (b) Because the price of 14 is above my value of 9, and the price is going up, I will not be able to buy and make money in this auction. If I do not click **Leave**, I will continue bidding, I may buy the good at a price below my value and lose money.

   [**Answer: (b)**]

Matching

You will be randomly re-matched with new participants in every auction. You will not be told which of the other participants you are matched with in any given auction, and they will not be told that you are matched with them. What happens in your auction has no effect on what happens in any other auction, and vice versa.

This will continue for a number of periods. Your total earnings in this part of the session are given by the sum of your period earnings. Your cumulative earnings in all parts will be displayed on your computer screen at all times during the auction.

The first **2** periods will be practice. You will receive no earnings for these periods. If you have a question, please raise your hand and I will answer your question in private.

ARE THERE ANY QUESTIONS?

1.B Experimental Auction Screenshots [5H treatment]

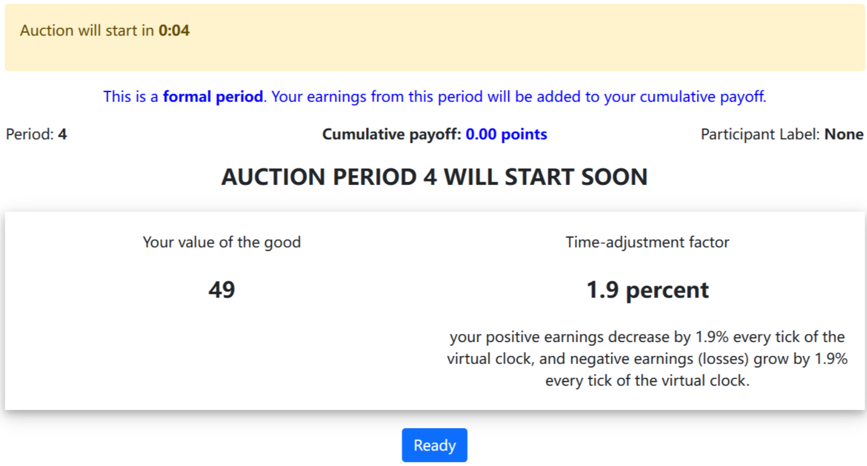


Figure 1.1: Auction start page

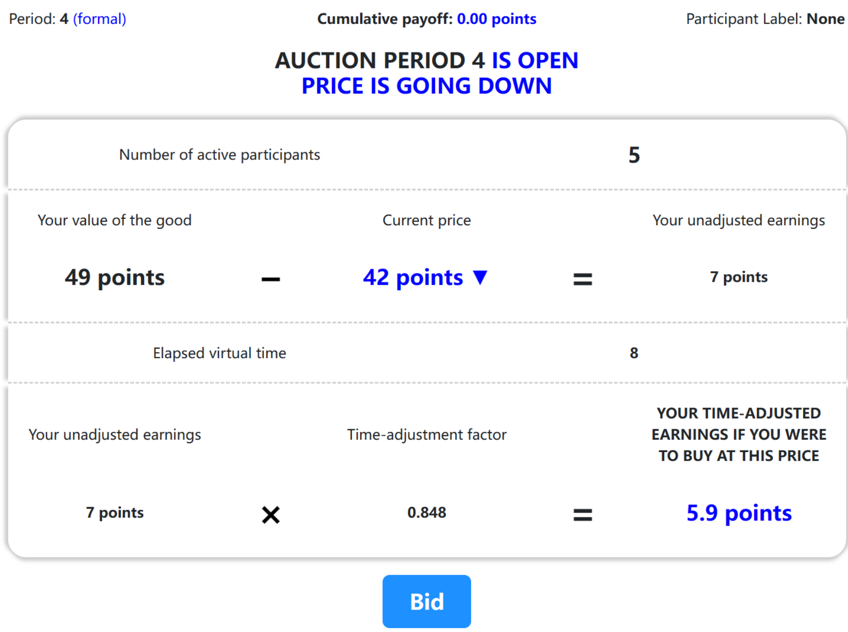


Figure 1.2: Dutch auction or Dutch Stage

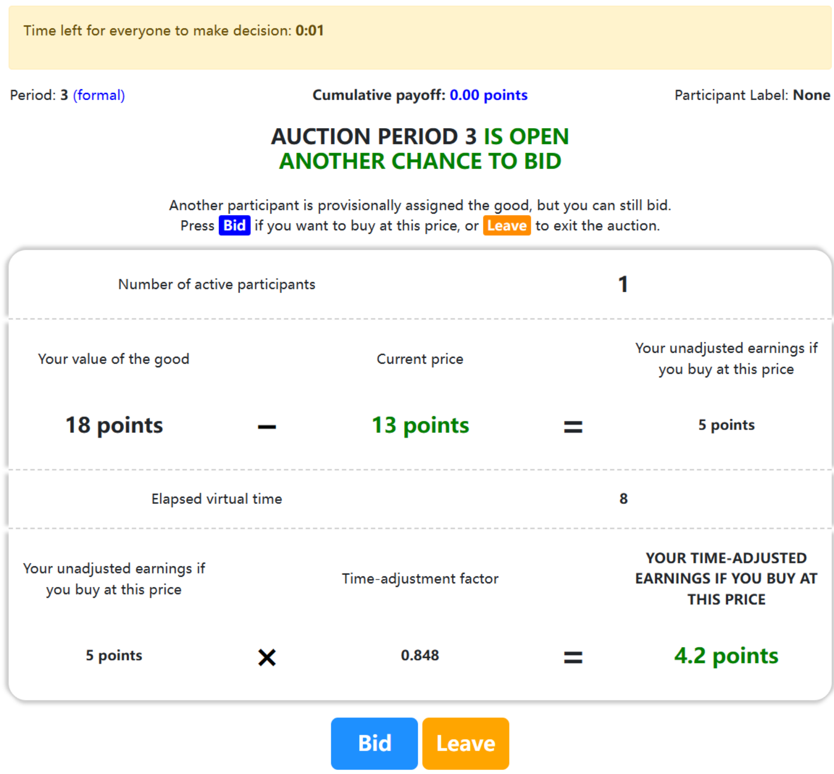


Figure 1.3: Contest Stage

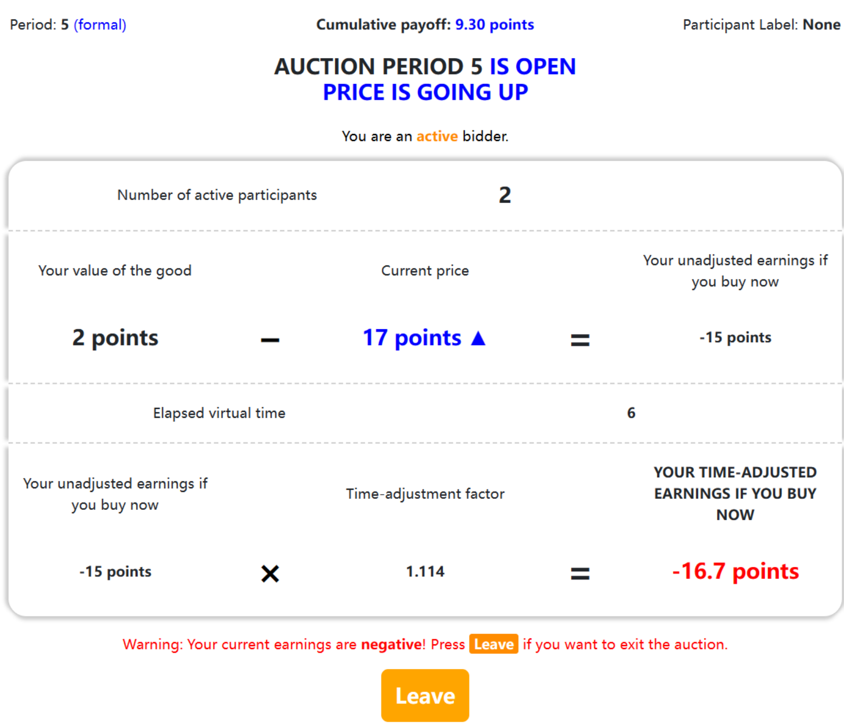


Figure 1.4: English Stage

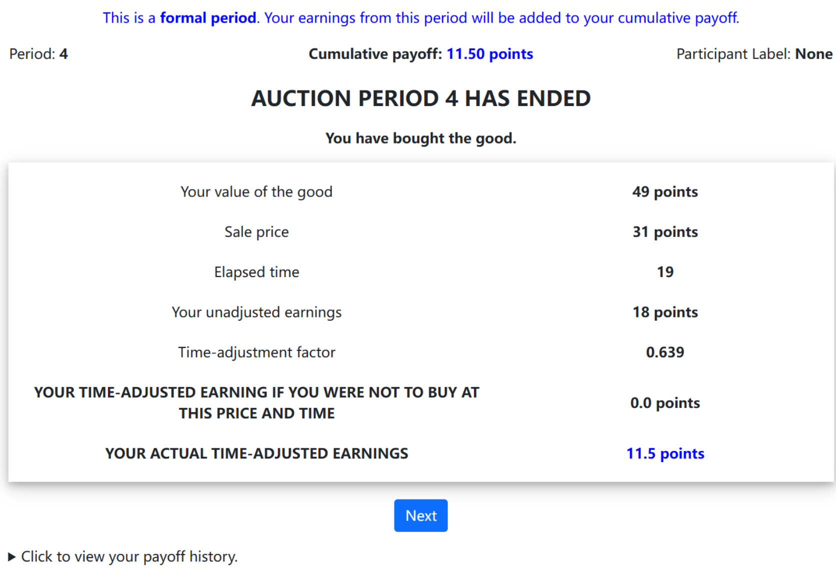


Figure 1.5: Auction result page

1.C Post-auction questionnaire

0. For every period in the auction, I

    1 always bought the object [– Hide questions 5 and 7]

    2 sometimes bought the object, and sometimes not.

    3 never bought the object [– Hide questions 4 and 6]

1. When I bid, all I cared about was buying as fast as possible.

    1 Completely Disagree

    2 Disagree

    3 Somewhat Disagree

    4 Neither Agree Nor Disagree

    5 Somewhat Agree

    6 Agree

    7 Completely Agree

2. When I bid, all I cared about was paying the lowest price possible.

    1 Completely Disagree

    2 Disagree

    3 Somewhat Disagree

    4 Neither Agree Nor Disagree

    5 Somewhat Agree

    6 Agree

    7 Completely Agree

3. When I bought the object, I often thought that I could have bought it at a lower price if I bid differently.

    1 Completely Disagree

    2 Disagree

    3 Somewhat Disagree

    4 Neither Agree Nor Disagree

    5 Somewhat Agree

    6 Agree

    7 Completely Agree

4. When I did not buy the object, I often thought that I could have bought it and made a profit if I bid differently.

1 Completely Disagree

2 Disagree

3 Somewhat Disagree

4 Neither Agree Nor Disagree

5 Somewhat Agree

6 Agree

7 Completely Agree

5. When I bought an object in the auction, I felt:

   1 Extremely Sad

   2 Sad

   3 Somewhat Sad

   4 Neither Happy Nor Sad

   5 Somewhat Happy

   6 Happy

   7 Extremely Happy

6. When I did not buy an object in the auction, I felt:

   1 Extremely Sad

   2 Sad

   3 Somewhat Sad

   4 Neither Happy Nor Sad

   5 Somewhat Happy

   6 Happy

   7 Extremely Happy

Chapter 2

Istanbul Flower Auctions: The Need for Speed

Joint work with Isa Hafalir, University of Technology Sydney; Onur Kesten, University of Sydney; Donglai Luo, University of Technology Sydney; Katerina Sherstyuk, University of Hawaii Manoa.

2.1 Introduction

The study of auction mechanisms has long been a subject of interest in the field of economics, given their extensive application in various markets for the sale of goods and services (Milgrom, 1989; Klemperer, 1999). Auctions are particularly vital in markets where goods need to be sold quickly and efficiently, such as those for perishable items that are auctioned off in numerous lots within a predetermined period. Fresh produce auctions, which deal with the sale of perishable goods like fruits, vegetables, fish, and flowers, are critical for ensuring these items reach consumers in a timely manner (Cassady, 1967). These markets are characterized by their need for rapid sales to facilitate transactions between a large number of sellers and buyers within very tight time frames.

Among the various auction formats, the Dutch auction has been widely adopted in markets dealing with perishable goods, praised for its speed and efficiency. In this format, the auctioneer sets a high starting price, which is progressively lowered until a bid is made, ensuring a swift sale of the item at hand. However, the Istanbul Flower Auction introduces a nuanced approach to the auction mechanism by incorporating elements of both the Dutch and English auction formats, adapting the mechanism based on the initial responses of the bidders. Specifically, given a starting price s, Istanbul Flower Auction operates as a Dutch auction if no one bids at s, and it operates as an English auction (among the initial bidders) if at least one bidder bids at s. This hybrid model is innovative in its flexibility, potentially offering a superior solution to the unique challenges faced in perishable goods markets.

In this paper, we study a comparative analysis of the Istanbul Flower Auction against the backdrop of traditional Dutch and English auctions. By focusing on the perishable goods market, particularly the sale of flowers in Istanbul, we explore how different auction formats can impact the outcomes and auctioneer's and bidders' utilities. The theoretical framework centers on the utility for both auctioneers and bidders, taking into account the time cost as a function of the total duration of the auction. Through this lens, we study the Istanbul Flower Auction, Dutch Auction and English Auction in terms of the utilities of the auctioneer and the bidders in perishable goods markets.

We study how the expected utility for the auctioneer in Istanbul Flower Auctions is affected by various factors, including the auction's starting price and the time cost functions for both the auctioneer and the bidders. We establish, especially in scenarios where any party incurs time costs, that the Istanbul Flower Auction offers superior utility for the auctioneer compared to traditional Dutch and English auctions (Proposition 2.2). This advantage is pronounced when the auctioneer values time highly, and the bidding strategy of the bidders is either non-increasing or invariant with respect to the starting price. We also explore how bidders with time costs can gain more from the Istanbul Flower Auction compared to Dutch and English auctions, emphasizing the auction's ability to adapt its starting price for optimized expected utility (Proposition 2.3). Our numerical analysis supports the theoretical insights, showing that the Istanbul Flower Auction format significantly benefits both auctioneers and bidders, particularly under conditions of high time costs. We find that the auctioneer's preference for the Istanbul format largely depends on her own time costs, with other auction characteristics playing a lesser role.

Our study is motivated by the premise that the choice of auction format can significantly affect market outcomes, especially in sectors where time is of the essence. By examining the Istanbul Flower Auction, we seek to contribute to the broader discourse on auction theory and practice, offering insights that could inform the design and selection of auction formats in various contexts. The findings of this research may have implications for auctioneers and market organizers worldwide, suggesting that flexibility in auction design could be key to optimizing sales processes, especially in markets where a large number of items need to be sold in very tight time frames.[1]

The rest of the paper is organized as follows. The introduction (Section 2.1) ends with a review of the related literature. Section 2.2 introduces our model and solves for the equilibrium of Istanbul Flower Auctions. Section 2.3 provides theoretical results on the

[1]These auctions are not only used for selling fresh produce. Used car auctions, which are used by dealerships or leasing companies, can sell hundreds to thousands of vehicles in a single event. Similarly, large wine auctions can involve the sale of thousands of bottles or cases, including rare and vintage wines, over the course of a single event.

payoff comparison of Istanbul Flower Auctions against the backdrop of Dutch and English Auctions. Section 2.4 provides our numerical results. Section 2.5 concludes. We provide all proofs in the Appendix.

2.1.1 Brief review of the literature

There is a long strand of literature on the descending Dutch and the ascending English auctions, exploring various analogs and extensions derived from the standard models. Comprehensive overviews of these studies can be found in Klemperer (2004) and Milgrom (2004). Our contribution to the literature is studying a unique hybrid auction format that has been implemented in Turkish flower markets for decades.

Hybrid auctions typically allow for both ascending and descending prices throughout the auction process, with many variants in practice. For example, eBay provides a selling mechanism that provides bidders with a "buy-it-now" option simultaneously with the English auction process. Mathews (2004) shows that under such circumstances, impatient sellers would gain extra revenue by setting an attractive "buy-it-now" price. Furthermore, Azevedo et al. (2020) consider a descending buyout price and attributed its advantage over standard formats to information acquisition costs. In another example, Katok and Roth (2004) study multi-unit Dutch auctions for homogeneous goods with divisible lots where the price can drop and then rise, and explained it by synergy effects between parts of the lot.

Recent studies have recognized the importance of auction speed in the design of mechanisms for real-world markets. Banks et al. (2003) document the trade-off between efficiency and speed in the Federal Communication Commission spectrum auctions, where speed and revenue can be enhanced through the improved design proposed by Kwasnica et al. (2005). Andersson and Erlanson (2013) numerically show that a hybrid Vickrey-English-Dutch algorithm is faster than the Vickrey-English or Vickrey-Dutch auctions. The role of speed in auctions has also been highlighted in experimental studies, where participants' impatience was linked to their enjoyment of participation (Cox et al., 1983) or designation costs (Katok and Kwasnica, 2008). However, none of these models formally incorporates the time cost of auction participants into the optimization problem, except for Hafalir et al. (2023).

A similar hybrid auction format is used in various fish markets (including Honolulu and Sydney fish markets,) for which a comprehensive analytical and experimental study on how the time costs of the participants affect the auction speed and revenue is provided in Hafalir et al. (2023). Unlike the fish auctions in Honolulu and Sydney, in Istanbul Flower Auctions the ascending or descending price is determined by the initial decision of bidders rather than occurring as serial stages. More specifically, the main difference between the Honolulu-

Sydney fish auction and the Istanbul flower auction is that in the former auction, the price can start going up after a bid in the Dutch phase (whereas this is not allowed in the latter auction.)

2.2 Model

We consider a single-item auction with n bidders in which the total time lapsed in the auction is important for both the auctioneer and the bidders. The private values of bidders are independently and identically distributed according to a twice differentiable cumulative distribution function $F(\cdot)$ over $[0, 1]$ with a corresponding density function $f(\cdot)$. As the clock ticks, the time cost is modeled as a decreasing and differentiable discounting multiplier $c_A(t)$ for the auctioneer and $c_B(t)$ for the bidders.

More specifically, if the auction ends when the bidder with value v wins at a price p after t units of time have passed, the auctioneer's utility is

$$U_A = p \cdot c_A(t)$$

and the winning bidder's utility is

$$U_B = (v - p) \cdot c_B(t)$$

where the time cost function of the auctioneer $c_A(t)$ and that of bidders $c_B(t)$ are strictly decreasing functions of time t (when time costs exist.) More specifically, for $i = A, B$, we have $c_i : \mathbb{R}_+ \to \mathbb{R}_+$, where $c_i(0) = 1$; moreover, $c_i'(t) < 0$ except when $c_i(t) = 1$ for all $t \geq 0$. We refer to $c_i(t) = 1$ as the "no time cost" case for the auctioneer ($i = A$) and the bidders ($i = B$), respectively.

Bidders who lose the auction would get a utility of 0. We refer to the gains of the auctioneer as utility as opposed to revenue because the auctioneer's utility is not necessarily equal to the revenue in the presence of time costs.

We also define several auxiliary functions which will be useful later. Let us denote the distribution of the highest value of $n-1$ random variables independently distributed according to $F(\cdot)$ by a cumulative distribution function $G(\cdot)$ with a corresponding density function $g(\cdot)$, i.e. $G(v) = F^{n-1}(v)$. Let us denote the joint density function of the highest value equals to v and the second highest value equals to x among n random variables identically and independently distributed according to $F(\cdot)$ by $h(v, x)$, we have $h(v, x) = n(n-1)f(v)f(x)F^{n-2}(x)$.

