

Evolutionary Big Data Analytics and Multi-Objective Optimization

by Iman Rahimi

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the degree of

Doctor of Philosophy

under the supervision of Professor Amir H. Gandomi

University of Technology Sydney
Faculty of Engineering and Information Technology

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CERTIFICATE OF ORIGINAL AUTHORSHIP

I, *Iman Rahimi*, declare that this thesis is submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the Faculty of Engineering and Information Technology at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Abstract- Optimization is a key concept in solving many industrial problems, often complicated by constraints that increase the complexity of finding feasible solutions. Many practical problems require optimization under constraints, a complex area extensively studied by researchers across various disciplines. This thesis introduces innovative approaches to handle constraints in single and multi-objective optimization tasks using population-based algorithms. The proposed methods focus on dynamically adjusting the boundaries of variables based on constraints, minimizing the generation of infeasible solutions, and guiding the search process toward feasible regions in the solution space.

These methods are applied to benchmarks and real-world optimization challenges with varying objectives and scales, employing state-of-the-art algorithms tailored for single-, multi- and many-objective scenarios. Examples include solving renowned engineering problems like the welded beam design and car side optimization problems using these newly proposed techniques. Moreover, the open-pit mining optimization problem is used as a large-scale optimization problem and is solved with the suggested method. The results demonstrate that the proposed methods outperform traditional techniques, improving the speed of finding feasible solutions and enhancing the objective space across both single- and multi-objective optimization tasks. Significant improvements are observed in handling multiple objectives and constraints, validated through comparative analyses with state-of-the-art algorithms. The findings have broad implications for the field of optimization, offering advancements in computational efficiency and practical applications in structural design, automotive engineering, and mining.

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Additionally, I want to convey my sincere thanks to my parents for their moral support, which was crucial in helping me finish my work within the allotted time.

Declaration

I, **Iman Rahimi**, hereby declare that the following chapters of this thesis are based on the research articles listed below. These articles have been published, accepted for publication, or are under review in peer-reviewed journals. I also declare the extent of contribution by myself and my co-authors, as well as the current status of each paper.

Chapter 2: A Review on Constraint Handling Techniques for Population-based Algorithms: from single-objective to multi-objective optimization

- **Authorship:** Iman Rahimi, Amir H. Gandomi, Fang Chen, and Efrén Mezura-Montes
- **Publication Outlet:** Archives of Computational Methods in Engineering
- **Status:** Published
- **Contribution:** Iman Rahimi : Conceptualization, Research design, literature review, writing the manuscript. Amir H. Gandomi : Supervision, conceptual advice. Fang Chen : Co-Supervision , Methodological guidance. Efrén Mezura-Montes : Technical editing and review.

Chapter 3: Extending Boundary Updating Approach for Constrained Multi-objective Optimization Problems

- **Authorship:** Iman Rahimi, Amir H. Gandomi, Mohammad Reza Nikoo, and Fang Chen
- **Publication Outlet:** International Conference on the Applications of Evolutionary Computation(Part of EvoStar)
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- **Contribution:** Iman Rahimi : Conceptualization, Development of methodology, data analysis, manuscript writing. Amir H. Gandomi : Conceptual advice and supervision. Mohammad Reza Nikoo : Technical contributions. Fang Chen : Co- Supervision , Review and editing.

Chapter 4: Efficient implicit constraint handling techniques for constrained optimization problems

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- **Contribution:** Iman Rahimi : Conceptualization, Algorithm development, experimental work, manuscript drafting. Amir H. Gandomi : Supervision and guidance. Mohammad Reza

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Chapter 5: A comprehensive Review of Mining Supply Chain Optimization Problems:Methods, Problems, and Technologies

- **Authorship:** Iman Rahimi, Amir H. Gandomi, Mohammad Reza Nikoo
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- **Contribution:** Iman Rahimi : Conceptualization, Comprehensive literature review, methodology development, manuscript preparation. Amir H. Gandomi : Supervision, conceptual advice, Mohammad Reza Nikoo:Reviewing, editing.

Chapter 6: The boundary updating approach coupled with Evolutionary Algorithm formixed integer programming optimization: An open pit mining optimization problem

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- **Publication Outlet:** Draft prepared
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Chapter 7: Variable Interaction and Social Network Analysis to Enhance Constrained Optimization via Boundary Update Method

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Production Note:
Amir H. Gandomi.....Signature removed prior to publication.

Navid Yazdanjue-----

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Conclusion

Chapter 1

Introduction and Background

Abstract- This chapter introduces the foundational concepts of constrained optimization and its challenges in real-world applications. The chapter emphasizes the importance of constraint-handling techniques (CHT) and their role in solving optimization problems effectively. The research questions and objectives are outlined, focusing on developing a novel methodology for improving the feasibility and computational efficiency of solutions in single- and multi-objective optimization tasks. Moreover, the chapter highlights the significance of this research in addressing engineering challenges, such as open-pit mining and structural design, and concludes with a detailed discussion of the research scope and methodology.

1. Background

Multi-objective optimization problems have been raised recently, and finding the solution approaches have been a challenge to researchers for a long time. To address constrained optimization in real-world problems, Constrained Evolutionary Algorithm Optimization (CEAO) integrates an evolutionary algorithm (EA) with a constraint handling technique (CHT). Various methods have been proposed to tackle these challenges, and CEO employs this combination to effectively manage constraints within the optimization process (Michalewicz et al., 1996; Ray et al., 2001; Y. Yang et al., 2019).

In a study by (Binh & Korn, 1997) an infeasible individual will be divided into different categories based on their distances to the feasible region, and ranking will be conducted according to the classes. The authors of (C. M. Fonseca & Fleming, 1998) introduced an approach that assigns high and low priorities to constraints and objective functions, respectively. (Jiménez et al., 1999) Proposed a CHT that only considers the inequality constraints, wherein the algorithm uses tournament selection that has better convergence properties in comparison

to the proportionate selection operator (Goldberg & Deb, 1991). However, the latter algorithm employs niche count for all populations that may increase the complexity of the computation. (Coello Coello & Christiansen, 1999) proposed an innovative method that disregards any solution failing to meet the specified constraints. Deb (2000) proposed the use of a genetic algorithm (GA) population-based approach plus a controlled mutation operator to keep diversity among feasible solutions. Ray et al., (2001) developed a CHT where three different non-dominated rankings of the population are performed, using objective function values, various constraints, and a combination of all objective functions and values.

Also, Hybrid methods integrate multiple CHTs to manage constraints effectively. Y. Yang et al., (2020) discussed four distinct hybrid approaches in their study:

- 1) The ensemble of constraint handling, which divides the population into three sub-populations (B. Y. Qu & Suganthan, 2011).
- 2) The adaptive trade-off model, incorporating two different CHTs (J.-P. Li et al., 2016);
- 3) The push and pull search, where the population is initially pushed to the unconstrained Pareto front and then pulled back to the Pareto front (Fan et al., 2019).
- 4) The two-phase framework (TPF), which first transforms the objective functions into a single objective function using the weighting method and then applies a constrained multiobjective evolutionary algorithm (MOEA) to achieve Pareto feasible solutions (Z.-Z. Liu & Wang, 2019).

Angantyr et al., (2003) combined a penalty function and multi-objective optimization technique, in which the ranking scheme is borrowed from the latter technique. Chafekar et al., (2003) suggested two approaches, namely Objective Exchange Genetic Algorithm and Objective Switching Genetic Algorithm, for solving constrained MOOPs.

(Garg, 2016) used a hybrid of PSO and GA algorithms to improve the balance between exploration and exploitation using genetic operators, namely crossover and mutation in PSO.

Gandomi & Deb, (2020) proposed a new approach, known as boundary update (BU) technique, which can handle constraints directly (i.e., updating variable bounds) and tested the proposed method on several single-objective optimization problems.

Big Data Analytics, characterized by the analysis and optimization of massive and complex datasets, plays a pivotal role in addressing modern optimization challenges (Emrouznejad, 2016). The integration of optimization techniques with Big Data Analytics is critical for solving large-scale, real-world problems across industries, including energy, healthcare, and supply chain management (Roy et al, 2018).

This study extends the work proposed by (Gandomi & Deb, 2020), which is applied to some multi-objective optimization problems. Most of the available constraint handling techniques handle the constraints explicitly, while the proposed approach in this thesis implicitly handles the constraints of an optimization problem and is coupled with an explicit method." On the other hand, the proposed method handle constraints directly rather than constraint handling by the penalty or other fix-ups. The remainder of the chapter is organized as follows. Section 2 presents the research question. Section 3 shows aims and research objectives. The Significance and scopes are provided in sections 4 and 5. Section 6 illustrates the proposed research methodology. Section 6 depicts the scope of this research. Section 7 shows the ethics and risk consideration. Section 8 presents research plan.

2. Research questions

Regarding the background introduced earlier, there exist some research questions as follows:

- Was the proposed BU algorithm, coupled with multi-objective evolutionary algorithms like NSGA-II or NSGA-III, gradually improving its performance across generations?
- Did the performance of the proposed algorithms improve in early generations and then slow down towards the end?
- How do the proposed constraint-handling methods solve large-scale problems effectively?
- Can the proposed constraint handling method solve mixed integer programming optimization problem?
- Which type of multi-objective evolutionary algorithms are suitable to couple with the proposed method?
- How can the BU method enhance the scalability and performance of evolutionary algorithms for data-intensive optimization problems in Big Data Analytics?
- What metrics best evaluate the performance of the suggested methods in data-driven optimization scenarios?
- What is the scalability of this method in solving complex multi-objective optimization problems?
- What specific constraints and metrics are critical for optimizing Big Data applications?
- What metrics are most effective for evaluating multi-objective optimization performance in data-driven applications?

3. Aims and Research Objectives

The aim of this research is to expand and develop a constraint handling technique for multi-objective constrained optimization problems. On the other hand, the aim of this research is eliminate the feasible space so that optimization algorithm could find the optimal solution much

faster than original technique without using constraint handling method. The research foundation is based on the BU method proposed by Gandomi and Deb (2020), which is an innovative technique designed for enhancing the handling of constraints within optimization problems. This method operates by dynamically adjusting the boundaries of variables based on the constraints present in the problem, thereby minimizing the generation of infeasible solutions and streamlining the search towards viable regions of the solution space. The BU method is showcased through various examples, illustrating its application to both mathematical and practical engineering optimization challenges. It proves to be particularly effective when integrated into population-based optimization algorithms, offering a more efficient and direct approach to managing constraints. This efficiency is evidenced by improved solution quality and reduced computational effort, as highlighted through comparative analysis with other constraint-handling techniques. The specific objectives are:

- Expanding a multi-objective mathematical optimization technique for constraint handling.
- Developing effective constraint handling techniques to tackle constrained single- and multi-objective optimization problems.
- Adjusting the constraint handling techniques mentioned in sections 1 and 2 for some constrained real-world engineering problems.
- Applying the constraint handling technique mentioned in sections 1 and 2 to large scale constrained open-pit mining optimization problem.

4. Significance

As a significance, the method narrows down variable space and then force the algorithm to focus its search in the feasible region by limiting the feasible search space for variable(s). The

proposed method can be used for any type of constraints and even problems having constraints defined in a black-box case. Also, the proposed method could be coupled with other constraint handling techniques and possesses the potential to handle constrained many-objective optimization problems.

Moreover, this proposed method could be applied to large scale optimization problem. One of the aim of this dissertation is to apply this solution approach on open-pit mining optimization problems. In 2019, Australia has been identified as the largest producer of "Iron Ore" in the world (37%) and fifth largest producer of Nickel (6.7), in 2019. There are other resources in Australia such as Aluminum, Copper, Gold, Silver, and Uranium possessing 27%, 5%, 10%, 5%, and 12% of world production.

Many solution approaches have been provided in the literature for the open pit mining planning problems (Caccetta and Hill, 1999). For example, Rivera Letelier et al., (2020) proposed a mixed-integer programming approach for strategic open pit mining planning problem. Therefore, the solution approach proposed in this dissertation is able to handle constraints of open-pit mining optimization problems, which are formulated as mixed-integer programming. Also, the methods developed in this research address the dual challenges of handling constraints and analyzing large-scale data, thus advancing the application of Big Data Analytics in optimization tasks.

5. Scope

Due to availability of resources, the scope of this research is focused on expanding a multi-objective mathematical optimization technique for constraint handling and developing constraint handling technique that can be applied for constrained single- and multi-objective

optimization problems. The mathematical technique is used in this research to optimize multi-objective model that can optimize some constrained real-world engineering problems. Then research can be applied into large scale constrained open-pit mining optimization problem. In this research, the expanded and developed technique is based on multi-objective mixed integer programming and is coded in Python software.

This research explores advanced constraint-handling techniques in multi-objective optimization, specifically focusing on large-scale computational challenges. While the term "Big Data Analytics" is not explicitly used in a separate chapter, its principles are inherently embedded within the methodologies developed throughout the thesis. The optimization problems addressed, such as open-pit mining and engineering design, require handling vast search spaces and computationally intensive decision-making, both of which align with the core characteristics of Big Data Analytics. The BU method, introduced in this work, enhances scalability and efficiency in solving these complex problems by dynamically refining search boundaries to navigate high-dimensional solution spaces effectively. Furthermore, the application of Social Network Analysis (SNA) in modeling variable interactions demonstrates an analytical approach commonly used in Big Data research. As such, the methodologies developed in this thesis contribute to the broader field of Big Data-driven optimization, even though the focus remains on improving constrained optimization techniques.

6. Proposed research methodology

The proposed method will be developed as a constraint handling technique to tackle constrained multi-objective optimization problems. To validate and verify the above-mentioned constraint handling technique, some constrained real-world engineering problems will be solved and adjusted with the suggested solution approaches. Also, the proposed method will be applied to

large scale constrained multi-objective openpit mining problem. Summary of the research methodology is as follows:

- a) Reviewing related papers in field of constraint handling and constrained multi-objective optimization. Selecting a proper solution method for multi-objective model is another finding of this step.
- b) Designing a framework based on the literature and real world requirements.
- c) Developing a constraint handling technique.
- d) Checking the feasibility of technique by testing different models.
- e) Verification and validation of the proposed methods using various case studies.

7. Ethics and risk consideration

It is noteworthy to mention that proposed methods in this research has been applied to evolutionary algorithms that are stochastic. However, it is necessary to check the effectiveness of this approach coupled with exact algorithms on optimization problems under uncertainty.

8. Research plan

The research plan proposed in the thesis has been proposed in Table 1.1.

Table 1.1 Research plan

Session	Task
Autumn Session 2021	• Understanding of key concepts, problems and issues in relevant literature. • Selecting, analyzing, and critically evaluating relevant material and literature pertaining to the field of study.

Spring session 2021	• Understanding of resource implications. • Justifying of proposed research design and methods. • Adopting of a suitable theoretical or conceptual framework for the proposed study.
Autumn session 2022	• Presenting emerging findings in a written form appropriate to the discipline. • Analyzing emerging findings in the research field. • Discussing of the outcome briefly.
Spring session 2022	• Validating the results from previous session • Comparing the proposed technique with the state-of-the-art techniques
Autumn-Spring session 2023	• Verifying the methods and the proposed model with the real data • Presenting the final discussion and conclusion
Spring session 2024	• Submitting the finalized thesis

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Chapter 2

A Review on Constraint Handling Techniques for Population-based Algorithms: from single-objective to multi-objective optimization

Iman Rahimi¹, Amir H. Gandomi^{1*}, Fang Chen¹ and Efrén Mezura-Montes²

¹ Data Science Institute, *Faculty of Engineering & Information Technology, University of Technology Sydney, Australia*; iman83@gmail.com, Gandomi@uts.edu.au, fang.chen@uts.edu.au

² *Artificial Intelligence Research Institute, University of Veracruz, MEXICO*, emezura@uv.mx

* Correspondence: Gandomi@uts.edu.au;

Abstract- Most real-world problems involve some type of optimization problems that are often constrained. Numerous researchers have investigated several techniques to deal with constrained single-objective and multi-objective evolutionary optimization in many fields, including theory and application. This presented study provides a novel analysis of scholarly literature on constraint-handling techniques for single-objective and multi-objective population-based algorithms according to the most relevant journals and articles. As a contribution to this study, the paper reviews the main ideas of the most state-of-the-art constraint handling techniques in population-based optimization, and then the study addresses the bibliometric analysis, with a focus on multi-objective, in the field. The extracted papers include research articles, reviews, book/book chapters, and conference papers published between 2000 and 2021 for analysis. The results indicate that the constraint handling techniques for multi-objective optimization have received much less attention compared with single-objective optimization. The most promising algorithms for such optimization were determined to be genetic algorithms, differential evolutionary algorithms, and particle swarm intelligence. Additionally, “Engineering,” “Computer Science,” and “Mathematics” were identified as the top three research fields in which future research work is anticipated to increase.

Keywords- Constraint handling technique, Evolutionary Computation, Multiobjective optimization, Population-based algorithms.

1. Introduction

Real-world problems involve some optimization problems that are often constrained, and most of these problems are considered multi-objective optimization problems (MOOPs). No single solution exists for a MOOP; instead, different solutions generate trade-offs for different objectives. Furthermore, MOOPs arise naturally in most fields, and solving them has been a challenging problem for researchers (Behmanesh et al., 2021a; Deb, 2014; Marler & Arora, 2004). Evolutionary computation (EC) methods have been identified as more effective methods to tackle the challenges that arise from the MOOPs, for which the form of the Pareto-optimal front (discontinuity, nonconvexity, etc.) is not important (Coello et al., 2007; Deb, 2011). Moreover, most multi-objective evolutionary algorithms (MOEAs) use the dominance concept (Corne et al., 2000; Horn et al., 1994, Gharehchopogh et al. 2020; Gharehchopogh & Gholizadeh, 2019; Ghafari & Gharehchopogh, 2021; Gharehchopogh, 2022;).

To solve the constrained optimization of all real-world problems, constrained evolutionary algorithm optimization (CEAO) implements an evolutionary algorithm (EA) combined with a constraint handling technique (CHT). In work by (Binh & Korn, 1997), an infeasible individual will be divided into different categories based on their distances to the feasible region, and ranking will be conducted according to the classes. (C. M. Fonseca & Fleming, 1998) introduced an approach that assigns high and low priorities to constraints and objective functions, respectively. Jiménez et al., (1999) proposed a CHT that only considers the inequality constraints, wherein the algorithm uses tournament selection that has better convergence properties in comparison to the proportionate selection operator (Goldberg & Deb, 1991b).

However, the latter algorithm employs niche count for all populations, which may increase the complexity of the computation.

Coello Coello & Christiansen, (1999) introduced a novel approach that ignores any solution that violates any of the assigned constraints. Deb, (2000) first proposed the use of a genetic algorithm (GA) population-based approach plus a controlled mutation operator to keep diversity among feasible solutions. The work of (Ray et al., 2001) proposed a CHT where three different non-dominated rankings of the population are performed using objective function values, different constraints, and a combination of all objective functions and values. Although the technique can handle infeasible solutions carefully and maintain diversity in the population, the algorithm performs poorly choosing parameter values and is computationally expensive. Jimenez et al., (2002) developed an EA based on the nondominated sorting concept that uses the min-max formulation for constraint handling.

Deb et al., (2002) ran the simulation of the non-dominated sorting genetic algorithm II (NSGAI) on a seven-constrained nonlinear problem, which exhibited better performance than Ray-Tai-Seow's algorithm. Coello, (2002) conducted an overview and analysis of the most popular CHTs using EAs along with pros and cons. Angantyr et al., (2003) combined a penalty function and multi-objective optimization technique, in which the ranking scheme is borrowed from the latter technique. Chafekar et al., (2003) suggested two approaches, namely Objective Exchange Genetic Algorithm and Objective Switching Genetic Algorithm, for solving constrained MOOPs. A new partial order relation from the constraint MOOPs was proposed by (Zou et al., 2004), under which the Pareto optimum set satisfies the constraints. Young, (2005) introduced the Blended Space EA, which uses a rank obtained by blending an individual's rank in the objective space to check dominance.

Venkatraman & Yen, (2005) introduced a two-phase algorithm, which separates the objective function and constraints. Cai & Wang, (2006) introduced a MOO-based EA (Cai and Wang method), abbreviated as CW, in addition to three other models for constrained optimization. In the proposed approach, the simplex crossover was used to enrich the exploitation and exploration abilities. Min et al., (2006) proposed an EA based on an evolutionary strategy for constrained multi-objective optimization problems. The method uses a min-max formulation for constraint handling in which feasible individuals and infeasible individuals evolve toward Pareto optimality and feasibility, respectively. Harada et al., (2007) suggested Pareto Descent Repair (PDR) to search for feasible solutions. Y. Wang et al., (2008) proposed an adaptive tradeoff model for constrained evolutionary optimization to address three main issues: evaluating an infeasible solution in case the population contains only infeasible solutions; achieving a balance between feasible and infeasible individuals when the population contains both solutions; and selecting the feasible solution in case the population possesses only feasible solutions.

Kaveh & Talatahari (2009) suggested a heuristic hybrid of particle swarm optimization (PSO) and ant colony optimization for the optimum design of trusses, which showed to handle the problem-specific constraints using a fly-back mechanism. The work of (Ray et al., 2009) suggested an infeasibility-driven EA (IDEA), which can retain a proportion of infeasible solutions among the population members and preserve diversity compared to NSGAI. Tulshyan et al., (2010) investigated an EA solution for approximate Karush-Kuhn-Tucker (KKT) conditions of smooth problems. The results of some test problems indicate that EA's operators lead the search process to a point close to the KKT point. Mezura-Montes & Coello (2011) discussed the most critical techniques, many of which were previously proposed (Coello,

2002; Michalewicz & Schoenauer, 1996). The previous work also addressed some state-of-the-art constrained handling techniques, including feasibility rules based on GA (Deb, 2000), epsilon-constrained method (Takahama et al., 2005), penalty functions (Deb & Datta, 2010; Xiao et al., 2007), and ensemble of constraint-handling methods (Mallipeddi & Suganthan, 2010b; Wolpert & Macready, 1997). Gandomi & Yang (2012) Introduced an evolutionary scheme for handling boundary constraints, combined it with differential evolution (DE) and compared the proposed method with other boundary constraints handling techniques. The results indicated the proposed approach is much better than the existing methods.

Eskandar et al. (2012) developed a water cycle algorithm, inspired by observations of the water cycle process that could be applied to a number of constraint optimization problems. Sadollah et al. (2013) introduced a population-based algorithm based on the mine blast explosion concept and then applied the proposed approach to some constraint optimization problems in comparison to other well-known optimizers. Hamza et al. (2014) used a constraint consensus method that helps an infeasible individual to move towards the feasible region and then combined the method with a memetic algorithm. The research conducted by (Jiao et al., 2014) developed a feasible-guiding strategy to guide the evolution of individuals, in which a revised objective function technique with a feasible guiding strategy based on NSGA-II is introduced to handle constrained MOOPs. The study proposed by (Padhye et al., 2015) proposed a class of constraint handling strategies, in which infeasible individuals are repaired when they are considered in the search space and explicitly preserve the feasibility of the solutions.

Garg (2016) used a hybrid of PSO and GA to improve the balance between exploration and exploitation by using genetic operators, namely crossover and mutation in PSO. A few years later, (Lu et al., 2018) extended the parameter-less CHT so as to provide a balance between the

feasible and infeasible solutions in a GA population. Gandomi & Deb (2020) proposed a new approach, known as the boundary update (BU) technique, which is able to handle constraints directly (i.e. updating variable bounds) and tested the proposed method on several constrained optimization problems. The method proposed by (Gandomi & Deb, 2020a) possesses the potential to couple with MOEA.

It is noteworthy to mention that the majority of the mentioned studies focused on CHTs for single-objective optimization with little attention to multi-objective optimization. This is attributed to the fact that most constraint handling methods developed for single-objective optimization could be modified for multi-objective optimization as well (Mezura-Montes & Coello, 2011). The main contribution of this work is reviewing the most state-of-the-art constraint handling techniques in population-based optimization for single- and multi-objective optimization problems, and then the study addresses the bibliometric analysis, with focus on multi-objective, in the field.

To attain a better understanding of the research field and to provide new insights from relevant publications, this work aimed to answer the following questions:

- RQ1: What are CHTs, and how are they important?
- RQ2: What are the disadvantages of the different CHTs?
- RQ3: What are the key subjects and keywords regarding constraint handling techniques?
- RQ4: Could we extract the most active journals, researchers, and countries in the field?
- RQ5: What are the basic statistics of constraint handling techniques for multi-objective population-based algorithms?

- RQ6: What are the most active countries and affiliations in the field?
- RQ7: What are the gaps found in literature and future trajectory in the area?

The reminder of the study is as follows. Section 2 describes the research methodology. Section 3 presents the CHTs in EAs. Section 4 describes the other approaches. Section 5 addresses the benchmark test problems. Section 6 discusses the scientometric analysis. Section 7 provides a summary of the study along with recommendations for future research. Concluding remarks are offered in the last section.

2. Research Methodology

The research methodology in this work was divided into several stages (Figure 2.1). First of all, documents from databases were gathered from databases namely, Scopus and Web of Science (WOS). For this aim, the authors used special keywords, namely (TITLE-ABS-KEY (constrained AND multi AND objective AND evolutionary AND optimization) OR TITLE-ABS-

KEY (constraint AND handling AND multi AND objective AND evolutionary AND optimization) OR TITLE-ABS-

KEY (constrained AND multi AND objective AND swarm AND optimization) OR TITLE-ABS-

KEY (constraint AND handling AND multi AND objective AND swarm AND optimization) to find the related articles published as of May 4, 2021. Supplementary A and B present the data extracted from Scopus and WOS, respectively. Since some of the articles were duplicates, they were identified and removed from the library in the next stage using Mendeley as a powerful reference manager. Also, some research questions for this study were designed. An

overview along with a general illustration of CHTs is provided in the next stage. A social network analysis, including co-occurrence, co-authorship, citation, and citation network analyses, is then conducted using VOSviewer (Van Eck & Waltman, 2010, 2013) and RStudio. Also, some interesting analytical features, such as number of pages and authors per article, were conducted in this stage. The last section required preparing the findings, identifying important gaps, and determining future research directions.

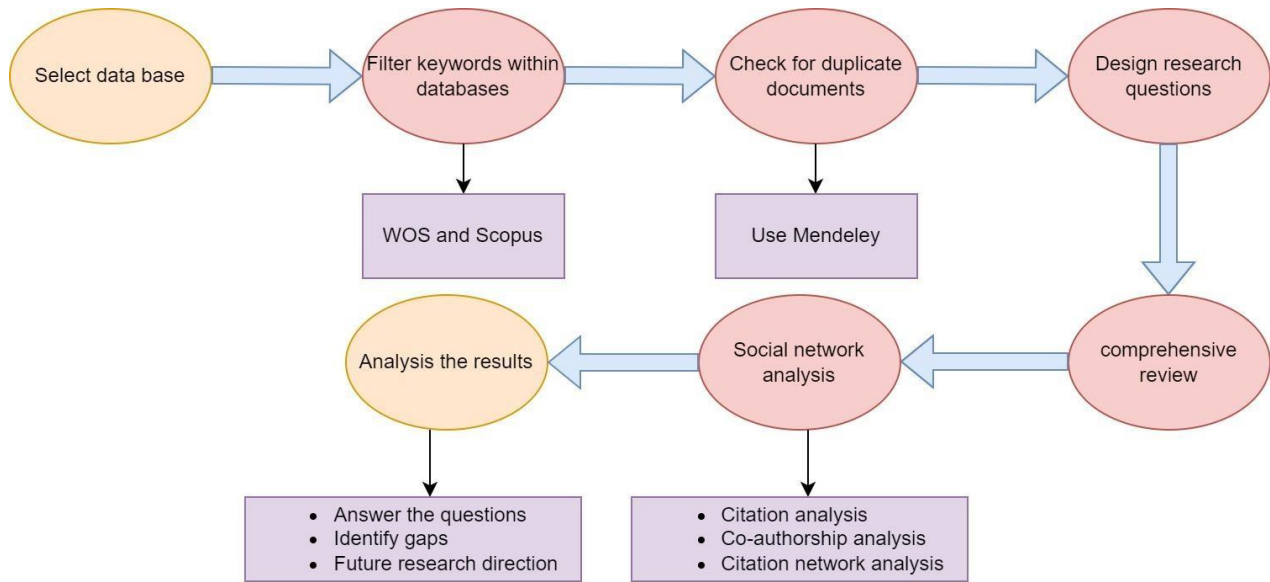


Figure 2.1 Research Procedure

3. Constraint Handling Methods in Evolutionary Algorithms (RQ1)

Almost all real-world problems are considered constraint problems. A general form of a constrained multi-objective optimization problem (CMOOP) is described as follows (Equations 1-3):

$$\text{Maximize (Minimize) } F(x) = (f_1(x), \dots, f_t(x)) \quad (1)$$

s.t.

$$h_i(x) = 0 \quad i = 1, \dots, n \quad (2)$$

$$g_j(x) \leq 0 \quad j = 1, \dots, m \quad (3)$$

Where $F(x)$ is the objective vector; and t , n and m are the number of objective function, equality and inequality constraints, respectively. There is no single solution for a MOOP that simultaneously optimizes each objective, instead there exists a number of Pareto optimal solutions. A Pareto front of possible solutions is called optimal or nondominated if improving anyone's objective further would lead to a decrease in other objectives. According to previous surveys (Coello, 2002; Michalewicz & Schoenauer, 1996), a simple taxonomy of the constraint handling methods in nature-inspired optimization algorithms is as follows:

- Penalty functions methods
- Decoders
- Special operators
- Separation techniques

The first and fourth techniques are discussed in detail later in the paper. As an example of decoders, (Koziel & Michalewicz, 1998) proposed a homomorphous mapping (HM) method between an n -dimensional cube and feasible space. The feasible region can be mapped onto a sample space where a population-based algorithm could run a comparative performance (Kim & Husbands, 1997, 1998; Koziel & Michalewicz, 1998, 1999). However, this method requires high computational costs. A special operator is used to preserve the feasibility of a solution or move within a special region (Michalewicz, 2013; Schoenauer & Michalewicz, 1996, 1997). Nevertheless, this method is hindered by the initialization of feasible solutions in the initial

population, which is challenging with highly-constrained optimization problems. In addition, (Coello, 2017) presented a taxonomy of CHTs in MOEA as follows (Figure 2.2):

- Penalty functions
- Separation method
- Retaining the infeasible solutions
- Hybrid methods

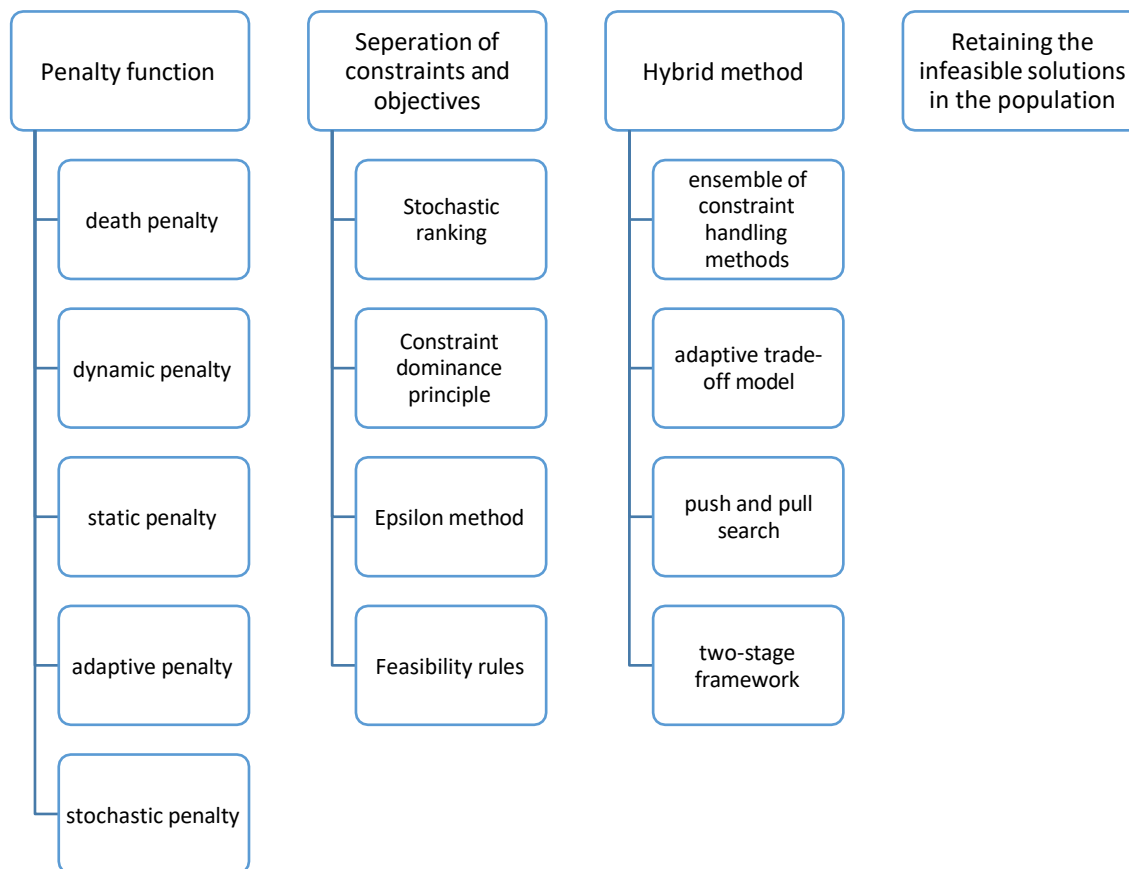


Figure 2.2 Taxonomy of different constraint handling methods in MOEA

Generally, penalty function techniques are one of the most simple CHTs. There are several types of penalty functions used with EAs, the most important ones include (Kramer, 2010):

- Death penalty
- Dynamic penalty
- Static penalty
- Adaptive penalty
- Stochastic ranking

Details regarding the penalty function methods will be discussed in the next section.

3.1 Penalty function approach

One of the easiest and most common ways to handle constraints in multi-objective evolutionary algorithms is the penalty function method.

From a mathematical point of view, two types of penalty functions could be considered as follows:

a) Interior methods

b) Exterior methods

In the first type of penalty function, interior methods, the penalty factor is selected such that the value will be small away from the constraint boundaries and need an initial feasible solution (Coello, 2017), whereas exterior methods do not need an initial feasible solution (Coello et al., 2007). Also, it should be noted that some of the infeasible solutions should be retained in the populations so that they are able to converge to a solution, which lies in the boundary between the feasible and infeasible regions (Mezura-Montes & Coello, 2005).

In the penalty function method, any infeasible solution is ignored (Coello et al., 1995). First, all constraints should be normalized, and for each solution, the constraint violations are calculated as follows (Equation 4):

$$w_j(x^i) = \begin{cases} |\bar{g}(x^i)|, & \text{if } \bar{g}(x^i) < 0 \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

where $\bar{g}(x^i)$ refer to the normalized values for a given constraint $\bar{g}_j(x^i) \geq 0, j = 1, \dots, J$.

Once the violations for the constraints are calculated, the values are added to determine the overall violation as follows (Equation 5):

$$\Omega(x^i) = \sum_{j=1}^J w_j(x^i) \quad (5)$$

Also, a penalty parameter is multiplied by the sum of constraint violations and then added to the objective function values. If a proper penalty parameter is selected, MOEAs will work well; otherwise, a set of infeasible solutions or poor distribution of solutions is possible.

3.1.1 Static penalty functions

In the static penalty proposed by (Homaifar et al., 1994), the penalty coefficient increases as a higher level of violation is reached. In fact, penalty functions do not change, a static penalty function is suggested, and several levels of violation are introduced in which the static penalty parameter is changed in case higher levels of violation are achieved (Jordehi, 2015). In the static penalty function, the expanded objective function is (Equation 6):

$$\varphi(x) = f(x) + \sum_{j=1}^p C_{kj} G_j \quad (6)$$

Where $G_j = \max\{0, g_j(x)\}^\beta$; and $k=1, \dots, l$ where l presents the number of levels of violation.

3.1.2 Dynamic penalty functions

In this category, functions are changed based on the iteration number. Joines & Houck (1994) proposed the following dynamic penalty function, in which the penalty increased when the iteration number increases.

Dynamic multi-objective optimization problems (DMOOPs) involve the simultaneous optimization of different objectives subject to a number of given constraints, where the objective functions, constraints, and/or dimensions of the objective space could change over time. EAs have acquired great attention among researchers for solving the above-mentioned problems.

3.1.3 Adaptive penalty functions

In this category, infeasible individuals are penalized according to the feedback taken from the search process (Ben Hadj-Alouane & Bean, 1997). Woldesenbet et al. (2009) proposed a CHT based on the adaptive penalty function and distance measure, which both change as the objective function value and constraint violations of an individual varies.

Penalty-based constraint handling for multi-objective optimization is similar to single-objective problems in which a penalty factor is added to all the objectives. (Woldesenbet et al., 2009) proposed a self-adaptive penalty function, which is suitable for solving constraint multi-objective optimization problems using evolutionary algorithms. In the self-adaptive penalty function method, the amount of penalty added to infeasible individuals are identified by tracking

the number of feasible individuals. Also, the method uses improved objective values instead of the original objective function values (B. Y. Qu & Suganthan, 2011).

3.1.4 Annealing-based penalty functions

Carlson & Shonkwiler (1998) introduced a multiplicative penalty function based on simulated annealing. In this type of penalty function, the temperature is decreased when the iteration number increases, which leads to an increased penalty.

3.1.5 Co-evolutionary-based penalty functions

Coello (2000) proposed a co-evolutionary approach in which the population is partitioned into two subpopulations. The first population evolves solutions, and the second population evolves penalty factors. In this approach, the penalty function considers information taken from the amount of constraint violations and a number of violations.

There are other types of penalty function methods, which Table 2.1 presents a summary and critique of the techniques for constraint handling.

Table 2.1 The important CHTs (penalty function)- *RQ2*

Method	Criticism	Consequences
Death Penalty (Coello & Christiansen, 1999)	No information is used from infeasible points. It may require the initialization of the population and lack of diversity.	Consumes many evaluations Low success rate
Static Penalty (Homaifar et al., 1994)	It is required to set up a high number of penalty parameters. It is also problem-dependent.	Time-consuming

Dynamic Penalty (Joines & Houck, 1994)	It is hard to drive good dynamic penalty functions in real cases. In some cases, this method converges to either an infeasible or feasible solution that is far from the global optimum (Michalewicz, 1995); (Dasgupta & Michalewicz, 2013)	Premature convergence or even an infeasible solution in some cases
Adaptive Penalty (Ben Hadj-Alouane & Bean, 1997)	Setting the parameters is difficult, such as determining the appropriate generational gap. It requires the definitions of additional parameters (Eiben & Van Der Hauw, 1997).	Time-consuming
Annealing Penalties (Watanabe & Hashem, 2004)	The main disadvantage is its sensitivity to the values of its factors. To handle linear constraints, the user should provide an initial feasible point to the algorithm.	The performance of the algorithms is not good.
Self-adaptive Penalty (Coello, 1999; Coello, 2000)	It defines four additional parameters that may affect the fitness function evaluations.	Time-consuming & weak or strong penalty during evolution
Segregated genetic algorithm (SGA) (Le Riche et al., 1995)	The main difficulty is selecting the penalties for each of the two sub-populations.	Time-consuming
Penalty function based on feasibility (Deb, 2000)	The main issue is maintaining diversity in the population, and in some cases, the use of a niching method combined with higher-than-usual mutation rates is essential.	Premature convergence

3.1.6 Separation of objective function and constraints

Unlike the penalty function technique, another approach exists that separates the values of objective functions and constraints in the nature-inspired algorithms (NIAs) (Powell & Skolnick, 1993), which is known as the separation of objective function and constraints. Hinterding & Michalewicz (1998) initially proposed the idea to divide the search space into two phases. In the first phase, feasible solutions, are found, and in the second phase, optimizing the objective function is considered.

Representative methods of this type of CHT are as follows:

- Constraint dominance principle (CDP)
- Epsilon CHT
- Feasibility rules

The next section provides further details about this type of constraint handling method.

3.1.7 Constraint dominance principle

In the constraint dominance principle (CDP), three feasibility rules are applied to compare any two solutions. If x^1 is feasible and x^2 is infeasible, then x^1 would be better than x^2 . If both solutions are infeasible, then the solution with a smaller constraint violation is better. If both are feasible, then the one dominating the other is better. Deb et al., (2002) adopted CDP to handle constraints in NSGAII (NSGAII-CDP), in which the population is divided into feasible and infeasible sub-populations. NSGAII-CDP first selects offspring from the feasible solutions and then selects solutions from the infeasible solutions. Jan & Khanum, (2013) also adopted CDP to handle constraints in the MOEA/D framework.

3.2.2 ε -constrained (EC) Method

The basic principle of the ε -constrained method, first introduced by (Takahama & Sakai, 2005), is similar to the superiority of feasible solution (SF) proposed by (Powell & Skolnick, 1993b) (Eqs. 12-13). The epsilon value is updated until the parameter k reaches the control generations T_c . Asafuddoula et al. (2012) embedded the epsilon CHT in MOEA/D so that the epsilon value is set adaptively of r comparison. Also, the violation threshold is based on the type of constraints, size of the feasible space, and the search outcome. In the method proposed by (Asafuddoula et al., 2012), the infeasible solutions with violations less than threshold are identified (Eqs. 7-8).

$$\varepsilon(0) = V(x_\theta) \quad (7)$$

$$\varepsilon(k) = \begin{cases} \varepsilon(0)(1 - \frac{k}{T_c})^{cp}, & 0 < k < T_c \\ 0, & k > T_c \end{cases} \quad (8)$$

where x_θ presents the top θ th individual at initialization; and the cp parameter is selected between $[2, 10]$ (B. Y. Qu & Suganthan, 2011).

3.2.3 Feasibility rules

The popularity of this method depends on its ability to be coupled to a range of algorithms without announcing new parameters (factors) (Mezura-Montes & Coello, 2011).

The feasibility rules proposed by (Deb, 2000) are simple, could be integrated into a variety of algorithms without adding new parameters, and thus, are largely used in the research field. Mezura-Montes et al. (2004, 2005, 2006) developed feasibility rules for the selection process, which have been adopted by different evolutionary algorithms such as DE, PSO, and GA.

According to the number of feasible solutions, the search space could be divided into three phases as follows (B. Y. Qu & Suganthan, 2011):

- No feasible solution is found.
- There exists at least one feasible solution.
- Integrating the parent-offspring population has more feasible solutions than the size of the next generation population.

The feasibility rules used in multi-objective optimization, also known as the superiority of feasible solution (SF), are addressed as follows (B. Y. Qu & Suganthan, 2011) (Equation 9):

$$fitness^m(x) = \begin{cases} f_m^m(x) & \text{if } x \text{ is feasible} \\ f_{worst}^m + v(x) & \text{otherwise} \end{cases} \quad (9)$$

where f_{worst}^m and $v(x)$ show the m th objective value of the worst feasible solution and the overall constraint violation, respectively.

3.2.4 Feasibility rules in Differential Evolution (DE)

Although the feasibility rules introduced by (Deb, 2000) have also been widely used by other researchers in DE (Jaimes et al., 2011; Kukkonen & Lampinen, 2006; Lampinen, 2002; Mezura-Montes et al., 2005, 2006; Mezura-Montes & Palomeque-Ortiz, 2009, 2009b; Zielinski, Vudathu, et al., 2008; Zielinski, Wang, et al., 2008), they have been rarely used in multi-objective differential evolution. Particularly, (Lampinen, 2002) used Pareto dominance in constrained space instead of the sum of constraint violations. Later, (Kukkonen & Lampinen, 2006) adopted the Pareto dominance in Generalized Differential Evolution (GDE), but encountered difficulties when there exist more than three constraints and/or objective functions.

Jaimes et al., (2011) proposed a scheme for partitioning the objective space using the conflict information for multi-objective optimization. Jiang, (2017) introduced an operational efficient model based on Data Envelopment Analysis (DEA) and introduced DE along with the feasibility rules to optimize the mentioned model. Si et al. (2017) proposed a combined constraint handling framework, known as CCHF, for solving constrained optimization problems, in which the features of two well-known CHTs (i.e. feasibility rules and multi-objective optimization) were addressed in three different situations (feasible situation, infeasible situation, and semi-feasible situation).

3.2.5 Feasibility rules in PSO

Mezura-Montes & Flores-Mendoza (2009) employed feasibility rules as a constraint handling technique to recognize the most competitive PSO variant when solving constrained numerical optimization problems (CNOPs). In the research by (Mezura-Montes & Flores-Mendoza, 2009), local-best was identified to be better than global best PSO. Karaboga & Akay (2011) adapted an artificial bee colony algorithm (ABC) to solve CNOPs by using feasibility rules by modifying the probability assignment for the roulette wheel selection. The authors of (Elsayed et al., 2010) compared different GA variants using the feasibility rules as the constraint handling method. In the work of (Y. Wang & Cai, 2009) a hybrid version of PSO to solve constrained optimization problems was introduced and the authors found that the swarm at each generation is split into several sub-swarms. Also, the hybrid version applied the feasibility rules to compare particles in the swarm.

3.2.6 Feasibility rules in GA

Elsayed et al. (2011) proposed a GA with a new multi-parent crossover for solving constraint optimization problems, in which the feasibility rules were added to handle the constraints. The latter authors also solved constraint numerical optimization problems by using different GA variants along with feasibility rules and found that all GAs perform equally. Venkatraman & Yen (2005) proposed a two-phase framework for solving constraint optimization problems was introduced. Specifically, the first phase ignores the objective function and the genetic algorithm minimizes the violation of the solutions, while the second phase optimizes bi-objective functions, including the original objective function and constraints satisfaction. Moreover, feasibility rules is applied to assign fitness values to the individuals.

3.2.7 Feasibility rules in other population-based algorithms

Feasibility rules have been adapted to other population-based algorithms, such as artificial immune systems (Aragón et al., 2007b, 2007a; Bernardino et al., 2007; Coello & Cortés, 2002; Cruz-Cortés, 2009; Hajela & Lee, 1996), organizational evolutionary algorithm (J. Liu et al., 2007), biogeography-based optimization (H. Ma & Simon, 2011), and bacterial foraging optimization (Mezura-Montes & Hernández-Ocaña, 2009).

3.2.8 Retaining infeasible solutions in the population

Another CHT is used to retain the infeasible individuals in the population. In other words, a constraint multi-objective optimization problem with m objective is transformed to an optimization problem with $m+1$ objectives, which could save the infeasible solution during the evolution process (C. Peng et al., 2017; Ray et al., 2009). Ning et al. (2017) proposed a

constraint handling technique so that the individuals that possess low Pareto rank and low constraint violation will be chosen.

3.2.9 Hybrid methods

Hybrid methods combine several CHTs to handle constraints. Y. Yang et al. (2020) addressed four different hybrid methods in their work: 1) the ensemble of constraint handling method (B. Y. Qu & Suganthan, 2011) separates the population into three sub-populations; 2) the adaptive trade-off model (J.-P. Li et al., 2016) contains two different CHTs; 3) in the push and pull search (Fan et al., 2019), the population is pushed to the unconstrained Pareto front (push) then the population is pulled back to the Pareto front (pull); 4) the two-phase framework (ToP) (Z.-Z. Liu & Wang, 2019) solves a constraint multi-objective optimization problem by first converting the objective functions into a single objective function via the weighting method then, in the second phase, a constrained MOEA is adopted to attain the Pareto feasible solutions. The disadvantages for each category are summarized in Figure 2.3.

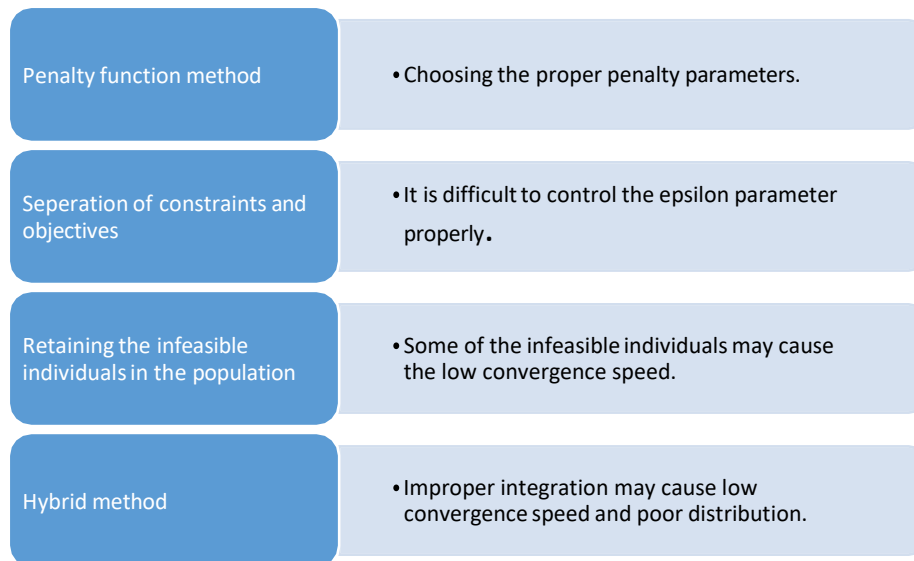


Figure 2.3 The main disadvantages of CHTs in MOEA- RQ2

Mezura-Montes & Coello (2011) presented a state-of-the-art taxonomy of CHTs, which is illustrated in Figure 2.4. Figure 2.5 presents the different state-of-the-art CHTs that have been used since 2000. As mentioned, Deb, (2000) first applied feasibility rules to the genetic algorithm. Runarsson & Yao (2000) introduced stochastic ranking, which employs a user-defined parameter instead of using penalty factors and is able to control the infeasible solutions based on the sum of constraint violation and objective function values. In a study of (Xiao et al., 2007) the epsilon-constraint method was proposed that transforms the constraint optimization problem into an unconstrained problem. Xiao et al., (2007) addressed a multi-constrained optimization problem based on the KS function. Leguizamon & Coello (2008) proposed a boundary search approach inspired by the ant colony metaphor based on conducting a boundary search between a feasible and infeasible solution. Ray et al., (2009) proposed an additional objective to solve a bi-objective optimization problem, where the first objective is the original problem and the second objective is the constraint violation measure. In the work of (Mallipeddi & Suganthan, 2010), a combination of four CHTs, namely feasibility rules, stochastic ranking, self-adaptive penalty function and the epsilon-constraint method to solve constraint numerical optimization problems was addressed.

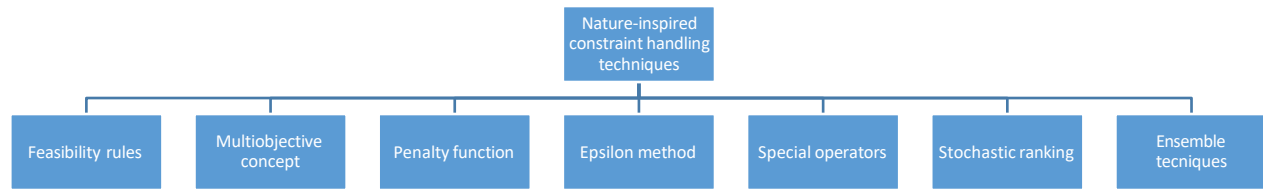


Figure 2.4 State-of-the-art CHTs (Mezura-Montes & Coello, 2011)

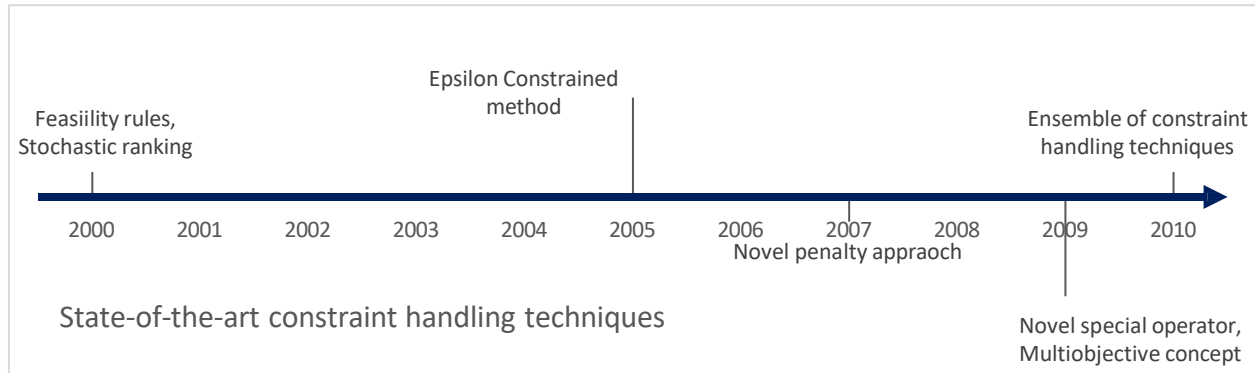


Figure 2.5 Timeline of different state-of-the-art CHTs

3.2.10 Stochastic ranking

Runarsson & Yao (2000) proposed the stochastic ranking (SR) approach to balance between the objective and penalty functions stochastically. The method was tested using a strategy evolution on several benchmarks, and the results showed that the method is able to improve the search performance with a user-defined parameter without introducing complicated variation operators. SR has also been coupled with other population-based algorithms, such as ant colony optimization (ACO) (L. G. Fonseca et al., 2007; Leguizamón & Coello, 2007), differential evolutionary (DE) (Fan et al., 2008; J. Liu et al., 2009; R. Liu et al., 2009), and evolutionary programming (EP) (Mallipeddi et al., 2009).

3.2.11 Ensemble techniques

Ensemble CHTs provide a new research platform to tackle constrained multi-objective optimization problems. Combining several CHTs could improve the capability of an approach compared with a single CHTs (Mezura-Montes & Coello, 2011; Vodopija et al., 2019). For instance, (Vodopija et al., 2019) proposed a combination of four CHTs, namely nondominated sorting, constrained-domination principle, multiple constraint ranking, and dynamic penalty function, and incorporated the proposed technique into an MOEA based on NSGAIL. Some other ensemble CHTs have been reported (Mallipeddi & Suganthan, 2010b, 2010a; Tasgetiren et al., 2010). Although the ensemble CHT has a competitive performance, it suffers from being parameter-dependent.

3.2.12 Multi-objective concept

Based on the multi-objective optimization concept, a constraint single-objective optimization problem is transferred to an unconstrained multi-objective optimization problem (Coello et al., 2007). The multi-objective version of the optimization problem possesses an extra objective function, which presents the sum of constraint violation (L. D. Li et al., 2008; Ray et al., 2009a; Reynoso-Meza et al., 2010; Y. Wang et al., 2007).

The authors of (Samanipour & Jelovica, 2020) presented a taxonomy for constraint handling strategies in multi-objective GA, which include:

- Penalty function methods
- Separation method
- Special operators
- Repair methods

Among these strategies, the penalty function method is not straightforward in multi-objective GA since the fitness assignment is based on the non-dominance rank of a solution rather than its objective function values (Long, 2014). Yet, the penalty function method is one of the most popular CHTs in constraint multi-objective optimization. Whenever the multi-objective function and constraint violation for each constraint are assessed, the sum of violations is added to each objective function value considering the multiplication of the penalty parameter (Deb, 1999; Srinivas & Deb, 1994). Chafekar et al., (2003) proposed two approaches, namely OEGADO and OSGADO. The OEGADO runs several GAs in parallel so that each GA optimizes one objective, whereas The OSGADO runs each objective sequentially with a common population for all objectives.

3.2.13 Repair approaches

There are several techniques used as repair algorithms, in which the search space is reduced (since only feasible individuals are considered):

- In the permutation encodings method, each solution of an EA population is simply signified as an ordered list (Bäck et al., 2018; Poon & Carter, 1995).
- Repair procedures in binary representations, which could be shown as fixing the number of 1s in binary representations and Hopfield networks (Ngo & Li, 1998; Salcedo-Sanz et al., 2004).
- Repair methods in graphs are represented as spanning trees and repairing graphs (Dengiz et al., 1997; Kou et al., 1981).
- Repair methods in grouping GAs, are proper for scenarios where a number of items should be assigned to a set of groups (Agustín-Blas et al., 2009; Falkenauer, 1993).

Pure EAs do not perform well in complex combinatorial problems with a high number of constraints (Krasnogor & Smith, 2005; Salcedo-Sanz, 2009). Single-solution-based algorithms (e.g. local search, simulated annealing) have good performance in exploitation, while population-based algorithms (e.g. swarm intelligence, EA) are exploration-oriented. In these problems, the hybridization of population-based algorithms with single-based algorithms can improve the power of both exploration and exploitation (Salcedo-Sanz, 2009; Suh & Van Gucht, 1987; Talbi, 2016). A memetic algorithm is a hybridization of an EA and a local search (LS) approach that LS is applied to improve the quality of the fitness function. On the other hand, LS could be used as a CHT (Salcedo-Sanz, 2009), i.e. the local repair algorithm only considers feasible individuals leading to reducing the search space. Repair methods could be applied to EAs in several ways, such as in permutation encodings (W. K. Lai & Coghill, 1996; Poon & Carter, 1995), in binary representation (Ngo & Li, 2003), and in graphs and trees (Dengiz et al., 1997; Esbensen, 1995). Although repair algorithms have numerous advantages, some disadvantages do exist. For instance, repair algorithms are problem-specific and must be designed for a specific problem (Coello & Carlos, 1999). Table 2.2 shows a summary of the disadvantages of the state-of-art CHTs.

Table 2.2 A summary of disadvantages of the state-of-art CHTs- RQ2

Method	Disadvantages
Ensemble method	Although the ensemble CHT has a competitive performance, the method is parameter-dependent.
Repair method	Repair algorithms are problem-specific and, thus, must be designed for a specific problem.
Feasibility rules	The method is likely to lead to premature convergence.
Stochastic ranking	Although the method has been employed in several nature-inspired algorithms, it is not often used for the multi-objective version of the algorithms.
Epsilon-constraint method	In some cases, premature convergence has been reported, while other works report that the method relies on gradient-based mutation.
Multi-objective concept	It may require gradient calculation (Mezura-Montes & Coello, 2011a).

3.2.13 Other approaches

Table 2.3 provides a summary of novel approaches proposed between 2020 and 2021 to tackle constrained multi-objective optimization problems. Based on Table 3, there are signs of a renewed interest in constrained multi-objective optimization, even the clear superior amount of research in constrained single-objective optimization.

Table 2.3 Novel approaches for constrained multi-objective optimization problems between 2020 and 2021

Method	Source
KKT points for constrained multi-objective optimization	(Constantin, 2020; Eichfelder & Warnow, 2021)
IoT and cloud computing	(G. Peng et al., 2021)
Indicator-based constrained handling technique	(Yuan et al., 2021)
Decomposition-based algorithm	(C. Peng et al., 2020; Y. Yang et al., 2020)
Push and pull search embedded	(Fan et al., 2020)

Multi-stage evolutionary algorithm	(H. Ma et al., 2021; Tian et al., 2021)
Partition selection	(Y. Yang et al., 2021)
Surrogate-assisted evolutionary algorithm	(Gu et al., 2021)
Purpose-directed two-phase multi-objective differential evolution	(K. Yu et al., 2021)
Directed Weight Vectors	(C. Peng et al., 2020)
Gradient-based repair method	(Cantú et al., 2021)
Detect and scape strategy	(Zhu et al., 2020)
Reference points-based method	(Abdel-Basset et al., 2021)
multi-objective wireless network optimization using the genetic algorithm	(Bouazid et al., 2020)

As a general, a taxonomy of CHTs in MOEAs could be summarized in Figure 6. The CHTs presented in Figure 2.6 have been explained in details in previous sections. As it is mentioned earlier, most of constraint handling techniques developed for single-objective optimization problems can be applied to multi-objective optimization problems. It is worthy to note that among them, stochastic ranking (Geng et al., 2006; Jan & Khanum, 2013; Ying et al., 2016; Z.-Z. Liu et al., 2019; Gu et al., 2021), penalty function (Chafekar et al., 2003; Woldesenbet et al., 2009), multi-objective method (Ray et al., 2009; Isaacs et al., 2008; Long, 2014; Zhou et al., 2018), Epsilon constrained method (Saxena et al., 2009; Zapotecas Martinez et al., 2014; Becerra et al., 2007; Z. Yang et al., 2014; Martinez & Coello, 2014; Fan et al., 2016; Y. Yang et al., 2019; Zapotecas-Martinez & Ponsich, 2020), transforming method (Y. Yang et al., 2020; Y. Yang et al., 2021; B. Liu et al., 2007; Tian et al., 2020; K. Li et al., 2018; J. Wang et al., 2021; Z.-Z. Liu et al., 2021; Fan et al., 2020; H. Wang et al., 2021), feasibility rules (with a modification) (Oyama et al., 2007; Jimenez et al., 2002), hybrid methods (Caraffini et al., 2013; Caraffini et al., 2014; Datta et al., 2017; Morovati & Pourkarimi, 2019; Uribe et al., 2020), and repair operators (Harada et al., 2007; Qian et al., 2012; X. Yu et al., 2018; B.-Y. Qu &

Suganthan, 2010; Y.-N. Wang et al., 2010; Y. Liu et al., 2019) have been addressed to multi-objective optimization problems.

4 Benchmark test problems

To measure the performance of the evolutionary algorithms, many benchmark or test problems have been suggested. On the other hand, the benchmark problems help researchers to better understand the strengths and weaknesses of an algorithm (Hobbie et al., 2021). These test problems are classified as single-objective such as Rosenbrock (Rosenbrock, 1960), G01-G09 (Michalewicz, 1995), Himmelblau's problem (Himmelblau, 1972), Welded Beam (Rao, 2019a), Pressure Vessel (Sandgren, 1990), Tension–Compression Spring (Arora, n.d.), Speed Reducer (Golinski, 1973), Corrugated bulkheads design (Kvalie, 1967), Heater exchanger (X.-S. Yang & Gandomi, 2012), Multiple disk clutch brake (Steven, 2002),

Rolling element bearing (Changsen & Wan, 1991), Car side design (Youn & Choi, 2004a), Stepped beam design problem (Vanderplaats, 2002), multi-objective including BNH (Binh & Korn, 1997), OSY (Osyczka & Kundu, 1995) , ZDT (Zitzler et al., 2000), BT (H. Li et al., 2016), Truss2D (Deb & Srinivasan, 2006), and many-objective optimization problems for example C-DTLZ (Jain & Deb, 2013), WFG(Huband et al., 2006) , DTLZ (Deb et al., 2005). Among the above-mentioned test problems, some of them are still unconstrained. More details are suggested in a review paper in the field by (Huband et al., 2006).



Figure 2.6 A generalized taxonomy of CHTS for multi-objective optimization problems

5 Scientometric Analysis (RQ3- RQ6)

A scientometric analysis is conducted to scientifically measure and analyze the literature in a particular field of study and has attracted much attention from researchers (Leydesdorff & Milojević, 2012; Childress, 2011; Estabrooks et al., 2008; Gandomi et al., 2020; Müßigmann et al., 2020; Neelam & Sood, 2020; Rahimi et al., 2017, Rahimi & Gandomi, 2021; Rahimi & Nematian, 2022, Rahimi et al., 2021). To perform the analysis in this work, VOSviewer (Van Eck & Waltman, 2013) and RStudio were used. The following sub-sections provide new insight into the scientometric analysis in the field.

6.1 Citation statistics

Figure 2.7 displays the trend of published documents, which shows that the number of documents in the field significantly increased from 2003 until the end of 2021 (just above 470 documents).

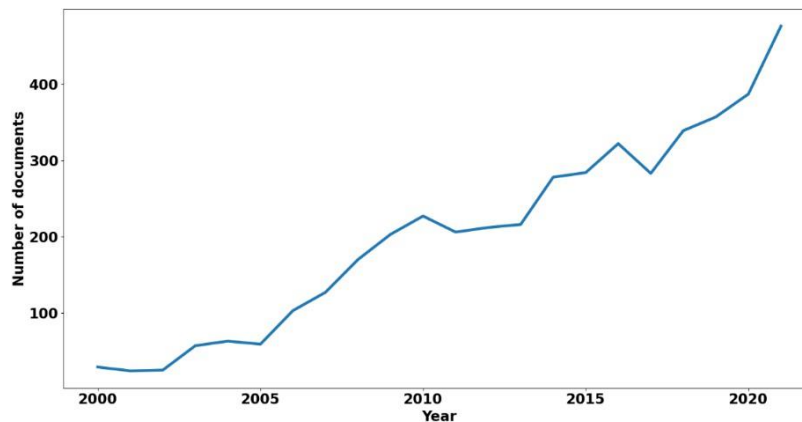


Figure 2.3 Trend of published documents

Figure 2.8 presents a combo chart of the number of documents vs. total citations. In 2002, most citations was achieved (the paper entitled: “A fast and elitist multi-objective genetic algorithm: NSGA-II” (Deb et al., 2002) has been received more than 29000 citations, according to the Scopus). It is apparent that the number of citations has increased dramatically according to the trend.

According to the WOS, the number of citations of the top articles in the field was analyzed and is presented in Supplementary C (Figure 1). Of the 735 related documents in WOS, about 45824 citations were identified from the related papers with an average of 1992.35 citations per year and an average of 62.35 citations per item. (Deb et al., 2002), (Das & Suganthan, 2010), and (Deb & Jain, 2013) are the top 3 cited articles with 20013, 2609, and 1591 citations in WOS, respectively.

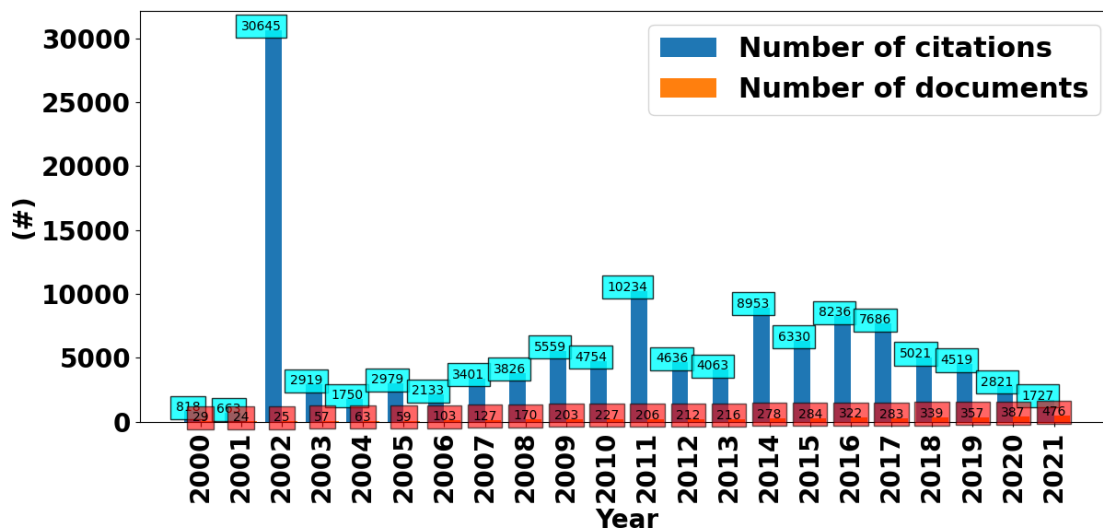


Figure 2.8 Combo chart of number of documents vs. total citations (Scopus)

6.2 Statistics based on document types

Among the document types, including article, proceedings paper, review, and other items indexed by WOS, a total of 735 publications on constraint handling multi-objective population-based optimization algorithms were found (Table 2.4). From the search, articles were the most popular document type, comprising a total of 522 articles (71.02% of 735 documents) with 2.60 authors per publication (APP). Also, articles as the document type had the highest CPP2021 of 84.10, followed by proceedings papers with TP of 220 (29.93% of contributions and APP=2.13). Moreover, there is a significant difference between the TC2021 article and that of the proceedings paper.

