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Ultra-high-dimensional multi-level optimisation strategies for electrical machines

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Abstract
Electrical machine optimisation is normally a high-dimensional non-linear multi-objective optimisation problem. A multi-level optimisation (MO) strategy is currently used to improve efficiency, where sensitivity analysis is required for dividing design parameters into different groups. However, the conventional MO strategy cannot handle ultra-high-dimensional optimisation problems. In this paper, a sensitivity analysis method with variable weighted intervals is proposed to calculate the sensitivity coefficient in the parameter design range. Moreover, three improved multi-level optimisation strategies based on different optimisation algorithms, sequential sensitivity strategies, and machine learning models are proposed, analysed, and compared with the conventional MO strategy. Through a case study of a synchronous reluctance machine, it can be seen that the proposed optimisation strategies can improve the optimisation results and efficiency of ultra-high-dimensional optimisation of electrical machines.

KEYWORDS
design, learning (artificial intelligence), optimisation, sensitivity analysis, synchronous machines

1 | INTRODUCTION

Electrical machines, such as induction machines, permanent magnet synchronous machines, and synchronous reluctance machines (SynRMs), have been widely used in industrial and home appliances. In the past decades, their performance improved in many ways, such as optimisation [1]. As electrical machines have many design parameters and can be evaluated by many performance indicators, the design optimisation of electrical machines is a high-dimensional, multi-objective non-linear problem [2]. For the design optimisation of a SynRM, the magnetic bridge shape plays a critical role, which is determined by many design parameters and its optimisation is time-consuming. Therefore, an appropriate optimisation strategy is needed to reduce the design optimisation time [3].

Both optimisation models and algorithms are the basis of electrical machine optimisation [4]. Regarding optimisation models, there are two main types in terms of the number of objectives: single-objective and multi-objective. The multi-objective optimisation model is widely used in literature. Average torque, power factor, efficiency, torque ripple, machine volume, and cost are normally selected for the optimisation objectives or constraints [5]. Regarding optimisation algorithms, intelligent optimisation algorithms are commonly used, such as genetic algorithm (GA), particle swarm optimisation, multi-objective genetic algorithm (MOGA), and non-dominated sorting GA [6–8].

In electrical machine optimisation, the modelling method is very important. The commonly used modelling methods are magnetic equivalent circuit (MEC) and finite element analysis

(FEA). MEC requires so many assumptions to simplify the problem that it is faster compared to FEA. However, the magnetic saturation and leakage cannot be considered fully. Therefore, FEA is normally used for the performance calculation of electric machines [9, 10]. As the simulation of FEA may be time-consuming for many situations, the total computational cost in the optimisation is normally huge, especially for high-dimensional problems.

The optimisation process can be sped up by combining surrogate models with optimisation strategies. Popular surrogate models include the response surface model (RSM), radial basis function (RBF) model, and kriging interpolation (KI) [11–16], which can construct approximate models with fewer FEA samples by using different designs of experiment (DOE) techniques. Conventional optimisation algorithms and approximation models struggle with high-dimensional optimisation problems for machines with more than 10 parameters. In this case, we face the following challenges:

- convergence risk
- huge computation cost
- low calculation accuracy for the approximation model

To improve the optimisation efficiency, a multi-level optimisation (MO) strategy has been developed for electrical machines [17–19]. However, only around 10 parameters were investigated in the conventional MO strategy. Literature [20] proposed an optimisation process of MO, where the design parameters are divided into three levels. Practically, there can be more than 20 parameters in an optimisation problem of electrical machines, which can be regarded as an ultra-high-dimensional optimisation problem.

The conventional MO strategy discussed in this paper faces challenges in handling ultra-high-dimensional optimisation problems. For a machine design problem with 36 optimisation parameters, it is solved using MO with three levels. Each level is still a high-dimensional problem with around 12 parameters, which is still a high-dimensional optimisation problem. The optimisation efficiency is still low when using intelligent algorithms, and the surrogate model has low precision when dealing with high-dimensional optimisation problems.

It is a challenge to efficiently handle ultra-high-dimensional optimisation, and one possible approach is to employ and improve MO. In MO, the design parameters are divided into three groups, and the high-dimensional problems are converted into several low-dimensional problems [21, 22]. By increasing the number of groups and selecting appropriate optimisation methods, MO can be extended to ultra-high-dimensional situations. However, even if the high-dimensional space is divided into low-dimensional spaces, the optimisation of the structural parameters in each space still requires a significant number of computational resources [23]. In order to improve the efficiency of optimisation, the hybrid optimisation algorithms are employed [24, 25].

Meanwhile, sensitivity analysis is often used to screen the design variables in order to ensure the accuracy of the approximate model and to improve the optimisation speed in previous MO studies. Only a few important design parameters were optimised while others are fixed to their initial values [26]. The computational effort usually becomes so large that it is difficult to solve for optimal results when the design parameters are increased. Moreover, the design parameters can be grouped through sensitivity sorting based on the sensitivity analysis; some grouping strategies were proposed in refs. [3, 27]. However, the conventional grouping method is only based on design experience or preliminary sensitivity analysis of the design parameters, and its efficiency is low when dealing with ultra-high-dimensional optimisation problems [28]. To solve the problem, all design parameters are divided into different optimisation spaces. The design parameters in each space are optimised in turn and the process is looped until the optimal solution is found [29]. Meanwhile, a new sensitivity analysis method is proposed to improve the optimisation efficiency of the machine. Moreover, three MO methods are proposed to enhance the efficiency and results of ultra-high-dimensional optimisation based on sensitivity analysis.