2.2.1 Istanbul Flower Auctions

The Istanbul flower auction proceeds as follows.[2] It begins with a starting price announced by the auctioneer and bidders' decisions to *bid* or *wait* at the starting price. Then, the auction turns into one of the following two formats conditional on the number of bidders bidding at the starting price:

(1) If no one bids at the starting price, the auction then operates as a Dutch auction: the price starts going down until someone bids (or the auction ends with no winner when the price drops to its minimum.)

(2) If at least one bids at the starting price, the auction then operates as an English auction for those initial bidders only: the price starts going up until the second last of the initial bidders leaves the auction (the rise of the price stops immediately at the starting price if there is only one bidder who bids at the starting price).

Bidders simultaneously make decisions on whether to bid at the starting price. We assume that decisions are made instantly, without the implied time cost. Moreover, in equilibrium, the decision to bid also reflects the bidder's preference for the auction format in a consistent way such that she will not join the other format.

We begin our analysis with the bidder's behavior. Consider a starting price $s \in [0, 1]$ and bidders implement symmetric increasing bidding strategies. They need to decide on (i) whether to bid at the starting price and (ii) at which price to bid or leave when they are active bidders in the subsequent Dutch or English auction.

We first discuss those bidders who choose to wait. Intuitively, it is always sub-optimal for a bidder with item value $v < s$ to bid at the beginning, because she can never acquire a positive payoff in the subsequent English auction. Therefore, as we restrict our attention to the monotonic increasing bidding strategy, we focus on the case featuring a cutoff $p(s) \in [s, 1]$, such that the bidder only bids at the starting price if her private value is greater than $p(s)$ and wait otherwise.

In the subsequent Dutch auction, given the starting price s, let us denote the symmetric equilibrium bid function by $b(v, s)$. Since this Dutch auction starts from s and not at 1, it could be possible that there is a cluster of bids at s: i.e., we may have $b(v, s) = s$ for all $s \in [\lambda(s), p(s)]$ for some $\lambda \in (0, p(s))$. We assume ties break evenly for those bidders potentially choosing to cluster their bids at s. Lemma 2.1 shows that, in equilibrium, there will be no cluster bids at s, and the value cutoff $p(s)$ is determined by the starting price s via the Dutch bidding function when $p(s) < 1$.

[2] For a more detailed description, see Öz and Çalışkan (2010).

Lemma 2.1 *In equilibrium, the probability that a tie occurs in the Dutch auction is zero. Moreover, the unique solution to the cutoff value $p(s)$ as a function of the starting price s is given by $b(p(s), s) = s$ when $p(s) < 1$.*

A necessary condition for $b(v, s)$ to be a symmetric equilibrium strategy is that the bidder maximizes her payoff at her own value v instead of any alternative v'. For $v, v' \leq p(s)$, it is the Dutch auction problem (in the presence of time costs):

$$v = \arg\max_{v'} G(v')(v - b(v', s))c_B(s - b(v', s))$$

which relates to the following ordinary differential equation derived from the first-order condition

$$\frac{\partial b(v,s)}{\partial v} = \frac{g(v)}{G(v)} \frac{(v - b(v,s))c_B(s - b(v,s))}{(v - b(v,s))c'_B(s - b(v,s)) + c_B(s - b(v,s))}. \quad (2.1)$$

Indeed, the Dutch bidding strategy $b(v, s)$ is defined by the differential equation 2.1 and the initial condition $b(0) = 0$.

Now, consider a bidder with value v who bids to initiate the English auction, i.e., $v \geq p(s)$. She will remain in the auction and compete with other bidders (if any) until the price reaches her value[3]. Her expected utility is given by [4]

$$EU_B^{FE}(v, s) = (v - s)G(p(s)) + \int_{p(s)}^{v} c_B(x - s)(v - x)dG(x)$$

where the first term is her expected utility from winning the item at the starting price when no other bidder is competing with her, and the second term is her expected utility from winning the item at the highest price among other competitors.

As such, when an Istanbul Flower Auction starts at s, the ex-ante expected utility for bidders is given by

$$\begin{aligned} EU_B^F(s) = &\int_0^{p(s)} c_B(s - b(v,s))(v - b(v,s))G(v)dF(v) \\ &+ \int_{p(s)}^{1} \left[(v - s)G(p(s)) + \int_{p(s)}^{v} c_B(x - s)(v - x)dG(x)\right] dF(v) \end{aligned}$$

[3]Even with the presence of time costs, since time costs are in the multiplicative form, it is easy to see that the weakly dominant strategy would be to remain in the auction till the price reaches the bidder's value.

[4]The superscript FE corresponds to the flower auction in the English auction phase. Similarly, FD: flower auction in Dutch auction phase, D: Dutch auction, E: English auction, F: Flower auction.

The resulting utility for the auctioneer is given by

$$EU_A^F(s) = \int_0^{p(s)} c_A(s - b(v,s))b(v,s)dF^n(v) + \int_{p(s)}^1 \left(\int_0^{p(s)} sh(v,x)dx + \int_{p(s)}^v c_A(x-s)xh(v,x)dx \right) dv$$

Now, the optimization problem for the auctioneer is to select the best initial price s^* in order to optimize her utility. that is

$$s^* = \arg\max_s EU_A^F(s)$$

We can argue that the equilibrium that we consider for the Istanbul Flower Auction is efficient. The reasons are given by: (i) the Dutch phase bidding function is assumed to be increasing in the bidders' item values, and (ii) each bidder remains in the auction until the price reaches her item value during the English phase. The following remark notes the efficiency of the Istanbul Flower Auction.

Remark 2.1 *In equilibrium, Istanbul Flower Auction is efficient, in the sense that the item is allocated to the bidder with the highest item value.*

Note that, in this setup, efficiency in the sense of "highest-valued bidder wins the item" does not imply social welfare maximization, since time costs also enter the bidders' and the auctioneer's utilities.

Dutch auctions and English auctions

Istanbul flower auction inherently contains elements of both English and Dutch auctions and can be explicitly converted to either format by setting the initial price at $s = 0$ or $s = 1$. When $s = 1$, as in the Dutch auctions, the price continuously descends from 1 until the first bidder stops the clock and takes it.

In this case, the expected utility for the auctioneer is given by

$$EU_A^D = EU_A^F(1) = \int_0^1 c_A(1 - b(v,1))b(v,1)dF^n(v).$$

The ex-ante expected utility for bidders is given by

$$EU_B^D = EU_B^F(1) = \int_0^1 c_B(1 - b(v,1))(v - b(v,1))G(v)dv.$$

The English auction is also incorporated into the Istanbul Flower Auction framework by setting the starting price at $s = 0$. In this case, the ex-ante expected utilities for the auctioneer and the bidders are given by

$$EU_A^E = EU_A^F(0) = \int_0^1 \int_0^v c_A(x) x h(v,x) dx dv,$$
$$EU_B^E = EU_A^F(0) = \int_0^1 \int_0^v c_B(x)(v-x) dG(x) dF(v).$$

Hence, the Istanbul flower auction can be viewed as a hybrid mechanism that is conducted as either an English or a Dutch auction. The initial price calibrates the occurrence of each format. In the following example, we show that the variability of the initial price at the Istanbul flower auction renders it a utility superiority of the auctioneer over Dutch and English auctions.

2.2.2 An illustrative example

Example 2.1 *Consider a 2-bidder Istanbul Flower Auction where both the auctioneer and the bidders share the same exponential time cost function $c_A(t) = c_B(t) = e^{-t}$. The private value of each bidder is independently and uniformly distributed on the unit interval $[0,1]$. The auctioneer sets the starting price s.*

The cutoff value $p(s)$ is determined by equating the expected utility of the cutoff bidder in the Dutch phase and the English phase. If the auction starts as a Dutch auction, the Dutch bidding strategy is determined by the differential equation below with the initial condition $b(0,s) = 0$:

$$\frac{\partial b(v,s)}{\partial v} = \frac{v - b(v,s)}{v} \frac{1}{1-(v-b(v,s))}.$$

The expected utility of the cutoff bidder with value p in the Dutch phase is given by

$$EU_B^{FD}(p,s) = e^{-(s-b(p,s)}(p-b(p,s))p.$$

If the auction starts as an English auction, the expected utility of the cutoff bidder is given by

$$EU_B^{FE}(v,s) = (p-s)p.$$

We solve p from $EU_B^{FD}(p,s) = EU_B^{FE}(p,s)$ numerically and observe that $p(s)$ increases from 0 to 1 in $(0, 0.592)$ and fixes at 1 in $(0.592, 1)$.

The expected utility of the auctioneer is given by

$$EU_A^F(s) = \int_0^p 2e^{s-b(v,s)} v b(v,s) dv - 2(p^2 + p + 1)e^{s-p} - 2(p-1)ps + 6e^{s-1}.$$

By plugging in the $p(s)$ *function calculated above, we can numerically find that the auctioneer's utility is maximized at the starting price* $s^* = 0.498$*. The cutoff value at the optimal starting price is* $p(s^*) = 0.861$*, which means that the bidder with item value* $v \in [0, 0.861]$ *will wait for the Dutch phase, while the bidder with item value* $v \in (0.861, 1]$ *will bid to start the English phase at the beginning of the auction.*

Figure 2.2 compares the revenue performance of the flower auction with the Dutch auction $(s = 1)$ *and the English auction* $(s = 0)$*. The auctioneer's utility is higher than the English auction as long as she sets the Istanbul Flower Auction differently. Although the Istanbul Flower Auction outperforms the Dutch auction in most cases, it is possible for the auctioneer to get a worse result by setting a sufficiently low starting price; in this example, it is* $s < 0.043$*. The expected utility for the auctioneer at the optimal starting price is* 0.340*, which is the peak of the utility curve. It outperforms the Dutch utility of* 0.216 *by* 57%*, and outperforms the English utility of* 0.207 *by* 64%*. This also outlines the importance of the auctioneer's decision on the starting price.*

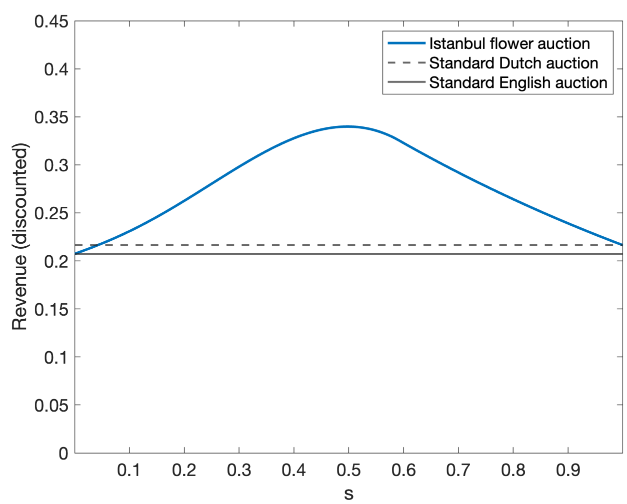


Figure 2.2: Example: Revenue Comparison

2.3 Payoff Comparison

In this section, we compare the payoffs of the auctioneer and bidders in the Istanbul flower auction, Dutch auction, and English auction formats.

2.3.1 Utility for the auctioneer

The first-order derivative of the expected utility for the auctioneer in Istanbul flower auctions is given by

$$\begin{aligned}
\frac{dEU_A^F(s)}{ds} = \;& \frac{dp(s)}{ds}[c_A(s-b(p(s),s))b(p(s),s)-s]nF^{n-1}(p(s))f(p(s)) \\
&+\frac{dp(s)}{ds}[s-c_A(p(s)-s)p(s)]n(n-1)f(p(s))F^{n-2}(p(s))[1-F(p(s))] \\
&+nF^{n-1}(p(s))[1-F(p(s))] \\
&-\int_{p(s)}^{1}\int_{p(s)}^{v} c_A'(x-s)xh(v,x)dxdv \\
&+\int_0^{p(s)}\left[c_A'(s-b(v,s))(1-\frac{\partial b(v,s)}{\partial s})b(v,s)+c_A(s-b(v,s))\frac{\partial b(v,s)}{\partial s}\right]dF^n(v).
\end{aligned} \tag{2.2}$$

It can be verified that this derivative is zero for all $s \in [0,1]$ when both the auctioneer and the bidders have no time cost. This is the classic revenue equivalence result (Myerson, 1981; Riley and Samuelson, 1981). We document it in the following proposition.

Proposition 2.1 *Istanbul flower auctions satisfy the Revenue Equivalence Principle when both the auctioneer and bidders have no time costs, i.e.,* $EU_A^F(s) = \int_0^1 \left[v - \frac{1-F(v)}{f(v)}\right] dF^n(v)$ *when* $c_A(t) = 1$ *and* $c_B(t) = 1$.

However, when either the auctioneer or the bidders incur time costs, the revenue equivalence principle no longer holds. In this more interesting case, we provide comparative static analyses based on the local properties of the auctioneer utility function in our time-costly setting.

Recall that $EU_A^D = EU_A^F(1)$, it is sufficient to claim that the Istanbul Flower Auction is a better choice for the auctioneer than the Dutch auction, by showing that the expected utility for the auctioneer is decreasing at $s = 1$ (or the derivative of the expected utility at $s = 1$ is negative), because then it must reach higher utility for some interior s. Similarly, the Istanbul Flower Auction is better than the English auction if the expected utility for the auctioneer is increasing at $s = 0$ (or the derivative of the expected utility at $s = 0$ is positive), because $EU_A^E = EU_A^F(0)$. Hence, by establishing the sign of the first-order derivative of the expected utility for the auctioneer with respect to the starting price in stylized scenarios, the revenue comparison follows naturally.

It is evident from the expression of the first-order derivative that the properties of the terms $\frac{\partial b(v,s)}{\partial s}$ and $\frac{dp(s)}{ds}$ are crucial in determining its sign.

We first note that $\frac{\partial b(v,s)}{\partial s}$ has an upper bound:

Lemma 2.2 $\frac{\partial b(v,s)}{\partial s} < 1$.

Intuitively, the variation of the starting price is only partly reflected in the variation of the Dutch bidding strategy.

Then, we describe how the Dutch-English cutoff value $p(s)$ in Istanbul Flower Auctions changes with the starting price s set by the auctioneer:

Lemma 2.3 *There exists a* $\tilde{s} \in (0, 1)$ *such that*

(a) $p(s) < 1$ *and* $\frac{dp(s)}{ds} > 0$ *when* $s \in [0, \tilde{s})$.

(b) $p(s) = 1$ *and* $\frac{dp(s)}{ds} = 0$*when* $s \in [\tilde{s}, 1]$.

The cutoff value is always increasing with the starting price, as long as Istanbul Flower Auctions can indeed proceed into either the Dutch or English phase. After the starting price exceeds a certain value, the Dutch-English cut-off is fixed at 1. That is, all bidders will wait for the Dutch phase in Istanbul Flower Auctions, so it simply becomes Dutch auctions starting at price s.

This lemma also helps us conceptualize the "triviality" of the equilibrium in the Istanbul Flower Auction: (i) the equilibrium is non-trivial when $s \in (0, \tilde{s})$ in that bidders differ in their decisions between waiting for the Dutch phase and bidding for the English phase at the beginning of the auction, while (ii) the equilibrium is trivial when $s = 0$ where all bidders bid in the English phase or $s \in \cup[\tilde{s}, 1]$ where all bidders bid at the Dutch phase.

So far, we have only established the upper bound of 1 for $\frac{\partial b(v,s)}{\partial s}$, which is insufficient for further analysis. Further insights into how the Dutch phase bidding function reacts to changes in the starting price can be obtained by imposing mild regularities on the bidder time cost function:

Assumption 2.1 *For any* $t \in [0, 1]$*, the time cost function for bidders* $c_B(t)$ *satisfies one of the following conditions:*

(a) $\frac{d}{dt}\left[\frac{c'_B(t)}{c_B(t)}\right] < 0$*, e.g. linear cost* $c_B(t) = 1 - \mu t$.

(b) $\frac{d}{dt}\left[\frac{c'_B(t)}{c_B(t)}\right] = 0.$*, e.g. exponential cost* $c_B(t) = e^{-\mu t}$.

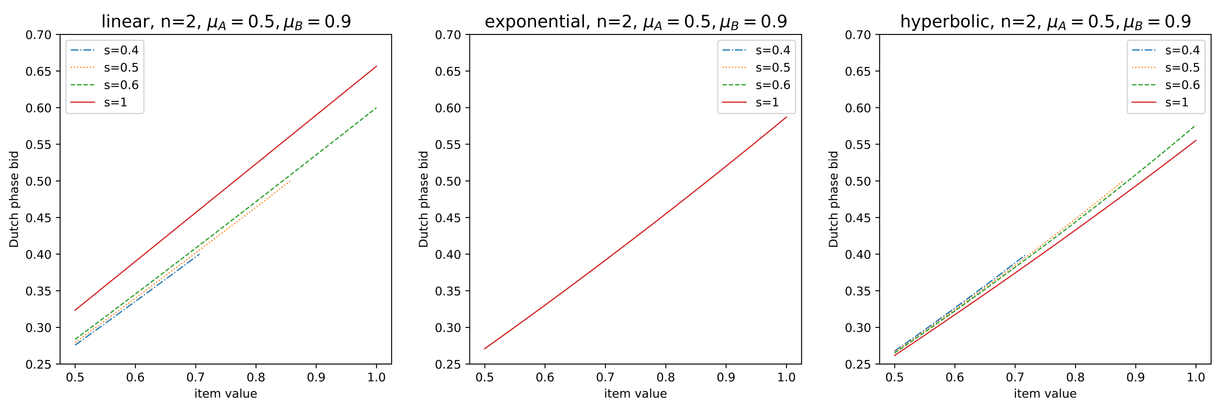
(c) $\frac{d}{dt}\left[\frac{c'_B(t)}{c_B(t)}\right] > 0.$*, e.g. the hyperbolic cost* $c_B(t) = \frac{1}{1+\mu t}$.

This assumption requires the relative decreasing rate of the cost function to change monotonically. Coincidentally, the three cases include three commonly used time cost (discounting) functions, linear, exponential, and hyperbolic, respectively.

The relationship between the sign of $\frac{\partial b(v,s)}{\partial s}$ and the assumed class of time cost functions is given by:

Lemma 2.4 *In equilibrium, the bidder's Dutch phase bidding function $b(v, s)$ in the Istanbul Flower Auction is (a) weakly increasing, (b) invariant, and (c) weakly decreasing in the starting price s, for each corresponding class of time cost functions in Assumption 2.1.*

To illustrate, we build numerical results for the three representative functions. The lemma 2.4 states that the Dutch phase bidding function will either shift upward (see Figure 2.3a), stay the same (see Figure 2.3b), or shift downward (see Figure 2.3c) when the starting price increases, depending on whether the cost functions are linear, exponential, or hyperbolic.



(a) Linear cost (b) Exponential cost (c) Hyperbolic cost

Figure 2.3: Lemma 2.4

With the assumption and lemmas introduced previously in this section, the revenue comparison between the Istanbul Flower Auctions and the Dutch or English auctions can be established in almost all scenarios. We now state our primary analytical finding on the general advantage of Istanbul Flower Auctions.

Proposition 2.2 *When at least one party incurs time cost, the comparison between the Istanbul Flower Auction and other standard auction formats from the perspective of the auctioneer can be established in the following scenarios:*

(a) Under Assumption 2.1(b) or 2.1(c), when the auctioneer has time cost, the Istanbul Flower Auction with starting price s^* is strictly better than the Dutch auction for the auctioneer. In this case, the equilibrium is non-trivial, i.e., $EU_A^F(s^*) > EU_A^D$ and $s^* \in (0, \tilde{s})$ when $c'_A(t) < 0$ and $\frac{\partial b(v,s)}{\partial s} \leq 0$.