Figure 2.9 presents the distribution of documents based on different types, according to WOS. It is clear from the figure that conference papers have the most contributions before 2010 followed by articles. However, since 2010, articles have the most contributions in the field. It is also interesting to note that book/book chapters have been published since 2000, however, the most number of book/book chapters have been published after 2010.

Table 2.4 Citations analysis based on document type

Document type	TP ¹	%	AU	APP	TC2021	CPP2021
Article	522	71.02	1362	2.60	43,904	84.10
Proceedings paper	220	29.93	469	2.13	1,543	7.01
Review	16	2.17	20	1.25	806	50.37
Other items	23	3.12	134	5.82	468	20.34

TP, AU, APP, TC2020, and CPP2020 present total number of articles; total number of ¹ authors; total number of authors for each publication; total citations from WOS since publication year to the end of 2020; total citations for each paper, respectively; Other items: early access and letters (H. Chen & Ho, 2015) (Ho, 2012).

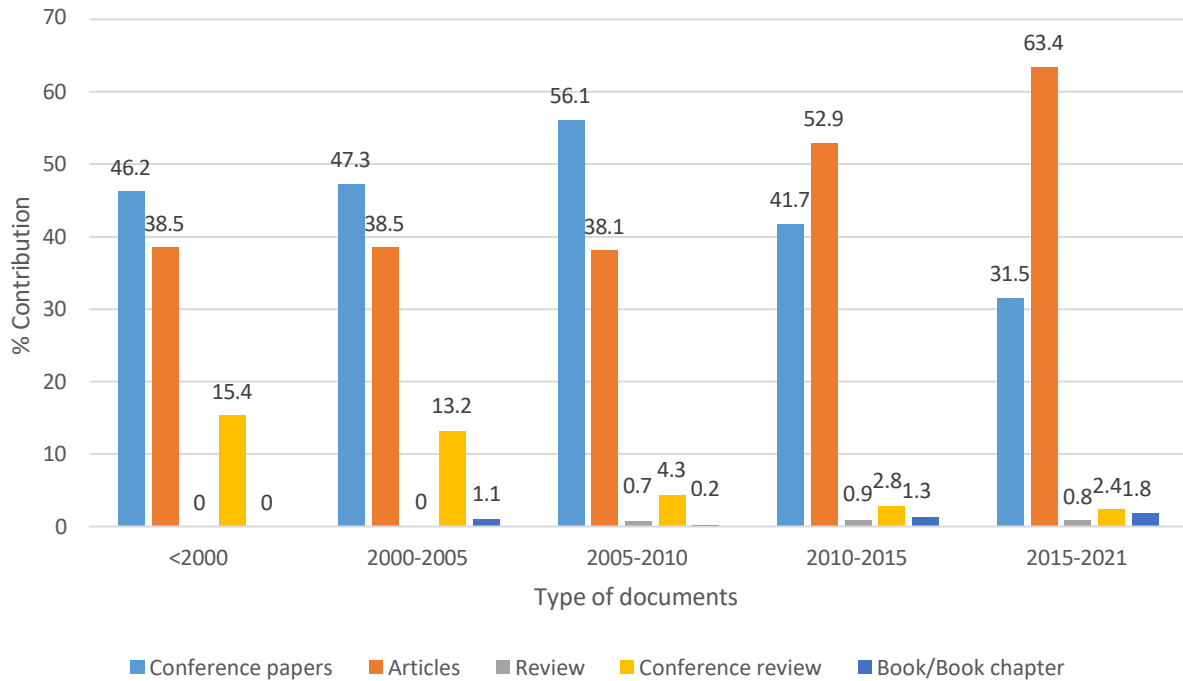


Figure 2.9 Type of research outputs

6.3 Publication statistics based on journal

Table 2.5 presents the top 20 journals that have published the greatest number of constraint handling multi-objective population-based algorithms papers based on Scopus. Accordingly, Lecture Notes In Computer Science (117), Applied Soft Computing Journal (57), and Swarm and Evolutionary Computation (30) are most are the most utilized, which predominate in the field of optimization and evolutionary computations.

A total of 735 articles were published in 399 journals, which are classified among the 51 WOS categories in SCI-EXPANDED. Table 2.6 lists the 10 most productive WOS categories. A total of 271 articles (36.87% of 735 articles) were published in the first category (Computer Science Artificial Intelligence), of which 83.39% were published in Engineering Electrical Electronic (23.26%) and Computer Science Theory Methods (23.26%). Comparing the top 10 categories,

the highest CPP2021 of articles published in the Computer Science Theory Methods category is 190.011, which includes the paper entitled: “A fast and elitist multi-objective genetic algorithm: NSGA-II” by (Deb et al., 2002a), and the highest APP for articles published in the Energy Fuels category is 2.97.

Table 2.5 The top 20 sources that have published the greatest number of constraint handling multi-objective population-based algorithms (Scopus)

#	Scopus	# of Documents	#	Scopus	# of documents
1	Lecture Notes In Computer Science	117	11	IEEE Access	32
2	Applied Soft Computing Journal	57	12	Swarm and Evolutionary Computation	30
3	“International Journal of Electrical Power	27	13	Engineering Optimization	21
4	“Kongzhi Yu Juece Control and Decision”	13	14	Soft Computing	16
5	Energy Conversion And Management	12	15	Studies In Computational Intelligence	16
6	IEEE Transactions On Cybernetics	12	16	“Advances In Intelligent Systems And	15
7	Structural And Multidisciplinary	13	17	“Communications In Computer And	14
8	IEEE Transactions On Evolutionary	27	18	Engineering Applications Of Artificial	14
9	Electric Power Systems Research	10	19	Energy	13
10	Applied Intelligence	12	20	Information Sciences	13

Table 2.6 The top 10 productive WOS categories

#	Web of Science category	TP	AU	APP	TC2021	CPP2021
1	“Computer Science” Artificial Intelligence”	271	628	2.31	35,074	129.42
2	“Engineering Electrical” Electronic”	171	463	2.70	3,688	21.56
3	“Computer Science” Interdisciplinary Applications”	92	243	2.64	2,691	29.25
4	“Operations Research” Management Science”	61	131	2.14	1,541	25.26
5	“Computer Science” Theory Methods”	171	385	2.25	32,492	190.011
6	“EngineeringMultidisciplinary”	64	167	2.60	2,377	37.14
7	“Mathematical interdisciplinary applications”	45	116	2.57	645	14.33
8	“Energy fuels”	41	122	2.97	950	23.17
9	“Computer Science Information Systems”	48	124	2.58	746	15.54
10	“Automation Control Systems”	60	155	2.58	1,178	19.63

Figure 2.10 provides a comparison of the development trends of the top four productive WOS categories, including “Computer Science Artificial Intelligence”, “Engineering Electrical Electronic”, “Computer Science Theory Methods”, and “Computer Science Interdisciplinary Applications”. Between 2001 and 2021, Computer Science Artificial Intelligence was the most predominant category and has possessed the highest number of publications since 2004, excluding the period between 2007 and 2008. The three other categories possess fluctuations between 2001 and 2021, and as of writing this paper, “Computer Science Theory Methods” and “Engineering Electrical Electronic” have the same TP of 171.

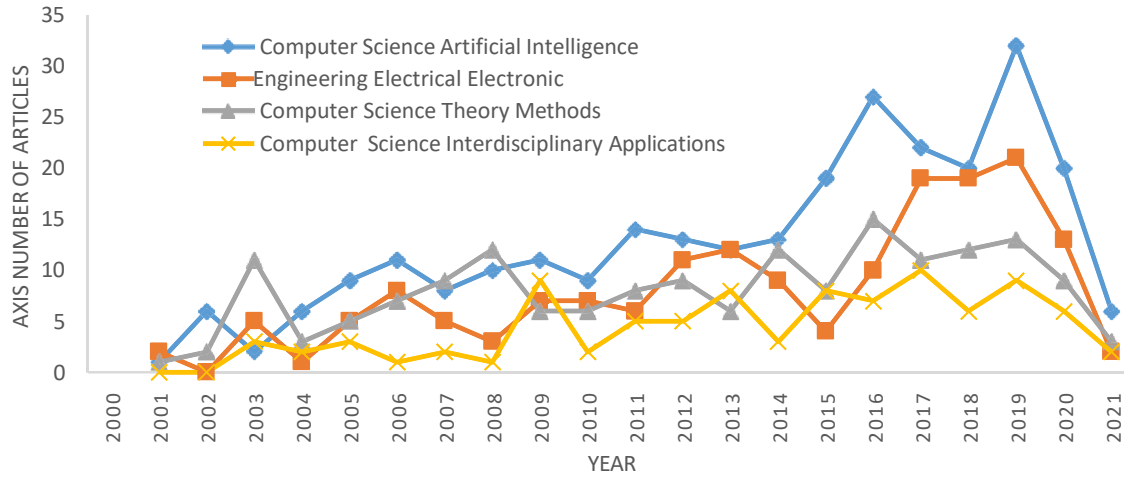


Figure 2.10 Comparison of the development trends of the top four productive WOS categories

6.4 Publication statistics by countries

From Figure 2.11 China, India, and the USA are the top three active countries in the field according to Scopus (respectively), while China, the USA, and India are the top 3 active territories in the field based on WOS, respectively. It is pertinent to mention that the USA is ranked second based on WOS, but India is ranked second according to Scopus. Also, it can be seen that there is a significant difference between the first rank (China) and second rank (India) based on the number of publications indexed by Scopus. Moreover, Figure 2.12 presents the collaboration among countries, where the links across the circles depict the collaborations, and the size of the circles represents the activities of the countries in the field. The green and yellow colors present the keywords that have been used recently, while the dark blue color indicates those that were used earlier (around 2008).

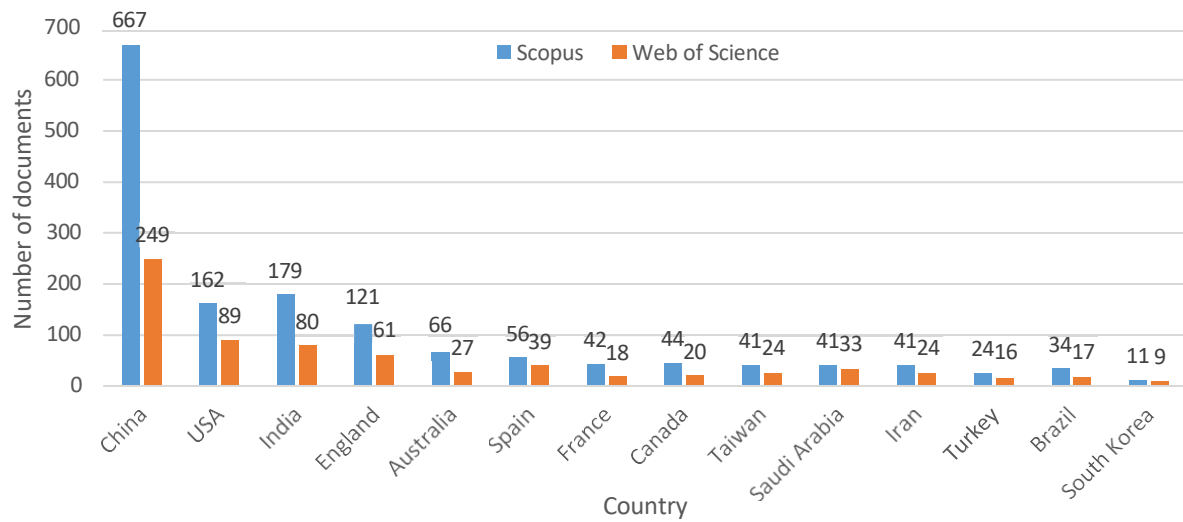


Figure 2.11 Research output of top 10 most productive countries across all databases

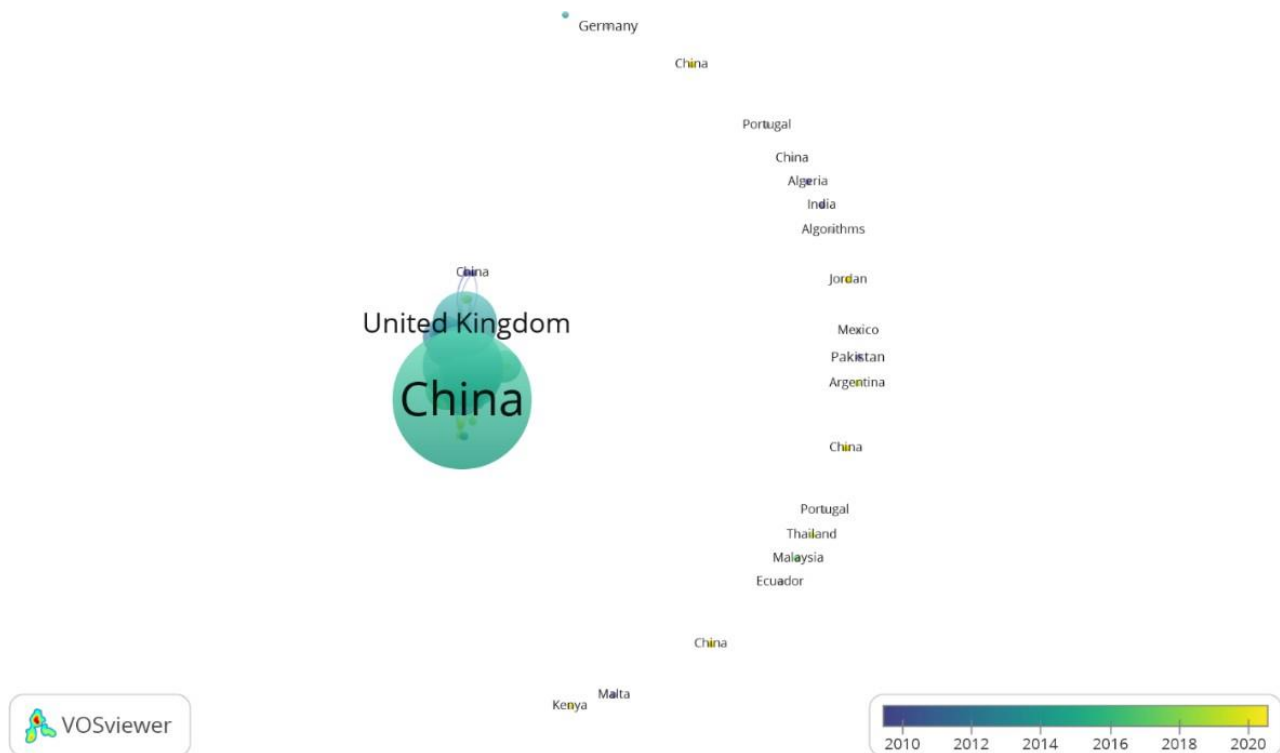


Figure 2.12 Country network visualization

Figure 2.13 displays the growth rate of the top 5 active countries in comparison to the world. While China and the USA have smooth trends between 2000 and 2021, India, the UK, and Australia show some fluctuations. Between 2002 and 2003, India presents the highest growth rate, then the trend continues smoothly until 2014 when it increases until 2015. The trend for the UK shows two growths between 2002-2003 and 2007-2008. While the number of articles published by Australia is much less than the four other countries, there is a significant rise between 2015 and 2016.

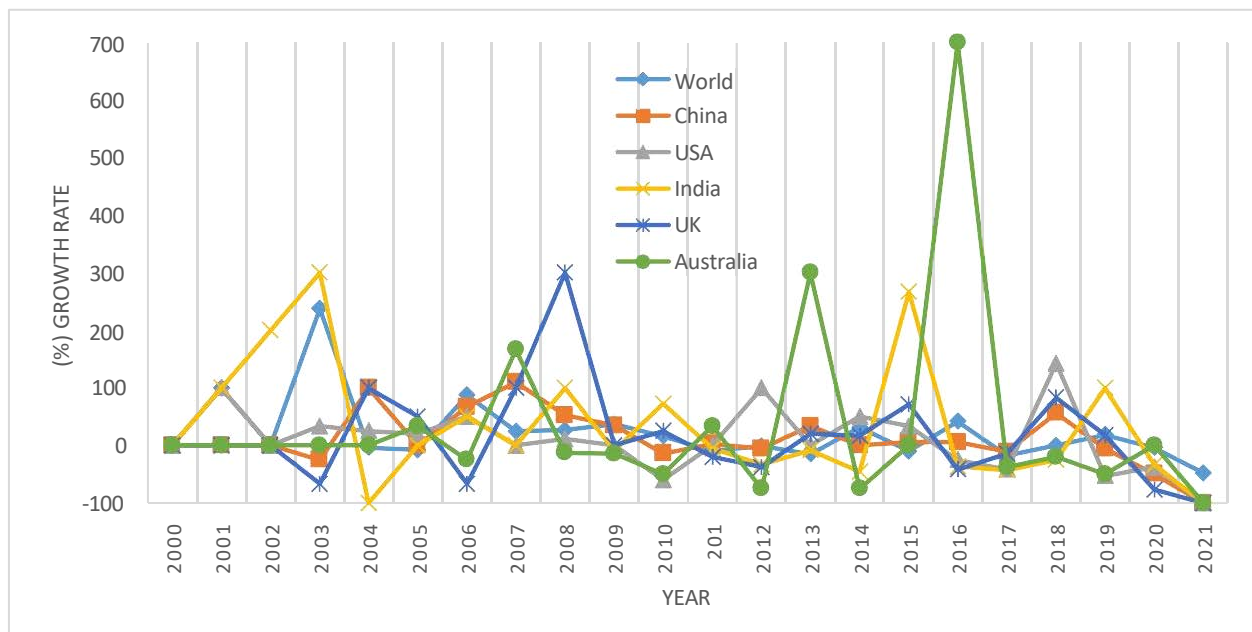


Figure 2.13 Growth rate of published documents for top 5 countries

6.5 Statistics based on the subject area

Figure 2.14 presents the distribution of articles based on the subject area. Computer science, Engineering, and Mathematics possess the most contributions with 936, 619, and 580 published articles, respectively. Comparatively, Pharmacology, Medicine, and Economics own the least contributions with 1, 4, and 6 published documents in the field, respectively.

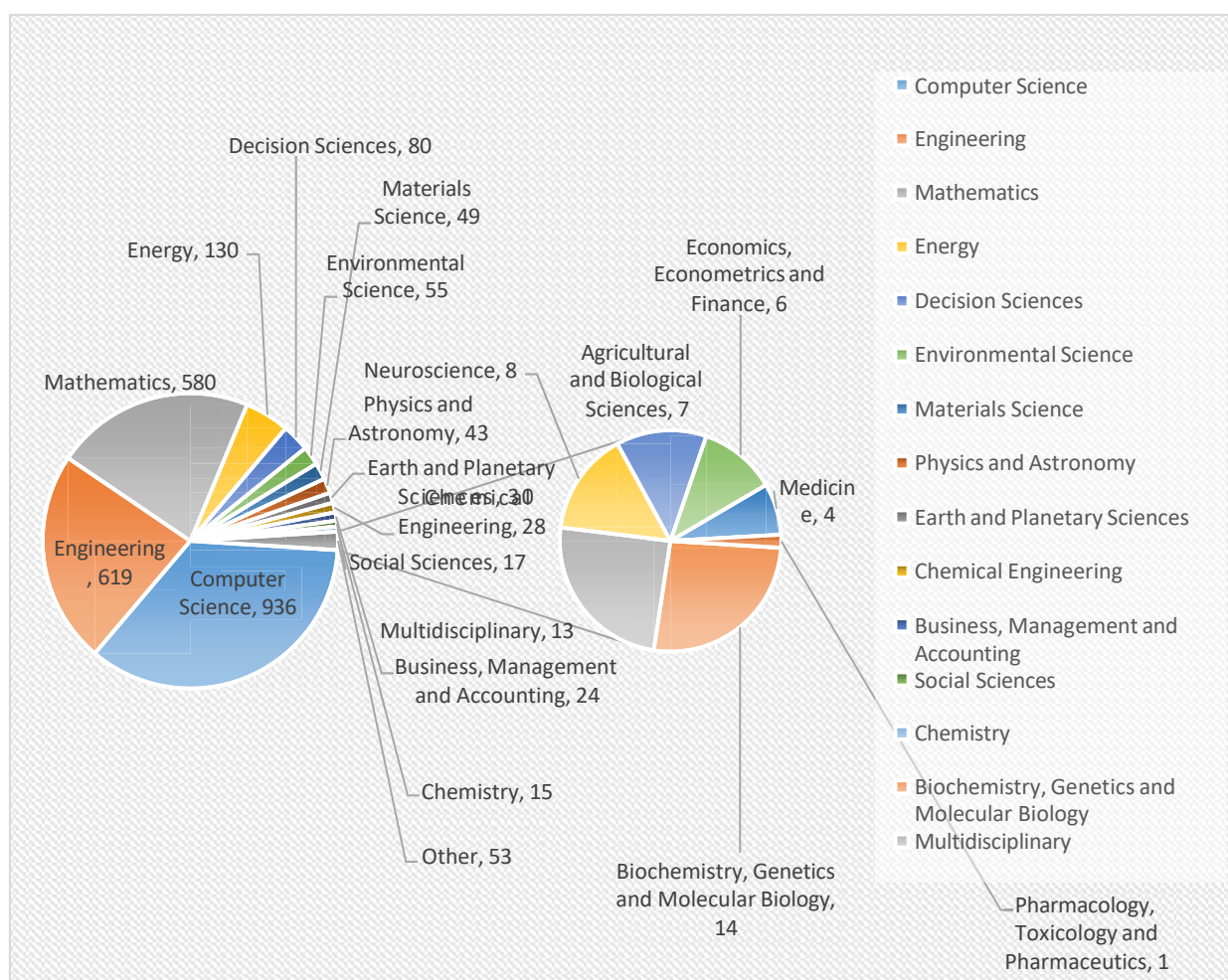


Figure 2.14 Distribution based on the subject area (Scopus)

6.6 Statistics based on authors

Figure 2.15 shows the top authors with the most publications according to Scopus. Kalyanmoy Deb from “Michigan State University (USA)”, Ray T. from “University of New South Wales (Australia)”, and Carlos A. Coello Coello from “Cinvestav-IPN (Mexico)” with 38, 32, and 28 publications are the top 3 authors in the field as indexed by Scopus. WANG Y from “City University Hong Kong (Hong Kong)”, Carlos A. Coello Coello from “Cinvestav-IPN (Mexico)”, and Ray T. from “University of New South Wales (Australia)” are the top 3 authors in the area with 21, 20, and 17 documents (indexed by WOS), respectively. According to WOS, 1717 authors have worked on constraint multi-objective population-based optimization.

In total, 0.5241% of authors own more than 10 documents; 1.6307% possess between 5 and 10 documents; 4.5428% have between 3 and 5 papers; 11.7647% own 2 papers; and 81.5377% possess 1 document (Figure 2.16). Figure 2.17 presents the collaboration among the authors, where links across the circles depict the collaborations, and the size of the circles shows the activities of the authors in the field. In addition, the yellow color represents recent activity, and the dark blue color depicts the contributions prior to 2014.

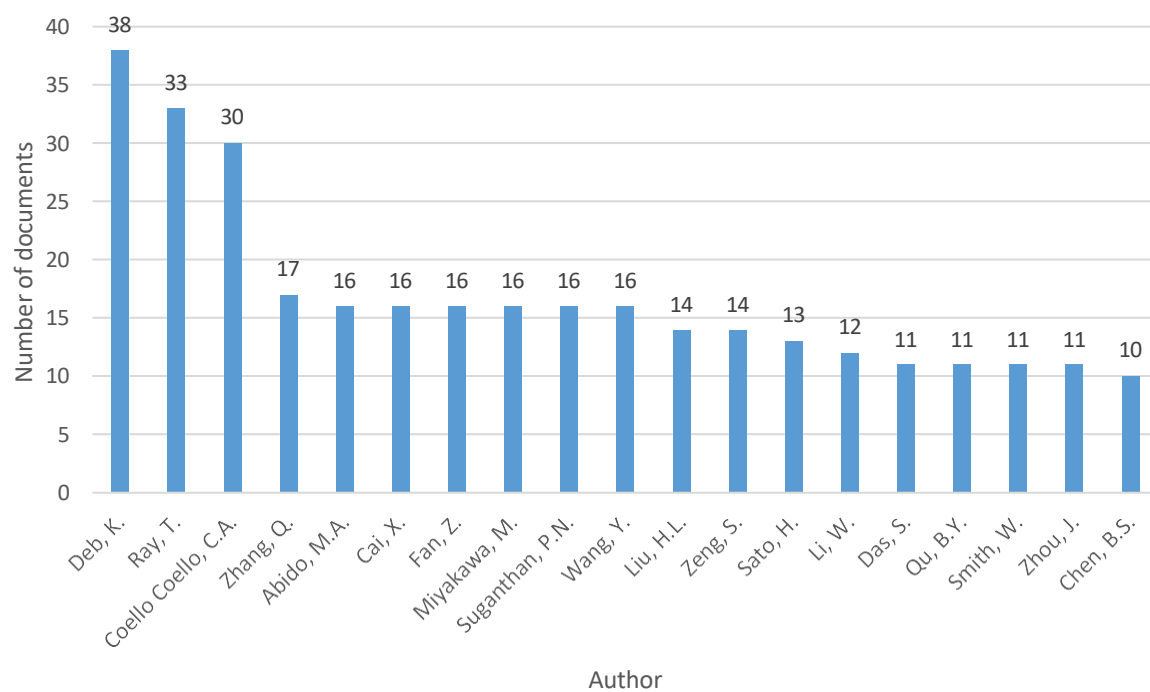


Figure 2.15 The most active authors in the field (Scopus)

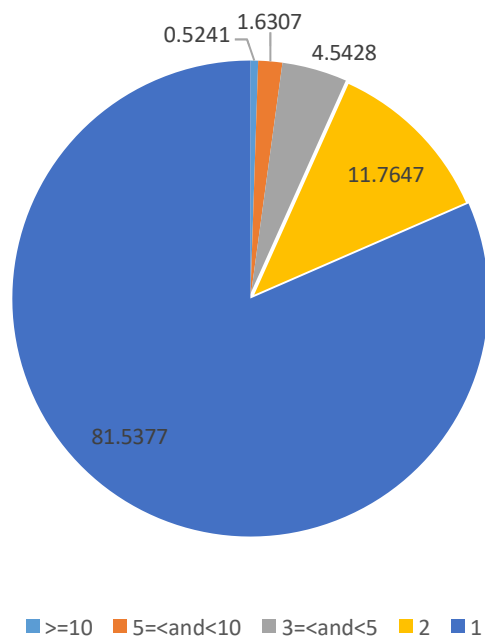


Figure 2.16 Contribution of authors based on the number of documents

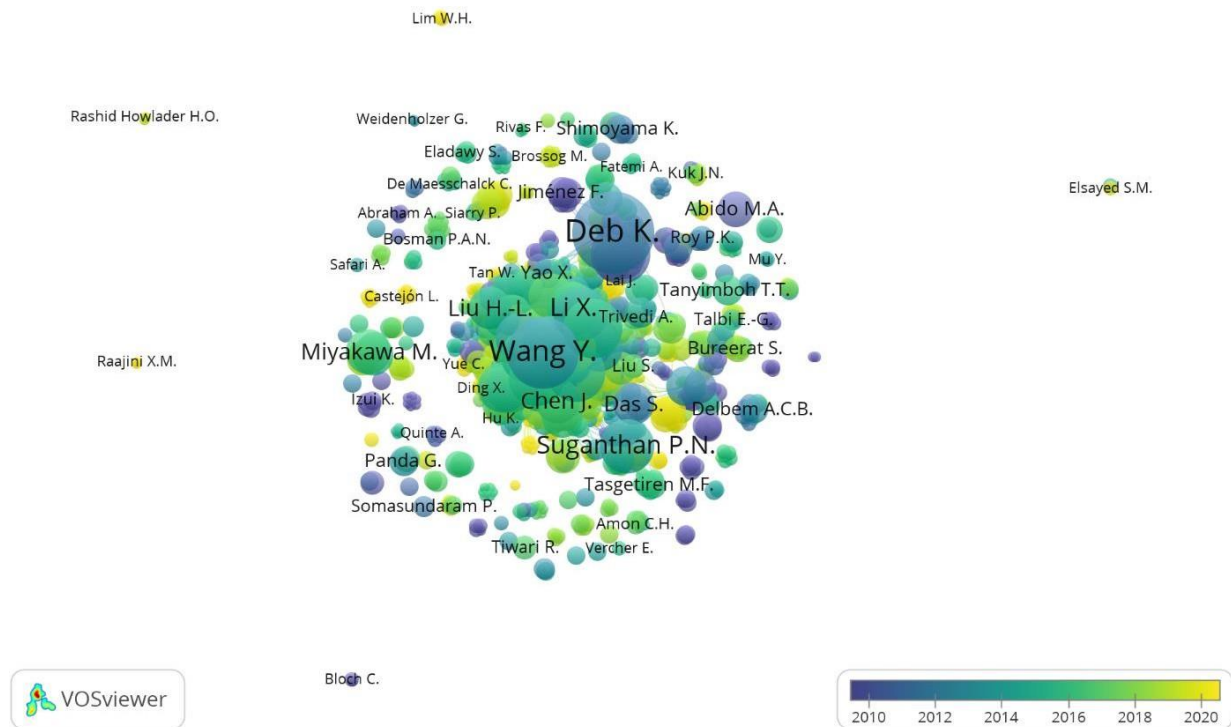


Figure 2.17 Collaboration among the authors (overlay visualization)

6.7 Statistics on keywords

Keywords indicate the basic parts of a certain field of research and could offer insight into the organization and knowledge provided in the articles. Figure 2.18 provides an overlay visualization of the co-occurrence analyses via a network map based on the Scopus database. Each node in the network represents a keyword, and the link between nodes indicates the co-occurrence of the keywords. The top keywords in Scopus include multi-objective optimization, Pareto optimal solution, evolutionary algorithm, artificial intelligence, machine design, stochastic systems, distributed power generation, and reliability. The color of each circle represents the identified cluster, and the size of each circle illustrates the importance of the keywords. Table 2.7 presents the top keywords of 1-word, 2-word, and 3-word lengths extracted from Scopus. Specifically, optimization, algorithm, and scheduling are the top 1-word length

Table 2.7 Top 1-,2-, and 3- word keywords used in the field

#	1-Word	Frequency	2-Word	Frequency	3-Word	Frequency
1	Optimization	680	Genetic Algorithms	367	Constraint-Handling Techniques	74
2	Algorithms	360	Constraint Handling	184	Particle Swarm Optimization (PSO)	565
3	Scheduling	141	Constrained Optimization	1071	Multi-objective Optimization	467
4	NSGA-II	97	Multi-objective Optimization	1339	Particle Swarm Optimization	205
5	Design	96	Evolutionary Algorithms	1081	Constrained multi-objective optimization	68
6	Algorithm	59	Differential evolution	208	Electric Load Dispatching	66
7	Reliability	33	Problem Solving	239	Multi-objective optimization problem	222
8	Investments	33	Multi objective	307	Differential evolution algorithms	71
10	Benchmarking	113	Decision making	135	Pareto optimal solutions	121
12	Costs	57	Pareto Principle	281	Constrained multi-objective optimizations	213

6.8 Publication statistics by number of pages (pages count)

As of writing this paper, May of 2021, approximately 22395.8 pages of papers on constraint handling multi-objective population-based algorithms were published, with an average of 13.0435 pages per paper. About 22.53931% of the articles possess between 10 and 15 pages; 12.17239% of the manuscripts are between 15 and 20 pages; 39.07979 % of the papers are between 5 and 10 pages; and 67.09377% of the manuscripts are between 5 and 20 pages. Figure 2.19 presents the distribution of the manuscripts based on page count.

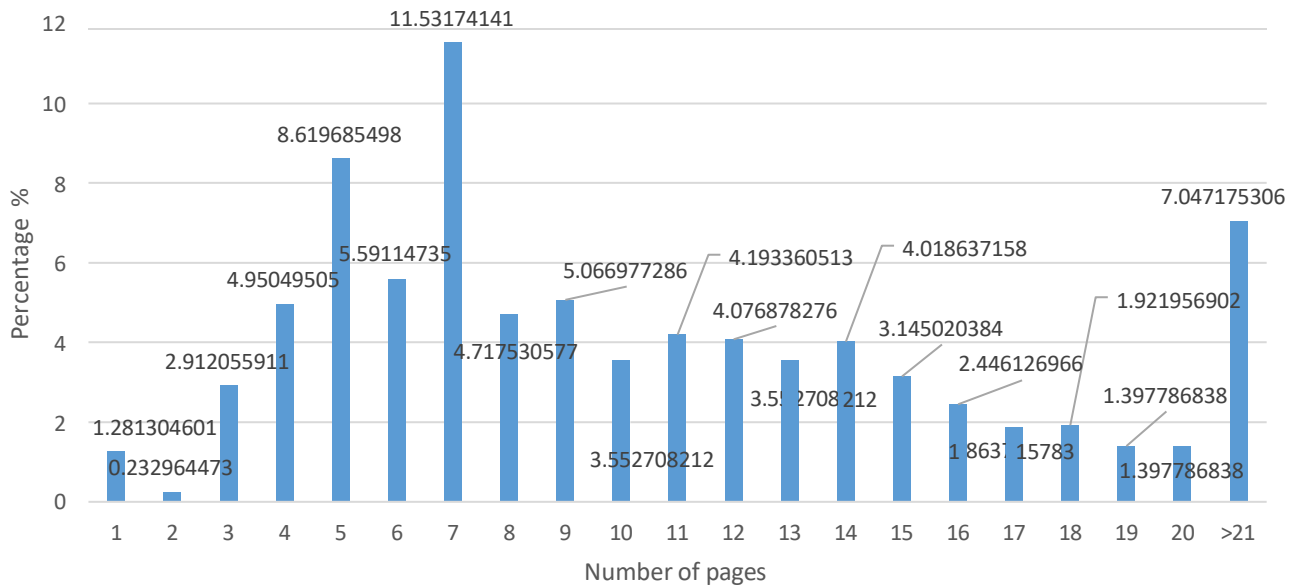


Figure 2.19 Distribution of documents based on page count

7 Summary and Future Research (RQ7)

The paper presents an analysis and overview of CHTs applied to multi-objective population-based algorithms. In the first part of the paper, the main idea of CHTs are defined, and the second part discusses a detailed scientometric analysis in the field. Some important technical points are extracted as follows:

- In the death penalty method, no information is used from infeasible points.
- Static penalty method is problem-dependent and may need several penalty parameters.
- Dynamic penalty method may converge to either an infeasible or feasible solution that is far from the global optimum.
- The main disadvantage of the annealing penalty method is the sensitivity to the values of its factors.
- Setting parameters in the adaptive penalty method is difficult and the method needs the definitions of additional parameters.
- In the self-adaptive penalty method, the additional parameters may affect the fitness function evaluations.
- The main difficulty in SGA is selecting the penalty factors for each sub-population.
- If the population is completely infeasible, choose the solutions that have a smaller overall constraint violation.
 - Retaining a proportion of infeasible solutions in the population may enhance the convergence and diversity of the algorithm.
 - Two types of CHTs, namely repair methods and special genetic operators, focus only on the feasible space.
 - Feasible solutions could be used to repair infeasible solutions (repairing population).

- According to the constraint dominance principle, the feasible solution is always preferred over the infeasible solution, which may cause loss of important information from infeasible individuals.
- Retaining a huge number of infeasible solutions may cause low convergence speed.
- Although special operators are known to be highly comparative CHT, their applicability is limited, which makes this technique difficult to run.
- Decoder is an interesting CHTs, but it involves a high computational cost and, thus, is now rarely used.
- Although the ensemble CHT has a competitive performance, the method is parameter-dependent.
- Although the stochastic ranking method has been employed in several nature-inspired algorithms, it is not often used for the multi-objective version of the algorithms.
- Epsilon constraint method has been known as a powerful CHT, however, in some cases, premature convergence has been reported, while other works report that the method relies on gradient-based mutation.
- Using multi-objective concept as a CHT may require gradient calculation.
- Recently, feasibility rules have been recognized as one of the most powerful CHTs, which are simple and flexible; however, one of the major disadvantages of this method is premature convergence since this technique favors feasible solutions.

As a future direction, the authors have identified the top 5 most-used keywords and research fields in the last three years (2019-2021) based on Scopus. Tables 9 and 10 show the mentioned keywords and research fields for this time period. It is obvious that multi-objective optimization, constraint optimization, and evolutionary algorithms are the most famous keywords in the last three years. It should be noted that CHTs for multi-objective optimization

have not received much attention compared with single-objective optimization. It is suggested that researchers focus on such methods in future works. Also, the BU technique, which is able to handle constraints directly possesses the potential to couple with a multi-objective evolutionary algorithm (MOEA) as well. Furthermore, it is suggested to focus on constraint handling techniques for many-objective optimization problems (with more than three objectives) as it is not received much attention. In addition, according to Tables 2.8 and 2.9, GA, DE, and PSO remain the top 3 algorithms, which are expected to be further explored in the future. Moreover, Engineering, Computer Science, and Mathematics have been the top 3 research fields in the last two years, and it is projected that research work will advance in these areas in the future. It is also recommended to review the applications of constrained multi-objective evolutionary algorithms in different sectors; including engineering design problems (Santana-Quintero et al., 2010) (Santana-Quintero et al., 2010); Tawhid & Savsani, 2018); Y. Yang et al., 2018), scheduling optimization problems (Z. Ma & Wang, 2021; X. Yu et al., 2018b; M.-R. Chen et al., 2019; M.-R. Chen et al., 2019; El-Shorbagy & Mousa, 2021), and resource optimization problems (K. Li et al., 2018b; C. Wang & Xu, 2020).

Table 2.8 Top 5 keywords in 2019 and 2021

#	Keywords (Scopus)	Frequency
1	Pareto principle	65
2	Genetic algorithms	72
3	Differential evolution	45
4	Particle swarm optimization (PSO)	132
5	Economic and social effects	34
6	Benchmarking	32
7	Decision making	36
8	Energy utilization	28
9	Scheduling	39
10	Pareto optimal solutions	24

Table 2.9 Top 5 research fields in 2019 and 2021

#	Research fields (Scopus)	(%) Contribution
1	Engineering	24.3
2	Computer Science	31.8
3	Mathematics	17.2
4	Energy	5.9
5	Decision Sciences	4.1
6	Materials Science	4.1
7	Business, Management and Accounting	1
8	Environmental Science	2.1
9	Physics and Astronomy	2.8
10	Earth and Planetary Sciences	1.4

8. Discussion and Conclusion

Constraint population-based optimization involves the use of a population-based algorithm in combining with a CHT to solve a constraint optimization problem. In the first part of the paper, the main idea of CHTs are defined, and the second part discusses a detailed scientometric analysis in the field. It is noteworthy to mention that the majority of the mentioned studies in the literature focused on CHTs for single-objective optimization with little attention to multi-objective optimization. This paper presents an analysis and evaluation of the CHTs with focus on multi-objective optimization population-based algorithms, which support evolutionary and swarm intelligence algorithms. To the best of our knowledge, this study is the first analysis of relevant journals evaluated over the most relevant journals, keywords, authors, and articles in this field. All related papers, including research articles, reviews, book/book chapters, conference papers, etc., as of writing this paper, were extracted and analyzed. Publication statistics by year, journal, country, affiliation, author, number of pages, number of authors, and keywords are discussed in this paper as follows:

- According to WOS, 45824 citations have been received by the related papers, which is an average of 1992.35 citations per year and an average of 62.35 citations per item in WOS.

- Based on WOS, articles were the most popular document type, with a total of 522 articles (71.02%), and 2.60 authors per publication.
- Articles as the document type had the highest CPP2021 of 84.10, followed by proceedings papers with TP of 220 (29.93% of contributions and APP=2.13).
- Conference papers have the most contributions before 2010 followed by articles. However, since 2010, articles have the most contributions in the field.
- A total of 271 articles (36.87% of the total), with 2.31 authors per publication (on average), were published in the category of Computer Science Artificial Intelligence, according to WOS.
- In total, 271 articles (36.87% of 735 articles) were published in the first category (Computer Science Artificial Intelligence) and a total of 83.39% were published in the first three categories: Engineering Electrical Electronic (23.26%) and Computer Science Theory Methods (23.26%).
- The highest CPP2021 of articles published in Computer Science Theory Methods is 190.011, which includes the paper “A fast and elitist multi-objective genetic algorithm: NSGA-II” by (Deb et al., 2002a), and the highest APP for articles published in ‘Energy fuels’ is 2.97.
- “Computer Science Artificial Intelligence,” “Engineering Electrical Electronic,” “Computer Science Theory Methods,” and “Computer Science Interdisciplinary Applications” were the top 4 productive WOS categories in the field.
- China, USA, and India were the top three active countries in the field according to WOS.
- Computer science, Engineering, and Mathematics have the most contributions with 936, 619, and 580 published articles, respectively. Pharmacology, Medicine, and Economics own the least contributions with 1, 4, and 6 published documents in the field, according to Scopus.

- Kalyanmoy Deb from “Michigan State University (USA)”, Ray T. from “University of New South Wales (Australia)”, and Carlos A. Coello Coello from “Cinvestav-IPN (Mexico)” are the top 3 authors in the field with 38, 32, and 28 publications (indexed by Scopus), respectively. WANG Y from “City University Hong Kong (Hong Kong)”, Carlos A. Coello Coello from “Cinvestav-IPN (Mexico)”, and Ray T. from “University of New South Wales (Australia)” are the top 3 authors in the area with 21, 20, and 17 documents (indexed by WOS), respectively.
- Almost 0.5241% of authors own more than 10 documents; 1.6307% possess between 5 and 10 documents; 4.5428% have between 3 and 5 papers; 11.7647% of authors own 2 papers; and 81.5377% of authors possess 1 document.
- Approximately 22.53931% of the articles possess between 10 and 15 pages; 12.17239% of the manuscripts are between 15 and 20 pages; 39.07979% of the papers are between 5 and 10 pages; and 67.09377% of the manuscripts are between 5 and 20 pages.

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Chapter 3

Extending Boundary Updating Approach for Constrained Multi-objective Optimization Problems

Iman Rahimi¹[0000-0003-3780-4726], Amir H. Gandomi^{1,2*} [0000-0002-2798-0104],
Mohammad Reza Nikoo³[0000-0002-3740-4389], and Fang Chen¹[0000-0003-4971-8729]

¹ Faculty of Engineering & Information Technology, University of Technology Sydney, Australia
iman83@gmail.com,*Gandomi@uts.edu.au, fang.chen@uts.edu.au

²University Research and Innovation Center (EKIK), Óbuda University, 1034, Budapest, Hungary

³ Department of Civil and Architectural Engineering, Sultan Qaboos University, Muscat, Oman
m.reza@squ.edu.om

Abstract. To date, several algorithms have been proposed to deal with constrained optimization problems, particularly multi-objective optimization problems (MOOPs), in real-world engineering. This work extends the 2020 study by Gandomi & Deb on boundary updating (BU) for the MOOPs. The proposed method is an implicit constraint handling technique (CHT) that aims to cut the infeasible search space, so the optimization algorithm focuses on feasible regions. Furthermore, the proposed method is coupled with an explicit CHT, namely, feasibility rules and then the search operator (here NSGA-II) is applied to the optimization problem. To illustrate the applicability of the proposed approach for MOOPs, a numerical example is presented in detail. Additionally, an evaluation of the BU method was conducted by comparing its performance to an approach without the BU method while the feasibility rules (as an explicit CHT) work alone. The results show that the proposed method can significantly boost the solutions of constrained multi-objective optimization.

Keywords: Constraint handling, Multi-objective Optimization, Evolutionary computation, NSGA-II

1 INTRODUCTION

Multi-objective optimization problems (MOOPs), are faced in several sectors of the engineering field. However, finding an optimal solution for these problems has been a challenge for scholars. Since

evolutionary algorithms (EAs) do not need concavity or convexity and can yield several alternative solutions in a single run (Poojari & Beasley, 2009), they are usually used for MOOPs (Rahimi et al., 2022). Additionally, EAs can be integrated with specific decomposition algorithms (Poojari & Beasley, 2009) and, generally, perform better in dealing with some MOOPs (Binh & Korn, 1997; Jiménez et al., 1999; Salcedo-Sanz, 2009).

Constraint handling technique (CHT) combined with an evolutionary algorithm is called constrained evolutionary algorithm optimization (CEAO) and has been used to tackle real-world constrained optimization problems (Rahimi et al., 2022).

In the literature, researchers have proposed numerous CHTs, for example, based on feasibility and infeasibility regions (Binh & Korn, 1997; Deb et al., 2002; Gandomi & Deb, 2020), priority assignment (Jiménez et al., 1999), tournament selection and selection operator (Jiménez et al., 1999; Salcedo-Sanz, 2009). Some studies in the literature have provided surveys for CHTs (Mezura-Montes & Coello, 2011; Rahimi et al., 2022; Salcedo-Sanz, 2009).

However, most studies are classified as explicit CHTs. In the work (Gandomi & Deb, 2020), the authors tested a method known as boundary updating (BU), as an implicit CHT, which updates variable bounds by directly using constraints and applied it to several single-objective optimization problems. In the current study, the BU method is coupled with an explicit CHT (here feasibility rules), and for the strategy without BU, an explicit CHT (feasibility rules) is still applied to solve the problem. As a contribution, in this paper, the latter work (Gandomi & Deb, 2020) is extended by applying the BU method to some MOOPs using non-dominated sorting genetic algorithm-II (NSGA-II), and also the BU is coupled with feasibility rules as it is superior to the penalty function approach. The NSGA-II is one of the most popular and efficient EAs, as it employs an elitist principle, a diversity mechanism (Deb et al., 2002), and has been widely used to solve constrained MOOPs

(Brownlee & Wright, 2015; Rahimi et al., 2022). This research is motivated by the lack of efficient constraint-handling techniques for multi-objective optimization.

The remainder of the study is organized as follows. Section 2 presents the proposed method. Section 3 illustrates the numerical study applied in this research. Finally, the last section gives the conclusion.

2 Proposed method

Equations (1-3) present a simplified constrained optimization problem:

$$\max(\min) F(x) = (f_1(x), \dots, f_t(x)) \quad (1)$$

$$h_i(x) = 0 \quad i = 1, \dots, n \quad (2)$$

$$g_j(x) \leq 0 \quad j = 1, \dots, m \quad (3)$$

where $F(x)$ is the objective vector that consists of several objectives (t is the number of objective functions); n and m are the numbers of inequality and equality constraints, and $x=(x_1, \dots, x_n)$ is a vector of decision variables that are subjected to lower bound (LB) and upper bound (UB) vectors. Rather than producing a single solution, these equations yield several Pareto optimal solutions instead (Deb, 2011).

Since optimization is an iterative process that the boundaries can be changed during the process, the i -th decision variable boundaries in each iteration own the dynamic nature; the proposed method uses the constraints to narrow down variable space and then forces the algorithm to focus its search in the feasible region by limiting the feasible search space for the variable (s). In the BU method, the boundaries are changing iteratively and are updated during the optimization procedure. Mathematically it could be written as follows (Gandomi & Deb, 2020):

$$\exists i \in \{1, \dots, m\}: [\forall j \in \{1, \dots, n\}: x_i \geq l_{i,j}(x_{\neq i}) \cup x_i \leq u_{i,j}(x_{\neq i})] \quad (4)$$

where $l_{i,j}$ and $u_{i,j}$ are dynamic lower bound and upper bound for i th decision variable. Regarding updating the bounds, there are some scenarios as follows:

If $lb_i = -\infty$ and $ub_i = +\infty$:

$$lb_i^u = l_{i,j}(x_{\neq i}) \quad (6)$$

$$ub_i^u = u_{i,j}(x_{\neq i}) \quad (7)$$

Else:

$$lb_i^u = \min(\max(l_{i,j}(x_{\neq i}), lb_i), ub_i) \quad (8)$$

$$ub_i^u = \max(\min(u_{i,j}(x_{\neq i}), ub_i), lb_i) \quad (9)$$

where (lb_i^u, ub_i^u) are updated boundaries. To start with the BU method, a repairing

variable, which can handle the greatest number of constraints without overlapping with other repairing variables, should be selected; in this case, if there is more than one candidate, the one that handles the greatest number of constraints is selected. Also, if there is still another candidate for variable selection, one variable is selected randomly (Gandomi & Deb, 2020).

If a repairing variable handles the first k_i constraints:

$$lb_i^u = \min(\max[l_{i,1}(x_{\neq i}), \dots, l_{i,k_i}(x_{\neq i}), lb_i], ub_i) \quad (10)$$

$$ub_i^u = \max(\min[u_{i,1}(x_{\neq i}), \dots, u_{i,k_i}(x_{\neq i}), ub_i], lb_i) \text{ (where } k_i \leq m) \quad (11)$$

If another repairing variable (x_r) is defined:

$$lb_i^u = \min(\max[l_{i,1}(x_{\neq i,r}), \dots, l_{i,k_i}(x_{\neq i,r}), lb_i], ub_i) \quad (11)$$

$$ub_i^u = \max(\min[u_{i,1}(x_{\neq i,r}), \dots, u_{i,k_i}(x_{\neq i,r}), ub_i], lb_i) \quad (12)$$

$$lb_r^u = \min(\max[l_{r,k_i+1}(x_{\neq r}), \dots, l_{r,k_i+k_r}(x_{\neq r}), lb_r], ub_r) \quad (13)$$

$$ub_r^u = \max(\min[u_{r,k_i+1}(x_{\neq r}), \dots, u_{r,k_i+k_r}(x_{\neq r}), ub_r], lb_r) \text{ where } r \in \{1, \dots, n\} \text{ and } r \neq i \quad (14)$$

$$x \in \{x_1, \dots, x_h, x_{h+1}, \dots, x_n\} \rightarrow mx \in \{p_1, \dots, p_h, x_{h+1}, \dots, x_n\} \quad (15)$$

$$x_i = lb_i^u + p_i(ub_i^u - lb_i^u) \text{ for } i=1, \dots, h \text{ where } 0 \leq p_i \leq 1 \quad (16)$$

In the BU approach, a repairing variable could be substituted with a generalized semi- independent variable (to avoid discontinuity among iterations (Gandomi & Deb, 2020)) and then rewritten with a lower and upper bound:

$$x \in \{x_1, \dots, x_h, x_{h+1}, \dots, x_n\} \rightarrow mx \in \{p_1, \dots, p_h, x_{h+1}, \dots, x_n\} \quad (17)$$

where $p_1, \dots, p_h$ are semi-independent variables.

After selecting repairing variables, the search operator, here NSGA-II, will be applied to the problem, the boundaries of non-repairing variables will be checked, and mx- vector will be updated. During the solution procedure, the boundaries of repairing variables will be updated, and then the semi-independent variables ($p_i, i = 1, \dots, h$) are remapped to the actual variables using updated boundaries. In the end, those constraints that were not involved in the repairing variable boundary along with fitness values will be evaluated using actual variables. The BU method is applied in the steps indicated in algorithm 1.

Algorithm 1. Implementation of the BU method in a MOOP

Initialization

While the criteria are not met

applying the search algorithm (here NSGA-II)
applying the BU method and updating the mx-vector
Check the boundaries of non-repairing variables
Update the boundaries of repairing variables
Remap p variable to the original variables using updated boundaries
Evaluate the constraint violations
Evaluating fitness function

End while

As an illustrative example, the following MOOP is considered to explain the BU.

$$\text{Min } f_1(x) = (x_1^2 + x_2^2) \quad (18)$$

$$\text{Min } f_2(x) = -(x_1 - 1)^2 - x_2^2 \quad (19)$$

$$g_1(x_1 + x_3) = x_1 + x_3 \leq 1000 \quad (20)$$

$$-2 \leq x_1 \leq 2 \quad (21)$$

$$-2 \leq x_2 \leq 2 \quad (22)$$

$$-2 \leq x_3 \leq 2 \quad (23)$$

Either x_1 or x_3 can be selected as a repairing variable of the single constraint. All variables are in the range of -2 to 2. From the variable selection strategy (Gandomi & Deb, 2020), as explained previously in this study, the repair variable, x_3 , is selected as a repairing variable. With respect to x_3 , the inequality constraint is solved, and the repairing variable, here x_3 , is used as the lower and upper bound functions. The repairing variable is substituted with the mapped variable, p , which is in the range of 0 to 1. Therefore, the lower and upper bounds of the variables are $\{-2, -2, 0\}$ and $\{2, 2, 1\}$, respectively. When the search algorithm is applied to the problem, the repairing variable is substituted with the mapped variable, and the boundary of the repairing variable is mapped for the rest of the search cycle (Gandomi & Deb, 2020). The original variable can be calculated by the following formula in order to compute the objectives and constraints:

$$x_3 = lb_3^u + p(ub_3^u - lb_3^u) \quad (24)$$

where ub_3^u and lb_3^u are the lower and upper bounds for repairing variable x_3 , respectively. The updated bounds can be written as follows:

$$lb_3^u = \min(\max(lb_3, 1000 - x_1), ub_3) \quad (25)$$

$$ub_3^u = \max(\min(ub_3, 1000 - x_1), lb_3) \quad (26)$$

In the end, the repairing variable is remapped to the actual boundary when the fitness values are evaluated. Fig. 1 compares Pareto solutions found by the BU method coupled with a CHT (here feasibility rules) and by the method without the BU method (only the feasibility rules approach is

considered as an explicit constraint handling technique) for a population size of 100. As a termination criterion, the number of generations was set to 100. From Figure 3.1, it is obvious that the BU method produced a greater number of non-dominated solutions than the approach without BU.

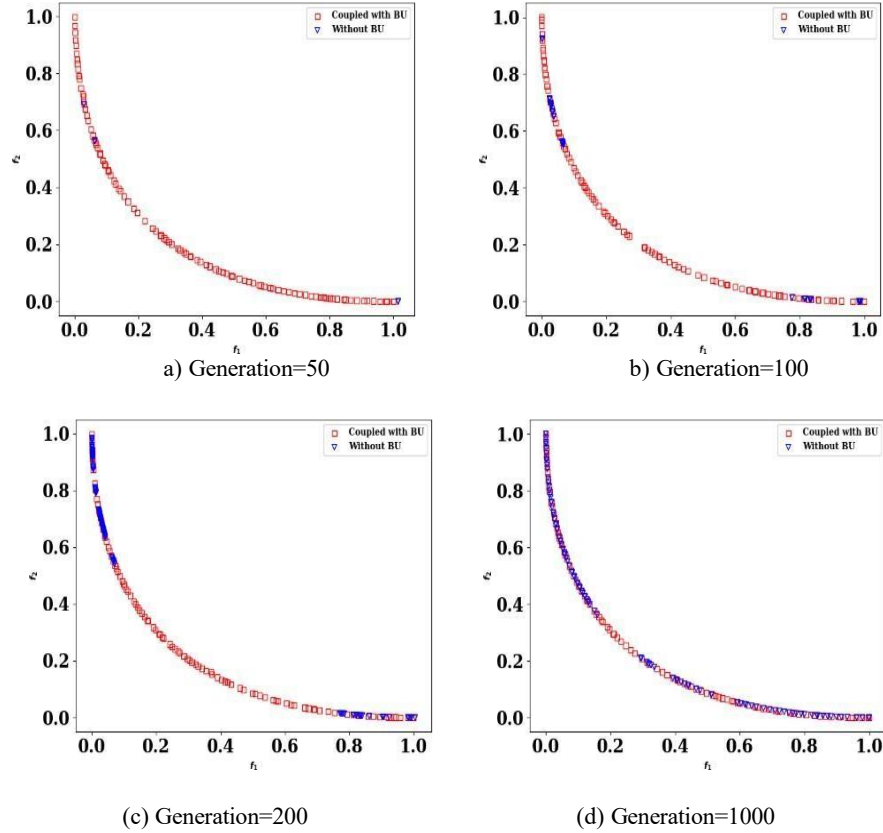


Figure 3.1. Pareto solutions obtained with and without the BU method

The BU method is designed to dynamically refine constraint boundaries during the optimization process, adapting them based on feasibility checks rather than relying on fixed constraint values. Unlike conventional constraint-handling techniques that strictly enforce predefined limits, the BU method progressively adjusts feasible search regions to enhance convergence and solution quality. Since the primary focus of this research is on improving the adaptability and efficiency of constraint-handling rather than solving specific constrained problems with static values, constraint boundaries are implicitly incorporated within the algorithm's mechanism rather than being explicitly defined in

the problem formulation. The effectiveness of the BU method is evaluated through comparative performance assessments across multiple constrained optimization problems, demonstrating its ability to navigate diverse constraint landscapes without the need for fixed constraint values.

3 Numerical study

This section presents two benchmark problems and two real-world problems to verify the performance of the proposed method, in which the BU method is implemented with NSGA-II. To evaluate the results, the following metrics were evaluated:

- The number of non-dominated solutions: The BU method was coupled with an explicit constraint handling technique and compared to the approach without BU (only feasibility rules were applied to the optimization problem). Since the BU method aims to reduce the search space, the number of non-dominated solutions was determined for comparison.
- Constraint violation (CV): Compared to unconstrained optimization problems, constrained MOOPs are more challenging since a large proportion of infeasibility regions appears in the search space (it means the hit ratio is low), which makes solving the constrained problem very challenging, especially for highly constrained and can lead to some difficulties such as convergence-, diversity-, and feasibility-hardness. Therefore, even finding one feasible solution is a major achievement. The BU method aims to reduce the search space by narrowing down variable space and cutting the infeasible search space. To compare the effectiveness of the BU method, the first feasible solution that could be found by the BU method was compared to the method without BU, and the whole population is tracked against generations as well.
- Performance indicators: Generational Distance (GD) (van Veldhuizen, 1999), Generational Distance Plus (GD+) (Ishibuchi et al., 2015), Inverted Generational Distance (IGD) (Coello Coello

& Reyes Sierra, 2004), Inverted Generational Distance Plus (IGD+) (Ishibuchi et al., 2015), and Hypervolume are five indicators that were analyzed.

- Running metric (Blank & Deb, 2020a): It is possible to trace the difference in the objective space followed by generations.
- The population and the maximum number of experiments were set to 100 and 31, respectively. All experiments in this study were performed in Python 3.10 and using the Pymoo library (Blank & Deb, 2020) (Table 3.1).

Table 3.1 Specific parameter settings for the algorithms used in this study

Parameter	Value
Number of Runs	31
Population Size	100
Number of Generations	500
Mutation Rate	0.1
Crossover Rate	0.9
Selection Method	Tournament Selection

3.1 Osyczka and Kundu (OSY)

The OSY problem, proposed by (Osyczka & Kundu, 1995), has two objective functions, six constraints, and six decision variables. For the OSY problem, x_1 , x_4 , and x_6 were selected as repairing variables as they can handle all constraints. To evaluate the performance of the BU method, we ran the NSGA-II algorithm and traced it in every generation. The algorithm without the BU method found the first feasible solution after 1000 evaluations (only feasibility rules are

applied to the optimization process), while the BU method coupled with the feasibility rules found the first feasible solution after only 250 evaluations (Fig. 3.2).

In addition, using the BU method, at generation 250, the whole population fell in the feasibility region, while without using the BU method after 1375 evaluations, the whole population converged in the feasibility area (Fig. 3.3). Fig. 3.4 illustrates the number of non-dominated solutions found by both methods from generations 1 to 40, indicating that the BU method found more Pareto solutions in the first 40 generations. Fig. 3.5 presents Pareto solutions obtained by the BU and without the BU in 20 generations.

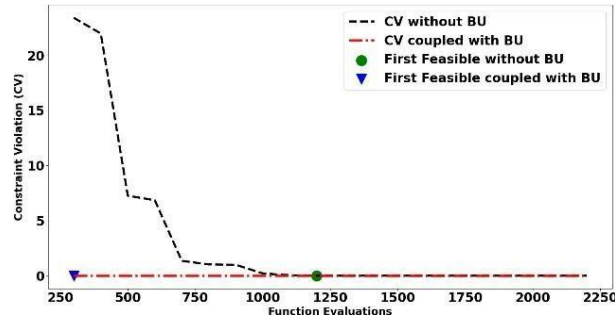


Fig. 3.2. First feasible solution found with and without the BU method (OSY problem)

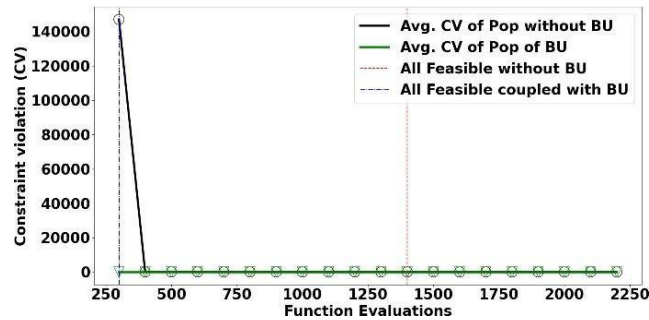


Fig.3.3. Convergence of the whole population to the feasible solution with and without the BU method (OSY problem)

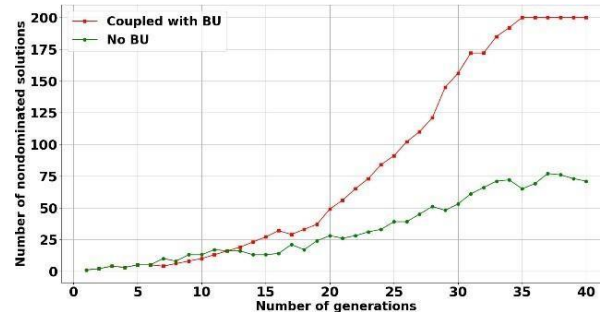


Fig.3.4. Pareto solutions obtained by NSGA-II with BU and without BU for the OSY problem

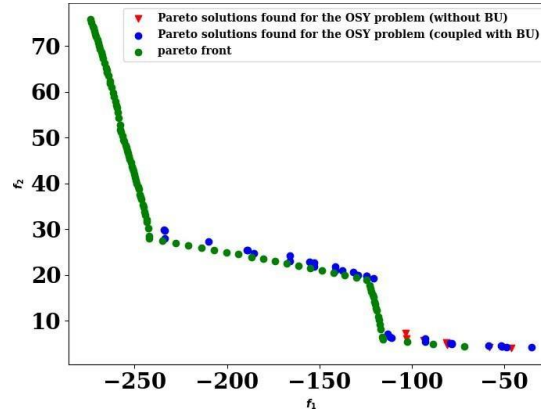


Fig 3.5. Pareto solutions obtained with and without the BU method

Figs. 3.6-3.9 display the performance indicator values obtained by the method with and without the BU method for the OSY problem. As mentioned, GD, GD+, IGD, and IGD+ were selected as the indicators. Sampling and crossover are two important factors for evolutionary algorithms, which strongly influence the convergence speed and are often used in noisy optimization, where reducing the negative impact of noise is a key matter; also, sampling is a popular strategy for dealing with noise. Therefore, experiments were conducted with different operators, namely real random sampling, Latin hypercube sampling, and also two different real-SBX crossover and uniform crossover (Qian et al., 2018). In the figures (3.6-3.9), the mean, median, standard deviation, and best, and worst values of the experiments using the BU method were much lower than the same method without BU. Table 3.2 presents the Summary of p-value of the Wilcoxon rank sum test ($\alpha = 0.05$) over all runs; as it can be seen, there is a significant difference overall run for all experiments excluding binary crossover.

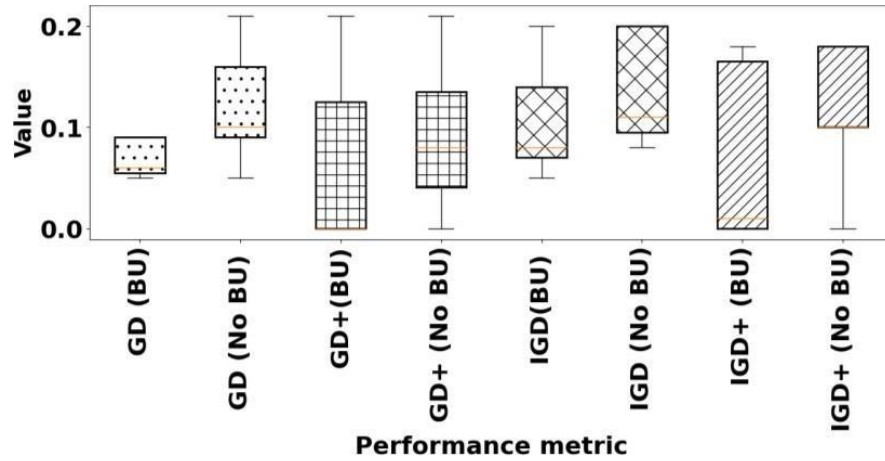


Fig. 3.64. Performance indicator values obtained with and without the BU method for the OSY problem (Sampling=Real-Random)

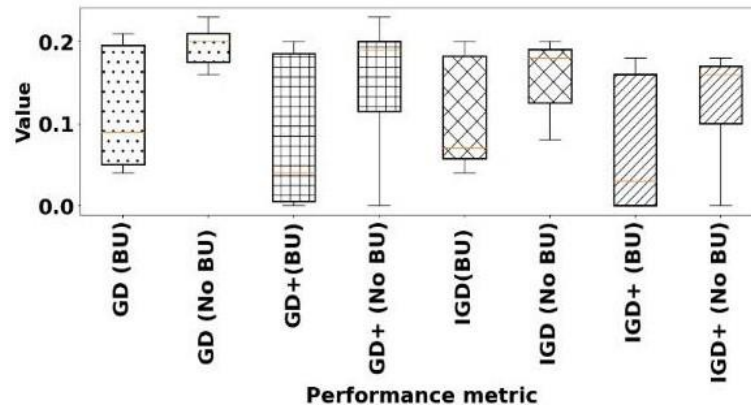


Fig.3.7. Performance indicator values obtained with and without the BU method for the OSY problem (Sampling=Latin hypercube sampling)

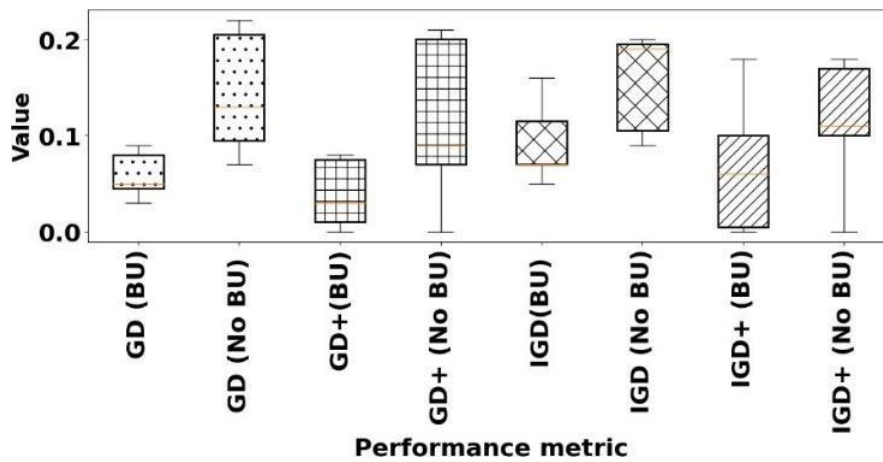


Fig.3.8. Performance indicator values obtained with and without the BU method for the OSY problem (Crossover=real_sbx)

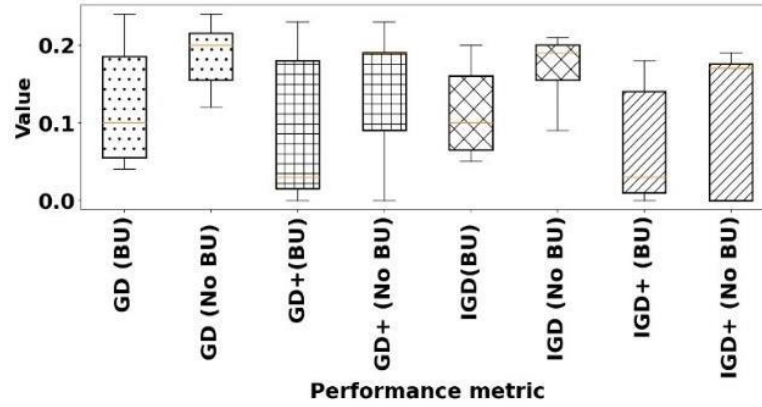


Fig. 3.95. Performance indicator values obtained with and without the BU method for the OSY problem (Crossover=bin_ux)

Table 3.2. *p*-value summary of the Wilcoxon rank sum test over all runs (BU vs. without BU)

Indicator	Sampling				Crossover			
	real-random		hypercube		real_sbx		bin_ux	
	<i>p</i> -value	h	<i>p</i> -value	h	<i>p</i> -value	h	<i>p</i> -value	h
GD	0.0068	+	0.0009	+	0.009	+	0.0050	+
IGD	0.009	+	0.009	+	0.004	+	0.0050	+
GD+	0.033	+	0.007	+	0.009	+	0.0656	~
IGD+	0.013	+	0.0356	+	0.12	+	0.0969	~

3.2 Bin and Korn Test Problem (BNH)

The BNH test problem, proposed by (Binh & Korn, 1997), has two objective functions, two constraints, and two decision variables. Fig. 3.10 presents the results found by selecting the repairing variable, x_1 , for this problem. However, because a smaller number of Pareto solutions is not always a good indicator for this problem, two other indicators, namely IGD and HV, were considered. Figs. 3.11-3.12 present the IGD and HV values found by the BU method in early generations compared with the original algorithm without BU (11 experiments have been conducted).