The remainder of this paper is organised as follows. Section 2 proposes a new sensitivity analysis method called local sensitivity analysis with variable weighted interval (LSA-VWI). Section 3 presents the optimisation processes of three different MO strategies, including the improved multi-level optimisation (IMO) strategy, the unequal-number parameters multi-level optimisation (UNPMO) strategy, and the sequential sensitivity analysis multi-level optimisation (SSAMO) strategy. Section 4 investigates a case study, and the rated parameters and structural parameters of a SynRM are introduced. Section 5 compares the simulation results and the efficiency of the proposed three MO strategies, followed by the conclusion. Section 6 concludes this paper.

2 | LOCAL SENSITIVITY ANALYSIS WITH VARIABLE WEIGHT INTERVAL (LSA-VWI)

During the optimisation process, several interpolation and modelling techniques are used to construct an approximate model of the objective function. Firstly, RSM fits polynomial models by designing experimental data to provide simplified representations of complex functions. Second, RBF is an interpolation method based on a distance metric, which smooths the estimated function values by weighting the distance and is suitable for multidimensional interpolation of non-linear problems. Finally, KI is based on a spatial correlation model that optimally predicts the values of unsampled points based on known data points. By combining these methods, it is possible to construct high-precision approximation models in high-dimensional optimisation problems and improve optimisation efficiency. To develop an efficient MO framework, the first step is the division of the ultra-high-dimensional parameter design space. A previous work has introduced four sensitivity analysis methods for permanent magnet machines, which are sizing equation, local sensitivity analysis (LSA), global sensitivity analysis (GSA), and analysis of variance (ANOVA) [30].

By utilising the current density in the sizing equation, the basic relationship between the objectives and parameters can be found. Considering that machine design optimisation is a non-linear and strong coupling problem, the accuracy of this method is not high. Moreover, the LSA and GSA are widely used in sensitivity analysis. As a reference point, that is, the initial design scheme is required in LSA, the sensitivity analysis results depend on the chosen reference point. This may affect the efficiency of the conventional MO method because it does not implement the sensitivity analysis for the optimal results obtained from the optimisation of each level. ANOVA is a technique based on DOE, which can be used to determine the significant factors from all design parameters. As the parameter increases, the samples required by ANOVA increase exponentially, which makes ANOVA rarely used in ultra-high-dimensional problems. Therefore, a new sensitivity analysis method, LSA-VWI, is proposed for sensitivity analysis of ultra-high-dimensional MO in this paper, which includes the following steps.

First, the sensitivity coefficients (SC) of the parameters to the objective function in a specific design interval are calculated. SC and weighting factor ω_i of each sample point in the interval are defined as follows:

$$\mathrm{SC} = \frac{1}{n-1}\sum_{i=2}^{n}\left(\omega_i \frac{|y_i - y_{i-1}| \cdot (x_U - x_L)}{(x_i - x_{i-1})}\right) \tag{1}$$

$$\omega_i = \begin{cases} \dfrac{1}{(y_{\mathrm{ave}} - y_{\mathrm{obj}})} & |y_{\mathrm{ave}} - y_{\mathrm{obj}}| > \varepsilon \\ 1 & |y_{\mathrm{ave}} - y_{\mathrm{obj}}| \le \varepsilon \end{cases} \tag{2}$$

where x_i and y_i are the values of the parameter and objective of the ith sample point, respectively, N is the number of sample points, x_U and x_L are the upper limit and lower limit of parameter design interval, respectively, y_{ave} is the average of the objective for all samples, y_{obj} is the objective value, and ε is a given constant with a small value.

As shown in Equation (2), when the absolute distance between y_{ave} and y_{obj} is larger than ε, the closer y_{ave} to y_{obj}, the larger the weighting factor and the greater the SC. Therefore, the parameter with a higher SC can be found more quickly in the initial analysis step. When the difference between y_{ave} and y_{obj} is within ε, the weighting factor is set to 1.

Based on Equations (1) and (2), the multi-objective sensitivity coefficient normalisation formula *MOSC(jk)* and the weighting factor ω_{ijk} of N variables and M objective functions optimisation problem are depicted as follows:

$$MOSC(jk) = \frac{1}{n-1}\sum_{i=2}^{n}\left(\omega_{ijk} \frac{|y_{ijk} - y_{(i-1)jk}| \cdot (x_{jkU} - x_{jkL})}{(x_{ijk} - x_{(i-1)jk})}\right)$$
$$(j = 1, 2, \ldots, M; k = 1, 2, \ldots, N) \tag{3}$$

$$\omega_{ijk} = \begin{cases} \dfrac{1}{(y_{jk\_\mathrm{ave}} - y_{jk\_\mathrm{obj}})} & |y_{jk\_\mathrm{ave}} - y_{jk\_\mathrm{obj}}| > \varepsilon \\ 1 & |y_{jk\_\mathrm{ave}} - y_{jk\_\mathrm{obj}}| \le \varepsilon \end{cases} \tag{4}$$

where, x_{ijk} and $x_{(i-1)jk}$ are the parameter values of the jth objective function of the kth parameter at the ith and $(i-1)$th sample points, respectively, x_{jkU} and x_{jkL} are the upper limit and lower limit of the parameter design interval of the kth parameter and the jth objective function, y_{ijk} and $y_{(i-1)jk}$ are the objective function values of the kth parameter and the jth objective function at the ith and $(i-1)$th sample points, respectively, and y_{jk_ave} and y_{jk_obj} are the average objective of the kth parameter and the jth objective function and the objective value.

