

A decision support system for open-pit mining optimisation with dynamic uncertainty and GPU-based parallel repair approach

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ABSTRACT

This paper presents an advanced Decision Support System (DSS) for long-term open-pit mine planning that integrates established optimization techniques—Large Neighborhood Search (LNS), Simulated Annealing (SA), and Dantzig-Wolfe decomposition—within a novel GPU-accelerated framework addressing geological uncertainty and computational complexity. The key methodological contributions include dynamic uncertainty modelling with time-dependent factors capturing geological confidence degradation and GPU-parallelized evaluation architecture enabling industrial-scale mine planning. Validation using 50,000 blocks across 10 geological scenarios demonstrates robust economic performance, achieving mean NPV of \$1.514 billion with limited variability (standard deviation \$16 million). The GPU-parallelized architecture achieves 29.6 % average computational speedup with peaks of 37 % compared to CPU implementations, enabling concurrent evaluation of 262,144 mining scenarios. Risk analysis reveals P90 Value-at-Risk of \$1.488 billion, indicating strong downside protection. The system maintains profit margins exceeding 95 % across all scenarios with cumulative cash flow reaching \$1.789.4 million by period 6. Narrow risk envelopes (P10-P90 spread <\$60 M) demonstrate robust performance under uncertainty, providing mining companies with practical tools for risk-informed strategic decision-making.

1. Introduction

Long-term open-pit mine planning involves determining the optimal sequence of ore extraction and processing over the mine's lifespan while addressing complex challenges such as geological uncertainty, operational constraints, and fluctuating economic conditions (Armstrong et al., 2021; Quelopana & Navarra, 2024; Tolouei et al., 2021). Metallurgical plants are crucial components of the mineral value chain, significantly influencing economic outcomes. For instance, extracting all valuable minerals like gold and silver from ore is not always practical. Such exhaustive extraction would require large quantities of chemical reagents (specifically cyanide), leading to higher operating costs and potentially lowering overall metal recovery rates, as some valuable minerals inevitably remain in the tailings (Ordenes J et al., 2021). Additionally, the comminution process—which involves all activities related to reducing the size or breaking down ores and rocky materials (Gupta CK, 2003)—is highly energy-intensive. This has a substantial impact on operational expenses, generally representing 30 %–60 % of the mine site's total energy usage, though in some cases this figure can reach as high as 80 % (Wang C et al., 2013; Lois-Morales P et al., 2022).

Plant reconfiguration must be approached as a tactical decision rather than one triggered by minor changes in ore feed. Instead, it should reflect the anticipated processing requirements of future ore streams (Navarra et al., 2017). For this purpose, the concept of an operational mode has been introduced, representing a set of parameters defining the metallurgical plant's configuration, including but not limited to metal recovery rates and processing capacities (Quelopana A et al., 2023). These operational modes are strategically developed to optimize the production of metals or alloys and to ensure efficient and safe processing for specific ore feeds, such as particular blends of rock types. Given that metal grades vary significantly across different rock types, having multiple operational modes available, when practical, enhances operational flexibility. This improved flexibility enables the plant to more effectively respond to varying ore characteristics and manage grade uncertainties (Quelopana A et al., 2023). To effectively optimize mine planning and metallurgical plant operations within a unified framework, a comprehensive analysis is essential. This involves careful model design, the development of efficient algorithms, and subsequent implementations that fully leverage existing computational resources.

The explosion of geological, operational, and market data demands

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innovative methodologies capable of efficiently handling this information for more accurate and adaptive mine planning strategies. The transition toward data-centric and computationally intensive approaches in mining has reshaped the landscape of long-term planning strategies, particularly in open-pit operations. Traditional mine planning frameworks often rely on deterministic models and fixed input data, which fail to accommodate the complexities introduced by real-world geological variability and operational uncertainty. As the mining industry moves into an era defined by digital transformation and sustainability imperatives, there is a pressing need to incorporate sophisticated optimisation methods that are not only computationally efficient but also adaptive to dynamic conditions. These methods must seamlessly integrate large volumes of heterogeneous data—from 3D geological models and market forecasts to equipment performance metrics and environmental impact indicators—to support actionable and resilient decision-making.

Quelopana & Navarra (2024) presented a two-stage optimisation method that combines Variable Neighborhood Descent (VND) metaheuristic with linear programming to integrate open-pit mine planning and metallurgical plant operations under geological uncertainty, allowing plants to switch between operational modes based on varying ore types.