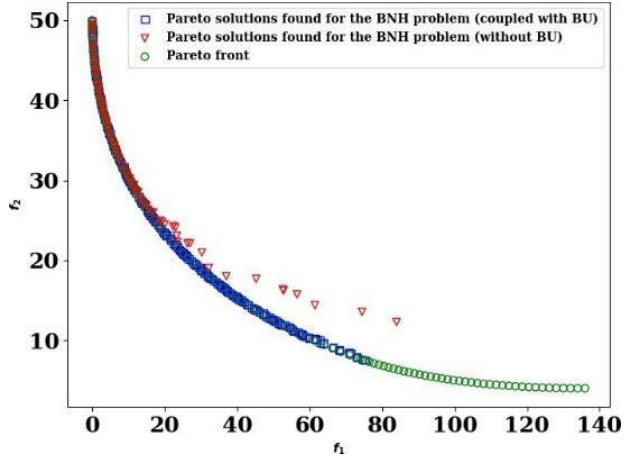


Fig.3.10. Pareto solutions found with and without the BU method for the BNH problem (generations = 20)

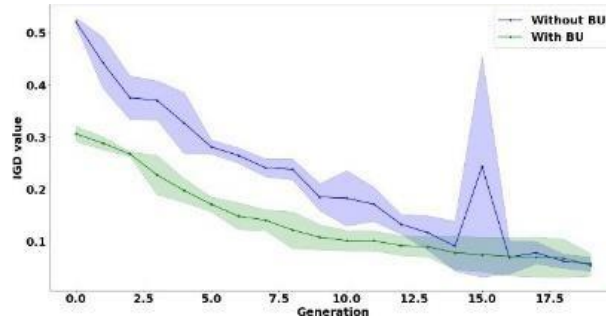


Fig. 3.116. IGD values for BNH problem

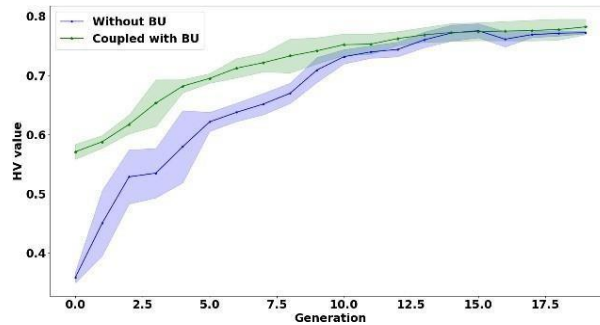


Fig.3.12. HV values for BNH problem

3.3 Welded Beam Design Problem

The welded beam design problem is a well-known engineering optimization benchmark problem (Deb & Sundar, 2006; Ray & Liew, 2002), which examines the tradeoff between the strength and cost of a beam. The two objective functions are minimizing the fabrication cost of the beam and minimizing the deflection of the end of the beam. This problem has four variables and four

constraints, where all constraints are handled by beam thickness. Herein, the NSGA-II was applied to this problem, and fitness values were evaluated.

Fig. 3.13 shows the Pareto optimal solutions found by the two approaches, with and without the BU method, considering 20 generations. It is apparent that NSGA-II using the BU method can find the Pareto solutions in the early generations (38) and, thus, can find the global optimum solutions faster than NSGA-II without BU. The results of IGD, IGD+, GD, and GD+ values for NSGA-II with and without BU are presented in Fig. 3.14.

As it was mentioned earlier, compared to unconstrained optimization problems, constrained MOOPs are more challenging since a large proportion of infeasibility regions appears in the search space (it means the hit ratio is low), which makes solving the constrained problem very difficult, especially for highly constrained and can lead to some issues such as convergence-, diversity-, and feasibility-hardness.

The results for welded beam design problem show that the BU approach found the first feasible solution and the whole feasible population after 300 evaluations. Comparatively, the NSGA-II algorithm without the BU method found the first feasible solution after 1100 evaluations, and the whole population is feasible after 1500 evaluations (Figs. 3.15 and 3.16). Herein, the performance of the running metric proposed in (Blank & Deb, 2020a) was evaluated, which illustrates the difference in the objective space and, specifically, the improvement from one generation to the next.

Fig. 3.17 demonstrates how the running metric could be applied to visualize and compare the performance of NSGA-II using the BU method, where $t=11$ to 13. Fig. 3.17 shows that NSGA-II using the BU method was improved from $10e9$ to $10e0$ in generation 6, which is a significant change, while the method without BU was improved almost from $10e2$ to $10e0$ in generation 12. On the other

hand, NSGA-II using the BU method converges to optimal solutions in the early generations, whereas the algorithm without BU needs to be further evaluated to find the optimal solutions.

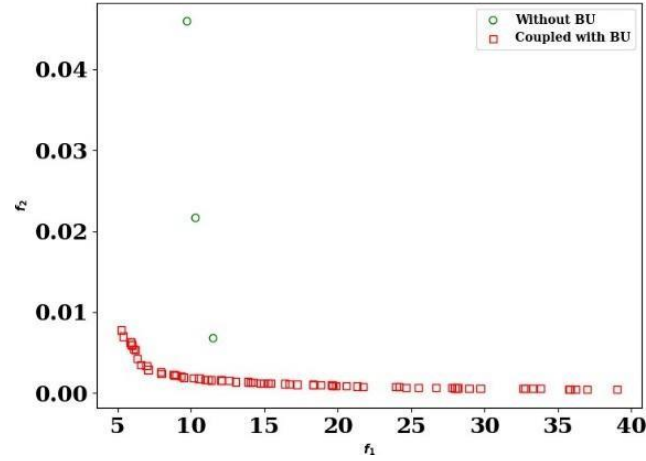


Fig. 3.13. Pareto solutions obtained for the welded beam design problem with and without the BU method

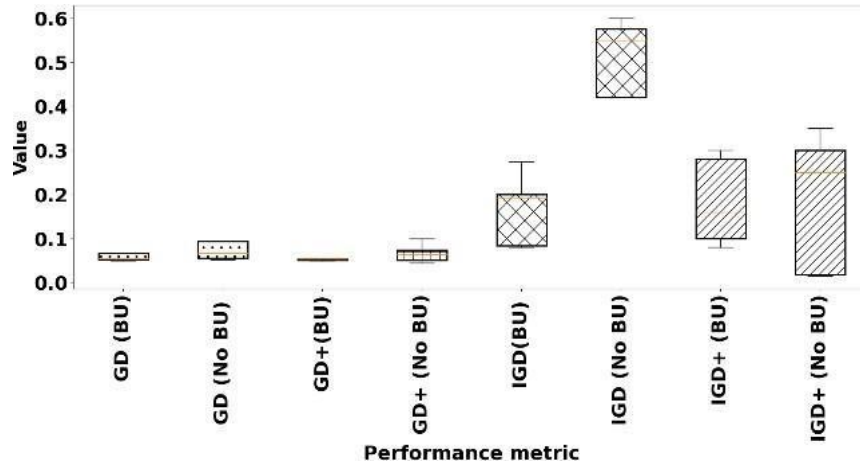


Fig. 3.14. Performance indicator values obtained with and without the BU method

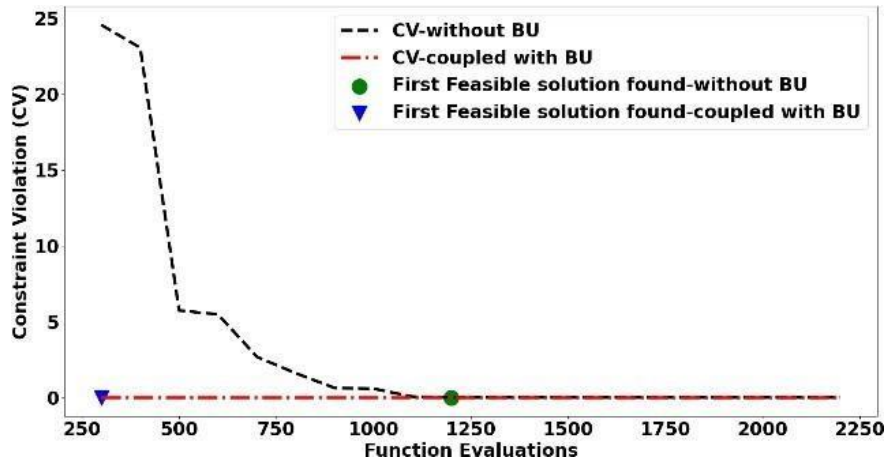


Fig. 3.15. Constraint violation evaluated by NSGA-II (first feasible solution)

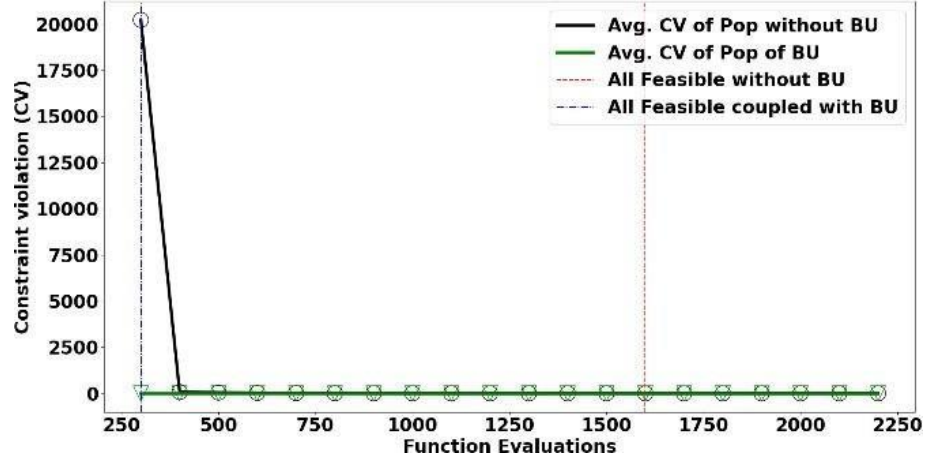


Fig. 3.16. Constraint violation evaluated by NSGA-II (convergence of the whole population to feasible solutions)

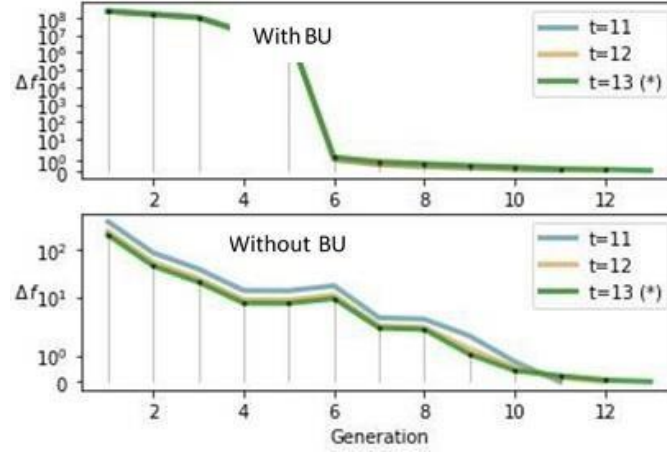


Fig. 3.17. Running metric for the multi-objective welded beam design problem with and without the BU method

3.4 Cantilevered Beam Design Problem

The Cantilevered design problem is an example of a large-scale size problem. In the cantilevered beam design problem, the stepped cantilever beam must be able to carry a prescribed end load (Vanderplaats, 2002). This problem is originally a single-objective optimization problem considering minimizing the beam volume; however, this study has extended the original problem and added one more objective function, which minimizes end deflection subject to various engineering design variables. The beam supports the given load, P , at a fixed distance L from the support. Besides, designers of the beam can vary the width (b_i) and height (h_i) of each section. The problem is highly

constrained and large-dimensional; however, for this example, five segments ($N=5$), including 10 dimensions (10 constraints) are considered. In this example, x_2 , x_4 , x_6 , x_8 , and x_{10} are considered as the repairing variables. This example illustrates how the BU method can be used to solve a constrained nonlinear optimization problem. Fig. 3.18 presents the optimal solutions found by NSGA-II with and without the BU method for the problems in generation 5.

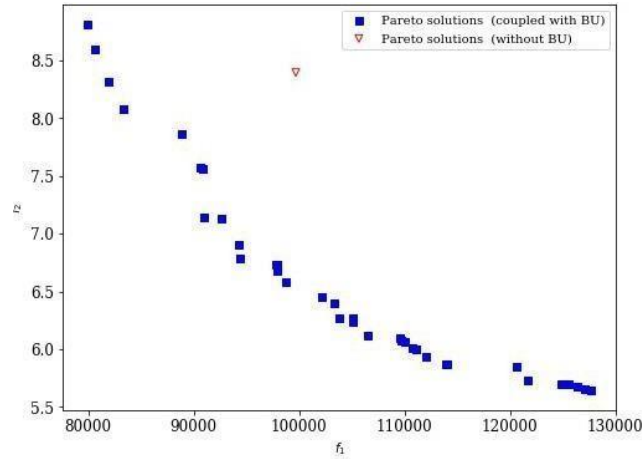


Fig. 3.187. Pareto solutions found for multi-objective cantilevered beam design problem using NSGA-II with and without the BU method (generations = 5)

Fig. 3.19 presents the feasibility of the first solutions and the whole population of the cantilevered beam design problem using both methods. The NSGA-II with the BU approach found the first feasible solution and the whole feasible population after 100 evaluations. Comparatively, the method without BU found the first feasible solution 500 evaluations and the whole population is feasible after 1100 evaluations.

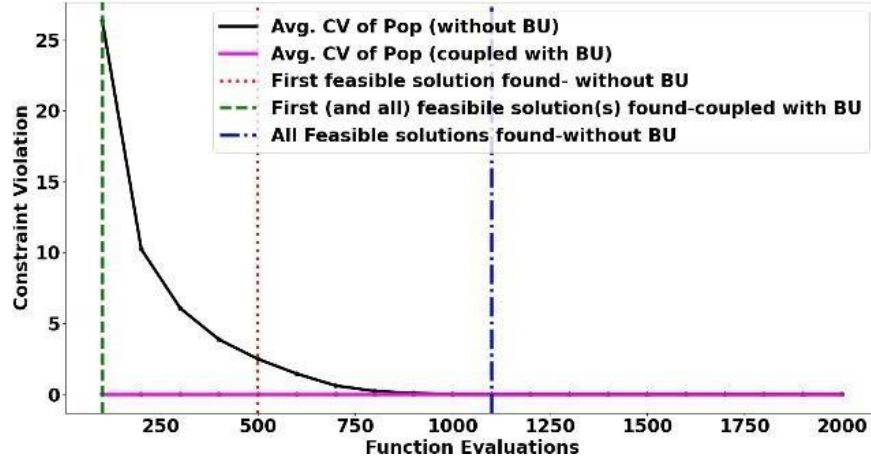


Fig. 3.198. Feasibility of first and whole population of cantilevered beam design problem using both methods

4 Conclusion

This study presents an extension of an implicit CHT called the BU method (Gandomi & Deb, 2020) for solving the MOOPs efficiently. Several performance indicators were obtained to evaluate the BU method. The results show that the NSGA-II algorithm using the BU method could find feasible solutions much faster than without the BU method (when only an explicit CHT is applied to an optimization process). Also, the performance indicators show significant improvement by implementing the optimization algorithm with the BU method in some cases. Moreover, the BU method found the first feasible solution and feasible solutions for the whole population in earlier generations.

As a future research direction, it is suggested to apply other population-based multi- and many-objective optimization algorithms to real-world optimization problems. Also, it is suggested to investigate the impact of binary crossover for other multi-objective evolutionary algorithms on other real-world problems. Also, it is suggested that the BU method is coupled with other constraint-handling techniques, which are available in the literature.

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Chapter 4

Efficient implicit constraint handling approaches for constrained optimization problems

Iman Rahimi¹, Amir H. Gandomi^{*1,2}, Mohammad Reza Nikoo³, Mohsen Mousavi¹, Fang Chen¹

¹*Faculty of Engineering and Information Technology, University of Technology Sydney, Australia (emails: iman83@gmail.com, gandomi@uts.edu.au, mohsen.mousavi@uts.edu.au, fang.chen@uts.edu.au)*

²*University Research and Innovation Center (EKIK), Óbuda University, 1034 Budapest, Hungary*

³*Department of Civil and Architectural Engineering, Sultan Qaboos University, Muscat, Oman (email: m.reza@squ.edu.om)*

Abstract— Many real-world optimization problems, particularly engineering ones, involve constraints that make finding a feasible solution challenging. Numerous researchers have investigated this challenge for constrained single- and multi-objective optimization problems. In particular, this work extends the boundary updating (BU) method proposed by Gandomi and Deb (2020) for the constrained optimization problem. BU is an implicit constraint handling technique that aims to cut the infeasible search space over iterations to find the feasible region faster. In doing so, the search space is twisted, which can make the optimization problem more challenging. In response, two switching mechanisms are implemented that transform the landscape along with the variables to the original problem when the feasible region is found. To achieve this objective, two thresholds, representing distinct switching methods, are taken into account. In the first approach, the optimization process transitions to a state without utilizing the BU approach when constraint violations reach zero. In the second method, the optimization process shifts to a BU method-free optimization phase when there is no further change observed in the objective space. To validate, benchmarks and engineering problems are considered to be solved with well-known evolutionary single- and multi-objective optimization algorithms. Herein, the proposed method is benchmarked using with and without BU approaches over the whole search process. The results show that the proposed method can significantly boost the solutions in both convergence speed and finding better solutions for constrained optimization problems.

Keywords— Constraint handling, Multi-objective optimization, Evolutionary algorithm, Boundary updating, Switching point.

1. Introduction

Constrained optimization problems arise naturally in most disciplines where finding optimized feasible solutions, especially with multiple objectives, is challenging. Despite the considerable variety of techniques developed in optimization fields and other disciplines to tackle these problems, the complexity of their solutions calls for alternative solution methods. Moreover, the computational cost has become another major concern due to the complexity of modern real-world problems. Also, the demand for optimal design and its applications in engineering and industry have become even more significant to satisfy the need for more strategic designs in modern engineering practices (Toloo et al., 2022). Engineering optimization problems usually consist of constraints that may be physical, geometrical, or operational; handling such constraints to find a single feasible solution is a challenging task (Rahimi, Gandomi, Chen, et al., 2023; Rahimi, Gandomi, Nikoo, et al., 2023). For multi-objective problems, constraint handling is critical since a set of feasible solutions, i.e., Pareto front set, is sought. To tackle real-world constrained optimization problems, the constraint handling technique (CHT) has been combined with an evolutionary algorithm (EA) to achieve constrained evolutionary algorithm optimization (CEAO) (Rahimi, Gandomi, Chen, et al., 2023). Most CHTs proposed in the literature are explicit methods, e.g., CHTs by penalty or other fix-ups (Rahimi, Gandomi, Chen, et al., 2023).

The CHTs come in two main forms: explicit and implicit. Explicit techniques involve the explicit definition of constraints as part of the problem formulation. They are designed to respect and enforce constraints explicitly, guiding the search towards feasible regions of the solution space and addressing violations directly.

For example, several existing CHTs in the literature are based on feasibility and infeasibility regions 4-6 , priority assignment (Jiménez et al., 1999) , and tournament selection and a selection operator(Jiménez et al., 1999; Jordehi, 2015) . Some studies also provide surveys of CHTs8– (Salcedo-Sanz, 2009) . In contrast, implicit techniques do not necessitate the explicit form of constraints. They handle constraints inherently during the search for the optimal solution, avoiding the need for additional objective function evaluations. Implicit CHT simplifies the problem formulation, but it may face challenges in handling complex or nonlinear constraints effectively.

For instance, (Diwekar & Rubin, 1993) proposed an implicit constraint handling technique for the ASPEN chemical process, which is based on a mixed integer mathematical programming approach consisting of master and subproblems. Since most of the relations in the above-mentioned problems are implicit,(Diwekar & Rubin, 1993) proposed a method for partitioning the variables to significantly decrease computational time.

Raghavan et al. (2012) developed an implicit constraint handling technique for optimization based on the proper orthogonal decomposition (POD) of shapes, further producing a bi-level reparameterization approach for structural geometries. Uemura et al. (2013) proposed a real-coded genetic algorithm for implicitly constrained black-box optimization, in which the method employs the weighted mean of the best individuals in a population to find the optimal solution.

Mirabel & Lamiraux (2018) proposed a method to handle all constraints explicitly and implicitly that deals with manipulation planning, where the constraints are solved explicitly as much as possible. Implicit handling is alternatively employed with few variables in case all constraints cannot be handled explicitly. Nomura et al. (2021) introduced a natural evolution strategy called DX-NES-IC for implicit handling of constrained black-box optimization, which demonstrated better performance than DX-NES, xNES, and CMA-ES.

The current study extends the work of (Gandomi & Deb, 2020) by extending and applying BU to single-objective and multi-objective optimization problems (MOOPs). As novel strategies, two switching mechanisms are suggested that transform the landscape and variables to the original problem when the feasible region is found. Then, the optimization process is continued without the BU method.

In the work by (Gandomi & Deb, 2020), the authors proposed a novel approach, boundary update (BU), as an implicit CHT that updates variable bounds by directly using the constraints and then applying them to several single-objective optimization problems. Since, the BU method is an implicit CHT, it should be coupled with an explicit CHT, that is feasibility rules in our study (Deb et al., 2002). As for the strategy without BU, only an explicit CHT (feasibility rules) is applied to solve the problem. The BU method is an implicit constraint handling approach that aims to cut the infeasible search space over iterations to find the feasible region faster. Although the BU method directs the search operators to the feasible space and reaches the first feasible solution reasonably fast, it twists the search space, making the optimization problem more challenging. The current study tries to tackle the above-mentioned problem by proposing two switching approaches.

The current study augments existing research by introducing two novel switching mechanisms. In the initial method, called Hybrid-cvtol, the BU approach is employed until constraint violations reach zero, ensuring zero violations across the entire population. Subsequently, the algorithm transitions to the optimization process without utilizing the BU approach. The second switching mechanism, called Hybrid-ftol, in this study involves employing the BU method during the initial phase until the objective space remains unchanged for a specified number of generations. At that point, the optimization problem shifts to a state without utilizing the BU approach.

The remainder of the paper is organized as follows. Section 2 presents the proposed approach. Section 3 discusses the research methodology. Section 4 provides numerical examples. Finally, Sections 5, 6, and 7 give the discussion, limitation, and conclusion.

2. Proposed Approach

A constrained optimization problem can be formulated in Equations 1-3 as follows:

$$\text{Maximize (Minimize) } F(x) = (f_1(x), \dots, f_t(x)) \quad (1)$$

$$\text{s.t. } h_i(x) \leq 0 \text{ for } i \in \{1, \dots, n\} \quad (2)$$

$$\text{for } j \in \{1, \dots, m\} g_j(x) = 0 \quad (3)$$

$$LB \leq x \leq UB \quad (4)$$

where $F(x)$ is the objective vector that consists of several objectives (t is the number of objective functions); n and m are the number of inequality and equality constraints, respectively; and x is the decision variable bounded by the lower bound (LB) and upper bound (UB) vectors. These equations yield several Pareto optimal solutions rather than producing a single solution (Deb, 2011). In the case of single-objective optimization, a single solution can be found as a solution to $F(x) = f_1(x)$. In the optimization process, the boundaries of decision variables, represented by lower bounds (LB) and upper bounds (UB) vectors, undergo dynamic changes as the algorithm progresses through iterations. This dynamic nature enables the optimization process to adapt and explore a continually evolving solution space, illustrating the inherent variability and adaptability of decision variable boundaries throughout the iterative stages. Hence, the iterative nature of optimization involves the dynamic evolution of the i th decision variable boundaries throughout each iteration.

The proposed method uses the constraints to narrow down variable space and then forces the algorithm to focus its search in the feasible region by limiting the viable search space for the

variable(s). In the BU method, the boundaries change iteratively and are updated during the optimization procedure. Mathematically, it could be written as follows (Gandomi & Deb, 2020):

$$\exists i \in \{1, \dots, m\}: [\forall j \in \{1, \dots, n\}: x_i \geq l_{i,j}(x_{\neq i}) \cup x_i \leq u_{i,j}(x_{\neq i})] \quad (4)$$

where $l_{i,j}$ and $u_{i,j}$ are dynamic lower bound and upper bound for the i th decision variable, respectively. Updating the bounds involves the following scenarios:

$$\text{If } lb_i = -\infty \text{ and } ub_i = +\infty: \quad (5)$$

$$lb_i^u = l_{i,j}(x_{\neq i}) \quad (6)$$

$$ub_i^u = u_{i,j}(x_{\neq i}) \quad (7)$$

Else:

$$lb_i^u = \min(\max(l_{i,j}(x_{\neq i}), lb_i), ub_i) \quad (8)$$

$$ub_i^u = \max(\min(u_{i,j}(x_{\neq i}), ub_i), lb_i) \quad (9)$$

where (lb_i^u, ub_i^u) are updated boundaries. BU begins by selecting a repairing variable that can handle the most significant number of constraints without overlapping other repairing variables. As such, if there is more than one candidate, the one that handles the greatest number of constraints is selected. Also, if there is still another candidate for variable selection, one variable is chosen randomly (Gandomi & Deb, 2020).

If a repairing variable handles the first k_i constraints, then:

$$lb_i^u = \min(\max[l_{i,1}(x_{\neq i}), \dots, l_{i,k_i}(x_{\neq i}), lb_i], ub_i) \quad (10)$$

$$ub_i^u = \max(\min[u_{i,1}(x_{\neq i}), \dots, u_{i,k_i}(x_{\neq i}), ub_i], lb_i) \quad (11)$$

(where $k_i \leq m$)

If another repairing variable (x_r) is defined:

$$lb_i^u = \min(\max[l_{i,1}(x_{\neq i,r}), \dots, l_{i,k_i}(x_{\neq i,r}), lb_i], ub_i) \quad (12)$$

$$ub_i^u = \max(\min[u_{i,1}(x_{\neq i,r}), \dots, u_{i,k_i}(x_{\neq i,r}), ub_i], lb_i) \quad (13)$$

$$lb_r^u = \min(\max[l_{r,k_i+1}(x_{\neq r}), \dots, l_{r,k_i+k_r}(x_{\neq r}), lb_r], ub_r) \quad (14)$$

$$ub_r^u = \max(\min[u_{r,k_i+1}(x_{\neq r}), \dots, u_{r,k_i+k_r}(x_{\neq r}), ub_r], lb_r) \text{ where } r \in \{1, \dots, n\} \text{ and } r \neq i \quad (15)$$

In the BU method, a repairing variable could be substituted with a generalized semi-independent variable (Gandomi & Deb, 2020) and then rewritten with lower and upper bounds as:

$$x \in \{x_1, \dots, x_h, x_{h+1}, \dots, x_n\} \rightarrow mx \in \{p_1, \dots, p_h, x_{h+1}, \dots, x_n\} \quad (16)$$

where $p_1, \dots, p_h$ are semi-independent variables; and mx indicates the set of selected repairing variables.

$$x \in \{x_1, \dots, x_h, x_{h+1}, \dots, x_n\} \rightarrow mx \in \{p_1, \dots, p_h, x_{h+1}, \dots, x_n\} \quad (17)$$

$$\text{for } i=1, \dots, h \text{ where } 0 \leq p_i \leq 1 \quad x_i = lb_i^u + p_i(ub_i^u - lb_i^u) \quad (18)$$

After selecting repairing variables, the search operator will be applied to the problem, the boundaries of non-repairing variables will be checked, and the mx -vector will be updated. During the solution procedure, the boundaries of repairing variables will be updated, and then the semi-

independent variables ($p_i, i = 1, \dots, h$) are remapped to the actual variables using updated boundaries. In the end, those constraints that were not involved in the repairing variable boundary, along with fitness values, will be evaluated using actual variables. Algorithm 1 illustrates the BU method in a constrained optimization problem.

Algorithm 1. Implementation of the BU method in a constrained optimization problem

Initialization

While the criteria are not met

Applying the search algorithm Applying the BU method and updating the mx-vector Check the boundaries of non-repairing variables Update the boundaries of repairing variables Remap p variable to the original variables using updated boundaries (lb_i^u, ub_i^u) Evaluate the constraint violations Evaluating fitness function

End while

As an illustrative example, the following multi-objective optimization problem (MOOP) is considered to explain BU:

$$\min f_1(x) = (x_1^2 + x_2^2) \quad (19)$$

$$\text{Min } f_2(x) = -(x_1 - 1)^2 - x_2^2 \quad (20)$$

$$g_1(x_1 + x_3) = x_1 + x_3 \leq 1 \quad (21)$$

$$-2 \leq x_1 \leq 2 \quad (22)$$

$$-2 \leq x_2 \leq 2 \quad (23)$$

$$-2 \leq x_3 \leq 2 \quad (24)$$

Either x_1 or x_3 can be selected as a repairing variable of the single constraint. All the variables are in the range of -2 to 2. From the variable selection strategy (Gandomi & Deb, 2020b), as explained

earlier, x_3 is selected as a repairing variable. The inequality constraint is solved with respect to x_3 as the lower and upper bound functions. The repairing variable (x_3) is substituted with the mapped variable, p , in the range of 0 to 1. Therefore, the lower and upper bounds of the variables are $\{-2, -2, 0\}$ and $\{2, 2, 1\}$, respectively. When the search algorithm is applied to the problem, the repairing variable is substituted with the mapped variable, and the boundary of the repairing variable is mapped for the rest of the search cycle (Gandomi & Deb, 2020). The original variable can be calculated by the following formula to compute the objectives and constraints:

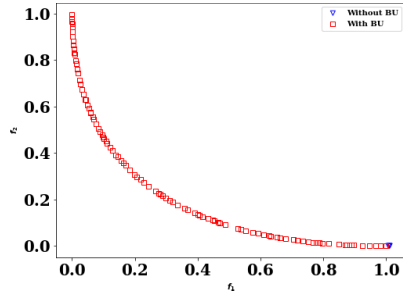
$$x_3 = lb_3^u + p_i \times (ub_3^u - lb_3^u) \quad (25)$$

where ub_3^u and lb_3^u are the lower and upper bounds for the repairing variable x_3 , respectively. The updated bounds can be written as follows:

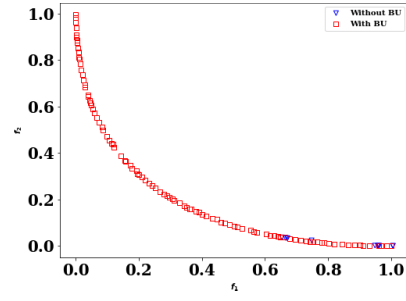
$$lb_3^u = \min(\max(lb_3, 1 - x_1), ub_3) \quad (26)$$

$$ub_3^u = \max(\min(ub_3, 1 - x_1), lb_3) \quad (27)$$

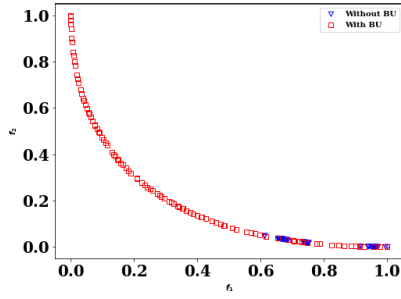
The repairing variable is remapped to the actual boundary when the fitness values are evaluated. Figures 1 (a-d) compare Pareto-optimal fronts found by the BU method coupled with a CHT (here, feasibility rules) and by the method without the BU method (only the feasibility rules approach is considered) for a population size of 100 for different generations. Figure 4.1 shows that the BU method produces more non-dominated solutions than the approach without BU. However, the number of non-dominated solutions is not always a good indicator for comparison. In the following sections, other indicators are explained in detail to compare the algorithms.



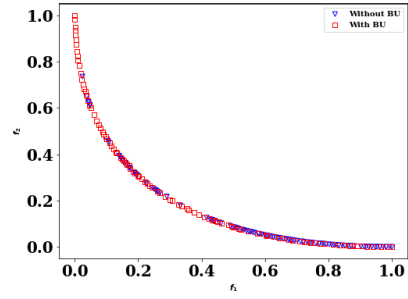
a) Generation=50



b) Generation=100



(c) Generation=200



(d) Generation=1000

Figure 4.1 Pareto-optimal fronts obtained with and without the BU method

Although the BU method directs the search operator to the feasible space and reaches the feasible region in a reasonable time, it twists the search space, which can make the optimization problem more challenging and, hence, does not always bear good results and could potentially result in premature convergence. Consider the following example:

$$\text{Max } f(x) = x_1^2 + x_2^2 - 2 \times x_1 - 2 \times x_2 + 2 \quad (28)$$

$$\text{s.t. } g_1(x): -(3 \times x_1 + x_2^2 - 5.5) \leq 0 \quad (29)$$

$$0 \leq g_2(x): -(x_1^2 + 2 \times x_2) - 4.5$$

$$g_3(x): 0.8 + x_1^3 - x_2 \leq 0$$

$$0 \leq x_1 \leq 4 \quad (30)$$

$$0 \leq x_2 \leq 4 \quad (31)$$

In case of selecting variable x_2 as a repairing variable, the design space for the original variables x_1 and x_2 (Figure 4.2 (a)) is converted to the new design space (Figure 4.2 (b)). Twisting landscape may result in challenges for optimization problems, such as premature convergence.

Some of these issues are presented in the following examples.

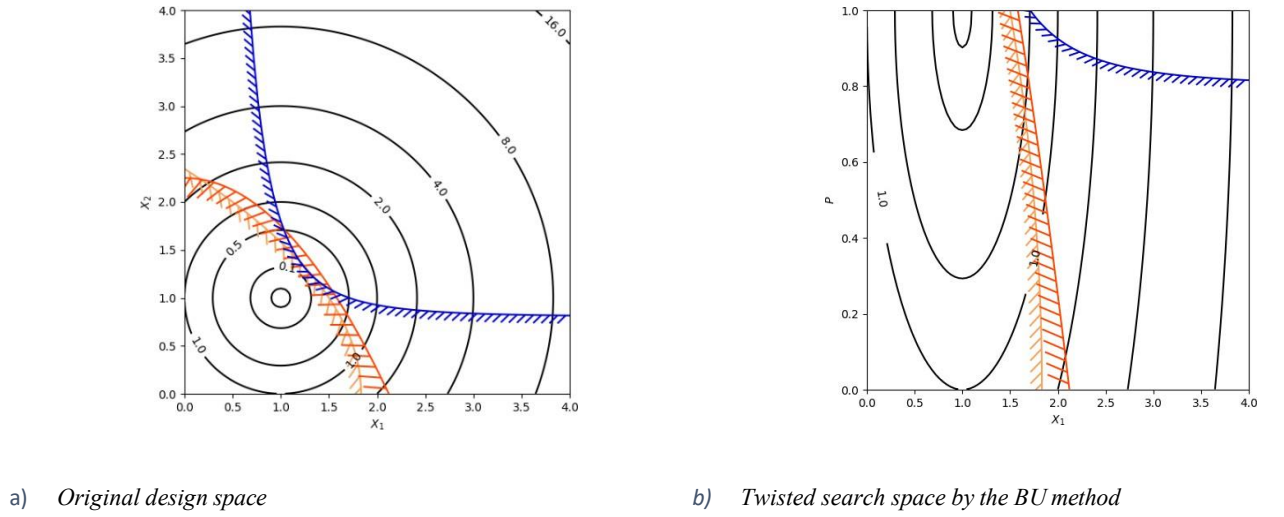


Figure 4.2 Contour plot of the search space

Therefore, two distinct hybrid methods are introduced, each incorporating its own unique switching mechanism. In the initial approach, the BU method is employed until all constraint violations are reduced to zero, ensuring the absence of violations across the entire population. Subsequently, the algorithm transitions to the optimization process, excluding the use of the BU approach. In the second switching mechanism, the BU method is utilized in the initial phase until the objective space remains unchanged for a specified number of generations. Following this, the optimization problem transitions to a state where the BU approach is no longer employed. In both approaches, the optimization process is halted once the criteria and threshold tolerance are met. In the subsequent phase, the optimization

problem is solved without the BU method, using the final population from the previous phase as the initial population for the new optimization problem. Figure 4.3 illustrates that the handling of boundaries and constraints occurs after the application of search operators.

During the first phase, the proposed methods simultaneously check constraints and examine variable boundaries, making the optimization process an iterative search procedure for handling constraints and boundaries concurrently. If the criteria are met, the second phase of the optimization process is conducted without utilizing the BU approach, constituting a sequential iterative search procedure where constraint handling occurs after boundary checking. The implementation of the proposed hybrid method is detailed in Algorithm 2.

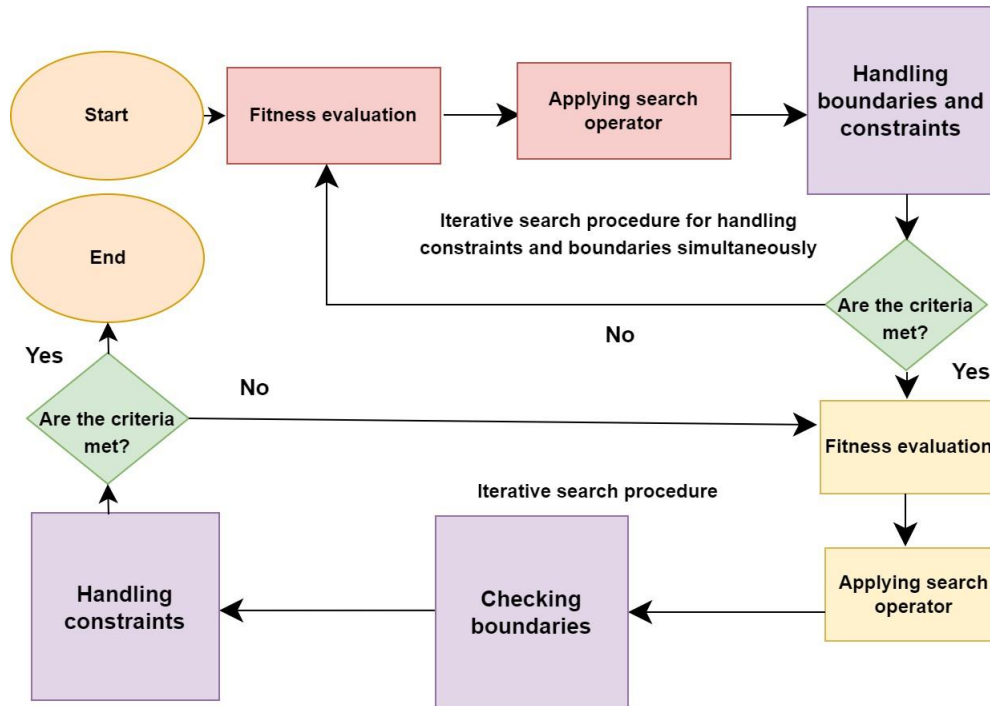


Figure 4.3 Optimization process of the proposed method

Algorithm 2. Implementation of the proposed method (hybrid methods)

```

Initialize the parameters
Set a threshold tolerance for constraint violation and objective space changes
While all constraints are handled
    Find the repairing variable (s)
    If there is more than one candidate
        Select the one (s) that handle/s the greatest number of constraints
        If there are still other candidates
            Choose randomly
        End if
    End if
End while

While the criteria are not met (here threshold tolerance: constraint violation should reach zero/ no change
in objective space in the last n generations)
    Applying the search algorithm
    Update mx-vector
    Check the boundaries of non-repairing variables ( $x_{h+1}, \dots, x_n$ )
    For  $i=1$  to  $h$  (repairing variables)
        Update the boundaries of repairing variables,  $lb_i$  and  $ub_i$ 
        Remap  $p$  variable to the original space for post-processing
    End for
    Check the constraint violation
End while

Stop the optimization process using the BU method
Initialize the algorithm using the final population of the previous optimization process
Set a termination criteria (e.g., number of evaluations, number of generations, time, etc.)
While the criteria are not met
    Apply the search algorithm
    Optimize the problem without the BU method
    Evaluate fitness function
End while

```

3. Research Methodology

To evaluate the proposed method, Python and Pymoo libraries (Blank & Deb, 2020) were used, which include well-stabilized algorithms. The specific algorithm parameters are presented in Table 4.1. Furthermore, because the implemented algorithms act stochastically, each of these algorithms were run 31 times with random initial point(s).

Several examples, including benchmarks and real-world problems, were solved using three methods (without the BU method, with the BU method, and hybrid method). The presented examples are of different types: constrained by one, two, and three objective functions. For the problem with one objective function, the proposed method was implemented using evolutionary strategy (ES), differential evolution (DE), stochastic ranking evolutionary strategy (SRES)(Runarsson & Yao, 2000), and biased random key genetic algorithm (BRKGA)(Gonçalves & Resende, 2011). For two objective functions, the BU method was applied using NSGA-II17,(Michalewicz, 1995) . For three and five objective functions, some other EAs were used. As such, NSGA-III(Deb & Jain, 2013) , U-NSGA-III(Seada & Deb, 2015) , AGE-MOEA (Panichella, 2019), NSDE(Angira & Babu, 2005) , and NSDER (Reddy & Dulikravich, 2019) were applied to optimization problems. The following metrics were evaluated to assess the accuracy of the obtained results:

- Number of non-dominated solutions: The BU method was coupled with an explicit constraint handling technique and compared to the approach without BU (only feasibility rules were applied to the optimization problem). Since the BU method aims to reduce the search space, the number of non-dominated solutions was determined for comparison.

Table 4.1 Specific parameter settings for the algorithms used in this study

Algorithm	Parameter settings
Evolutionary strategy	Number of offspring=200, rule=1.0 / 7.0
Differential evolution	Variant="DE/rand/1/bin", crossover constant (CR)=0.3, Weighting factor (F)=0.8
Stochastic ranking evolutionary strategy	Number of offspring=200, rule=1.0 / 7.0, gamma=0.85, alpha=0.2
Biased random key genetic algorithm	Number of elite=200, number of offspring=700, number of mutant=100, bias=0.7

- Constraint violation (CV): Compared to unconstrained optimization problems, constrained optimization problems are more challenging since a large proportion of infeasible regions appears in the search space (meaning the hit ratio is low), especially for highly constrained problems where these regions lead to some difficulties, such as feasibility-hardness and convergence-hardness. Therefore, even finding one feasible solution is a significant achievement. To test the effectiveness of the proposed method, the first feasible solutions found by the methods were compared and the whole population was tracked against generations.
- Performance indicators: Generational Distance (GD) (van Veldhuizen, 1999), Generational Distance Plus (GD+) (Ishibuchi et al., 2015), Inverted Generational Distance (IGD)(Coello Coello & Reyes Sierra, 2004) , Inverted Generational Distance Plus (IGD+) (Ishibuchi et al., 2015), and Hypervolume (HV) are the indicators that were analyzed.
- Running metric(Blank & Deb, 2020a) : It is possible to trace the difference in the objective space followed by generations.

Moreover, the population was fixed at 100, and the maximum number of experiments was set to 11.

4. Numerical Examples

The proposed approach was applied to several single-objective and multi-objective optimization examples, including benchmarks and real-world problems. The presented examples include problems constrained by one, two, and three objective functions. The performance of the three strategies was evaluated using an example that has linear constraints. Following this numerical example, the proposed approach was evaluated to solve a surrogate model for a car-side impact design problem. This design problem consists of two versions of optimization, i.e., single- and multiobjective optimizations, and is an example of a black-box optimization problem. The speed reducer problem, a single-objective constrained engineering optimization problem, was also considered. For the multi-

objective optimization problem, two benchmarks, namely OSY and BNH, and three real-world constrained optimization problems, including welded beam design, Cantilevered Beam design, and multi-objective car-side design problems were solved via the three strategies.

Several metaheuristic methods have been developed and implemented to reduce the total runtime of the optimization problems (Behmanesh et al., 2021; Rahimi, Gandomi, Chen, et al., 2023), but still produce results that are almost as accurate and precise as conventional solving methods. Also, real-world engineering problems involve some type of optimization that is often constrained, most of which are considered MOOPs. No single solution exists for a MOOP; instead, different solutions generate trade-offs for various objectives. Furthermore, MOOPs arise naturally in most fields, and solving them has been a challenging problem for researchers (Behmanesh et al., 2021b; Deb, 2014). EA methods have been identified as more effective in tackling the challenges that arise from MOOPs, for which the form of the Pareto-optimal front (discontinuity, nonconvexity, etc.) is not important (Coello, 2007; Deb, 2011). Moreover, most multiobjective evolutionary algorithms (MOEAs) use the dominance concept (Corne et al., 2000b; Horn et al., 1994). In this work, for the single-objective optimization problems, Genetic Algorithm (GA), Differential Evolution (DE) (Price et al., 2006), Evolutionary Strategy (ES) (Rechenberg, 1989), Stochastic Ranking Evolutionary Strategy (SRES) (Runarsson & Yao, 2000), and Biased Ranking Key Genetic Algorithm (BRKGA) (Gonçalves & Resende, 2011) were considered. NSGA-II (Deb, 2000) is applied to bi-objective and multi-objective optimization problems with three objective functions and to multi-objective optimization problems with more than three objective functions, which are called many-objective optimization problems. NSGA-III (Deb & Jain, 2013), U-NSGA-III (Seada & Deb, 2015), and Adaptive Geometry Estimation-based Multi-objective Evolutionary Algorithm (AGE-MOEA) (Panichella, 2019) were further implemented for optimizing the problems. For all problems, the BU method was coupled with feasibility rules proposed by (Deb, 2000). Table 4.2 presents the

studied examples in this work to assess the effectiveness of the proposed method in comparison with the other methods.

Table 4.2 Test problems considered in this study

Problem	Type of problem	Example	No. of objective/s	No. of constraints	No. of variables
1	Benchmark	G1 problem(Michalewicz, 1995c)	1	9	13
2	Real-world	Single-objective car-side	1	10	11
3	Real-world	Speed reducer(Golinski, 1973b)	1	11	7
4	Benchmark	BNH(Binh & Korn, 1997c)	2	2	2
5	Benchmark	OSY(Osyczka & Kundu, 1995b)	2	6	6
6	Real-world problem	Welded beam problem(Rao, 2019a)	2	4	4
7	Real-world problem	Cantilevered Beam design Problem	2	10	10
8	Real-world problem	Multi-objective car-side problem50	3	10	7

4.1 Single-objective optimization problems

In this section, three single-objective optimization problems, namely G1, car-side design problem, and speed reducer, are considered to be solved using the different strategies discussed above.

4.1.1 G1 problem

The G1 problem is a famous mathematical constrained optimization problem that includes 13 decision variables, a quadratic objective function, and nine linear constraints. The problem is an excellent example of a highly linearly constrained problem, with a feasibility ratio of 0.111%(Gandomi & Deb, 2020) (see the Appendix). In other words, it is a highly challenging to find a feasible solution for this constrained problem. The three different strategies were applied to this problem. According to the variable selection strategy, x_{10} , x_{11} , and x_{12} were selected as repairing variables for handling all nine constraints. The global optimum for this problem is -15, and the

optimization results by using GA, DE, ES, SRES, and BRKGA with the BU method, without the BU method, and hybrid methods are presented in Tables 4.3-4.6 and Figure 4. The Friedman rank test at a significant level of 5% was used to detect differences in treatments across test attempts (Table 4.4). Also, the Post-hoc Nemenyi Friedman test was performed to identify exactly which groups have different means (Tables 4.5-4.6).

The comprehensive analysis of algorithm performance on the G01 optimization problem, characterized by 13 continuous decision variables, nine linear constraints, and a quadratic objective function, unveils intricate relationships between problem features and algorithmic behavior. Each algorithm's response to the problem's unique attributes provides valuable insights into their adaptability and efficacy.

The hybrid methods, consistently showcasing superior performance across diverse algorithms, reflect an inherent versatility that aligns with the multi-faceted nature of the G01 problem. The distinct advantage of the hybridization approach becomes particularly evident in the handling of constraints. The BU method, serving as an implicit constraint-handling technique, demonstrates a keen ability to navigate the infeasible search space effectively. This characteristic proves invaluable in problems like G01, where constraints play a crucial role in shaping the feasible region. The hybrid methods, by strategically integrating the BU method, adeptly leverage this constraint-handling mechanism, enabling faster convergence to the feasible area.

Considering the explicit constraints and the quadratic objective function in G01, it becomes apparent that the algorithmic adaptation to these features significantly impacts performance. Genetic algorithms (GA), when combined with the BU method (as hybrid methods), exhibit a noteworthy convergence pattern (as illustrated in Figure 4.5). The initial phase, focused on constraints handling, ensures the algorithm converges to the feasible area efficiently. Subsequently, the transition to the

optimization process without BU capitalizes on the algorithm's newfound knowledge of the feasible region, resulting in a streamlined path to the final solution. This adaptive approach to the problem's structure showcases how algorithmic behavior is intricately linked to specific features, allowing for a dynamic and context-aware optimization process.

The inadequacy of the SRES algorithm in this context sheds light on the sensitivity of algorithmic performance to the problem's characteristics. The SRES algorithm, designed for stochastic optimization, might face challenges when confronted with the structured nature of the G01 problem. The lack of significant improvement suggests that algorithmic suitability is not universal but rather contingent on the problem's intricacies. This observation underscores the importance of aligning algorithmic choices with the specific features and constraints of the optimization problem at hand. Furthermore, the reduction in standard deviation observed in the hybrid methods implies a more stable and reliable convergence behavior. This heightened stability is particularly valuable in the presence of complex problem features, such as the quadratic objective function, where rapid fluctuations in convergence patterns might hinder progress. The hybrid methods' ability to maintain a consistent trajectory towards the optimal solution underscores their resilience in handling the intricacies introduced by specific problem characteristics. In summary, the algorithmic performance on the G01 problem intricately relates to its unique features. The hybrid methods showcase adaptability and versatility in addressing constraints, demonstrating a keen understanding of the problem structure. The nuanced convergence patterns, strategic switching mechanisms, and sensitivity to problem-specific attributes provide valuable insights into the dynamic interplay between algorithmic behavior and the intricacies of the optimization problem.

Table 4.3 Statistical results of different methods on G1 problem (The best performing method (objective function values) is marked in bold)

Algorithm	BU step	Best	Mean	Median	Worst	St. Dev.
GA	None ²	-13.83	-12.87	-13.07	-11.99	0.68
	BU	-14.43	-13.94	-14.17	-13.02	0.48
	Hybrid-cvtol	-15.0	-14.87	-14.95	-14.58	0.15
	Hybrid-ftol	-15.0	-14.99	-14.99	-14.99	0.00
DE	None	-10.08	-8.83	-8.74	-8.10	0.66
	BU	-14.02	-12.61	-13.46	-9.55	1.62
	Hybrid-cvtol	-14.80	-14.718	-14.710	-14.66	0.047
	Hybrid-ftol	-14.35	-14.02	-14.05	-13.68	0.21
ES	None	-10.75	-9.24	-9.38	-8.13	0.89
	BU	-10.89	-9.89	-9.99	-8.98	0.65
	Hybrid-cvtol	-13.41	-10.47	-9.84	-8.63	1.63
	Hybrid-ftol	-10.77	-9.92	-10.0	-8.98	0.57
SRES	None	-7.40	-5.68	-5.85	-3.63	1.24
	BU	-9.05	-8.27	-8.18	-1.04	0.44
	Hybrid-cvtol	-7.96	-5.10	-5.04	-1.04	1.68
	Hybrid-ftol	-8.38	-5.51	-4.54	-3.27	1.84
BRKGA	None	-13.83	-12.46	-12.20	-11.30	0.89
	BU	-14.43	-13.56	-13.85	-11.54	1.03
	Hybrid-cvtol	-14.90	-14.88	-14.88	-14.86	0.01
	Hybrid2-ftol	-14.93	-14.87	-14.88	-14.85	0.02

Table 4.4 Summary of the p-value of the Friedman rank test over all runs

Algorithm	p-value ³	statistic value
GA	0.002	14.62
DE	0.002	14.03
ES	0.06	7.28
SRES	0.25	4.07
BRKGA	0.005	12.59

Table 4.5 Summary of the p-value of the posthoc_nemenyi_friedman over all runs ⁴

Algorithm	Hybrid-cvtol vs. BU		Hybrid-cvtol vs. Without BU		BU vs. Without BU	
	p-value	h	p-value	h	p-value	h
GA	0.45	~	0.03	+	0.59	~
DE	0.12	~	0.001	+	0.45	~
ES	0.90	~	0.25	~	0.15	~

² "None" means no BU method or hybrid is used and only feasibility rules as an explicit constraint handling method is worked alone.

³ All statistical test in this study was performed at a significant level of 5%.

⁴ Posthoc_nemenyi_friedman: +, ~, and – presents the first method performs statistically significantly better, equal, and worse than the second approach.

SRES	0.22	~	0.59	~	0.90	~
BRKGA	0.12	~	0.01	+	0.87	+

Table 4.6 Summary of the p-value of the posthoc_nemenyi_friedman over all runs (continue)

Algorithm	Hybrid-ftol vs. BU		Hybrid-ftol vs. Without BU		Hybrid-ftol vs. Hybrid-cvtol	
	p-value	h	p-value	h	p-value	h
GA	0.12	~	0.003	~	0.87	~
DE	0.87	~	0.12	~	0.45	~
ES	0.90	~	0.06	~	0.90	~
SRES	0.59	~	0.90	~	0.90	~
BRKGA	0.20	~	0.03	+	0.90	~

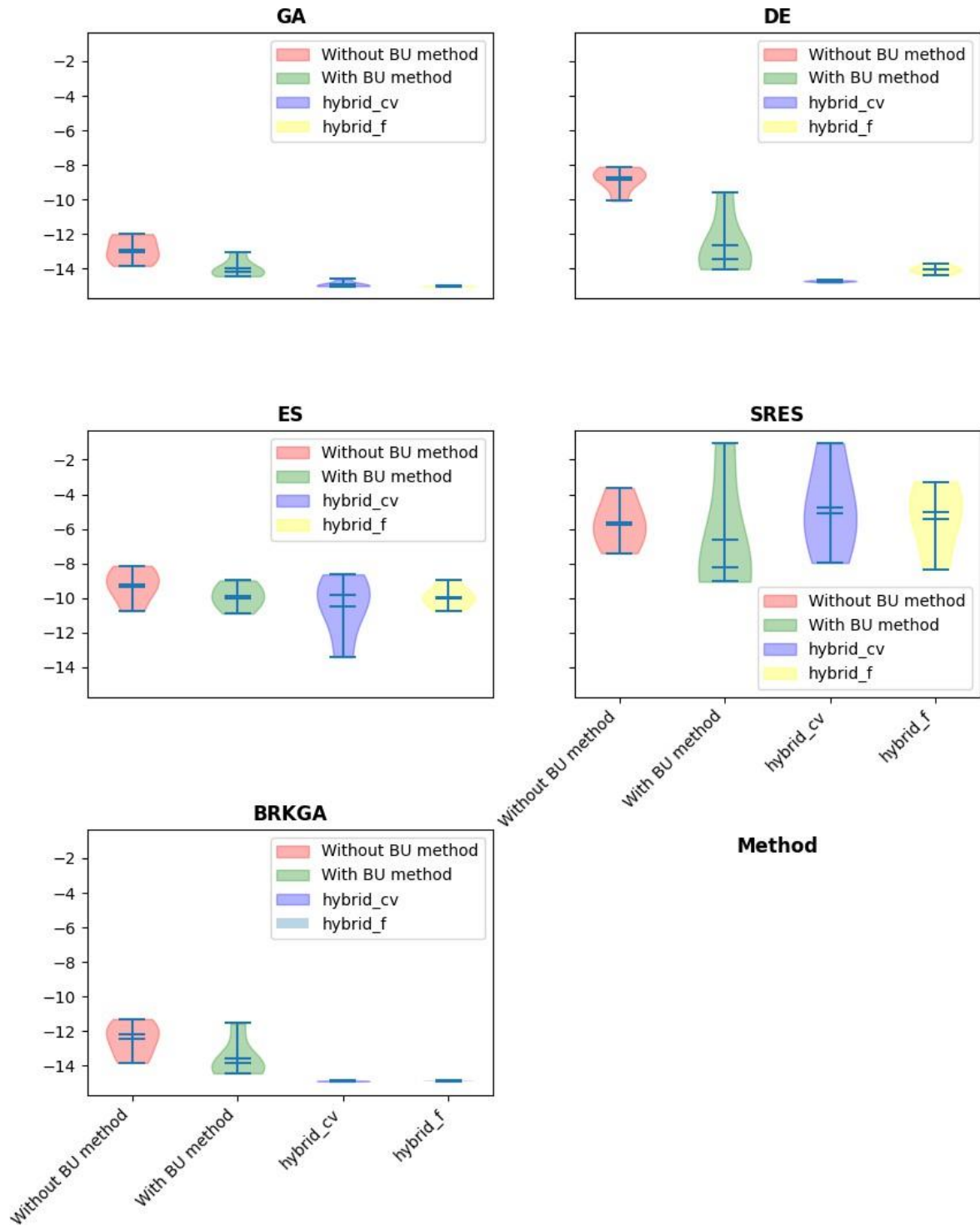


Figure 4.4 Violin plot of implementation of using different algorithms constraint handling methods (G01 problem)

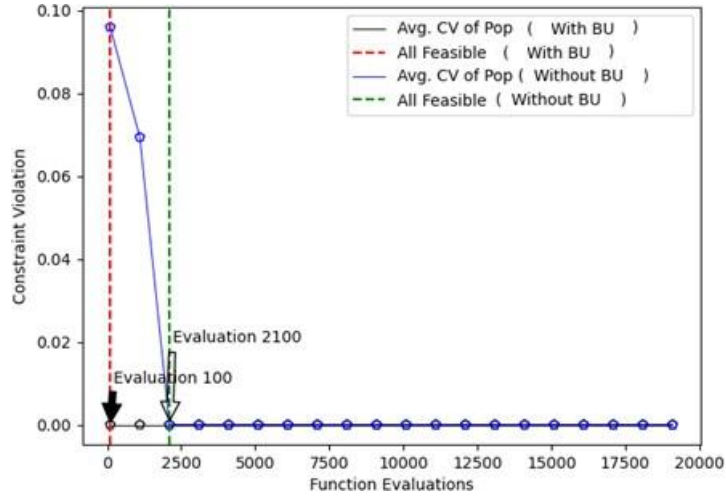


Figure 4.5 Constraint violation implemented by GA on the G01 test problem

4.1.2 Car-side problem

The car-side impact design problem, a notorious challenge in engineering, stands out as a complex and time-consuming optimization task, as depicted in Figure 6. This problem, adaptable to both single-objective and multi-objective formulations, adds layers of intricacy due to its quadratic objective function and explicit models. Specifically, when addressing the single-objective version with the goal of minimizing car weight, the constraints and objective function take on quadratic forms, amplifying the intricacies involved. The thicknesses of the inner floor side (X_3) are strategically selected as decision variables to solve the explicit models, and their bounds are dynamically adjusted to accommodate the interconnected variables, adding an additional layer of complexity.

The results of this formidable optimization challenge are presented in Tables 4.7-4.10 and Figure 4.7, offering a quantitative assessment of algorithmic performance. Statistical tests (Tables 4.8-4.10) and visualizations in Figure 4.7 highlight the superiority of the hybrid methods, particularly when combined with the genetic algorithm (GA). This dominance is

established through the comparative analysis, showcasing that the hybrid approach consistently outperforms both the individual methods and, notably, requires fewer function evaluations (FE) to attain the final solution.

Significantly, the examination of the results reveals nuanced differences between the differential evolution (DE) and evolutionary strategy (ES) methods, with the hybrid method showcasing slightly superior outcomes. The incorporation of the BU method, along with the hybrid methods, demonstrates a tangible advantage, resulting in enhanced solution quality. The visual representation of the switching point for GA in Figure 8 provides further clarity on the optimization process. The algorithm strategically transitions from the BU-assisted phase to the standard optimization phase, showcasing an adaptive mechanism designed to optimize the convergence path. However, the study identifies limitations within the context of the car-side impact design problem. While GA, BU, and hybrid methods exhibit notable improvements, the performance of biased random key genetic algorithm (BRKGA) and stochastic ranking evolution strategy (SRES) fails to show enhancement when incorporated into the hybrid framework. This suggests that BRKGA and SRES may not be suitable for addressing the intricacies of this particular problem, underscoring the need for algorithmic adaptability in the face of diverse engineering challenges. In conclusion, the car-side impact design problem proves to be a formidable optimization challenge, and the hybrid methods, particularly in conjunction with GA, emerge as the most effective approach. The adaptability of the hybrid methods, strategic switching mechanisms, and the incorporation of the BU method collectively contribute to enhanced solution quality and computational efficiency. Nonetheless, the study sheds light on

the context-specific nature of algorithmic suitability, emphasizing the importance of tailoring optimization strategies to the unique characteristics of complex engineering problems.

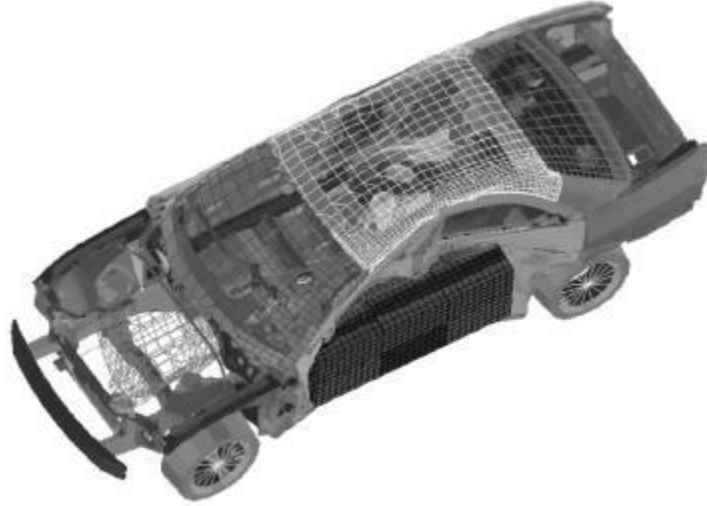


Figure 4.6 Schematic of car side impact design problem (Youn & Choi, 2004)

Table 4.7 Statistical results of different methods on car side design impact problem

Algorithm	BU step	Best	Mean	Median	Worst	St. Dev.
GA	None ⁵	23.889	24.474	24.246	25.295	0.486
	BU	23.538	23.558	23.563	23.568	0.011
	Hybrid-cvtol	23.522	23.551	23.557	23.566	0.015
	Hybrid-ftol	23.566	23.714	23.602	24.189	0.238
DE	None	21.920	22.370	22.142	24.389	0.661
	BU	20.797	21.842	21.786	23.128	0.696
	Hybrid-cvtol	20.750	21.784	21.782	23.067	0.618
	Hybrid-ftol	21.318	21.714	23.602	22.556	0.238
ES	None	20.432	20.770	20.815	21.168	0.251
	BU	20.147	20.453	20.540	20.585	0.161
	Hybrid-cvtol	20.147	20.584	20.434	21.386	0.422
	Hybrid-ftol	20.446	20.746	20.727	21.064	0.234
SRES	None	20.518	20.673	20.728	20.760	0.092
	BU	20.246	20.424	20.436	20.528	0.101
	Hybrid-cvtol	20.249	20.425	20.436	20.528	0.100
	Hybrid-ftol	20.395	20.477	20.432	20.580	0.077
BRKGA	None	18.165	18.904	18.90	19.572	0.493
	BU	20.088	20.246	20.202	20.590	0.178
	Hybrid-cvtol	19.631	20.082	20.124	20.639	0.346
	Hybrid-ftol	19.359	20.127	20.20	20.534	0.411

⁵ “None” means no BU method or hybrid is used and only feasibility rules as an explicit constraint handling method is worked alone.