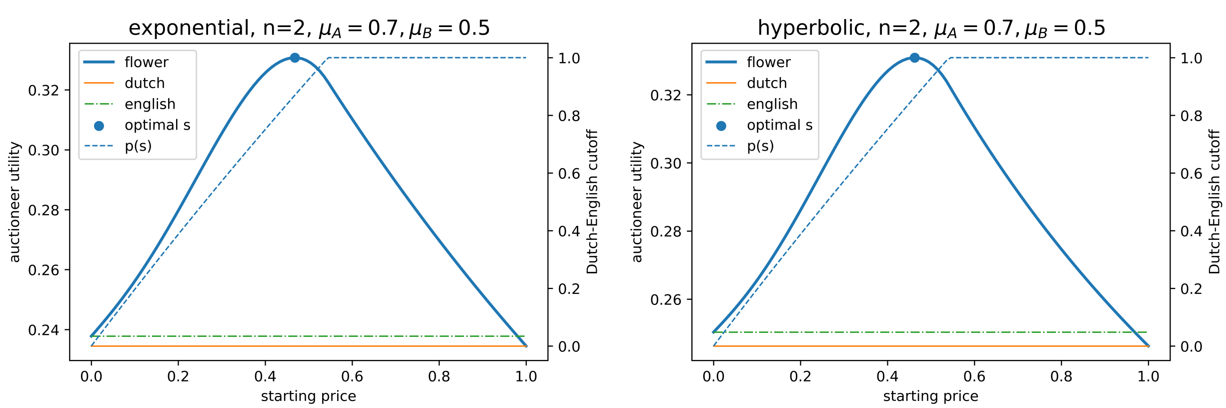
(b) *Under Assumption 2.1(a) or 2.1(b), when the auctioneer has no time cost, the Istanbul Flower Auction with starting price s^* gives the same expected utility to the auctioneer as the Dutch auction. In this case, the equilibrium is trivial, i.e.,* $EU_A^F(s^*) = EU_A^D$ *and* $s^* \in [\tilde{s}, 1]$ *when* $c_A(t) = 1$ *and* $\frac{\partial b(v,s)}{\partial s} \geq 0$.

(c) *Under Assumption 2.1(c), when the auctioneer has no time cost, the Istanbul Flower Auction with starting price s^* is strictly better than the Dutch auction for the auctioneer, i.e.,* $EU_A^F(s^*) > EU_A^D$ *when* $c_A(t) = 1$ *and* $\frac{\partial b(v,s)}{\partial s} < 0$.

(d) *The Istanbul Flower Auction with starting price s^* is strictly better than the English auction for the auctioneer, i.e.,* $EU_A^F(s^*) > EU_A^E$.

This proposition demonstrates the general superiority of Istanbul Flower Auctions compared to the two conventional auction formats: Dutch and English. The "strictly better" result for the comparison with English auctions is established for all time-costly scenarios, where the additional assumption on the time cost function is redundant. The "strictly better" or "indifferent" result for the comparison with Dutch auctions depends on whether the auctioneer incurs a time cost and on how the cost function is specified by Assumption 2.1.

Istanbul Flower Auction outperforms the Dutch auctions when the auctioneer has time cost and the Dutch phase bidding strategy of the bidder is non-increasing in the starting price. Specifically, the auctioneer with time cost strictly prefers Istanbul Flower Auctions when bidders face exponential, hyperbolic, or no time cost (see Figure 2.4).

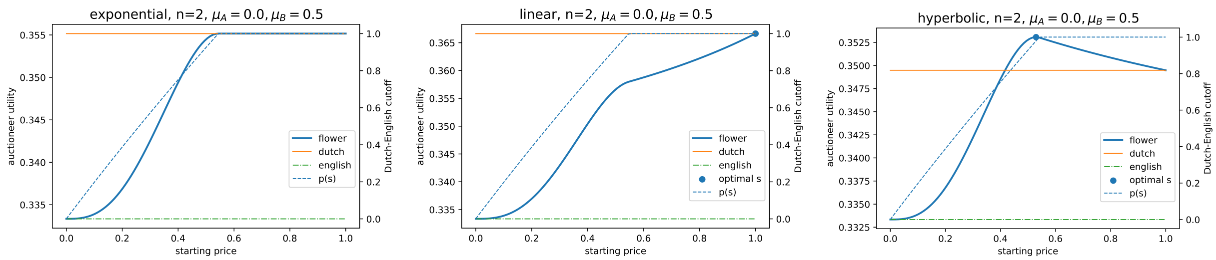


(a) Exponential cost

(b) Hyperbolic cost

Figure 2.4: A numerical example for Proposition 2.2 with auctioneer with time cost

When the auctioneer has no time cost, the Istanbul flower auction might appear as a revenue equivalent but a more complicated choice compared to the Dutch auction (see Figure 2.5a and 2.5c), though it strongly depends on the assumed shape of the cost function (see the counterexample in Figure 2.5b). By contrasting "no time cost" scenario in Figure 2.5a with the costly scenario in Figure 2.4a, we observe a close connection between the auctioneer's time cost and her preference for Istanbul Flower Auctions. Moreover, it is worth noting that Dutch auctions outperform English auctions when only bidders incur time costs.



(a) Exponential cost (b) Linear cost (c) Hyperbolic cost

Figure 2.5: A numerical example for Proposition 2.2 with the auctioneer with no time cost

We summarise the auctioneer's preference for the three auction formats in Table 2.2.

Table 2.2: Auctioneer's preference for auction formats

	Assumption on bidder time cost			
Auctioneer time cost	1(b) no cost	1(a)	1(b) costly	1(c)
No cost	**IFA** = **DA** = **EA**	**IFA** = **DA** > EA	**IFA** = **DA** > EA	**IFA** > DA > EA
Costly	**IFA** > DA IFA ≥ DA **IFA** > EA	**IFA** > DA IFA > EA	**IFA** > DA **IFA** > EA	**IFA** > EA

* IFA — optimal Istanbul Flower auctions, DA — Dutch auctions, EA — English auctions. The most preferred auction format is marked by bold text. The non-trivial IFA is marked by boxed text.

In addition, note that two functional forms, the exponential cost and the hyperbolic cost, are typically employed to model the time cost (or impatience) in the literature. They will be the primary focus of our numerical analysis in the next section. Therefore, we restate our findings in the main proposition for these two scenarios below.

Remark 2.2 *The English auction is always strictly suboptimal for the auctioneer, except for the no time cost case for the auctioneer and the bidders.*

Remark 2.3 *Under exponential time cost, Istanbul Flower Auctions are weakly better than Dutch auctions and strictly better than English auctions for the auctioneer when at least one party incurs time cost. The indifference can only occur when the auctioneer has no time cost.*

Remark 2.4 *Under hyperbolic time cost, Istanbul Flower Auctions are strictly better than Dutch auctions and English auctions for the auctioneer when at least one party incurs time cost.*

2.3.2 Expected utilities for the bidders

Analogously, we can compare the ex-ante expected utility for bidders by studying its first-order derivative at $s = 0$ and $s = 1$. Although the optimality of the starting price is usually not aligned between the auctioneer and the bidders,[5] we demonstrate that bidders with time costs can benefit from a certain starting price in $(0, 1)$; that is, they prefer some specifications of Istanbul Flower Auctions over the Dutch and English auctions.

Proposition 2.3 *We have:*

(a) When bidders have time costs that satisfy Assumption 2.1(b) or 2.1(c), there exists some starting price $\hat{s}_1 < 1$ such that Istanbul flower auction is strictly better than Dutch auctions for the bidders, and this equilibrium is non-trivial, i.e., there exists $\hat{s}_1 < \tilde{s}$ such that $EU_B^F(\hat{s}_1) > EU_B^D$ when $c'_B(t) < 0$.

(b) When bidders have time costs, there exists some starting price $\hat{s}_2 < 1$ such that Istanbul flower auction is strictly better than English auctions for the bidders, i.e., there exists $\hat{s}_2 > 0$ such that $EU_B^F(\hat{s}_2) > EU_B^E$ when $c'_B(t) < 0$.

We verify stronger results regarding the bidder's preference of Istanbul Flower Auctions over Dutch auctions numerically in the next section, for the two functional formats of the time cost that we focus on.[6] In fact, the utility function of the bidders with respect to the starting price s always follows the same pattern of increasing and then decreasing. We know that $s = 0$ is equivalent to the English auction and $s = 1$ is equivalent to the Dutch auction; therefore, the Istanbul Flower Auction is the preferred choice for the bidders in these scenarios.

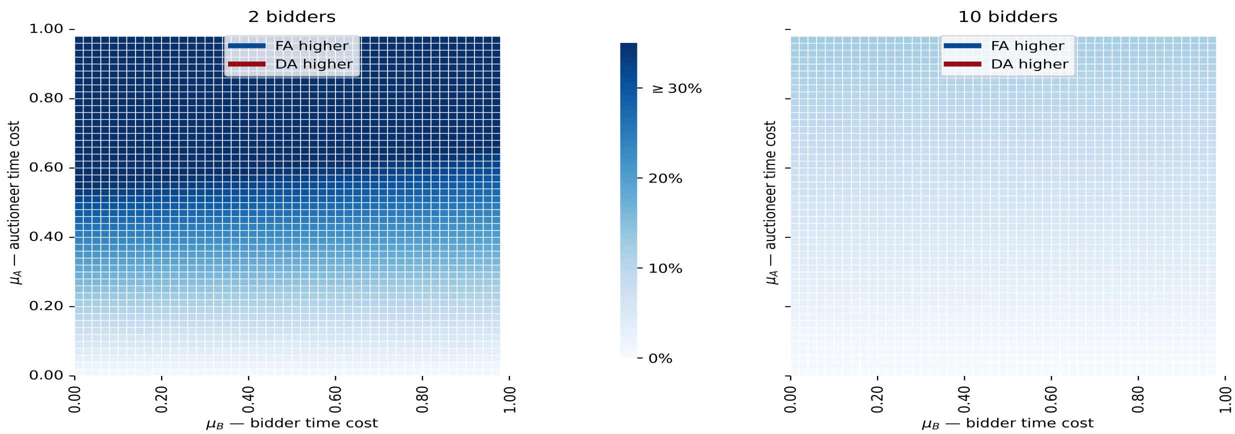
2.4 Numerical Analysis

We conduct a numerical analysis to evaluate the performance of Istanbul Flower Auctions and Dutch auctions, focusing on two specific time cost functions—exponential cost and hyperbolic cost, varying time costs of the auctioneer and the bidders. Our findings indicate that opting for the Istanbul Flower Auction format is more advantageous for both the auctioneer and the bidders, and it becomes significantly more favorable when both the auctioneer and the bidders have time costs. Furthermore, the main factor driving the preference for Istanbul

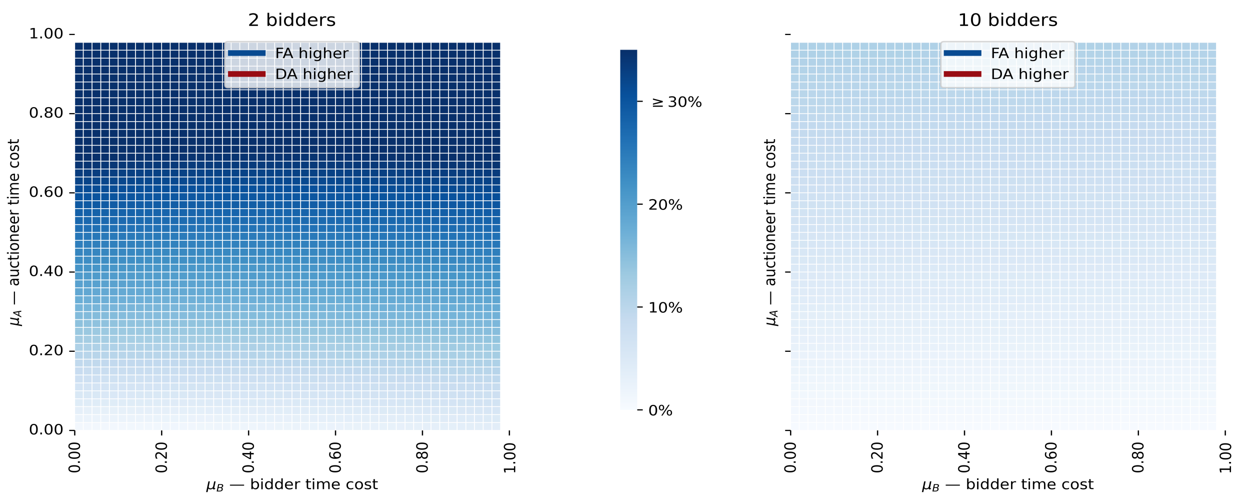
[5]In the optimal Istanbul flower auction, the auctioneer would choose the starting price to maximize her own utility, not the bidders' expected utilities.

[6]In the next section, we calculate the expected utilities of the bidders for the starting price that is chosen to maximize the auctioneers expected utility.

Flower Auctions for the auctioneer (bidders) stems from the auctioneer's (bidders') own time costs, showing minimal influence from other auction characteristics. Lastly, we observe that Istanbul flower auctions' relative advantage over Dutch auctions does not quickly fade when we have more bidders.



(a) Exponential time cost



(b) Hyperbolic time cost

Figure 2.6: Expected auctioneer utility differences of Istanbul Flower Auctions and Dutch auctions

2.4.1 Utility for the auctioneer

As illustrated in Figure 2.6, Istanbul Flower Auctions is strictly better than Dutch auctions in terms of the expected utility for the auctioneer, except for the auctioneer with no time cost (who is indifferent between the two auction formats.) The relative difference between Istanbul Flower Auctions and Dutch auctions becomes significantly larger when the time

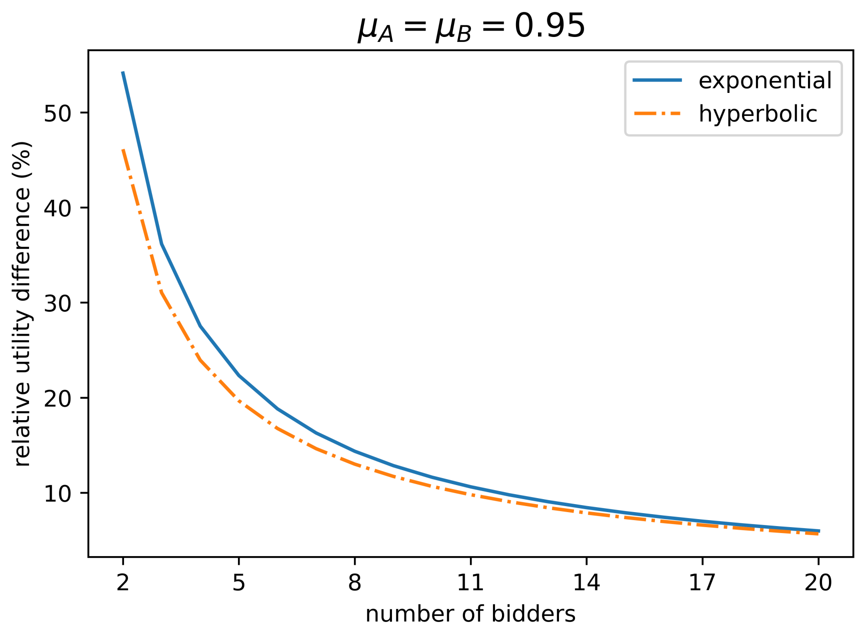


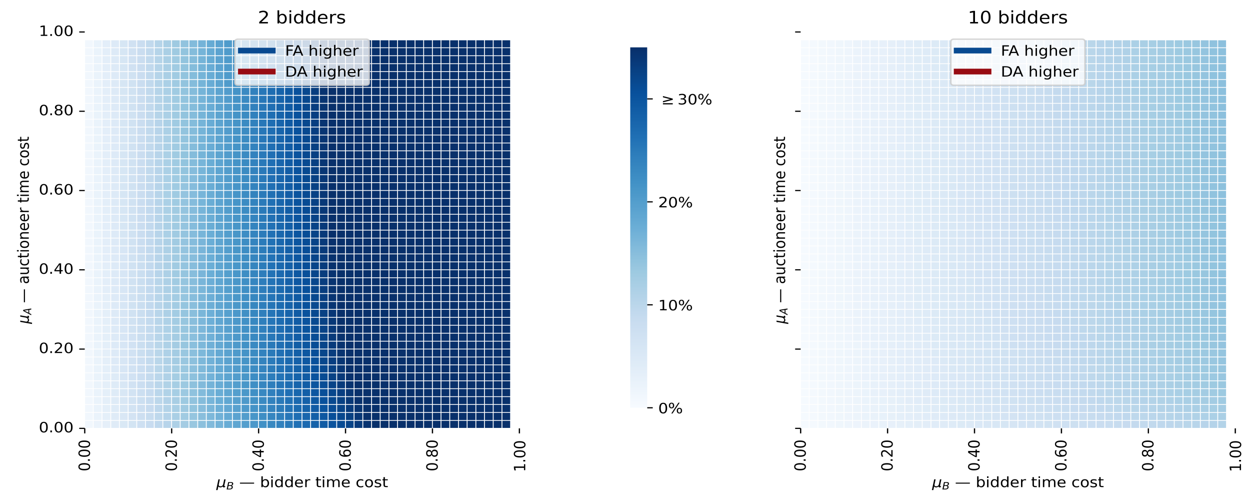
Figure 2.7: FA auctioneer benefit under high time cost

cost of the auctioneer is higher.

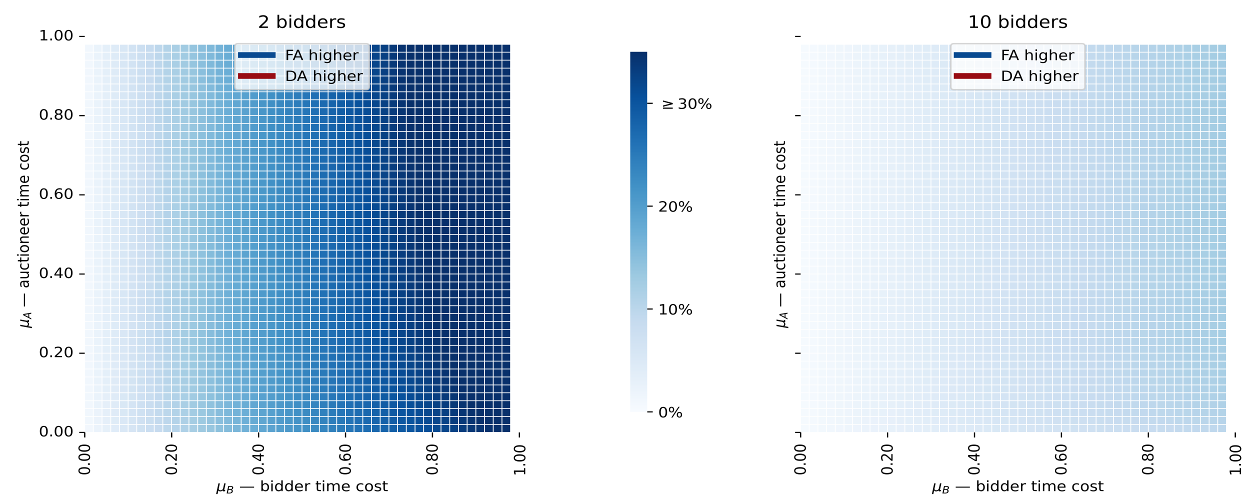
When there is more competition in the auction, i.e., more bidders, we have the common observation that the relative advantage of Istanbul Flower Auctions diminishes, since more competition leads to quicker bids, and therefore mitigates the effect of time cost. However, the relative advantage of Istanbul Flower Auctions does not quickly attenuate to near zero within a practical range of bidder numbers, as long as the auctioneer time cost is high. For example (see Figure 2.7), when the time cost parameters are set to 0.95, which means that a unit of value depreciates almost half after a unit of time elapsed under hyperbolic cost and depreciates even more under exponential cost, the auctioneer utility is about 50% higher in 2-bidder auctions and is still at least 10% higher in 10-bidder auctions when compared to Dutch auctions.