To address this, Graphics Processing Unit (GPU)-accelerated computing offers a transformative solution. GPU-accelerated optimization has gained momentum across various domains, from mining optimization to medical applications such as federated learning frameworks for disease detection (Mukhopadhyay et al., 2025), though application to constrained mine scheduling remains underexplored. With the capability to execute thousands of threads in parallel, GPUs significantly reduce evaluation time for large datasets, making real-time or near-real-time planning feasible. When integrated with search heuristics such as Large Neighborhood Search (LNS) and Simulated Annealing (SA), GPU-based evaluation can support rapid convergence to high-quality solutions, even in the presence of noisy or incomplete data. Ferreira et al. (2013) built a highly optimized CUDA implementation of simulated annealing on Nvidia GPUs, comparing a sequential baseline with asynchronous and synchronous multi-Markov-chain variants; the synchronous design gives the best balance of convergence and accuracy, while the asynchronous one is a bit faster but less accurate. Li et al., (2017) applied a GPU-parallel genetic algorithm using an island model in which the mutation step is replaced by simulated annealing, and further introduced a cyclic “spheroidizing” annealing temperature schedule to help escape local optima; they evaluated the approach on the Traveling Salesman Problem, on an NVIDIA Titan Black GPU, the method achieved about $300\times$ speedup over a single-thread CPU and about $100\times$ over a multi-thread CPU, while also converging faster than a CPU master-slave GA. Moreover, the explicit incorporation of dynamic uncertainty in both mining and processing stages represents a major methodological advancement. In this framework, uncertainty is not treated as static or exogenous; rather, it evolves over time and across scenarios, influenced by the planning horizon, the spatial location of mining blocks, and the accuracy of geological models. This dynamic uncertainty modelling allows the optimisation process to prioritize more reliable information in early planning periods while adopting a risk-adjusted approach in later periods where uncertainty is higher. Such adaptive strategies are crucial for maintaining economic viability and operational safety in long-term projects.

Existing mine planning frameworks lack integrated approaches that simultaneously handle (1) dynamic temporal uncertainty modelling, (2) GPU-accelerated large-scale optimization, and (3) real-time operational mode selection under uncertainty. Current literature addresses these challenges in isolation but not as an integrated system capable of industrial-scale application. This study contributes to the growing body of knowledge in computational mine planning by proposing an integrated decision support system (DSS) that synergizes dynamic uncertainty modelling with GPU-parallelized optimisation through three

novel methodological advances with clear physical interpretations. First, dynamic uncertainty modelling introduces time-dependent factors that capture geological confidence degradation over planning horizons, reflecting mining reality where geological estimates become less reliable with distance from drill holes and time from estimation. Second, GPU-accelerated parallel repair employs hierarchical CUDA-based architecture enabling real-time assessment of 262,144 mining scenarios, mimicking simultaneous evaluation by multiple planning teams and dramatically accelerating traditionally sequential decision-making processes. Third, integrated operational mode selection synthesizes strategic mine planning with real-time operational mode switching, reflecting actual metallurgical plant operations where processing configurations must adapt to varying ore characteristics and recovery optimization. The framework comprehensively addresses both strategic and operational planning objectives by first generating an initial mining schedule satisfying precedence and capacity constraints, then systematically refining this schedule through scenario-driven evaluations and advanced repair mechanisms. This sophisticated approach ensures that despite perturbations introduced by the hybrid LNS-SA metaheuristic combination, where SA governs solution acceptance to escape local optima, the resulting mining plan maintains both operational executability and economic viability throughout the optimization process.

Beyond the technical innovation, this work aligns with broader industry trends toward digital mine planning, sustainability, and risk-informed decision-making. By enabling fast, adaptive, and scalable optimisation, the proposed system supports mining companies in achieving key performance indicators such as maximizing Net present value (NPV), improving plant utilization, reducing environmental impact, and managing operational risk. The inclusion of multiple figures and metrics—ranging from NPV risk profiles to convergence behaviour and sustainability scorecards—provides a comprehensive view of planning performance across different scenarios and time horizons.

This study presents a novel and practically viable optimisation system for long-term open-pit mine scheduling under uncertainty. It builds upon the latest advancements in mathematical programming, high-performance computing, and data-driven modelling to deliver a flexible, scalable, and intelligent decision support system. The approach is validated through rigorous computational experiments using a large-scale mining dataset and is implemented on modern GPU hardware to demonstrate real-world applicability. The insights gained from this research have implications not only for the mining industry but also for other domains where large-scale, uncertainty-aware scheduling is essential, such as energy systems, transportation, and infrastructure management.