Table 4.8 Summary of p value of the Friedman rank test over all runs

Algorithm	p-value	statistic value
GA	0.001	15.00
DE	0.025	9.339
ES	0.162	5.125
SRES	0.0175	10.125
BRKGA	0.020	9.734

Table 4.9 Summary of the p -value of the posthoc_nemenyi_friedman over all runs

Algorithm	Hybrid-cvtol vs. BU		Hybrid-cvtol vs. Without BU		BU vs. Without BU	
	p-value	h	p-value	h	p-value	h
GA	0.59	~	0.001	+	0.06	+
DE	0.72	~	0.02	+	0.26	~
ES	0.90	~	0.31	~	0.20	~
SRES	0.87	~	0.20	~	0.03	+
BRKGA	0.87	~	0.15		0.02	+

Table 4.10 Summary of the p -value of the posthoc_nemenyi_friedman over all runs (continue)

Algorithm	Hybrid-ftol vs. BU		Hybrid-ftol vs. Without BU		Hybrid-ftol vs. Hybrid-cvtol	
	p-value	h	p-value	h	p-value	h
GA	0.59	~	0.59	~	0.06	~
DE	0.90	~	0.09	~	0.90	~
ES	0.59	~	0.87	~	0.73	~
SRES	0.90	~	0.03	+	0.87	~
BRKGA	0.90	~	0.06	~	0.90	~

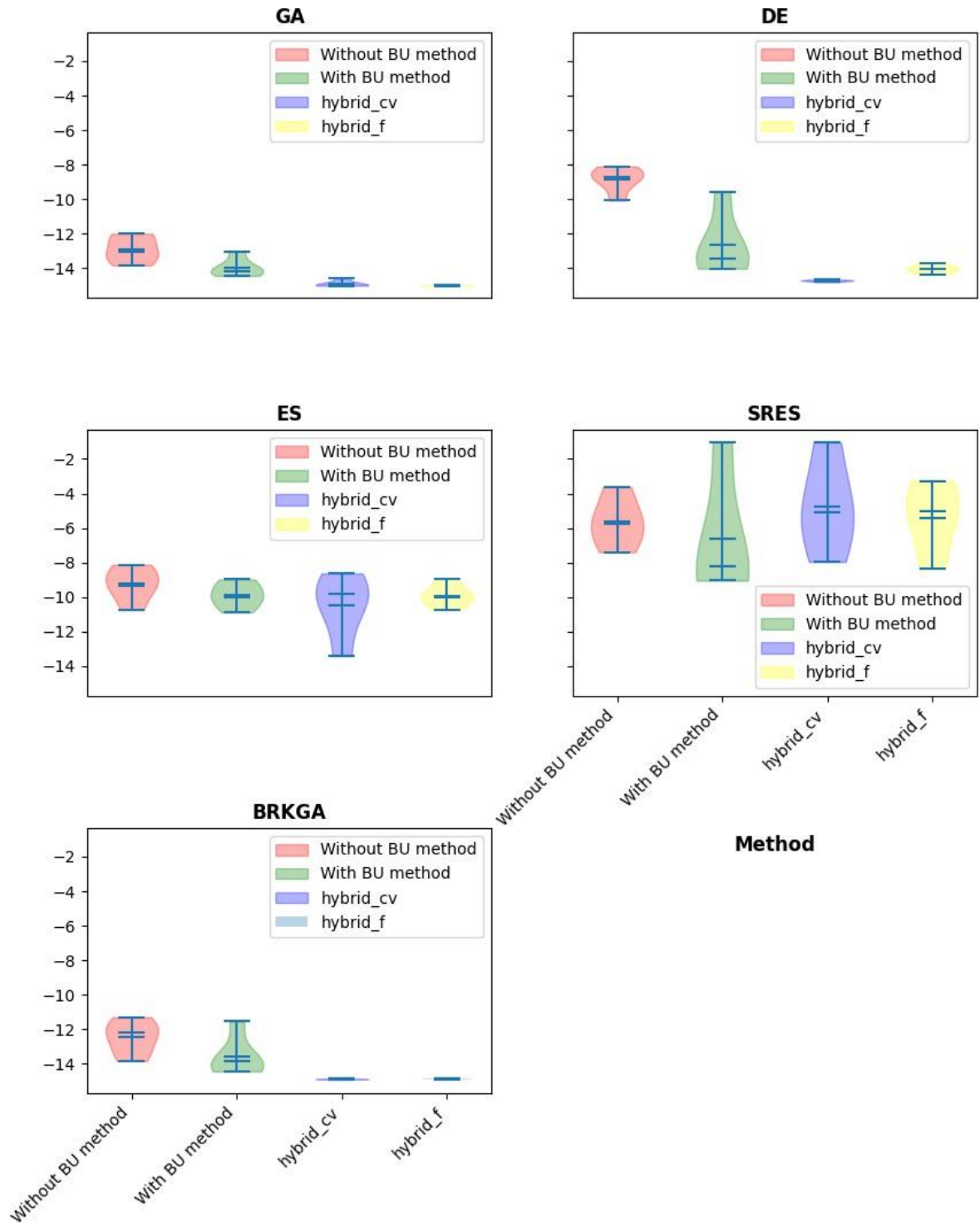


Figure 4.7 Violin plot of implementation of using different algorithms constraint handling methods (Carside problem)

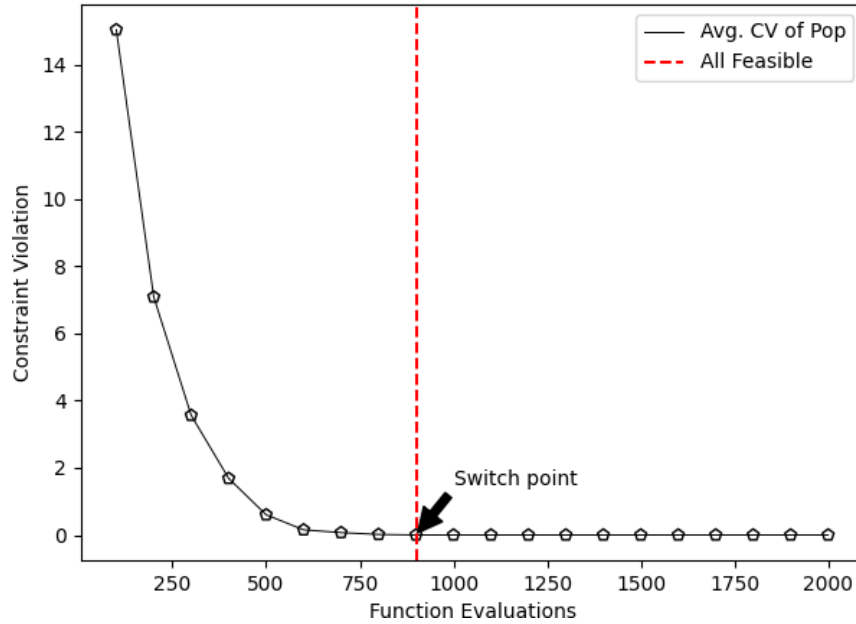


Figure 4.8 Switch point for GA to normal optimization without the BU method

4.1.3 Speed Reducer

The examination of the speed reducer design problem, featuring one objective function and seven continuous decision variables, has yielded multifaceted insights into the interplay between algorithmic approaches and problem-specific characteristics. With the overarching goal of minimizing the total weight of the speed reducer, the complexity of this task is compounded by a combination of linear and nonlinear constraints. Strategically leveraging three repairing variables—specifically, the face width and shaft diameters (x_1 , x_6 , and x_7)—proves instrumental in managing these constraints effectively.

The results, meticulously presented in Tables 4.11-4.14 and Figure 4.9, reveal a consistent trend: the hybrid approaches consistently outperform standalone methods, yielding the best objective function values across all algorithms considered. This dominance suggests that the hybridization process

contributes to algorithmic robustness, offering a versatile strategy to navigate the intricate optimization landscape presented by the speed reducer design problem.

Notably, the algorithms (GA, DE, and BRKGA) exhibit improved performance when integrated into hybrid frameworks, signifying a synergistic effect where the strengths of individual algorithms complement one another. The statistical tests conducted (Tables 4.12-4.14) affirm the significance of these improvements, providing a robust validation of the observed enhancements and instilling confidence in the findings.

A closer examination of GA within the hybrid methods reveals its particular suitability for this problem, consistently outperforming standalone and BU-assisted methods. This positions GA as a robust choice for addressing the complexities inherent in the optimization landscape of the speed reducer design problem.

The incorporation of the BU method consistently leads to a substantial reduction in the number of function evaluations required to reach the final solution. This reduction in computational cost is a valuable contribution, emphasizing the efficiency gains afforded by the BU method, a crucial consideration in engineering optimization scenarios.

Despite the positive outcomes, the study highlights that algorithms like BRKGA and SRES do not exhibit improved performance within the hybrid framework, suggesting their unsuitability for this particular problem. This observation underscores the importance of algorithmic adaptability and tailoring strategies to specific problem characteristics.

In conclusion, the insights gained from the speed reducer design problem analysis underscore the effectiveness of hybrid approaches in enhancing algorithmic performance. The adaptability of GA, efficiency gains with the BU method, and the nuanced trade-offs between algorithms offer valuable

guidance for tackling real-world engineering optimization challenges. These findings contribute not only to the understanding of algorithmic behavior in this specific context but also pave the way for broader applications in engineering design optimization.

Table 4.11 Statistical results of different methods on speed reducer problem

Algorithm	BU step	Best	Mean	Median	Worst	St. Dev.
GA	None ⁶	2997.353	2998.827	2998.933	2999.647	0.645
	BU	3119.764	3126.639	3127.78	3129.697	2.792
	Hybrid-cvtol	2997.002	2997.964	2998.023	2998.779	0.539
	Hybrid-ftol	2996.371	2996.422	2996.404	2996.542	0.055
DE	None	2847.208	2982.631	2996.641	2997.054	42.848
	BU	2706.172	2900.272	2881.668	3131.199	157.048
	Hybrid-cvtol	2706.141	2838.565	2828.955	2996.729	108.803
	Hybrid-ftol	2718.692	2996.422	2996.404	2742.44	0.055
ES	None	3001.0	3023.363	3025.0	3039.00	12.694
	BU	3120.0	3123.727	3123.0	3129.00	2.561
	Hybrid-cvtol	3007.0	3018.000	3018.0	3030.00	6.281
	Hybrid-ftol	3005	3015.428	3015	3028.00	6.694
SRES	None	3012.0	3031.272	3025.0	3063.00	17.669
	BU	3108.0	3123.181	3125.0	3128.00	5.236
	Hybrid-cvtol	3015.0	3039.000	3038.0	3076.00	17.832
	Hybrid-ftol	3108	3122.57	3125	3128	6.298
BRKGA	None	2997.996	3001.692	3001.084	3006.936	2.761
	BU	3117.466	3123.452	3123.855	3127.798	3.386
	Hybrid-cvtol	2997.312	3000.977	3000.109	3006.877	3.254
	Hybrid-ftol	2875.699	2946.35	2951.593	3036.718	56.361

Table 4.12 Summary of the p-value of the Friedman rank test over all runs

Algorithm	p-value	statistic value
GA	0.0001	21.0
DE	0.0018	15.000
ES	0.0031	13.799
SRES	0.0003	18.71
BRKGA	0.0031	13.799

⁶ "None" means no BU method or hybrid is used and only feasibility rules as an explicit constraint handling method is worked alone.

Table 4.13 Summary of the p-value of the posthoc_nemenyi_friedman over all runs

Algorithm	Hybrid-cvtol vs. BU		Hybrid-cvtol vs. Without BU		BU vs. Without BU	
	p-value	h	p-value	h	p-value	h
GA	0.019	+	0.46	~	0.46	~
DE	0.29	~	0.06	~	0.87	~
ES	0.06	~	0.90	~	0.03	~
SRES	0.014	+	0.90	~	0.026	+
BRKGA	0.03	+	0.90	~	0.06	~

Table 4.14 Summary of the p-value of the posthoc_nemenyi_friedman over all runs (continue)

Algorithm	Hybrid-ftol vs. BU		Hybrid-ftol vs. Without BU		Hybrid-ftol vs. Hybrid-cvtol	
	p-value	h	p-value	h	p-value	h
GA	0.001	+	0.019	+	0.46	~
DE	0.03	+	0.003	+	0.76	~
ES	0.002	+	0.82	~	0.70	~
SRES	0.90	~	0.02	+	0.014	+
BRKGA	0.002	+	0.70	~	0.82	~

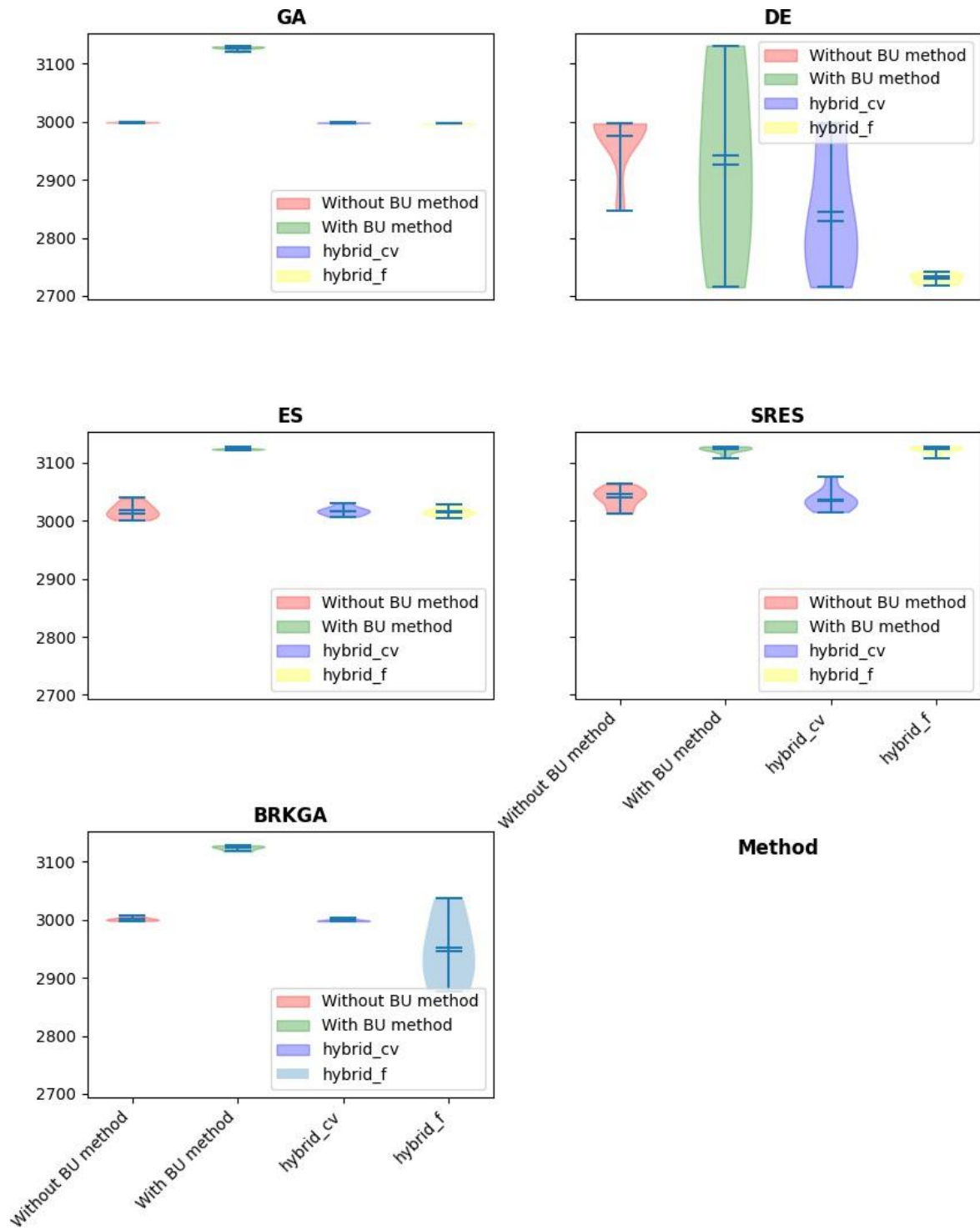


Figure 4.9 Violin plot of implementation of using different algorithms constraint handling methods (speed reducer problem)

4.2 Multi-objective optimization problems

The following subsections provide two benchmarks and two real-world optimization problems to assess the effectiveness of the proposed hybrid method for multi-objective optimization problems.

4.2.1 Osyczka and Kundu (OSY)

Osyczka and Kundu (Osyczka & Kundu, 1995) introduced the OSY test problem, a widely recognized benchmark featuring two nonlinear objective functions and six linear and nonlinear constraints. As highlighted earlier, the BU method is a central focus of this work, particularly in its ability to directly handle constraints and its subsequent integration with other explicit constraint-handling techniques. The BU approach strategically reduces the feasible search space by altering one or more variables, compelling the algorithm to concentrate on exploring regions that adhere to the defined constraints. For the OSY problem, a specific repairing variable selection strategy is employed, targeting variables x_1 , x_4 , and x_6 identified as effective for handling all constraints inherent in the problem.

Following the application of the BU approach, NSGA-II is employed to navigate the complex landscape of the OSY problem. NSGA-II, known for identifying viable Pareto fronts, initially achieves an entirely feasible population after 1500 evaluations. In contrast, the coupling of the BU method with the algorithm accelerates this process, achieving a fully feasible population after only 300 evaluations. Subsequently, the optimization process seamlessly transitions to routine optimization without the continued use of the BU method.

The evaluation of performance indicators, presented in Tables 4.15-4.18 and Figure 4.10, offers a comprehensive understanding of the algorithmic performance. It becomes evident that the hybrid methods, particularly when incorporating the BU approach, consistently outperform other methods.

Statistical analyses, including the Friedman rank and posthoc tests, validate this observation by demonstrating a significant improvement in all performance indicators (excluding IGD+) when the hybrid approach is implemented. Examining Figure 10 through the lens of a violin plot further illustrates the superiority of the hybrid methods. The best values for all performance indicators, with the exception of IGD, show marked improvement when the BU and hybrid methods are employed. This enhancement in performance underscores the effectiveness of the BU method in combination with the hybrid approach, providing valuable insights into the algorithmic strategies that lead to improved convergence and solution quality for the OSY problem. In summary, the OSY problem, coupled with the BU method and hybrid approach, serves as a compelling case study. The specific variable selection strategy, the seamless transition between BU-assisted and routine optimization, and the consistent improvement in performance indicators highlight the adaptability and effectiveness of the proposed approach. These findings contribute not only to the optimization of the OSY problem but also offer broader insights into the potential of the BU method and hybridization in addressing complex, constrained optimization scenarios.

Table 4.15 Statistical results of different methods on the OSY problem

Performance Indicator	BU step	Best	Mean	Median	Worst	St. Dev.
IGD	None ⁷	0.354	0.652	0.697	0.727	0.12
	BU	0.680	0.712	0.698	0.772	0.032
	Hybrid-cvtol	0.636	0.655	0.651	0.677	0.014
	Hybrid-ftol	0.412	0.484	0.499	0.503	0.030
IGD+	None	0.504	0.520	0.526	0.549	0.013
	BU	0.503	0.573	0.506	0.754	0.107
	Hybrid-cvtol	0.491	0.495	0.495	0.500	0.003
	Hybrid-ftol	0.400	0.433	0.439	0.439	0.013
GD	None	0.031	0.042	0.046	0.050	0.007
	BU	0.002	0.114	0.007	0.403	0.17
	Hybrid-cvtol	0.006	0.009	0.010	0.013	0.002
	Hybrid-ftol	0.032	0.057	0.038	0.176	0.048
GD+	None	0.022	0.034	0.037	0.043	0.006
	BU	0.000	0.113	0.006	0.403	0.172

⁷ "None" means no BU method or hybrid is used and only feasibility rules as an explicit constraint handling method is worked alone.

Hybrid-cvtol	0.001	0.004	0.005	0.009	0.002
Hybrid-ftol	0.030	0.055	0.037	0.175	0.048

Table 4.16 Summary of the p-value of the Friedman rank test over all runs

Performance Indicator	p-value	statistic value
IGD	0.0006	17.22
IGD+	0.0003	18.44
GD	0.0005	17.38
GD+	0.0006	17.26

Table 4.17 Summary of the p-value of the posthoc_nemenyi_friedman over all runs

Performance Indicator	Hybrid-cvtol vs. BU		Hybrid-cvtol vs. Without BU		BU vs. Without BU	
	p-value	h	p-value	h	p-value	h
IGD	0.03	+	0.005	+	0.90	~
IGD+	0.10	~	0.005	+	0.70	~
GD	0.019	+	0.010	+	0.90	~
GD+	0.04	+	0.026	+	0.90	~

Table 4.18 Summary of the p-value of the posthoc_nemenyi_friedman over all runs (continue)

Performance Indicator	Hybrid-ftol vs. BU		Hybrid-ftol vs. Without BU		Hybrid-ftol vs. Hybrid-cvtol	
	p-value	h	p-value	h	p-value	h
IGD	0.06	~	0.01	+	0.90	~
IGD+	0.06	~	0.002	+	0.90	~
GD	0.035	+	0.019	+	0.90	~
GD+	0.014	+	0.007	+	0.90	~

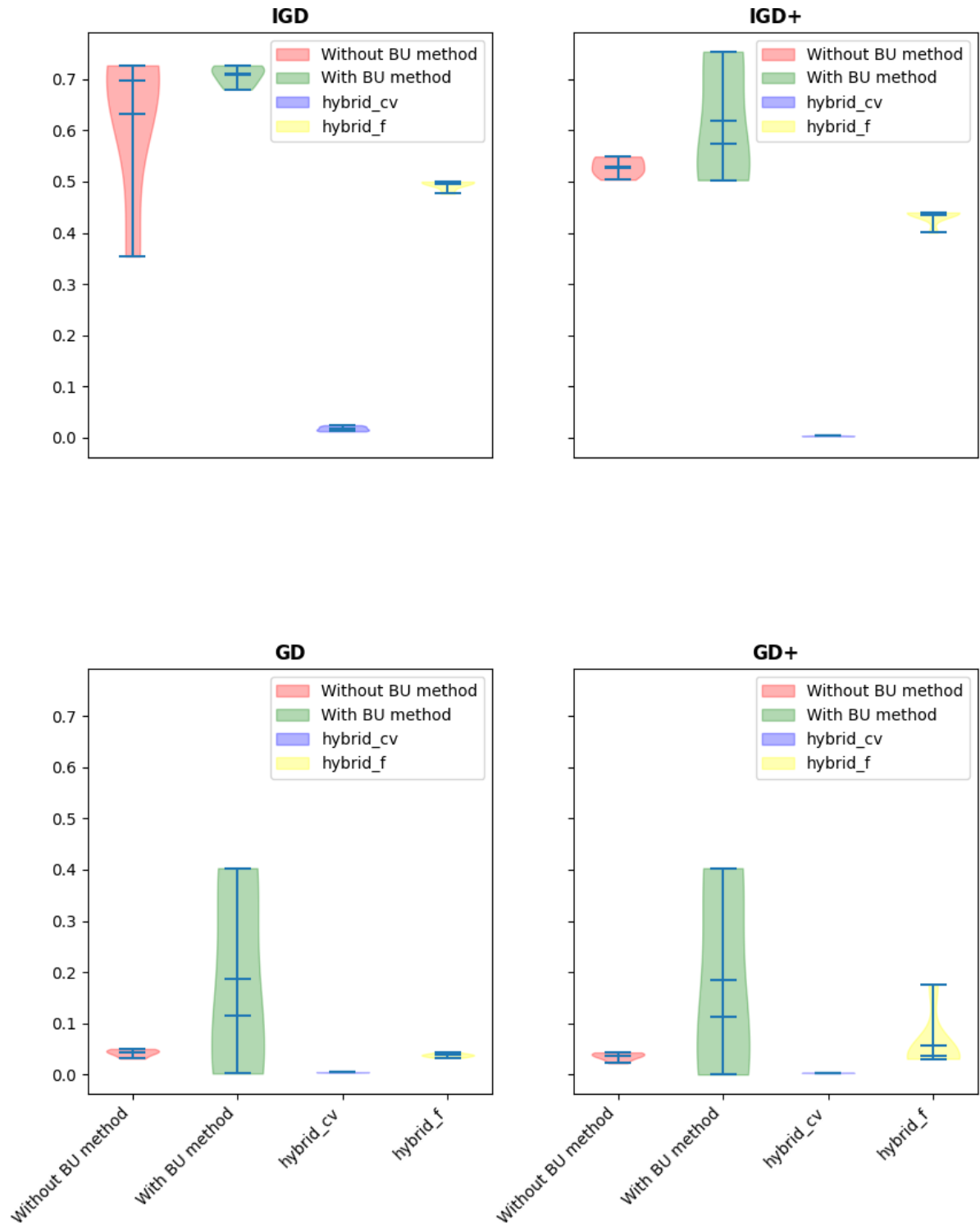


Figure 4.10 Violin plot of implementation of using different indicators (OSY problem)

4.2.3 Bin and Korn Test Problem (BNH)

BNH is a test problem proposed by Binh and Korn (Binh & Korn, 1997) , characterized by two conflicting objectives and two constraints, introduces a challenging landscape for optimization. The nonlinear nature of the objectives and constraints adds an additional layer of complexity, requiring sophisticated algorithmic strategies to navigate the intricate solution space effectively. The objectives and constraints may exhibit intricate relationships, posing a considerable challenge to traditional optimization approaches.

In addressing this nonlinear bi-objective optimization problem, the algorithmic focus will be on developing robust strategies that balance the optimization of both objectives while adhering to the imposed constraints. The inherent trade-offs between conflicting objectives necessitate a careful exploration of the Pareto front to identify solutions that represent optimal compromises. This problem involves two continuous variables, x_1 and x_2 , which can be strategically chosen as repairing variables. The findings presented in Tables 4.19-4.25 and Figure 4.11 illuminate a substantial disparity between the hybrid method and alternative approaches. Particularly noteworthy is the superior performance of the hybrid method, especially when objective function tolerance is considered, showcasing noteworthy enhancements over other methods across various cases. The results unequivocally demonstrate the hybrid approaches' consistent outperformance in tackling this bi-objective optimization problem characterized by nonlinear constraints and objective functions. Furthermore, the performance indicator values associated with the hybrid approaches are significantly smaller compared to their counterparts, underscoring the effectiveness of the proposed methodology in achieving superior convergence and solution quality. These insights not only contribute to the optimization of the BNH problem but also offer valuable considerations for addressing the nuances of nonlinear bi-objective optimization challenges across diverse applications.

Table 4.19 Statistical results of different methods on the BNH problem

Performance Indicator	BU step	Best	Mean	Median	Worst	St. Dev.
IGD	None ⁸	0.0423	0.0803	0.0834	0.1281	0.0253
	BU	0.0255	0.0535	0.0426	0.0896	0.0272
	Hybrid-cvtol	0.0123	0.0289	0.0209	0.0660	0.0172
	Hybrid-ftol	0.0110	0.0183	0.0200	0.0222	0.0038
IGD+	None	0.0170	0.0268	0.0247	0.0433	0.0084
	BU	0.0112	0.0222	0.0179	0.0419	0.0111
	Hybrid-cvtol	0.0024	0.0061	0.0048	0.0143	0.0040
	Hybrid-ftol	0.0029	0.0035	0.0033	0.0043	0.0005
GD	None	0.0045	0.0052	0.0050	0.0065	0.0006
	BU	0.0061	0.0073	0.0066	0.0098	0.0013
	Hybrid-cvtol	0.0044	0.0046	0.0046	0.0049	0.0001
	Hybrid-ftol	0.0040	0.0044	0.0046	0.0047	0.0002
GD+	None	0.0029	0.0034	0.0033	0.0048	0.0005
	BU	0.0039	0.0053	0.0049	0.0079	0.0013
	Hybrid-cvtol	0.0026	0.0027	0.0027	0.0029	0.00009
	Hybrid-ftol	0.0020	0.0028	0.0030	0.0031	0.0003

Table 4.20 Summary of the p-value of the Friedman rank test over all runs

Performance Indicator	p-value	statistic value
IGD	0.0006	17.22
IGD+	0.0003	18.44
GD	0.0005	17.38
GD+	0.0006	17.26

Table 4.21 Summary of the p-value of the posthoc_nemenyi_friedman over all runs

Performance Indicator	Hybrid-cvtol vs. BU		Hybrid-cvtol vs. Without BU		BU vs. Without BU	
	p-value	h ⁹	p-value	h	p-value	h
IGD	0.244362	~	0.003837	+	0.244362	~
IGD+	0.244362	~	0.003837	+	0.244362	~
GD	0.001488	+	0.376245	~	0.082380	~
GD+	0.001000	+	0.147296	~	0.147296	~

⁸ “None” means no BU method or hybrid is used and only feasibility rules as an explicit constraint handling method is worked alone.

⁹ Posthoc_nemenyi_friedman: +, ~, and – presents the first method performs statistically significantly better, equal, and worse than the second approach.

Table 4.22 Summary of the p-value of the posthoc_nemenyi_friedman over all runs- IGD value comparison

Method	Hybrid-cvtol	Hybrid-ftol	BU	Without BU
Hybrid-cvtol	-	0.9000	0.0358	0.0051
Hybrid-ftol	0.9000	-	0.06243	0.01026
BU	0.0358	0.06243	-	0.9000
Without BU	0.0051	0.01026	0.9000	-

Table 4.23 Summary of the p-value of the posthoc_nemenyi_friedman over all runs- IGD+ value comparison

Method	Hybrid-cvtol	Hybrid-ftol	BU	Without BU
Hybrid-cvtol	-	0.900000	0.103359	0.005121
Hybrid-ftol	0.900000	-	0.062433	0.002444
BU	0.103359	0.062433	-	0.704147
Without BU	0.005121	0.002444	0.704147	-

Table 4.24 Summary of the p-value of the posthoc_nemenyi_friedman over all runs- GD value comparison

Method	Hybrid-cvtol	Hybrid-ftol	BU	Without BU
Hybrid-cvtol	-	0.9000	0.019641	0.010266
Hybrid-ftol	0.9000	-	0.035858	0.019641
BU	0.019641	0.035858	-	0.9000
Without BU	0.010266	0.019641	0.9000	-

Table 4.25 Summary of the p-value of the posthoc_nemenyi_friedman over all runs- GD+ value comparison

Method	Hybrid-cvtol	Hybrid-ftol	BU	Without BU
Hybrid-cvtol	-	0.9000	0.047574	0.026690
Hybrid-ftol	0.9000	-	0.014282	0.007293
BU	0.047574	0.014282	-	0.9000
Without BU	0.026690	0.007293	0.9000	-

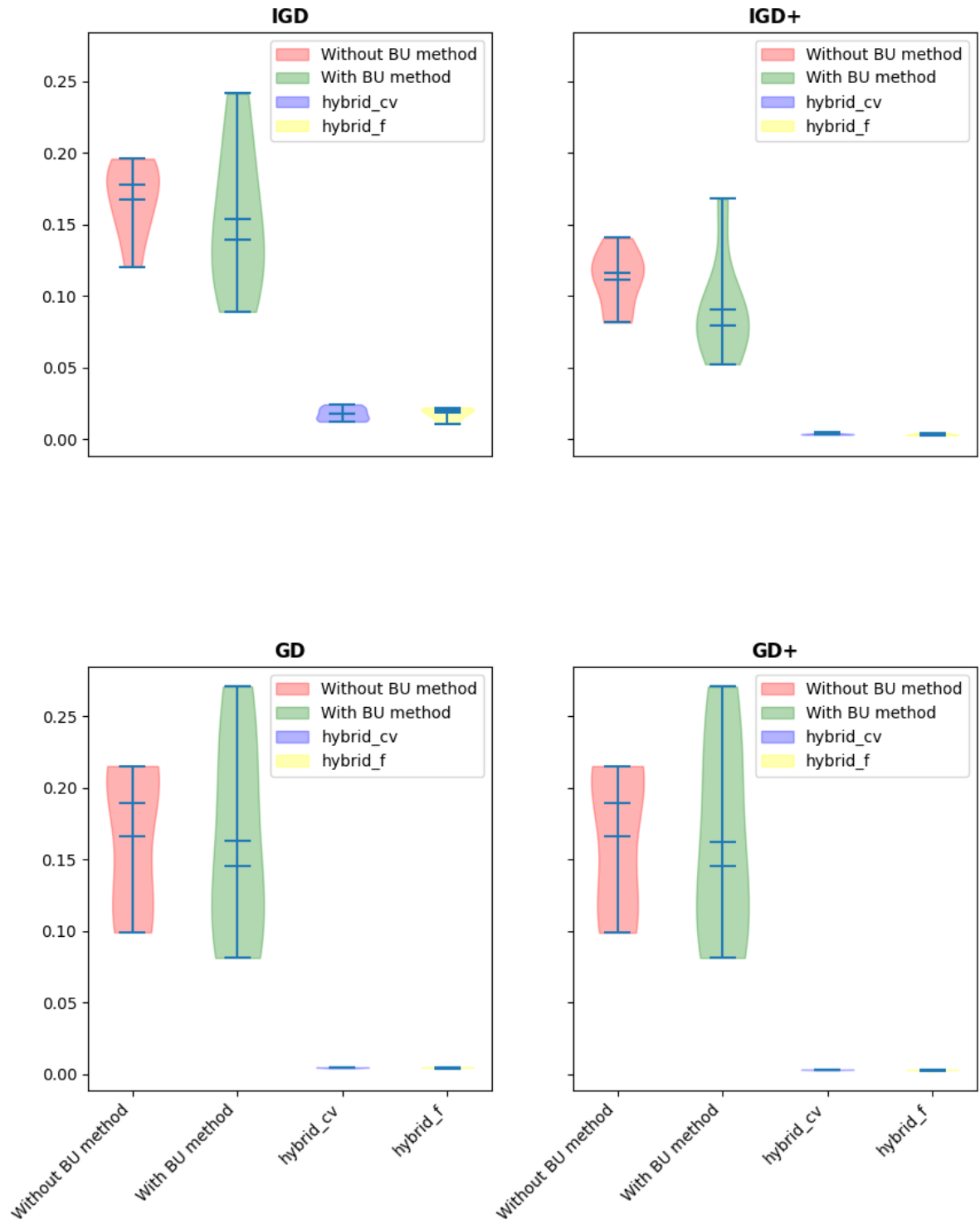


Figure 4.11 Violin plot of implementation of using different indicators (BNH problem)

4.2.4 Welded beam design problem

The welded beam fabrication is a well-known engineering optimization benchmark problem (Deb & Sundar, 2006) that examines the trade-off between the strength and cost of a beam. The problem includes four constraints (Eqs. 32-35) and four design variables, which are:

- $x_1 = h$, the thickness of the welds
- $x_2 = l$, the length of the welds
- $x_3 = t$, the height of the beam
- $x_4 = b$, the width of the beam

The constraints are:

$$g_1 = \left(\frac{1}{t_{max}}\right) \times (t - t_{max}) \leq 0 \quad (32)$$

$$g_2 = \left(\frac{1}{s_{max}}\right) \times (s - s_{max}) \leq 0 \quad (33)$$

$$g_3 = \left(\frac{1}{5 - 0.125}\right) \times (x_1 - x_4) \leq 0 \quad (34)$$

$$g_4 = \left(\frac{1}{P}\right) \times (P - P_c) \leq 0 \quad (35)$$

where $t_{max} = 13600, s_{max} = 30000, P = 6000, P_c = 64746.022 \times (1 - 0.0282346 \times x_3) \times x_3 \times x_4^3, s = 6 \times P \times 14 / (x_4 \times x_3^2)$. The two objective functions are to minimize the fabrication cost

of the beam and minimize the deflection of the end of the beam:

$$f_1 = 1.10471 \times x_1^2 \times x_2 + 0.04811 \times x_3 \times x_4 \times (14.0 + x_2) \quad (36)$$

$$f_2 = 2.1952 / (x_4 \times x_3^3) \quad (37)$$

The four variables of this problem include: 1) thickness of the weld, 2) welded joint's length, 3) beam's width, and 4) beam's thickness. Using the three strategies, NSGA-II was applied to this problem, and fitness values were evaluated.

The exploration of sampling and crossover operators in EAs stands as a critical endeavor, influencing the convergence speed and efficacy of these algorithms, particularly in the context of noisy optimization. In the context of the Welded Beam Design problem, where the impact of noise is significant, the experimentation involved diverse operators, including real random sampling, Latin hypercube sampling, and two distinct crossovers: real-SBX crossover and uniform crossover. Four performance indicators, namely GD, GD+, IGD, and IGD+, were meticulously evaluated against these operators to glean insights into their comparative effectiveness. The compelling findings, detailed in Tables 4.26-4.32 and illustrated in Figure 4.12, underscore the superiority of the hybrid approach over the two alternative methods.

Examining the Pareto optimal solutions generated by the hybrid methods across different generations reveals a consistent outperformance, particularly noteworthy when considering constraint violations as tolerance with real random sampling. This suggests that the hybrid approach, when coupled with appropriate operators, significantly improves the optimization process for the Welded Beam Design problem. Notably, the hybrid method's effectiveness is evident with Latin hypercube sampling, showcasing its compatibility and superior performance within this optimization context. Furthermore, the hybrid method incorporating objective function tolerance using uniform crossover emerges as a robust option, outperforming optimization without the BU approach.

In contrast to initial expectations regarding the BU approach as a constraint handling technique, the results indicate that it may not be the optimal choice for this specific nonlinear bi-objective model. However, the hybrid approaches, through strategic integration of various operators, demonstrate a

substantial enhancement in performance. This nuanced observation highlights the importance of selecting appropriate operators within hybrid approaches, suggesting that the effectiveness of the BU method can be context-dependent.

The statistical analyses, including the Friedman rank test and the Posthoc_Nemenyi_Friedman, contribute to the robustness of the findings. These tests not only affirm the significant differences between the hybrid approach and the two other methods but also reinforce the superiority of the proposed hybrid method for solving the Welded Beam Design problem. The identification of the switching point for the second phase of the optimization process at evaluation 300 further exemplifies the efficiency of the hybrid approach compared to the without BU approach, where the feasible region is reached at evaluation 1600 (Figure 4.13).

In conclusion, the investigation into sampling and crossover operators, coupled with the nuanced insights from performance indicators and statistical analyses, positions the hybrid approach as the preferred strategy for solving the Welded Beam Design problem. The adaptability of the hybrid method, evident in its performance across various operators, reinforces its efficacy in addressing the challenges posed by the specific characteristics of this nonlinear bi-objective optimization scenario.

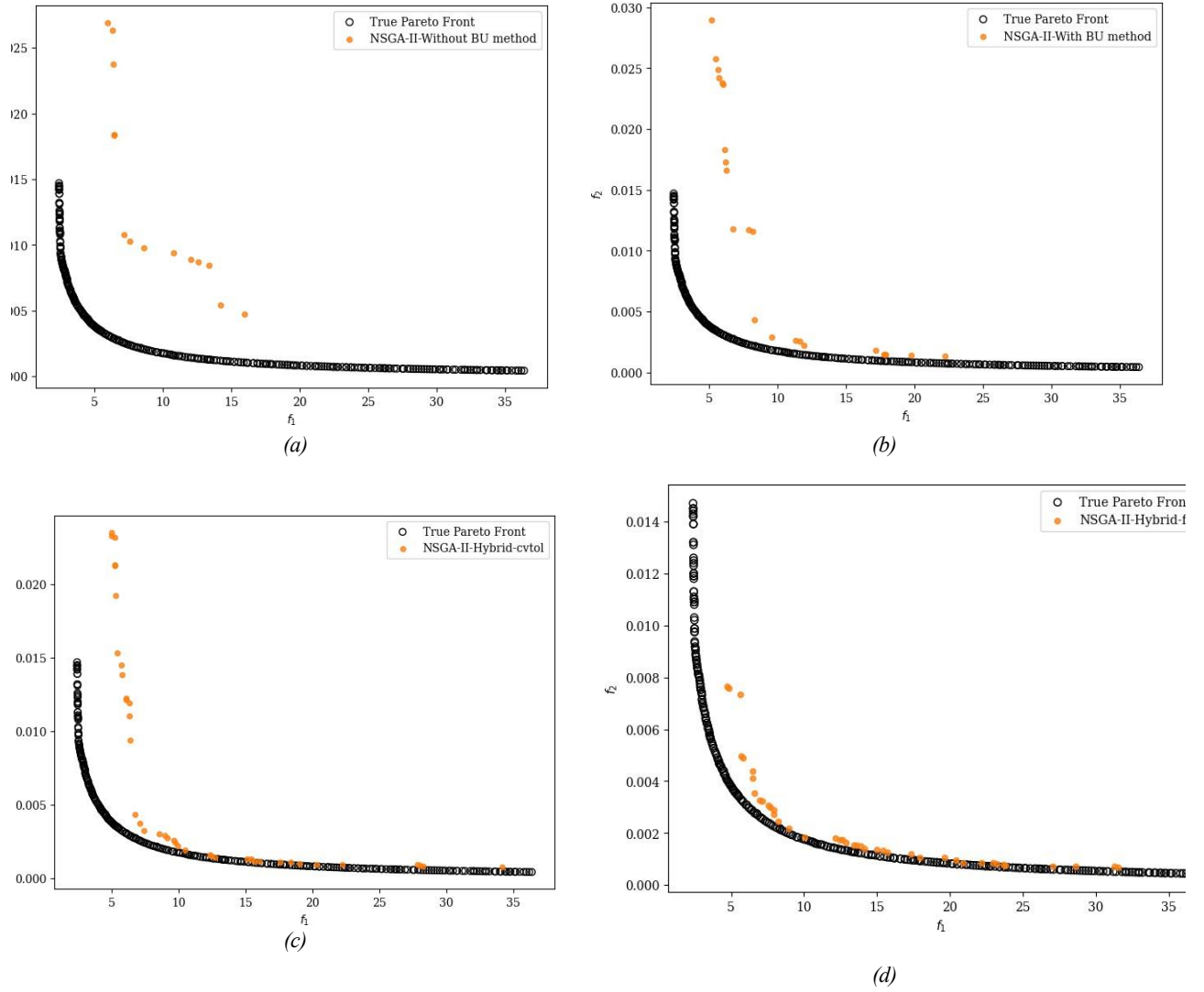


Figure 4.12 Pareto-optimal fronts obtained for the welded beam design problem for both BU and without BU compared with the true Pareto-optimal front (Generation 20)

Table 4.26 Statistical results of different methods on the welded beam design problem (sampling)

Performance Indicator	Sampling											
	Real_lhs						Real_random					
	BU step	Best	Mean	Median	Worst	St. Dev.	BU step	Best	Mean	Median	Worst	St. Dev.
IGD	None ¹⁰	0.12	0.15	0.147	0.179	0.021	None	0.12	0.167	0.178	0.19	0.024
	BU	0.05	0.15	0.161	0.239	0.051	BU	0.08	0.153	0.139	0.24	0.048
	Hybrid-cvtol	0.01	0.03	0.021	0.089	0.028	Hybrid-cvtol	0.01	0.033	0.034	0.05	0.010
	Hybrid-ftol	0.04 4	0.05 8	0.058	0.069	0.009	Hybrid-ftol	0.02 8	0.057	0.061	0.08 4	0.0190
IGD+	None	0.05	0.08	0.071	0.119	0.023	None	0.08	0.111	0.116	0.14	0.019
	BU	0.03	0.08	0.070	0.138	0.034	BU	0.05	0.090	0.079	0.16	0.034
	Hybrid-cvtol	0.00	0.00	0.003	0.016	0.004	Hybrid-cvtol	0.00	0.005	0.004	0.00	0.002
	Hybrid-ftol	0.01 5	0.01 8	0.018	0.020	0.001	Hybrid-ftol	0.01 2	0.018	0.018	0.27	0.004
GD	None	0.05	0.11	0.113	0.196	0.037	None	0.09	0.166	0.189	0.21	0.048
	BU	0.02	0.13	0.128	0.282	0.071	BU	0.08	0.162	0.145	0.27	0.067
	Hybrid-cvtol	0.00	0.00	0.002	0.004	0.000	Hybrid-cvtol	0.00	0.002	0.002	0.00	0.001
	Hybrid-ftol	0.00 4	0.00 8	0.008	0.011	0.002	Hybrid-ftol	0.00 4	0.008	0.005	0.03 0	0.008
GD+	None	0.05	0.11	0.113	0.196	0.037	None	0.09	0.166	0.189	0.21	0.048
	BU	0.02	0.13	0.128	0.282	0.071	BU	0.08	0.162	0.145	0.27	0.067

¹⁰ “None” means no BU method or hybrid is used and only feasibility rules as an explicit constraint handling method is worked alone.

Hybrid -cvtol	0.00	0.00	0.001	0.003	0.000	Hybrid d- cvtol	0.00	0.001	0.001	0.00	0.000
Hybrid -ftol	0.00 1	0.00 7	0.007	0.012	0.004	Hybrid d-ftol	0.00 3	0.008	0.005	0.03 0	0.009

Table 4.27 Statistical results of different methods on the welded beam design problem (crossover)

Performance Indicator	Crossover											
	Uniform						real_sbx					
	BU step	Best	Mean	Median	Worst	St. Dev.	BU step	Best	Mean	Median	Worst	St. Dev.
IGD	None	0.14	0.16	0.16	0.19	0.01	None	0.08	0.14	0.15	0.19	0.03
	BU	0.08	0.12	0.12	0.19	0.03	BU	0.13	0.16	0.16	0.18	0.01
	Hybrid -cvtol	0.00	0.05	0.05	0.09	0.02	Hybrid d- cvtol	0.00	0.03	0.02	0.07	0.02
	Hybrid -ftol	0.03	0.05	0.05	0.06	0.01	Hybrid d-ftol	0.01	0.03	0.03	0.04	0.01
IGD+	None	0.07	0.09	0.09	0.11	0.01	None	0.03	0.07	0.06	0.10	0.02
	BU	0.03	0.07	0.07	0.10	0.02	BU	0.05	0.09	0.08	0.13	0.02
	Hybrid -cvtol	0.00	0.00	0.00	0.01	0.00	Hybrid d- cvtol	0.00	0.00	0.00	0.00	0.00
	Hybrid -ftol	0.00	0.00	0.00	0.01	0.00	Hybrid d-ftol	0.00	0.00	0.00	0.00	0.00
GD	None	0.08	0.15	0.176	0.21	0.04	None	0.03	0.10	0.08	0.18	0.05
	BU	0.04	0.14	0.123	0.31	0.08	BU	0.06	0.19	0.18	0.41	0.09
	Hybrid -cvtol	0.00	0.00	0.00	0.00	0.00	Hybrid d- cvtol	0.00	0.00	0.00	0.00	0.00

	Hybrid -ftol	0.00	0.00	0.00	0.00	0.00	Hybrid d-ftol	0.00	0.00	0.00	0.00	0.00
GD+	None	0.08	0.15	0.17	0.21	0.04	None	0.03	0.10	0.08	0.18	0.05
	BU	0.04	0.14	0.12	0.31	0.08	BU	0.06	0.19	0.18	0.41	0.09
	Hybrid -cvtol	0.00	0.00	0.00	0.00	0.00	Hybrid d- cvtol	0.00	0.00	0.00	0.00	0.00
	Hybrid -ftol	0.00	0.00	0.00	0.00	0.00	Hybrid d-ftol	0.00	0.00	0.00	0.00	0.00

Table 4.28 Summary of the p-value of the Friedman rank test over all runs

Performance Indicator	Sampling				Crossover			
	Real_lhs		Real_random		Uniform		real_sbx	
	p-value	statistic value	p-value	statistic value	p-value	statistic value	p-value	statistic value
IGD	0.16	10.23	0.01	11.09	0.01	9.89	0.01	10.20
IGD+	0.01	11.15	0.03	8.39	0.01	11.15	0.009	11.36
GD	0.01	11.09	0.01	11.09	0.01	10.78	0.008	11.75
GD+	0.01	11.09	0.1	11.09	0.01	10.78	0.008	11.75

Table 4.29 Summary of the p-value of the posthoc_nemenyi_friedman test evaluated over all runs (sampling)

Performance Indicator	Sampling											
	Real_lhs				Real_random							
	Hybrid-cvtol vs. BU		Hybrid-cvtol vs. Without BU		BU vs. Without BU		Hybrid-cvtol vs. BU		Hybrid-cvtol vs. Without BU		BU vs. Without BU	
	p-value	h ¹¹	p-value	h	p-value	h	p-value	h	p-value	h	p-value	h
IGD	0.04	+	0.09	~	0.90	~	0.06	~	0.01	+	0.90	~
IGD+	0.04	+	0.02	+	0.90	~	0.12	~	0.03	+	0.90	~
GD	0.01	+	0.06	~	0.90	~	0.06	~	0.01	+	0.90	~
GD+	0.01	+	0.06	~	0.90	~	0.06	~	0.01	+	0.90	~

¹¹ Posthoc_nemenyi_friedman: +, ~, and – presents the first method performs statistically significantly better, equal, and worse than the second approach.

Table 4.30 Summary of the p-value of the posthoc_nemenyi_friedman test evaluated over all runs (sampling-Continue)

Performance Indicator	Sampling											
	Real_lhs				Real_random							
	Hybrid-ftol vs. BU		Hybrid-ftol vs. Without BU		Hybrid-ftol vs. Hybrid-cvtol		Hybrid-ftol vs. BU		Hybrid-ftol vs. Without BU		Hybrid-ftol vs. Hybrid-cvtol	
	p-value	h ¹²	p-value	h	p-value	h	p-value	h	p-value	h	p-value	h
IGD	0.04	+	0.09	~	0.90	~	0.51	~	0.22	~	0.67	~
IGD+	0.43	~	0.28	~	0.67	~	0.90	~	0.67	~	0.35	~
GD	0.22	~	0.51	~	0.67	~	0.51	~	0.22	~	0.67	~
GD+	0.22	~	0.51	~	0.67	~	0.51	~	0.22	~	0.67	~

Table 4.31 Summary of p value of the posthoc_nemenyi_friedmantest evaluated over all runs (crossover)

Performance Indicator	Crossover											
	Uniform				real_sbx							
	Hybrid-cvtol vs. BU		Hybrid-cvtol vs. Without BU		BU vs. Without BU		Hybrid-cvtol vs. BU		Hybrid-cvtol vs. Without BU		BU vs. Without BU	
	p-value	h ¹³	p-value	h	p-value	h	p-value	h	p-value	h	p-value	h
IGD	0.22	~	0.06	~	0.90	~	0.03	+	0.12	~	0.90	~

¹² Posthoc_nemenyi_friedman: +, ~, and – presents the first method performs statistically significantly better, equal, and worse than the second approach.

¹³ Posthoc_nemenyi_friedman: +, ~, and – presents the first method performs statistically significantly better, equal, and worse than the second approach.

IGD+	0.43	~	0.04	+	0.67	~	0.03	+	0.35	~	0.67	~
GD	0.28	~	0.09	~	0.90	~	0.04	+	0.43	~	0.67	~
GD+	0.28	~	0.09	~	0.90	~	0.04	+	0.43	~	0.67	~

Table 4.32 Summary of *p* value of the posthoc_nemenyi_friedmantest evaluated over all runs (crossover-Continue)

Performance Indicator	Crossover											
	Uniform						real_sbx					
	Hybrid-ftol vs. BU		Hybrid-ftol vs. Without BU		Hybrid-ftol vs. Hybrid-cvtol		Hybrid-ftol vs. BU		Hybrid-ftol vs. Without BU		Hybrid-ftol vs. Hybrid-cvtol	
	p-value	h ¹⁴	p-value	h	p-value	h	p-value	h	p-value	h	p-value	h
IGD	0.22	~	0.06	~	0.90	~	0.12	~	0.35	~	0.90	~
IGD+	0.28	~	0.02	+	0.90	~	0.03	+	0.35	~	0.67	~
GD	0.16	~	0.04	+	0.90	~	0.02	+	0.28	~	0.90	~
GD+	0.16	~	0.04	+	0.90	~	0.02	+	0.28	~	0.90	~

¹⁴ Posthoc_nemenyi_friedman: +, ~, and – presents the first method performs statistically significantly better, equal, and worse than the second approach.

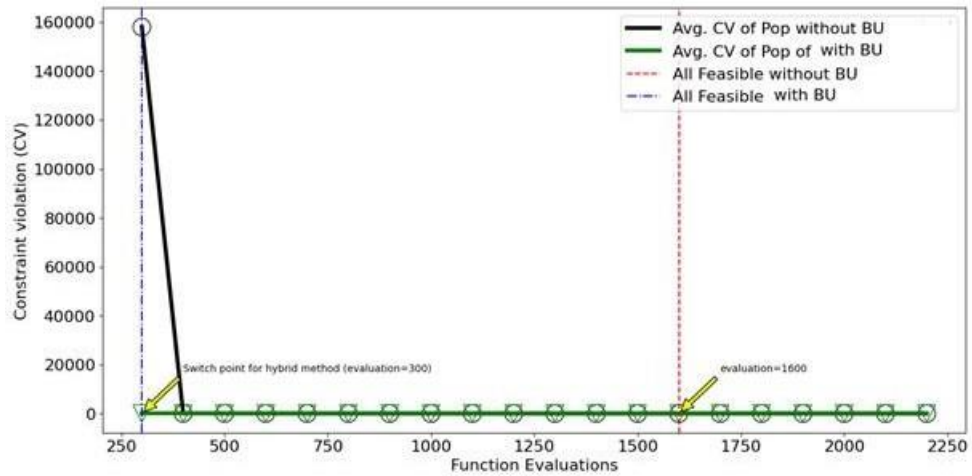


Figure 4.13 Constraint violation evaluated by NSGA-II (convergence of the whole population to feasible solutions)

4.2.5 Cantilevered Stepped Beam Design Problem

The cantilevered stepped design problem is an example of large-scale size problem. In the cantilevered beam design problem, the stepped cantilever beam must be able to carry a prescribed end load 49. This problem is originally a single-objective optimization problem considering minimizing the beam volume; however, this study has extended the original problem and added one more objective function, which minimize end deflection subject to various engineering design variables (see Appendix). The beam supports the given load, P , at a fixed distance L from the support. Besides, designers of the beam can vary the width (b_i) and height (h_i) of each section (Figure 14). The problem is highly constraint and large-dimensional; however, for this example five segments ($N=5$) including 10 dimensions (10 constraints) are considered. In the pursuit of solving both the single- and multi-objective versions of this intricate problem, repairing variables x_2 , x_4 , x_6 , x_8 , and x_{10} have been identified. The application of the BU method to this constrained nonlinear optimization problem is showcased, providing a valuable demonstration of the BU method's utility. Figure 15 (a-d) visually presents

the optimal solutions discovered by NSGA-II employing different methods across various generations.

To assess the performance of the proposed approaches, the hypervolume (HV) indicator is utilized, offering a comprehensive measure of the quality of solutions. The results, detailed in Tables 4.35-4.39 and visualized in Figure 16, unequivocally demonstrate the efficacy of the hybrid approaches in enhancing solution quality. This improvement is particularly noteworthy in the case of Latin hypercube sampling and simulated binary crossover (SBX), where the hybrid approaches exhibit a significant enhancement in solution quality.

This example provides valuable insights into the application of the BU method in addressing complex, large-scale, constrained optimization problems. The extension of the original single-objective problem to a multi-objective setting showcases the flexibility of the proposed methodology. The utilization of hypervolume as a performance indicator reaffirms the substantial improvements achieved by the hybrid approaches. Overall, this study contributes not only to the optimization of the Cantilevered Stepped Design problem but also sheds light on the broader potential of the BU method in tackling diverse engineering optimization challenges.

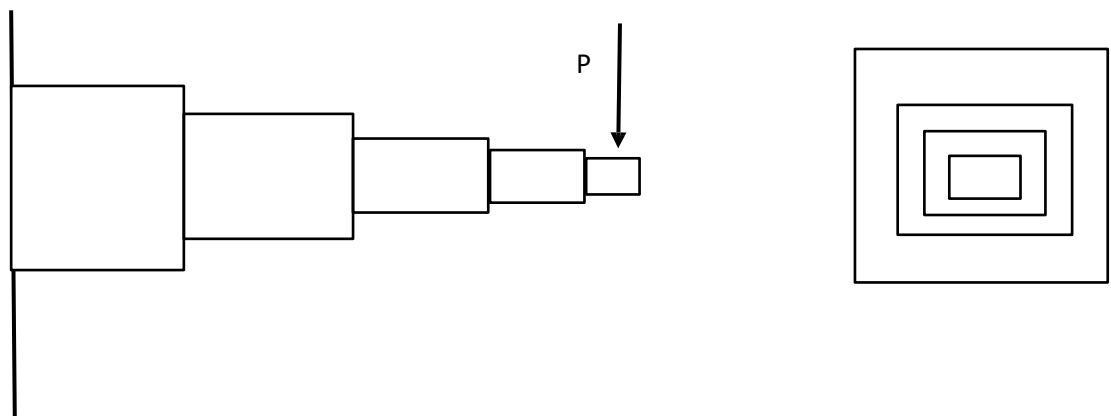
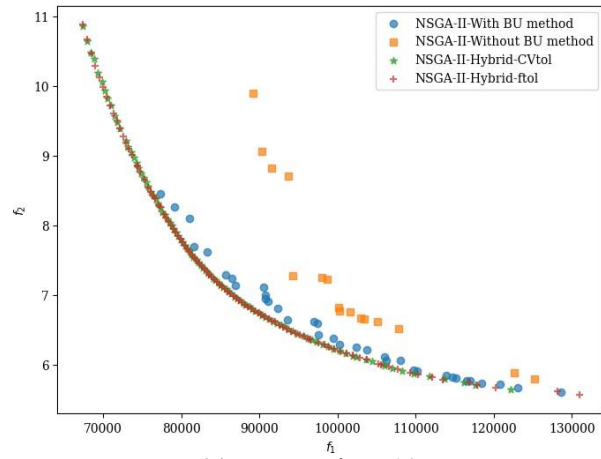
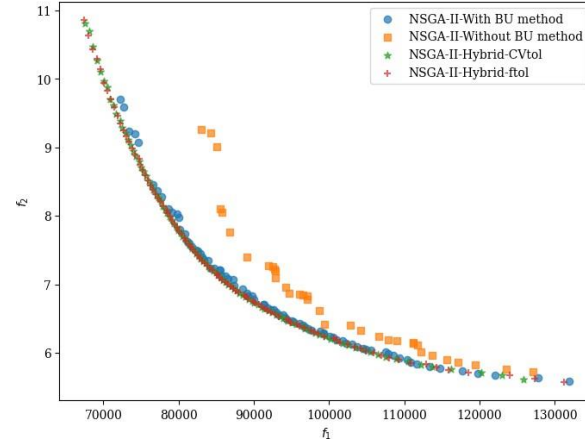


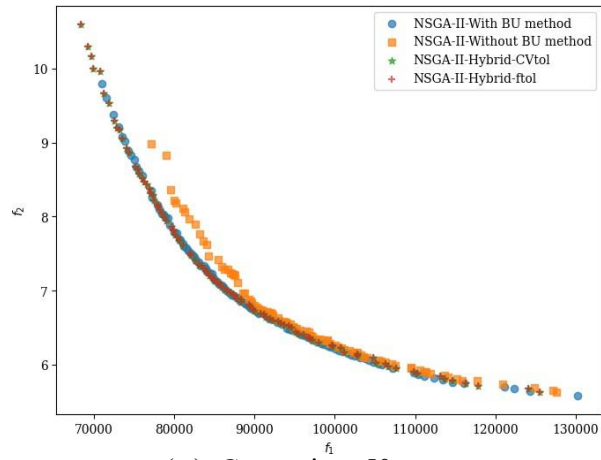
Figure 4.14 Schematic of the stepped beam design problem



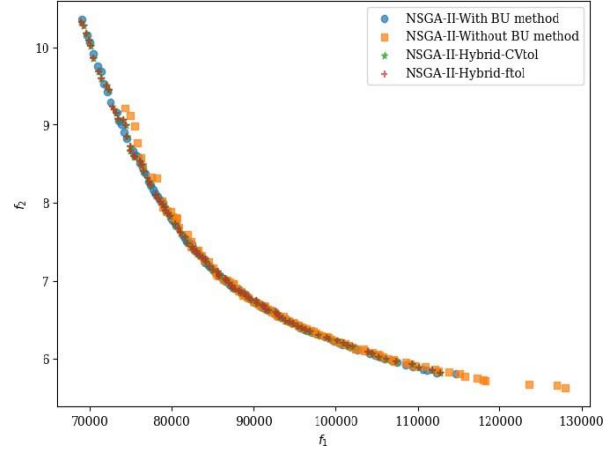
(a) Generation=10



(b) Generation=20



(c) Generation=50



(d) Generation=100

Figure 4.15 Pareto solutions found by NSGA-II by different approaches and generations

Table 4.33 Results of different methods on the cantilevered stepped design problem (HV values-sampling)

Algorithm	Sampling											
	real_random						Real_lhs					
	BU step	Best	Mean	Median	Worst	St. Dev.	BU step	Best	Mean	Median	Worst	St. Dev.
NSGA-II	None ¹⁵	0.6960	0.6344	0.6131	0.6118	0.0325	None	0.7324	0.6585	0.6396	0.6118	0.0478
	BU	0.7415	0.7309	0.7355	0.7144	0.0102	BU	0.7428	0.7313	0.7329	0.7156	0.0103
	Hybrid-cvtol	0.7734	0.7731	0.7730	0.7729	0.0002	Hybrid-cvtol	0.7737	0.7731	0.7731	0.7727	0.0003
	Hybrid-ftol	0.7734	0.7730	0.7729	0.7728	0.0002	Hybrid-ftol	0.7737	0.7729	0.7727	0.7723	0.0004

Table 4.34 Results of different methods on the the cantilevered stepped design problem (HV values-crossover)

Algorithm	Crossover											
	real_sbx						Uniform					
	BU step	Best	Mean	Median	Worst	St. Dev.	BU step	Best	Mean	Median	Worst	St. Dev.
NSGA-II	None ¹⁶	0.6544	0.6243	0.6396	0.5762	0.0277	None	0.6950	0.6849	0.6941	0.6539	0.0157
	BU	0.7428	0.7315	0.7347	0.7156	0.0089	BU	0.7428	0.7285	0.7347	0.7106	0.0129
	Hybrid-cvtol	0.7737	0.7733	0.7733	0.7727	0.0003	Hybrid-cvtol	0.7733	0.7728	0.7730	0.7721	0.0004
	Hybrid-ftol	0.7737	0.7732	0.7733	0.7729	0.0002	Hybrid-ftol	0.7734	0.7732	0.7732	0.7730	0.0001

Table 4.35 Summary of the p-value of the Friedman rank test over all runs

Algorithm	Sampling				Crossover			
	Real_random		Real_lhs		real_sbx		Uniform	
	p-value	statistic value	p-value	statistic value	p-value	statistic value	p-value	statistic value
NSGA-II	0.002	14.61	0.003	13.77	0.002	14.03	0.002	14.03

Table 4.36 Summary of the p-value of the posthoc_nemenyi_friedman test evaluated over all runs (sampling)

Algorithm	Sampling

¹⁵ “None” means no BU method or hybrid is used and only feasibility rules as an explicit constraint handling method is worked alone.

¹⁶ “None” means no BU method or hybrid is used and only feasibility rules as an explicit constraint handling method is worked alone.

	Real_lhs						Real_random					
	Hybrid-cvtol vs. BU		Hybrid-cvtol vs. Without BU		BU vs. Without BU		Hybrid-cvtol vs. BU		Hybrid-cvtol vs. Without BU		BU vs. Without BU	
	p-value	h ¹⁷	p-value	h	p-value	h	p-value	h	p-value	h	p-value	h
NSGA-II	0.16	~	0.008	+	0.67	~	0.16	~	0.008	+	0.67	~

Table 4.37 Summary of the p-value of the posthoc_nemenyi_friedman test evaluated over all runs (sampling-Continue)

Algorithm	Sampling											
	Real_lhs						Real_random					
	Hybrid-ftol vs. BU		Hybrid-ftol vs. Without BU		Hybrid-ftol vs. Hybrid-cvtol		Hybrid-ftol vs. BU		Hybrid-ftol vs. Without BU		Hybrid-ftol vs. Hybrid-cvtol	
	p-value	h ¹⁸	p-value	h	p-value	h	p-value	h	p-value	h	p-value	h
NSGA-II	0.59	~	0.09	~	0.82	~	0.59	~	0.09	~	0.82	~

Table 4.38 Summary of p value of the posthoc_nemenyi_friedmantest evaluated over all runs (crossover)

Algorithm	Crossover											
	Uniform						real_sbx					
	Hybrid-cvtol vs. BU		Hybrid-cvtol vs. Without BU		BU vs. Without BU		Hybrid-cvtol vs. BU		Hybrid-cvtol vs. Without BU		BU vs. Without BU	
	p-value	h ¹⁹	p-value	h	p-value	h	p-value	h	p-value	h	p-value	h
NSGA-II	0.67	~	0.12	~	0.67		0.35	~	0.03	+	0.67	~

¹⁷ Posthoc_nemenyi_friedman: +, ~, and – presents the first method performs statistically significantly better, equal, and worse than the second approach.

¹⁸ Posthoc_nemenyi_friedman: +, ~, and – presents the first method performs statistically significantly better, equal, and worse than the second approach.

¹⁹ Posthoc_nemenyi_friedman: +, ~, and – presents the first method performs statistically significantly better, equal, and worse than the second approach.

Table 4.39 Summary of p value of the *posthoc_nemenyi_friedmantest* evaluated over all runs (crossover-Continue)

Algorithm	Crossover											
	Uniform				real_sbx							
	Hybrid-ftol vs. BU		Hybrid-ftol vs. Without BU		Hybrid-ftol vs. Hybrid-cvtol		Hybrid-ftol vs. BU		Hybrid-ftol vs. Without BU		Hybrid-ftol vs. Hybrid-cvtol	
	p-value	h ²⁰	p-value	h	p-value	h	p-value	h	p-value	h	p-value	h
NSGA-II	0.12	~	0.005	+	0.67	~	0.35	~	0.03	+	0.90	~

²⁰ Posthoc_nemenyi_friedman: +, ~, and – presents the first method performs statistically significantly better, equal, and worse than the second approach.

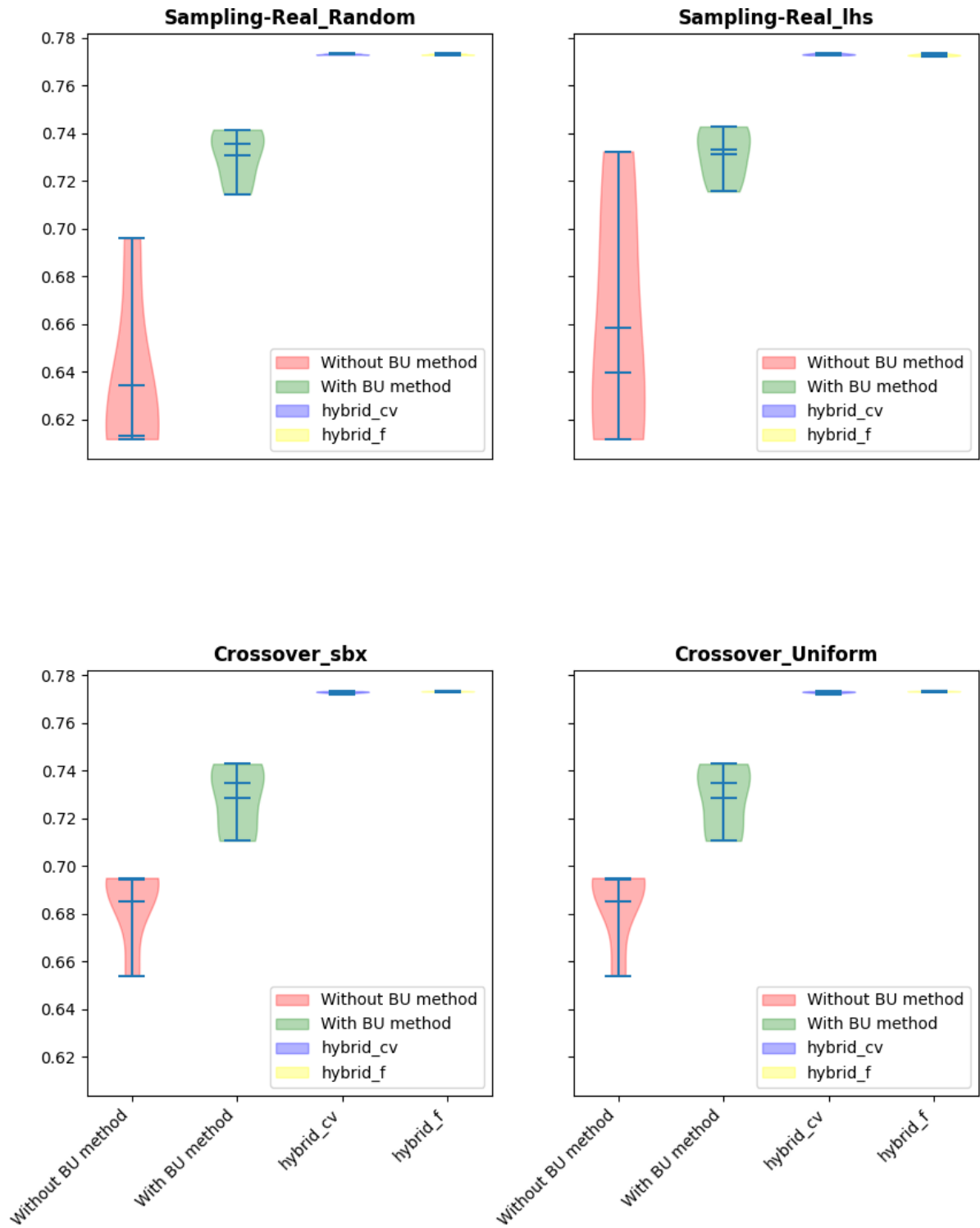


Figure 4.16 Violin plot of implementation of using different methods (Stepped beam design problem problem)

4.2.6 Multi-Objective Car-Side Impact Problem Formulation

In scenarios where the black-box constraints of an optimization problem exhibit highly complex behavior, the direct evaluation of these constraints can be prohibitively time-consuming. In such instances, the use of surrogates becomes a valuable strategy to mitigate the complexity and expedite the optimization process. Surrogates, effectively approximating the actual constraints, enable the optimization algorithm to learn from previous evaluations and make informed decisions. The surrogate model, if reducible to a single variable, provides an opportunity to apply the BU approach to solve the constrained surrogate model efficiently, as detailed in the appendix.