The impact of the bidder time cost on the difference in utility for the auctioneer is minimal. If we keep the auctioneer time cost constant, the range of variation in the auctioneer utility difference due to different bidder time costs is always less than 25% of the maximum difference within the parameter range $\mu \in [0, 1]$ that we are examining. The profit that the auctioneer gains from Istanbul Flower Auctions consistently decreases as the bidders' costs increase under exponential cost. However, under hyperbolic cost, the auctioneer's benefit may either decrease or first decrease and then increase.

These observations suggest that the auctioneer's time cost significantly influences their preference for Istanbul Flower Auctions. If the auctioneer values the speed of the auctions,



(a) Exponential time cost



(b) Hyperbolic time cost

Figure 2.8: Expected bidder utility differences of Istanbul Flower Auctions and Dutch auctions

they are inclined to choose Istanbul Flower Auctions over Dutch auctions, irrespective of the number or type of bidders present.

2.4.2 Utility for bidders

Although the initial price set by the auctioneer may not be ideal for the bidders, Istanbul Flower Auctions are consistently more advantageous than Dutch auctions in terms of the expected utility for bidders in all scenarios depicted in Figure 2.8, except for bidders with no time costs, who are indifferent between the two auction formats. The relative advantage of Istanbul Flower Auctions significantly increases when the time cost of bidders is higher.

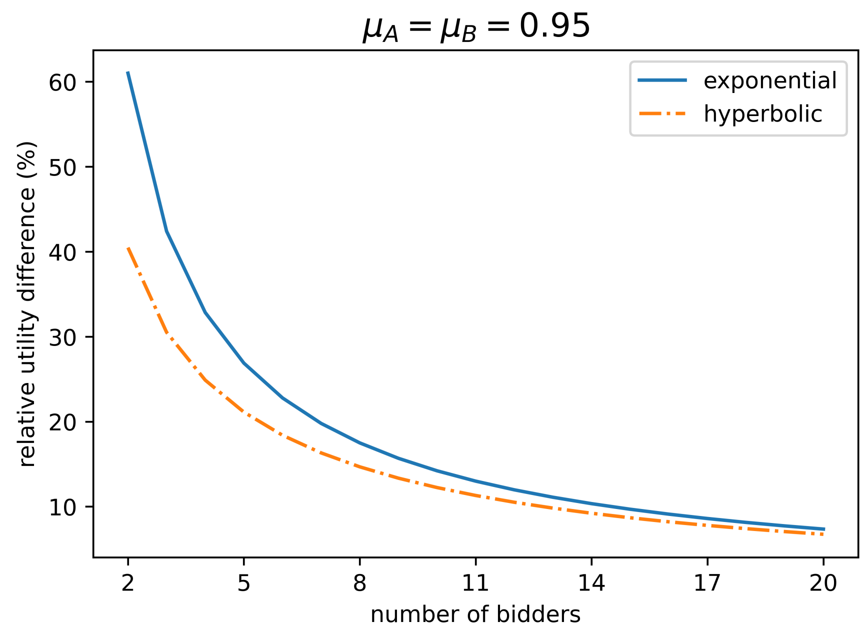


Figure 2.9: FA bidder benefit under high time cost

When varying the number of bidders, we observe that the relative advantage of Istanbul Flower Auctions becomes smaller in a more competitive market. It is important to highlight that this benefit remains significant within a practical range of bidders, provided that the cost of time for bidders is substantial. For example, using the time cost scenario from the preceding subsection, the bidder's benefit exceeds 40% in 2-bidder auctions and remains above 10% even in 10-bidder auctions when switching from the Dutch auction format to the Istanbul Flower Auction format.

The time cost of the auctioneer does not play an important role in the bidder utility difference. When fixing the bidder time cost, the range of variation in the bidder utility difference due to different auctioneer time costs is always within 25% of the maximum difference within the parameter range $\mu \in [0, 1]$. Moreover, the difference in bidder utility consistently becomes smaller as the time cost of the auctioneer increases under either time discounting function we consider.

Similar to our findings on the auctioneer utility difference, these observations suggest that the time cost of the bidder herself is the primary factor that influences her preference for Istanbul Flower Auctions, regardless of the number of bidders or the time cost of the auctioneer.

2.5 Conclusion

Our analysis reveals that the Istanbul Flower Auction format, with its innovative approach to combining Dutch and English auction elements, offers distinct advantages in achieving higher utilities for both auctioneers and bidders. This format proves particularly beneficial in markets characterized by high time sensitivity, such as perishable goods auctions. The flexibility to switch between auction types based on initial bidding activity allows for more efficient price discovery and less time costs. It can lead to higher utility for all parties involved, especially under conditions of time cost. The numerical findings suggest that the auctioneer's or bidder's preference for the Istanbul Flower Auction largely depends on their own time costs. This insight aligns with real-world scenarios where sellers need to sell goods swiftly. Their urgency justifies the use of the Istanbul Flower Auction regardless of the buyers' perspectives. Moreover, the auction is beneficial for all parties: it does not disadvantage bidders even when they incur no cost by participating, supporting the adoption of the Istanbul Flower Auction in the Istanbul flower market.

These findings suggest that adopting flexible auction formats like the Istanbul Flower Auction could enhance outcomes for all the parties involved in various auction-based markets, particularly those where the speed of the auction is important due to the large number of items needing to be sold in a fixed amount of time.

Appendix

2.A Omitted proofs

Proof of Lemma 2.1. In contrast to the Dutch auction setting where the price continuously declines from 1 and we can solve for a strictly increasing bidding function; the bidding function here may be capped by the starting price s when the bidder's private value exceeds certain cutoff $\lambda(s) \geq s$, i.e., Therefore, the bidding function for the subsequent Dutch auction is given by

$$\beta(v,s) = \begin{cases} b(v,s) & v \leq \lambda(s) \\ s & \lambda(s) \leq v \leq p(s) \end{cases}$$

and the bidder's expected utility is given by

$$EU_B^{FD}(v,s) = \begin{cases} c_B(s - b(v,s))(v - b(v,s))G(v) & v \leq \lambda(s) \\ \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{v-s}{k+1} F^{n-1-k}(\lambda(s))[F(p(s)) - F(\lambda(s))]^k & \lambda(s) \leq v \leq p(s) \end{cases}$$

where $b(\cdot, s) \to [0, s]$ is a strictly increasing and differentiable function with $b(0) = 0$ given the starting price s.

Step 1 We prove $\lambda(s) = p(s)$ in two cases, depending on whether the subsequent auction is possible to be an English auction.

Let's start from the case $p(s) < 1$, where both Dutch and English can be the subsequent auction format. Suppose $\lambda(s) < p(s)$, a bidder with value $v = p(s)$ should be indifferent between initially not bidding or bidding at the starting price s in equilibrium, that is

$$EU_B^{FD}(p(s), s) = EU_B^{FE}(p(s), s).$$

We know that

$$EU_B^{FE}(p(s), s) = (p(s) - s)G(p(s))$$

and

$$\begin{aligned} EU_B^{FD}(p(s), s) &= \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{p(s) - s}{k+1} F^{n-1-k}(\lambda(s))[F(p(s)) - F(\lambda(s))]^k \\ &= \frac{p(s) - s}{n} \frac{F^n(p(s)) - F^n(\lambda(s))}{F(p(s)) - F(\lambda(s))} \end{aligned}$$

Let's define an auxiliary function

$$k(x) = \frac{1 - x^n}{n(1-x)} = \frac{1 + x + \cdots + x^{n-1}}{n}.$$

It is obvious that $k'(x) > 0$ for $x \in (0,1)$, and $k(1) = 1$. Then notice that $\frac{F(\lambda(s))}{F(p(s))} \in (F(\lambda(s)), 1)$ and

$$k\left(\frac{F(\lambda(s))}{F(p(s))}\right) = \frac{EU_B^{FD}(p(s), s)}{EU_B^{FE}(p(s), s)},$$

we conclude that there is no equilibrium such that $\lambda(s) < p(s)$ when $p(s) < 1$.

Now let's look into the case $p(s) = 1$, where Istanbul flower auctions become Dutch auctions starting at the given price s. Suppose $\lambda(s) < p(s)$, any Dutch bidder whose bid is capped at s will deviate to bidding at the beginning, because she will win the item at the price s for sure in the English auction instead of possibly competing with other bidders in the Dutch auction.

Step 2 Given that $\lambda(s) = p(s)$ in equilibrium, when $p(s) < 1$, use the indifference condition at $v = p(s)$ again, we have

$$\begin{aligned} EU_B^{FD}(p(s), s) &= EU_B^{FE}(p(s), s) \\ c_B(s - b(p(s), s))(p(s) - b(p(s), s))G(p(s)) &= (p(s) - s)G(p(s)) \\ c_B(s - b(p(s), s))(p(s) - b(p(s), s)) &= p(s) - s \end{aligned}$$

By definition of the time cost function $c_B(0) = 1$, so $b(p(s), s) = s$ is a solution. Let's define an auxiliary function

$$l(p) = p - s - c_B(s - b(p, s))(p - b(p, s)).$$

Compute its first-order derivative and reformat it with the ordinary differential equation derived for $b(\cdot, s)$, we have

$$\begin{aligned} l'(p) &= 1 - \left[-c_B'(s - b(p, s))\frac{\partial b(p, s)}{\partial p}(p - b(p, s)) + c_B(s - b(p, s))(1 - \frac{\partial b(p, s)}{\partial p})\right] \\ &= 1 - c_B(s - b(p, s))\left[1 - \frac{g(p)}{G(p)}(p - b(p, s))\right] \\ &> 0 \end{aligned}$$

because $c_B(\cdot) \in [0,1]$ and $p \geq b(p, s)$. Therefore, $l(p)$ is strictly increasing in p, and the solution to $l(p) = 0$ is unique, which is implicitly given by $b(p, s) = s$.

When $p(s) = 1$, then we do not necessarily have $b(p, s) = s$, but the following must be true $b(p, s) \leq s$.

□

Proof of Lemma 2.2. Starting from the Dutch bidding strategy in equation 2.1, we take the partial derivative with respect to s on both sides, we get:

$$\frac{\partial^2 b(v,s)}{\partial s \partial v} = \frac{g(v)}{G(v)}\Big(\frac{-\frac{\partial b}{\partial s}}{(v-b(v,s))\frac{c_B'(s-b(v,s))}{c_B(s-b(v,s))}+1} + (v-b(v,s))\Big(-\frac{-\frac{\partial b}{\partial s}\frac{c_B'(s-b(v,s))}{c_B(s-b(v,s))} + (v-b(v,s))\cdot\frac{\partial \frac{c_B'(s-b(v,s))}{c_B(s-b(v,s))}}{\partial s}}{[(v-b(v,s))\frac{c_B'(s-b(v,s))}{c_B(s-b(v,s))}+1]^2}\Big)\Big)$$

To simplify, let $\gamma(v,s) := \frac{c_B'(s-b(v,s))}{c_B(s-b(v,s))}$, the above equation turns into:

$$\frac{\partial^2 b(v,s)}{\partial s \partial v} = \frac{g(v)}{G(v)}\left(\frac{-\frac{\partial b}{\partial s}}{(v-b(v,s))\gamma(v,s)+1} + (v-b(v,s))\left(-\frac{-\frac{\partial b}{\partial s}\gamma(v,s) + (v-b(v,s))\cdot\frac{\partial\gamma(v,s)}{\partial s}}{[(v-b(v))\gamma(v,s)+1]^2}\right)\right)$$

and

$$\frac{\partial\gamma(v,s)}{\partial s} = (1-\frac{\partial b}{\partial s})\frac{d\gamma}{d(s-b(v))}$$

Then,

$$\frac{\partial^2 b(v,s)}{\partial s \partial v} = \frac{g(v)}{G(v)}\frac{-\frac{\partial b}{\partial s} - (v-b(v))^2(1-\frac{\partial b}{\partial s})\frac{d\gamma}{d(s-b(v))}}{[(v-b(v))\gamma(v,s)+1]^2}$$

Note $\frac{\partial b}{\partial s}$ is a function of v, assuming $b(v, s)$ is well-behaved such that we could interchange the order of partial derivative, then $\frac{\partial b}{\partial s}$ is determined by a differential equation:

$$\frac{d\frac{\partial b(v,s)}{\partial s}}{dv} = \frac{g(v)}{G(v)}\frac{-\frac{\partial b}{\partial s} - (v-b(v))^2(1-\frac{\partial b}{\partial s})\frac{d\gamma}{d(s-b(v))}}{[(v-b(v))\gamma(v,s)+1]^2}$$

Since we focus on the class of symmetric increasing bidding function, we have $b(0) \equiv 0$ as the bidder with the lowest type has no interest in either overbidding or underbidding. It follows that $\frac{\partial b(v,s)}{\partial s}|_{v=0} = 0$ is the initial condition of the differential equation. It is obvious when $\frac{\partial b(v)}{\partial s} \to 1$ at certain point, $\frac{d\frac{\partial b(v,s)}{\partial s}}{dv} < 0$ at that point. Therefore, the curve $\frac{\partial b}{\partial s}(v)$ could never reach the border of $\frac{\partial b(v)}{\partial s} = 1$ when the initial condition starts under the line.

□

Proof of Lemma 2.3. The starting price s functions as a cap on the Dutch phase bids. By Lemma 2.2, we know that the Dutch phase bidding curve will never move upward faster than the cap s. It is also evident that $b(1, s) < 1$. Therefore, when increasing s from 0 to 1, it departs upwards from the Dutch phase bidding curve and finally becomes non-restrictive to the Dutch phase bidding strategy after reaching some interim $\tilde{s}$. By Lemma 2.1, we can solve $\tilde{s}$ from $b(1, \tilde{s}) = \tilde{s}$, and use the property of the $l(p)$ function defined in the proof of the lemma to prove that $p(s) \equiv 1$ for all $s \in [\tilde{s}, 1]$.

Then, let us consider $s \in [0, \tilde{s})$ where the Dutch phase bids of high-value bidders are capped. By Lemma 2.1 we know that

$$b(p(s), s) = s.$$

Taking the derivative with respect to s on the above equation, we have

$$\frac{\partial b(v,s)}{\partial v}\frac{dp(s)}{ds} + \frac{\partial b(v,s)}{\partial s} = 1.$$

We are looking for a strictly increasing Dutch bidding strategy, which means $\frac{\partial b(v,s)}{\partial v} > 0$. By Lemma 2.2, we have $\frac{\partial b(v,s)}{\partial s} < 1$. Therefore, we know that

$$\frac{dp(s)}{ds} > 0.$$

□

Proof of Lemma 2.4. We first investigate the properties of the Dutch phase bidding function at the intersection point if two bidding functions with different s possibly intersect with each other. Let us write the Dutch ODE in the following form:

$$\frac{G(v)}{g(v)} = \frac{v - b(v,s)}{\frac{\partial b(v,s)}{\partial v}} \frac{1}{(v - b(v,s))\frac{c_B'(s - b(v,s))}{c_B(s - b(v,s))} + 1}.$$

Recall that we focus on such a Dutch phase bidding strategy that is strictly increasing in the item value v. Therefore, the second denominator must be positive for any v and s in equilibrium. Consider two starting prices s_1 and $s_2 > s_1$. Suppose there exists a $v_0 \in (0, 1]$ such that $b(v, s_1) = b(v, s_2) = b_0$, then we have

$$\frac{v_0 - b_0}{\frac{\partial b(v_0,s_1)}{\partial v}} \frac{1}{(v_0 - b_0)\frac{c_B'(s_1 - b_0)}{c_B(s_1 - b_0)} + 1} = \frac{v_0 - b_0}{\frac{\partial b(v_0,s_2)}{\partial v}} \frac{1}{(v_0 - b_0)\frac{c_B'(s_2 - b_0)}{c_B(s_2 - b_0)} + 1}.$$

Then, the comparison between $\frac{\partial b(v_0,s_1)}{\partial v}$ and $\frac{\partial b(v_0,s_2)}{\partial v}$ is determined by the relationship between $\frac{c_B'(s_1-b_0)}{c_B(s_1-b_0)}$ and $\frac{c_B'(s_2-b_0)}{c_B(s_2-b_0)}$. We also know that $s_1 < s_2$, so we have the following result:

1. $\frac{\partial b(v_0,s_1)}{\partial v} < \frac{\partial b(v_0,s_2)}{\partial v}$ under condition (a).

2. $\frac{\partial b(v_0,s_1)}{\partial v} = \frac{\partial b(v_0,s_2)}{\partial v}$ under condition (b).

3. $\frac{\partial b(v_0,s_1)}{\partial v} > \frac{\partial b(v_0,s_2)}{\partial v}$ under condition (c).

Then, we investigate the properties of the Dutch phase bidding function at the left end 0. Apply Euler's method to the ODE with the initial condition $b(0,s) = 0$, for $h \to 0$ we have

$$b(h,s) \approx b(0,s) + h\frac{\partial b(0,s)}{\partial v} = 0.$$

Plug this into the ODE we have

$$\frac{\partial b(h,s)}{\partial v} \approx \frac{g(h)}{G(h)} \frac{1}{\dfrac{c_B'(s)}{c_B(s)} + \dfrac{1}{h}}.$$

Taking the limit we get $\frac{\partial b(0,s)}{\partial v}$. Similarly, the comparison between $\frac{\partial b(0,s_1)}{\partial v}$ and $\frac{\partial b(0,s_2)}{\partial v}$ is determined by the relationship between $\frac{c_B'(s_1)}{c_B(s_1)}$ and $\frac{c_B'(s_2)}{c_B(s_2)}$. Given that $s_1 < s_2$, we have the following result:

1. $\frac{\partial b(0,s_1)}{\partial v} < \frac{\partial b(0,s_2)}{\partial v}$ under condition (a).

2. $\frac{\partial b(0,s_1)}{\partial v} = \frac{\partial b(0,s_2)}{\partial v}$ under condition (b).

3. $\frac{\partial b(0,s_1)}{\partial v} > \frac{\partial b(0,s_2)}{\partial v}$ under condition (c).

Finally, use the ODE initial condition $b(0,s_1) = b(0,s_2) = 0$ together with the above results, we conclude that:

1. $b(v,s_1) \leq b(v,s_2)$ for any $v \in (0,1]$, or equivalently $\frac{\partial b(v,s)}{\partial s} \geq 0$, under condition (a).

2. $b(v,s_1) = b(v,s_2)$ for any $v \in (0,1]$, or equivalently $\frac{\partial b(v,s)}{\partial s} = 0$, under condition (b).

3. $b(v,s_1) \geq b(v,s_2)$ for any $v \in (0,1]$, or equivalently $\frac{\partial b(v,s)}{\partial s} \leq 0$, under condition (c).

Note that we only have a weak monotonicity here because we cannot exclude the possibility that the two bidding functions are tangent to each other at some point.