The rest of the paper is organized as follows. Section 2 shows the related works. Section 3 presents problem formulation. Section 4 shows the proposed DSS for the case study. Section 5 depicts the research methodology. Sections 6 and 7 illustrates results and discussion. The final section provides the conclusion and future works.

2. Related works

Contemporary research in mine planning increasingly emphasizes the role of uncertainty in shaping extraction and processing strategies. Geological uncertainty, in particular, presents a formidable challenge, as inaccurate grade estimates or mischaracterization of rock properties can lead to suboptimal extraction paths and economic underperformance (Rim  l   et al., 2020). Traditional approaches are predominantly deterministic in nature, as they overlook uncertainty in modelling (Hustrulid et al., 2013; Osanloo et al., 2008; Boland et al., 2009; Ramazan, 2007).

In response, researchers have advanced scenario-based and stochastic models to accommodate multiple realizations of uncertain parameters (Upadhyay and Askari-Nasab, 2018; MacNeil & Dimitrakopoulos, 2017). Cutler & Dimitrakopoulos (2024) Introduced a two-stage stochastic integer program that jointly optimizes long-term open-pit schedules with ramp/road design, enforcing block

accessibility via checkpoints, ramps and intra-bench roads; on a gold case it yields a coherent ramp layout while keeping head grades within target bands. Muke et al., (2025) proposed a mixed-integer programming model solved with a genetic algorithm (GA) to dynamically integrate long- and medium-term open-pit schedules, aligning rates/grades and enforcing temporal & spatial consistency through penalty terms. Chen et al. (2025) address open-pit production planning under dual geological-economic uncertainty by generating Monte Carlo grade/cost scenarios and optimizing with NSGA-II plus weighted grey relational analysis; a market-price-fluctuation constraint drives a second iteration to adapt extraction plans to price bands and then rank Pareto solutions. Jiang & Dimitrakopoulos (2024) extended the framework by embedding stochastic equipment-capacity simulations and penalties into the two-stage model, producing capacity-respecting life-of-asset schedules and delivering ~2 % higher NPV in a copper complex case study.

However, such models often suffer from scalability issues and computational bottlenecks, especially when applied to large-scale deposits with thousands of blocks and dozens of planning periods. The growing complexity of these models has necessitated the adoption of high-performance computing and algorithmic innovation, including the use of metaheuristics, hybrid optimisation methods, and parallelization techniques.

Decomposition strategies also widely used by researchers in optimisation of open-pit mining, which there are some issues (Blom et al., 2016; Fathollahzadeh et al., 2021). One critical development lies in the use of decomposition strategies, particularly Dantzig–Wolfe decomposition, which enables the division of a large-scale mine planning problem into smaller, more tractable subproblems. This layered optimisation structure allows for more targeted decision-making at both strategic and operational levels. The master problem governs the global extraction schedule, while subproblems evaluate localized mining configurations under various operational modes and uncertainty realizations. This decomposition not only enhances computational efficiency but also provides greater flexibility in modelling interdependencies between extraction blocks, equipment constraints, and processing plant capacities.

In addition to advanced optimisation strategies, modern mine planning frameworks must address the computational intensity associated with evaluating a vast number of potential mining scenarios. This challenge is particularly acute in stochastic and robust optimisation settings, where multiple geological scenarios must be analysed simultaneously. Open-pit mine production scheduling (OPMPS) has been extensively researched over the past several decades due to its critical role in optimizing the profitability of mining operations. Traditional approaches initially applied linear programming techniques, such as those by Johnson (1969), who introduced a method based on Dantzig–Wolfe decomposition and network flow algorithms.

Other researchers like Dagdelen (1986) utilized integer programming and Lagrangian relaxation, while Caccetta and Hill (2003) applied the branch-and-cut algorithm for medium-sized problems. Dynamic programming approaches were also tested, though their scalability remains limited (Elevli, 1995; Erarslan & Çelebi, 2001). Gershon (1983) and others developed Mixed-Integer Programming (MIP) models, but these approaches often become computationally prohibitive for large-scale problems. Various efforts have been made to reduce computational complexity, including block aggregation techniques (Ramazan et al., 2005) and advanced decomposition or cutting plane algorithms (Bley et al., 2010; Askari-Nasab et al., 2011; Lambert & Newman, 2014). Despite these developments, many of these methods remain limited to simplified or small-to-medium-sized problems due to the combinatorial nature of OPMPS and its large number of constraints and variables.