The car side impact problem stands out as a prime example of a complex and time-consuming optimization challenge, focusing on the multi-objective optimization of a vehicle's side impact crashworthiness [50]. Repairing variables X_2 and X_3 are identified for this problem. Various evolutionary algorithms, including NSGA-II, NSGA-III, UNSGA-III, and AGEMOEA, are implemented to explore the solution space. Tables 4.40-4.46 present the results of the non-dominated solutions obtained by these methods.

The results underscore the superiority of the hybrid methods, implemented with the proposed evolutionary algorithms, in terms of diversity. The hybrid approaches exhibit a well-distributed population across the entire search space, leading to enhanced performance. While the application of the BU approach alone may face efficiency challenges, particularly in the context of a three-objective optimization problem, the hybrid approach proves adept at overcoming these difficulties. Statistical analyses of the results reveal that, in some cases and with different algorithms, the BU method may not be as efficient, yet the hybrid approach consistently outperforms it, signifying the added benefits of the hybridization strategy.

Comparative statistical tests emphasize the significant advantage of the hybrid approaches over the BU method. Notably, although some statistical tests indicate no significant difference between the hybrid and BU methods, the hybrid method consistently achieves higher hypervolume (HV) values. This suggests that, even when statistical significance is not apparent, the hybrid approach provides a qualitative improvement in solution quality.

In conclusion, the exploration of the car side impact problem demonstrates the efficacy of hybrid approaches in addressing complex, multi-objective optimization challenges with intricate black-box constraints. The incorporation of surrogates and the application of the BU method within a hybrid framework contribute to improved diversity and solution quality. These findings not only advance the optimization of the car side impact problem but also offer valuable insights into the broader applicability of hybrid evolutionary algorithms in addressing real-world, complex optimization scenarios.

Table 4.40 Results of different methods on the multi-objective car-side impact problem (HV values-sampling)

Algorithm	Sampling											
	real_random						Real_lhs					
	BU step	Best	Mean	Median	Worst	St. Dev.	BU step	Best	Mean	Median	Worst	St. Dev.
NSGA-II	None ²¹	0.249	0.244	0.2466	0.2398	0.0038	None	0.260	0.217	0.213	0.180	0.0283
	BU	0.770	0.756	0.76	0.733	0.0132	BU	0.767	0.743	0.750	0.700	0.0228
	Hybrid-cvtol	0.818	0.800	0.7985	0.790	0.0094	Hybrid-cvtol	0.814	0.795	0.799	0.768	0.0151
	Hybrid-ftol	0.803	0.803	0.803	0.802	0.0004	Hybrid-ftol	0.806	0.797	0.797	0.785	0.0075
NSGA-III	None	0.259	0.165	0.1333	0.1225	0.0520	None	0.263	0.186	0.154	0.136	0.0505
	BU	0.818	0.803	0.805	0.778	0.0141	BU	0.824	0.809	0.813	0.785	0.0132
	Hybrid-cvtol	0.850	0.826	0.826	0.810	0.0141	Hybrid-cvtol	0.853	0.829	0.824	0.810	0.0162
	Hybrid-ftol	0.858	0.852	0.852	0.845	0.004	Hybrid-ftol	0.841	0.833	0.837	0.819	0.008
UNSGA-III	None	0.214	0.191	0.1996	0.1575	0.0195	None	0.182	0.141	0.149	0.074	0.0361
	BU	0.815	0.802	0.80	0.788	0.0090	BU	0.833	0.805	0.803	0.790	0.0146
	Hybrid-cvtol	0.823	0.811	0.816	0.791	0.0110	Hybrid-cvtol	0.830	0.819	0.827	0.796	0.0133
	Hybrid-ftol	0.816	0.810	0.810	0.803	0.004	Hybrid-ftol	0.829	0.822	0.826	0.811	0.006
AGEMOEA	None	0.272	0.226	0.222	0.1937	0.0265	None	0.276	0.224	0.251	0.163	0.0481
	BU	0.849	0.801	0.794	0.771	0.0257	BU	0.804	0.777	0.773	0.763	0.0140
	Hybrid-cvtol	0.849	0.828	0.819	0.812	0.0145	Hybrid-cvtol	0.853	0.833	0.833	0.816	0.0147
	Hybrid-ftol	0.835	0.827	0.826	0.816	0.006	Hybrid-ftol	0.841	0.834	0.838	0.823	0.006
NSDE	None	0.825	0.803	0.811	0.770	0.018	None	0.830	0.773	0.782	0.707	0.045
	BU	0.812	0.780	0.771	0.747	0.026	BU	0.777	0.757	0.767	0.734	0.018
	Hybrid-cvtol	0.818	0.810	0.816	0.794	0.008	Hybrid-cvtol	0.827	0.813	0.811	0.803	0.008
	Hybrid-ftol	0.808	0.798	0.793	0.791	0.007	Hybrid-ftol	0.823	0.809	0.809	0.791	0.011
NSDER	None	0.782	0.737	0.737	0.704	0.029	None	0.816	0.805	0.812	0.774	0.015
	BU	0.790	0.759	0.767	0.734	0.021	BU	0.794	0.782	0.781	0.776	0.006
	Hybrid-cvtol	0.829	0.816	0.817	0.809	0.006	Hybrid-cvtol	0.840	0.809	0.805	0.791	0.016
	Hybrid-ftol	0.823	0.806	0.809	0.791	0.104	Hybrid-ftol	0.808	0.798	0.793	0.791	0.007

²¹ “None” means no BU method or hybrid is used and only feasibility rules as an explicit constraint handling method is worked alone.

Table 4.41 Results of different methods on the multi-objective car side design problem (HV values-crossover)

Algorithm	Crossover											
	real_sbx						Uniform					
	BU step	Best	Mean	Median	Worst	St. Dev.	BU step	Best	Mean	Median	Worst	St. Dev.
NSGA-II	None	0.23	0.18	0.18	0.17	0.02	None	0.30	0.26	0.27	0.16	0.05
	BU	0.76	0.74	0.75	0.69	0.02	BU	0.77	0.75	0.76	0.73	0.01
	Hybrid-cvtol	0.81	0.79	0.79	0.77	0.01	Hybrid-cvtol	0.80	0.79	0.79	0.78	0.00
	Hybrid-ftol	0.80	0.80	0.80	0.79	0.00	Hybrid-ftol	0.80	0.79	0.80	0.79	0.00
NSGA-III	None	0.25	0.18	0.18	0.12	0.05	None	0.23	0.20	0.21	0.16	0.02
	BU	0.81	0.80	0.81	0.79	0.00	BU	0.81	0.78	0.77	0.76	0.02
	Hybrid-cvtol	0.85	0.82	0.82	0.78	0.02	Hybrid-cvtol	0.84	0.82	0.82	0.81	0.00
	Hybrid-ftol	0.83	0.82	0.83	0.81	0.00	Hybrid-ftol	0.83	0.82	0.83	0.82	0.00
UNSGA-III	None	0.18	0.18	0.18	0.17	0.00	None	0.25	0.22	0.21	0.19	0.02
	BU	0.79	0.78	0.78	0.76	0.01	BU	0.82	0.80	0.80	0.77	0.01
	Hybrid-cvtol	0.85	0.83	0.82	0.81	0.01	Hybrid-cvtol	0.83	0.82	0.82	0.80	0.00
	Hybrid-ftol	0.84	0.83	0.83	0.82	0.00	Hybrid-ftol	0.83	0.82	0.82	0.81	0.00
AGEMOEa	None	0.26	0.22	0.21	0.19	0.03	None	0.27	0.25	0.27	0.19	0.03
	BU	0.81	0.78	0.79	0.72	0.02	BU	0.79	0.78	0.79	0.77	0.00
	Hybrid-cvtol	0.85	0.83	0.84	0.82	0.01	Hybrid-cvtol	0.85	0.83	0.85	0.81	0.01
	Hybrid-ftol	0.84	0.83	0.83	0.83	0.00	Hybrid-ftol	0.84	0.83	0.83	0.82	0.00
NSDE	None	0.81	0.80	0.80	0.79	0.00	None	0.81	0.80	0.80	0.79	0.00
	BU	0.79	0.78	0.78	0.76	0.01	BU	0.78	0.78	0.78	0.77	0.00
	Hybrid-cvtol	0.81	0.81	0.81	0.80	0.00	Hybrid-cvtol	0.81	0.81	0.81	0.80	0.00
	Hybrid-ftol	0.80	0.79	0.79	0.79	0.00	Hybrid-ftol	0.79	0.79	0.79	0.79	0.00
NSDER	None	0.75	0.73	0.73	0.71	0.01	None	0.74	0.74	0.73	0.73	0.00
	BU	0.77	0.76	0.76	0.74	0.00	BU	0.76	0.76	0.76	0.75	0.00
	Hybrid-cvtol	0.81	0.81	0.81	0.81	0.00	Hybrid-cvtol	0.81	0.81	0.81	0.81	0.00
	Hybrid-ftol	0.87	0.81	0.81	0.73	0.04	Hybrid-ftol	0.84	0.82	0.83	0.78	0.02

Table 4.42 Summary of the p-value of the Friedman rank test over all runs

Algorithm	Sampling				Crossover			
	Real_lhs		Real_random		Uniform		real_sbx	
	p-value	statistic value	p-value	statistic value	p-value	statistic value	p-value	statistic value
NSGA-II	0.0067	10.0	0.0067	10.0	0.0067	10.0	0.0067	10.0
NSGA-III	0.0149	8.40	0.0149	8.40	0.0067	10.0	0.0149	8.40
UNSGA-III	0.0223	7.60	0.02237	7.60	0.0067	10.0	0.0067	10.0
AGEMOEA	0.0223	7.60	0.0149	8.40	0.0067	10.0	0.0067	10.0
NSDE	0.032	8.75	0.050	7.73	0.0018	15.0	0.002	14.03
NSDER	0.054	7.62	0.001	15.0	0.003	13.56	0.006	12.11

Table 4.43 Summary of the p-value of the posthoc_nemenyi_friedman test evaluated over all runs (sampling)

Algorithm	Sampling											
	Real_lhs						Real_random					
	Hybrid-cvtol vs. BU		Hybrid-cvtol vs. Without BU		BU vs. Without BU		Hybrid-cvtol vs. BU		Hybrid-cvtol vs. Without BU		BU vs. Without BU	
	p-value	h ²²	p-value	h	p-value	h	p-value	h	p-value	h	p-value	h
NSGA-II	0.31	~	0.01	+	0.59	~	0.51	~	0.06	~	0.67	~
NSGA-III	0.51	~	0.06	~	0.67	~	0.67	~	0.12	~	0.67	~
UNSGA-III	0.82	~	0.06	~	0.35	~	0.22	~	0.01	+	0.67	~
AGEMOEA	0.51	~	0.06	~	0.67	~	0.67	~	0.04	+	0.43	~
NSDE	0.03	+	0.35	~	0.67	~	0.03	+	0.82	~	0.22	~
NSDER	0.04	+	0.74	~	0.35	~	0.12	~	0.005	+	0.67	~

Table 4.44 Summary of the p-value of the posthoc_nemenyi_friedman test evaluated over all runs (sampling)

Algorithm	Sampling											
	Real_lhs						Real_random					
	Hybrid-ftol vs. BU		Hybrid-ftol vs. Without BU		Hybrid-cvtol vs. Hybrid-ftol		Hybrid-ftol vs. BU		Hybridftol vs. Without BU		Hybrid-cvtol vs. Hybrid-ftol	
	p-value	h ²³	p-value	h	p-value	h	p-value	h	p-value	h	p-value	h
NSGA-II	0.20	~	0.007	+	0.90	~	0.22	~	0.01	+	0.90	~
NSGA-III	0.22	~	0.01	+	0.90	~	0.12	~	0.005	+	0.67	~
UNSGA-III	0.82	~	0.06	~	0.90	~	0.51	~	0.90	~	0.06	~

²² Posthoc_nemenyi_friedman: +, ~, and – presents the first method performs statistically significantly better, equal, and worse than the second approach.

²³ Posthoc_nemenyi_friedman: +, ~, and – presents the first method performs statistically significantly better, equal, and worse than the second approach.

AGEMOE A	0.22	~	0.01	+	0.90	~	0.74	~	0.06	~	0.90	~
NSDE	0.35	~	0.90	~	0.67	~	0.03	+	0.82	~	0.22	~
NSDER	0.59	~	0.90	~	0.51	~	0.67	~	0.12	~	0.67	~

Table 4.45 Summary of the p-value of the posthoc_nemenyi_friedman test evaluated over all runs (Crossover)

Algorithm	Crossover											
	Uniform						real_sbx					
	Hybrid-cvtol vs. BU		Hybrid-cvtol vs. Without BU		BU vs. Without BU		Hybrid-cvtol vs. BU		Hybrid-cvtol vs. Without BU		BU vs. Without BU	
	p-value	h ²⁴	p-value	h	p-value	h	p-value	h	p-value	h	p-value	h
NSGA-II	0.45	~	0.03	+	0.59	~	0.45	~	0.03	+	0.59	~
NSGA-III	0.51	~	0.06	~	0.67	~	0.82	~	0.12	~	0.51	~
UNSGA-III	0.67	~	0.12	~	0.67	~	0.82	~	0.06	~	0.35	~
AGEMOE A	0.28	~	0.02	+	0.67	~	0.22	~	0.01	+	0.67	~
NSDE	0.005	+	0.67	~	0.12	~	0.005		0.51	~	0.22	~
NSDER	0.51	~	0.06	~	0.67	~	0.35	~	0.013		0.51	~

²⁴ Posthoc_nemenyi_friedman: +, ~, and – presents the first method performs statistically significantly better, equal, and worse than the second approach.

Table 4.46 Summary of the *p*-value of the *posthoc_nemenyi_friedman* test evaluated over all runs (Crossover)

Algorithm	Crossover											
	Uniform						real_sbx					
	Hybrid-ftol vs. BU		Hybrid-ftol vs. Without BU		Hybrid-cvtol vs. Hybrid-ftol		Hybrid-ftol vs. BU		Hybridfvtol vs. Without BU		Hybrid-cvtol vs. Hybrid-ftol	
	p-value	h ²⁵	p-value	h	p-value	h	p-value	h	p-value	h	p-value	h
NSGA-II	0.12	~	0.003	+	0.87	~	0.12	~	0.003	+	0.87	~
NSGA-III	0.22	~	0.01	+	0.90	~	0.35	~	0.01	+	0.82	~
UNSGA-III	0.12	~	0.005	+	0.67	~	0.82	~	0.06	~	0.90	~
AGEMOEA	0.43	~	0.04	+	0.90	~	0.51	~	0.06	~	0.90	~
NSDE	0.67	~	0.67	~	0.12	~	0.51	~	0.90	~	0.22	~
NSDER	0.22	~	0.013	+	0.90	~	0.82	~	0.12	~	0.82	~

²⁵ Posthoc_nemenyi_friedman: +, ~, and – presents the first method performs statistically significantly better, equal, and worse than the second approach.

5. Discussion

The BU method is an implicit constraint-handling technique coupled with an explicit constraint-handling technique that aims to cut the infeasible search space over iterations to find the feasible region faster. Since optimization is an iterative process whose boundaries can be changed during the process, the i^{th} decision variable boundaries in each iteration exhibit a dynamic nature. By applying this philosophy to boost the optimization process in the BU method, the bounds of selected variable (s) are repaired (repairing variables) to satisfy the constraints. As a result, new solutions are created within the updated boundary of the latest search space. The above-mentioned process helps the algorithm search for a solution within the feasible region instead of the whole area. Although the BU method directs the search operator to the feasible space and reaches the feasible region in a reasonable time, it twists the search space, making the optimization problem more challenging. As such, the results are not always accurate, and the algorithm may result in premature convergence. Therefore, in this paper, two “hybrid” switching mechanisms are proposed. In the proposed approach, the BU method is applied to the problem after initializing the algorithm, and in this phase, a threshold tolerance is set. The BU method optimizes the problem based on the threshold tolerances so that the population has zero constraint violations or there is no more change in the objective space. Then, the optimization problem is continued with the BU method.

The results suggest that:

- Advantageously, the hybrid method can be used with and without the BU method.

In the first phase of the hybrid method, the BU method helps the optimization

algorithm to avoid searching inside the infeasible region and boosts it to explore the feasible area more efficiently. This way, the optimization algorithm finds the feasible area with fewer FEs. However, because the BU brings about changes to the landscape of the search space, it may result in premature convergence. Hence, in the second phase of the hybrid method, the optimization algorithm searches the feasible area without using the BU approach.

- GA has the least standard deviation among the other algorithms for the single-objective function optimization problem.
- The statistical tests for single-objective optimization problems show there is a significant difference between hybrid methods and two other methods (with BU and without BU) using GA. Moreover, GA works well for this method and boosts optimization problems. Due to the fact that GA can benefit from different operator settings, such as crossover and sampling, better results by choosing the appropriate operator setting may be achieved.
- Regarding the multi-objective optimization problems, the hybrid method could find more non-dominated solutions with better distribution compared to with and without the BU methods. The BNH test problems indicated that the hybrid method produced promising results for IGD and HV values compared to with BU and without BU approaches.
- Although the BU and hybrid methods outperformed the other algorithms in single- and multi-objective optimization, they are still case-dependent, and there are EAs that are not suitable in some cases.

- By boosting the optimization process, the hybrid method finds the first and whole populations much faster than the classic BU and without BU methods for most problems (i.e., the single- or multi-objective optimization problems of this paper). Moreover, regarding the car side impact design problem, the hybrid method obtained good diversity, whereas without the BU method searched for solutions in a specific feasible space.
- For some case studies, although the hybrid method resulted in the same (or close-to-final) solution as the two other methods, it requires fewer FEs to converge to a solution.
- Compared to unconstrained optimization problems, constrained optimization problems are more challenging since many infeasible regions appear in the search space (meaning the hit ratio is low). This makes solving a constrained problem more challenging, especially highly constrained problems that can lead to difficulties such as convergence-, diversity-, and feasibility-hardness. Therefore, finding just one feasible solution is a significant achievement. The primary goal of the BU method is to minimize the search space by narrowing down the variable range and eliminating infeasible options. To compare the effectiveness of the proposed approach, the first feasible solution found by different methods is compared to one another, and the whole population is tracked against generations. The hybrid method uses this philosophy to switch the optimization process and avoid reaching premature convergence. On the other hand, once the whole feasible area is found by the BU method, the final solution (global) could be achieved with fewer FEs in a reasonable time.

- Regarding the multi-objective car side impact design problem, the hybrid approach works significantly better than without the BU method when incorporated in the employed multi-objective algorithms. It is worth mentioning that although there is no significant difference in the final HV values between the hybrid and BU methods, as mentioned earlier, the hybrid method could achieve the final solution in fewer FEs.
- In scenarios where explicit formulation of constraints is challenging, the use of surrogate models, also known as metamodels, proves to be a valuable solution. Surrogate models have demonstrated effectiveness in situations where deriving explicit formulations for constraints is impractical. These models serve as approximations of intricate simulations, providing an efficient means to evaluate constraints during optimization without the computational overhead of running resource-intensive simulations. For instance, consider optimization problems associated with finite element models, such as those involving nodal displacements and bar stresses in truss optimization. In these cases, surrogate models can be strategically trained to emulate the intricate behavior observed in simulation results. This approach becomes particularly advantageous when the direct formulation of these constraints proves to be impractical due to their complexity or reliance on computationally expensive procedures.

In EAs, sampling and crossover are critical factors that can strongly affect convergence speed and are often used in noisy optimization problems where reducing the negative impact of noise is crucial. Moreover, sampling is a popular strategy for dealing with noise. Therefore, experiments with different sampling operators, including real random

sampling, Latin hypercube sampling, and two different real-SBX and uniform crossovers, were conducted to reduce the noise impact.

6. LIMITATION

Although this study provides valuable insights into optimization problems, there are certain limitations to this work. One significant limitation of this work lies in the relatively small size of variables in the problems. It should be noted that this study aims to contribute by addressing a specific aspect of the optimization problem. In this study, the benchmark libraries have not been used and some specific problems with solvable constraints with respect to at least one variable have been used. While the limited number of variables in our optimization problems may restrict the immediate applicability of our findings to large-scale size problems, it is clear the insights gained from this research serve as a valuable foundation for further research in the field.

Another limitation of this study is related to the experimental section, where the algorithms compared with the proposed algorithm are not the most recent. It is important to note that the aim of this work is not focused on proposing and developing state-of-the-art algorithms; rather, the advancement of constraint handling techniques and the development of the BU approach are being concentrated on. Consequently, well-known algorithms such as NSGA-II in the analysis have been included. However, as a future study, it would be valuable to consider incorporating state-of-the-art algorithms to provide a more comprehensive evaluation of our proposed approach in comparison to the latest advancements in the field. Moreover, while the current selection of repairing variables is designed to be applicable even in scenarios with multiple disjoint feasible regions, the evaluation of constraints can pose challenges in situations where the behavior is non-trivial. In such cases, alternative

approaches to repairing variable selection, involving book-keeping or linking of these disjoint feasible ranges, may be necessary. This intriguing aspect has been highlighted and it is proposed as a subject for future investigation, intending to delve deeper into the nuanced evaluation of constraints in conditions assessment.

7. Conclusion

This study presents an extension of the work proposed by (Gandomi & Deb, 2020) for solving constrained optimization problems efficiently. The proposed methods use an implicit constraint handling approach called BU, which focuses on directly handling constraints and is coupled with other explicit constraint-handling techniques (here, feasibility rules).

In this study, two augmented versions of the BU method were applied to single-objective, multi-objective, and many-objective constrained optimization problems, including benchmarks and real-world problems. To accomplish this goal, two thresholds are considered, each representing different switching method. In the initial method, the optimization process shifts to a state that doesn't involve the BU approach once constraint violations reach zero. In the second approach, the optimization process enters a phase without BU methods when there is no longer any observed change in the objective space. For single-objective constrained optimization problems, popular algorithms like GA, DE, ES, SRES, and BRKGA were employed, and NSGA-II, a renowned algorithm, was used for bi-objective problems. For the multi-objective constrained optimization problems, NSGA-III, UNSGA-III, and AGEMOEA were applied. The results show that the hybrid approach performs significantly better than with- and without the BU methods, and can find better solutions with fewer iterations.

As a future study, it is suggested that the proposed BU method be coupled with other constraint-handling techniques, for example different types of penalty methods with different objective functions. Also, additional test problems in the literature, such as many-objective and dynamic functions, could be considered for further validating the proposed hybrid approach. After BU generalization, it can also be applied to benchmark libraries such as CEC, NEVERGRAD, BBOB, GNBG, and DIRECTGOLib. Although BU is not defined for any specific optimization algorithms, implementing BU to a wider range of optimization algorithms can be investigated in another future study. Moreover, as a prospective avenue for future research, exploring the applicability of the proposed approach to constrained problems featuring multiple disconnected regions is a promising direction. Many constrained optimization scenarios exhibit this characteristic, with only certain regions potentially close to the true optimum. Investigating the effectiveness and adaptability of our approach in addressing such complex problem structures holds significant potential for enhancing the robustness and versatility of the proposed methodology. This important consideration will be a key focus in our forthcoming studies.

Data availability

All data used in this study are simulated and not publicly available. The datasets were generated solely for the purpose of conducting the research outlined in this manuscript. Simulated data files are not deposited in any public repository or third-party platform. However, upon reasonable request, and subject to any applicable restrictions, the

corresponding author can provide the simulated data used in this study. Also, our code has been executed using the open-source Python library, Pymoo²⁶ and PymooDE²⁷.

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²⁶ <https://pymoo.org/index.html>

²⁷ <https://pymooDE.readthedocs.io/en/latest/>

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Appendix-

G01 formulation:

$$f_1(x) = 5 \times (x_1 + x_2 + x_3 + x_4) - 5 \times (x_1^2 + x_2^2 + x_3^2 + x_4^2) \quad (38)$$

$$-(x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13})$$

S.t.

$$g_1(x) = 2 \times x_1 + 2 \times x_2 + x_{10} + x_{11} \leq 10 \quad (39)$$

$$g_2(x) = 2 \times x_1 + 2 \times x_3 + x_{10} + x_{12} \leq 10 \quad (40)$$

$$g_3(x) = 2 \times x_2 + 2 \times x_3 + x_{11} + x_{12} \leq 10 \quad (41)$$

$$g_4(x) = -8 \times x_1 + x_{10} \leq 0 \quad (42)$$

$$g_5(x) = -8 \times x_2 + x_{11} \leq 0 \quad (43)$$

$$g_6(x) = -8 \times x_3 + x_{12} \leq 0 \quad (44)$$

$$g_7(x) = -2 \times x_4 - x_5 + x_{10} \leq 0 \quad (45)$$

$$g_8(x) = -2 \times x_6 - x_7 + x_{11} \leq 0 \quad (46)$$

$$g_9(x) = -2 \times x_8 - x_9 + x_{12} \leq 0 \quad (47)$$

$$0 \leq x_1 \dots x_9 \leq 1, \quad 0 \leq x_{10}, x_{11}, x_{12} \leq 100, \quad 0 \leq x_{13} \leq 1 \quad (48)$$

Car-side problem formulation:

$$f_1(x) = 1.98 + 4.90 \times x_1 + 6.67 \times x_2 + 6.98 \times x_3 + 4.01 \times x_4 + 1.78 \times x_5 + 2.73 \times x_7 \quad (49)$$

$$g_1(x) = 1.16 - 0.3717 \times x_2 \times x_4 - 0.00931 \times x_2 \times x_{10} - 0.484 \times x_3 \times x_9 \\ + 0.01343 \times x_6 \times x_{10} \quad (50)$$

$$g_2(x) = 46.36 - 9.9 \times x_2 - 12.9 \times x_1 \times x_8 + 0.1107 \times x_3 \times x_{10} \quad (51)$$

$$g_3(x) = 33.86 + 2.95 \times x_3 + 0.1792 \times x_{10} - 5.057 \times x_1 \times x_2 - 11.0 \times x_2 \times x_8 - 0.021 \times x_5 \\ \times x_{10} - 9.98 \times x_7 \times x_8 + 22.0 \times x_8 \times x_9 \quad (52)$$

$$g_4(x) = 28.98 + 3.818 \times x_3 - 4.2 \times x_1 \times x_2 - 5.0207 \times x_5 \times x_{10} + 6.63 \times x_6 \times x_9 - 7.7 \\ \times x_7 \times x_8 + 0.32 \times x_9 \times x_{10} \quad (53)$$

$$g_5(x) = 0.261 - 0.0159 \times x_1 \times x_2 - 0.188 \times x_1 \times x_8 - 0.019 \times x_2 \times x_7 + 0.0144 \times x_3 \times x_5 \\ + 0.0008757 \times x_5 \times x_{10} + 0.08045 \times x_6 \times x_9 + 0.00139 \times x_8 \times x_{11} \\ + 0.00001575 \times x_{10} \times x_{11} \quad (54)$$

$$g_6(x) = 0.214 + 0.00817 \times x_5 - 0.0131 \times x_1 \times x_8 - 0.0704 \times x_1 \times x_9 + 0.03099 \times x_2 \times x_6 \\ - 0.018 \times x_2 \times x_7 + 0.0208 \times x_3 \times x_8 + 0.121 \times x_3 \times x_9 - 0.00364 \times x_5 \times x_6 \\ + 0.0007715 \times x_5 \times x_{10} - 0.0005354 \times x_6 \times x_{10} + 0.00121 \times x_8 \times x_{11} \\ + 0.00184 \times x_9 \times x_{10} - 0.02 \times x_2^2 \quad (55)$$

$$g_7(x) = 0.74 - 0.61 \times x_2 - 0.163 \times x_3 \times x_8 + 0.001232 \times x_3 \times x_{10} - 0.166 \times x_7 \times x_9 \\ + 0.227 \times x_2^2 \quad (56)$$

$$g_8(x) = 4.72 - 0.5 \times x_4 - 0.19 \times x_2 \times x_3 - 0.0122 \times x_4 \times x_{10} + 0.009325 \times x_6 \times x_{10} \\ + 0.000191 \times x_{11}^2 \quad (57)$$

$$g_9(x) = 10.58 - 0.674 \times x_1 \times x_2 - 1.95 \times x_2 \times x_8 + 0.02054 \times x_3 \times x_{10} - 0.0198 \times x_4 \times x_{10} + 0.028 \times x_6 \times x_{10} \quad (58)$$

$$g_{10} = 16.45 - 0.489 \times x_3 \times x_7 - 0.843 \times x_5 \times x_6 + 0.0432 \times x_9 \times x_{10} - 0.0556 \times x_9 \times x_{11} - 0.000786 \times x_{11}^2 \quad (59)$$

$$\begin{aligned}
0.5 \leq x_1, x_3, x_4 \leq 1.5, \quad 0.45 \leq x_2 \leq 1.35, \quad 0.875 \leq x_5 \leq 2.625, \quad 0.4 \leq x_6, x_7 \leq 1.2, \\
0.19 \leq x_8, x_9 \leq 0.35, \quad -30 \leq x_{10}, x_{11} \leq 30
\end{aligned} \tag{60}$$

Speed reducer formulation:

$$\begin{aligned}
\min z = & 0.785 \times x_1 \times x_2^2 \times (3.33 \times x_3^2 + 14.9334 \times x_3 - 43.0934) - \\
& 1.508 \times x_1 \times (x_6^2 + x_7^2) + 7.4777 \times (x_6^3 + x_7^3) + 0.7854 \times (x_4 \times x_6^2 + \\
& x_5 \times x_7^2)
\end{aligned} \tag{61}$$

$$\text{S.t.} \tag{62}$$

$$27/(x_1 \times x_2^2 \times x_3) \leq 1 \tag{63}$$

$$397.5/x_1 \times x_2^2 \times x_3^2 \leq 1 \tag{64}$$

$$1.93 \times x_4^3/x_2 \times x_3 \times x_6^4 \leq 1 \tag{65}$$

$$1.93 \times x_5^3/(x_2 \times x_3 \times x_7^4) \leq 1 \tag{66}$$

$$\sqrt{(745 \times \frac{x_4}{x_2 \times x_3})^2 + 1.69 \times 10^7/(110 \times x_6^3)} \leq 1 \tag{67}$$

$$\sqrt{(745 \times \frac{x_5}{x_2 \times x_3})^2 + 1.575 \times 10^8/(85 \times x_7^3)} \leq 1 \tag{68}$$

$$(x_2 \times x_3)/40 \leq 1 \tag{69}$$

$$(5 \times x_2)/x_1 \leq 1 \quad (70)$$

$$x_1/(12 \times x_2) \leq 1 \quad (71)$$

$$(1.5 \times x_6 + 1.9)/x_4 \leq 1 \quad (72)$$

$$(1.1 \times x_7 + 1.9)/x_5 \leq 1 \quad (73)$$

$$, 0.7 \leq x_2 \leq 0.8, 17 \leq x_3 \leq 28, 2.6 \leq x_1 \leq 3.6 \quad (74)$$

$$, 2.9 \leq x_6 \leq 3.9, 7.3 \leq x_4 \leq 8.3, 7.8 \leq x_5 \leq 8.3$$

$$5.0 \leq x_7 \leq 5.5$$

OSY formulation:

$$\min f_1(x) = -[25 \times (x_1 - 2)^2 + (x_2 - 2)^2 + (x_3 - 1)^2 \quad (75)$$

$$]+(x_4 - 4)^2 + (x_5 - 1)^2$$

$$\min f_2(x) = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 \quad (70)$$

$$\text{S.t.} \quad (76)$$

$$C_1(x) = x_1 + x_2 - 2 \geq 0 \quad (77)$$

$$C_2(x) = 6 - x_1 - x_2 \geq 0 \quad (78)$$

$$C_3(x) = 2 - x_2 + x_1 \geq 0 \quad (79)$$

$$C_4(x) = 2 - x_1 + 3 \times x_2 \geq 0 \quad (80)$$

$$C_5(x) = 4 - (x_3 - 3)^2 - x_4 \geq 0 \quad (81)$$

$$C_6(x) = (x_5 - 3)^2 + x_6 - 4 \geq 0 \quad (82)$$

$$1 \leq x_3, x_5 \leq 5 \quad 1 \leq x_4 \leq 60 \leq x_1, x_2, x_6 \leq 10 \quad (83)$$

BNH formulation:

$$\min f_1(x) = 4 \times x_1^2 + 4 \times x_2^2 \quad (84)$$

$$\min f_2(x) = (x_1 - 5)^2 + (x_2 - 5)^2 \quad (85)$$

$$\text{S.t.} \quad (86)$$

$$+x_2^2 \leq 25C_1(x) = (x_1 - 5)^2 \quad (87)$$

$$C_2(x) = (x_1 - 8)^2 + (x_2 + 3)^2 \geq 7.7 \quad (88)$$

$$0 \leq x_2 \leq 30 \leq x_1 \leq 5 \quad (89)$$

Cantilevered Beam Design Problem:

$$\text{Min } f_1(x) = 100 \times (x_5 \times x_6 + x_7 \times x_8 + x_9 \times x_{10} + x_1 \times x_2 + x_3 \times x_4) \quad (90)$$

$$\text{Min } f_2(x) = 10000 \times \left(\frac{244}{x_5 \times x_6^3} + \frac{148}{x_7 \times x_8^3} + \frac{76}{x_9 \times x_{10}^3} + \frac{28}{x_1 \times x_2^3} + \frac{4}{x_3 \times x_4^3} \right) \quad (91)$$

Subject to

$$g_1(x) = 10.7143 - \frac{x_5 \times x_6^2}{1000} \leq 0 \quad (92)$$

$$g_2(x) = 8.5714 - \frac{x_7 \times x_8^2}{1000} \leq 0 \quad (93)$$

$$g_3(x) = 6.4286 - \frac{x_9 \times x_{10}^2}{1000} \leq 0 \quad (94)$$

$$g_4(x) = 4.2857 - \frac{x_1 \times x_2^2}{1000} \leq 0 \quad (95)$$

$$g_5(x) = 2.1429 - \frac{x_3 \times x_4^2}{1000} \leq 0 \quad (96)$$

$$g_6(x) = x_6 - 20 \times x_5 \leq 0 \quad (97)$$

$$g_7(x) = x_8 - 20 \times x_7 \leq 0 \quad (98)$$

$$g_8(x) = x_{10} - 20 \times x_9 \leq 0 \quad (99)$$

$$g_9(x) = x_2 - 20 \times x_1 \leq 0 \quad (100)$$

$$g_{10}(x) = x_4 - 20 \times x_3 \leq 0 \quad (101)$$

$$x_1 = [1, 30, 2.4, 45, 2.4, 45, 1, 30, 1, 30] \quad (102)$$

$$[5, 65, 3.1, 60, 3.1, 60, 5, 65, 5, 65] = x_u \quad (103)$$

Multi-objective car side formulation: (104)

$$f_1(x) = 1.98 + 4.9 \times x_1 + 6.67 \times x_2 + 6.98 \times x_3 + 4.01 \times x_4 + 1.78 \times x_5 \\ + 0.00001 \times x_6 + 2.73 \times x_7$$

$$f_2(x) = 4.72 - 0.5 \times x_4 - 0.19 \times x_2 \times x_3 \quad (105)$$

$$f_3(x) = 0.5 \times (10.58 - 0.674 \times x_1 \times x_2 - 0.67275 \times x_2 + 16.45 - 0.489 \\ \times x_3 \times x_7 - 0.843 \times x_5 \times x_6 \quad (106)$$

$$g_1(x) = 1.16 - 0.3717 \times x_2 \times x_4 - 0.0092928 \times x_3 \leq 1 \quad (107)$$

$$g_2(x) = 0.261 - 0.0159 \times x_1 \times x_2 - 0.06486 \times x_1 - 0.019 \times x_2 \times x_7 \\ + 0.0144 \times x_3 \times x_5 + 0.0154464 \times x_6 \leq 0.32 \quad (108)$$

$$g_3(x) = 0.214 + 0.00817 \times x_5 - 0.045195 \times x_1 - 0.0135168 \times x_1 \\ + 0.03099 \times x_2 \times x_6 - 0.018 \times x_2 \times x_7 + 0.007176 \times x_3 \\ + 0.023232 \times x_3 - 0.00364 \times x_5 \times x_6 - 0.018 \times x_2^2 \leq 0.32 \quad (109)$$

$$g_4(x) = 0.74 - 0.61 \times x_2 - 0.31296 \times x_3 - 0.031872 \times x_7 + 0.227 \times x_2^2 \\ \leq 0.32 \quad (110)$$

$$g_5(x) = 28.98 + 3.818 \times x_3 - 4.2 \times x_1 \times x_2 + 1.2729 \times x_6 - 2.68065 \times x_7 \\ \leq 32 \quad (111)$$

$$-5.057 \times x_1 \times x_2 - 3.795 \times x_2 - 3.4431 \times x_7 + g_6(x) = 33.86 + 2.95 \times x_3 \\ 1.45728 \leq 32 \quad (112)$$

$$g_7(x) = 46.36 - 9.9 \times x_2 - 4.4505 \times x_1 \leq 32 \quad (113)$$

$$g_8(x) = 4.72 - 0.5 \times x_4 - 0.19 \times x_2 \times x_3 \leq 4 \quad (114)$$

$$10.58 - 0.674 \times x_1 \times x_2 - 0.672 \times x_2 \leq 9.9 g_9(x) = \quad (115)$$

$$g_{10}(x) = 16.45 - 0.489 \times x_3 \times x_7 - 0.843 \times x_5 \times x_6 \leq 15.7 \quad (116)$$

$$\begin{aligned} 0.5 \leq x_1, x_3, x_4 \leq 1.5, \quad 0.45 \leq x_2 \leq 1.35, \quad 0.875 \leq x_5 \leq 2.625, \quad 0.4 \\ \leq x_6, x_7 \leq 1.2, \end{aligned} \quad (117)$$

Chapter 5

A Comprehensive Review of Mining Supply Chain Optimization:

Methods, Problems, and Technologies

Iman Rahimi¹, Amir H. Gandomi^{*1,2}, Mohammad Reza Nikoo³

¹*Faculty of Engineering and Information Technology, University of Technology Sydney, Australia*
(emails: iman83@gmail.com, gandomi@uts.edu.au, mohsen.mousavi@uts.edu.au,
fang.chen@uts.edu.au)

²*University Research and Innovation Center (EKIK), Óbuda University, 1034 Budapest, Hungary*

³*Department of Civil and Architectural Engineering, Sultan Qaboos University, Muscat, Oman*
(email: m.reza@squ.edu.om)

Abstract

Various mathematical models and algorithms have been developed in the field of mining supply chain optimization. This study reviews the most state-of-the-art papers in the mining supply chain optimization field, which covers methods, problems, and technologies. The study also conducts a bibliometric analysis, which includes citations, keywords, authors, institution, and publisher analysis. The review paper overviews the most recent applications that have been widely used in the field, as well as the integration of advanced methodologies in optimizing the mining supply chain. Furthermore, the review paper investigates the challenges and limitations of these models and the algorithms in real case studies. In the end, comprehensive answers to the research questions are provided. The study concludes with some insights into future works.

Keywords - Mining, Supply Chain, Optimization, Review.

1. Introduction

In the ever-evolving world of business, enhancing supply chains has claimed a prime spot on the priority list. This has spurred the pursuit of advancement in supply chain mining research. Supply Chain Mining Optimization (SCMO) is the ultimate convergence of multiple domains, including supply chain management, data mining, and optimization methods. In this thorough review paper, our aim is to delve into the complexities of SCMO, offering a comprehensive view of its genesis, methodologies, and present state.

The importance of optimization in the realm of supply chain management cannot be emphasized enough. In our ever-evolving global markets, companies constantly seek ways to enhance customer satisfaction, reduce costs, and boost efficiency. Given the intricacies of these modern markets, optimization techniques serve as a valuable tool for streamlining decision-making processes. This research investigates the application of deterministic, stochastic, and dynamic optimization models within the context of supply chain mining, shedding light on their practical implementation.

The evolution of SCMO strategies mirrors broader trends in commerce and technology. This field has progressed from traditional linear programming methods to more advanced heuristic approaches to meet the constantly changing needs of businesses and navigate the challenges presented by increasingly complex supply chains.

In the mining industry, the foremost objective of optimization is to maximize the financial returns derived from mining operations while carefully considering various constraints and factors. These factors encompass aspects such as the layout and design of the excavation site, the planning of mining activities, equipment allocation, waste management, and the mitigation of environmental concerns. Achieving optimal results in open pit mining necessitates skillful management of the delicate equilibrium between extracting the most

valuable materials from the earth and minimizing operational expenses, safety hazards, and environmental impact.

To tackle the intricacies arising from the intricate interplay of geological, operational, and economic factors, a plethora of mathematical and computational tools are employed. These techniques encompass a spectrum of options, ranging from conventional methods like integer and linear programming to more advanced ones such as mixed-integer programming, dynamic programming, and optimization through simulation. Moreover, the toolbox for optimizing open-pit mining operations has expanded considerably thanks to technological advancements like integrating geographic information systems (GIS), geological modeling software, and sophisticated data analytics.

Mining enterprises and researchers employ optimization techniques to make well-informed choices concerning pit design, production timetables, and resource allocation. These decisions have the potential to result in higher revenues, reduced waste production, enhanced safety protocols, and minimized adverse environmental impacts. As mining operations evolve and encounter growing complexities, the realm of open pit mining optimization will maintain its pivotal role in guiding the sustainable and efficient extraction of minerals from the Earth's surface layer.

This study proposes a comprehensive review and analysis of the advantages and disadvantages of various methods, provides an overview of the literature, and addresses an in-depth analysis of notable studies within the field of the mining supply chain. In addition, a close exploration of real-world examples from various industries is undertaken with the goal of addressing the practical aspects of SCMO. These case studies demonstrate the use of optimization methods and provide insight into outcomes,

operational enhancements, and implementation obstacles faced. Despite progress in SCMO, challenges and limitations remain. These include data reliability issues, costly computing, limited scalability, and practical barriers posed by current models.

The remainder of this paper is organized as follows:

The next section provides research methodology. Section 3 illustrates the fundamental concept. Section 4 presents a literature review of the field. Section 5 shows the research questions. In section 6, a summary of other reviews is provided. In section 7, supply chain mining optimization techniques are shown. Section 8 displays case studies and applications. Section 9 demonstrates technologies. Sections 10, 11, and 12 demonstrate a bibliometric analysis, answers to research questions; discussion, future trends, and conclusion, respectively.

2. Research Methodology

Google Scholar and Scopus are two main sources that were used to find the articles. For this aim, the keywords “supply chain”, “optimization” and “mining” have been used. More than 350 articles have been found. From the title, abstract, and index Keywords, the articles are filtered based on the following sub-categories:

In Table 5.1, different categories cover a diversity of different fields, including computer science, mathematics, supply chain management, and engineering. The review starts with a comprehensive literature review after filtering based on the keywords. Supply chain mining optimization techniques also cover the works with a brief review of implemented techniques, key findings, application areas, and limitations have been provided. Case

studies and applications are the next section of this study, which are proposed. Moreover, challenges that arise from the literature are reviewed. Bibliometric analysis is another part of the review. The bibliometric analysis covers citation analysis over time, publication trend analysis over time, keywords analysis, Journal analysis, Research keyword analysis over time, and prolific institutions.

Table 5.2 Various categories of mining optimization models

Models									
Blending	Capacity Expansion Problems	Vessel Allocation and Scheduling Models	Supply Chain Production and Scheduling	Operational Equipment Allocation Models	Train scheduling problems	Stockyard planning Models	Conveyor Routing Models	Layout and design models	Mine production scheduling
Ore capacity expansion	Short-term mine optimization	Mine to mill optimization	Mining production scheduling	Coal supply chain	Mine capacity	Mine facility location	Mine production	Artificial Intelligence in mine	Coal blending
Ore Rail Scheduling	Ultimate pit optimization	Mine truck allocation	Mining rail scheduling	Conveyor routing	Mine complex	Mine haulage system	Mine pushback	Belt conveyor transport	Coal capacity expansion
Pit limit optimization	Underground mine	Mine truck dispatch	Mining shipping optimization	In-pit crushing scheduling	Mine Design	Mine phases optimization	Mine rail scheduling		Coal production planning
Short-range mine optimization	Optimization Underground	Mining blending	Mining stochastic planning	In-pit dumping scheduling	Mine equipment allocation	Mine planning optimization	Mine shipping		Coal production scheduling
Short-term mine optimization	Mining optimization	Mining conveyor routing	Mining stochastic scheduling	Machine learning in mine opt.	Mine equipment dispatch		Mine scheduling		Coal rail scheduling
		Mining layout optimization	Mining stockpile blending	Mine block sequence	Mine equipment selection				Coal stockpiling
		Mining production planning	Mining stockyard optimization						Coal stockyard

3. Fundamental Concept

The supply chain of mining optimization can be classified into various stages (Leal Gomes Leite et al., 2020), namely:

Mine, Railway, Port, Shipping, Client. Mine involves operations research problems related to open-pit and underground mines. Decisions such as operational equipment allocation, scheduling, mine layout, and design are classified into this category. Railways are decisions related to railway scheduling problems. Port and shipping covers routing and scheduling of cargo ships, respectively (Figure 5.1).

Table 5.2 provides an overview of research literature pertaining to optimization strategies within the mining supply chain. It categorizes various studies into different aspects of mine operations, including layout models, scheduling problems, operational equipment and allocation models, railway operations, port management, conveyor routing, shipping logistics, and supply chain management. Each category encompasses a range of studies addressing specific challenges and solutions within the mining industry.

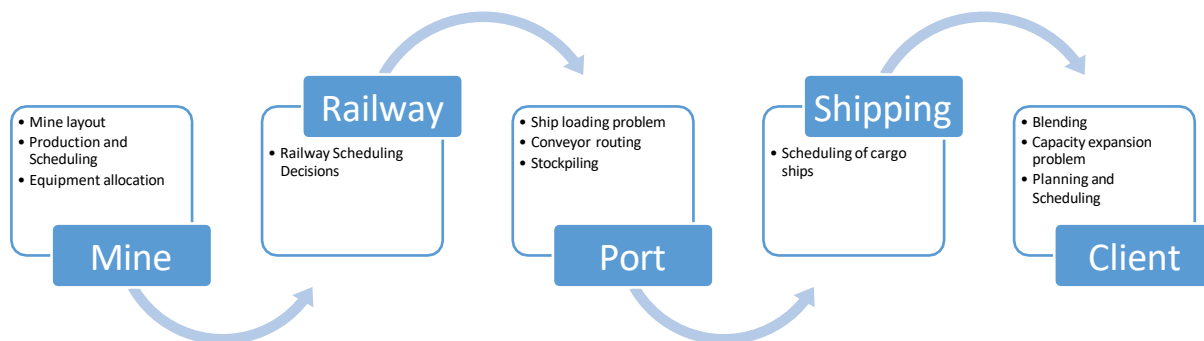


Figure 5.9 A Classification of the mining supply chain(Leal Gomes Leite et al., 2020)

Table 5.3 Summary of mining supply chain optimization classification

Mine	Layout and design models	(Caccetta, 2007; Kozan & Liu, 2011; Pimentel et al., 2010)(Bjørndal et al., 2012)(Ataee-Pour, 2005;Newman et al., 2010; Alford et al., 2007; Musingwini, 2016; Meagher et al., 2014; Nhleko et al., 2018;Osanloo & Ataei, 2003;Hochbaum & Chen, 2000; Underwood & Tolwinski, 1998;Klingman & Phillips, 1988; Ahmadi & Bazzazi, 2019; Asad & Topal, 2011; Sari & Kumral, 2018;Rahmanpour & Osanloo, 2017; Topal & Sens, 2010; Sandanayake et al., 2015; Grieco & Dimitrakopoulos, 2007; King et al., 2017; Whittle et al., 2018)
	Production and Scheduling Problems	(Bjørndal et al., 2012; Caccetta, 2007; Kozan & Liu, 2011; Pimentel et al., 2010; Askari-Nasab et al., 2011;COSTA, 2005; Mining, n.d.; Gershon, 1983; Askari-Nasab et al., 2010)
	Operational Equipment and Allocation Models	(Bjørndal et al., 2012;Pimentel et al., 2010; Kozan & Liu, 2011; Newman et al., 2010; Johnson, 1968; Musingwini, 2016; Munirathinam & Yingling, 1994; Alarie & Gamache, 2002)
Railway	Railway scheduling decision	(Kozan & Liu, 2011; Pimentel et al., 2010;Kozan & Liu, 2012; S. Q. Liu & Kozan, 2009;S. Q. Liu & Kozan, 2011; Salim & Cai, 1997; Singh et al., 2015)
Port	Stockyard planning models	(de Aguiar Nobrega, 1997; Ago et al., 2007;N. Boland et al., 2012; N. Boland et al., 2011;Belov et al., 2014, 2015; Hanoun et al., 2013)
	Conveyor routing models	(Fioroni et al., 2007; van Vianen et al., 2016; Que et al., 2016; MCNEARNY & NIE, 2000; Fagerholt, 1999; Xiao-ping et al., 2015; Que et al., 2016)
Shipping	Vessel allocation and scheduling models	(Shih, 1997; Q. Zeng & Yang, 2007;C. Liu, 2008; Christiansen et al., 2011; N. L. Boland & Savelsbergh, 2011; Arigoni et al., 2017)
Supply chain	Blending	(Ravindran & Hanline, 1980; Sherali & Puri, 1993; Chanda & Dagdelen, 1995; Lyu et al., 1995; J.-W. Lai & Chen, 1996;C.-M. Liu & Sherali, 2000)
	Planning and scheduling	(Askari-Nasab et al., 2010; Askari-Nasab et al., 2011; Samanta et al., 2005; Eivazy & Askari-Nasab, 2012; Fytas et al., 1993; Yashar et al., 2013)
	Capacity expansion problem	(Singh et al., 2012; Benalcazar et al., 2017)

In mine planning, several objectives could be considered as follows:

- Max total product volume (e.g. (Y. Li et al., 2016; Y. Li et al., 2018))
- Max value/ profit (e.g. (Topal & Sens, 2010; Sandanayake et al., 2015))
- Max revenue (e.g. (Dimitrakopoulos et al., 2007; King et al., 2017))
- Min operational cost (e.g. (Dimitrakopoulos et al., 2007; King et al., 2017))
- Min investment cost (e.g. (Groeneveld & Topal, 2011; Ahmadi & Shahabi, 2018))
- Min transport time (e.g. (Vagenas, 1991; Gamache et al., 2005)
- Min energy consumption/CO2 emission (e.g. (Patterson et al., 2017))
- Min volume deviations (penalties) (e.g. (R. C. Goodfellow & Dimitrakopoulos, 2016; Kumar & Chatterjee, 2017)
- Min operational rules deviations (penalties) (e.g. (Lamghari & Dimitrakopoulos, 2016),
- Min quality deviations (penalties) (e.g. (Nehring et al., 2012; Sarin & West-Hansen, 2005)
- Min idle time (e.g. (Singh et al., 2015; N. L. Boland & Savelsbergh, 2011)
- Min time schedule deviations (Penalties) (e.g. (Thomas et al., 2016; Abdekhodae et al., 2004)
- Max stockyard availability (e.g. (Abdekhodae et al., 2004))
- Min inventory average age (e.g. (Hanoun et al., 2013))
- Min energy consumption (CO2 emission) (e.g. (Patterson et al., 2017))

Constraints are addressed as follows:

- Equipment and Facilities Volume Capacity (e.g. (Grieco & Dimitrakopoulos, 2007; (King et al., 2017))
- Demand (e.g. (Montiel & Dimitrakopoulos, 2015; Moreno et al., 2017)
- Blending/Quality (e.g. (R. C. Goodfellow & Dimitrakopoulos, 2016; Kumar & Chatterjee, 2017)
- Pit Geometry (e.g. (Giannini, 1990; Underwood & Tolwinski, 1998)
- Equipment Allocation (e.g. (Y. Li et al., 2016), (Y. Li et al., 2016))
- Flow/ Inventory Balance (e.g. (Askari-Nasab et al., 2010; Askari-Nasab et al., 2011)
- Train Availability (e.g. (Singh et al., 2012))
- Berth Use (e.g. (Singh et al., 2012))
- Routing (e.g. (Vagenas, 1991; Beaulieu & Gamache, 2006)
- Frame Size (e.g. (Sari & Kumral, 2018))
- Capital Budget (e.g. (Y. Liu et al., 2009))

Almost the majority of the articles have focused on deterministic models. However, recently, researchers have concentrated on stochastic approaches such as discrete event simulation.

4. Literature Review

This section will explain supply chain mining and its role in modern business, the evolution of optimization techniques in supply chain management, and the aim and contribution of the paper.

H. Peng et al. (2009) presented a model for optimizing coal supply chains. It focuses on maximizing system profit while ensuring customer satisfaction. The model integrates various aspects of the supply chain, including production, washing, transportation, and sales. Applied to the Xuzhou coal mines, it makes references for a more systematic use of resources. However, a potential disadvantage is its specific focus on coal supply chains, which might limit its applicability to other industries or contexts.

Sandeman et al. (2010) explored the benefits of combining optimization models with simulation in mining operations. Two case studies demonstrate the effectiveness of this approach, emphasizing blending optimization in supply chains. The first study utilizes a fixed optimization model, while the second employs a dynamic model with ongoing adjustments. This integration enhances decision-making and system understanding. The complexity and time-intensive nature of integrating optimization with simulation models could be a drawback, especially in rapidly changing or less predictable operational environments.

Bodon et al. (2011) defined an approach for modeling a complex mining export supply chain using optimization and Discrete Event Simulation (DES) methods. The paper includes a case study of PT Kaltim Prima Coal in Indonesia, demonstrating the successful implementation of this approach. Additionally, the approach may require extensive data and expertise to accurately model and interpret the results.

Dimitrakopoulos (2013) focused on optimizing mining supply chains amidst geological uncertainties. It introduces a particle swarm optimization algorithm and uses k-means clustering to manage complex supply chain issues, including blending operations. A case study at Vale's Onça Puma nickel laterite mine is used to illustrate the methodology. However, the paper's complex models and heavy reliance on specific data and geological simulations could limit its practical applicability and make implementation challenging in various mining contexts.

Montiel & Dimitrakopoulos (2015) presented a method for optimizing mining complexes. It incorporates geological uncertainty through stochastic orebody simulations and considers multiple processing transportation systems. The approach aims to minimize deviations from capacity targets while enhancing the expected net present value (NPV). A case study demonstrates the method's effectiveness in a multi-pit copper operation. The computational intensity of the model might be challenging for some operations to implement effectively.

Canales-Bustos et al. (2017) suggested a multi-objective model focusing on economic, environmental, and technological aspects of decarbonized supply chains in mining. The above-mentioned paper introduced a multi-objective hybrid particle swarm optimization

(MOHPSO) for problem-solving and compares its performance with an epsilon-constraint-based algorithm (AUGMECON). Key areas include supply chain network design, emissions reduction, and optimization of processing plant efficiency. The complexity of the MOHPSO method might pose challenges in practical implementation, especially in scenarios with limited computational resources.

Barakos et al. (2016) critically examined the rare earth elements (REE) industry, particularly focusing on the global supply chain, market evaluation, resource classification, and legislative optimization. It discusses the past and current challenges in the REE industry, including market analysis errors and supply chain issues. The paper proposes an assessment tool for optimizing mining processes and highlights the need for a stronger global REE supply chain, emphasizing sustainable practices and international cooperation. However, the paper may include a heavy focus on theoretical models and a lack of practical, real-world implementation examples.

R. C. Goodfellow & Dimitrakopoulos (2016) presented a two-stage stochastic global optimization model for open pit mining complexes, integrating geological uncertainties into production scheduling. The model uses metaheuristics like simulated annealing, particle swarm optimization, and differential evolution. A case study demonstrates its effectiveness in enhancing expected net present value. The model's complexity and reliance on specific and accurate geological data might be a disadvantage, which could limit practical application and adaptability in varied mining contexts.

R. Goodfellow & Dimitrakopoulos (2017) presented a novel approach to mining optimization. The study introduces a two-stage stochastic approach, integrating various

aspects of mining operations into a single model. This includes managing geological uncertainties and optimizing multiple interrelated components of a mining complex. The paper demonstrates its methodology through case studies, showing significant improvements compared to traditional methods. Its complex mathematical and computational requirements which might limit its accessibility and practical implementation in smaller or less technologically advanced mining operations.

Del Castillo & Dimitrakopoulos (2019) focused on improving strategic planning in mining complexes. It introduces a dynamic stochastic optimization method to address supply uncertainty and incorporates capital expenditure decisions dynamically. The process uses an adapted multistage stochastic programming model, allowing for flexible mine designs. A case study on a copper mining complex demonstrates the model's capability to increase net present value significantly compared to traditional methods. Nevertheless, the paper's specific focus on a single mining complex may limit its applicability to other types of mining operations or industries.

Saliba & Dimitrakopoulos (2019) presented a sophisticated approach to optimizing mining operations under supply and market uncertainty conditions. The study proposes a framework that considers both supply uncertainty (geological) and market uncertainty (commodity prices), leading to more efficient and profitable operations. The study's results demonstrate significant improvements in metal production and net present values compared to traditional methods. The study focuses on a single gold mining complex may limit the generalizability of the findings to other types of mining operations or different commodities. The complexity and computational intensity of the approach might also pose practical challenges for wider adoption in the industry.

Shishvan & Benndorf (2019) presented a new approach for optimizing material dispatch in coal mines. It combines a deterministic optimization model with a stochastic simulation model, addressing the complexity of different material types in coal mining and their dispatch to spreaders. The approach is tested on a large mine, demonstrating its effectiveness in minimizing idle time due to the unavailability of dumping space. The complexity could limit its applicability in smaller mining operations or those with limited technological capabilities.

Dorrington & Olsen (2019) explored the design of a supply chain network for delivering asteroid-derived resources to Earth orbit customers. It focuses on maximizing the sellable mass or the total net present value of the mining venture. The reliance on advanced space technology and the need for extensive data about asteroid locations and compositions also present challenges.

Paithankar et al. (2020) presented a mathematical model combining extraction sequencing and cut-off grade policy optimization in open-pit mining. It considers grade uncertainty and stockpiling, aiming to maximize the discounted value of a mining operation. The model uses genetic algorithms and the maximum flow method. A notable limitation is the model's complexity and potential computational intensity, which may hinder its practical application in varying mining contexts.

J. Li et al. (2020) focused on optimizing coal supply chains considering various carbon emission policies. It proposes new optimization models integrating these policies, aiming to reduce carbon emissions while minimizing costs. The paper uses a coal supply chain network as a case study to evaluate the impact of emission cap, carbon tax, carbon trade,

and carbon offset policies. However, the reliance on detailed and accurate data for effective implementation could be a challenge.

Ospina-Mateus et al. (2020) introduced a mathematical model for the supply chain of non-metallic mining, accounting for uncertainties in materials and multi-level, multi-product capacity planning. It focuses on collaborative profit maximization within the supply chain, applied to the limestone industry in the Calamarí-Sucre mining district, Colombia. The model, implemented using GAMS software with RMINLP libraries and the DICOPT solver, reveals that higher raw material availability yields greater benefits. It also highlights a trend towards vertical integration in the sector, underscoring the model's utility in optimizing strategies across different supply chain levels.

Kalinowski et al. (2020) addressed the complex task of planning maintenance for Australia's largest coal rail network. It introduces a Mixed Integer Programming (MIP) model and a metaheuristic approach to efficiently schedule maintenance access windows (MAWs). This model aims to optimize maintenance resource allocation while minimizing the network's throughput impact. A key challenge in this approach is the balancing act between maintaining the rail network's condition and minimizing disruption to the transportation of goods.

Yaping & Bossman (2021) investigated the logistics and supply chain operations of Ghana's mining industry. It identifies challenges and inefficiencies within these operations and suggests strategies for optimization. The study employs a Vehicle Routing Problem (VRP) Optimization Model to enhance transportation efficiency for mining firms in Ghana. The research identifies key challenges like poor road infrastructure, variable lead times, and strict governmental regulations. However, the paper's focus on Ghana's mining

industry may limit its applicability to other regions or industries. Additionally, the complex nature of the VRP model might pose implementation challenges for some firms.

Fattahi et al. (2021) focused on creating a sustainable mining supply chain that incorporates wind and solar generators at mining sites. The paper introduces a cost-efficient multi-stage stochastic program that integrates strategic and tactical planning decisions. This approach aims to balance economic goals with greenhouse gas emission mitigation and social impacts, using a case study in Iran related to zinc concentrate production. The study might be its specific focus on the zinc industry in Iran, which could limit the generalizability of the findings to other mining sectors or geographical regions.

LaRoche-Boisvert & Dimitrakopoulos (2021) presented a case study on applying simultaneous stochastic optimization (SSO) to the Rosebel Gold Mines complex. The study focuses on maximizing net present value (NPV) and optimizing production schedules across three deposits, considering geological uncertainties. The approach identifies the semi-autogenous grinder (SAG) mill as a bottleneck and includes material hardness management. The study results in an 18-year production plan, emphasizing SSO's effectiveness in complex mining operations. The study focuses on a single mining complex, which might limit the generalizability of the results to other mining operations with different characteristics or challenges.

Isnafitri et al. (2021) investigated factors influencing the performance of coal supply chains in Indonesia. It employs Structural Equation Modeling (SEM) to analyze the relationships between variables like supply chain integration, information sharing, supplier relationship management, and supply chain performance. A survey of coal mining companies provides

empirical data. A limitation of this study could be its specific focus on Indonesia's coal industry, potentially limiting its general applicability to other regions or sectors.

Soleimani (2021) presented a new closed-loop supply chain model tailored for the mining industry, focusing on sustainable. The model is applied to a travertine quarry as a case study, demonstrating its effectiveness in enhancing sustainability in the mining sector. The research integrates transportation and operational aspects to address the unique challenges of the mining industry, aiming to balance profitability with environmental responsibility. A possible limitation of the study could be its specific application to a travertine quarry, which may not fully represent other types of mining operations or generalize to different contexts in the mining industry.

Haonan et al. (2021) focused on optimizing the blending process in open-pit mining. It extends a decomposition-based algorithm to solve the blending problem involving multiple mines and ports over multiple periods. The paper introduces new heuristics like Prune Routine and Ranked TopoSort, significantly improving solution quality and reducing computation time. The research aims to balance grade and quality targets in mining products. A disadvantage might be its complex mathematical models, which might be challenging to implement in practical, real-world mining operations.

LaRoche-Boisvert et al. (2021) presented a novel approach to optimizing industrial mining complexes. It introduces a stochastic simultaneous optimization (SSO) model that simultaneously optimizes the schedule of production and mines-to-port transportation of mining complexes, particularly under uncertain material supply. This method is particularly applied to iron ore mining complexes and aims to minimize deviations from product requirements while effectively utilizing mine-to-port transportation equipment.

The study primarily focuses on long-term strategic planning, which may not fully address short-term operational challenges. Additionally, the model's complexity could present difficulties in practical implementation, and its reliance on accurate data input might be challenging in dynamically changing mining environments.

Rahimi et al. (2022) applied two metaheuristics, one as a trajectory-based algorithm known as simulated annealing and the other one as a population-based algorithm known as a genetic algorithm for a constrained open-pit mining problem considering three benchmark datasets and compared the results of these algorithms with each other.

5. Research Questions

Mining supply chain problems are crucial as they have financial benefits for countries. Many mathematical models and algorithms have been developed in the field and exist in the literature. To achieve the goals of this review to overview the current problems along with the challenges, the following research questions are raised:

RQ1: What are the current gaps in the literature in mining supply chain optimization?

RQ2: What algorithms are mostly used to optimize mining supply chain problems?

RQ3: What are the challenges in the implementation of the mining models and implementation of the algorithms?

RQ4: What are the trends and the emerging research topics under the scope of this review?

RQ5: What are the key challenges and limitations faced in applying optimization algorithms in mining supply chain management?

RQ6: How can the decreasing trend in average citations from 2020 be interpreted in terms of the field's research impact?

RQ7: what can the peaks in publication trends show about the field of specific productivity or focus within the field during those years?

RQ8: How can the fluctuation in the occurrence of keywords like “Supply Chain” and “Optimization” be correlated with technological advancements, industry changes, or academic trends?

These research questions are answered at the end of this study and in the discussion section.

6. Other Reviews

Bjørndal et al. (2012) discussed several applications of operations research in the natural resource industry. The aforementioned study discusses topics like agro-ecological land use planning in Kenya, and optimization in underground mine design.

Kozan & Liu (2011) provided a comprehensive review of operations research methodologies, which are applied in mining industries, and categorizes these methodologies into four main ones: design of mine, mine production, mine transportation, and mine evaluation.

Meagher et al. (2014) reviewed existing methods of pushback design in open pit mining and discussed the issue of gap problems that refer to inconsistent sizes between successive pushbacks.

Ataee-Pour (2005) addressed a critical review of the optimization of stope layout, which is classified using available literature, was addressed. In the above-mentioned review, the capabilities, restrictions and applications of these algorithms are evaluated and compared. Alford et al. (2007) addressed the challenges for optimization in the design of underground mines, which is known as a complex task because of a wide range of mining strategies and solutions for each deposit.

Zeng et al. (2021) provided an in-depth analysis of mine supply chain management (MSCM) research, evaluating over 200 papers. The study organizes these papers into nine distinct categories, including capacity planning, logistics, inventory control, network architecture, economic considerations, performance assessment, risk management, environmental conservation, and scheduling of production. The above-mentioned work offers insights into potential research opportunities and managerial implications in MSCM, focusing on various qualitative and quantitative methodologies across these categories. This interdisciplinary approach highlights the complexity and evolving nature of MSCM research.

Leal Gomes Leite et al. (2020) addressed a comprehensive bibliographical review of operational research applications in the mining industry. It maps applications within each isolated link of the supply chain, including mine, railway transportation, port, and maritime shipping. The study also explores interlinking research connecting multiple stages of the mining chain. It highlights the importance of state-of-the-art optimization tools in managing complex supply chains, offering insights and future research directions in this domain.

7. Supply Chain Mining Optimization Techniques

Mining supply chain optimization is a critical challenge that requires innovative approaches to enhance efficiency, sustainability, and profitability. A global demand for minerals and resources escalates, so does the imperative for mining operations to transcend traditional boundaries, leveraging state-of-the-art optimization techniques to overcome the multifaceted challenges they face. This subsection collates an array of seminal works spanning from advanced algorithmic solutions to pragmatic application insights, which are presented in Tables 5.3-5.11.

The advent of algorithms has heralded a new field in the optimization of mining supply chains, enabling stakeholders to navigate the intricate web of production scheduling, material transportation, and resource allocation with unprecedented precision and agility.

Looking ahead, this subsection not only covers the achievements to date but also casts a forward gaze toward future horizons in mining supply chain optimization. It beckons the integration of cutting-edge technologies such as artificial intelligence, blockchain, and the Internet of Things (IoT), promising to revolutionize mining operations by enhancing real-time decision-making, transparency, and traceability. Furthermore, it advocates for a multidisciplinary approach, drawing from fields as diverse as operations research, environmental science, and data analytics, to forge comprehensive solutions that are not only economically viable but also environmentally sustainable and socially responsible.