□

Proof of Proposition 2.2. **(a)** By Lemma 2.1 and Lemma 2.3, we either have

$$s \in [0, \tilde{s}), p(s) < 1, \frac{dp(s)}{ds} > 0, b(p(s), s) = s$$

or have

$$s \in (\tilde{s}, 1], p(s) = 1, \frac{dp(s)}{ds} = 0, b(p(s), s) < s.$$

Specifically, $p(s)$ is not differentiable at $\tilde{s}$: the left-sided limit belongs to the first case, and the right-sided limit belongs to the second case. Plugging these into equation 2.2, the first-order derivative of the auctioneer utility for $s \in [\tilde{s}, 1)$ is given by

$$\frac{dEU_A^F(s)}{ds} = \int_0^1 \left[c_A'(s - b(v,s))(1 - \frac{\partial b(v,s)}{\partial s}) b(v,s) + c_A(s - b(v,s)) \frac{\partial b(v,s)}{\partial s} \right] dF^n(v).$$

Given that the auctioneer has a time cost, that is, $c_A'(t) < 0$, and our assumption that $\frac{\partial b(v,s)}{\partial s} \leq 0$, we finally have

$$\frac{dEU_A^F(s)}{ds} < 0, s \in [\tilde{s}, 1).$$

Therefore, we must have $EU_A^F(s^*) > EU_A^F(s) \geq EU_A^F(1) = EU_A^D$ for any $s \in [\tilde{s}, 1]$. It is obvious that $s^* < \tilde{s}$, and $s^* > 0$ is proved in **(d)**, so the optimal Istanbul Flower Auction is non-trivial.

(b) Again, by Lemma 2.1 and Lemma 2.3, we can compute that for the auctioneer with no time cost $c_A(t) \equiv 1$, when $s \in (0, \tilde{s})$, the first and fourth terms on the right-hand side of equation 2.2 are zeros, the sum of the second and third terms is positive because we can prove that $\frac{dp(s)}{ds}(p - s) < \frac{F(p(s))}{(n-1)f(p(s))}$ by taking the derivative of s on both sides of the equation $b(p(s), s) = p(s) - \int_0^{p(s)} \frac{G(y)c_B(s-b(y,s))}{G(p)} dy = s$, and the last term is simplified to

$$\int_0^{p(s)} \frac{\partial b(v,s)}{\partial s} dF^n(v).$$

Given our assumption that $\frac{\partial b(v,s)}{\partial s} \geq 0$, we finally have

$$\frac{dEU_A^F(s)}{ds} > 0, s \in (0, \tilde{s}).$$

When $s \in (\tilde{s}, 1)$, the only difference is that the first four terms are all zeros, and finally we have

$$\frac{dEU_A^F(s)}{ds} \geq 0, s \in (\tilde{s}, 1).$$

Therefore, we must have $EU_A^D = EU_A^F(1) = EU_A^F(s^*) > EU_A^F(s)$ for any $s \in [0, \tilde{s})$. It is obvious that $s^* \geq \tilde{s}$, which means that the optimal Istanbul Flower Auction is trivial.

(c) Again, by Lemma 2.1 and Lemma 2.3, we can compute that for the auctioneer with no time cost $c_A(t) \equiv 1$, when $s \in (\tilde{s}, 1)$, the first four terms on the right-hand side of equation 2.2

are all zeros, and the last term becomes

$$\int_0^{p(s)} \frac{\partial b(v,s)}{\partial s} dF^n(v).$$

Given our assumption that $\frac{\partial b(v,s)}{\partial s} < 0$, we finally have

$$\frac{dEU_A^F(s)}{ds} < 0, s \in (\tilde{s}, 1).$$

Therefore, we must have $EU_A^F(s^*) > EU_A^F(s) \geq EU_A^F(1) = EU_A^D$ for any $s \in (\tilde{s}, 1]$.
(d) When the auctioneer has time cost, that is, $c'_A(t) < 0$, using equation 2.2 we can compute that

$$\frac{dEU_A^F(0)}{ds} = -\int_0^1 c'_A(x,s)xh(v,x)dxdv < 0.$$

Therefore, we must have

$$EU_A^F(s^*) > EU_A^F(0) = EU_A^E.$$

When the auctioneer has no time cost, that is, $c_A(t) \equiv 1$, we can write the Dutch phase bidding function as

$$b(v,s) = v - \int_0^v \frac{G(y)c_B(s - b(y,s)}{G(v)c_B(s - b(v,s)} dy.$$

Let us denote the special case of no bidder time cost by subscript 0. We have

$$b_0(v,s) = v - \int_0^v \frac{G(y)}{G(v)} dy < b(v,s).$$

Then, for the Dutch auction, we have

$$EU_A^D = EU_A^F(1) = \int_0^1 b(v,1)dF^n(v) > \int_0^1 b_0(v,1)dF^n(v) = EU_{A0}^D.$$

For the English auction, since the winner will always pay the second highest item value and the auctioneer has no time cost, the expected utility of the auctioneer is always the same, which is given by

$$EU_A^E = EU_A^F(0) = \int_0^1 \int_0^v xh(v,x)dxdv = EU_{A0}^E.$$

By the Revenue Equivalence Principle, we have

$$EU_{A0}^E = EU_{A0}^D = \int_0^1 \left[v - \frac{1-F(v)}{f(v} \right] dF^n(v).$$

Therefore, we must have

$$EU_A^F(s^*) \geq EU_A^F(1) > EU_A^F(0) = EU_A^E.$$

$\square$

Proof of Proposition 2.3. **(a)** For $s \in (\tilde{s}, 1)$, we can compute that

$$\frac{dEU_B^F(s)}{ds} = \int_0^1 \Big[c_B'(s - b(v,s))(1 - \frac{\partial b(v,s)}{\partial s})(v - b(v,s)) - c_B(s - b(v,s))\frac{\partial b(v,s)}{\partial s} \Big] G(v)dF(v).$$

Using the assumption together with Lemma 2.2, we know that $\frac{\partial b(v,s)}{\partial s} \in [0,1)$. Then, for bidders with time costs, we finally have

$$\frac{dEU_B^F(1)}{ds} < 0, s \in (\tilde{s}, 1).$$

Therefore, there exists $\hat{s_1} < \tilde{s}$ such that $EU_B^F(\hat{s_1}) > EU_B^F(1) = EU_B^D$.
(b) We can compute that

$$\frac{dEU_B^F(0)}{ds} = -\int_0^1 \int_0^v c_B'(x)(v - x)dG(x)dF(v).$$

Then, for bidders with time costs, we always have

$$\frac{dEU_B^F(0)}{ds} > 0.$$

Therefore, there exists $\hat{s_2} > 0$ such that $EU_B^F(\hat{s_2}) > EU_B^F(0) = EU_B^E$. $\square$

Chapter 3

Repeated Gambles with Bundled Options: A Behavioural Model of Probability Misperception

3.1 Introduction

Gacha games are video games that use a toy vending machine mechanic where players typically use real money to purchase random draws of virtual items. These free-to-play games with in-game purchases currently dominate the mobile games market. As stated in the recent report *Digital Games and Interactive Media: 2020 Year in Review* [1], free-to-play games account for 78% of digital games revenue, with mobile games contributing 75% of the total $98.4 billion revenue generated by free-to-play games. The top two free-to-play titles, both published by Tencent, implement the Gacha mechanic in purchasing character outfits and accessories. In fact, the list of popular mobile games is dominated by those published by Asian companies, with Gacha mechanics being implemented in almost all popular mobile games from Chinese and Japanese developers. The question arises as to how Gacha mechanics attract players, generate profits and become prevalent in mobile games. To answer this question, I use a behavioural model that captures several important features in Gacha games and looks for possible explanations.

These games can be modelled as repeated fair gambles. The fairness comes from the rapid development of online trading platforms, where players can easily trade their game accounts or even in-game items, and evaluate those virtual items accurately through a large number of available quotes. According to early behavioural research, such as the experimental results in Tversky and Kahneman (1992), fair gambles are usually rejected. However, this is not reflective of the reality in Gacha games, as numerous players engage in excessive draws to obtain desired items, resulting in regretful outcomes and a negative net gain. This

[1] published by *SuperData*

inconsistency suggests that these players may be irrational (naive) during gameplay, and their behavioural pattern can be linked to the gambler's fallacy or, more generally, the belief in "the law of small numbers" (Tversky and Kahneman, 1971), which is supported by a wealth of evidence in the literature. The gambler's fallacy is the mistaken belief that past outcomes affect future probabilities in a game of chance. This can lead naive players to perceive winning as more likely after a losing streak, encouraging them to continue playing in order to not miss out on a chance to win. This misperception can also be seen as an attempt to bring the sample distribution back to the population distribution, in accordance with the law of small numbers. Thus, I propose several justifiable patterns for the perceived probabilities of naive players during the game without specifying their functional forms.

Gacha games usually provide two features: (1) free draws at the start of the game, and (2) the option to make a certain number of consecutive draws or a single draw on the virtual vending machine interface. This model explains how these features generate profits for the game provider.

Firstly, to entice naive players to start a game that may otherwise be unattractive, a sample that sufficiently does not resemble the population distribution is necessary. Free initial rounds can effectively generate such a sample, resulting in highly misperceived probabilities for some naive players. By transforming non-participants into participants of the game, and considering the low cost of providing virtual rewards, offering free initial rounds can be a profitable choice for the game provider.

Secondly, as the optimisation problem for this game is highly complex, players may only be able to consider a limited number of future rounds. This narrow-bracketing behaviour further categorises them.

My analysis begins with the simplest scenario where the naive 1-planning player is going to participate in 50-50 fair gambles with 2-draw bundles. Proposition 3.1 gives the baseline gain-exit behavioural pattern (continue gambling when overall winning but exit the game when losing) when the bundled option is not introduced, which holds true for all reasonable parameter specifications. Proposition 3.2 then examines the effect of making the 2-draw bundles available in the game, indicating that the naive 1-planning player will be incentivised to continue playing for a longer period of time compared to the baseline result.

Then, I demonstrate that the two propositions for 50-50 fair gambles can each be extended to a typical class of skewed fair gambles, in which the player gains m with probability $\frac{1}{m+1}$ and loses 1 with probability $\frac{1}{m+1}$, at the cost of imposing a restriction on the loss-aversion parameter. However, the generality of these two follow-up propositions is only minimally affected, in that almost all parameter estimations in the literature satisfy the assumption.

The increase in the game provider's profit after introducing the 2-draw bundles is naturally deduced from the aforementioned behaviour patterns of the player. Bundling two rounds together increases the likelihood that a naive 1-planning player continues to play by exacerbating their probability misperception. However, this effect does not apply to other players who plan a longer period of time.

For better understanding, all of these findings are further examined through numerical analysis.

The final step involves examining scenarios in which the naive player plans for more rounds or the game provider offers the option for more bundled rounds. I find hierarchical relationships among players grouped by different bracketing levels. Broader-bracketing naive players are more likely to continue playing, making them more lucrative for the game provider. This is because all possible plans considered by the player who plans for a short term will be encompassed by the player who plans for a longer term.

The paper is organised as follows. In Section 3.2, I present the theoretical model of Gacha games with naive players and bundled options. The main body of my analysis, including both analytical and numerical findings for naive 1-planning players provided with 2-draw bundles, is presented in Section 3.3. The general patterns for naive N-planning players are presented in Section 3.4. I conclude in Section 3.5.

3.1.1 Literature review

My model is largely inspired by Barberis (2012). In his model of casino gambling, a player optimises a plan of actions for all possible subsequent gambles using cumulative prospect theory (CPT) value functions (Tversky and Kahneman, 1992) to evaluate outcomes. Depending on the specification of CPT parameters, the player can either start with a loss-exit plan (continue gambling when overall winning but exit the game when losing) and end up playing a gain-exit strategy, or the reverse, in 50-50 fair gambles. Ebert and Strack (2015) pointed out that the CPT probability weighting leads to unrealistic never-stopping results for small and skewed gambles. My model produces a middle-ground result between ambiguous parameter dependency and strict never-stopping behaviour by first proposing properties based on the law of small numbers to replace the CPT probability weighting, and second simplifying the planning process by narrow bracketing.

There is a substantial body of literature on "the law of small numbers" originating from Tversky and Kahneman (1971). This law essentially describes the tendency to overestimate how a probability distribution in a small sample will resemble the population distribution, and has been linked to the gambler's fallacy later. Clotfelter and Cook (1993) and Terrell (1994) provided compelling evidence for the existence of the gambler's fallacy by examining state-run lotteries. Rabin (2002) modelled a believer in the law of small numbers who mistakenly assumes that when drawing from finite signals representing the population distribution, some draws are taken with replacement while others are not. However, there is no definite evidence on the functional form of misperceived probabilities. My model allows for flexibility by assuming only several justifiable properties based on these evidences.

The concept of "narrow barcketing" is firstly introduced in Tversky and Kahneman (1981) to describe the mental procedure of a decision maker who faces multiple decisions and handles them separately. In my model, the planning and optimisation processes can be extremely complex. Therefore, players are assumed to separate future decisions into those that are close and far away, handling the former first by only planning for a limited number of rounds. Empirical research suggests that individuals' decision-making behaviour can vary depending on whether they are making decisions in a combined or separated manner (Simonson, 1990; Gourville, 1998; Gneezy and Potters, 1997). I examine the effect of offering a broader plan to the narrow-bracketing player, and find that it does change the player's behavioural pattern.

The mechanism of Gacha games is not well-studied, as the mobile game industry has only thrived recently. To the best of my knowledge, Gan (2022) is the only existing analytical study based on prospect theory preferences that discusses the optimality of this mechanism. Gan demonstrated that, in the presence of probability weighting, the optimality of selling "loot boxes" with or without the worst-case insurance is determined by whether the player is sophisticated or naive about her own inconsistency. Chen and Fang (2023) modelled the Gacha game differently as a Stackelberg game and demonstrated that the revenue-optimal design is equivalent to the single-item single-bidder Myerson auction.

3.2 Model

The game conforms to the following structure: there exists a constraint T specifying the maximum number of rounds that can be undertaken, with each round of gamble adhering to identical parameters. At the beginning of each round, the player can opt for the acceptance or rejection of the current round of gamble without any cost. Termination of the game occurs if the player declines participation in the current round or if the maximum allowable rounds have been exhausted. Alternatively, the game proceeds if the player opts to engage in current round, requiring a payment of $\$a$ to participate. There are two potential outcomes for each gamble: either a win U indicating an upward movement in the game tree representation, or a loss D indicating a downward movement. The monetary rewards associated with these outcomes are represented as $B_U > 0$ for a win and $B_D = 0$ for a loss, respectively. Let us denote such a gamble by $h_U, p^U; h_D, p^D$, where p_i is the true probability of outcome $i \in U, D$, yielding a monetary outcome h_i for the player. It is evident that $h_U = B_U - a > 0$ and $h_D = B_D - a < 0$. The gamble is fair from the expectational perspective, demanding that $p^U h_U + p^D h_D = 0$, or equivalently, $p^U B_U + p^D B_D = a$.

A natural and clear representation of such game is a game tree. In the graph, the total monetary outcome is plotted against the number of rounds $t \in \{0, 1, \cdots, T\}$ played. The possible states of the player during the game are represented by tree nodes. The potential evolutionary paths of these states are represented by directed edges connecting tree nodes. Moreover, the node representing a player's state after winning k_U times and losing k_D times can be uniquely determined by the vector (k_U, k_D), since $k_U + k_D = t$ is the number of completed rounds and $k_U h_U + k_D h_D$ is the total monetary gain. Therefore, it is convenient to refer to each node by its unique state vector $\boldsymbol{k}^t = (k_U, k_D)$.

3.2.1 Probability misperception

A common mistake made by players in repeated gambles is "the gambler's fallacy," which is the non-Bayesian belief that a certain outcome which has just occurred is less likely to happen subsequently. These players are referred to as "naive," and I propose some justifiable patterns of their probability misperception. The explanations of these patterns are based on the more general behavioural concept of "the law of small numbers" (Tversky and Kahneman, 1971): probability distributions in small samples are expected to resemble the population distribution.

Consider a naive player who is losing (winning) overall and believes that small samples resemble the population. Winning (losing) the next round is "expected" because it will bring the sample closer to the population distribution, while losing (winning) the next round is "unexpected" because it further unbalances the sample. This incorrect expectation can lead to biased perceptions of probabilities, and the extent of this misperception depends on how much the current sample deviates from the expectation, i.e., the population distribution. However, bias can be absent when the sample perfectly matches the population distribution or at the beginning of the game when no sample is available, since the naive player does not misinterpret other aspects of the game. Let us denote the naive player's perceived probabilities for next round of gamble on node $\boldsymbol{k}^t$ by $p^U_{\boldsymbol{k}^t}$ and $p^D_{\boldsymbol{k}^t}$, the first property of the naive player's probability misperception is given by:

Property 3.1 *For any $\boldsymbol{k}^t = (k_U, k_D)$, the naive player's perceived probabilities satisfy*

$$h_U k_U + h_D k_D < 0 \Rightarrow p^U_{\boldsymbol{k}^t} > p^U,$$
$$h_U k_U + h_D k_D = 0 \Rightarrow p^U_{\boldsymbol{k}^t} = p^U,$$
$$h_U k_U + h_D k_D > 0 \Rightarrow p^U_{\boldsymbol{k}^t} < p^U.$$

Apart from the basic property mentioned above, there may be cases where one sample is perceived to have more or less resemblance to the population distribution than the other by a naive player. As a result, the perceived probabilities on certain node should be adjusted accordingly. The next two properties are derived in this way.

Let us consider the scenario where a naive player on $\boldsymbol{k}^t = (k_U, k_D)$ is experiencing a losing streak. The sample distribution will vary according to the following sequence:

$$(\frac{k_U}{t}, \frac{k_D}{t}), (\frac{k_U}{t+1}, \frac{k_D+1}{t+1}), (\frac{k_U}{t+2}, \frac{k_D+2}{t+2}), \cdots$$

If $k_U \neq 0$, the direction of change along the sequence is monotonic when compared to the population distribution (p^U, p^D). However, the degree of change is attenuating, and the distribution converges to $(0, 1)$ for a sufficently long losing streak. The implications of this observation are substantial. Firstly, the losing streak may lead a naive player to believe that they are more likely to win the next round. Secondly, the naive player may perceive the samples as less different from the previous sample along the streak, resulting in smaller further adjustments needed to be made in her mind. Finally, the naive player may think that they are almost sure to win the next round after sufficient losses. For $k_U = 0$, it is possible to retain monotonicity and convergency. This is because a large sample with a distribution of $(0, 1)$ is less likely to occur than a small sample with the same distribution. Therefore, the naive player will be more biased and will consider herself almost ensured to win the next round if they have never won for a sufficient number of rounds. These properties of monotonicity, convergency and convexity are summarised as follows:

Property 3.2 *For any node (k_U, k_D) and any $i \in \{U, D\}$, let us denote the other outcome by $-i$, the naive player's perceived probabilities satisfy*

$$p^i_{(k_i+1, k_{-i})} - p^i_{(k_i, k_{-i})} < 0$$
$$\lim_{k_i \to \infty} p^i_{(k_i, k_{-i})} = 0.$$

If $k_{-i} \neq 0$, they further satisfy

$$p^i_{(k_i+2,k_{-i})} - p^i_{(k_i+1,k_{-i})} > p^i_{(k_i+1,k_{-i})} - p^i_{(k_i,k_{-i})}.$$

Then, consider the scenario where a naive player on node $\boldsymbol{k}^t = (k_U, k_D)$ plays for the next $t^{'}$ rounds, and the aggeregate outcomes in these rounds perfectly match the population distribution, i.e., she wins $t^{'}p^U \in \mathbb{N}^+$ rounds and loses $t^{'}p^D \in \mathbb{N}^+$ rounds. The player receives the same net payoff as on node $\boldsymbol{k}$, but the sample distribution changes from $(\frac{k_U}{t}, \frac{k_D}{t})$ to $(\frac{k_U+t^{'}p^U}{t+t^{'}}, \frac{k_D+t^{'}p^D}{t+t^{'}})$. The new distribution is always closer to the population distribution (p^U, p^D), as long as the player does not have a perfectly matched sample on $\boldsymbol{k}^t$. Furthermore, for sufficiently large values of $t^{'}$, the latter distribution converges to the population distribution. This is because a larger sample size makes the same absolute difference between expected and realized wins or losses less significant. When adding minor differences to a sufficiently large sample that perfectly matches the population distribution, the naive player may feel that the new sample is almost identical to what is expected. These properties are summaries as follows:

Property 3.3 *For any $i \in \{U, D\}$ and any $\boldsymbol{k}^t = (k_U, k_D)$ such that $k_U h_U + k_D h_D \neq 0$, given any $\boldsymbol{k}^{t^{'}} = (k^{'}_U, k^{'}_D)$ such that $k^{'}_U h_U + k^{'}_D h_D = 0$, the naive player's perceived probabilities satisfy*

$$\left|p^i_{\boldsymbol{k}^t} - p^i\right| > \left|p^i_{\boldsymbol{k}^t+\boldsymbol{k}^{t^{'}}} - p^i\right|$$
$$\lim_{t^{'}\to\infty} p^i_{\boldsymbol{k}^t+\boldsymbol{k}^{t^{'}}} = p^i$$

3.2.2 Planning and optimisation

The decision problem is to determine the choice on each node that may be reached in the future, given that a player always has the freedom to accept or reject the next round. A plan is a specification of choices for all possible nodes. The player then identifies the optimised plan among all possible plans based on an evaluation process of expected outcomes and implements it in the next round. Barberis developed the initial model for casino gambling, but I made two significant modifications.