To address these limitations, researchers have increasingly explored heuristic and metaheuristic methods for OPMPS. Techniques such as Genetic Algorithms (Denby & Schofield, 1994; Alipour et al., 2019), Tabu Search and VND (Lamghari & Dimitrakopoulos, 2012), Ant Colony Optimisation (Soleymani Shishvan & Sattarvand, 2015), Particle Swarm

Optimisation (Khan & Niemann-Delius, 2014, 2018; Khan, 2018), and Differential Evolution have shown effectiveness in generating near-optimal solutions for large-scale problems. Navarro et al., (2024) Defined a parallel genetic algorithm (PGA) for a geometrically constrained production scheduling problem. In the above-mentioned work, individuals are centroid sets (genotype) clustered by MeanShift into truncated-cone phases (phenotype), with fitness computed by a fast best-case sequential scheduling heuristic to approximate NPV under precedence/capacity, however, GA's fitness relies on a best-case sequential scheduling proxy (not fully operational), results are sensitive to geometric parameters and can miss ideal solutions. Stochastic optimisation approaches, such as those presented by Dimitrakopoulos & Sénécal (2019) and Montiel & Dimitrakopoulos (2013), have been employed to address geological uncertainty and multiple processing routes. Moreover, hybrid approaches combining Lagrangian Relaxation with algorithms like Firefly and Bat Algorithm (Tolouei, Moosavi, Bangian Tabrizi, & Afzal, 2020; Tolouei, Moosavi, Tabrizi, et al., 2020; Tolouei, Moosavi, Bangian Tabrizi, Afzal, et al., 2020) have emerged to improve solution quality under uncertainty. Simulated Annealing (Kumral & Dowd, 2005; Kumral, 2013) has been applied for various scheduling problems, including ore-waste discrimination and stochastic long-term scheduling, often enhanced by memory-based strategies to improve computational efficiency (Sari & Kumral, 2016). Despite these advances, most existing approaches do not adequately incorporate stockpiling into OPMPS models, highlighting the need for methods that can realistically handle such complexities.

Blom, M., et al, (2024) presented a LNS based approach for solving complex long-term open-pit mine planning problems. It starts with an initial feasible solution generated by a sliding windows heuristic and iteratively improves it by solving smaller, restricted mixed-integer subproblems, where only selected decision variables are allowed to change. The LNS exploits problem structure to form effective neighbourhoods, enabling the solution of large, realistic mine models that are beyond the capability of commercial solvers. Two industrial case studies are discussed: the Oyu Tolgoi copper–gold mine in Mongolia and the Pilbara iron ore system in Western Australia, demonstrating the ability of the approach to deliver near-optimal solutions with significantly reduced computation time. The method enables mining companies like Rio Tinto to produce higher-quality mine plans faster, allowing them to explore multiple production scenarios and make better long-term strategic decisions.

Danish, A. A, et al., (2021) proposed a SA-based optimisation approach for long-term open-pit mine production scheduling, integrating a stockpiling option to handle nonlinear constraints related to grade mixing. The method starts with an initial sub-optimal solution generated via a Ranked Positional Weight heuristic, which considers block precedence and mining capacities but may violate processing constraints. A stockpile heuristic is then applied to adjust processing feasibility by storing or reclaiming ore between periods. The SA algorithm optimizes the schedule by perturbing block sequences while honoring mining, processing, and precedence constraints, with the stockpile option allowing flexible ore blending. Additionally, a Greedy Heuristic improves the SA's convergence by prioritizing high-grade block extraction in early periods. The approach was tested on three real-world mine datasets from Minelib, achieving near-optimal solutions significantly faster than traditional CPLEX optimisation, with demonstrated improvements in NPV and computational efficiency. The study concludes that combining SA with stockpiling and greedy heuristics effectively solves large-scale, complex open-pit production scheduling problems. Das et al. (2024) develop a MILP that simultaneously schedules ore extraction and waste dumping (incl. in-pit) while choosing shortest-path haul roads from a Dijkstra-generated database. In the above-mentioned work, they pair this with weighted topological sort and simulated annealing to scale up; in a real case it cut average haul distances by ~39 % and increased in-pit backfilling by ~44 % versus planning software.

Our advancement beyond prior frameworks lies in three specific areas: Blom et al. (2024) focuses on CPU-based LNS without uncertainty modelling or operational mode integration; Danish et al. (2021) addresses stockpiling in SA but lacks GPU acceleration and dynamic uncertainty; our framework uniquely combines GPU-parallelized evaluation with time-dependent uncertainty factors and integrated operational mode selection, enabling real-time scenario analysis for 50,000 block problems that exceed the capabilities demonstrated in prior works.