Table 5.4 Summary of techniques applied to mining supply chain optimization problems

Authors	Algorithm	Key findings	Application areas	Limitation/Disadvantages
(Haonan et al., 2021)	Decomposition-based algorithm (DA) with prune routine, ranking-based topological sorting heuristic, adaptive algorithm	Enhanced efficiency in production scheduling and blending optimization	blending problems involving multiple mines and ports	Complexity of model and algorithms, potential limited applicability in diverse mining contexts
(Dorrington & Olsen, 2019)	Location-routing problem solving, integer programming	Supply chain network for asteroid mining	Design of a supply chain network for asteroid-derived resources delivery to Earth orbit customers.	High complexity and significant computational resources required, reliance on advanced space technology and extensive data
(Soleimani, 2021)	Sustainable closed-loop supply chain (CLSC) model with fixed-charge transportation	Integration of sustainability into the mining supply chain with a focus on profit maximization, energy consumption, and service levels	Optimization of supply chains in the travertine quarry sector	High complexity and specificity to travertine quarries, which may limit broader applicability
(Goettsch et al., 2020)	Dynamic simultaneous optimization, multistage stochastic programming	Substantial improvement in expected net present value, dynamic investment decisions over the production schedule	Strategic planning of open pit mining complexes	Complexity of the model, potential challenges in data requirements and computational resources

Table 5.5 Summary of techniques applied to mining supply chain optimization problems (Continue)

Authors	Algorithm	Key findings	Application areas	Limitation/Disadvantages
(Paithankar et al., 2020)	Genetic algorithms, maximum flow method	Improved optimization considering grade uncertainty and stockpiling, demonstrating significant value increase in mining operations	Optimization of extraction sequences and cut-off grade policies in open-pit mining	Complexity and computational intensity of the model, challenges in practical application due to specific mining scenarios
(LaRoche-Boisvert & Dimitrakopoulos, 2021)	Stochastic simultaneous optimization	Minimizing deviations from product requirements while maximizing mines-to-port equipment utilization	Production scheduling and mines-to-port transportation in iron ore mining complexes	Model complexity and reliance on accurate data for uncertain material supply
(Montiel & Dimitrakopoulos, 2015)	Stochastic orebody simulations, optimization of processing paths and transportation systems	Reduction of deviations from capacity targets, improvement in expected NPV	Mining complexes, specifically a multipit copper operation	Complexity of the model and reliance on detailed geological data, which could limit practical applicability in varied mining contexts
(Shishvan & Benndorf, 2019)	Combination of deterministic optimization and stochastic simulation	Minimization of equipment idle time, enhanced efficiency in dispatching	Material dispatch in coal mines	Complexity and computational intensity, potential limitation in practical application

Table 5.6 Summary of techniques applied to mining supply chain optimization problems (Continue)

Authors	Algorithm	Key findings	Application areas	Limitation/Disadvantages
(J. Li et al., 2020)	Salp swarm algorithm based on differential evolution (DE-SSA)	Developed a model to minimize low-carbon production costs under various carbon emission policies	Coal supply chain network optimization	The model's complexity and potential computational intensity might hinder its practical application in varying contexts
(Goodfellow & Dimitrakopoulos, 2017)	Two-stage stochastic global optimization model, metaheuristics (simulated annealing, particle swarm optimization, differential evolution)	The model maximizes economic value under geological uncertainty, enhances risk management in mining operations	Production scheduling in open pit mining complexes	Complexity and reliance on detailed geological data, potential computational intensity
(D. Wang et al., 2015)	Linear programming combined with Monte Carlo simulation	Improved decision-making for production scheduling considering uncertainties in coal quality and market demand	Production scheduling in coal mines	Complexity of the simulation model and potential computational intensity, possibly limiting scalability and adaptability to different mining operations
(Barakos et al., 2016)	Review of concentration processes in the REE supply chain	Importance of efficient concentration processes for environmental and economic sustainability in the REE supply chain	Rare Earth Elements supply chain	The paper primarily provides a review rather than presenting new empirical data or modeling approaches

Table 5.7 Summary of techniques applied to mining supply chain optimization problems (Continue)

Authors	Algorithm	Key findings	Application areas	Limitation/Disadvantages
(Canales-Bustos et al., 2017)	Multi-objective MILP, Multi-objective Hybrid Particle Swarm Optimization (MOHPSO)	Integrated approach for optimizing economic and environmental objectives in mining supply chains	Sustainable mining supply chain design	Complexity, specific data requirements, and significant computational resources needed
(Kalinowski et al., 2020)	Mixed Integer Programming (MIP) model, Matheuristic approach	Efficient scheduling of maintenance resources in rail network to minimize throughput impact	Maintenance scheduling for the coal rail network	Complexity and specific data requirements, potentially limited to larger-scale applications
(Yaping & Bossman, 2021)	Vehicle Routing Problem (VRP) Optimization using Microsoft Excel Solvers	Identified and addressed challenges in logistics and supply chain operations in Ghana's mining industry	Transportation efficiency in mining firms in Ghana	Focus primarily on Ghana, which may limit the model's applicability in other regions or industries
(Dimitrakopoulos, 2013)	Particle swarm optimization, k-means clustering	Enhanced efficiency in solving complex non-linear optimization problems in the mining supply chain	Mining supply chain optimization, blending operations under geological uncertainty	The complexity of the models, reliance on specific geological data for optimization

Table 5.8 Summary of techniques applied to mining supply chain optimization problems (Continue)

Authors	Algorithm	Key findings	Application areas	Limitation/Disadvantages
(Sandeman et al., 2010)	Integration of optimization models with simulation models	Demonstrates advantages of integrating optimization with simulation for dynamic and accurate mine planning	Mine planning and optimization	Complexity of integrating models; potentially high computational requirements
(Isnafitri et al., 2021)	Optimization model, Oracle Crystal Ball Software	Optimization of truck selection for different routes to minimize total investment and transportation costs	Truck allocation in open pit mining	The complexity of the model and the requirement for extensive iterations for optimization may limit its practical applicability.
(Fattahi et al., 2021)	Multi-stage stochastic programming	Enhanced sustainability through economic, environmental, and social considerations in mining supply chains	Integration of renewable energy in mining supply chains	Complexity of the stochastic model, specific focus on zinc mining in Iran, which may limit generalizability
(LaRoche-Boisvert et al., 2021)	Simultaneous Stochastic Optimization (SSO)	Maximization of net present value and gold production over an 18-year period; identification of the SAG mill as a bottleneck	Optimization of various components of a gold mining operation, including production scheduling and resource allocation	High complexity of the model; requirement of extensive data for accurate optimization; bottleneck challenges at the SAG mill

Table 5.9 Summary of techniques applied to mining supply chain optimization problems (Continue)

Authors	Algorithm	Key findings	Application areas	Limitation/Disadvantages
(Oyola & Amaya-Mier, 2019)	Mixed Integer Linear Programming (MILP)	Proposed efficient and cost-effective methods for OTR tire recycling and disposal	Reverse logistics network for managing OTR tire waste in the mining industry	Specific focus on the Colombian mining industry; may require adaptation for broader applicability
(Bodon et al., 2011)	Combination of Optimization and Discrete Event Simulation (DES)	Demonstrated successful application in PT Kaltim Prima Coal's supply chain for capacity analysis and evaluating expansion options	Export supply chain of the mining industry, focusing on capacity analysis and expansion options	The complexity of integrating optimization with DES and potential challenges in data requirements and computational resources
(Saliba & Dimitrakopoulos, 2019)	Stochastic optimization framework integrating supply and market uncertainties	Enhanced decision-making in mining optimization by considering uncertainties in both supply and market factors	Optimization of mining, destination, and processing decisions in a multi-pit gold mining complex	Complexity of the model and the requirement for detailed data and simulations for accurate implementation
(Epstein et al., 2012)	Capacitated multicommodity network flow formulation	Integrated optimization of mining and processing operations, enhancing overall efficiency	Long-term planning in underground and open-pit copper mines	High complexity and computational demands; challenges in solving large-scale instances effectively

Table 5.10 Summary of techniques applied to mining supply chain optimization problems (Continue)

Authors	Algorithm	Key findings	Application areas	Limitation/Disadvantages
(Both & Dimitrakopoulos, 2020)	Stochastic optimization, simulated annealing metaheuristic	Reduction in costs related to shovel movements, improved truck utilization, significant operational cost reductions	Short-term production scheduling and fleet management in open-pit mining	Complexity and potential computational intensity; specific focus on open-pit mining which may limit broader applicability
(Kusi-Sarpong et al., 2016)	Fuzzy Theory, DEMATEL, ANP	Developed a framework to assess the impact of GSCM practices on sustainability performance in mining	Evaluation of GSCM practices in the Ghanaian mining industry	Focus on the Ghanaian context may limit generalizability to other regions or industries
(Ozdemir & Kumral, 2019)	Simulation-based optimization, linear programming	Improved utilization of truck-shovel systems, addressing uncertainties in mining operations	Optimization of truck-shovel material handling in surface mining	Complexity of the model and the need for accurate data to simulate real-world scenarios
(Gupta et al., 2021)	AHP for vehicle prioritization, DEA for efficiency evaluation	Optimizes transportation costs and efficiency, minimizes GHG emissions	Sustainable transportation in the mining industry	Complexity in integrating AHP and DEA, reliance on extensive data for accurate results

Table 5.11 Summary of techniques applied to mining supply chain optimization problems (Continue)

Authors	Algorithm	Key findings	Application areas	Limitation/Disadvantages
(Azzamouri et al., 2021)	Dynamic blending approach combined with alternative routings and security stocks	Introduces flexibility in blending operations by incorporating alternative routings and security stocks, enhancing risk management in the supply chain	Phosphate supply chain, specifically in blending operations	Complexity in implementation and potential challenges in adapting to different types of supply chains or mining operations
(Thomas et al., 2013)	Mixed Integer Programming, Lagrangian relaxation-based distributed algorithm	Improved computational efficiency in scheduling with limited resources in coal supply chains	Multi-resource constrained scheduling in coal supply chains	Complexity of the distributed algorithm and its applicability to specific scenarios of coal supply chains
(Farghaly et al., 2021)	Linear programming for blending and production optimization in mining	Developed a model to efficiently manage production and distribution in mining, optimizing costs and assays	Optimizing plant operations, including transportation and metallurgical processing in mining	Complexity in implementation and requirement of detailed data from various mining locations
(Akbari-Kasgari et al., 2022)	Mathematical modeling for sustainable supply chain design	Importance of backup suppliers for resilience, trade-offs in economic, environmental, and social performances	Copper industry supply chain	Specific focus on copper industry, complexity in balancing sustainability and resilience, applicability in different contexts

Table 5.12 Summary of techniques applied to mining supply chain optimization problems (Continue)

Authors	Algorithm	Key findings	Application areas	Limitation/Disadvantages
(Gupta et al., 2021; Kumar & Dimitrakopoulos, 2019)	Simultaneous stochastic optimization with geo-metallurgical decisions	Improved decision-making by incorporating grade, material type uncertainty, and geo-metallurgical properties; higher NPV	Optimization of copper-gold mining complex	The complexity of the model and the need for detailed data, potential challenges in practical implementation
(Njuaem & Ogundare, 2023)	Combining qualitative and quantitative approaches within a unified framework for assessment	The need for a systemic approach in assessing sustainability in supply chains, acknowledging globalization complexities	Mining and manufacturing industries' supply chains, with a focus on sustainability amid globalization	challenges in integrating global aspects into traditional supply chain models, the complexity of the proposed mathematical model
(Strang, 2012)	Sustainable closed-loop supply chain (CLSC) model with fixed-charge transportation	Integration of sustainability into the mining supply chain with a focus on profit maximization, energy consumption, and service levels	Optimization of supply chains in the travertine quarry sector	High complexity and specificity to travertine quarries, which may limit broader applicability

8. Case Studies and Applications

Optimization plays a crucial role in improving efficiency and sustainability within the mining industry. The case studies on optimization in the area of mining operations provide insights into challenges and lessons that can be learned from implementing various optimization strategies within the industry.

In the area where the pressure to maximize productivity is paralleled by an equally compelling to minimize environmental impact and ensure sustainable practices, optimization methods are not merely a tool for improvement but a necessity. The presented case studies present rich evidence showcasing how diverse optimization approaches have been successfully applied to achieve remarkable efficiencies, enhance decision-making processes, and secure economic viability, all while adhering to the principles of sustainability.

Tables 12-24 collectively emphasize the significance of innovative optimization approaches and adaptive strategies to address the various challenges in mining operations and supply chain management and highlight different optimization strategies. Moreover, the tables provide attention to the strategies that are essential for the mining industry's resilience.

The tables highlight how the industry has included change, responding to pressures such as fluctuating market demands, stringent environmental regulations, and the advent of new technologies with agility and innovation. From the optimization of truck-shovel operations

for improved material handling to the strategic design of mine-to-port supply chains and the integration of green supply chain practices, each case study contributes a piece to the puzzle of how optimization can lead to more informed, efficient, and responsible mining practices.

Moreover, these studies are considered an example of future explorations in the mining optimization domain. They not only reveal the current state of play but also pave the way for emerging research areas, such as the application of artificial intelligence and machine learning in predictive analytics, the exploration of sustainable and circular economy models within mining operations, and the development of more robust risk management frameworks to better anticipate and mitigate uncertainties.

Table 5.13 Consolidated Summary of Case Studies on Optimization in Mining Operations

Authors	Case Study Focus	Key Outcomes	Challenges Faced	Lessons Learned
(Ozdemir & Kumral, 2019)	Optimization of truck-shovel material handling in a multi-pit surface mine	A 9.4% increase in production per shift, with a total increase of 6.0 K tonnes.	Handling uncertain mining conditions and parameters like loading time and truck cycle time	Importance of simulation-based optimization and real-time truck dispatching to enhance production and equipment utilization
(Haonan et al., 2021)	Optimization for a multi-pit mine-to-port supply chain	Implementation of simulation-based optimization led to more efficient planning over a 5-week horizon, considering various quality attributes and blending targets	Balancing numerous operational constraints (e.g., transport capacity, processing plant capacity) and precedence constraints in a complex mine-to-port network	Demonstrated the effectiveness of simulation in managing complex logistics and operational constraints in mining supply chains, highlighting the importance of flexibility in planning and optimization strategies
(Dorrington & Olsen, 2019)	Design of an asteroid mining supply chain network	The optimal route and depot location could deliver between 33.15% and 52.63% of the extracted mass in a single trip	Formulating and solving a complex location-routing problem for optimal facility location and vehicle routing	Highlighted the complexity of a two-echelon, four-layer supply chain network in space and the importance of heuristic algorithms in solving such problems

Table 5.14 Consolidated Summary of Case Studies on Optimization in Mining Operations (Continue)

Authors	Case Study Focus	Key Outcomes	Challenges Faced	Lessons Learned
(Gupta et al., 2021)	Developing a multi-objective optimization model for sustainable transportation in the coal mining industry	The model maximizes efficiency, minimizes costs, and reduces greenhouse gas emissions.	Addressing the complexity of transportation in the mining industry, particularly in remote areas with poor infrastructure.	Demonstrated the importance of combining AHP and DEA methods for efficient and sustainable transportation in the mining sector, highlighting the need for optimizing multiple conflicting objectives.
(Soleimani, 2021)	Development of a sustainable closed-loop supply chain model for the mining industry, with a focus on a travertine quarry in Iran	The study demonstrated the feasibility and profitability of a closed-loop supply chain in decorative stone quarries.	The primary challenge was addressing sustainability issues in mining, particularly in the context of developing countries with less systemic reverse logistic activities.	The study highlighted the importance of incorporating sustainability into the mining industry, emphasizing on energy consumption, profit maximization, and service level requirements.
(Del Castillo & Dimitrakopoulos, 2019)	Optimizing annual production scheduling of an open pit mining complex under supply uncertainty	The case study of a Cu-Au mine showed an increased expected net present value of over US\$170M compared to the two-stage stochastic formulation.	Managing the complexities of multiple mines and mineral processing streams, each governed by internal and external uncertainties.	The model effectively integrates new information and capital investment decisions over time.

Table 5.15 Consolidated Summary of Case Studies on Optimization in Mining Operations (Continue)

Authors	Case Study Focus	Key Outcomes	Challenges Faced	Lessons Learned
(Paithankar et al., 2020)	Optimization of production sequence and dynamic cut-off grades in open pit mining operations	Demonstrated up to a 29% increase in discounted value through stockpiling and risk quantification under uncertainty.	Managing the complexity of open pit mining operations, including the uncertainty in the orebody's grade and the nonlinearity in mixing/blending behaviors in stockpiles.	Showed the effectiveness of a genetic maximum flow and cut-off grade algorithm framework to address the nonlinear problem of mining operation optimization.
(LaRoche-Boisvert & Dimitrakopoulos, 2021)	Optimizing scheduling and transportation	The model aims to minimize mining and mine-to-port transportation costs, manage risks associated with meeting product demand at the port, and ensure the yearly use of mine-to-port transportation equipment	Balancing mining extraction constraints, such as mining rates and block connectivity, with the capacity and maintenance of transportation equipment. Managing the risk of not meeting product targets due to material supply uncertainty.	Demonstrated the importance and effectiveness of incorporating mine-to-port transportation into the overall long-term production planning of mining complexes.
(Haonan et al., 2021)	Optimizing blending within a mining supply chain through integration	The enhancements produced solutions of higher quality than the original algorithm and significantly reduced computation time.	The complexity of solving the integrated blending optimisation due to the scale of the problem and the bilinear constraints for blending behaviours.	Demonstrated the effectiveness of extending existing algorithms with new techniques to address complex optimization problems in mining.

Table 5.16 Consolidated Summary of Case Studies on Optimization in Mining Operations (Continue)

Authors	Case Study Focus	Key Outcomes	Challenges Faced	Lessons Learned
(Montiel & Dimitrakopoulos, 2015)	Optimization of mining complexes considering geological uncertainty	Developed a risk-based optimization method using stochastic simulations and simulated annealing, improving NPV and reducing deviations in mining operations.	Complex interrelations in mining activities and geological uncertainties.	Importance of considering geological uncertainty and effectiveness of risk-based approaches and stochastic simulations in mining optimization.
(Shishvan & Benndorf, 2019)	Optimization of material dispatching in coal mines with a continuous material flow paradigm	Developed a simulation-based optimization for dispatch in coal mines, improving idle time management.	Complexities due to varied material types, geotechnical requirements, and operational constraints.	Effectiveness of simulation and optimization integration in complex mining operations.
(Kusi-Sarpong et al., 2016)	Green supply chain practices in the Ghanaian mining industry	Developed a framework to evaluate green supply chain practices using fuzzy-DEMATEL and ANP methods.	Environmental and social challenges in mining supply chains, especially in Ghana.	Highlighted the utility of hybrid MCDM methodologies for improving green supply chain practices in mining, underscoring the role of sustainable development.

Table 5.17 Consolidated Summary of Case Studies on Optimization in Mining Operations (Continue)

Authors	Case Study Focus	Key Outcomes	Challenges Faced	Lessons Learned
(J. Li et al., 2020)	Optimization of coal supply chain under carbon emission policies	Developing models for supply chain optimization integrating carbon policies; Introduced DE-SSA algorithm for enhanced efficiency	Balancing environmental sustainability and economic development in coal power; Adapting to global carbon emission policies	Effective integration of carbon policies with supply chain optimization; The DE-SSA algorithm improves accuracy and speed in supply chain management.
(H. Peng et al., 2009)	Optimization of an integrated coal supply chain system	Creating a dynamic optimization model for a unified coal supply chain, attaining maximum profit and ensuring customer satisfaction	Balancing customer satisfaction with system benefits in a complex, multi-tiered coal supply chain	Importance of a systematic approach to managing coal supply chains and the value of dynamic optimization models in decision-making for complex coal enterprises
(Barakos et al., 2016)	Optimization of mining process and REE supply chain	Developing a comprehensive assessment tool for REE mine ability; Analysis of past mistakes in REE industry for future improvement.	Managing environmental impacts; Navigating legislative complexities.	Importance of sustainable practices and strengthening the REE supply chain.

Table 5.18 Consolidated Summary of Case Studies on Optimization in Mining Operations (Continue)

Authors	Case Study Focus	Key Outcomes	Challenges Faced	Lessons Learned
(Ramlall, 2020)	Measurement and modeling of entrainment in flotation	Developed a new model for predicting recovery by entrainment in flotation	Understanding the complexities of the entrainment process in mineral processing	Importance of accurate modeling in enhancing mineral recovery efficiency in mining operations.
(Oyola & Amaya-Mier, 2019)	Optimization of reverse logistics network for residual OTR tires in the mining industry: Colombian case study	Developed a Mixed Integer Linear Programming (MILP) model to optimize the reverse logistics network for tire disposal at minimal cost. Demonstrated the feasibility of a power generation plant for tire recycling	Handling and management complexities of scrap tires; logistical and economic challenges in recycling large OTR tires	Importance of reverse logistics in waste management; potential cost-effectiveness of converting waste into energy; Compliance with environmental regulations.
(Kumar & Dimitrakopoulos, 2019)	Stochastic optimization in copper-gold mining with geo-metallurgical decisions	Implementation of a model increasing metal production and NPV;	Modeling non-additive geo-metallurgical properties;	Incorporating geo-metallurgical factors and uncertainty is crucial for optimization in mining.

Table 5.19 Consolidated Summary of Case Studies on Optimization in Mining Operations (Continue)

Authors	Case Study Focus	Key Outcomes	Challenges Faced	Lessons Learned
(Kalinowski et al., 2020)	Australia's role as the world's lithium supplier	Australia's leading position in the global lithium market due to its large reserves and mining capabilities.	Market fluctuations, international competition, environmental concerns.	The need for sustainable mining, technological innovation, and strong international trade relations.
(Fiscor, 2020)	Optimization of Pit-to-Port Supply Chain in Mining	Implementation of BOLT platform for supply chain optimization, resulting in increased efficiency and profitability.	Complexity of supply chain management; Need for real-time response to disruptions.	Importance of automating decision processes and integrating systems for effective supply chain management in mining.
(Thomas et al., 2013)	Distributed optimization of mining complex for climate change mitigation	Development of a distributed optimization model for mining operations with a focus on reducing climate impact.	Balancing environmental sustainability with economic viability in mining operations.	The importance of incorporating environmental considerations in mining operations for climate change mitigation.

Table 5.20 Consolidated Summary of Case Studies on Optimization in Mining Operations (Continue)

Authors	Case Study Focus	Key Outcomes	Challenges Faced	Lessons Learned
(Yaping & Bossman, 2021)	Efficiency Strategy for Ghana's Mining Industry	Explored the logistics and supply chain processes of mining corporations in Ghana, recommending optimization models for efficiency.	Challenges include poor road networks, unreliable lead times, high transport costs, and policy restrictions.	significant of implementing effective service delivery and strategies for minimizing transportation expenses in the mining sector.
(Epstein et al., 2012)	Optimization of fleet size and mix in copper mines	Improved net present value and resource allocation; Successful application of a mixed-integer programming model	Handling complex, large-scale mining operations	Effectiveness of mathematical programming in mining optimization; importance of detailed modeling in mining decisions.
(Azzamouri et al., 2021)	Flexibility of dynamic blending in the phosphate mining supply chain	Developed a model optimizing blending processes with alternative routings and security stocks, improving supply chain flexibility and efficiency.	Balancing blending process complexity with real-world applications; managing uncertainty in ore characteristics and order changes.	Integrating risk management strategies, like security stocks, is vital for handling supply chain uncertainties and enhancing operational efficiency.

Table 5.21 Consolidated Summary of Case Studies on Optimization in Mining Operations (Continue)

Authors	Case Study Focus	Key Outcomes	Challenges Faced	Lessons Learned
(Azzamouri et al., 2021)	Flexibility of dynamic blending in the phosphate mining supply chain	Developed a model optimizing blending processes with alternative routings and security stocks, improving supply chain flexibility and efficiency.	Balancing blending process complexity with real-world applications; managing uncertainty in ore characteristics and order changes.	Integrating risk management strategies, like security stocks, is vital for handling supply chain uncertainties and enhancing operational efficiency.
(R. Goodfellow & Dimitrakopoulos, 2017)	Mining Supply Chain Optimization under Geological Uncertainty	Developed a particle swarm optimization and k-means clustering method for supply chain optimization.	Managing the complexity of the supply chain and non-linear relationships in mining operations.	Incorporating geological uncertainty into mining supply chain management is crucial for efficiency and viability.
(Njuaem & Ogundare, 2023)	Sustainability in globalized mining supply chains	Developed a model assessing the impact of globalization on supply chains, focusing on sustainability.	Balancing economic, social, and environmental aspects in a globalized context.	Importance of considering all sustainability dimensions (economic, social, environmental) in global supply chain management.

Table 5.22 Consolidated Summary of Case Studies on Optimization in Mining Operations (Continue)

Authors	Case Study Focus	Key Outcomes	Challenges Faced	Lessons Learned
(Sandeman et al., 2010)	Lihir Gold Limited and PT Kaltim Prima Coal (KPC) Mine Planning	Maximizing autoclave throughput (Lihir) and understanding throughput-quality interaction (KPC) with simulation models.	Complexities in modeling mining operations and operational uncertainties.	Strategic planning and simulation are crucial in managing real-life uncertainties and operational variabilities in mine planning.
(Isnafitri et al., 2021)	Open Pit Mining - Truck Allocation	Developed a model to minimize investment and transportation costs in open pit mining by optimizing truck allocation. The model selects specific trucks for different routes based on cost-effectiveness.	Balancing investment costs with operational efficiency in truck allocation.	Effective truck allocation is crucial for cost optimization in mining operations, emphasizing the need for strategic planning and model-based decision-making.
(Dorrington& Olsen, 2019)	Design of an Asteroid Mining Supply Chain	Developed a model for optimizing the supply chain network for asteroid-derived resources to Earth orbit, focusing on minimizing costs and maximizing efficiency in space logistics.	Balancing the complex orbital mechanics and logistics of space transportation with economic viability.	Strategic planning in space logistics is crucial for the efficient and cost-effective transportation of asteroid-derived resources.

Table 5.23 Consolidated Summary of Case Studies on Optimization in Mining Operations (Continue)

Authors	Case Study Focus	Key Outcomes	Challenges Faced	Lessons Learned
(LaRoche-Boisvert & Dimitrakopoulos, 2021)	Optimization in Open-Pit Gold Mining	Developed a simultaneous stochastic optimization model to enhance NPV and gold production in a mining complex. Identified SAG mill as operation bottleneck.	Managing uncertainty in mineral supply and material hardness; optimizing under constraints like mill capacity.	Strategic optimization and consideration of uncertainty are key in maximizing the value and efficiency of mining operations.
(Fattahi et al., 2021)	Sustainable Mining Supply Chain with Renewable Energy	Developed a multi-stage stochastic program for sustainable planning in mining supply chains, integrating renewable energy sources to achieve low-carbon operations.	Balancing economic goals with environmental performance and social responsibility impacts.	Strategic integration of renewable energy and comprehensive planning are crucial for sustainable mining supply chain management.
(Farghaly et al., 2021)	Blending and Production in Mining	Developed an optimization model for blending and production in mining, considering various mines and metallurgical units, using linear programming to minimize costs.	Balancing the complexity of multiple source mines and metallurgical processes; aligning with economic efficiency.	Effective use of linear programming in complex mining operations can significantly enhance production efficiency and cost management.
(Saliba& Dimitrakopoulos, 2019)	Open Pit Gold Mining with Uncertainties	Developed a stochastic optimization framework for an open-pit gold mining complex, addressing supply and market uncertainties.	Integrating supply and market uncertainties into the optimization process; managing geological and price-related risks.	The importance of simultaneous stochastic optimization in mining for managing uncertainties and improving operational decision-making.

Table 5.24 Consolidated Summary of Case Studies on Optimization in Mining Operations (Continue)

Authors	Case Study Focus	Key Outcomes	Challenges Faced	Lessons Learned
(Strang, 2012)	Marketing and SCM in Mining	The study presents how logistics optimization reduces the need for marketing in mining, focusing on prioritization and supply chain logistics as key marketing functions.	Balancing marketing and logistics in a resource-based industry overshadowed by distribution logistics.	Emphasizing logistics and supply chain management as pivotal in marketing functions in mining, highlighting the integration of these elements for effective business operations.
(Muduli&Barve, 2011)	Green Issues in Mining Supply Chain	The study emphasizes the importance of addressing environmental concerns in the mining supply chain for sustainable development.	Balancing economic goals with environmental sustainability; resistance to change in traditional mining practices.	Integrating green practices into mining supply chains is essential for sustainable development, requiring a holistic approach that considers environmental, economic, and social aspects.
(Akbari-Kasgari et al., 2022)	Copper Industry Supply Chain	Developed a resilient and sustainable closed-loop supply chain network model for the copper industry,	Balancing the resilience and sustainability of the supply chain amidst disruption risks like earthquakes.	Importance of integrating sustainability and resilience in supply chain design, particularly in industries susceptible to disruptions like mining.
(Saliba& Dimitrakopoulos, 2019)	Open Pit Gold Mining with Uncertainties	Developed a stochastic optimization framework for an open-pit gold mining complex, addressing uncertainties.	Integrating supply and market uncertainties into the optimization process; managing geological and price-related risks.	The importance of simultaneous stochastic optimization in mining for managing uncertainties and improving operational decision-making.

Table 5.25 Consolidated Summary of Case Studies on Optimization in Mining Operations (Continue)

Authors	Case Study Focus	Key Outcomes	Challenges Faced	Lessons Learned
(Goodfellow& Dimitrakopoulos, 2017)	Optimization of Mining Complexes and value chains in minerals	Developed a simultaneous stochastic optimization model to address limitations in mining complexes and mineral value chains, integrating and managing geological uncertainty through geostatistical simulation.	handling geological unpredictability and enhancing multiple interconnected elements of mineral value chains within a unified model.	Importance of using simultaneous stochastic optimization to integrate various components of a mineral value chain, enhancing overall performance and managing risks.
(Both& Dimitrakopoulos, 2020)	Short-term Production Scheduling and Fleet Management	Designing an innovative stochastic optimization model to address simultaneous short-term extraction sequencing, scheduling shovel relocation, and managing diverse hauling fleets and downstream allocation	Integrating geological uncertainty and equipment performance in a comprehensive optimization model.	Importance of simultaneous stochastic optimization for efficient short-term planning in mining, considering multiple uncertainties.
(Bodon et al., 2011)	Mining Supply Chain from Mine to Port	Developed a combined optimization and simulation model for mining supply chains, enabling capacity analysis and evaluation of expansion options, with a case study of PT Kaltim Prima Coal in Indonesia.	Integrating complex export supply chains and addressing uncertainties in mining operations.	Effective integration of optimization and simulation is crucial for comprehensive analysis and strategic planning in mining supply chains.

9. Technologie

In mining optimization supply chain systems, various technologies can be used. These technologies could cover fields like HxGN MinePlan Software, Fleet Management Systems, Inventory and Warehouse Management, Demand Forecasting and Inventory Optimization, Transportation Management Systems, Blockchain Technology, Predictive Analytics and AI, IOT Sensors and Telematics, Geographic Information Systems (GIS), Environmental and Compliance Software, 3D Printing and Additive Manufacturing, Remote Monitoring and Control, Robotics and Automation, Advanced Data Analytics Platforms (Table 5.25).

MineSight is related to Geological Modeling, mine design, and optimization plans. GEOVIA Surpac provides 3d mine planning and geological modeling software for delivering advanced data management and statistical analyses for either open pit or underground operations. As a fleet management system, MineStar is the industry's most comprehensively integrated mine operations system. Wenco fleet management system is known as a real-time monitoring and control of mining equipment.

Table 5.26 Mining Technology Classification

Mine planning software	Fleet Management Systems	Supply Chain Management	Demand Forecasting and Inventory Optimization	Transportation Management Systems (TMS)	Blockchain and Compliance	Advanced Technologies	Data Analytics and Visualization
HxGN MinePlan	MineStar	SAP Extended Warehouse Management (EWM)	Kinaxis RapidResponse	Manhattan Associates TMS	Blockchain Technology	Predictive Analytics and AI	Tableau
GEOVIA Surpac	Wenco Fleet Management System	Oracle SCM Cloud	ToolsGroup SO99+	MercuryGate TMS	Environmental and Compliance Software	IoT Sensors and Telematics	Power BI

10. Bibliometric Analysis

The following sub-sections provide a bibliometric analysis of published documents in the fields. This analysis includes citations, publication trends, keywords, journal and conference venues, institutions, impact factors, and authors.

10.1 Citation Analysis

Figure 5.2 indicates a fluctuating trend in citation impact over the years. A notable peak in average citations was observed around 2005, suggesting a high impact of publications during that year. Following this peak, there is a sharp decline with a few intermittent increases, although none reach the height of 2005. Recent years, from 2020 onwards, show a downward trend, with the lowest average citations occurring in the latest year, 2023. This could suggest either a decrease in the impact of publications or insufficient time for newer articles to accumulate citations. The data represented in the chart provides a visual representation of the changing influence of research in the field over nearly four decades.

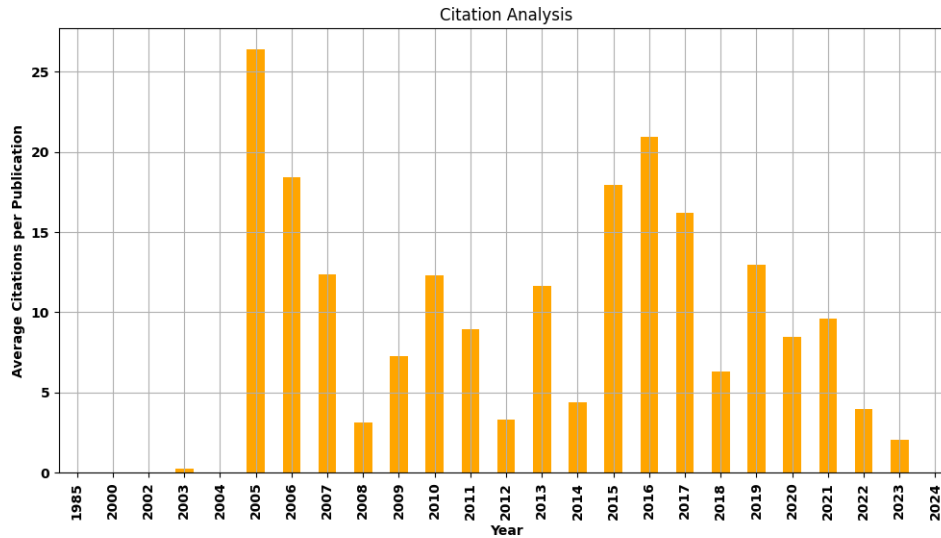


Figure 5.2 Citation Analysis over Years

The line graph, Figure 5.3, provides a temporal overview of the citation impact of the body of work encompassed by the dataset. The x-axis indicates the progression of years from 1985 to 2024, while the y-axis quantifies the total citations received each year.

From the early period of 1985 until about the year 2000, the graph shows a baseline with minimal citation activity, suggesting that the works from this period either experienced delayed recognition or were part of a nascent field gradually gaining attention. A sharp rise in citations begins post-2000, reflecting an increasing acknowledgment of the research. This uptick may correlate with seminal works becoming foundational in the field, the advent of digital accessibility to scholarly works, or a combination of both factors. The peak of citation activity occurs between 2010 and 2020, with the highest point nearing 500 total citations in a single year. This peak can indicate prolific research output, pivotal publications that captured the research community's interest, or an overall increase in the field's activity and visibility.

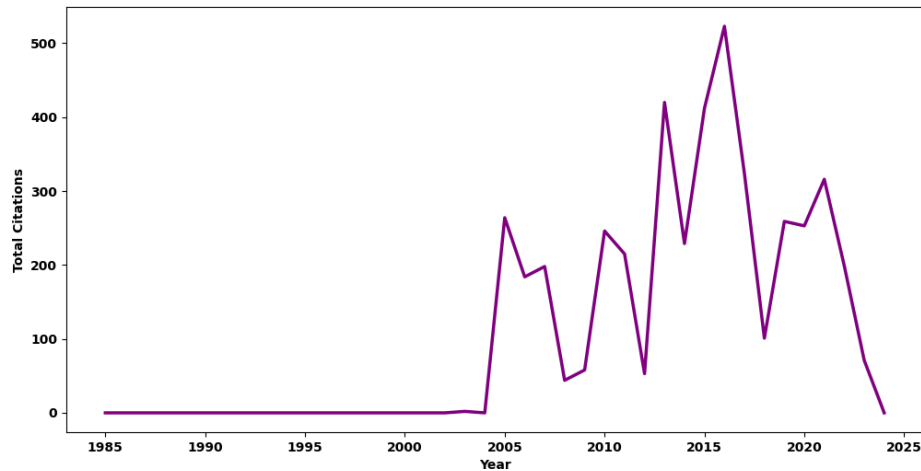


Figure 5.3 Temporal Citation Analysis

10.2 Publication Trend Analysis

In Figure 5.4, the number of publications appears relatively low at the beginning of the period, with a slight increase leading up to the year 2012. In 2012, there was a significant peak, indicating a substantial increase in the number of publications that year, which is the highest point in the entire dataset. Following this peak, there is a sharp decline until 2014, after which the number of publications gradually increases again. Another notable peak occurs in 2021, though it does not surpass the 2012 peak. The final year, 2024, shows a decrease compared to the previous year. The data represented in the chart provides insights into the research activity over time, highlighting years of particular productivity or focus within the field.

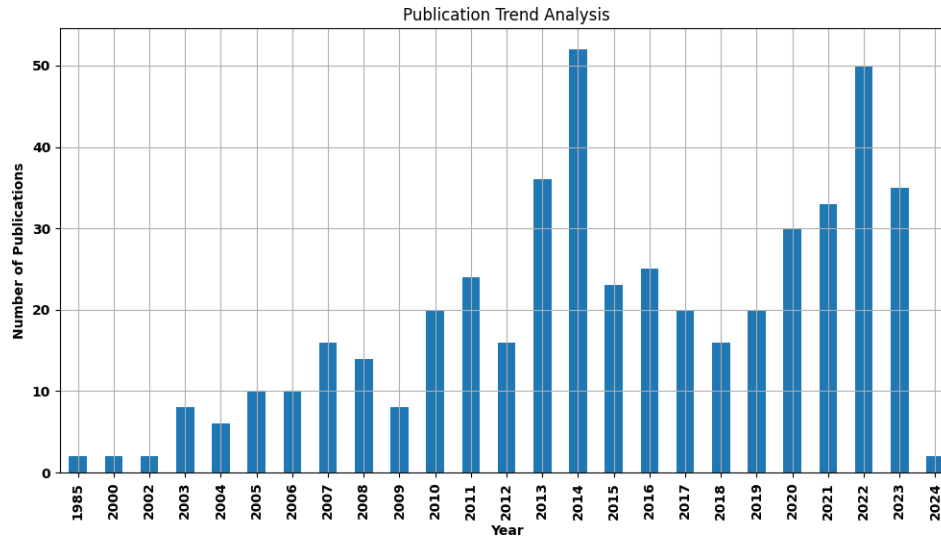


Figure 5.4 Publication Trend Analysis over Years

10.3 Keywords Analysis

In Figure 5.5, the keyword "Data mining" has the highest frequency, indicating it is a prevalent topic in this research corpus. Following closely are "Supply chains" and "Optimization," which are also significant themes, suggesting a strong focus on these areas within the field. Other notable keywords include "Supply chain management," "Decision making," and "Mining," all of which have substantial representation, pointing to their relevance in the research. Keywords like "Artificial intelligence," "Coal," and "Optimisation" appear as well, though with lower frequencies. This chart reveals the primary areas of focus and interest within the dataset, highlighting the importance of data analysis and supply chain optimization in the current research landscape.

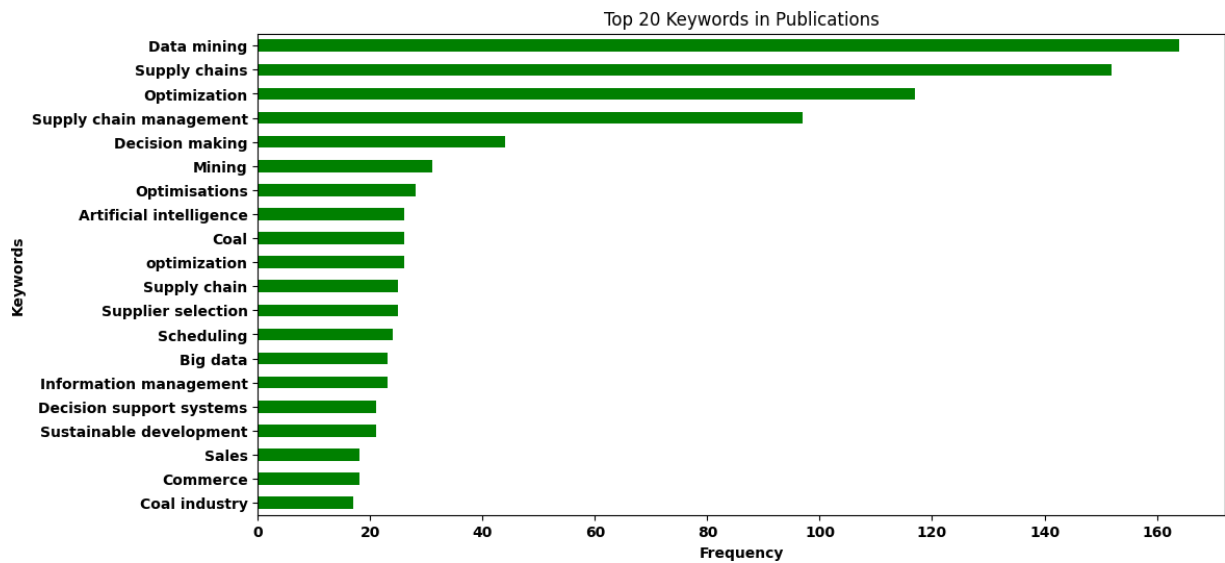


Figure 5.5 Top 20 Keywords in Publications

Figure 5.6 presents the temporal trends in the prevalence of selected research keywords within the dataset, spanning from 2005 to the present. The x-axis marks the years, while the y-axis quantifies the frequency of occurrence for each keyword, providing a longitudinal perspective on the research topics' popularity and relevance.

The lines represent individual keywords with their color coding indicated in the legend. Notably, "Supply chain" and "data mining" appear as distinct yet related threads, both demonstrating variability over the years, with peaks suggesting periods of heightened focus or relevance in the research community. The "Optimization" keyword exhibits a significant peak, indicating a time when this topic was of particular interest, possibly reflecting technological advancements or shifts in industry practices that called for enhanced optimization techniques.

The "artificial intelligence" keyword also shows substantial fluctuation, with a notable increase in recent years, likely due to the growing emphasis on big data analytics and its applications across various domains, including supply chain management.

The graph reveals that while some keywords maintain a steady presence, others fluctuate more dramatically, indicating shifts in research focus, which may correspond to emerging technologies, industry demands, or global economic and social trends that drive academic inquiry.

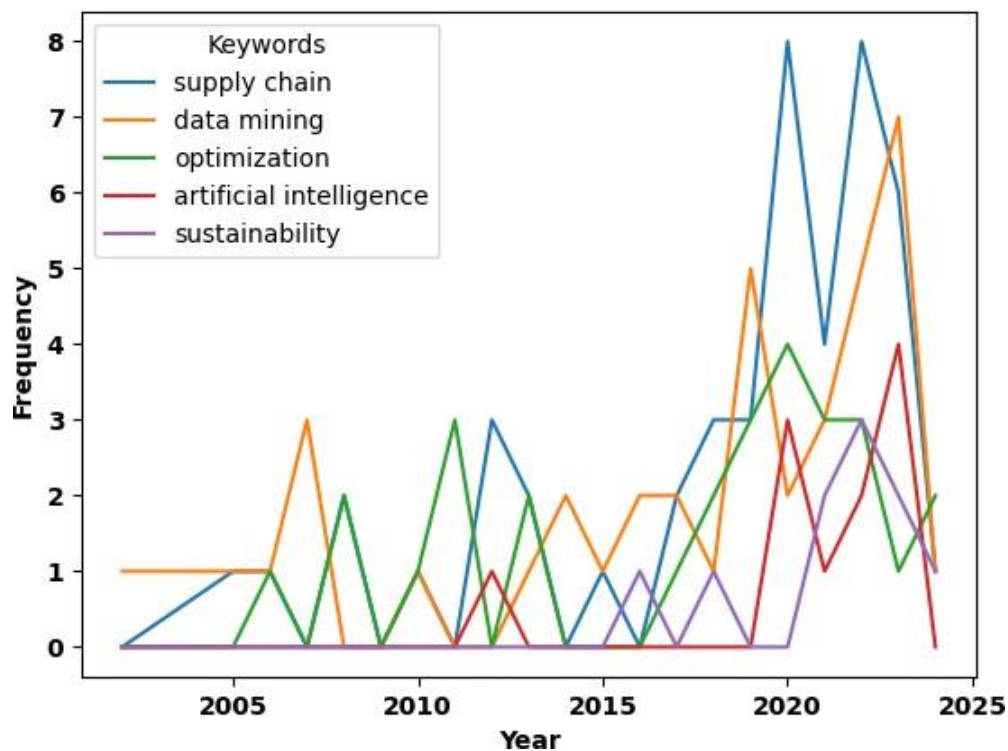


Figure 5.6 Top 5 Research Keywords over time

10.4 Publisher Analysis

The provided donut chart, Figure 5.7, illustrates the distribution of publications across various scholarly journals and conference proceedings within the dataset. It showcases the percentage share of total publications attributed to each venue, offering insight into the

most frequented outlets for disseminating research in this field. The “Applied Mechanics and Materials” holds 14.8% of the publication share, reflecting a strong focus on applied research in the mechanical and material sciences. “Advanced Materials Research” also constitutes a significant fraction, with 11.3%, which suggests a research trend leaning towards the development and application of advanced material technologies. Following closely, the “Lecture Notes in Computer Science” series emerge as the leading publication venue, accounting for 12.7% of the total publications. This prominence underscores the series’ pivotal role in the computer science domain, particularly due to its inclusivity of subseries such as “Lecture Notes in Artificial Intelligence” and “Lecture Notes in Bioinformatics,” which are central to the advancing interdisciplinary nexus of computer science and life sciences.

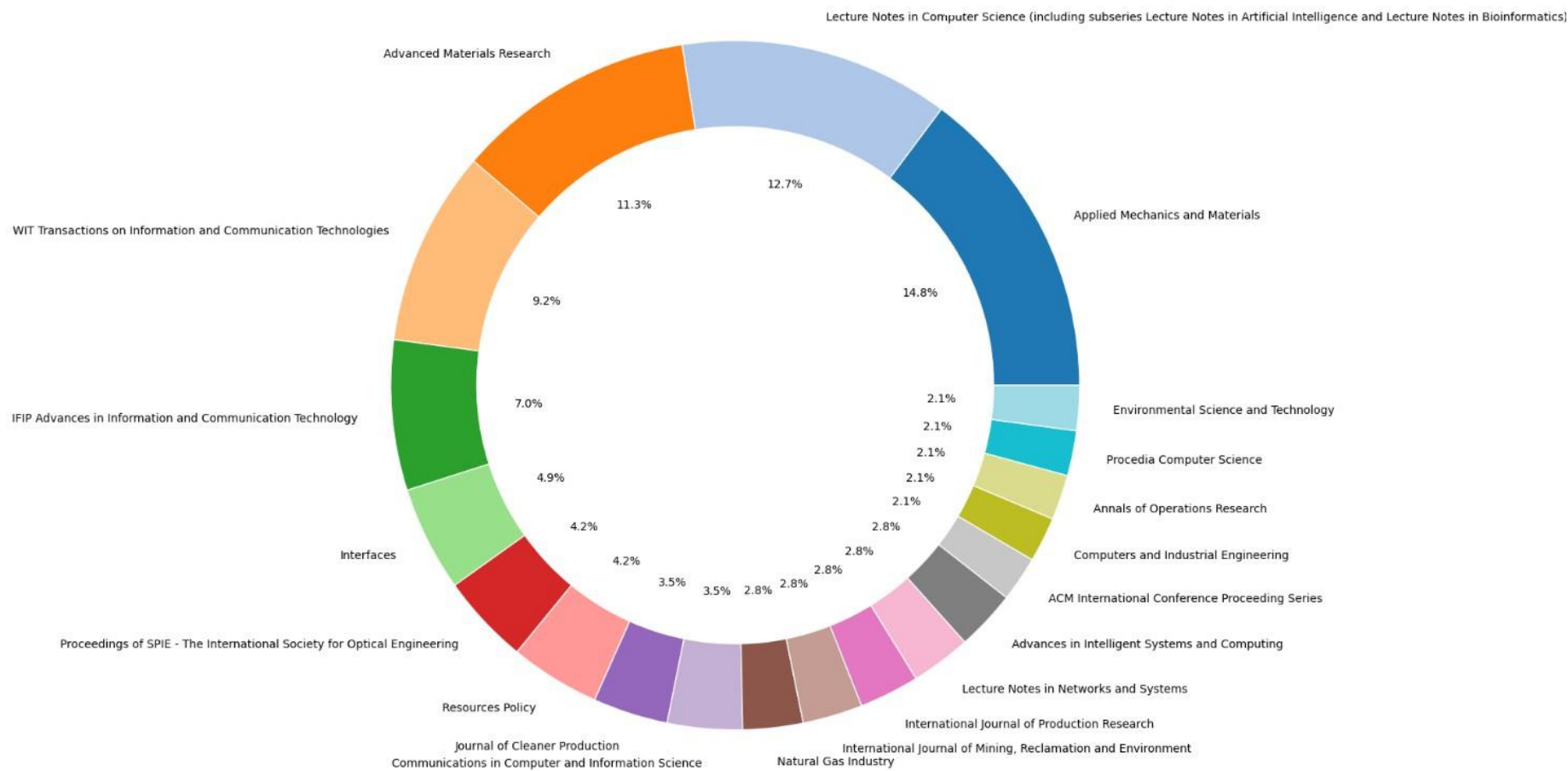


Figure 5.7 Top 20 Journals and Conference Venues

Other notable outlets include "IFIP Advances in Information and Communication Technology" and "WIT Transactions on Information and Communication Technologies," representing Significant contributions. These figures indicate a robust engagement with information technology research, highlighting its evolving and interdisciplinary nature.

The chart further includes a diverse array of specialized journals, such as "Environmental Science and Technology" and "Resources Policy," each contributing to a smaller yet significant portion of the research landscape. These journals signify the field's diverse interests, encompassing environmental impact studies and resource management, which are increasingly relevant in today's technology-driven ecological discourse.

10.5 Impact Factor Analysis

This horizontal bar chart, Figure 5.8, provides an impact analysis of research publications by displaying the average number of citations per publication across a selection of journals and conference proceedings. Each bar's length represents the average citation count, a proxy for the relative impact and influence of research disseminated through these channels.

At the forefront of this analysis is "Maintenance, Replacement, and Reliability: Theory and Applications, Second Edition," with an impressive average citation count nearing 190, indicative of its seminal role in the maintenance and reliability engineering field. Similarly, "Energy Conversion and Management" demonstrates a substantial impact with an average citation count of 95, reflecting its significance in the energy sector's ongoing discussions and advancements.

Publications in "Expert Systems with Applications" and "Applied System Innovation" also show noteworthy citation averages, underscoring their relevance in applied research and the practical application of expert and innovative systems in various domains. The chart includes a range of specialized and interdisciplinary outlets, such as "Environmental Science and Technology" and "Cognitive Systems Research," highlighting diverse research foci within the dataset. This diversity points to a broad interest in integrating environmental considerations and cognitive approaches into technological research. The color gradient, ranging from dark blue to yellow, visually differentiates between the journals based on their average citation counts, facilitating a quicker grasp of the data's hierarchy.



Figure 5.8 Impact Factor Analysis

10.6 Institute Analysis

This radar chart, Figure 5.9, presents a comparative analysis of the impact of the research across various academic institutions. The axes emanating from the center represent different research impact or output metrics, which could include publication count, citation count, h-index, or other relevant performance indicators.

Each institution's profile is plotted on this multi-axial graph, with points connected to form a polygon, illustrating the institution's performance across the considered metrics. The chart includes the Department of Industrial Engineering at Qazvin Branch, Islamic Azad University in Iran; West Virginia University in the United States; Ton Duc Thang University in Ho Chi Minh City, Vietnam; and the University of Newcastle, School of Mathematical and Physical Sciences in Australia.

The overlay of institutional profiles allows for an at-a-glance comparison, revealing strengths and weaknesses relative to each metric. For instance, one institution may show a pronounced presence in citation count, while another may excel in publication volume. The radar chart facilitates an intuitive understanding of how each institution fares in a multidimensional research landscape.

This visualization underscores the diversity of research profiles, suggesting areas where institutions may focus their strategic development or seek collaborative partnerships to leverage complementary strengths. It is an insightful tool for benchmarking and fostering competitive and collaborative academic endeavors.

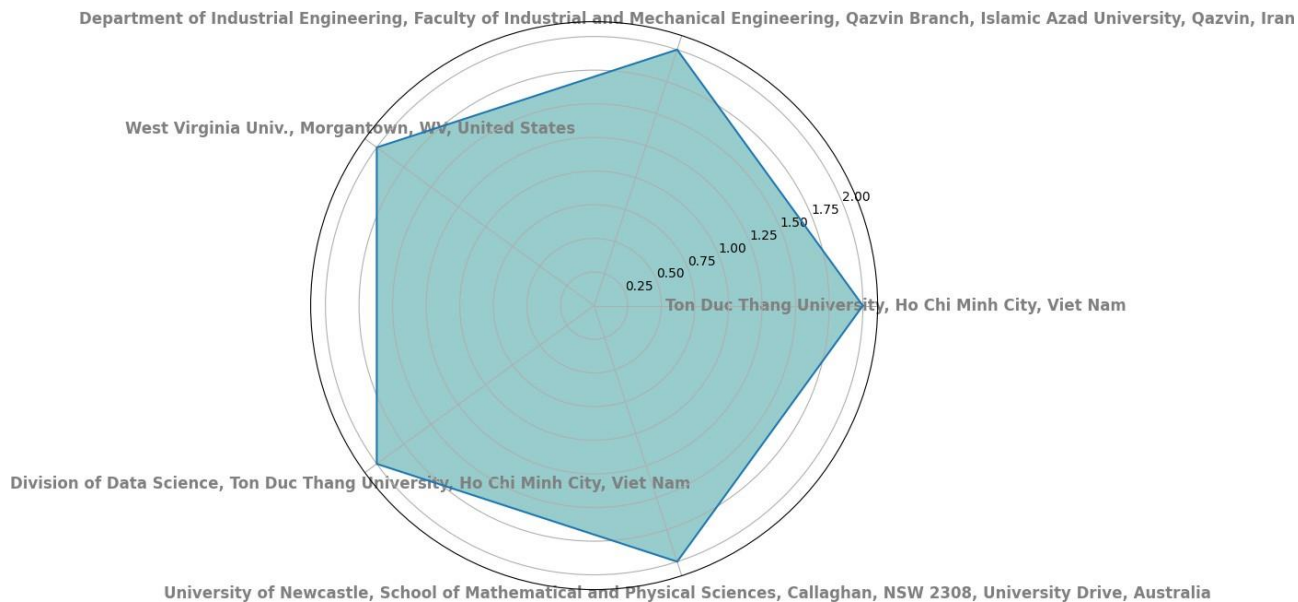


Figure 5.9 Top 5 Most Prolific Institutions

10.7 Authors Analysis

The word cloud, Figure 5.10, represents the frequency of author names within the dataset, with the prominence of each name corresponding to the number of occurrences. Such visualizations are particularly useful for quickly identifying key contributors and influential researchers in a specific field or dataset.

Central and larger in size are names like "Dimitrakopoulos", "Stanford C." and "Sandeman T.," suggesting these authors have a substantial presence in the dataset, possibly indicating their significant contributions to the research area. Similarly, "Pamparana G." and "Stanford C." highlight their recurring involvement in the publications analyzed.

Smaller, yet still noticeably represented, are names like "Osborne D.," "Boland N.," and "Liu M.," among others, which underscores their active participation and suggests a strong network of collaboration or a series of influential works that warrant attention. This word cloud not only serves as a testament to the individual contributions within the scholarly

community but also aids in discerning the collaborative landscape of the field. It offers a snapshot of the most prolific authors, potentially guiding readers toward key authors whose work may provide foundational insights or represent significant advancements in the domain. As is observed from Figure 5.11, the majority of publications possess 3 and 4 authors, and also it is interesting that the number of publications with a single author (only one author) is notable. However, this number is significantly less than publications with 3 or 4 authors. Moreover, it is clear that as the number of authors per publication increases, the frequency of such publications decreases. Publications with five or more authors are less common. In this pattern, it is seen that collaborative research involving 3 to 4 authors is common, while both single research and large team research are less common.



Figure 5.10 Word Cloud of Most Productive Authors

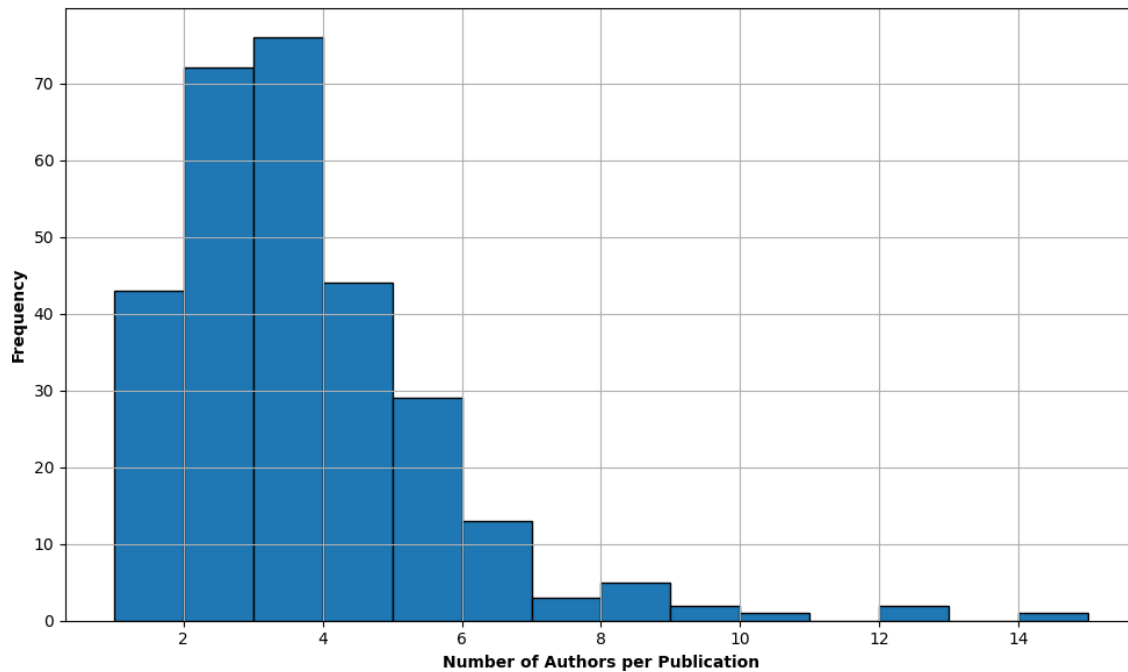


Figure 5.11 Distribution of number of authors per publication

10.8 Publication Statistics by Number of Pages (page count)

As of writing this paper, March 2024, approximately 32,519 pages of documents on mining supply chain were published, with an average 8.928 pages per paper. The bar chart presented in Figure 5.12 shows the distribution of number of pages in percentages in different ranges. As it is clear from Figure 5.12, the majority papers (about 30%) possess between 6-10 pages while 1.87 % of documents have 26-30 pages. The presented chart shows that the distribution of page count is not uniform. The largest portion of papers belongs to the range 6-10 pages and the percentage generally decreases as the page count range increases, with a noticeable increase again for articles with more than 30 pages. Figure 5.13 shows fluctuations in average page count per publication, which suggests that the length of research articles have varied year to year without a consistent pattern of becoming longer or shorter over two decades. There are two notable peaks in the average

page count, one in 2010 and the other in 2023. These could be potentially because of publishing particularly comprehensive studies due to major events or breakthroughs in research that needed more detailed documentation and analysis.

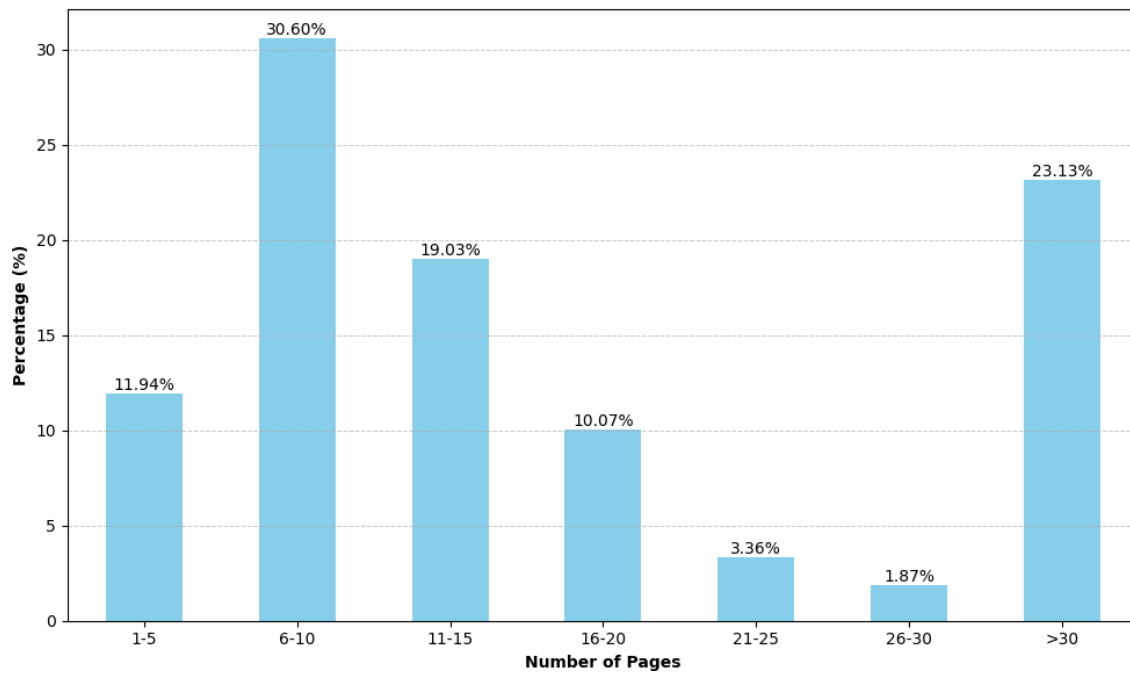


Figure 5.12 Distribution of documents based on page count

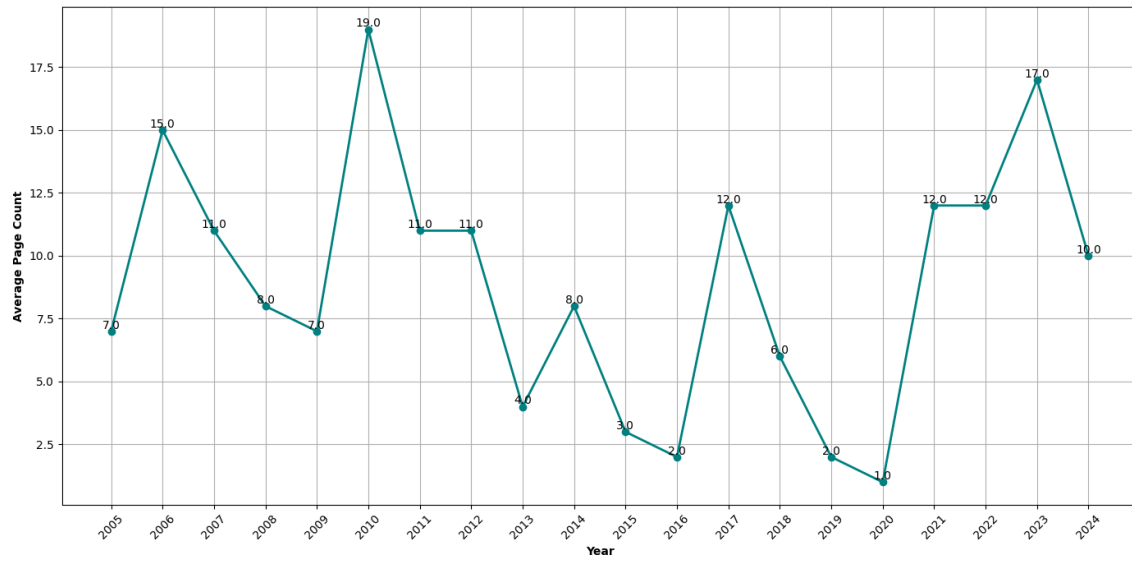


Figure 5.13 Average page count per publication per year

11. Answer to Research Questions

RQ1: What are the current gaps in the literature in mining supply chain optimization?

Answer: the study reviews several challenges in the available literature, for example:

- The need for enhanced supply chains which is raised from evolving business dynamics and convergence of several domains, such as supply chain management, data mining, and optimization approaches.
- The importance of optimization approaches in the mining supply chain is emphasized, specifically in response to global market evolutions and the initiative to improve customer satisfaction.
- Also, the study recognized some challenges and limitations in the mining supply chain management field, such as data reliability issues, costly computing, limited scalability, and practical barriers.

RQ2: What algorithms are mostly used to optimize mining supply chain problems?

The study covers various optimization algorithms in the mining supply chain, expressing implemented algorithms, key findings, application areas, and limitations. Various algorithms have been used in this area:

- Decomposition-based algorithms with prune routine
- Ranking-based topological sorting heuristic adaptive algorithms
- Integer programming for location-routing problems
- Dynamic simultaneous optimization with multistage stochastic programming

These algorithms are applied to enhance computational efficiency, manage complex supply chain operations, and improve the expected net present value in decision-making of the mining supply chain.

RQ3: What are the challenges in the implementation of the mining models and the implementation of the algorithms?

Some key challenges have been identified by reviewing the literature. These challenges include:

- Handling the complexity and scalability of models
- Managing uncertainty in ore characteristics
- Ensuring detailed modeling for effective decision-making
- Addressing the limitations of algorithms in diverse mining contexts.
- Reliance on advanced technology
- Specify models for certain mining sectors

RQ4: What are the trends and the emerging research topics under the scope of this review?

The study potentials insights into emerging trends and future mining supply chain optimization research areas.

These topics include various categories like:

- Capacity planning
- Logistics
- Inventory control
- Network design
- Environmental protection

RQ5: What are the key challenges and limitations faced in applying optimization algorithms in mining supply chain management?

According to the literature, the challenges in the area include:

- Complexity and scalability of models
- Specificity to mining contexts
- Dependency on advanced technology and data

- Real-world application challenges
- Environmental and sustainability considerations

RQ6: How can the decreasing trend in average citations from 2020 be interpreted in terms of the field's research impact?

Some scenarios can be considered for this as follows:

- It may be possible that there has not been sufficient time for newer papers to accumulate citations. This suggests that these recent publications are still in the early stages of recognition within the research community.
- Another interpretation is a decrease in the impact of publications in the field. This may be interpreted as a shift in research area focus, and newer topics are receiving less attention compared to the earlier years.
- It could also suggest that the current field of study is already saturated, where the most influential research has already been addressed, making it challenging for new publications to gain the same level of impact.

RQ7: what can the peaks in publication trends show about the field of specific productivity or focus within the field during those years?

The insights could be analyzed as follows:

- A specific event or development within the field of research has happened.
- Also, this could happen since a specific topic or research area gained prominence during this time.

RQ8: How can the fluctuation in the occurrence of keywords like “Supply Chain” and “Optimization” be correlated with technological advancements, industry changes, or academic trends?

Possible scenarios are as follows:

- Fluctuations in keyword occurrence may be caused by introducing new technology or development in a specific field.
- Keywords may also present the industry trends. For example, an economic pandemic may increase focus on a specific field of study.
- Academic trends and shifts in a specific research area can lead to fluctuations in keyword usage. For example, shifts in funding priorities may influence the choice of keywords in research publications.

12. Discussion, Future Trends, and Conclusion

The study provided a comprehensive review of mining supply chain optimization, covering methods, applications, case studies, problems, and technologies. The key aspects of this study are classified as follows:

- The study explored the evolution of supply chain mining optimization (SCMO) strategies, highlighting development from classic approaches to heuristics and metaheuristics.
- The paper also discussed various objective functions in designing the supply chain of mining, including maximizing profit and revenue, reducing costs, and environmental impacts, as well as constraints such as equipment capacity and blending quality.
- Various optimization algorithms have been proposed in the literature. These algorithms cover linear programming, mixed-integer programming, and machine learning.
- The paper also discussed case studies demonstrating the practical application of the implemented algorithms in real-world cases.