Firstly, Barberis assumes that a player will always make a plan for subsequent rounds of gambles until the end of the game. However, this assumption is flawed as even small-sized repeated gambles can involve heavy computational load due to the exponential increase in the number of possible plans as the number of rounds increases. A practical solution is to introduce narrow-bracketing behaviour. This means that an N-planning player will only plan for a maximum of N rounds to limit the the complexity of the optimisation problem. The value of N sets an upper bound on the computational complexity, and categorises players based on their planning and optimisation ability. In this paper, I mainly focus on the $N = 1$ case. It is worth noting that Barberis's planning is equivalent to $N \to \infty$.

Secondly, in Barberis's model for casino gambling, the player is assumed to refer to their original wealth level when evaluating a plan. However, in Gacha games, this is different. While bidirectional conversion between real currency and tokens is always feasible in casino gambling, Gacha game players can only top up their virtual account and are unable to convert in-game currency to real money. Therefore, a player with a budget of \$100 can confidently bring \$100 worth of tokens into the casino, but is more likely to only top up for the \$5 draw that they are going to take in Gacha games. When checking their account during the game, a gambler can clearly see their change in wealth relative to their original level, while a Gacha game player is more likely to focus on the gain and loss within their current gamble. Based on this, it can be assumed that the player refers to their current wealth level when making plans.

The planning and optimisation procedures in my model is formally stated below.

An N-planning player with inital wealth w_0 and current state $\boldsymbol{k}^t = (k_U, k_D)$ is faced with the decision of making a plan s^N. This plan is a mapping of each possible node between round t and round $t+N-1$ to two possible actions for the next round: "reject" or "accept". A path of future realisation of outcomes

$$\varphi = (i_{t+1}, i_{t+2}, \cdots, i_{t+\theta}, \cdots, i_{t+\Theta})$$
$$\Theta \leq N, i_{t+\theta} \in \{U, D\}$$

is considered valid for a given plan s^N if the choice on every passed node, starting from $\boldsymbol{k}^t$ and moving according to the sequence of $i_{t+\theta}$ until $\boldsymbol{k}^{t+\Theta-1}$, is to accept next round. If we denote the set of all valid paths for plan s^N by Φ_{s^N}, then the distribution of monetary outcomes from implementing s^N can be written as a random variable

$$X_{s^N} \sim \{H_1, P_1; H_2, P_2; \cdots; H_j, P_j; \cdots\}$$

where H_j represents the possible monetary outcome relative to the current wealth level $w_{\boldsymbol{k}^t} = w_0 + k_U h_U + k_D h_D$, and

$$P_j = \sum_{\substack{\varphi=(i_{t+1}, i_{t+2}, \cdots) \in \Phi_{s^N} \\ h_{i_{t+1}} + h_{i_{t+2}} + \cdots = H_j}} p_{\boldsymbol{k}^t}^{i_{t+1}} p_{\boldsymbol{k}^{t+1}}^{i_{t+2}} \cdots$$

represents the perceived probability of this outcome, which is calculated based on the dynamically perceived probabilities of the player at each node.

To gain a better understanding of these notations for the player's plan, a simple example is given below.

Example 3.1 *Let us consider a 2-planning player who is currently on node (k_U, k_D). One possible plan for her is to accept the first round and then accept the second round only if she is winning. This plan and its outcome distribution can be expressed using the notations provided above as*

$$s^2 = \{(k_U, k_D), \textit{Accept}; (k_U + 1, k_D), \textit{Accept}; (k_U, k_D + 1), \textit{Reject}\}$$
$$X_{s^2} \sim \{2h_U, p^U_{(k_U,k_D)} p^U_{(k_U+1,k_D)}; h_U + h_D, p^U_{(k_U,k_D)} p^D_{(k_U+1,k_D)}; h_D, p^D_{(k_U,k_D)}\}.$$

Now we can proceed to the optimisation problem for an N-planning player. Let us denote the set of all possible plans s^N by S^N, the player's optimisation problem is given by

$$\max_{s^N \in S^N} V(X_{s^N}; w_{\boldsymbol{k}^t})$$

where $V(\cdot)$ is the value fuction in the following form

$$V(X_{s^N}; w_{\boldsymbol{k}^t}) = \sum_j P_j U(H_j; w_{\boldsymbol{k}^t}),$$

and $U(\cdot)$ is the utility function in the following form

$$U(x; w) = \alpha(w) u(x)$$
$$\alpha(w) > 0, \alpha^{'}(w) < 0$$
$$u(x) = \begin{cases} x^r, & x \geq 0 \\ -\lambda(-x)^r, & x < 0 \end{cases}$$
$$\lambda > 1, 0 < r < 1.$$

Here, it is assumed that the level of wealth only affects the utility function as a positive scalar that decreases as wealth increases. This assumption is based on the fact that the relative change in wealth is smaller for a richer person, even if the absolute change is the same. As a result, each player calculates her unadjusted utility using the same method, but adjusts it based on her current wealth level in a way that the player with less wealth experiences a greater change in utility from the same amount of monetary gain or loss. The unadjusted utility function $u(\cdot)$ follows the conventional form in prospect theory (Kahneman and Tversky, 1979). Its two parameters, λ and r, indicate the degree of loss aversion and diminishing sensitivity, respectively.

Finally, the player should follow the optimal plan

$$s^N_{Optimal} = \arg\max_{s^N \in S^N} V(X_{s^N}; w_{\boldsymbol{k}^t}),$$

to make the decision on current node $\boldsymbol{k}^t$. If the decision is to reject, she exits the game. Otherwise, she should a single draw and move to a new state, where she will face a new planning and optimisation problem.

In addition, it is useful to evaluate the acceptability of the gamble by solving the switching probability γ from the following equation

$$\gamma u(h_U) + (1-\gamma)u(h_D) = 0.$$

The unique solution is

$$\gamma = \frac{\lambda}{\lambda + (-\frac{h_U}{h_D})^r} \in (0,1).$$

A player on node $\boldsymbol{k}^t$ will accept the next round of gamble if and only if her perceived probability of winning is greater than the switching probability, that is $p^U_{\boldsymbol{k}^t} \geq \gamma$, or equivalently $p^D_{\boldsymbol{k}^t} \leq 1-\gamma$.

3.2.3 Bundled options

With the single-draw option only, a player can exit the game at the end of any round. In contrast, bundled options function as several consecutive rounds of gambles where the player cannot quit in the middle. In Gacha games, it is common for single draw and consecutive draws to be displayed as two option buttons on the toy vending machine interface. Bundled options can impact the player's planning and optimisation behaviour.

An option of M consecutive gambles is referred to as an M-draw bundle. An N-planning player considers it as an additional possible plan s^M_{Bundle}. The optimisation problem then becomes

$$\max_{s^N \in S^N \cup \{s^M_{Bundle}\}} V(X_{s^N}; w_{\boldsymbol{k}^t}).$$

The only difference from the no-bundle scenario is the set of plans that will be optimised. For a narrow-bracketing player who has $N < M$, the bundled option forces her to consider one specified larger plan. Otherwise, for a broader-bracketing player who has $N \geq M$, she has already planned for longer term and the bundled option does not introduce anything unforeseen. The reason is, the expected outcomes of the bundle are the same as a no-bundle plan $s^N_{M,accept} \in S^N$, where the player always accepts the next round of gamble for the first M rounds and rejects on other nodes. It follows that $X_{s^M_{Bundle}} = X_{s^N_{M,accept}}$ for $N \geq M$. One remaining ambiguity is the case when the bundled option is optimal. I simply assume that the player will always choose the no-bundle equivalent $s^N_{M,accept}$ instead of s^M_{Bundle}. Finally, the optimisation result remains unchanged for the player with $N > M$ after introducing bundled options.

Echoing the previous discussion on computational complexity in Barberis's model, the bundled option in my model only extends the optimisation problem by at most one additional plan. This minor extra effort can always be assumed to be within the player's ability. The impact of this additional plan on the optimisation results is of interest. In the remaining sections, I will focus on how long-term bundles impact the behaviour of players who can only plan for the short term, as well as the behaviour of others.

3.2.4 Game provider's profit

The cost of providing rewards for game producers is almost negligible since the rewards consist of virtual in-game items. Allowing for the inclusion of other repeated gambles, it is sufficient to require the cost for the provider be strictly lower than the value for the player. Then, for example, my model could also be applied to a promotional event for an industrial producer, where the rewards consist of their own products whose market values exceed their production costs.

Formally, the game provider incurs costs of C_U and C_D to provide winning and losing rewards for providing winning and losing rewards respectively, such that

$$C_D = B_D = 0$$
$$0 \leq C_U < B_U.$$

Use the fair gamble condition $p^U B_U + p^D B_D = a$, the game provider's expected revenue from one round of gamble is positive, that is

$$a - (p^U C_U + p^D C_D) > 0.$$

The total expected revenue is calculated by multiplying the expected revenue per round by the expected number of rounds. Therefore, the game provider has an incentive to encourage players to continue playing in order to increase profits. Additionally, the game provider may offer several free rounds at the start of the game, which will reduce her profit by a fixed amount but will not alter her incentive to encourage players to continue playing.

Note that this model does not apply to casino games where rewards are in real currency; in such cases, the cost of provision always equals the monetary values, and the provider's expected revenue is always zero if games are fair.

3.3 Analysis on Naive 1-planning Players

The distinguishing feature of a naive 1-planning player, as opposed to other naive N-planning players, is that any plan she devises will be executed in full. This is because she is only responsible for making decisions regarding the current gamble. The player's behaviour is entirely driven by her misperception of probabilities, which provides a clear explanation for the results.

As explained in Section 3.2.4, it is desirable for the game provider to keep players engaged for as long as possible. However, even a naive player is unbiased at the start of the game and will not accept a fair gamble, because she has prior experience and therefore no source of bias. This raises the question for the game provider: how can naivety be exploited to increase profits? A simple answer is, the game provider can probably "send" the player to the state where she would be willing to accept the next round by offering her several free draws at the beginning to help her build up a sample and thus a source of bias. For simplicity, I assume $p^U_{(0,1)} \geq \gamma$, that is a naive player who loses in the initial free round will accept the next round. It is worth noting that "sending" the player to other state is costly for the game provider in general, so it is feasible only if the costs to offer free rounds could be fully compensated. In Gacha games, it may be reasonable for virtual items to have a zero providing cost. Therefore, to ensure feasibility, I assume zero cost for most of the results. The discussion of providing cost will be presented in the concluding section.

The game provider may wonder if the narrow-bracketing characteristic can be used to increase profits. The 1-planning player gives up the opportunity to improve her payoff by considering larger plans due to the complexity of the optimisation problem. However, she may still assess any specific larger plan presented to them. Then, "forcing" this player to take a more attractive and previously unconsidered plan into consideration may bring her back to the game if she were planning to exit according to the original plan. In order to generate a specific larger plan, the game provider may only be able to bundle several rounds together, as the drawing options are typically not conditional. In this section, I demonstrate that bundling two rounds together increases the likelihood of a naive 1-planning player continuing to play the game, which in turn effectively increases the game provider's overall profit for a typical class of gambles.

3.3.1 50-50 fair gambles

This is the game setting in Barberis's model of casino gambling. Close connections and key differences can be identified by contrasting the results from my model with his. Without loss of generality, I consider the following repeated gambles:

$$p^U = \frac{1}{2}, h_U = 1,$$
$$p^D = \frac{1}{2}, h_D = -1,$$
$$\gamma = \frac{\lambda}{\lambda + 1}.$$

I start with two lemmas which describe the patterns of acceptance state for next gamble.

Lemma 3.1 *For a naive* 1*-planning player, if the next round of gamble is accepted on node* (k_U, k_D)*, then the node* $(k_U, k_D + 1)$ *will also be accepted; if the next round of gamble is rejected on node* (k_U, k_D)*, then the node* $(k_U, k_D - 1)$ *was also rejected.*

This lemma is based on the gambler's fallacy, which suggests that a naive player should have a higher expectation of winning in the next round of gamble compared to their current perceived gamble if they were to lose the current one. Therefore, a player should accept the gamble after one more loss if the gamble before losing is already sufficiently appealing, but she should reject the gamble before one more loss if the gamble after losing is still insufficiently appealing.

Lemma 3.2 *For any given* $k_0 \in \mathbb{N}$*, there exists a unique* $\delta_{k_0} \in \mathbb{N}^+$ *such that, a naive* 1*-planning player on node* $(\delta, k_0 + \delta), \delta \in \mathbb{N}$ *accepts the next round of gamble for any* $\delta < \delta_{k_0}$ *while rejects it for any* $\delta \geq \delta_{k_0}$*, and* δ_{k_0} *is non-decreasing in* k_0*.*

This lemma examines the acceptability of nodes in the lower half of the tree where the player has a non-positive net gain. Nodes are categorised based on the player's net gain, determined by k_0. The sequence of nodes on the same iso-net-gain line, i.e., $\delta \in \{0, 1, 2, \cdots\}$ for a given k_0, can always be divided into two parts: all nodes prior to the cut point determined by δ_{k_0} are accepted, and the remaining nodes are rejected. Furthermore, the cut point will not appear in an earlier round when the net loss becomes higher, i.e., non-increasing δ_{k_0}. If these accepted nodes can all be reached during the game, a naive 1-planning player who has lost more would stop later in the game.

The reachability of accepted nodes can actually be proved. Let us first formalise the node reachability using the concept of "valid paths" in Section 3.2.2 as follows:

1. Reachable: there exists a valid path from $(0, 0)$ to the node, and the node itself is accepted.

2. Stop point: there exists a valid path from $(0, 0)$ to the node, and the node itself is rejected.

3. Unreachable: there is no path from $(0, 0)$ to the node.

Then, the behavioural pattern of the player can be established. For a naive 1-planning player, she must be on either a reachable node or a stop point at any stage of the game. And the player should only exit if she is on a stop point. It is formally stated below:

Proposition 3.1 *Consider a naive* 1*-planning player who is to play the game described in this subsection with the single-draw option only.*

1. *In the non-positive-net-gain part of the game tree, every accepted node in Lemma 3.2 is reachable. Among the remaining nodes, those with a reachable predecessor node are stop points, while the others are unreachable nodes.*

2. *In the positive-net-gain part of the game tree, every node is unreachable except for* $(1, 0)$ *which is a stop point.*

The above proposition demonstrates that the reachable area in the game tree, defined as the smallest area encompassing all reachable nodes and stop points, takes on a stair-step shape (see Figure 3.1a). This can be viewed as a stronger version of the "gain-exit" outcome documented by Barberis. In addition to finding that the expected length of the game, conditional on exiting with a gain, is less than exiting with a loss, the player never continues to play when she is overall winning. However, she always stays longer if she has lost more. Moreover, a crucial distinction from Barberis's findings is that my results are solely based on probability misperception and one of the properties of prospect theory, namely that losses have a greater impact than gains. Therefore, it will never switch to the "loss-exit" pattern for any reasonable parameter setting in the utility function. In my model, the naive player who has lost more believes that she has been accumulating "good luck" for the future. As a result, she perceives her imaginary lucky streak as long-lasting and is less likely to give up this "opportunity".

Proposition 3.1 is referred to as the baseline result. The game provider did not consider offering bundles in this game to potentially alter the behavioural patterns of some naive players and increase her profit. The impact of bundled options is revealed in comparison with this baseline result.

The following proposition demonstrates that the naive 1-planning player can be encouraged to continue playing for a longer duration through the use of 2-draw bundles:

Proposition 3.2 *Let the option of the 2-draw bundle available to the naive 1-planning player, and refer to the results in Proposition 3.1 as "previous":*

1. *The player will accept the bundle at some previous stop points in the negative-net-gain part of the game tree. If there is no limit on the maximum number of rounds, there are infinitely many such stop points.*

2. *The player will accept the bundle at every previous reachable node, unless it is not possible to take two more draws on the reachable node.*

The above proposition illustrates the impact of an additional option of the 2-draw bundle on the behaviour of a naive 1-planning player. The reachable area is expanded compared to the no-bundle case (see figure 3.1b), meaning that the player will be incentivised to continue playing for a longer period of time. It is obvious that the reachable area will never be reduced, because the single-draw option is always available, regardless of the attractiveness of the bundle. The reachable area expands when the accepted bundle's outcome area includes a node that was previously unreachable. The node where the bundled option is accepted must be an adjacent node to a stop point that was previously reachable, or a previous stop point. Intuitively, since the naive 1-planning player underestimates the probability of losing more severely as she is on a losing streak, if she is overall losing now, the worst result is underestimated twice in the bundle but only once in a single draw, while the best result is overestimated twice in the bundle, which misleads her to be more confident about the occurrence of satisfactory results in the bundle than in a single draw. Besides, note that only sufficient conditions are used to prove that the bundle will be accepted on some nodes. The actual reachable area extended by the bundled option may be larger.

3.3.2 A typical class of gambles

Now, I consider a typical class of repeated gambles as given below:

$$p^U = \frac{1}{m+1}, h_U = m$$
$$p^D = \frac{m}{m+1}, h_D = -1$$
$$\gamma = \frac{\lambda}{\lambda + m^r}$$
$$m \in \mathbb{N}, m \geq 2.$$

It represents the common prize pool in Gacha games where valuable rewards have a low probability of being drawn.

Analogous to Proposition 3.1, the baseline result for the behaviour pattern of a naive 1-planning player is presented below:

Proposition 3.3 *Consider a naive* 1*-planning player who is to play the repeated gambles described in this subsection with the single-draw option only.*

1. *In the non-positive-net-gain part of the game tree, every accepted node is reachable. Among the remaining nodes, those with a reachable predecessor node are stop points while the others are unreachable nodes.*

2. *In the positive-net-gain part of the game tree,* $(1, 0)$ *and adjacent nodes to a reachable node are stop points, while the remaining nodes are unreachable.*

The proof follows exactly the same procedure: we first derive patterns of acceptance state, and then demonstrate the reachability of accepted nodes. However, note that there may be stop points other than $(1, 0)$ in the positive-net-gain part of the game tree. This is due to the presence of asymmetric winning and losing payoffs.