To evaluate the sensitivity of a parameter, it is necessary to integrate the sensitivity of this parameter to all objective functions, and the combined sensitivity coefficient *MOSC(k)* is depicted as follows:

$$MOSC(k) = \sum_{j=1}^{M} MOSC(jk) \tag{5}$$

The greater the *MOSC*, the more sensitive the parameter is to all the objective functions. On the other hand, the smaller the *MOSC*, the less sensitivity of this parameter to all the objective functions.

3 | MULTI-LEVEL OPTIMISATION STRATEGIES

In this paper, three MO strategies, IMO, UNPMO and SSAMO, are proposed. The advantages and disadvantages of the above methods are listed in Table 1. Meanwhile, the detailed description of the methods is elaborated below.

3.1 | IMO and UNPMO strategies

After *MOSCs* of all design parameters are obtained, the design parameters are grouped according to the sorting results in IMO and UNPMO strategies. For the conventional MO method, most optimisation cases have more than three levels, and all levels are optimised using the same optimisation algorithm. The detailed process of MO can be found in the literature [20].

The IMO strategy uses the same optimisation process as the conventional MO strategy. However, the number of groups can be adjusted according to parameter dimensions and some parameters with low sensitivity can be discarded in IMO. The flowcharts of MO and IMO are shown in Figure 1.

In MO strategies, the transfer of design points between different levels is an important problem. Each level selects a design point as the initial design point of the next optimisation level. The commonly used selection method is to divide the

TABLE 1 Advantages and disadvantages of IMO, UNPMO and SSAMO methods.

Methods	Advantages	Disadvantages
IMO	Suitable for low to medium dimensional problems with high computational efficiency	Difficulty in dealing effectively with ultra-high dimensional problems
UNPMO	Good global search capability	High computational complexity
SSAMO	Effective adaptive strategies with good optimisation results	Dependent on initial conditions and parameter settings

Abbreviations: IMO, improved multi-level optimisation; SAMO, sequential sensitivity analysis multi-level optimisation; UNPMO, unequal-number parameters multi-level optimisation.

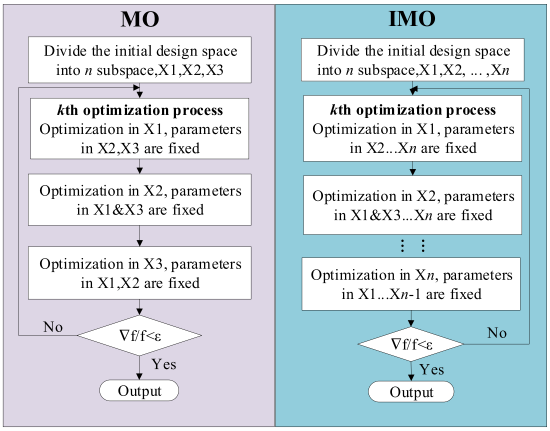

FIGURE 1 Flowcharts of MO and IMO. IMO, improved multi-level optimisation.

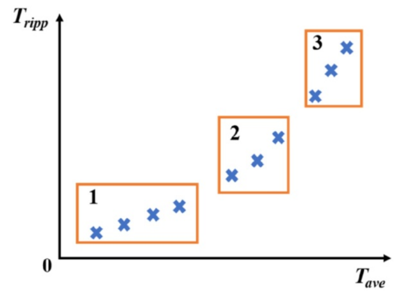

FIGURE 2 Selection of design points.

Pareto front into three sections, as shown in Figure 2. Select a point in a region based on the compromise between several objective functions, which is highly dependent on the experience of the designer.

For the IMO strategy and other MO strategies in this paper, the designer can select the design points of each level according to the change of the most sensitive parameter of points on the Pareto front in this iteration. Assume that the most sensitive parameter in this iteration is P, whose value of the initial design is P_0, and the values of this parameter for other points on the Pareto front are P_i ($i = 1, 2, \ldots, n$). The point with max $|P_i - P_0|$ should be selected as the design point, which can make the most sensitive parameter in this iteration remain near the optimum to improve the convergence rate. Therefore, a design point is selected as the optimal design of this level and transferred to the next level based on the above principle.

In the UNPMO strategy, different optimisation algorithms are used for various numbers of parameters in different groups. The principle of grouping and selecting optimisation algorithms in the UNPMO strategy is as follows: Group A contains parameters in the top 25% of the *MOSC* rank; group B contains parameters in the 25%–75% of the *MOSC* rank; group C contains parameters in the 75%–100% of the *MOSC* rank. Different optimisation methods can be applied to different groups considering the optimisation efficiency and accuracy.

Firstly, group A parameters have a great influence on performance and need more accurate optimisation. Therefore, the MOGA is adopted for the optimisation of parameters in group A. The accuracy of MOGA decreases with the increase of the parameter for machine optimisation due to the non-linearity. Several subspaces (with 3 or 4 parameters in each subspace) can be defined according to the *MOSC* rank. Secondly, group B usually includes more parameters than groups A and C according to design experience. The sequential Taguchi method can be employed due to its efficiency in dealing with high-dimensional optimisation problems [31]. Thirdly, the parameters in group C can be optimised by using the surrogate model based on DOE technology, which can be divided into secondary groups of four to six parameters.

The optimisation efficiency can be improved greatly by using the UNPMO strategy. Meanwhile, a globally optimal solution can be achieved by distributing all the parameters into each group reasonably and adopting appropriate optimisation algorithms in each level and each secondary group.

Figure 3 shows a flowchart for the UNPMO strategy, which consists of the following main steps.