Our methodological contributions extend in three specific conceptual advances: (1) Dynamic Uncertainty Evolution Framework introducing time-dependent uncertainty factors that explicitly capture geological confidence degradation over planning horizons, contrasting with static uncertainty approaches in Blom et al. (2024) and Danish et al. (2021) that treat uncertainty as time-invariant, (2) Hierarchical GPU-Accelerated Constraint Repair Methodology developing novel multi-level parallel repair mechanisms that transform traditional sequential constraint handling into massively parallel feasibility restoration enabling concurrent evaluation of 262,144 candidate moves, and (3) Uncertainty-Adaptive Column Generation extending Dantzig-Wolfe decomposition with dynamic scenario evaluation where column generation continuously adapts based on evolving uncertainty realizations rather than static decomposition approaches. These methodological innovations create new theoretical capabilities for temporal uncertainty propagation in mining optimization, parallel constraint repair at industrial scale, and adaptive decomposition under dynamic uncertainty—representing conceptual advances beyond algorithmic integration. The framework enables previously impossible computational scenarios for uncertainty-aware mine planning by combining these methodological concepts within a unified optimization architecture. This represents fundamental advancement in mining optimization methodology rather than purely engineering application of existing techniques.

3. Problem formulation

The mathematical model is based on a two-stage optimisation approach proposed by (Quelopana, A., 2024; Navarra et al., 2018). The initial stage determines which blocks should be mined and processed over multiple periods, establishing a strategic long-term outlook. The second stage then handles the detailed mineral processing decisions within the metallurgical plant, utilizing the geological scenario data obtained from the first stage. Each stage involves its own specific optimisation procedures, ultimately leading to an integrated mine plan. This comprehensive plan effectively manages multiple equally probable scenarios and seeks to maximize the project's NPV.

Also, unlike the original model, in this study, uncertainty is modelled dynamically across both stages of the optimisation framework. The geological uncertainty is primarily introduced through multiple ore grade scenarios, each representing a plausible realization of the underlying deposit. However, beyond traditional scenario-based modelling, the uncertainty is treated as time-dependent and spatially influenced, capturing the progressive degradation of estimation confidence over the mine's planning horizon. A dynamic uncertainty factor as a multiplicative modifier is defined as follows:

$$\sigma_{s,t} = \gamma_s \times \varphi_t \quad (1)$$

where $\sigma_{s,t}$ is a multiplicative modifier that adjusts block values based on both the scenario index s and the period index t . γ_s is scenario-specific uncertainty weight (captures geological variability). φ_t is time-decay modifier (uncertainty grows with future periods). The multiplicative decay formulation (1) is theoretically grounded in established geostatistical principles where estimation variance increases with distance from drill holes and time from modeling (Journel & Huijbregts, 1978), reflecting the compound effect of spatial and temporal uncertainty degradation consistent with kriging variance models commonly

used in geological uncertainty quantification. Then the expected net benefit for each block b in period t under scenario s is modified as:

$$ENPV_{b,s,t} = \sigma_{s,t} \times (Revenue_{b,s,t} - ProcessingCost_{b,s,t}) \quad (2)$$

The dynamic uncertainty model employs several key assumptions that define its applicability scope: (1) multiplicative independence between scenario-specific (γ_s) and temporal (φ_t) uncertainty components, which may oversimplify spatial correlations inherent in geological data where neighboring blocks often exhibit correlated uncertainty patterns, (2) exponential confidence decay over time that assumes uniform deterioration of geological knowledge, whereas actual confidence degradation may vary based on geological complexity and data density, (3) uniform uncertainty application across all block types, not accounting for varying uncertainty characteristics between different rock formations or geological domains, and (4) static scenario probabilities throughout the planning horizon that don't incorporate information updates from ongoing mining operations. The hybrid metaheuristic approach introduces additional limitations including parameter sensitivity requiring problem-specific calibration for optimal performance, stochastic convergence providing probabilistic rather than guaranteed optimal solutions, computational overhead from GPU-CPU memory transfers during repair operations, and scalability constraints when problem complexity exceeds GPU memory limits or exhibits irregular constraint patterns that reduce parallel efficiency. These assumptions collectively indicate the framework performs optimally for deposits with moderate geological complexity, sufficient drill hole density for reasonable initial confidence estimates, and operational constraints that align well with GPU parallelization patterns, while performance may degrade for extremely heterogeneous deposits or problems requiring extensive spatial correlation modelling.