- Moreover, the paper reviewed the field's most significant challenges and limitations. These challenges included data reliability, computational costs, scalability issues, and practical barriers in implementing the models.

Accordingly, the future work suggestions could be addressed as follows:

- Implementation of advanced AI and machine learning for enhanced forecasting and real-time decision-making.
- Developing advanced technologies for more sustainable mining practices, focusing on energy efficiency and waste reduction.
- Exploring blockchain technology for improving security in the mining supply chain.
- Implementing big data analytics for more accurate and efficient supply chain management.
- Investment in robotics and automation for the goal of safety and efficient mining operations.

This work provided a detailed review and analysis of state-of-the-art studies and developments in mathematical modeling and algorithms in the area of mining supply chain optimization. The study addresses an in-depth review of research papers in the field, which covers methods, applications, challenges, and advanced technologies. Furthermore, this study includes a bibliometric analysis, which assesses aspects of citations, keywords, authorship, institutional contributions, and publishers. This study comprehensively reviews current technologies applied in the field of mining supply chain optimization. Moreover, this study provided a critical analysis of challenges and limitations in practical applications of the available models and algorithms. In the end, the study provided answers to the formulated research questions and gives some suggestions and future directions in the field of mining supply chain optimization.

Conflict of interest: The authors declare no conflict of interest to disclose.

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Chapter 6

The boundary update approach coupled with Evolutionary Algorithm for mixed integer programming optimization:

An open pit mining optimization problem

Iman Rahimi¹, Amir H. Gandomi^{*1,2}, Mohammad Reza Nikoo³

¹*Faculty of Engineering and Information Technology, University of Technology Sydney, Australia (emails: iman83@gmail.com, gandomi@uts.edu.au, mohsen.mousavi@uts.edu.au, fang.chen@uts.edu.au)*

²*University Research and Innovation Center (EKIK), Óbuda University, 1034 Budapest, Hungary*

³*Department of Civil and Architectural Engineering, Sultan Qaboos University, Muscat, Oman (email: m.reza@squ.edu.om)*

Abstract-

Enhancing open-pit mining efficiency presents a complex task with a multitude of intricate considerations and factors to address. In this study, we use an implicit constraint-handling method to address the optimization challenge in open-pit mining, specifically focusing on addressing the restricted pit issue (CPIT). Furthermore, we extend our approach to a multi-objective optimization problem and introduce the boundary updating method to address the aforementioned issue comprehensively. Our results demonstrate the effectiveness of the Boundary Update (BU) approach when coupled with the Evolutionary Algorithms, providing efficient solutions to the optimization problem. This research contributes to the advancement of open-pit mining optimization methods, offering valuable insights and solutions for enhanced operational efficiency.

Keywords- Evolutionary Algorithm, Optimization, Boundary Updating, Integer Programming.

1. Introduction

The field of open pit mining optimization encompasses both academic research and practical applications, focusing on enhancing the efficiency and effectiveness of open pit mining operations. Open pit mining, a widely used method for extracting valuable resources and minerals from the Earth's surface, involves creating large excavations to access and retrieve these reserves. As mining operations become increasingly complex and resource-intensive, the importance of strategic decision-making and optimization techniques grows significantly (Dimitrakopoulos, 2018; Gilani & Sattarvand, 2016).

The main objective of optimizing open-pit mining is to enhance the economic value derived from the mining site while considering various factors and constraints. These factors include the layout and design of the excavation, the scheduling of mining operations, equipment allocation, waste management, and environmental considerations. Achieving optimal solutions in open pit mining requires effectively balancing the extraction of maximum valuable resources with minimizing operational costs, safety risks, and environmental impacts.

To tackle the complexities resulting from the intricate interplay of geological, operational, and economic factors, a range of mathematical and computational techniques are employed. These methods span from traditional strategies like linear programming and integer programming to more advanced approaches such as mixed-integer programming, dynamic programming, and simulation-based optimization. Additionally, technological advancements, including the integration of geographic information systems (GIS), geological modeling software, and advanced data analytics, have significantly expanded the array of tools available for optimizing open pit mining operations.

By employing optimization methodologies, mining enterprises and researchers aim to make well-informed decisions regarding pit layout, production scheduling, and resource allocation. These decisions can lead

to increased profitability, reduced waste production, improved safety protocols, and minimized environmental impacts. As mining activities continue to evolve and face growing challenges, the field of open pit mining optimization remains crucial in guiding the sustainable and efficient extraction of resources from the Earth's surface.

An optimization challenge, especially prominent in the realm of mixed-integer programming (MIP), becomes demanding in terms of computational resources for traditional algorithms to handle. This problem becomes more severe as the number of variables and constraints grows. As a result, metaheuristics are proposed as a solution for managing these complexities (Behmanesh et al., 2021; Gandomi et al., 2020).

The remainder of the paper is as follows. Section 2 introduces the literature review. Section 3 presents the terminology. Sections 4 and 5 show boundary update and results. Section 6 illustrates the conclusion.

2. Literature review

H. Peng et al. (2009) presented a model for optimizing coal supply chains, focusing on maximizing system profit while ensuring customer satisfaction. The model incorporates various aspects of the supply chain, including production, washing, transportation, and sales. Applied to the Xuzhou coal mines, it offers recommendations for a more systematic use of resources. However, a potential drawback is its specific focus on coal supply chains, which may limit its applicability to other industries or contexts.

Sandeman et al. (2010) acknowledges the advantages of integrating optimization models and simulation in mining activities. Through two case studies, they demonstrate the efficacy of this integration, particularly in optimizing supply chains. While the first study employs a static optimization model, the second utilizes a dynamic model that continuously adjusts. This integration improves decision-making and comprehension of the system. However, it's worth noting that the complexity and time-consuming

nature of integrating optimization and simulation models may pose challenges, particularly in swiftly evolving or unpredictable operational contexts.

Bodon et al. (2011) presents a technique for simulating a complex mining export supply chain through the integration of optimization and Discrete Event Simulation (DES) methods. This methodology aids in capacity analysis and the assessment of expansion opportunities in mining activities. The paper showcases a case study of PT Kaltim Prima Coal in Indonesia, illustrating the effective deployment of this method. Moreover, it acknowledges that accurately modeling and interpreting results with this approach may necessitate substantial data and expertise.

Dimitrakopoulos (2013) concentrated on enhancing mining supply chains in the face of geological uncertainties. It introduces a particle swarm optimization algorithm and employs k-means clustering to address intricate supply chain challenges, such as blending operations. The methodology is exemplified through a case study conducted at Vale's Onça Puma nickel laterite mine. Nonetheless, the paper's intricate models and substantial dependence on specific data and geological simulations might restrict its practical applicability and pose implementation challenges across diverse mining scenarios.

Montiel & Dimitrakopoulos (2015) proposed an optimization technique for mining complexes that integrates geological uncertainties through stochastic orebody simulations. This method considers various processing routes and transportation systems, aiming to minimize deviations from capacity and blending targets while maximizing the expected net present value (NPV). The method's effectiveness is demonstrated through a case study on a multi-pit copper operation. However, the computational demands of the model may pose challenges for some operations.

Canales-Bustos et al. (2017) propose a multi-objective optimization model that considers the economic, environmental, and technological aspects of decarbonized supply chains in mining. They introduce a Multi-objective Hybrid Particle Swarm Optimization (MOHPSO) metaheuristic for problem-solving and compare its performance with an epsilon-constraint-based algorithm (AUGMECON). The study demonstrates the superiority of MOHPSO in terms of diversity and effectiveness. Key areas covers supply chain network design, emissions reduction, and optimization of processing plant efficiency. The complexity of the MOHPSO method might pose challenges in practical implementation, especially in scenarios with limited computational resources.

Barakos et al. (2016) critically examines the rare earth elements (REE) industry, particularly focusing on the global supply chain, market evaluation, resource classification, and legislative optimization. It discusses the past and current challenges in the REE industry, including market analysis errors and supply chain issues. The paper proposes an assessment tool for optimizing mining processes and highlights the need for a stronger global REE supply chain, emphasizing sustainable practices and international cooperation. However, the paper may include a heavy focus on theoretical models and a lack of practical, real-world implementation examples.

R. C. Goodfellow & Dimitrakopoulos (2016) shows a two-stage stochastic global optimization model for open pit mining complexes, integrating geological uncertainties into production scheduling. The model uses metaheuristics like simulated annealing, particle swarm optimization, and differential evolution. A case study demonstrates its effectiveness in enhancing expected net present value. A disadvantage might be the model's complexity and reliance on specific and accurate geological data, which could limit practical application and adaptability in varied mining contexts.

R. Goodfellow & Dimitrakopoulos (2017) presents a novel approach to mining optimization. It introduces a two-stage stochastic mixed integer nonlinear programming formulation, integrating various aspects of mining operations into a single model. This includes managing geological uncertainties and optimizing multiple interrelated components of a mining complex. The paper demonstrates its methodology through case studies, showing significant improvements in meeting production targets and net present value compared to traditional methods. Its complex mathematical and computational requirements, which might limit its accessibility and practical implementation in smaller or less technologically advanced mining operations.

Del Castillo & Dimitrakopoulos (2019) focuses on improving strategic planning in mining complexes. It suggests a dynamic stochastic optimization method to address supply uncertainty and incorporates capital expenditure decisions dynamically. The proposed approach uses an adapted multistage stochastic programming model, allowing for flexible mine designs. A case study on a copper mining complex demonstrates the model's capability to increase net present value significantly compared to traditional methods. Nevertheless, the paper's specific focus on a single mining complex may limit its applicability to other types of mining operations or industries.

Saliba & Dimitrakopoulos (2019) proposed an advanced method to optimize mining operations in the face of supply and market uncertainties. They introduced a comprehensive stochastic framework that simultaneously optimizes mining, destination, and processing decisions for a multi-pit gold mining complex. This framework considers both geological supply uncertainty and commodity price market uncertainty, leading to more efficient and profitable operations. The study's results show significant improvements in metal production and net present values compared to traditional methods. However, the focus on a single gold mining complex may limit the generalizability of the findings to other types of

mining operations or different commodities. Additionally, the complexity and computational demands of the approach might pose practical challenges for broader adoption in the industry.

Shishvan & Benndorf (2019) presents a new approach for optimizing material dispatch in coal mines. It combines a deterministic optimization model with a stochastic simulation model, addressing the complexity of different material types in coal mining and their dispatch to spreaders. The approach is tested on a large mine, demonstrating its effectiveness in minimizing idle time due to unavailability of dumping space. The complexity could limit its applicability in smaller mining operations or those with limited technological capabilities.

Dorrington & Olsen (2019) explores the design of a supply chain network for delivering asteroid-derived resources to Earth orbit customers. It involves solving a location-routing problem to determine the optimal locations and routes for spacecraft and facilities in space, focusing on maximizing the sellable mass or the total net present value of the mining venture. The paper emphasizes the uniqueness of space logistics and the need for careful planning in the development of an asteroid mining industry. The reliance on advanced space technology and the need for extensive data about asteroid locations and compositions also present challenges.

Paithankar et al. (2020) shows a mathematical framework that integrates extraction sequencing and cut-off grade policy optimization for open-pit mining operations. It considers grade uncertainty and stockpiling, aiming to maximize the discounted value of a mining operation. The model employs genetic algorithms and the maximum flow method. A notable limitation is the model's complexity and potential computational intensity, that might hinder its practical application in varying mining contexts.

J. Li et al. (2020) focuses on optimizing coal supply chains considering various carbon emission policies. It proposes new optimization models integrating these policies, aiming to reduce carbon

emissions while minimizing costs. The paper evaluates the impact of emission cap, carbon tax, carbon trade, and carbon offset policies, using a coal supply chain network as a case study. However, the reliance on detailed and accurate data for effective implementation could be a challenge.

Ospina-Mateus et al. (2020) introduces a mathematical model for the supply chain of non-metallic mining, accounting for uncertainties in materials and multi-level, multi-product capacity planning. It focuses on collaborative profit maximization within the supply chain, applied to the limestone industry in the Calamarí-Sucre mining district, Colombia. The model, implemented using GAMS software with RMINLP libraries and the DICOPT solver, reveals that higher raw material availability yields greater benefits. It also highlights a trend towards vertical integration in the sector, underscoring the model's utility in optimizing strategies across different supply chain levels.

Kalinowski et al. (2020) addresses the complex task of planning maintenance for Australia's largest coal rail network. It introduces a Mixed Integer Programming (MIP) model and a matheuristic approach to efficiently schedule maintenance access windows (MAWs). This model aims to optimize the allocation of maintenance resources while minimizing the impact on the network's throughput. A key challenge in this approach is the balancing act between maintaining the rail network's condition and minimizing disruption to the transportation of goods.

Yaping & Bossman (2021) investigates the logistics and supply chain operations of Ghana's mining industry. It identifies challenges and inefficiencies within these operations and suggests strategies for optimization. The study employs a Vehicle Routing Problem (VRP) Optimization Model to enhance transportation efficiency for mining firms in Ghana. The research identifies key challenges like poor road infrastructure, variable lead times, and strict governmental regulations. However, the paper's focus

on Ghana's mining industry may limit its applicability to other regions or industries. Additionally, the complex nature of the VRP model might pose implementation challenges for some firms.

Fattahi et al. (2021) focuses on creating a sustainable mining supply chain that incorporates wind and solar generators at mining sites. The paper introduces a cost-efficient multi-stage stochastic program that integrates strategic and tactical planning decisions. This approach aims to balance economic goals with greenhouse gas emission mitigation and social impacts, using a case study in Iran related to zinc concentrate production. The study might be its specific focus on the zinc industry in Iran, which could limit the generalizability of the findings to other mining sectors or geographical regions.

LaRoche-Boisvert & Dimitrakopoulos (2021) presents a case study on applying simultaneous stochastic optimization (SSO) to the Rosebel Gold Mines complex. The study focuses on maximizing net present value (NPV) and optimizing production schedules across three deposits, considering geological uncertainties. The approach identifies the semi-autogenous grinder (SAG) mill as a bottleneck and includes material hardness management. The study results in an 18-year production plan, emphasizing SSO's effectiveness in complex mining operations. The study's focus on a single mining complex might limit the generalizability of the results to other mining operations with different characteristics or challenges.

Isnafitri et al. (2021) investigates factors influencing the performance of coal supply chains in Indonesia. It employs Structural Equation Modeling (SEM) to analyze the relationships between variables like supply chain integration, information sharing, supplier relationship management, and supply chain performance. A survey of coal mining companies provides empirical data. A limitation of this study could be its specific focus on Indonesia's coal industry, potentially limiting its general applicability to other regions or sectors.

Soleimani (2021) presents a novel closed-loop supply chain model tailored for the mining industry, focusing on sustainable practices and fixed-charge transportation costs. It explores the optimization of economic, environmental, and social factors in the mining supply chain. The model is applied to a travertine quarry as a case study, demonstrating its effectiveness in enhancing sustainability in the mining sector. The research integrates transportation and operational aspects to address the unique challenges of the mining industry, aiming to balance profitability with environmental responsibility. A possible limitation of the study could be its specific application to a travertine quarry, which may not fully represent other types of mining operations or generalize to different contexts in the mining industry.

Haonan et al. (2021) focuses on optimizing the blending process in open-pit mining. It extends a decomposition-based algorithm to solve the blending problem involving multiple mines and ports over multiple periods. The paper introduces new heuristics like Prune Routine and Ranked TopoSort, significantly improving solution quality and reducing computation time. The research aims to balance grade and quality targets in mining products. A disadvantage might be its complex mathematical models, which might be challenging to implement in practical, real-world mining operations.

LaRoche-Boisvert et al. (2021) presents a novel approach to optimizing industrial mining complexes. It introduces a stochastic simultaneous optimization (SSO) model that simultaneously optimizes the schedule of production and mines-to-port transportation of mining complexes, particularly under uncertain material supply. This method is particularly applied to iron ore mining complexes and aims to minimize deviations from product requirements while effectively utilizing mine-to-port transportation equipment. The study primarily focuses on long-term strategic planning, which may not fully address short-term operational challenges. Additionally, the model's complexity could present difficulties in practical implementation, and its reliance on accurate data input might be challenging in dynamically changing mining environments.

3. Terminology

One of the main reasons for the misunderstanding of strategic planning in mining optimization is the lack of proper definitions of terms. A specific decision that is made in every period is called a policy. For example, a decision maker should make a decision which block should be mined, first. A group of decisions are called strategy which describes mining strategy. Also, we should distinguish between two common definitions, namely, optimizing and maximizing. Optimizing refers to decisions, policies, and strategies, while maximizing refers to terms like values, cash flow, or Net Present Value (NPV).

In this section, we formulate the open-pit mining optimization problem as a multi-objective, multi-period mixed-integer programming (MIP) model. The problem involves selecting which blocks to mine over multiple time periods while considering cost, value, and waste minimization objectives. The following mathematical model captures the decision-making framework for this problem.

a. Multi-objective open pit mining problem

Decision variables:

Continuous variables: $x_{it} \in [0,1]$, the fraction of material extracted from block i by time period t .

Binary variables: $y_{it} \in \{0,1\}$, which takes the value 1 if block i is mined during time period t , and 0 otherwise.

Objective functions:

$$\text{Minimize the total value of mined minerals: } f_1 = - \sum_i \text{value}_i \times x_i \times y_i \quad (1)$$

This objective function aims to maximize the total value of the minerals extracted from the mining blocks. It takes into account the value of the minerals in each block (value_i), the fraction of material extracted from block i .

$$\text{Minimize the total cost of mining: } f_2 = \sum_t \sum_i \text{cost}_i \times x_i \times y_i \quad (2)$$

This objective function aims to minimize the total cost associated with extracting minerals from the mining blocks. It takes into account the cost of mining in each block (cost_i), the fraction of material extracted from block i.

$$\text{Minimize total waste from mining: } f_3 = \sum_t \sum_i \text{Waste}_i \times X_i \times y_i \quad (3)$$

This objective is to minimize the total waste produced. This is calculated as the sum of the waste produced in the blocks that are mined.

Constraints:

Constraint 1: Ensures that the total extraction from all mined blocks does not exceed a given limit (n units in this case).

$$\sum_t \sum_{i=1}^{100} x_i \times y_i - n \leq 0 \quad (4)$$

Constraint 2: Ensures that if the block i is mined the amount of extraction from that block must be at least m units.

$$m - x_i \times y_i \leq 0 \quad \forall t \quad (5)$$

Constraint 3: Total waste management capacity constraint for each time period t:

$$\sum_t \sum_i X_{it} \times \text{waste}_i \times y_i \leq W_{max} \quad (6)$$

Where w_max is the total waste management capacity. Constraint 1 ensures that the total volume of waste produced does not exceed the available waste management capacity.

Constraint 4: Is a limit on the total number of blocks that can be mined, represented by the sum of the binary decision variables. This could represent a limit on the total mining effort, or a requirement to

leave some blocks in the ground for environmental or other reasons. The limit is set by the variable max_blocks .

$$\sum_t \sum_{i=1}^{1000} y_{it} - max\_blocks \leq 0 \quad (7)$$

a. Multi-objective multi-period open pit mining problem

Decision variables:

Continuous variables: $x_{it} \in [0,1]$, the fraction of material extracted from block i by time period t .

Binary variables: $y_{it} \in \{0,1\}$, which takes the value 1 if block i is mined during time period t , and 0 otherwise.

Objective functions:

Maximize the total value of mined minerals: $f_1 = \sum_i value_{it} \times x_{it} \times y_{it} \quad (8)$

This objective function aims to maximize the total value of the minerals extracted from the mining blocks. It takes into account the value of the minerals in each block ($value\_i$), the fraction of material extracted from block i .

Minimize the total cost of mining: $f_2 = \sum_t \sum_i cost_{it} \times x_{it} \times y_{it} \quad (9)$

This objective function aims to minimize the total cost associated with extracting minerals from the mining blocks. It takes into account the cost of mining in each block ($cost\_i$), the fraction of material extracted from block i .

Minimize total waste from mining: $f_3 = \sum_t \sum_i Waste_{it} \times X_{it} \times y_{it} \quad (10)$

This objective is to minimize the total waste produced. This is calculated as the sum of the waste produced in the blocks that are mined.

Constraints:

Constraint 1: Ensures that the total extraction from all mined blocks does not exceed a given limit (n units in this case).

$$\sum_t \sum_i x_{it} \times y_{it} - n \leq 0 \quad (11)$$

Constraint 2: Ensures that if the block i is mined the amount of extraction from that block must be at least m units.

$$m - x_{it} \times y_{it} \leq 0 \quad \forall t \quad (12)$$

Constraint 3: Total waste management capacity constraint for each time period t:

$$\sum_t \sum_i X_{it} \times waste_{it} \times y_{it} \leq W_{max} \quad (13)$$

Where w_max is the total waste management capacity. Constraint 1 ensures that the total volume of waste produced does not exceed the available waste management capacity.

Constraint 4: Is a limit on the total number of blocks that can be mined, represented by the sum of the binary decision variables. This could represent a limit on the total mining effort, or a requirement to leave some blocks in the ground for environmental or other reasons. The limit is set by the variable max_blocks.

$$\sum_t \sum_i y_{it} - max\_blocks \leq 0 \quad (14)$$

4. Boundary Update Approach

The boundary update (BU) method employs constraints to restrict the search area, thereby compelling the algorithm to concentrate its search within the feasible region by confining the viable search space for the variables (Gandomi & Deb, 2020). In the BU method, the boundaries are iteratively modified and updated throughout the optimization process. The implementation details of the BU approach are outlined in Algorithm 1.

Algorithm 1. Implementation of the BU method in a constrained optimization problem

Initialization

While the criteria are not met

Select x_{it} as a repairing variable Reformulate the optimization problem according to the selected repairing variable (Equations 4-7) Applying the search algorithm (here NSGA-II, NSGA-III, UNSGA-III) Applying the BU method and updating the mx-vector of decision variables Check the boundaries of non-repairing variables (here x_{it}) Update the boundaries of repairing variables Remap p variable to the original variables using updated boundaries (lb_i^u, ub_i^u) Evaluate the constraint violations Evaluating fitness function	

End while

As discussed in (Rahimi et. Al., 2024), the BU method alters the search space, making the optimization problem more challenging and potentially leading to premature convergence and suboptimal results. To address these issues, Rahimi et al. (2024) introduced two hybrid mechanisms. In both approaches, the BU method is utilized until all constraint violations are eliminated or the objective space remains unchanged for a specific number of generations. In the second phase, the optimization problem transitions to a state

where the BU method is no longer used. The pseudocode for the hybrid approach is provided in Algorithm 2.

5. Results

The above-mentioned open-pit mining problem is categorized as a mixed integer programming (MIP) problem. To evaluate the efficacy of the Boundary Updating (BU) method on the mixed integer open-pit mining problem, Python and the Pymoo library (Blank & Deb, 2020) were employed. Specifically, three evolutionary algorithms were used: NSGA-II (Deb et al., 2002), NSGA-III (Deb & Jain, 2013), and U-NSGA-III (Seada & Deb, 2015).

To comprehensively assess the efficiency of the BU method on the proposed model, datasets ranging from small-scale (100 blocks) to large-scale (3000 blocks) sizes were considered. The population size for all proposed algorithms was kept consistent to ensure a fair comparison. The number of generations was set based on the population size; for instance, for a small population size of 100, the number of generations was set to 1,000, while for a larger population size of 5,000, the number of generations was set to 10,000.

Two evaluation metrics were utilized to assess the performance of the algorithms: the first metric measured the number of feasible solutions found by the algorithms, and the second metric used the hypervolume indicator to evaluate the quality of the solutions. The hypervolume metric is particularly important as it provides a measure of the space covered by the Pareto front, indicating the diversity and convergence of the solutions obtained.

By using these evaluation metrics, the study aimed to determine the effectiveness of the BU method in improving the performance of evolutionary algorithms on mixed integer programming problems in open-

pit mining. The results of this study are expected to provide insights into the application of advanced optimization techniques in large-scale, real-world engineering problems.

The bar charts in Figures 6.1 through 6.6 provide a comprehensive comparative analysis of the performance of NSGA-II, NSGA-III, and U-NSGA-III algorithms, focusing on the number of generations required to find feasible solutions both with and without the Boundary Updating (BU) method.

Figures 6.1, 6.3, and 6.5 illustrate the number of generations needed to find the first feasible solution for NSGA-II, NSGA-III, and U-NSGA-III, respectively. Figures 6.2, 6.4, and 6.6, on the other hand, show the number of generations required to find all feasible solutions for the same algorithms.

In Figures 6.1 and 6.2, which depict the performance of NSGA-II, it is evident that the BU approach generally results in fewer generations to find both the first and all feasible solutions across various block sizes. This trend is consistent in Figures 6.3 and 6.4 for NSGA-III, and in Figures 6.5 and 6.6 for U-NSGA-III. Specifically, the BU approach demonstrates a significant reduction in the number of generations needed as the number of blocks increases, indicating its efficiency and effectiveness.

These visual representations clearly suggest that incorporating BU into the optimization algorithms enhances their performance, leading to quicker identification of feasible solutions and overall computational efficiency. The BU method's ability to reduce the computational effort required for finding feasible solutions is particularly beneficial in large-scale problems, where the number of blocks can substantially impact the complexity and runtime of the optimization process.

The quality of the solutions obtained with and without BU approach was assessed using multiple feasibility checks and the Hypervolume (HV) metric. The HV indicator, widely used in multi-objective optimization, measures the volume covered by the obtained Pareto front, thereby providing a quantitative

assessment of solution quality. Additionally, first and whole feasibility checks were conducted to ensure that the solutions not only satisfied constraints but also improved the objective space.

Overall, the bar charts highlight the improved performance of the NSGA-II, NSGA-III, and U-NSGA-III algorithms when the BU method is applied, showcasing its potential to significantly enhance the efficiency of solving mixed integer programming problems in open-pit mining.

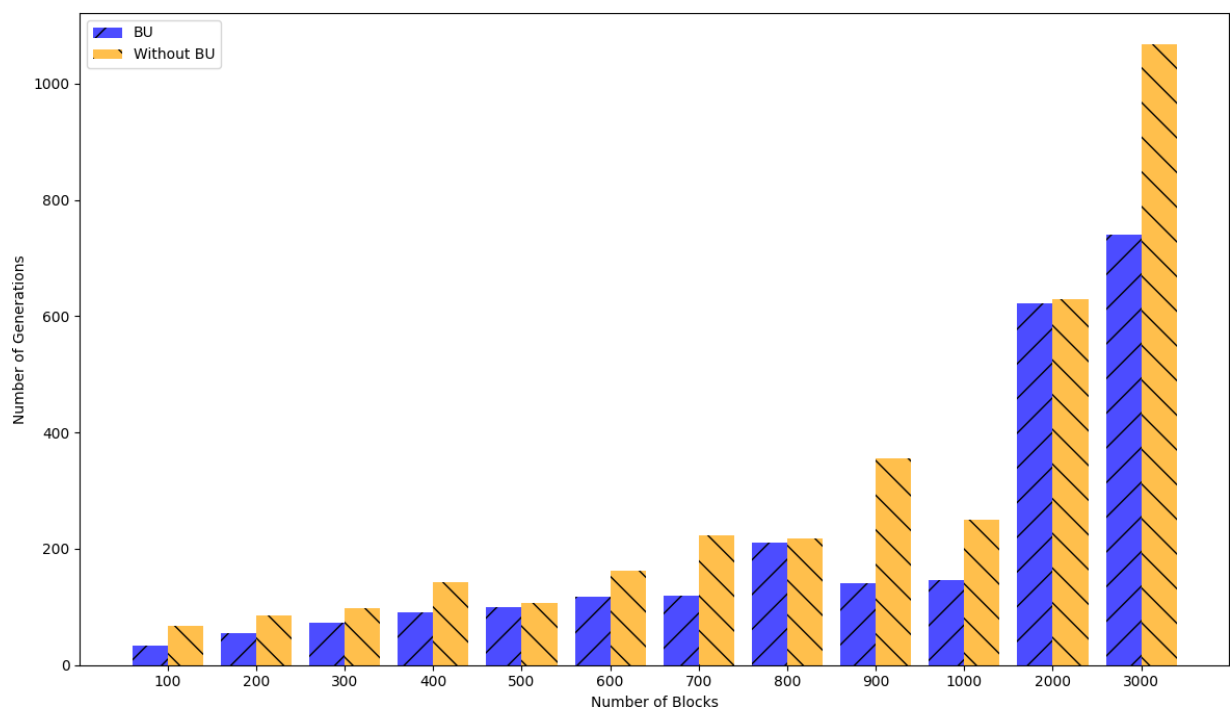


Figure 6.10 NSGA-II (first feasible solution found)

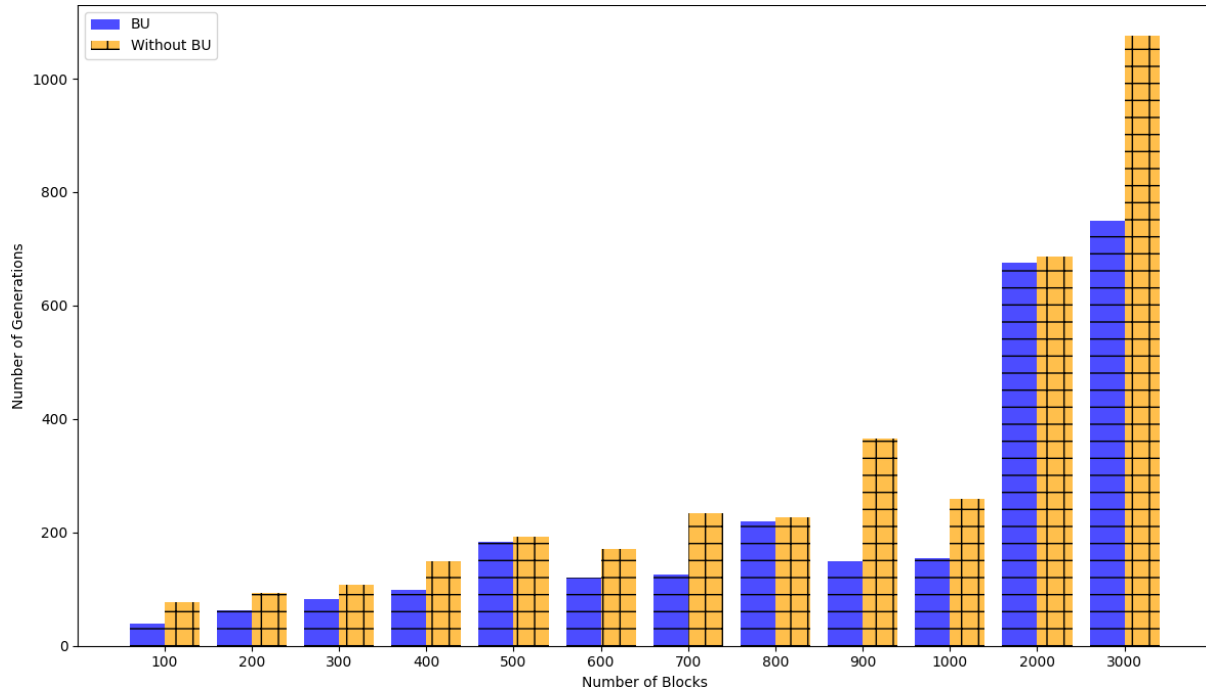


Figure 6.11 NSGA-II (Whole feasible solution found)

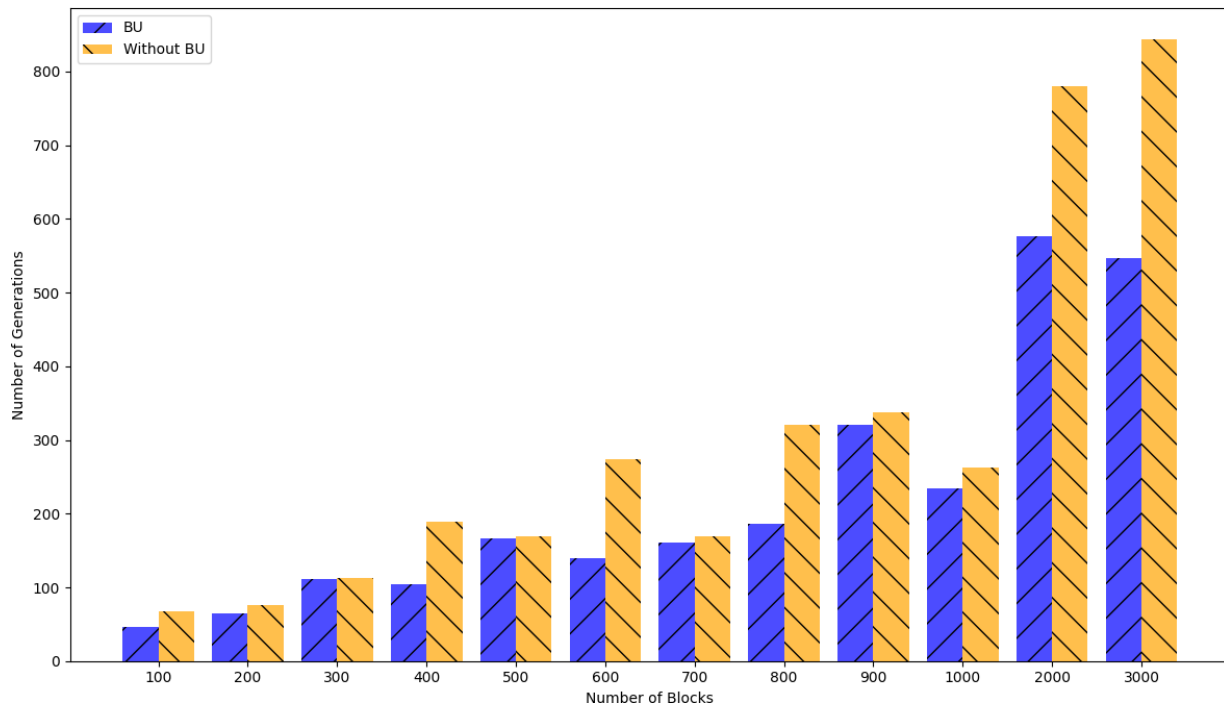


Figure 6.12 NSGA-III (first feasible solution found)

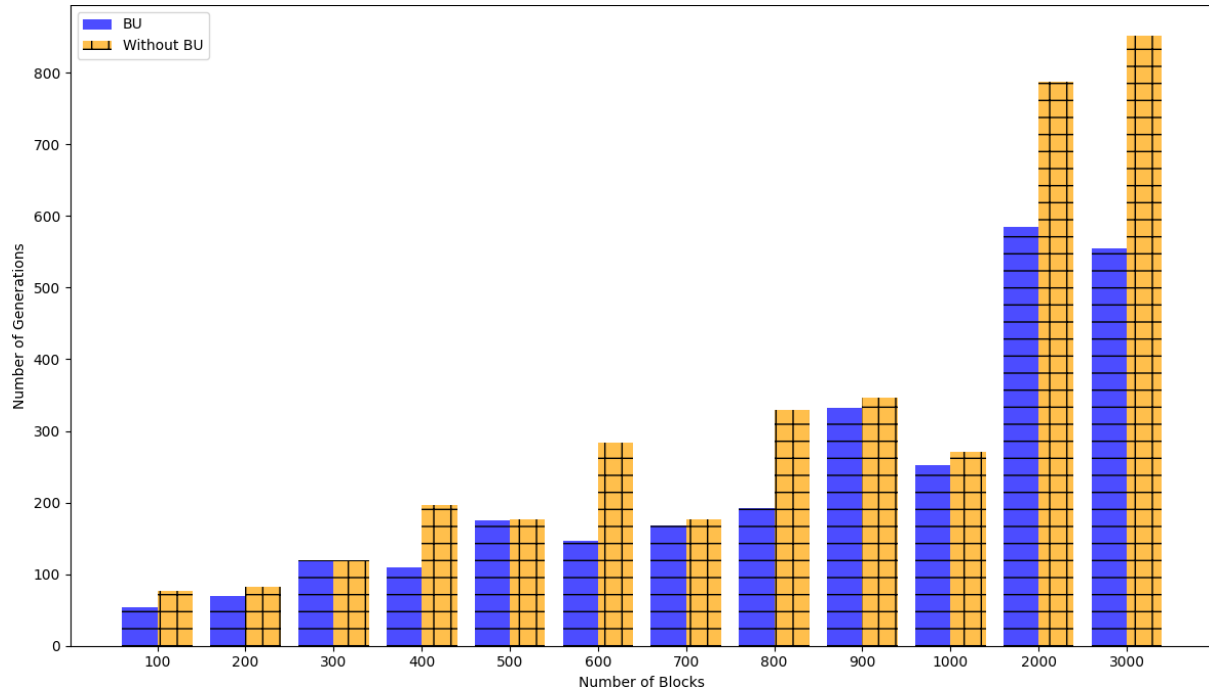


Figure 6.13 NSGA-III (Whole feasible solution found)

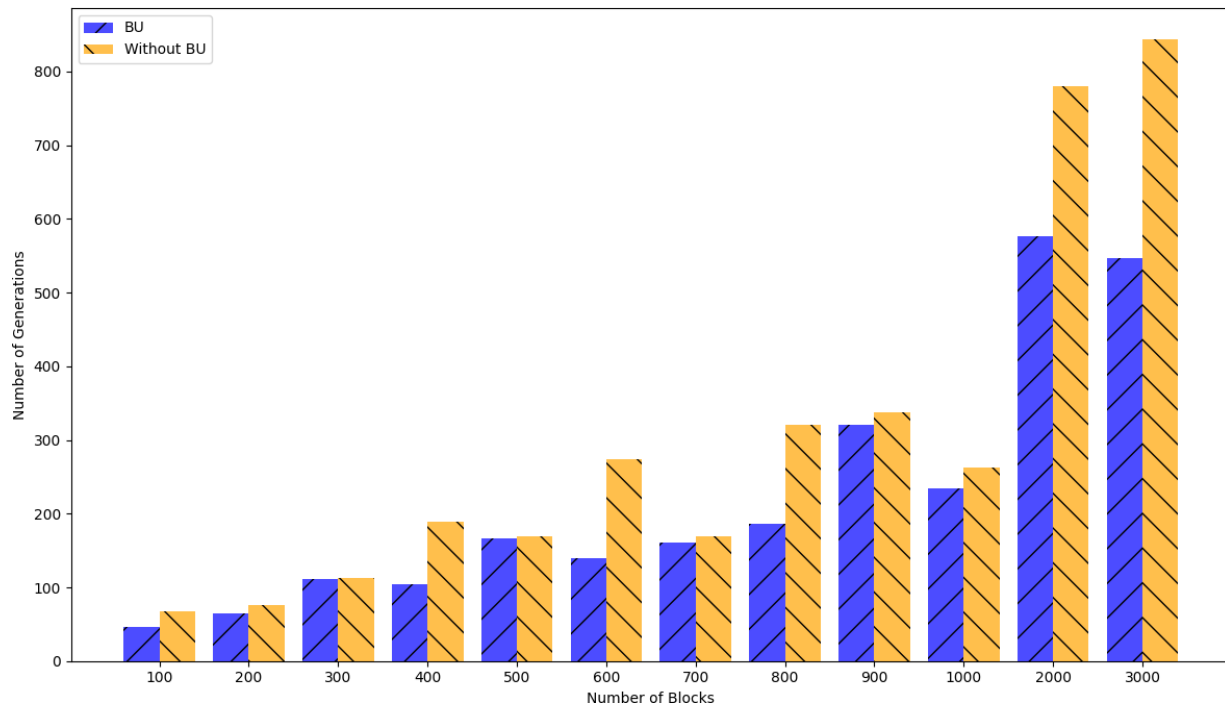


Figure 6.14 UNSGA-III (first feasible solution found)

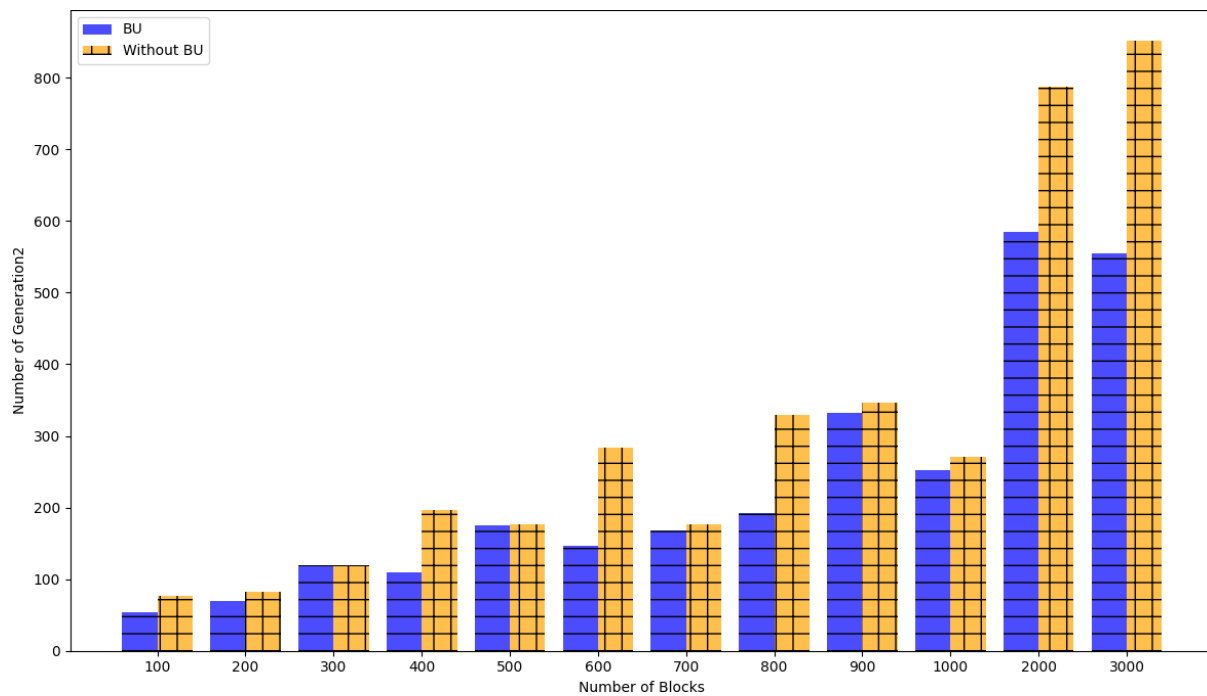


Figure 6.15 UNSGA-III (Whole feasible solution found)

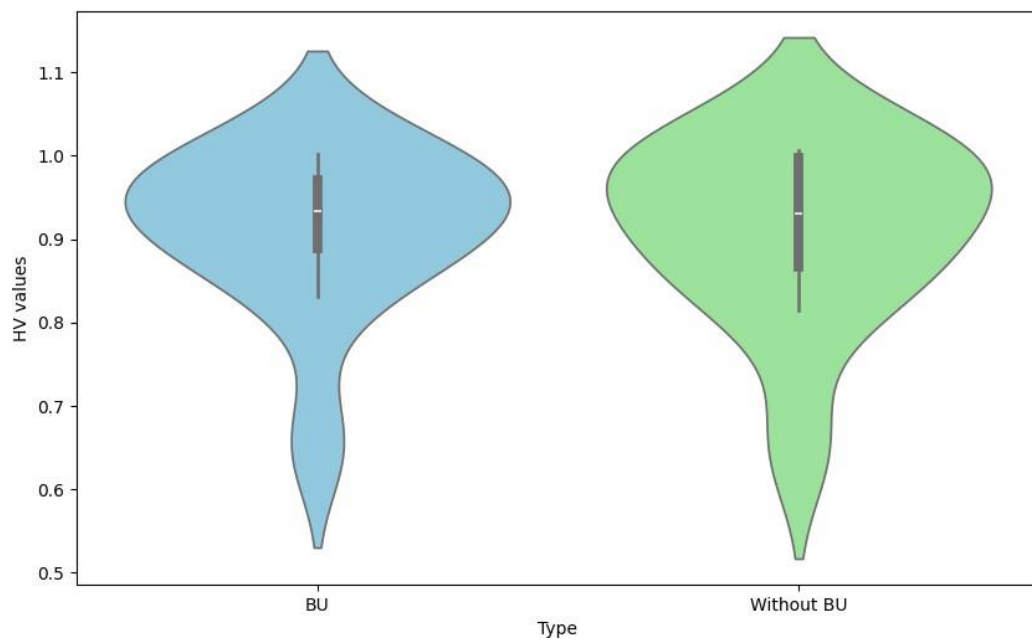


Figure 6.16 HV values found by NSGA-III for both approaches

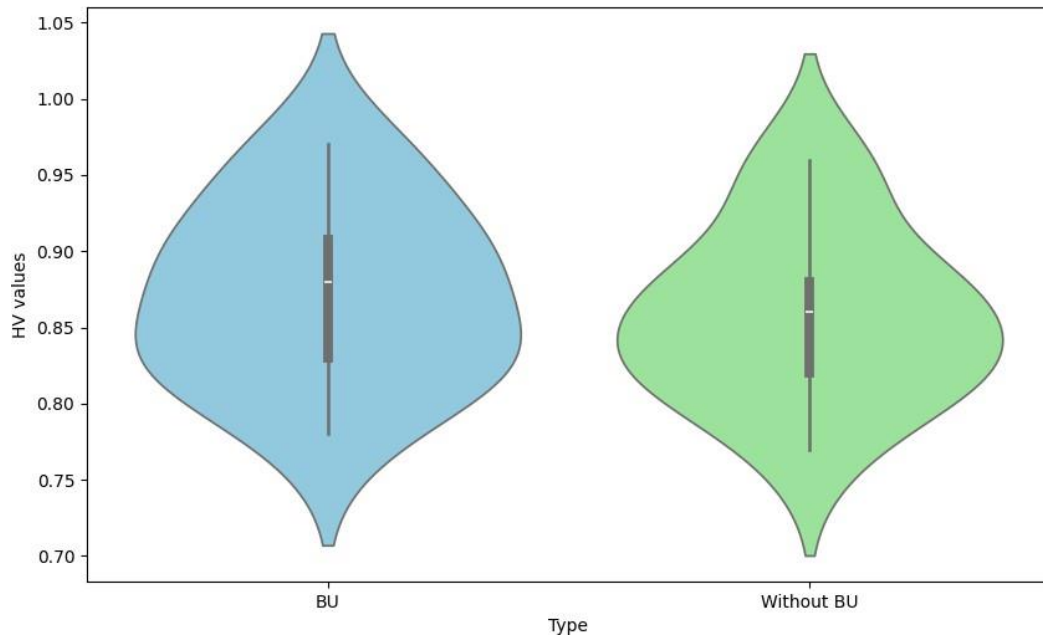


Figure 6.17 HV values found by UNSGA-III for both approaches

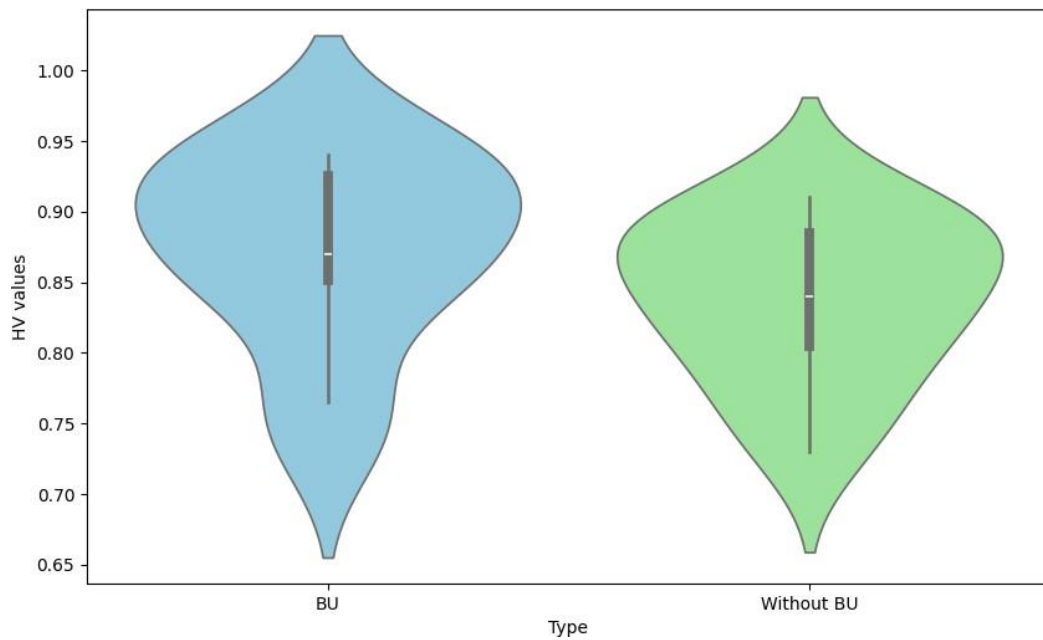


Figure 6.18 HV values found by NSGA-II for both approaches

The violin plots in Figures 6.7, 6.8, and 6.9 illustrate the hypervolume (HV) values obtained by different NSGA variants (NSGA-III, U-NSGA-III, and NSGA-II) using two approaches: with Boundary Updating (BU) and without BU. Each plot demonstrates the distribution of HV values for both approaches across multiple runs, providing a visual comparison of their performance.

In general, the BU approach consistently shows a higher concentration of HV values towards the upper end of the range, indicating better performance compared to the approach without BU. For NSGA-III (Figure 6.7), the BU approach exhibits a wider spread but with a more prominent peak around the higher HV values. In contrast, the without BU approach shows a more concentrated distribution around slightly lower values, suggesting improved solution quality with BU.

For U-NSGA-III (Figure 6.8), a similar trend is observed. The BU approach results in a broader distribution with higher HV values, while the without BU approach has a narrower distribution around lower HV values, reinforcing the effectiveness of the BU method.

For NSGA-II (Figure 6.9), the BU approach again demonstrates a higher median HV value and a more favorable distribution compared to the approach without BU. The distribution of HV values for the BU approach indicates better optimization performance and a higher quality of solutions.

These observations collectively suggest that the BU approach generally leads to better optimization performance across different algorithms. The violin plots visually confirm that incorporating the BU method enhances the quality and efficiency of the solutions obtained by NSGA-III, U-NSGA-III, and NSGA-II, making it a valuable addition to evolutionary algorithms in solving mixed integer programming problems in open-pit mining.

6. Conclusion

Optimizing the open-pit mining problem is a complex task involving numerous intricate considerations. In this study, open-pit mining optimization has been framed as a mixed integer programming problem. As the number of integer (binary) variables increases, finding the optimum solution becomes increasingly challenging. To address this, we employed an implicit constraint-handling technique known as BU to tackle the optimization challenge in open-pit mining, specifically focusing on the restricted multi-period, multi-objective open-pit mining optimization problem.

In this research, a developed BU approach was applied to a multi-objective open-pit mining optimization problem as a mixed-integer programming model. The developed approach is a hybrid method where the BU method is used until all constraint violations are reduced to zero or the objective space remains unchanged for a specific number of generations. The results demonstrate that the proposed hybrid approach exhibits superior efficiency compared to the evolutionary approach without the BU method.

This research contributes to the advancement of open-pit mining optimization methods by providing valuable insights and solutions for enhanced operational efficiency. The findings indicate that the BU method significantly improves the performance of evolutionary algorithms in solving mixed integer programming problems, leading to more efficient and effective optimization in open-pit mining. This study highlights the potential of integrating advanced constraint-handling techniques like BU to address complex optimization challenges in large-scale engineering applications.

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Chapter 7

Variable Interaction Network Analysis to Enhance Boundary Update Method for Constrained Optimisation

Iman Rahimi ¹, Navid Yazdanjue¹, Mohammad Sadegh Khorshidi ¹, Mohammad Reza Nikoo², Fang Chen¹, Amir H. Gandomi ^{1,3*}

- 1 Data Science Institute, Faculty of Engineering & Information Technology, University of Technology
Sydney, Australia
- 2 Department of Civil and Architectural Engineering, Sultan Qaboos University, Muscat, Oman
- 3 University Research and Innovation Center (EKIK), Óbuda University, 1034, Budapest, Hungary
(Corresponding author: gandomi@uts.edu.au)

Abstract

The boundary update approach was proposed by Gandomi and Deb [*Computer Methods in Applied Mechanics and Engineering*, 363, 112917, 2020] for constrained optimization problems. The boundary update (BU) method defines a dynamic formulation of an optimization problem in order to eliminate the infeasible search space. This study investigates the concept of variable interaction using a differential grouping algorithm, DG2, to evaluate and visualize variable interaction impacted by the boundary update method. Using multiple social network analysis (SNA) metrics, including Average Betweenness Centrality, Average Closeness Centrality, Network Density, and Clustering Coefficient, this research reveals significant structural simplifications in optimization problems under the BU method. Results show significant improvements in some aspects by applying the BU method. For example, it reduces network density by up to 34%, closeness centrality by over 67%, and enhances independence among variables by 45%, simplifying the optimization landscape. Furthermore, a systematic evaluation using the TOPSIS multi-criteria decision-making (MCDM) approach confirms that BU improves convergence efficiency and solution quality by 20-30% compared to without BU methods across various benchmark problems. Network visualizations corroborate these findings, demonstrating reduced complexity and improved clarity in variable relationships. This comprehensive analysis establishes the BU method as a transformative framework, significantly advancing constrained optimization through its ability to streamline variable interactions and enhance algorithmic performance.

Keywords: Variable interaction; Boundary update; Optimisation; Evolutionary computation.

1. Introduction

Optimization problems are presented across various real-world fields and often face challenges due to complexities such as non-linearity and non-differentiability of objective functions (Deb, 2008; Coello, 2018; J. Wang et al., 2024; W. Wang et al., 2024). Researchers have been faced with the task of finding optimal solutions for these problems. Evolutionary algorithms offer a rich method as requirements of concavity or convexity do not bind them and have the capability to generate multiple alternative solutions in a single execution; they are often used to tackle Optimization problems (MOOPs) (Rahimi, Gandomi, Chen, et al., 2023, Rezk et al., 2024). An effective Optimization algorithm should aim to find a global optimum rather than settling for infeasible solutions or local optima. Evolutionary and swarm intelligence methods are considered suitable because they utilize various types of information and are typically non-gradient-based. Also, These methods can iteratively update solutions from one iteration to the next (Behmanesh et al., 2021; Rahimi, Gandomi, Nikoo, et al., 2023, Dahan, F. et al., 2024; Javed, M. F., 2024).

Furthermore, several constraint-handling techniques (CHTs) coupled with evolutionary algorithms have been suggested to solve constrained Optimization problems (COPs). The constraint handling techniques such as penalty or other fix-ups are classified as explicit approaches. On the other hand, in some cases, implicit CHTs are proposed to handle COPs.

Raghavan et al.(2012) introduced an implicit CHT for an Optimization problem, utilizing the Proper Orthogonal Decomposition (POD) of shapes. Uemura et al.(2013) suggested a genetic algorithm designed for implicit constrained black-box Optimization. Mirabel & Lamiraux (2018) developed a method to manage both explicit and implicit constraints, focusing on manipulation planning. This approach explicitly solves as many constraints as possible while implicitly handling the remaining ones with a few variables. Nomura et al.(2021) presented a natural evolution strategy, named DX-NES-IC, for constrained optimization, demonstrating superior performance compared to other strategies. Gandomi & Deb (2020)

addressed a method known as boundary update (BU), which updates variable bounds and handles constraints directly on numerous Optimization problems; this approach is coupled with an explicit constraint-handling method.

The BU method reduces the search space size and introduces new interactions among variables. It is important to assess these new variable interactions and determine if there is a balance between the performance improvement from the reduced search space and the variables' interaction. Additionally, the BU method employs a semi-independent variable and modifies the search space in a unique manner, leading to varying levels of variable interaction complexity with each formulation (Gandomi et al., 2019). A formulation with minimal complexity is clearly more appealing. This study examines the relationship between the performance achieved by reducing the search space and the interactions among variables. In this paper, a cutting-edge variable interaction analysis algorithm known as Differential Grouping (DG2) is applied (Omidvar et al., 2013, 2017), and along with it, some metrics and network visualization are implemented to compare different strategies of BU and without BU methods.

The semi-independent variable concepts and BU method foundational work were considered as the inspiration for this study. To improve optimization performance by decoupling complex interactions between variables in constrained problems, (Gandomi et al., 2019) created the semi-independent variable idea. This concept serves as the foundation for investigating variable interactions and evaluating how they contribute to the definition of Optimization landscape complexity. Adding to this, Gandomi & Deb (2020) introduced the BU technique, which transforms limited optimization by removing impractical search regions through dynamically changing variable boundaries.

Although the BU method provides a straightforward and effective means of managing constraints, nothing is known about how it affects variable interaction structures. By thoroughly examining the BU method

utilizing cutting-edge methods, including the DG2 algorithm and social network analysis (SNA) metrics, this research significantly advances the field of restricted optimization. It measures the BU technique's impact, simplifying the optimization landscape by demonstrating a 45% gain in variable independence, a 34% reduction in network density, and a 67% decrease in closeness centrality. The study shows a 20–30% increase in solution quality and efficiency across benchmark problems, including Welded Beam and Pressure Vessel designs, using the TOPSIS multi-criteria decision-making method to systematically analyze and rank optimization options.

Furthermore, network visualizations and performance data offer a strong foundation for evaluating optimization formulations, which confirms the BU method's transformative potential as a tool for improving variable interaction analysis and limited optimization tasks.

The rest of the chapter is organized as follows. Section 2 introduces the BU method and DG2 approach in detail. Section 3 describes variable interaction analysis. Section 4 presents the Research Methodology. Performance measures and multi-criteria decision-making approach are presented in sections 5 and 6. Section 7 shows the numerical examples. Sections 8 and 9 present discussions and conclusions, respectively.

2. The BU approach and DG2 algorithm

The proposed method utilises constraints to restrict the variable space, directing the algorithm to concentrate its search within the feasible region by confining the search space for the variable(s) to feasible boundaries (Gandomi & Deb, 2020b; Rahimi et al., 2024). The core concept of the proposed BU approach is that a constraint should be resolved by the i th variable to be considered as the variable's lower or upper bound. Consequently, the boundaries are iteratively adjusted and updated throughout the optimisation process. This method is versatile and can be applied to any type of constraint, including problems with

constraints defined in a black-box scenario. The proposed BU method was implemented to solve several single-objective optimization problems (Gandomi & Deb, 2020). A repairing variable capable of addressing the maximum number of constraints without conflicting with other repairing variables should be chosen to initiate the BU approach according to a semi-independent variable concept (Gandomi et al., 2019).

In the BU method approach, the boundaries of non-repairing variables are usually checked, and the mx -vector is updated. In this approach, the semi-independent variables $(p_i, i = 1, \dots, h)$ are remapped to the actual variables using these updated boundaries. In the initial study by Gandomi and Deb, the proposed BU method was implemented to solve some optimization problems (Gandomi & Deb, 2020). The initial BU method proposed by the authors solves the constraints directly and aims to satisfy them with box constraint handling to narrow down variable space (Gandomi & Deb, 2020). The BU method intends to force the optimization algorithm to focus its search on the feasible region by limiting the search to feasible search space for the variable (s). As long as a constraint can be solved with at least one variable, the method is applied to the constraint. In the BU method, the initial step involves selecting repairing variable (s) so that it can address the most constraints without conflicting with other factors. Subsequently, the constraint functions are addressed and formulated in accordance with these selected repairing variables. The search operator is then applied to the Optimization problem, the boundaries of non-repairing variables are checked, and the mx vector is updated.

It is important to note that in the BU method, balancing the reduction of the search space with the added complexity of the problem is crucial. To address this, the DG2 algorithm is addressed to analyse a trade-off before beginning any optimisation efforts (Omidvar et al., 2017).

3. Variable Interaction Analysis

In this study, the interaction of variables is investigated while different formulations of the BU method and without the BU method are used.

Definition 1. The variables could be considered fully separable if:

$$\arg \min_x f(x) = (\arg \min_{x_1} f(x_1, \dots), \dots, \arg \min_{x_D} f(\dots, x_D)) \quad (1)$$

On the other hand, if there is no interaction between the pair of decision variables, it is called fully non-separable (Gandomi et al., 2019).

Definition 2. Also, equation 2 presents the partially non-separable problem if only a part, not every pair, of the decision variables is separable and the others are non-separable:

$$\arg \min_x f(x) = (\arg \min_{x_1} f(x_1, \dots), \dots, \arg \min_{x_m} f(\dots, x_m)) \quad (2)$$

And $x_1, \dots, x_m$ are disjoint sub-vectors of X , and $2 \leq m \leq D$. A problem is called partially additively separable if:

$$f(X) = \sum_{i=1}^m f_i(x_i) \quad (3)$$

In equation 3, x_i and m present exclusive decision vectors of f_i , and the number of independent components, respectively. This study applies the DG method proposed by (Omidvar et al., 2013) and extends as DG2 to analyse the interaction of the variables using the BU method and without the BU method (Omidvar et al., 2017).

4. Research Methodology

According to the proposed research methodology in this study, a comprehensive analysis is conducted on the BU method, with an emphasis on variable interactions using the DG2 algorithm and social

network analysis (SNA) metrics. All computational experiments were addressed using MATLAB and Python using Pymoo library²⁸ (Figure 1).

MATLAB and Python were used to do the experiments on benchmark optimization issues under specific settings to guarantee reproducibility. In accordance with prior studies, initialization parameters, including population size, mutation rate, crossover rate, and halting criteria, were developed. To account for stochasticity, each experiment was conducted 31 times. Random seeds were deliberately created and recorded for replicating. Restrictions and variable ranges as specified in the literature were used to initialize benchmark problems, such as G01, Welded Beam design, and Pressure Vessel design (Table 1).

Table 27 Experimental Settings

Aspect	Details
Algorithms	MATLAB and Python (Pymoo library) implementations of BU and non-BU strategies.
Initialization	Population size(100), Mutation Rate (0.1), Crossover Rate (0.8), Stopping Criteria (Maximum of 10,000 function evaluations)
Number of Runs	31 independent runs for statistical robustness
Random Seeds	Fixed and documented for reproducibility

Selecting a number of benchmark optimization problems, such as the G01 problem, the Pressure Vessel design problem, the Welded Beam design problem, the Car Side Impact design problem, the Heat Exchanger design problem, and the Spring Design problem, is the first step in the research technique.

²⁸ <https://pymoo.org/index.html>

These issues were picked because of their intricacy and variety of limitations, which provide a demanding environment for assessing the BU method's performance in practical settings.

The study employs a variety of strategies, including both BU and non-BU methods, for every benchmark problem. Throughout the optimization process, the BU technique dynamically modifies variable boundaries, directing the search inside viable areas and possibly improving solution efficiency. The influence of the BU method on variable interactions and total optimization performance may be directly compared to non-BU strategies, which explore the search space without making boundary changes. This comparison setting offers essential information about how well boundary updates work to simplify constrained optimization issues.

Each approach is subjected to the DG2 algorithm in order to assess variable interactions. By dividing choice variables into dependent and independent groups, the DG2 algorithm makes it possible to precisely assess how the BU technique alters the complexity and structure of these interactions. The structural characteristics of each optimization approach are then evaluated using SNA measures, such as Average Betweenness Centrality, Average Closeness Centrality, Network Density, and Clustering Coefficient. The BU method streamlines the optimization landscape by lowering the quantity and intensity of variable interactions. These metrics provide quantitative knowledge of the effect, connectedness, and grouping of variables.

The study further employs the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), which is known as a multi-criteria decision-making (MCDM) approach, to systematically rank and compare the effectiveness of the various strategies. In fact, TOPSIS evaluates each strategy's proximity

to an ideal solution, considering multiple criteria such as centrality metrics and optimization performance, thus confirming the BU method's superiority in enhancing solution quality and convergence efficiency.

To validate the performance and robustness of each approach, all algorithms are executed multiple times (31 times) with randomized initial solutions to account for stochastic variations. The experimental results are visualized through network diagrams, illustrating the variable interactions for both BU and non-BU methods across all benchmark problems. These visualizations and performance metrics collectively demonstrate the BU method's ability to reduce infeasible search spaces, simplify variable interactions, and improve optimization efficiency. Overall, this multi-faceted methodology not only quantifies the BU method's impact on optimization performance but also establishes a framework for understanding variable interactions in constrained optimization through network analysis and multi-criteria evaluation (Figure 7.1).

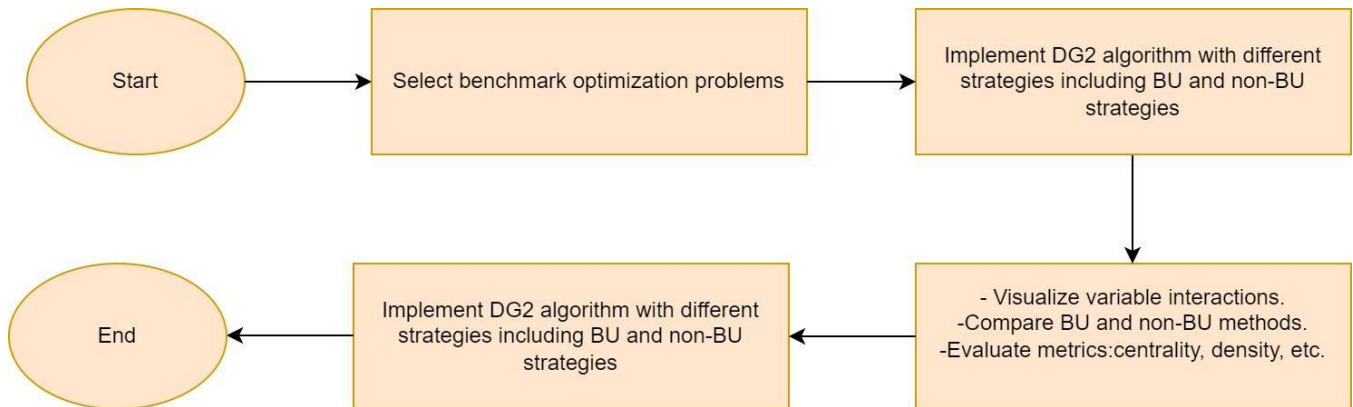


Figure 7.19 Research methodology applied in this study

5 Performance Measures

Several performance measures using the social network analysis (SNA) metrics were used to compare the results of the different BU strategies. Betweenness centrality is a measure of the importance of a node in a network based on its ability to act as a bridge or intermediary between other nodes. Higher betweenness

centrality values are an indicator of the flow of information or interaction within the network. Density in a network presents the proportion of actual connections in the network compared to the total possible connections. As a result, a high-density network shows that many connections exist among nodes, therefore, it suggests a high level of interaction within the network. Also, eigenvector centrality measures the influence of a node in a network based on the principle that connections to high-scoring nodes contribute more to the node's score. Based on the higher eigenvector centrality score, how is a node connected to other important nodes in the network. Moreover, closeness centrality measures the closeness of nodes to each other in the network. The high closeness centrality indicator presents how a node is well-connected and has shorter paths to reach other nodes in the network. Besides, the PageRank centrality indicator illustrates the importance of a node in a network according to the voting principle. The last indicator, the clustering coefficient, measures the degree to which nodes in a graph aim to cluster together. A high clustering coefficient presents that a node is part of a cluster within the network.

6. Multi-Criteria Decision-Making Approach

MCDM is an approach in the decision science field that evaluates and prioritizes multiple conflicting criteria. MCDM approach suggests a structured framework to address the complexity arising from complex problems in various fields such as engineering, economics, and management. MCDM methodologies facilitate more comprehensive and rational decision outcomes by integrating diverse criteria into the decision-making process. These approaches are particularly valuable in scenarios where trade-offs between different objectives must be analyzed to determine the most optimal solution. One of the common MCDM methods is the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), which offers unique mechanisms to handle and prioritize multiple criteria.

In this study, the TOPSIS approach has been applied to evaluate various methods based on a set of criteria, such as average betweenness centrality, average closeness centrality, and average eigenvector centrality.