1) Use LSA-VWI to calculate the *MOSC* of parameters to all objective functions, and sort the parameters by *MOSC* from largest to smallest.
2) Group A contains the top 25% of *MOSC* sorted parameters, group B contains 25%–75% of *MOSC* sorted parameters, and group C contains the remaining parameters. If there are too many parameters in groups A and C, they can be divided into secondary groups.
3) Optimise the design parameters in each group by using the conventional MO strategy and optimisation algorithms.
4) The super volume index (I_h) between the Pareto front and the reference point (such as an initial point) can be calculated for the last obtained optimised parameters in each group. Figure 4 shows the schematic diagram of the I_h

calculating method of two targets with five points on the Pareto front. I_h is the area of the shaded area as shown in Figure 4, which is used to evaluate whether the iteration converges.

5) Compare the I_h of the last group of two adjacent iteration processes. If the change is within γ (the value of γ can be selected from 2.5% to 5%), then the optimisation is considered to be converged. Meanwhile, the iteration process is completed. Otherwise, repeat steps 3 and 4 until the convergence criteria are met.

3.2 | SSAMO strategy

In the IMO and UNPMO strategies, design parameters in different groups are optimised sequentially, and the sensitivity analysis is conducted only once for the entire design optimisation process. Therefore, the most reasonable sensitivity ranking cannot be obtained and the grouping of parameters may not be the most appropriate for each iteration process. When the number of design parameters is small, keeping the design parameters in fixed groups does not have a significant impact on the optimisation results and computation time. However, the convergence speed is slow when the IMO and UNPMO strategies are used for ultra-high-dimensional problems. For an optimisation problem with 12 parameters, which can be divided into three levels with four parameters in each level, 625 FEA calculations are required for each level in the solving process. With the increase of parameters, the number of parameters of each level in each iteration increases, and the FEA computation times of each level increase exponentially.

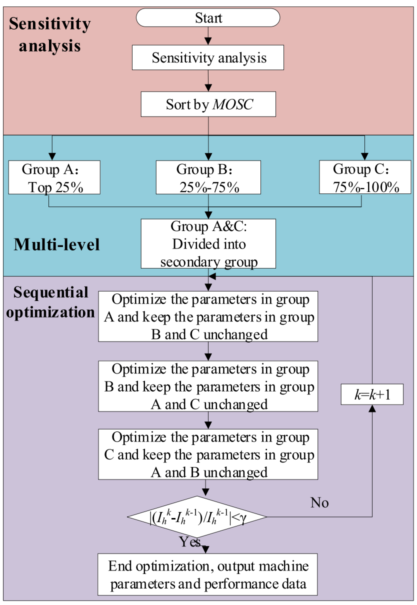

FIGURE 3 The flowchart of UNPMO optimisation. UNPMO, unequal-number parameters multi-level optimisation.

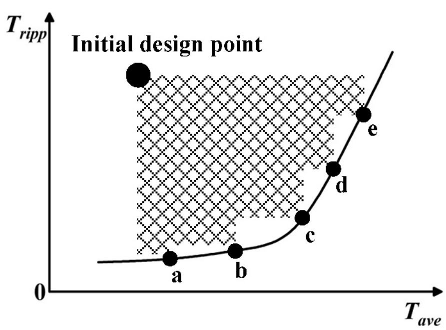

FIGURE 4 A schematic diagram of the super volume index of two targets.

The SSAMO strategy divides the design parameters into groups A and B based on *MOSC*. Group A contains the top 20% parameters of *MOSC* rank and group B contains all remaining parameters. Meanwhile, only the parameters in group A are optimised in each design optimisation process. After that, the design space of the optimised design parameters is narrowed for sensitivity analysis and regrouped. The flowchart of the SSAMO strategy is shown in Figure 5.

The main steps of the SSAMO strategy are as follows:

1) Use the LSA-VWI method to calculate the *MOSC* of design parameters, and sort them by *MOSC* from largest to smallest.
2) Divide design parameters into groups A and B. Group A contains the top 20% of *MOSC* sorted parameters, and group B contains the remaining parameters.
3) Optimise the design parameters in group A using an intelligent optimisation algorithm. In this process, keep the design parameters in group B fixed.

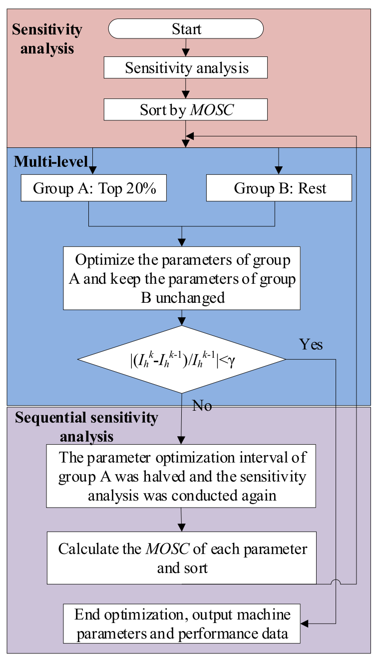

FIGURE 5 The flowchart of SSAMO optimisation. SSAMO, sequential sensitivity analysis multi-level optimisation.

4) Calculate the super volume index (I_h^k) of the Pareto front for group A and compare it with the super volume index (I_h^{k-1}) obtained from the previous iteration. If the relative error in the absolute value of the difference between the different super volume indices of the two iterations is less than γ, the optimisation for this time is considered to have converged, and the optimisation results are outputted. Otherwise, perform step 5.
5) Reduce the interval of design parameters in group A to half of the original by using the space reduction technique, and recalculate *MOSC*.
6) Calculate the *MOSC* of each parameter and sort them. Repeat the steps 2–6.