Then, analogous to Proposition 3.2, the naive 1-planning player can be encouraged to continue playing for a longer duration through the use of 2-draw bundles:

Proposition 3.4 *Let the option of the* 2*-draw bundle available to the naive* 1*-planning player, and refer to the results in Proposition 3.3 as "previous". Under the assumption that* $\lambda \leq m^{1+r}$ *holdes for the utility function:*

1. *The player will accept the bundle at some previous stop points where the net loss is greater than or equal to* m*. If there is no limit on the maximum number of rounds, there are infinitely many such stop points.*

2. *The player will accept the bundle at every previous reachable node where the net loss is greater than or equal to m, unless it is not possible to take two more draws on the reachable node.*

This proposition yields a weaker result than Proposition 3.2, as it requires additional restrictions. However, given conventional estimations that λ is around 2 and r is not too close to zero, for instance Tversky and Kahneman (1992) estimate $\lambda = 2.25, r = 0.88$, I argue that the results are widely applicable, because the restriction $\lambda \leq m^{1+r}$ applies to a wide range of parameters. For instance, if we reasonably assume $\lambda \leq 3$, then $r > 0.59$ is sufficient for the restriction to hold for any $m \geq 2$; if $m \geq 3$, the restriction holds for any $0 < r < 1$. It is important to note that the requirement for the net loss to be greater than or equal to m can be relaxed to any negative net gain if $\lambda \leq m^r$. This tighter restriction still holds for many reasonable λ and r. For instance, $m \geq 3$ is sufficient for the previously mentioned estimations by Tversky and Kahneman. Again, the proof only uses sufficient conditions, so the actual reachable area extended by the bundled option may be larger.

In summary, the findings and explanations for 50-50 fair gambles are mostly applicable to this typical class of repeated gambles.

3.3.3 Game providers' profits

The change in the expected profit for the game provider resulting from introducing bundled options is directly derived from the behavioural patterns proposed in previous subsections.

Corollary 3.1 *In repeated gambles* $\{+m, \frac{1}{m+1}; -1, \frac{m}{m+1}\}, m \in \mathbb{N}^+$*, the introduction of the* 2*-draw bundle in addition to the single-draw option, increases the expected profit of the game provider generated from naive* 1*-planning players.*

The naive 1-planning player's behaviour is affected in a clear way: she will not stop at previous reachable nodes, but may stay longer on previous stop points when the 2-draw bundle is available. Therefore, the expected number of rounds played must increase. Along with the assumption of a positive expected revenue from one draw, the game provider's profit will be higher. These findings indicate that the bundled option is effective in generating higher profits from the most narrowly bracketing naive 1-planning players.

However, from the game provider's perspective, earning more from one group of players does not necessarily increase their overall profit. The provider should consider whether offering a bundled option will have undesirable effects on other players' behaviour. The following result reassures the provider that offering such a bundle is feasible and profitable, as other players who are planning for a longer future have already taken such an option into account.

Corollary 3.2 *In repeated gambles* $\{+m, \frac{1}{m+1}; -1, \frac{m}{m+1}\}, m \in \mathbb{N}^+$, *the introduction of the 2-draw bundle does affect the decisions made by players with a higher order of planning behaviour. In other words, for any* $N \geq 2$, *the* N*-planning player's behaviour is exactly the same in both games with and without.*

The result follows immediately from the player's planning and optimisation process described in Sections 3.2.2 and 3.2.3. Therefore, the game provider's expected profit in repeated gambles $\{+m, \frac{1}{m+1}; -1, \frac{m}{m+1}\}, m \in \mathbb{N}^+$ will increase if the option of the 2-draw bundle is available in addition to the single-draw option.

To summarise, the analytical evidence indicates that offering free initial rounds together with bundled options can increase the game provider's profits. The results support the prevalence of these features in the design of toy vending machine mechanics in video games.

3.3.4 Evidences from numerical analysis

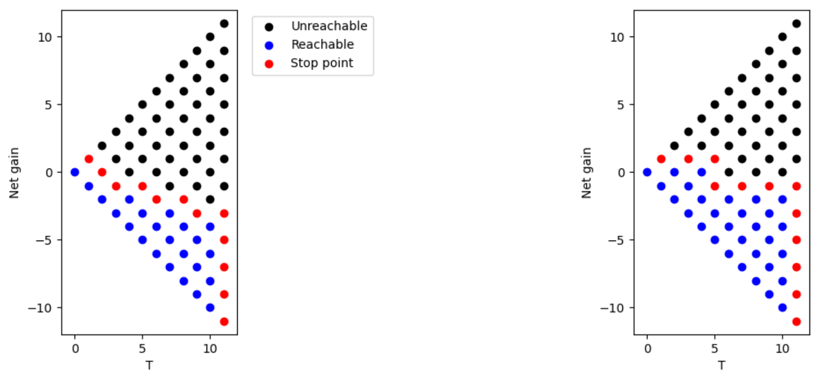
In the previous part of this section, I have explained how two common features in real-world Gacha games, initial free rounds and bundled options, may help the game provider increase its profit. For a better understanding of this mechanism, I conduct a numerical analysis to further support the feasibility and profitability of such a design. The following notations will be used:

Notation	Explanation
NumFree	Number of initial free rounds.
T	Maximum number of rounds that can be played (NumFree excluded).
ExpNB	Expected number of rounds played without bundles (NumFree excluded).
ExpB	Expected number of rounds played with bundles (NumFree excluded).

The numerical analysis focuses on the following specification of repeated 50-50 fair gambles:

$$m = 1,$$
$$p^U_{(k_U,k_D)} = \frac{k_D + 1}{k_U + k_D + 2},$$
$$p^D_{(k_U,k_D)} = \frac{k_U + 1}{k_U + k_D + 2},$$
$$NumFree = 1,$$
$$\lambda = 2, r = 0.88.$$

The figures below show the behavioural patterns of a 1-planning player before and after the introduction of 2-draw bundles:



(a) No bundle (b) 2-draw bundle

Figure 3.1: Naive 1-planning player's behavioural patterns; $\lambda = 2$, $NumFree = 1$.

The figure below quantifies the effect of 2-draw bundles on the game provider's expected profit from this player, measured by the expected number of rounds played:

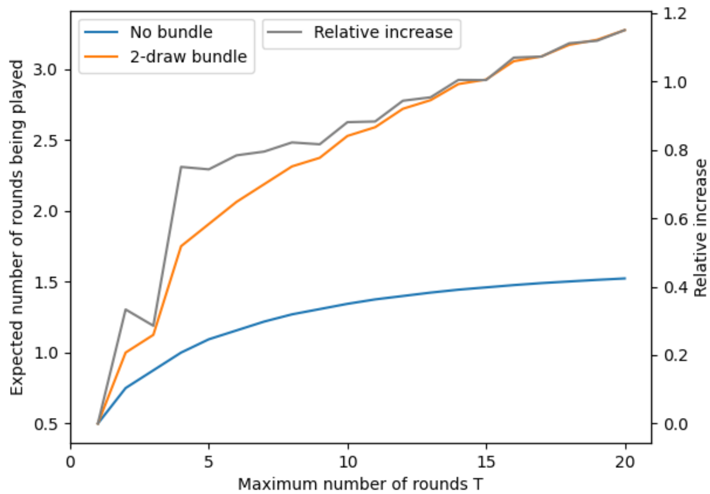


Figure 3.2: Expected number of rounds being played by a naive 1-planning player with 2-draw bundles; $\lambda = 2$, $NumFree = 1$.

It is evident that the expected number of rounds played by a naive 1-planning player increases logarithmically with or without bundles as the maximum number of rounds increases. The speed of increase is much greater with bundles, resulting in a significant increase in the game provider's profit when 2-draw bundles are offered. A profit increase of over 75% can be achieved if at least 5 rounds are played. Additionally, a relative increase in profit of about 10% can be made each time if 5 more rounds are possible. The relative increase in profit fluctuates but shows an overall upward trend, supporting the behavioural pattern proposed in Proposition 3.2. The overall upward trend is typically interrupted by a minor downward movement at odd values of T. This happens because, for odd T, the last round must be a single draw if the naive 1-planning player kept taking 2-draw bundles before. Indeed, the naive 1-planning player will keep accepting 2-draw bundles. However, the game restriction prevents the reachable area for the last round from expanding, even if the player would accept the bundle if it were offered. These blocked willing expansions then slightly reduce the relative advantage of the bundled options. However, the overall trend is primarily driven by more expansions that occur an the maximum number of rounds increases.

Based on the analysis of expected profit above, I can discuss the costs of providing rewards. As the cost of the initial free rounds is fixed, and the game provider's expected return from the 1-planning player's later participation in the game increases with the maximum number of rounds, the return will eventually exceed the cost if the game can last longer, under the assumption on lower providing cost than its value. For the numerical setting here, a cost that is less than 50% of the value is enough to make $T = 4$ profitable without bundles.

For $T = 10$, it is profitable even if the cost is 70% as high as the value.

Now, let us examine the impact of the bundle size. The figure below illustrates how the game provider's expected profit from a naive 1-planning player changes with the number of rounds that is bundled in the extra option:

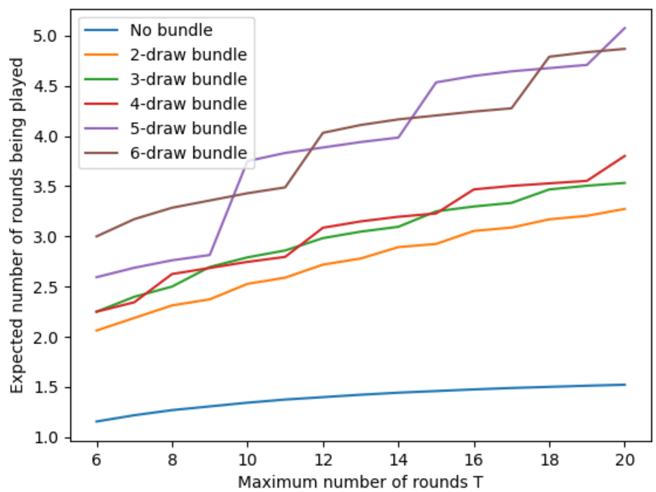


Figure 3.3: Expected number of rounds being played by a naive 1-planning player with M-draw bundles; $M \in \{2, 3, 4, 5, 6\}$, $\lambda = 2$, $NumFree = 1$.

In general, the number of rounds played by a naive 1-planning player tends to increase when offered larger bundles, such as 2-draw bundles compared to 3-draw or 4-draw bundles, or 3-draw or 4-draw bundles compared to 5-draw or 6-draw bundles. However, the relationship between bundle size and game provider profit is not always clear, as seen in the case of 3-draw versus 4-draw bundles, or 5-draw versus 6-draw bundles. Therefore, increasing the bundle size may not always result in a noticeable improvement in profit. This may explain why real-world Gacha games typically bundle no more than ten rounds together: bundling more rounds together may not significantly increase profits, but it can make the bundle unaffordable for low-wealth players.

3.4 General Patterns for Naive N-planning Players

The behavioural patterns in the general case of offering M-draw bundles to N-planning players may not be easily described as in previous propositions. However, hierarchical relationships exist among players grouped by different bracketing levels.

The key to the proofs are the following facts derived from the model setting in Section 3.2.2. Firstly, for any $N \in \mathbb{N}^+$, there exists a unique plan in S^N that results in the naive player choosing to exit the game at current node. This plan is denoted by s^N_{Reject}, because the choice at each node in the plan is to reject. It is evident that $X_{s^N_{Reject}} = (0,1)$. If a naive player is to follow any plan $s^N \in S^N_{Continue} = S^N \setminus \{s^N_{Reject}\}$, her current decision must be to continue. Secondly, for $N^{'} > N$, any N-round plan s^N can be mapped one-to-one to its unique N'-round equivalent $s^{N'}(s^N)$ by rejecting nodes from round $N+1$ to round N'. In mathematical terms, the following facts hold: (1) $X_{s^{N'}(s^N)} = X_{s^N}$ (2) $s^{N'}(s^N_{Reject}) = s^{N'}_{Reject}$; and (3) $\{s^{N'}(s^N) \mid s^N \in S^N_{Continue}\} \subset S^{N'}_{Continue}$.

I start by further investigating the behavioural patterns outlined in Proposition 3.1 and 3.3. Suppose that the game offered effectively induces the naive 1-planning player to participate, i.e., the free initial round is offered and the associated condition is satisfied, then the following relationship holds among naive players with varying levels of narrow bracketing behaviour:

Proposition 3.5 *In repeated gambles* $\{+m, \frac{1}{m+1}; -1, \frac{m}{m+1}\}, m \in \mathbb{N}^+$*, for any* $N, N' \in \mathbb{N}^+$ *such that* $N' > N$*, the expected number of rounds played by a naive* N'*-planning player is always greater than or equal to that played by a naive* N*-planning player.*

This proposition suggests that a broader-bracketing naive player may behave like a wiseacre. Although she may expect to outperform others by taking a broader view of the future, she is unaware of her naivety. As a result, planning for more rounds only misleads her further.

Then, consider offering an additional option of the general M-draw bundle instead of the 2-draw bundle studied in Proposition 3.2 and 3.4. The following relationship holds among naive players with different levels of narrow bracketing behaviour:

Proposition 3.6 *In repeated gambles* $\{+m, \frac{1}{m+1}; -1, \frac{m}{m+1}\}, m \in \mathbb{N}^+$*, for any* $M, N, N^{'} \in \mathbb{N}^+$ *such that* $N^{'} < N < M$*, if a naive* N*-planning player accepts the* M*-draw bundle at a certain node, then a naive* $N^{'}$*-planning player also accepts it at the same node.*

Proposition 3.7 *In repeated gambles* $\{+m, \frac{1}{m+1}; -1, \frac{m}{m+1}\}, m \in \mathbb{N}^+$*, for any* $M, N, N^{'} \in \mathbb{N}^+$ *such that* $N < M$ *and* $N < N^{'}$*, if a naive* N*-planning player rejects the* M*-draw bundle at a certain node, then a naive* $N^{'}$*-planning player also rejects it at the same node.*

As we already know that planning for more rounds accumulates probability misperception for a naive player, the intuition for such hierarchical structure is quite clear. When comparing it to one's original plan, the more additional misperception brought by the bundle, the more likely it is to be accepted by the naive player. A naive narrow-bracketing player has relatively less misperception than a naive broader-bracketing player before introducing the bundle, so the former is more likely to be affected by the additional bundled option.

Now, let us discuss the implication on the game provider's profit.

According to the characteristics of different players, the Bayesian players are not likely to be misled, so the game provider focuses on exploiting those who believe in the law of small numbers, i.e., the naive players. The probability misperception is triggered by initial free rounds, but later participation is determined by the naive player's personal characteristics. It turns out that those broader-bracketing players are more likely to continue playing, making them a more profitable group.

Note that naivety is a common source of bias. Also, note that narrow-bracketing players only lack the ability to plan for longer periods. Naive players in those less profitable groups are simply not "wise" enough to be wiseacres in those more profitable groups. Then, bundled options are designed to exploit these differences among naive players. If a naive N-planning player finds an M-draw bundle attractive, she can be considered a quasi-M-planning player. This is because her current choice, i.e., the bundle, is a plan with M rounds, although it may not necessarily be optimal for an M-planning player. Therefore, offering bundled options can be viewed as an attempt by the game provider to homogenise the expected profits of naive players, and as a result, the additional improvement in the game provider's profit comes mainly from naive players who merely plan ahead. The corollaries in this section support the idea that these players should be the focus when designing bundled options. It is important to make the bundles attractive to the most narrowly bracketing players, i.e., the naive 1-planning players, otherwise offering bundled options are unlikely to make any profit improvement.

3.5 Discussion and Conclusion

This paper explains the mechanics of toy vending machines commonly found in mobile games using a behavioural model. In this model, a player with utility based on prospect theory acts according to an optimised plan for a limited number of future rounds in repeated gambles. However, the player may hold a naive belief in the law of small numbers when considering probabilities in future gambles.

I demonstrate that naive players who perceive independent probabilities in a history-dependent way can be induced to play for longer than optimal, resulting in increased profits for the game provider. Two common features in real-world Gacha games — free initial rounds and bundled options, both play important roles in exploiting the naivety of players.

Firstly, offering free initial rounds "entices" naive 1-planning players to start a game that

that may ultimately harm them. Those who quit the game later may suffer more. The game provider can exploit the naivety of players by creating a sample through free draws that triggers probability misperception. The costs of providing free draws can be offset by the later game participation of these naive players.

Secondly, bundling two rounds increases the likelihood that naive 1-planning players will continue to play by reinforcing their probability misperception. In addition, this design has the benefit of not affecting the behaviour of other type of players, resulting in an overall increase in profit.

Numerical examples further support these findings, providing quantitative results on the feasibility and profitability of this mechanic. The increasing expected profit in the maximum number of rounds suggests that offering free rounds is likely to be more profitable than its cost. Furthermore, the game provider's profit can be significantly increased by introducing bundled options.

Finally, I demonstrate that some behavioural patterns for general naive M-planning players can be derived from the results for naive 1-planning players. There exists a hierarchical structure of the effects of free initial rounds and bundled options, which is determined by the time range M bracketed by the naive player when planning. Free initial rounds trigger the probability misperception, but naive broader-bracketing players are more likely to continue playing, thereby generating higher profits for the game provider. Bundled options increase the game provider's profits by homogenising naive players and broadening their bracketing range, resulting in higher profits from previously less profitable groups of players.

However, there is still much work to be done in studying mechanisms such as the one used in Gacha games. While playing Gacha games is often equated with gambling, few models have been presented by researchers from this perspective. However, no empirical evidence has been provided to support any of these analytical results. Conducting laboratory experiments or analysing real-world player data in Gacha games will help to examine the theory. Furthermore, this mechanism may be motivated by various behavioural features, such as loss aversion, probability misperception, and narrow bracketing. It will be beneficial to investigate the impact of each of these features in more detail.

Appendix

3.A Omitted proofs

Proof of Lemma 3.1. By monotonicity in property 3.2, a losing streak across $(k_U k_D - 1)$ implies that

$$p^D_{(k_U,k_D-1)} > p^D_{(k_U,k_D)} > p^D_{(k_U,k_D+1)}.$$

If (k_U, k_D) is accepted, that is

$$p^D_{(k_U,k_D)} \leq \frac{1}{\lambda+1},$$

then we have

$$p^D_{(k_U,k_D+1)} < p^D_{(k_U,k_D)} \leq \frac{1}{\lambda+1}$$

which means $(k_U, k_D + 1)$ will also be accepted. Similarly, if (k_U, k_D) is rejected, then we have

$$p^D_{(k_U,k_D-1)} > p^D_{(k_U,k_D)} > \frac{1}{\lambda+1}$$

which means $(k_U, k_D + 1)$ will also be rejected. □

Proof of Lemma 3.2. For $k_0 = 0$, given that $h_U = -h_D$, by property 3.1 we know that every node $(\delta, 0+\delta), \delta \in \mathbb{N}^+$ will be rejected because

$$p^D_{(\delta,\delta)} = \frac{1}{2} < \frac{1}{\lambda+1}.$$

But $(0, 0)$ will be accepted by the assumption on initial free round. So, we know that $\delta_0 = 1$.