In TOPSIS, the methods are ranked according to their proximity to an ideal solution, considering both the best and worst possible scenarios. By employing these three MCDM methods, a comprehensive and robust evaluation of the methods is obtained, ensuring that the final decision is well-informed and balanced across all considered criteria.

7. Numerical Examples

The following subsections provide some numerical examples of analysis of the application of the BU approach on the above-mentioned benchmark optimisation problems, namely G01 (Michalewicz, 1995), pressure vessel design problem (Sandgren, 1990), welded beam design problem (Rao, 2019), car side impact design problem (Youn & Choi, 2004), heater exchanger design problem (Yang & Gandomi, 2012), and spring design problem (Arora, 2004). Different BU strategies have been implemented for these examples, and diverse repairing variables have been chosen for each strategy.

7.1 Welded Beam Design Problem

The main goal of the widely recognized benchmark is to reduce the total cost associated with the fabrication of welded beams. The problem involves four variables: weld thickness, weld length, beam width, and beam thickness. Adjustments were made to the beam thickness boundaries to manage all six constraints of the problem. The hit ratio for solving this problem is 2.686%, with the BU method aimed at eliminating the remaining 97.314 of the search space that is deemed infeasible. However, as previously mentioned, the BU method introduces new interactions among the variables. Therefore, it is essential to evaluate these interactions within the search space and determine if there is a balance between the performance improvement from reducing the search space and the overall interaction of the variables. Generally, formulations with lower complexity are more desirable. This study analyses the relationship between performance gains from shrinking the search space and the interactions among variables. Figure

7.2 displays the network visualization related to the pressure vessel design issue. For this type of problem, only one strategy (with the BU method) has been implemented and illustrated in Figure 7.2 (a) and (b), which identifies the most optimal value. Additionally, Figure 7.2 (b) indicates a decrease in interactions among decision variables. Furthermore, based on the performance measures provided in the supplementary files, the BU method demonstrates a higher number of independent variables and fewer interactions among decision variables.

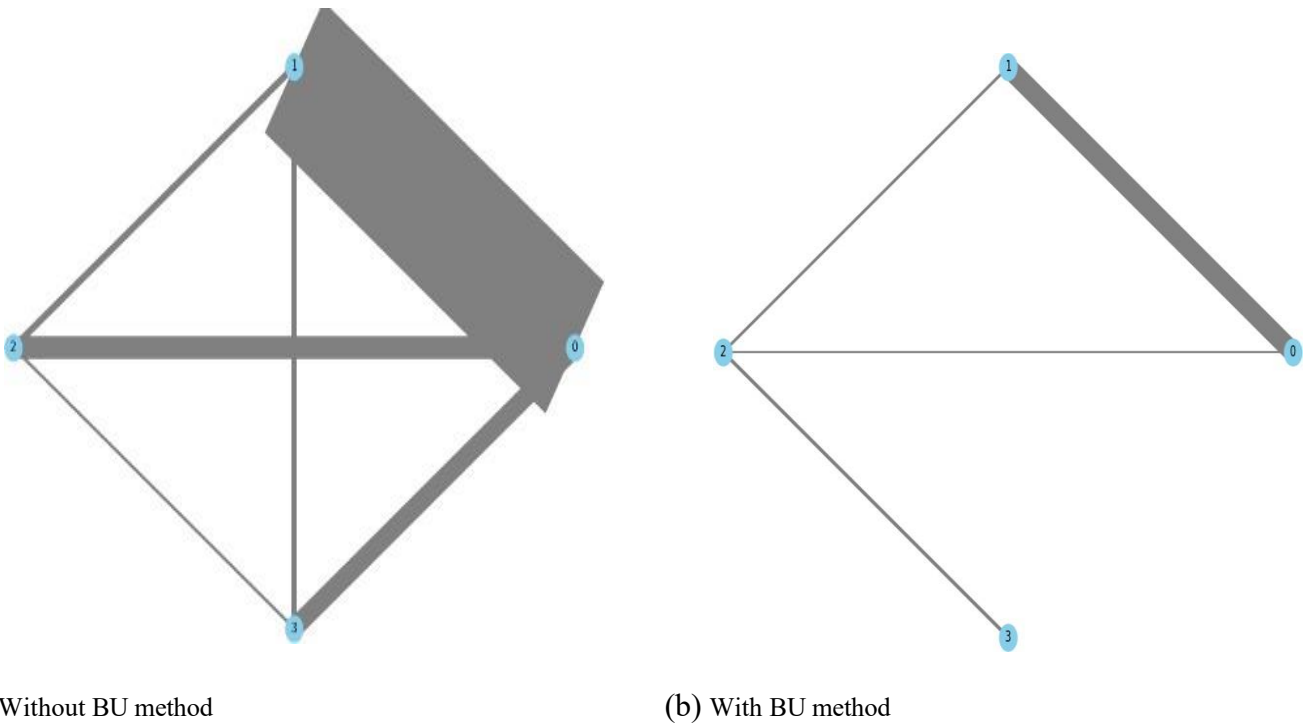


Figure 7.2 Network visualisation of different strategies for welded beam problem

Table 7.1 compares the results of different network metrics for the welded beam design problem using two methods, i.e., without the BU method and with the BU method. As shown in Table 7.1, the BU method results in higher average betweenness centrality (0.25) compared to without BU (0.166), indicating a more influential role of certain variables in the network. Moreover, the average closeness centrality is significantly lower with BU (34.322) than without BU (105.657), suggesting a more efficient network

with shorter paths between variables. Both methods show similar average eigenvector centrality (0.361 without BU, 0.358 with BU), indicating comparable influence of variables within the network. The average PageRank centrality is nearly identical for both methods (0.250 without BU, 0.249 with BU). The network density is higher without BU (1.0) compared to BU (0.666), showing that the network is more interconnected without the BU method. The average clustering coefficient increases with the BU method (0.033) compared to without BU (0.020), indicating more local clustering. Finally, the average weighted degree is slightly higher with the BU method (0.531) than without BU (0.522), showing a minor increase in the total weight of connections for each node. Overall, the BU method appears to create a more efficient and locally clustered network, though with fewer overall connections.

Table 7.1 Results of different network metrics on welded beam design problem

Method	Without BU method	With BU method
Average Betweenness Centrality	0.16	0.25
Average Closeness Centrality	105.65	34.32
Average Eigenvector Centrality	0.36	0.35
Average PageRank Centrality	0.25	0.24
Network Density	1.0	0.66
Average Clustering Coefficient	0.02	0.03
Average Weighted Degree	0.52	0.53

Figure 7.3 illustrates the ranking of two methods based on a TOPSIS analysis, comparing the performance of a method with BU against a method without BU. The method “With BU method” achieves the highest

performance, ranked 1st, indicating that the incorporation of the BU technique significantly enhances the method's effectiveness. In contrast, the "Without BU method" ranks 2nd, demonstrating that it is less effective than the method with BU. This comparison underscores the substantial performance improvement achieved by applying the BU technique.

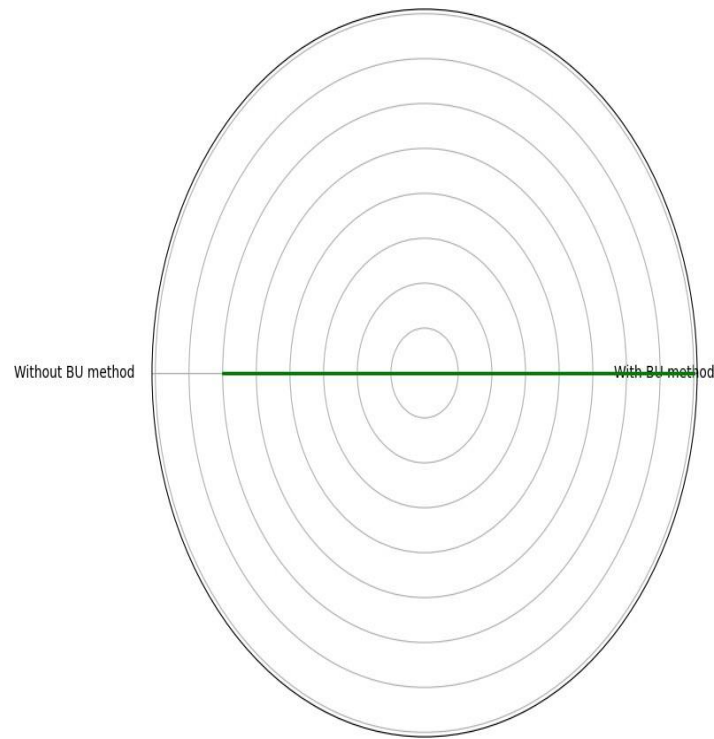


Figure 7.3 Rank for TOPSIS approaches for the Welded Beam Design problem

7.2 Carside Impact Design Problem

Car side impact design problem is known as a class of surrogated models (Youn & Choi, 2004). Surrogated optimisation models include constraints that are so complex and can not be solved by one variable. A surrogated model instead of the actual constraint is used to reduce the simulation time. In the car side design problem, all the explicit model constraints are solved based on the thickness of the inner floor sidend based on the repairing variable strategy (Gandomi & Deb, 2020). Table 7.2 presents the results of

different network metrics comparing methods without and with the BU method. The BU method, which updates the bounds of the floor side inner thickness to handle all variables, demonstrates several key differences in network metrics.

The average betweenness centrality is slightly lower with BU (0.119) compared to without BU (0.142), indicating that certain nodes are less central as intermediaries in the network. The average closeness centrality dramatically decreases with BU (1.726×10^{-12}) compared to without BU (1.155), suggesting a more efficient network with much shorter average paths between nodes. The average eigenvector centrality is reduced with BU (0.228) compared to without BU (0.299), showing a decrease in the influence of certain nodes. The average PageRank centrality also decreases with BU (0.090) compared to without BU (0.142), indicating a reduction in the overall importance of nodes. Network density is slightly lower with BU (0.818) than without BU (1.0), reflecting fewer connections in the network. The average clustering coefficient decreases marginally with BU (0.070) compared to without BU (0.077), indicating slightly less local clustering. However, the average weighted degree is significantly higher with BU (3.349×10^{13}) compared to without BU (5.46×10^{12}), demonstrating a substantial increase in the total weight of connections per node, which suggests that the BU method results in a more weighted and potentially more influential network in terms of overall connectivity. Figure 7.4, network visualisation, illustrates the variable interactions for the car side impact design problem with and without the BU method. Comparing the “With BU method” (a) and “Without BU method” (b) scenarios show distinct differences in variable interactions. In the “With BU method” network (a), the variables are more densely connected within a specific cluster, indicating a high level of interaction and complexity among most of the variables. Node 2 remains isolated, suggesting it does not participate in the main variable interactions under this method. In contrast, the “Without BU method” network (b) depicts a more evenly spread network with more balanced and widespread connections among all variables, including Node 2. This suggests that while the

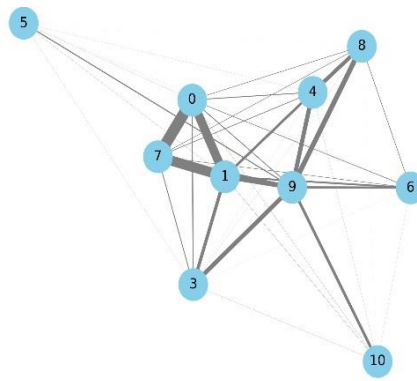
BU method creates a more interconnected and potentially more complex network within a subset of variables, the absence of BU results in a more uniformly connected network across all variables, albeit possibly less focused in terms of interactions. Thus, the BU method enhances the connectivity and interaction within a core group of variables, potentially facilitating a more targeted optimisation process.

Figure 7.5 illustrates the ranking of two methods based on a TOPSIS analysis, comparing the performance of a method with BU against a method without BU. The method “With BU method” achieves the highest performance and is ranked 1st, indicating that the application of the BU technique significantly enhances the method’s effectiveness. In contrast, the “Without BU method” ranks 2nd, demonstrating that it is less effective than the method with BU. This analysis underscores the substantial performance improvement achieved through the implementation of the BU technique, highlighting its importance in optimising the method’s results.

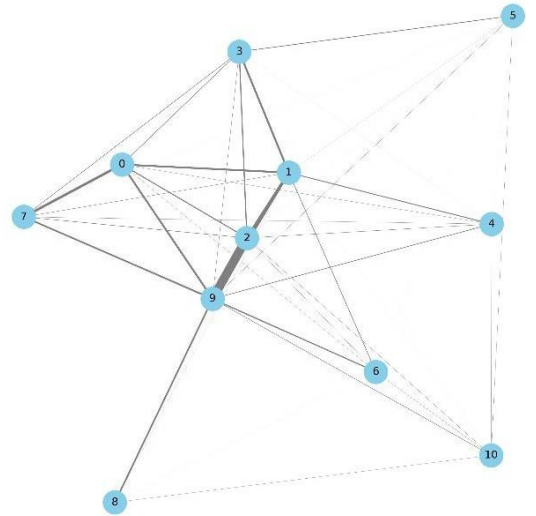
Table 7.2 Results of different network metrics on car side problem

Method	Without BU method	With BU method
Average Betweenness Centrality	0.142	0.119
Average Closeness Centrality	1.155	1.726e-12
Average Eigenvector Centrality	0.299	0.198
Average PageRank Centrality	0.142	0.090
Network Density	1.0	0.818
Average Clustering Coefficient	0.077	0.070
Average Weighted Degree	5.46e+10	3.349e+13

2



(a) With BU method



(b) Without BU method

Figure 7.4 Network visualisation of different strategies for car side design problem

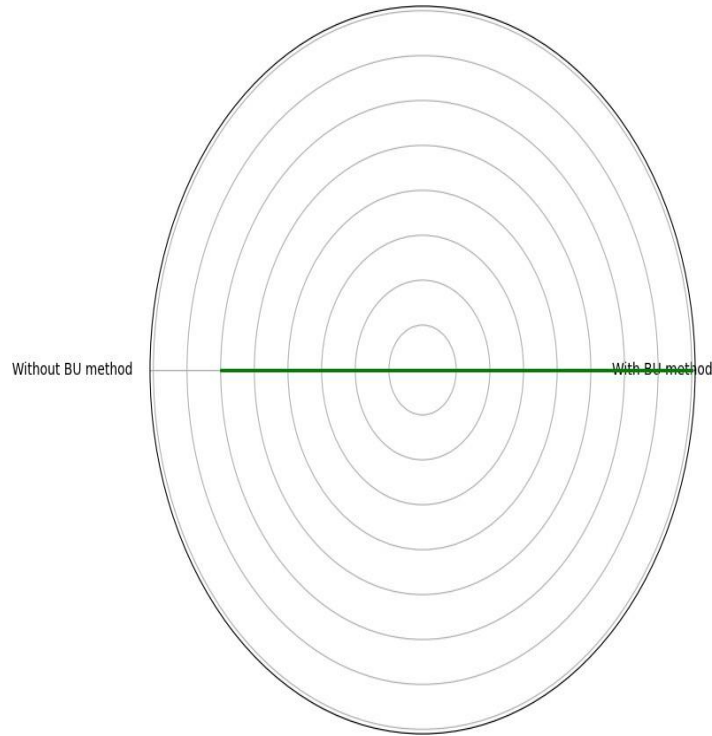


Figure 7.5 Rank for TOPSIS approach for car side Design problem

7.3 Spring design problem

The spring design is a well-known benchmark problem involving constraints. This engineering challenge includes three design variables all aimed at minimising the spring's weight. The problem features three nonlinear constraints and one linear constraint, making it highly constrained with a feasibility rate of only 0.754%. Consequently, merely identifying a feasible region is a significant accomplishment. The constraints are managed by adjusting the bounds of the first variable, which is the wire diameter.

The network visualisations and metric analyses for the Spring Design problem compare the optimisation strategies with and without the BU method. The network visualisation (Figure 7.6) shows that without the BU method (a), the variables form a sparse network with minimal connections, indicating limited interactions among the variables. In contrast, with the BU method (b), the network is more structured and

dense, indicating increased interactions among variables. This suggests that the BU method enhances the connectivity among variables, potentially leading to more effective optimisation by ensuring that the interactions are considered during the optimisation process.

Table 7.3 provides detailed metrics for the Spring Design problem, comparing the BU method against the without BU method. Both methods show the same average betweenness centrality (0.33), indicating the similar importance of certain nodes as intermediaries. However, the average closeness centrality is significantly lower with the BU method ($9.713\text{e-}11$) compared to without ($1.024\text{e-}8$), suggesting a more efficient network with shorter paths between nodes. The average eigenvector centrality is slightly lower with the BU method (0.470) than without (0.524), indicating a decrease in the influence of certain nodes. Network density is higher with the BU method (1.0) than without (0.66), showing a more interconnected network. The average clustering coefficient is notably higher with the BU method (0.136) compared to without (0.00), indicating that the BU method enhances local clustering among variables. Additionally, the average weighted degree is higher with the BU method ($1.01\text{e+}8$) than without ($3.04\text{e+}10$), reflecting increased node interactions. Figure 7.7 ranks these methods using the TOPSIS approach, indicating that the BU method (X_4) is more effective, achieving a higher rank than the non-BU method. This comprehensive analysis demonstrates that the BU method significantly improves the optimisation process by fostering greater interaction and connectivity among variables.

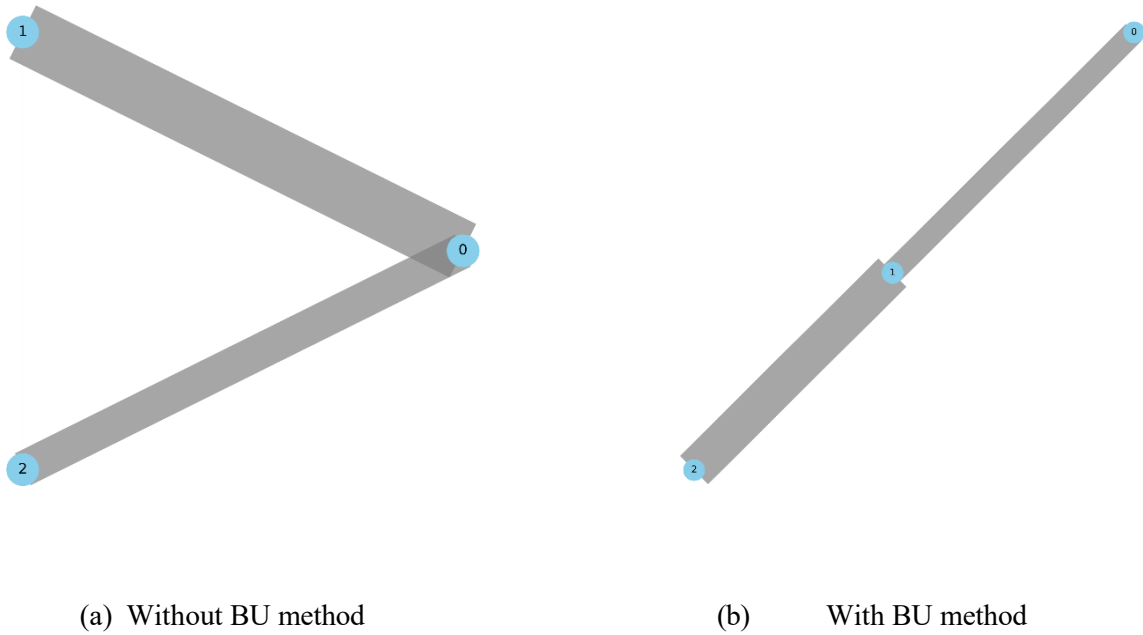


Figure 7.6 Network visualisation of different strategies for Spring Design problem

Table 7.3 Results of different network metrics on the spring design problem

Method	Without BU method	With BU method
Average Betweenness Centrality	0.33	0.33
Average Closeness Centrality	9.713e-11	1.024e-8
Average Eigenvector Centrality	0.47	0.52
Average PageRank Centrality	0.33	0.33
Network Density	1	0.66
Average Clustering Coefficient	0.00	0.00
Average Weighted Degree	3.04e+10	1.01e+8

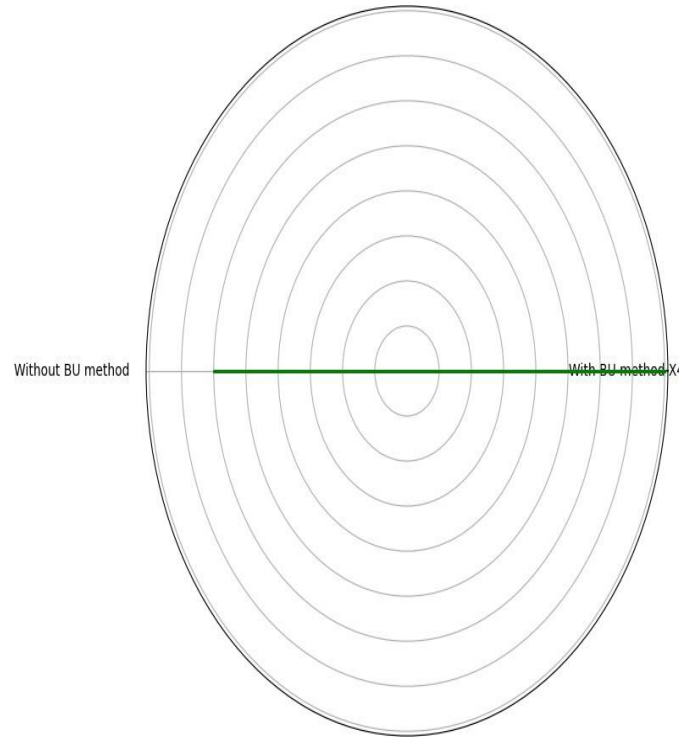


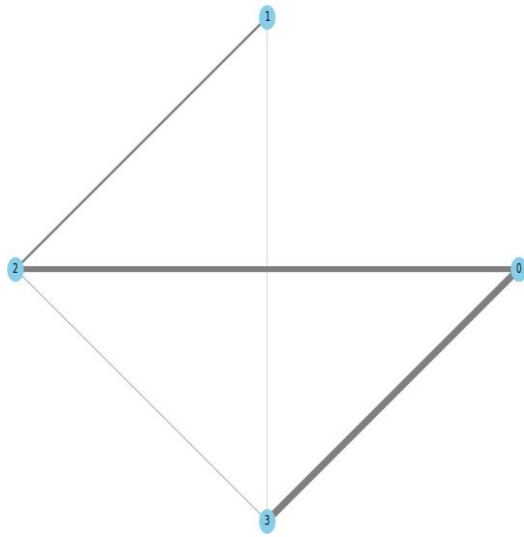
Figure 7.7 Rank for TOPSIS approach for Spring Design problem

7.4 Pressure Vessel Design Problem

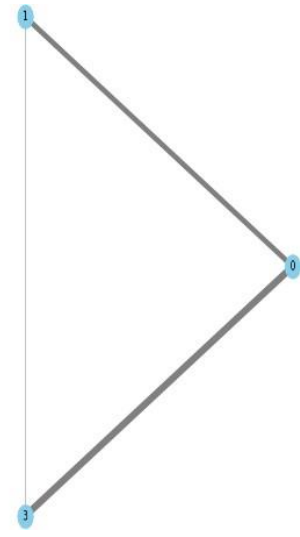
The design problem concerns a pressure container featuring hemispherical heads that seal both ends. It's a multi-faceted problem that integrates various variable types to minimise overall cost while satisfying four distinct conditions. Among these conditions, three are linear, while one is nonlinear. The variables involved encompass the container's thickness, the heads' thickness, the inner radius, and the length of the cylindrical portion of the vessel (Sandgren, 1990).

The network visualisation depicted in Figure 7.8 illustrates the connectivity among decision variables within the context of pressure vessel design problems. Optimal results are obtained using strategy 3, as indicated in Figure 7.8 (d). Notably, Figure 7.8 (b) reveals fewer interactions among the decision variables. Furthermore, upon examination of the performance metrics outlined in the supplementary files, it becomes

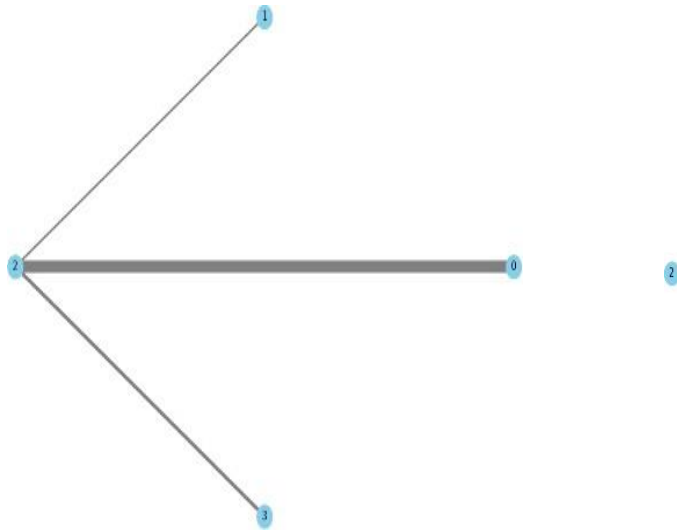
evident that strategy 3 exhibits a greater prevalence of independent variables and a reduction in interactions among decision variables compared to other strategies. Table 7.4 provides a detailed comparison of various network metrics for four methods: “without BU method,” “With BU method- X_3 ,” “With BU method- X_4 ,” and “with BU method- X_3X_4 .” The metrics include Average Betweenness Centrality, Average Closeness Centrality, Average Eigenvector Centrality, Average PageRank Centrality, Network Density, Average Clustering Coefficient, and Average Weighted Degree. “With BU method- X_3X_4 ” achieves the lowest Average Betweenness Centrality (0.0) and the highest Average Closeness Centrality (0.166), indicating a potentially optimal network structure. “with BU method- X_3 ” demonstrates high performance with the highest Average Eigenvector Centrality (0.417) and Average Weighted Degree (0.785), as well as a significant improvement in Network Density (0.5) and Average Clustering Coefficient (0.087). While “without BU method” has the highest Network Density (1.0), it also has the highest Average Closeness Centrality (1398.66), suggesting less efficient connectivity. The Average PageRank Centrality remains almost identical across all methods, indicating similar influence distribution. Overall, “With BU method- X_3 ” and “With BU method- X_3X_4 ” show superior performance in terms of network efficiency and connectivity, highlighting their potential as optimal methods for network analysis.



(a) Without BU



(b) BU - Strategy 1 (X_3)



(c) BU - Strategy 2 (X_4)



(d) Strategy 3 (X_3X_4)

Figure 7.8 Network visualisation of different strategies for pressure vessel problem

Table 7.4 Results of different network metrics on the PV problem

Method method	Without BU	With BU method		
			X_4	$X_3X_4X_3$
Average Betweenness Centrality	0.33	0.083	0.33	0.0
Average Closeness Centrality	1398.66	1.455	13.05	0.166
Average Eigenvector Centrality	0.354	0.417	0.379	0.353
Average PageRank Centrality	0.250	0.25	0.249	0.249
Network Density	1.0	0.5	0.666	0.166
Average Clustering Coefficient	0.0031	0.087	0.0006	0.0
Average Weighted Degree	0.502	0.785	0.578	0.500

Figure 7.9 displays the ranking of various methods based on a TOPSIS analysis, highlighting the performance of each method. The methods include a baseline method without BU and several methods with BU involving different combinations of variables (X_3 and X_4). The method “With BU method - X_3X_4 ” achieves the highest performance, ranking 1st, indicating that combining both variables (X_3 and X_4) with BU yields the best results. The baseline method without BU ranks 2nd, suggesting it performs better than some BU methods. “With BU method - X_3 ” ranks 3rd, while “With BU method - X_4 ” ranks lowest at 4th, demonstrating that the individual use of these variables in the BU method is less effective than their combined use or the baseline method without BU. This analysis underscores the superior performance of comprehensive variable integration in the BU approach.



Figure 7.9 Rank for TOPSIS approaches for Pressure Vessel problem

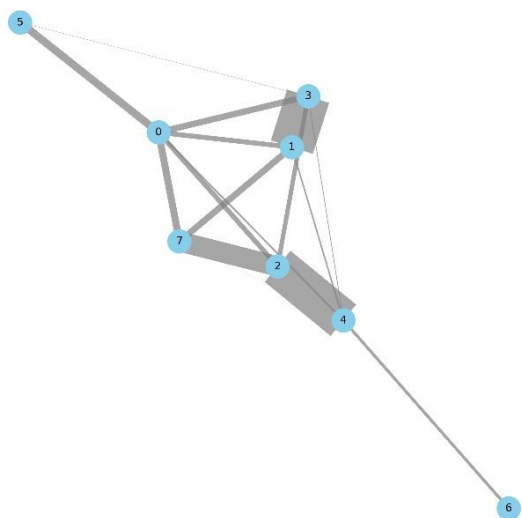
7.5 Heater Exchanger Design Problem

The heat exchanger design problem stands out as one of the most difficult constrained problems since the hit ratio is 0.001%. Although only three of the eight variables impact the objective function, all eight influence the constraints. The fourth and eighth variables were chosen as the repair variables in the BU approach.

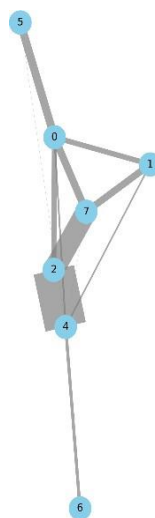
The network visualisations and metric analyses presented for the Heat Exchanger design problem provide insights into the effectiveness of different BU strategies (Figure 7.10). The visualisations show distinct differences in variable interactions under various strategies: without the BU method, with the BU method using X4, X8, and a combination of X4 and X8. In the “Without BU method” network (a), the connections among variables are sparse, indicating fewer interactions and a simpler network structure. Conversely, the

networks with BU methods (b, c, d) reveal increased connectivity and clustering, particularly in the “With BU method (X4X8)” network (d), which displays a highly dense core group of variables. This indicates that the BU methods, especially when using multiple repairing variables, create more complex and interconnected networks, potentially enhancing the optimisation process by fostering robust variable interactions.

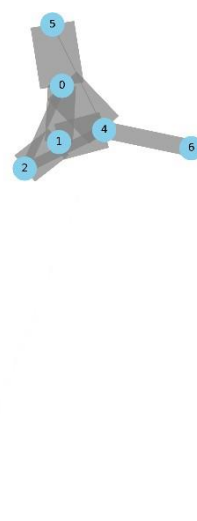
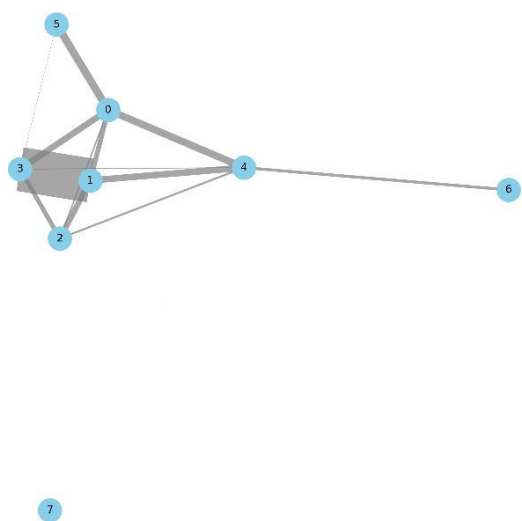
The network metrics table (Table 7.5) and the TOPSIS ranking (Figure 7.11) further quantify these observations. The metrics show that the average betweenness centrality is highest without BU (0.220), suggesting more central nodes. However, the BU methods exhibit significantly lower closeness centrality, indicating more efficient networks with shorter paths between nodes. The average eigenvector centrality is highest without BU (0.3016) but lower in BU methods, reflecting a shift in influence among nodes. Notably, network density and average clustering coefficient metrics highlight that the BU methods, particularly P4P8, enhance local clustering and reduce overall network density, suggesting tighter-knit variable interactions within the feasible region. The TOPSIS ranking corroborates these findings, with the “With BU method (X_4)” strategy ranking highest, indicating its superior performance in handling complex constraints effectively. In summary, the BU methods, especially X_4 and X_4X_8 , enhance the optimisation process by creating more interconnected and efficient networks, outperforming the strategy without BU.



(a) Without BU method



(b) With BU method (X4)



(c) With BU method (X8)

(d) With BU method (X4X8)

Figure 7.20 Network visualisation of different strategies for the heater exchanger problem

Table 7.5 Results of different network metrics on the Heater Exchanger problem

Method	Without BU method	With BU method		
		X4	X ₈	X ₄ X ₈
Average Betweenness Centrality	0.220	0.148	0.202	0.148
Average Closeness Centrality	2.384e-19	2.394e-20	5.311e-18	4.426e-20
Average Eigenvector Centrality	0.3016	0.243	0.235	0.280
Average PageRank Centrality	0.125	0.124	0.1249	0.125
Network Density	0.821	0.607	0.7857	0.571
Average Clustering Coefficient	0.0325	0.0292	0.0299	0.0811
Average Weighted Degree	1.339e+22	8.490e+21	8.22e+21	3.318e+21

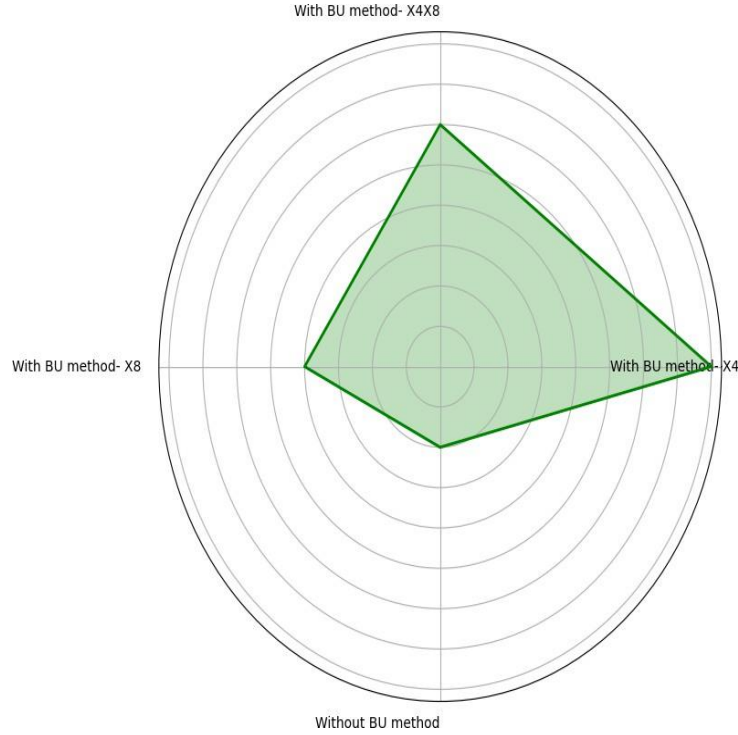


Figure 7.21 Rank for TOPSIS approach for the Heather Exchanger problem

7.6 G01

The G1 optimisation problem involves 13 decision variables and it includes a quadratic objective function and 9 linear constraints, making it an exemplary case of a problem with a high level of linear constraints and a feasibility ratio of only 0.0111% (Gandomi & Deb, 2020). This makes it particularly difficult to find a feasible solution, highlighting its suitability for testing constrained optimisation methods. The BU approach is applied to the G1 problem to reduce the search space. Eight different strategies are selected, starting with one approach without the BU method and the other seven approaches with the BU method. For the BU method, the strategies involve using the following variables as repairing variables respectively: X_{10} , X_{11} , X_{12} , $X_{10}X_{11}$, $X_{10}X_{12}$, $X_{11}X_{12}$, and $X_{10}X_{11}X_{12}$. Table 7.6 shows different network centrality and connectivity metrics on G01. Generally, applying the BU method impacts the metrics, sometimes reducing them to zero (e.g.,

Average Betweenness Centrality, Network Density, and Average Clustering Coefficient in some cases). The most noticeable changes occur in the average closeness centrality and average weighted degree, which vary widely depending on the specific BU method. The average PageRank centrality remains constant across all methods, indicating that the importance of nodes, as calculated by PageRank, is unaffected by the BU method in this instance.

Table 7.6 Results of different network metrics on the G01 problem

Method	Without BU method	With BU method						
		X_{10}	X_{11}	X_{12}	$X_{10}X_{11}$	$X_{10}X_{12}$	$X_{11}X_{12}$	$X_{10}X_{11}X_{12}$
Average Betweenness Centrality	0.045	0.055	0.052	0.053	0.052	0.052	0.052	0.0
Average Closeness Centrality	4.267e-12	370.195	6.260	74.975	1.656	0.889	0.844	0.0
Average Eigenvector Centrality	0.156	0.121	0.109	0.117	0.205	0.205	0.206	0.277
Average PageRank Centrality	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076
Network Density	0.384	0.269	0.128	0.269	0.128	0.128	0.128	0
Average Clustering Coefficient	0.116	0.012	0.0	0.030	0.0	0.0	0.0	0
Average Weighted Degree	1.717e+12	0.160	0.156	0.177	0.392	0.729	0.764	0

Figure 7.12 illustrates the ranking of various methods based on multi-criteria decision-making analysis, highlighting the performance impact of incorporating the BU technique with different combinations of variables. The method “With BU method - $X_{10}X_{11}X_{12}$ ” ranks the highest, suggesting that integrating all three variables (X_{10} , X_{11} , X_{12}) with BU yields the best results.

High-performing methods also include “With BU method - $X_{11}X_{12}$ ” and “With BU method - $X_{10}X_{11}$.” The middle range includes “With BU method - $X_{10}X_{12}$ ” and “Without BU method,” indicating that the baseline method without BU outperforms some BU methods. Lower ranks are seen for “With BU method - X_{10} ,” “With BU method - X_{12} ,” and “With BU method - X_{11} ,” showing less effectiveness when each variable is used individually in the BU method. This analysis underscores the superior performance of comprehensive variable integration in the BU approach. Also, as it is clear from Figure 7.13, applying the BU method leads to fewer interactions between variables and results in more independent variables.

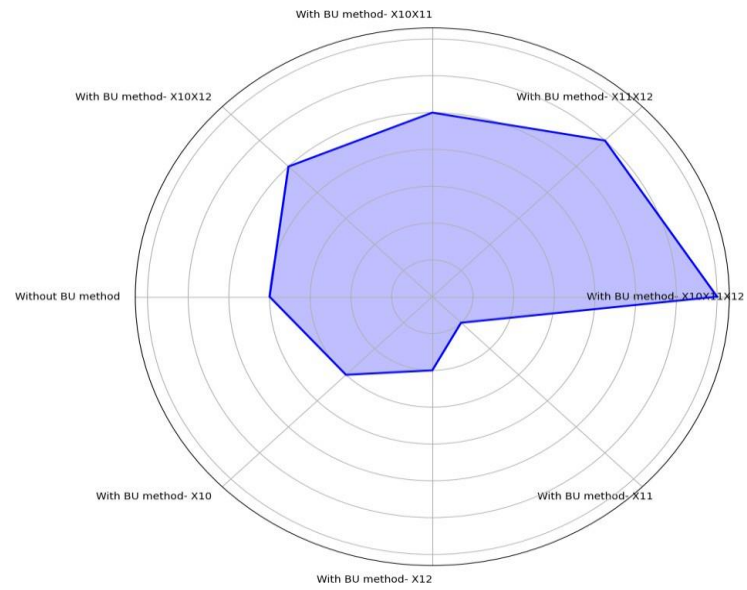
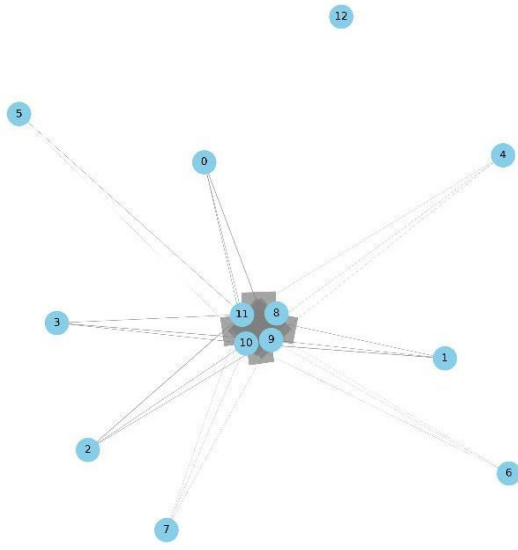
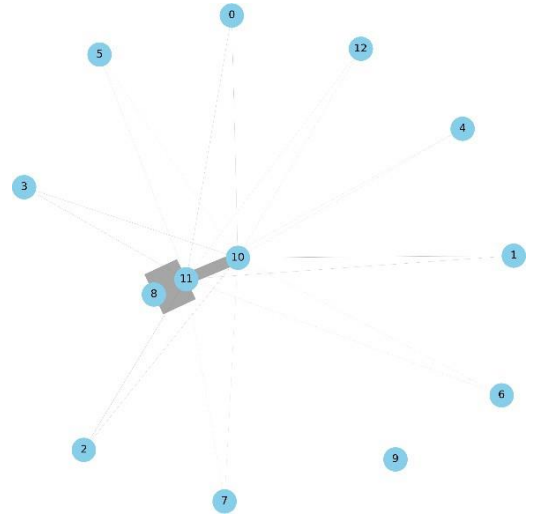


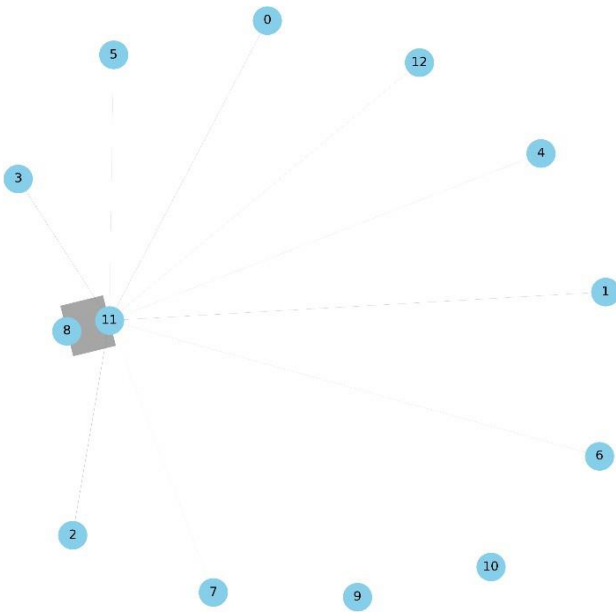
Figure 7.12 Rank for TOPSIS approach for G01 problem



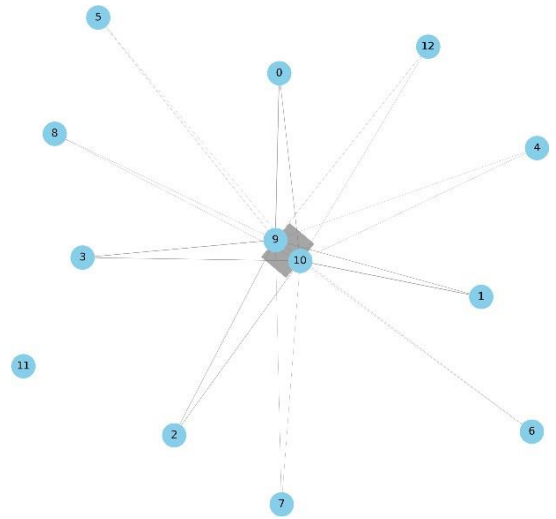
(a) G01-Without BU method



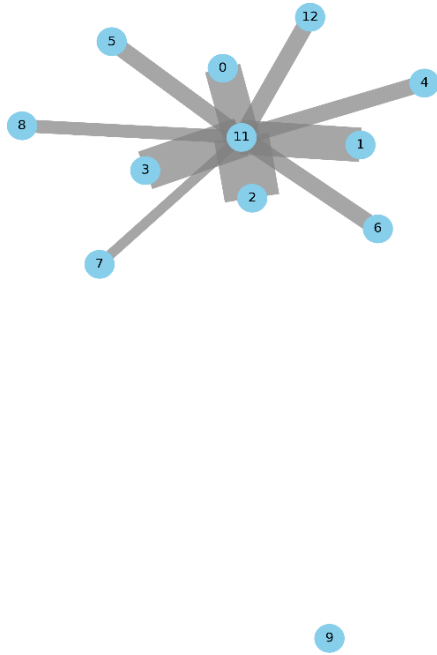
(b) G01-With BU method (X10)



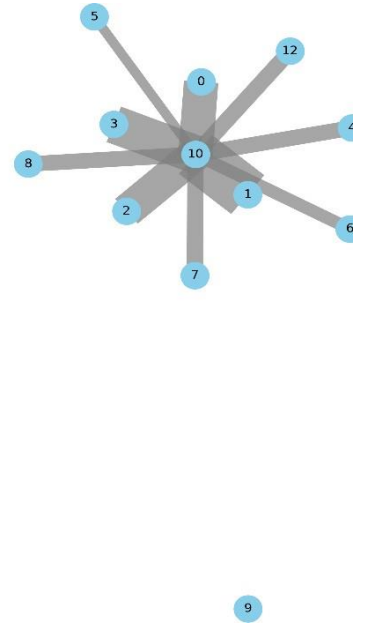
(c) G01- With BU method (X11)



(d) G01- With BU method (X12)



(e) G01-With BU method (X10X11)



(f) G01- With BU method (X10X12)

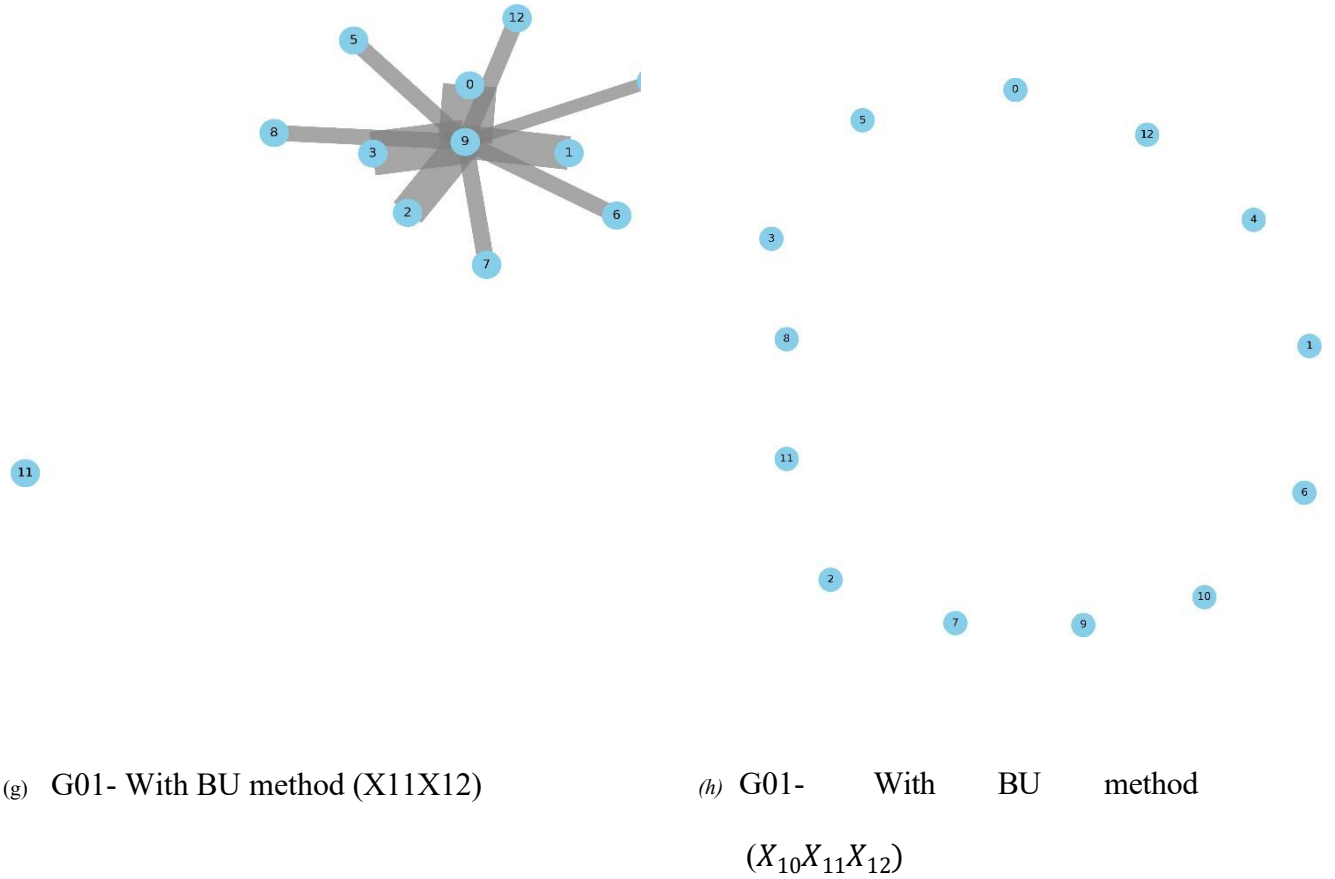


Figure 7.122 Network visualisation of different strategies on the G01 problem

8 Discussion

This study evaluates the impact of the BU method on constrained optimization problems using SNA metrics and a systematic performance evaluation. Instead of providing explicit mathematical formulations of SNA, we leverage key SNA indicators such as Betweenness Centrality, Closeness Centrality, Network Density, and Clustering Coefficient to quantify the impact of BU on variable interactions. Additionally, rather than performing traditional sensitivity analysis, the robustness of the BU method is validated across multiple benchmark problems, including the Welded Beam, Car Side Impact, and Pressure Vessel Optimization problems. The results indicate that BU significantly enhances solution feasibility and optimization performance, as further confirmed by the TOPSIS

multi-criteria decision-making approach. This approach ensures a practical, application-driven validation of the method's effectiveness without requiring additional sensitivity analysis.

The findings of this research show significant insights into the BU method's effectiveness when applied to constrained optimisation problems. The detailed analysis of variable interactions using the DG2 algorithm and various network metrics has shed light on the structural properties of optimisation problems under different formulations. Several key points emerge from the discussion as follows:

Impact of BU method: Applying the BU method for the studied numerical examples consistently leads to a more simplified optimisation landscape, which results in a reduction in infeasible search space. This reduction and simplification in search space complexity was evident across all benchmark problems, resulting in more independent variables and fewer interactions among them.

Variable interaction analysis: The DG2 algorithm was used to identify the interactions between variables in both BU and non-BU scenarios. The metrics, such as average betweenness centrality, closeness centrality, eigenvector centrality, and network density, provided comprehensive insights into the variable interactions. The BU method generally resulted in a more efficient network by analysing the lower betweenness and closeness centrality values, which indicates fewer critical intermediary nodes and shorter paths between variables.

MCDM Approach: The superiority of the BU method is further validated using the TOPSIS approach. The TOPSIS analysis ranked various BU strategies according to their proximity to an ideal solution, and the strategies involve multiple repairing variables that consistently rank higher. This method highlighted the robustness and reliability of the BU method, which handles complex constraints effectively.

Trade-offs and Balance: A critical aspect of this study is finding the balance between shrinking the search space and managing the complexity of variable interactions. The BU method's ability to twist

the search space while maintaining an efficient optimisation process was observed in the findings. This balance is essential for ensuring that the performance gains from the BU method are not offset by increased problem complexity.

9 Conclusion

This study endeavours to implement the BU method across various benchmark problems. The BU method enhances the optimisation search by eliminating the infeasible search space, by applying this approach, the search area is twisted, and empirical results on this problem suggest that the BU method leads to a simpler variable interaction structure, which has positive impacts on the optimisation algorithm's performance. This is because of the shrinkage of the search area due to the BU approach and the reduction of the complexity of the optimisation problem. Moreover, the reduction of the search space due to the BU method, the complexity of the optimisation problem, and the search behaviour of different optimisers may possess different responses to different BU formulations. In this research, various formulations of the BU method are conducted, and the DG2 algorithm analyses the interaction of decision variables.

The analysis for the studied numerical examples, supported by various SNA metrics, illustrates that strategies with the highest number of repairing variables outperform other strategies. This superior performance presences more independent decision variables and fewer interdependencies among them, facilitating the discovery of optimal solutions.

Furthermore, the application of the MCDM approach, TOPSIS, enables a robust evaluation and ranking of different boundary update formulations. The results show that the boundary update method simplifies variable interactions and improves the optimisation algorithm's overall effectiveness, resulting in the best optimal values as initially obtained (Gandomi & Deb, 2020).

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Chapter 8

Conclusions

And Recommendation

Abstract- This chapter summarizes the key contributions of the research, including the development and validation of innovative constraint-handling techniques. It reflects on the practical implications of these advancements for industries such as mining and structural engineering. Limitations of the current work are discussed, along with opportunities for future research in areas like many-objective optimization and integration of machine learning with optimization algorithms. The chapter concludes with recommendations for applying the above-mentioned methods to emerging challenges in optimization.

Introduction- Many industry problems include optimization concept. Many practical optimization problems are bounded by constraints, which may result in complexity. This thesis proposes an innovative method for handling constrained optimization problems using population-based algorithms. The approach is applied to benchmarks and real-world optimization problems with varying objectives and scales. Examples include solving single- and multi-objective renowned engineering problems such as welded beam design problems, spring design problem, heater exchanger design problem, and car side optimization problems, which are used and solved with the suggested algorithms. Furthermore, the open-pit mining optimization problem is addressed in this study as a mixed integer optimization problem. The results and analysis show the proposed method outperforms the classic approaches. The summary of the findings are as follows:

- **Innovative constraint handling technique:**
 - The proposed methods enhance constraint handling in large-scale single- and multi-objective optimization tasks using population-based algorithms. These approaches consist several positive points, which include dynamically adjusting the boundaries of variables based on the constraints, effectively minimizing the generation of infeasible solutions and streamlining the search process toward viable regions of the solution space among them.
- **Application to Benchmarks and Real-World Problems:**

- The techniques were applied to various benchmarks and real-world optimization challenges such as the welded beam design, car side optimization, and large-scale open-pit mining optimization. In each case, the proposed methods demonstrated superior performance in finding feasible solutions faster and improving the objective space compared to traditional approaches.
- **Improved Objective Space:**
 - The results from applying the proposed constraint-handling techniques (CHTs) showed significant improvements in the objective space. The methods' ability to handle multiple objectives and constraints efficiently have been validated through comparative analysis with existing state-of-the-art algorithms. The open-pit mining optimization problem, in particular, highlighted the method's capability to manage large-scale mixed integer optimization problems effectively.
- **Implications of the Study**

The findings from this research have several important implications for the field of optimization and beyond:

- **Advancement in Optimization Techniques:**

The development of new CHTs for single- and multi-objective optimization represents a significant advancement in the field. By integrating these methods with population-based algorithms, the study provides a robust framework that can be adapted to various complex optimization problems across different domains.
- **Practical Applications:**

The practical applications of the proposed approaches extend to several engineering fields, including structural design, automotive engineering, and mining. The success of the methods

in these fields suggests us its potential for using in other areas that need efficient and effective optimization solutions.

- **Enhanced Computational Efficiency:**

The proposed techniques are able to find feasible solutions faster and improve the objective space to enhance computational efficiency. This is particularly important for large-scale problems where traditional optimization approaches may struggle with computational complexity and time constraints.

- **Future Research Directions**

While this study has made substantial contributions to the field of constrained optimization, several areas warrant further investigation:

- **Extension to Many-Objective Optimization Problems:**

Future research could extend the proposed method to handle many-objective optimization problems, involving more than three objectives that are called many-objective optimization problems. This would require further refinement of the CHTs to manage the increased complexity and ensure efficient convergence to Pareto-optimal solutions.

- **Integration with other Optimization Algorithms:**

The integration of the proposed CHTs with other algorithms, such as machine learning-based approaches or hybrid algorithms combining evolutionary strategies with exact methods, could be explored. This would potentially enhance the method's versatility and effectiveness across different engineering problem domains.

- **Additional Validation**

While this thesis focuses on multi-objective optimization, the BU method could be adapted to single-objective constrained optimization problems, such as those in the IEEE CEC2006 benchmark suite. Investigating its effectiveness in this context may provide further insights into its adaptability and efficiency.

- **Real- World Case Studies:**

Conducting additional real-world case studies in other real-case studies such as healthcare, logistics, and energy could provide further validation of the method's practical applicability. These studies could also uncover new challenges and opportunities for refining the technique.

- **Scalability and Adaptability:**

Investigating the scalability and adaptability of the proposed method for extremely large-scale problems, such as those encountered in big data analytics, would be a valuable direction for future research. This could involve developing parallel computing strategies or leveraging cloud-based platforms to manage computational demands.

This thesis has introduced novel constraint-handling methodologies aimed at enhancing multi-objective optimization in large-scale computational environments. While the research primarily focused on improving the efficiency and feasibility of evolutionary algorithms, the integration of these methods into real-world, data-intensive applications positions this work within the broader context of Big Data Analytics. The optimization of large-scale mining problems and engineering designs inherently deals with vast and complex datasets, requiring scalable and computationally efficient approaches. The BU method and its integration with Social Network Analysis (SNA) provide a structured approach to handling high-dimensional optimization landscapes, demonstrating methodologies that align with Big Data principles. Future research will further explore the application of these techniques to real-world datasets, extending the impact of this work to

practical Big Data-driven decision-making problems in areas such as energy management, logistics, and predictive analytics.

Conclusion- In conclusion, this thesis has introduced and validated novel methods to handling constraints in single and multi-objective constrained optimization tasks using population-based algorithms. The proposed methods have shown significant improvements in finding feasible solutions faster and enhancing the objective space compared to existing methods. The study's findings have important implications for advancing optimization methodologies and offer a promising foundation for future research and practical applications in various fields. Through continued exploration and refinement, the proposed techniques have the potential to make a substantial impact on the efficiency and effectiveness of optimization processes across a wide range of complex, real-world problems.

Appendix-

1. G01 Problem:

$$f_1(x) = 5 \times (x_1 + x_2 + x_3 + x_4) - 5 \times (x_1^2 + x_2^2 + x_3^2 + x_4^2) - (x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}) \quad (1)$$

$$g_1(x) = 2 \times x_1 + 2 \times x_2 + x_{10} + x_{11} \leq 10 \quad (2)$$

$$g_2(x) = 2 \times x_1 + 2 \times x_3 + x_{10} + x_{12} \leq 10 \quad (3)$$

$$g_3(x) = 2 \times x_2 + 2 \times x_3 + x_{11} + x_{12} \leq 10 \quad (4)$$

$$g_4(x) = -8 \times x_1 + x_{10} \leq 0 \quad (5)$$

$$g_5(x) = -8 \times x_2 + x_{11} \leq 0 \quad (6)$$

$$g_6(x) = -8 \times x_3 + x_{12} \leq 0 \quad (7)$$

$$g_7(x) = -2 \times x_4 - x_5 + x_{10} \leq 0 \quad (8)$$

$$g_8(x) = -2 \times x_6 - x_7 + x_{11} \leq 0 \quad (9)$$

$$g_9(x) = -2 \times x_8 - x_9 + x_{12} \leq 0 \quad (10)$$

$$0 \leq x_1 \dots x_9 \leq 1, \quad 0 \leq x_{10}, x_{11}, x_{12} \leq 100, \quad 0 \leq x_{13} \leq 1 \quad (11)$$

2. Welded Beam Design Problem

Objective Function (Minimize Cost):

$$f(x) = 1.1047 \times x_1^2 \times x_2 + 0.04811 \times x_3 \times x_4 \times (14 + x_2)$$

Where x_1, x_2, x_3, x_4 are design variables.

Constraints:

$$g_1(x) = \frac{6000}{\sqrt{2} \times x_1 \times x_2} - 13600 \leq 0$$

$$g_2(x) = \frac{6000}{x_3 \times x_4} - 30000 \leq 0$$

$$g_3(x) = x_2 - x_3 \leq 0$$

$$g_4(x) = 0.10471 \times x_1^2 + 0.04811 \times x_3 \times x_4 \times (14 + x_2) - 5 \leq 0$$

Variable Bounds:

$$0.1 \leq x_1 \leq 2.0, 0.1 \leq x_2 \leq 10.0, 0.1 \leq x_3 \leq 10.0, 0.1 \leq x_4 \leq 2.0$$

3. Car Side Impact Problem

$$f_1(x) = 1.98 + 4.90 \times x_1 + 6.67 \times x_2 + 6.98 \times x_3 + 4.01 \times x_4 + 1.78 \times x_5 + 2.73 \times x_7 \quad (43)$$

$$g_1(x) = 1.16 - 0.3717 \times x_2 \times x_4 - 0.00931 \times x_2 \times x_{10} - 0.484 \times x_3 \times x_9 + 0.01343 \times x_6 \times x_{10} \quad (44)$$

$$g_2(x) = 46.36 - 9.9 \times x_2 - 12.9 \times x_1 \times x_8 + 0.1107 \times x_3 \times x_{10} \quad (45)$$

$$g_3(x) = 33.86 + 2.95 \times x_3 + 0.1792 \times x_{10} - 5.057 \times x_1 \times x_2 - 11.0 \times x_2 \times x_8 - 0.021 \times x_5 \times x_{10} \\ - 9.98 \times x_7 \times x_8 + 22.0 \times x_8 \times x_9 \quad (46)$$

$$g_4(x) = 28.98 + 3.818 \times x_3 - 4.2 \times x_1 \times x_2 - 5.0207 \times x_5 \times x_{10} + 6.63 \times x_6 \times x_9 - 7.7 \times x_7 \times x_8 \\ + 0.32 \times x_9 \times x_{10} \quad (47)$$

$$\begin{aligned}
g_5(x) = & 0.261 - 0.0159 \times x_1 \times x_2 - 0.188 \times x_1 \times x_8 - 0.019 \times x_2 \times x_7 + 0.0144 \times x_3 \times x_5 \\
& + 0.0008757 \times x_5 \times x_{10} + 0.08045 \times x_6 \times x_9 + 0.00139 \times x_8 \times x_{11} + 0.00001575 \\
& \times x_{10} \times x_{11}
\end{aligned} \tag{48}$$

$$\begin{aligned}
g_6(x) = & 0.214 + 0.00817 \times x_5 - 0.0131 \times x_1 \times x_8 - 0.0704 \times x_1 \times x_9 + 0.03099 \times x_2 \times x_6 - 0.018 \\
& \times x_2 \times x_7 + 0.0208 \times x_3 \times x_8 + 0.121 \times x_3 \times x_9 - 0.00364 \times x_5 \times x_6 + 0.0007715 \times x_5 \\
& \times x_{10} - 0.0005354 \times x_6 \times x_{10} + 0.00121 \times x_8 \times x_{11} + 0.00184 \times x_9 \times x_{10} - 0.02 \times x_2^2
\end{aligned} \tag{49}$$

$$g_7(x) = 0.74 - 0.61 \times x_2 - 0.163 \times x_3 \times x_8 + 0.001232 \times x_3 \times x_{10} - 0.166 \times x_7 \times x_9 + 0.227 \times x_2^2 \tag{50}$$

$$\begin{aligned}
g_8(x) = & 4.72 - 0.5 \times x_4 - 0.19 \times x_2 \times x_3 - 0.0122 \times x_4 \times x_{10} + 0.009325 \times x_6 \times x_{10} + 0.000191 \\
& \times x_{11}^2
\end{aligned} \tag{51}$$

$$\begin{aligned}
g_9(x) = & 10.58 - 0.674 \times x_1 \times x_2 - 1.95 \times x_2 \times x_8 + 0.02054 \times x_3 \times x_{10} - 0.0198 \times x_4 \times x_{10} + 0.028 \\
& \times x_6 \times x_{10}
\end{aligned} \tag{52}$$

$$\begin{aligned}
g_{10} = & 16.45 - 0.489 \times x_3 \times x_7 - 0.843 \times x_5 \times x_6 + 0.0432 \times x_9 \times x_{10} - 0.0556 \times x_9 \times x_{11} - 0.000786 \\
& \times x_{11}^2
\end{aligned} \tag{53}$$

$$\begin{aligned}
0.5 \leq x_1, x_3, x_4 \leq 1.5, \quad 0.45 \leq x_2 \leq 1.35, \quad 0.875 \leq x_5 \leq 2.625, \quad 0.4 \leq x_6, x_7 \leq 1.2, \\
0.19 \leq x_8, x_9 \leq 0.35, \quad -30 \leq x_{10}, x_{11} \leq 30
\end{aligned} \tag{54}$$

4. Speed Reducer Problem:

$$\begin{aligned}
f(x) = & 0.785 \times x_1 \times x_2^2 \times (3.33 \times x_3^2 + 14.9334 \times x_3 - 43.0934) - 1.508 \times x_1 \times (x_6^2 + x_7^2) + 7.4777 \\
& \times (x_6^3 + x_7^3) + 0.7854 \times (x_4 \times x_6^2 + x_5 \times x_7^2)
\end{aligned} \tag{55}$$

$$\text{S.t.} \tag{56}$$

$$27/(x_1 \times x_2^2 \times x_3) \leq 1 \tag{57}$$

$$397.5/x_1 \times x_2^2 \times x_3^2 \leq 1 \quad (58)$$

$$1.93 \times x_4^3/x_2 \times x_3 \times x_6^4 \leq 1 \quad (59)$$

$$1.93 \times x_5^3/(x_2 \times x_3 \times x_7^4) \leq 1 \quad (60)$$

$$\sqrt{(745 \times \frac{x_4}{x_2 \times x_3})^2 + 1.69 \times 10^7/(110 \times x_6^3)} \leq 1 \quad (61)$$

$$\sqrt{(745 \times \frac{x_5}{x_2 \times x_3})^2 + 1.575 \times 10^8/(85 \times x_7^3)} \leq 1 \quad (62)$$

$$(x_2 \times x_3)/40 \leq 1 \quad (63)$$

$$(5 \times x_2)/x_1 \leq 1 \quad (64)$$

$$x_1/(12 \times x_2) \leq 1 \quad (65)$$

$$(1.5 \times x_6 + 1.9)/x_4 \leq 1 \quad (66)$$

$$(1.1 \times x_7 + 1.9)/x_5 \leq 1 \quad (67)$$

$$2.6 \leq x_1 \leq 3.6, 0.7 \leq x_2 \leq 0.8, 17 \leq x_3 \leq 28, \quad (68)$$

$$7.3 \leq x_4 \leq 8.3, 7.8 \leq x_5 \leq 8.3, 2.9 \leq x_6 \leq 3.9,$$

$$5.0 \leq x_7 \leq 5.5$$

5. Spring Design Problem

Objective Function (Minimize Weight):

$$f(x) = (x_3 + 2) \times x_2 \times x_1^2$$

Constraints:

$$g_1(x) = 1 - \frac{x_2^3 \times x_3}{71785 \times x_1} \leq 0$$

$$g_2(x) = \frac{4 \times x_2^2 - x_1 \times x_2}{12566 \times (x_2 \times x_1^3 - x_1^4)} + \frac{1}{5108 \times x_1^2} - 1 \leq 0$$

Variable Bounds:

$$0.05 \leq x_1 \leq 2.0, 0.2 \leq x_2 \leq 1.0, 2.0 \leq x_3 \leq 15.0$$

6. Pressure Vessel Problem

Objective Function (Minimize Cost):

$$f(x) = 0.6224 \times x_1 \times x_3 \times x_4 + 1.7781 \times x_2 \times x_3^2 + 3.1661 \times x_1^2 \times x_4 + 19.84 \times x_1^2 \times x_3$$

Constraints:

$$g_1(x) = -x_1 + 0.0193 \times x_3 \leq 0$$

$$g_2(x) = -x_2 + 0.00954 \times x_3 \leq 0$$

Variable Bounds:

$$0.0625 \leq x_1, x_2 \leq 99$$

$$10 \leq x_3, x_4 \leq 200$$

7. Heat Exchanger Problem

Objective Function (Maximize Heat Exchange Efficiency):

$$f(x) = x_1 \times x_2 - x_3 \times x_4$$

Constraints:

$$g_1(x) = x_1 + x_2 - x_3 \leq 0$$

Variable Bounds:

$$1.0 \leq x_1, x_2, x_3, x_4 \leq 10.0$$