The space reduction technique used by SSAMO can keep the ratio of design points to the endpoints in the optimisation space constant, and scale down the optimisation space. Moreover, the schematic diagram of optimising space reduction for a parameter is shown in Figure 6. As shown, B_{k-1} and C_{k-1} are the endpoints of the $(k-1)$th optimisation interval, A_k is the design point of the $(k-1)$th optimisation, B_k and C_k are the endpoints of the kth optimisation interval. B_k is the mid-point between A_k and B_{k-1}, and C_k is the mid-point between A_k and C_{k-1}. Therefore, the use of the space reduction technique shortens the design interval and greatly reduces the optimisation time.

4 | OPTIMISATION MODEL OF A SynRM

4.1 | Design of a SynRM

In this paper, a SynRM designed with three-layer flux barriers and 24-slot 4-pole is adopted for the case study, as shown in Figure 7. Figure 7b shows the design parameters of the SynRM. The distance between the third layer flux barrier and the shaft is w_1, the thicknesses of the rotor flux barrier and the magnetic bridge are w_2–w_6, respectively, the width of the flux barrier is L_1–L_6, respectively, the deflection angles are β_1–β_6, respectively, and the radius of the end arc of the three-layer flux barriers are R_1–R_3, respectively. Table 2 lists the main parameters and dimensions of the studied SynRM. Table 3 lists the parameters to be optimised for the parametric model. Changing these 21 design parameters can make the SynRM with various U-shaped flux barriers, one of which provides the best performance [32].

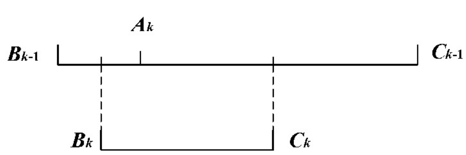

FIGURE 6 The reduction of kth optimisation space.

4.2 | Optimisation model

The high torque ripple generated by the SynRM is a limitation as it leads to vibration and noise, restricting its application range. The torque ripple can be calculated as follows:

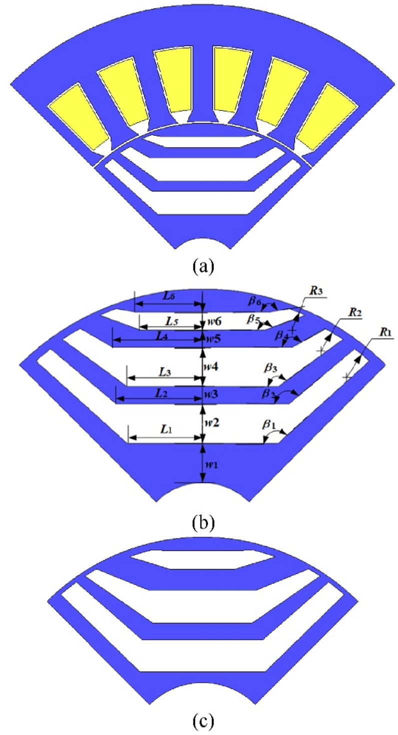

FIGURE 7 The structures of SynRM, (a) the 1/4 model of SynRM, (b) rotor design parameters of SynRM, (c) the final optimised rotor structure of the SynRM. SynRM, synchronous reluctance machine.

TABLE 2 Main parameters and dimensions of synchronous reluctance machine.

Parameter	Value
Rated current density (A/mm^2)	6
Rated power (kW)	2.5
Rated phase voltage (V)	187
Rated speed (r/min)	3000
Max speed (r/min)	6000
Efficiency	>85%
Slot number	24
Pole pairs	2
Flux barrier layers	3
Number of turns per slot	30
Space factor	53%
The outer diameter of the stator (mm)	70
Stack length (mm)	61

TABLE 3 Parameters to be optimised.

Description	Parameter
The distance between the third layer flux barrier and the shaft	w_1
The thickness of the rotor flux barrier and the magnetic bridge	w_2–w_5
The width of the flux barrier	L_1–L_6
The deflection angle	β_1–β_6
The radius of the end arc of the three-layer flux barrier	R_1–R_3

$$T_{\text{ripp}} = \frac{T_{\max} - T_{\min}}{T_{\text{ave}}} \times 100\% \tag{6}$$

where $T_{\max}$ is the maximum torque, $T_{\min}$ is the minimum torque, and T_{ave} is the average torque.

Efficiency is another important performance index in machine design and it can be calculated as follows:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{em}} - P_{\text{core}} - P_0}{P_{\text{em}} + P_{\text{copper}}} \tag{7}$$

where P_{em} is the calculated electromagnetic power, P_{core} is core loss, P_{copper} is copper loss, and P_0 is the mechanical loss and stray loss.

The electromagnetic power, copper loss, and core loss can be expressed in Equations (8)–(10), respectively.

$$P_{\text{em}} = \frac{T_{\text{em}} \times n}{9.55} \tag{8}$$

$$P_{\text{copper}} = 3I^2R \tag{9}$$

$$P_{\text{core}} = P_h + P_e + P_a \tag{10}$$

where T_{em} is the calculated electromagnetic torque using FEA, n is the speed of SynRM, I is phase current, R is phase resistance, P_h is hysteresis loss, P_e is eddy current loss, and P_a is additional loss.