For $k_0 \in \mathbb{N}^+$, firstly the assumption on perceived probabilities after the initial free round indicates that

$$p^D_{(0,1)} \leq \frac{1}{\lambda+1},$$

then by monotonicity in property 3.2, for any $k_0 \in \mathbb{N}^+$, we have

$$p^D_{(0,k_0)} \leq p^D_{(0,1)} \leq \frac{1}{\lambda+1} < \frac{1}{2}.$$

Secondly, decomposing $(\delta, k_0 + \delta)$ to $(0, k_0) + (\delta, \delta)$, it is easy to verify $0 \cdot h_U + k_0 h_D \neq 0$ and $\delta h_U + \delta h_D = 0$, then using the result from property 3.1 that

$$p^D_{(\delta,k_0+\delta)} < p^D, \delta \in \mathbb{N},$$

we can derive from property 3.3 that $p^D_{(\delta,k_0+\delta)}$ is increasing in δ and

$$\lim_{\delta\to\infty} p^D_{(\delta,k_0+\delta)} = p^D = \frac{1}{2}.$$

Thus, there exists a unique $\delta_{k_0} \in \mathbb{N}$ for given k_0 such that

$$p^D_{(0,k_0)} < p^D_{(1,k_0+1)} < \cdots \\ < p^D_{(\delta_{k_0}-1,k_0+\delta_{k_0}-1)} \leq \frac{1}{\lambda+1} < p^D_{(\delta_{k_0},k_0+\delta_{k_0})} \\ < p^D_{(\delta_{k_0}+1,k_0+\delta_{k_0}+1)} < \cdots$$

which means the naive 1-planning player will accept node $(\delta, k_0+\delta), \delta \in \mathbb{N}$ for any $\delta < \delta_{k_0}$ while reject it for any $\delta \geq \delta_{k_0}$.

Now let $k_0^{'} = k_0 + 1$, by definition of $\delta_{k_0^{'}}$, we know that $(\delta_{k_0^{'}}, k_0^{'} + \delta_{k_0^{'}})$ is rejected. Then by lemma 3.1, $(\delta_{k_0^{'}}, k_0^{'} + \delta_{k_0^{'}} - 1)$ will also be rejected. Note that this node can be written as $(\delta_{k_0^{'}}, k_0 + \delta_{k_0^{'}})$, suppose $\delta_{k_0^{'}} < \delta_{k_0}$, by definition of δ_{k_0}, this node must be accepted, a contradiction. So, $\delta_{k_0+1} \geq \delta_{k_0}$ always holds, or equivalently, δ_{k_0} is non-decreasing in k_0. □

Proof of Proposition 3.1. We know that a node (k_U, k_D) has two possible predecessor nodes $(k_U - 1, k_D)$ (does not exist if $k_U = 0$) and $(k_U, k_D - 1)$ (does not exist if $k_D = 0$). We also know that for $h_U = -h_D$, the player's net gain on node (k_U, k_D) is positive (zero/negative) if and only if k_U is greater than (equal to/less than) k_D.

Suppose the player is on a node (k_U, k_D) with positive net gain, that is $k_U > k_D$. It is obvious that $(1, 0)$ can be reached after initial free round. For any other node, there exist $n_0 \in \mathbb{N}^+$ such that

$$k_U - n_0 = k_D$$

and $n_1 \in \mathbb{N}^+$ such that

$$k_U = k_D + n_1 - 1.$$

Since the player is break-even on both $(k_U - n_0, k_D)$ and $(k_U, k_D + n_1 - 1)$, by property 3.1, we have

$$p^U_{(k_U-n_0,k_D)} = \frac{1}{2}$$
$$p^D_{(k_U,k_D+n_1-1)} = \frac{1}{2}.$$

Then by monotonicity in property 3.2, we have

$$p^U_{(k_U-1,k_D)} \le p^U_{(k_U-n_0,k_D)} = \frac{1}{2} < \frac{\lambda}{\lambda+1}$$
$$p^D_{(k_U,k_D-1)} > p^D_{(k_U,k_D+n_1-1)} = \frac{1}{2} > \frac{1}{\lambda+1}.$$

Therefore, the player would have rejected either possibile predecessor node (if it exists) and is never able to reach (k_U, k_D).

The restriction on maximum number of rounds T only forces the player to stop on some otherwise accepted nodes after the last round of gamble. So, we only need to derive results for infinitely many rounds, and the restriction is implemented by changing those reachable nodes representing the player's state at the beginning of round $T+1$ to stop points.

Suppose the player is on a node (k_U, k_D) with non-positive net gain, that is $k_U \le k_D$. This node can be represented in another form $(\delta, k_0+\delta)$ by setting $k_0 = k_D - k_U$ and $\delta = k_U$, where $k_0 \in \mathbb{N}$ and $\delta \in \mathbb{N}$. We first prove that every accpeted node in lemma 3.2 is actually reachable. For any accepted node $(\delta, k_0+\delta)$, consider the following path

$$(0,0) \to (0,1) \to (0,2) \to \cdots \to (0,k_0)$$
$$(0,k_0) \to (0,k_0+1) \to (1,k_0+1)$$
$$(1,k_0+1) \to (1,k_0+2) \to (2,k_0+2)$$
$$\cdots$$
$$(\delta-1,k_0+\delta-1) \to (\delta,k_0+\delta-1) \to (\delta,k_0+\delta)$$

By the assumption on initial free round and related perceived probabilities, $(0,0)$ and $(0,1)$ are accepted. Other nodes in this path will also be accpeted by lemma 3.1 and 3.2. So, this is a valid path, and by definition this accepted node $(\delta, k_0+\delta)$ is reachable. Secondly, for a rejected node with a reachable predecessor node, appending it to the end of the above path for its reachable predecessor still produces a valid path, then by definition it is a reachable stop point. Unreachable nods are therefore those rejected nodes with no reachable predecessor node. □

Proof of Proposition 3.2. Again we prove the unrestricted case and take the restriction on maximum number of rounds into consideration when necessary.

We first claim that there exist infinitely many previous stop points (k_U, k_D) whose successor node (k_U, k_D+1) is accepted and

$$p^U_{(k_U,k_D)} > \frac{1}{2} + \frac{1}{2}\left(\frac{\lambda}{\lambda+1} - \frac{1}{2}\right).$$

This is because, firstly, by property 3.1, and also by monotonicity and convergency in property 3.2, for any $k_U \in \mathbb{N}^+$, there exists an interger $k_D > k_U$ such that

$$p^D_{(k_U,0)} > \cdots > p^D_{(k_U,k_U)} = \frac{1}{2} > \cdots > p^D_{(k_U,k_D)} > \frac{1}{\lambda+1} \geq p^D_{(k_U,k_D+1)},$$

so (k_U, k_D+1) is accepted and (k_U, k_D) is rejected, and (k_U-1, k_D)—which is a predecessor node of (k_U, k_D)—is of the same net gain as (k_U, k_D+1), then by lemma 3.2, (k_U, k_D) is a stop point whose successor node (k_U, k_D+1) is accepted. Secondly, suppose on this stop point (k_U, k_D) we have

$$p^U_{(k_U,k_D)} \leq \frac{1}{2} + \frac{1}{2}\left(\frac{\lambda}{\lambda+1} - \frac{1}{2}\right),$$

by convexity in property 3.3, we have

$$p^U_{(k_U,k_D+1)} - p^U_{(k_U,k_D)} < p^U_{(k_U,k_D)} - p^U_{(k_U,k_D-1)},$$

notice that $k_D > k_U$ implies $k_D - 1 \geq k_U$, which means

$$p^U_{(k_U,k_D-1)} \leq \frac{1}{2},$$

then we can compute that

$$p^U_{(k_U,k_D+1)} < 2p^U_{(k_U,k_D)} - p^U_{(k_U,k_D-1)} \leq \frac{\lambda}{\lambda+1}$$

which means (k_U, k_D+1) is rejected, a contradiction and we must have

$$p^U_{(k_U,k_D)} > \frac{1}{2} + \frac{1}{2}\left(\frac{\lambda}{\lambda+1} - \frac{1}{2}\right).$$

Finally, we can find such a (k_U, k_D) for every $k_U \in \mathbb{N}^+$.

Then consider a naive 1-planning player on any stop point (k_U, k_D) with above properties. We know that (k_U, k_D+1) is accepted, that is

$$p^D_{(k_U,k_D+1)} \leq \frac{1}{\lambda+1}.$$

We also know that $k_D - 1 \geq k_U$, so (k_U+1, k_D) is of non-positive net gain, by property 3.1, we have

$$p^U_{(k_U+1,k_D)} \geq \frac{1}{2}.$$

Thus, note that $u(2) > 0$ and $u(-2) < 0$, this player's value V for a 2-draw bundle on

(k_U, k_D) satisfies

$$\begin{aligned}\frac{V}{\alpha(w_{(k_U,k_D)})} &= p^U_{(k_U,k_D)} p^U_{(k_U+1,k_D)} u(2) + p^D_{(k_U,k_D)} p^D_{(k_U,k_D+1)} u(-2) \\ &\geq \left[\frac{1}{2} + \frac{1}{2}\left(\frac{\lambda}{\lambda+1} - \frac{1}{2}\right)\right] \cdot \frac{1}{2} \cdot u(2) + \left[\frac{1}{2} - \frac{1}{2}\left(\frac{\lambda}{\lambda+1} - \frac{1}{2}\right)\right] \cdot \frac{1}{\lambda+1} \cdot u(-2) \\ &= \frac{3\lambda+1}{4(\lambda+1)} \cdot \frac{1}{2} \cdot u(2) + \frac{\lambda+3}{4(\lambda+1)} \cdot \frac{1}{\lambda+1} \cdot u(-2).\end{aligned}$$

We can compute that the coefficient of $u(2)$ is always more than λ times as large as the coefficent of $u(-2)$, that is

$$\frac{\frac{3\lambda+1}{4(\lambda+1)} \cdot \frac{1}{2}}{\frac{\lambda+3}{4(\lambda+1)} \cdot \frac{1}{\lambda+1}} - \lambda = \frac{(\lambda-1)^2}{2(\lambda+3)} > 0$$

always holds for any $\lambda > 1$. By the assumption on the utility function we have $\lambda u(2) + u(-2) = 0$, therefore

$$\frac{V}{\alpha(w_{(k_U,k_D)})} \geq 0$$

or simply $V > 0$, that is a 2 draw bundle will be accepted on (k_U, k_D).

The acceptance of a 2-draw bundle on a previously reachable node (k_U, k_D) is obvious from the fact that

$$p^U_{(k_U,k_D)} \geq \frac{\lambda}{\lambda+1} > \frac{1}{2} + \frac{1}{2}\left(\frac{\lambda}{\lambda+1} - \frac{1}{2}\right).$$

If there is a restriction on the maximum number of rounds that can be played, it is clear that the 2-draw bundle is available as long as it has not come to the last possible round. □

Proof of Proposition 3.4. Analogous to the proof of Proposition 3.2, for each $k_U \in \mathbb{N}^+$, we can find a previous stop point (k_U, k_D) in the non-potisitive-net-gain part of the tree whose predecessor node $(k_U, k_D + 1)$ is accepted and

$$p^U_{(k_U,k_D)} > p^U + \frac{1}{2}\left(\frac{\lambda}{\lambda+m^r} - p^U\right)$$

Additionally, we only consider those nodes with sufficiently large net loss (greater than or euqal to m) such that $(k_U + 1, k_D)$ is still in the non-positive-net-gain part of the game tree. This is true for all k_U greater than some positive integer beacuse previouly reachable nodes form a stair-step shape, so there are still infinitely many such stop points in unrestricted case.

Then consider a naive 1-planning player on any of the above (k_U, k_D), her value for a

2-draw bundle is

$$\begin{aligned}\frac{V}{\alpha(w_{(k_U,k_D)})} &= p^U_{(k_U,k_D)}p^U_{(k_U+1,k_D)}u(2m)\\ &+ p^U_{(k_U,k_D)}p^D_{(k_U+1,k_D)}u(m-1) + p^D_{(k_U,k_D)}p^U_{(k_U,k_D+1)}u(m-1)\\ &+ p^D_{(k_U,k_D)}p^D_{(k_U,k_D+1)}u(-2).\end{aligned}$$

By property 3.1, since (k_U+1,k_D) is of non-positive net gain, we have $p^U_{(k_U+1,k_D)} \geq p^U$ so that

$$p^U_{(k_U,k_D)}p^U_{(k_U+1,k_D)}u(2m) \geq p^U_{(k_U,k_D)}p^U(2m)^r.$$

By monotonicity in property 3.2, we have

$$p^U_{(k_U,k_D)}p^D_{(k_U+1,k_D)}u(m-1) > p^U_{(k_U,k_D)}p^D_{(k_U,k_D)}(m-1)^r$$

and

$$p^D_{(k_U,k_D)}p^U_{(k_U,k_D+1)}u(m-1) > p^D_{(k_U,k_D)}p^U_{(k_U,k_D)}(m-1)^r.$$

Accepting next round on (k_U, k_D+1) indicates $p^U_{(k_U,k_D+1)}U(m) + p^D_{(k_U,k_D+1)}U(-1) \geq U(0)$, this gives

$$\lambda \leq \frac{p^U_{(k_U,k_D+1)}}{p^D_{(k_U,k_D+1)}}m^r,$$

so we have

$$\begin{aligned}p^D_{(k_U,k_D)}p^D_{(k_U,k_D+1)}u(-2) &= -\lambda p^D_{(k_U,k_D)}p^D_{(k_U,k_D+1)}2^r\\ &\geq -p^D_{(k_U,k_D)}p^U_{(k_U,k_D+1)}(2m)^r\\ &> -p^D_{(k_U,k_D)}p^U_{(k_U,k_D)}(2m)^r\end{aligned}$$

Combining these ineqalities, we have

$$\begin{aligned}\frac{V}{\alpha(w_{(k_U,k_D)})} &> p^U_{(k_U,k_D)} \cdot \left[p^U(2m)^r + 2p^D_{(k_U,k_D)}(m-1)^r - p^D_{(k_U,k_D)}(2m)^r\right]\\ &= p^U_{(k_U,k_D)} \cdot (p^D_{(k_U,k_D)} - p^U)(2m)^r \cdot \left[\frac{2p^D_{(k_U,k_D)}}{p^D_{(k_U,k_D)} - p^U}(\frac{m-1}{2m})^r - 1\right]\\ &= p^U_{(k_U,k_D)} \cdot (p^D_{(k_U,k_D)} - p^U)(2m)^r \cdot \left[\frac{(1-\frac{1}{m})^r}{1-\frac{p^U}{p^D_{(k_U,k_D)}}}2^{1-r} - 1\right]\end{aligned}$$

We know that $0 < r < 1$ and $m \geq 2$, then

$$(1 - \frac{1}{m})^r > 1 - \frac{1}{m}.$$

We also know that (k_U, k_D) is a stop point and

$$p^U_{(k_U,k_D)} > p^U + \frac{1}{2}\left(\frac{\lambda}{\lambda + m^r} - p^U\right),$$

the rejection of this node together with the additional restriction $\lambda \leq m^{1+r}$ indicate that

$$p^D_{(k_U,k_D)} > \frac{m^r}{\lambda + m^r} = \frac{1}{\frac{\lambda}{m^r} + 1} \geq \frac{1}{m+1} = p^U,$$

while the inequality is equivalent to

$$p^D_{(k_U,k_D)} < p^D - \frac{1}{2}\left(\frac{\lambda}{\lambda + m^r} - p^U\right),$$

which implies that

$$\begin{aligned}
1 - \frac{p^U}{p^D_{(k_U,k_D)}} &< 1 - \frac{p^U}{p^D - \frac{1}{2}\left(\frac{\lambda}{\lambda+m^r} - p^U\right)} \\
&= 1 - \frac{1}{m - \frac{m+1}{2}\left(\frac{\lambda}{\lambda+m^r} - \frac{1}{m+1}\right)} \\
&< 1 - \frac{1}{m - \frac{m+1}{2}\left(1 - \frac{1}{m+1}\right)} \\
&< 1 - \frac{1}{m}.
\end{aligned}$$

So, we know that

$$\frac{(1 - \frac{1}{m})^r}{1 - \frac{p^U}{p^D_{(k_U,k_D)}}} > 1,$$

and obviously $2^{1-r} > 1$, then

$$\frac{(1 - \frac{1}{m})^r}{1 - \frac{p^U}{p^D_{(k_U,k_D)}}} 2^{1-r} - 1 > 0.$$

Therefore, we finally conclude that

$$\frac{V}{\alpha(w_{(k_U,k_D)})} > 0$$

or simply $V > 0$, that is a 2-draw bundle will be accepted on (k_U, k_D).

If the net loss on a previously reachable node (k_U, k_D) is greater than or equal to m, the properties for above found stop points are all satisfied from the fact that

$$p^U_{(k_U,k_D)} \geq \frac{\lambda}{\lambda + m^r} > p^U + \frac{1}{2}\left(\frac{\lambda}{\lambda + m^r} - p^U\right),$$

then the acceptance of the 2-draw bundle is obvious. □

Proof of Proposition 3.5. This can be proved by induction.

At any reachable node for a naive 1-planning player, it is obvious that $s^1_{Optimal} \in S^1_{Continue}$. Then, $s^2(s^1_{Optimal}) \in S^2_{Continue}$, and it must be better than s^2_{Reject} because $V(X_{s^2(s^1_{Optimal})}) = V(X_{s^1_{Optimal}}) > V(X_{s^1_{Reject}}) = V(X_{s^2_{Reject}})$. Therefore, $s^2_{Optimal} \in S^2_{Continue}$, which means that every reachable node for the naive 1-planning player is still reachable for a naive 2-planning player.

However, at any stop point for a naive 1-planning player, although s^2_{Reject} is better than any $s^2 \in \{s^2(s^1) \mid s^1 \in S^1_{Continue}\}$, it is still possible that $s^2_{Optimal} \in S^2_{Continue} \setminus \{s^2(s^1) \mid s^1 \in S^1_{Continue}\}$. Therefore, the stop point for the naive 1-planning player is either a reachable node or a stop point for a naive 2-planning player.

Finally, the reachable area for a naive 2-planning player cannot be smaller than that for a naive 1-planning player, and she is expected to stay in the game for at least the same expected length.

Similarly, the node reachability for naive $(N+1)$-planning players can be derived from that for naive N-planning players. Reachable nodes will always remain reachable, but stop points may also be reached. Therefore, the expected number of rounds played cannot be reduced. □

Proof of Proposition 3.6 and 3.7. It simply comes from the fact that $\{s^{N'}(s^N) \mid s^N \in S^N\} \in S^{N'}$ for any $N' > N$, which means a broader-bracketing player will consider all possible plans that have been considered by a narrow-bracketing player.

If the M-draw bundle is accepted by a naive N-planning player, then $V(X_{s^M_{Bundle}}) > V(X_{s^N(s^{N'})}) = V(X_{s^{N'}})$ for any $s^{N'} \in S^{N'}$ such that $N' < N$, which means the bundle is also optimal for a naive N'-planning player.

If the M-draw bundle is rejected by a naive N-planning player, then $s^N_{Optimal} \in S^N$, which implies $s^{N'}(s^N_{Optimal}) \in S^{N'}$ for any $N' > N$. Then, we have $V(X_{s^{N'}_{Optimal}}) \geq V(X_{s^{N'}(s^N_{Optimal})}) = V(X_{s^N_{Optimal}}) \geq V(X_{s^M_{Bundle}})$, which means the optimal plan for a naive N'-planning player cannot be the bundle. Note that the equal sign could hold because we have assumed that the bundled option is suboptimal to its single-draw equivalent. □

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