Taking the average torque T_{ave} and torque ripple T_{ripp} as the objective functions, efficiency η and back electromotive force V_φ as constraint conditions, a MO model can be constructed as given in Equation (11).

$$\begin{cases} \max: T_{\text{ave}} \quad \min: T_{\text{ripp}} \\ \text{s.t.}: \quad \eta - 0.86 > 0 \\ \qquad V_\varphi - 200 < 0 \end{cases} \tag{11}$$

5 | OPTIMISATION RESULTS AND COMPARATIVE ANALYSIS

In this section, the MOGA, IMO, UNPMO, and SSAMO strategies are used to optimise the SynRM for the 21 design parameters listed in Table 2, respectively. The results are shown in Figures 8–11, where the optimisation results and efficiencies

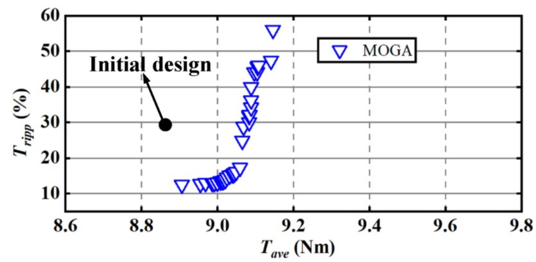

FIGURE 8 The Pareto front of MOGA. MOGA, multi-objective genetic algorithm.

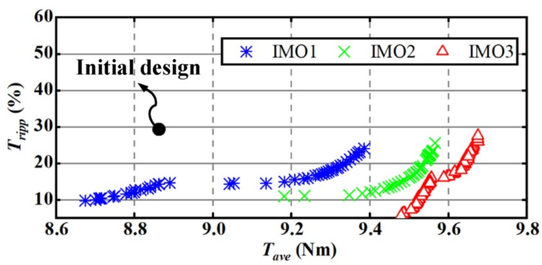

FIGURE 9 The Pareto fronts of IMO. IMO, improved multi-level optimisation.

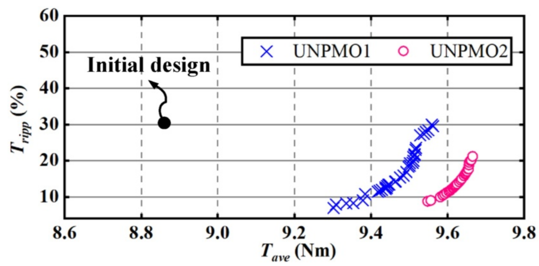

FIGURE 10 The Pareto fronts of UNPMO. UNPMO, unequal-number parameters multi-level optimisation.

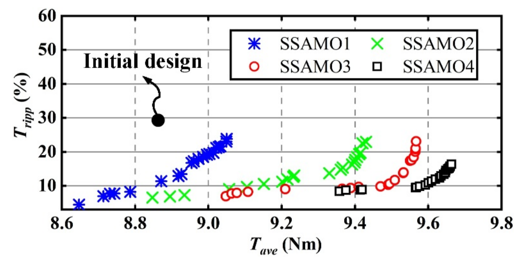

FIGURE 11 The Pareto fronts of SSAMO. SSAMO, sequential sensitivity analysis multi-level optimisation.

are compared and discussed. Moreover, the performance of the machine is best after SSAMO optimisation and the rotor structure of the optimised SynRM is shown in the Figure 7c.

FEA calculations are performed on a workstation with a processor (Intel CPU 8 cores, base speed 2.9 GHz), memory (32 GB RAM), and hard disk (10T), and it takes about 35 s to calculate one FEA analysis of the SynRM. MOGA requires 300 generations, with 80 individuals per generation. Therefore, the MOGA optimisation process requires a substantial number of FEA calculations, which can be time-consuming, taking about 233 h to obtain the optimal Pareto front in the case study discussed in the paper. Meanwhile, the MOGA was optimised 10 times repeatedly and the optimal Pareto front is shown in Figure 8. As shown, the T_{ripp} of all optimised points on the Pareto front is greater than 10% and T_{ave} is less than 9.2 Nm. The MOGA cannot guarantee that the optimal solution can be obtained every time due to the strong non-linearity of high-dimensional problems, and it is very likely to fall into the local optimal solution. Therefore, the optimisation accuracy of MOGA is a big problem, and multiple optimisation and selection also bring obstacles to optimisation efficiency.

When γ is set to 5%, the IMO iterates three times in total, and the Pareto fronts of the optimisation at the last layer of each iteration are shown in Figure 9. Meanwhile, the UNPMO iterates twice, and the Pareto fronts of the optimisation at the last layer of each iteration are shown in Figure 10. Figure 11 shows the Pareto fronts of each iteration of SSAMO, which the blue dots, green dots, red dots, and black dots represent the Pareto fronts of the first, second, third, and fourth rounds of SSAMO optimisation, respectively. It can be seen that the SSAMO optimisation process converges after four iterations.

Figure 12 shows the sensitivity analysis results of 21 design parameters in four iterations of SSAMO using LSA-VWI. As shown, the design parameters L_5, β_5, R_2, and R_3 appear repeatedly in group A. However, some design parameters do not occur in group A all the time.

Figure 13 shows the Pareto front of MOGA, IMO, UNPMO, and SSAMO for the final optimised results. The optimisation results obtained by UNPMO and SSAMO are quite similar and are better than those obtained by IMO. Meanwhile, the Pareto front obtained by MOGA is worse than the Pareto fronts of the other three MO strategies. Therefore, the SynRM optimised by the SSAMO strategy can be determined as a representative case of MO strategies. Considering operation robustness and manufacturing process, two design points B and C in Figure 13 are determined as the optimal points by using the MOGA and SSAMO strategies. Moreover, the FEA was used to evaluate the performance of the two optimised SynRMs.

Figure 14 shows the torque waveform of three different SynRMs. Table 4 lists the comparison of the average torque and torque ripple of three SynRMs. As shown, the average torque and torque ripple of SynRM optimised by SSAMO strategy is 9.42 Nm and 8.81%, respectively, which are the best ones in the table. The average torque of the SSAMO design (9.42 Nm) has been improved by 6.32% and 4.43% compared with those of the initial design (8.86 Nm) and MOGA

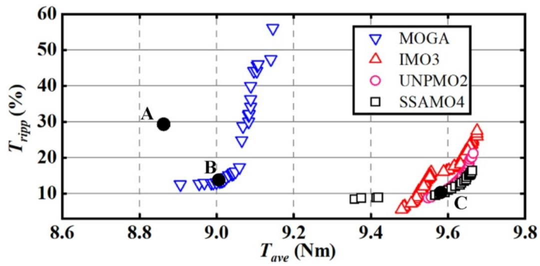

FIGURE 13 The Pareto front of the last optimisation of the four optimisation algorithms. IMO, improved multi-level optimisation; MOGA, multi-objective genetic algorithm; SAMO, sequential sensitivity analysis multi-level optimisation; UNPMO, unequal-number parameters multi-level optimisation.

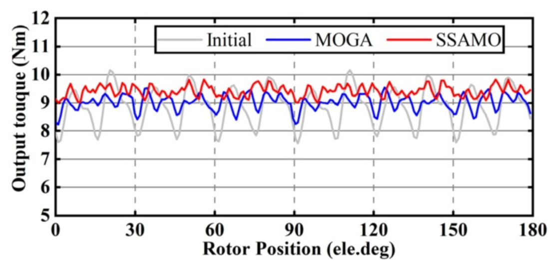

FIGURE 14 Comparison of torque waveform. MOGA, multi-objective genetic algorithm; SAMO, sequential sensitivity analysis multi-level optimisation.

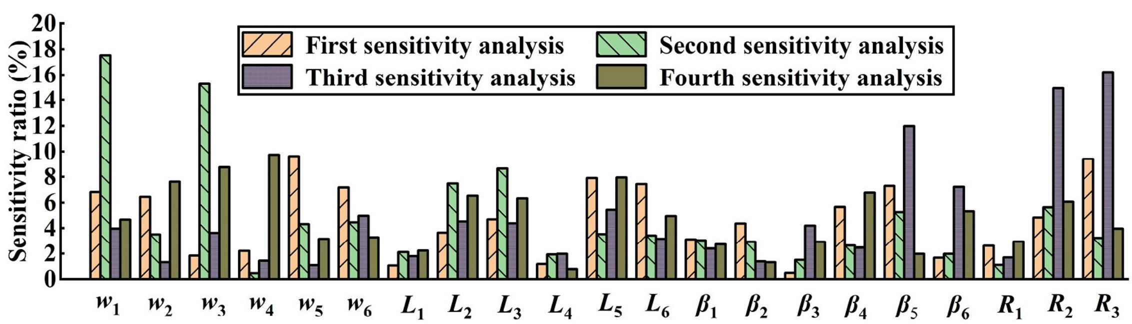

FIGURE 12 Four sensitivity analyses of SSAMO in group A. SSAMO, sequential sensitivity analysis multi-level optimisation.

optimised design (9.02 Nm), respectively. The torque ripple (8.81%) of the SynRM optimised by SSAMO strategy is decreased by 69.90% and 39.07% compared with the initial design (29.28%) and MOGA optimised design (14.47%), respectively. The torque performance of SynRM optimised by the MOGA strategy does not meet the requirements of practical industrial applications, which indicates that the MOGA strategy falls into a locally optimal solution due to many design parameters being optimised. Meanwhile, the average torque and torque ripple coefficient of the SynRM optimised by SSAMO meet the performance requirement of the machine.

Moreover, it can be seen that the proposed three MO strategies in this paper have good performance for optimising the electrical machine with ultra-high-dimensional design parameters compared to the MOGA strategy. Table 5 lists the comparison of these four optimisation strategies in terms of three aspects, the number of iterations, the number of finite element calculations, and the optimisation time.

In the IMO strategy, 21 design parameters are divided into 4 groups with 5 parameters in each group, with 1 design parameter discarded, and Latin hypercube DOE design is adopted for the sampling. The number of FEA calculations of the IMO strategy was 9375, and the consumed optimisation time of the IMO strategy was about 91 h. Moreover, the time of establishing the surrogate model has been ignored since it was very short compared with the FEA calculation time.

In the UNPMO strategy, group A has 6 parameters, group B has 11 parameters, and group C has 4 parameters. Meanwhile, group A is further divided into two secondary groups. The MOGA is used for optimising the parameters in two secondary groups, and 1800 FEA calculations are required. For the parameters in group B, 500 FEA calculations are required using the orthogonal table of L50 (5^{11}). For group C, the Kriging surrogate model is adopted, and 625 FEA calculations are performed. Therefore, 5850 FEA calculations are required for UNPMO, and the optimisation time was 57 h.

TABLE 4 Comparison of torque and torque ripple coefficient.

	Initial	MOGA	SSAMO
T_{ave} (Nm)	8.86	9.02	9.42
T_{ripp} (%)	29.28	14.47	8.81

Abbreviations: MOGA, multi-objective genetic algorithm; SAMO, sequential sensitivity analysis multi-level optimisation.

TABLE 5 Comparison of four optimisation strategies.

	MOGA	IMO	UNPMO	SSAMO
Number of iterations	—	3	2	4
FEA calls	24,000	9375	5850	4440
Opt. time (h)	233	91	57	43

Abbreviations: FEA, finite element analysis; IMO, improved multi-level optimisation; MOGA, multi-objective genetic algorithm; SAMO, sequential sensitivity analysis multi-level optimisation; UNPMO, unequal-number parameters multi-level optimisation.

The number of iterations of SSAMO is the largest among the three strategies. However, only the design parameters in group A are iterated. The number of design parameters in group A is varied from 5 to 6 each time, and MOGA is used for optimisation. A total of 4440 FEA calculations are required for SSAMO, and the optimisation time was 43 h.

For the optimisation time, the SSAMO strategy takes the shortest optimisation time, while the MOGA strategy takes the longest. Although the IMO strategy and UNPMO strategy have similar optimisation effects, the optimisation time required by the UNPMO strategy has been saved by about 37.4% compared with that required by the IMO strategy. Compared with the MOGA strategy, the optimisation time required by the IMO, UNPMO, and SSAMO strategies has been improved by 60.94%, 75.53%, and 81.54%, respectively.

The optimisation results achieved by the UNPMO and SSAMO strategies are better than those of the IMO strategy. The optimisation result achieved by the MOGA strategy is the worst one in ultra-high-dimensional optimisation problems, which cannot meet the actual requirements of engineering.

To sum up, several points should be noted for the three MO strategies introduced in this paper:

a) The IMO, UNPMO, and SSAMO strategies are much more effective and efficient than the MOGA strategy for ultra-high-dimensional optimisation problems, though MOGA has good performance for conventional low-dimensional optimisation problems.
b) The IMO, UNPMO, and SSAMO strategies can be adopted for multi-objective design optimisation. In this paper, two objectives are optimised. The above optimisation strategy can also be used for optimisation problems with three to five objectives. However, the optimisation time will increase as well with the increase of the optimisation objectives.
c) The optimisation efficiency of the UNPMO and SSAMO strategies is related to γ. If the γ is reduced, more iterations and longer optimisation time will be required. On the other hand, decreasing γ will improve the optimisation results. Moreover, a reasonable γ is recommended to be between 2.5% and 5%.

6 | CONCLUSION

In this paper, the three MO strategies (IMO, UNPMO, and SSAMO) are proposed, which have obvious advantages over MOGA for ultra-high-dimensional optimisation. Meanwhile, the variable-weighted interval sensitivity analysis method is proposed, which can reduce the optimisation time and accelerate convergence. The optimisation results and efficiency of the MOGA and three MO strategies are compared and analysed based on the SynRM case study.

The following conclusions can be drawn from the case of SynRM analysed in this paper. Firstly, the optimisation results obtained by the IMO, UNPMO, and SSAMO strategies are much better than those by the MOGA strategy. The Pareto fronts of the last layer of UNPMO and SSAMO strategies are almost overlapping and slightly better than the IMO strategy. Secondly, the optimisation time of IMO, UNPMO, and SSAMO is significantly reduced compared with MOGA, and SSAMO takes the shortest time.

Moreover, the MOGA method has a long optimisation time and is not suitable for dealing with high-dimensional optimisation problems. The IMO method shows high stability and efficiency in dealing with low and medium dimensional problems. However, it is easy to fall into local optimal solutions in ultra-high-dimensional parameter optimisation. UNPMO and SSAMO methods show better global search capability in solving ultra-high-dimensional problems by adopting hierarchical optimisation strategies with different optimisation algorithms. Despite the higher computational complexity, they are more applicable in complex optimisation scenarios. On the other hand, the proposed methods provide a flexible multi-level solution scheme when dealing with multi-objective optimisation problems. However, the effectiveness of the methods is highly dependent on the initial conditions and parameter settings in practical applications. Future research will further optimise the algorithm design of these methods to enhance their application efficiency and accuracy in practical engineering.

AUTHOR CONTRIBUTIONS

Chengcheng Liu: Writing—review & editing. **Shiwei Zhang**: Software; writing—review & editing. **Hongming Zhang**: Methodology; writing—review & editing. **Youhua Wang**: Conceptualization; writing—original draft; Writing—review & editing. **Lin Liu**: Conceptualization; formal analysis; writing—original draft; writing—review & editing.

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NOMENCLATURE

I	phase current
I_h	super volume index
L_1–L_6	the width of the flux barrier
P_0	mechanical loss and stray loss
P_a	additional loss
P_{copper}	copper loss
P_{core}	core loss
P_e	eddy current loss
P_{em}	electromagnetic power
P_h	hysteresis loss
P_{in}	input power
P_{out}	output power
R	phase resistance
R_1–R_3	the radius of the end arc of the three layer flux barrier
T_{ave}	average torque
T_{em}	electromagnetic torque
T_{max}	maximum torque
T_{min}	minimum torque
T_{ripp}	torque ripple
V_φ	back electromotive force
w_1	the distance between the third layer flux barrier and the shaft
w_2–w_5	the thickness of the rotor flux barrier and the magnetic bridge
y_{ave}	average of the objective
y_{obj}	objective value of optimisation
β_1–β_6	the deflection Angle
η	efficiency

CONFLICT OF INTEREST STATEMENT

The authors have no relevant financial or non-financial interests to disclose. The authors have no competing interests to declare that are relevant to the content of this article. All authors certify that they have no affiliations with or involvement in any organisation or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript. The authors have no financial or proprietary interests in any material discussed in this article.

DATA AVAILABILITY STATEMENT

Research data are not shared.

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