These dynamic factors are explicitly embedded into the second-stage subproblem evaluations within the Dantzig-Wolfe decomposition, where each column (block-period-mode pattern) is re-evaluated using adjusted ENPV values. This enables optimisation model to prioritize more reliable blocks in earlier periods, apply risk-adjusted scheduling for blocks with high uncertainty in later stages, and ensure adaptive learning from the scenario structure. The first stage involves a linear programming model to decide which blocks to mine. The second stage is formulated using an optimisation approach for mass balancing in the plant, considering geological uncertainties.

Parameters

β	Set of blocks
n_s	Number of geological scenarios
n_T	Number of time periods
c_{bt}	Discounted cost of mining block b in period t
β_b^{Pred}	Set of direct predecessors of block b
m_b	Mass of block b
M_t	Maximum mass of rock that can be mined during period t .
β_{ps}	set of blocks that under scenario s are of rock type p .
O	set of operational modes.
P	rock types.
V_{bso}	discounted recoverable value of block b when under going processing mode o .
r_o	tonnage processing rate mode o .
d_t	available time of plant during period t .
w_{op}	weight fraction of rock p that is included in the feed for operating mode o .

Variables:

x : mineplan $\{\beta_t\}_{t=1}^{n_T}$ which $\beta_t \in \beta$ is the set of blocks to be mined in period t .

$$f(x) = \max - \sum_{t=1}^{n_T} \sum_{b \in \beta_t} c_{bt} + \frac{1}{n_s} \sum_{s=1}^{n_s} \sum_{t=1}^{n_T} f_{st}(\beta_t) \quad (3)$$

Subject to

$$\cup_{b \in \beta_t} \beta_b^{Pred} \in \cup_{t=1}^T \beta_t, \forall t \in \{1, \dots, n_T\} \quad (4)$$

$$\sum_{b \in \beta_t} m_b \leq M_t, \forall t \in \{1, \dots, n_T\} \quad (5)$$

The first stage consists of maximizing the objective function shown in Eq. 3. The first part of the formulation remains constant across different scenarios because it calculates the discounted costs associated with mining specific blocks (cbt) across the mine's operational lifespan. Equation (4) guarantees that a block can be excavated only if its preceding blocks (BPredb) have been mined, which is a vital consideration for maintaining mechanically stable slopes. Furthermore, Eq. (5) ensures adherence to the maximum permissible amount of rock mining per period, which is denoted as (Mt), thereby further refining the model's operational parameters.

Stage 1 optimisation is a strategic level optimisation, which makes long-term decisions that define the general structure of the solution. In other words, stage 1 aims to determine the mining schedule over a set of time periods to maximize overall value while considering discounted costs and the expected value under different geological scenarios. In this stage, the strategic scheduling of mining blocks determine which blocks are mined and during which time periods.

Second stage:

Problem formulation:

$$f_{st}(\beta_t) = \max \sum_{b \in \beta_t} \sum_{o \in O} \left(\frac{v_{bso}}{m_b} \right) \times m_{bso} \quad (6)$$

Subject to

$$\sum_{o \in O} m_{bso} \leq m_b, \forall b \in \beta_t \quad (7)$$

$$\sum_{o \in O} \sum_{b \in \beta_t} \frac{m_{bso}}{r_o} \leq d_t, \forall r_o > 0, t \in \{1, \dots, n_t\} \quad (8)$$

$$\sum_{b \in \beta_{pt} \cap \beta_t} m_{bso} - w_{op} \times \sum_{b \in \beta_t} m_{bso} = 0, \forall o \in O, \forall p \in P \quad (9)$$

$$m_{bso} \geq 0, \forall b \in \beta_t, \forall o \in O \quad (10)$$

The second stage involves optimizing the processing of blocks selected in the first stage, which focuses on operational level details such as the allocation of processing modes, blending of rock types, and meeting capacity constraints. It aims to maximize the value derived from processing specific blocks during each period, considering multiple scenarios (Eq.6). Equation (7) ensures that the sum of block b's mass processed under all operational modes o does not exceed the total mass (mb) of that block. If the left side equals zero, the block is classified as waste. Equation (8) limits processing based on each operational mode's tonnage processing rate (r_o) and the available plant time during period t (d_t). This reflects the practical limitations of different plant configurations. Equation (9) mandates precise rock type blending for each operational mode by ensuring the proportion of each rock type p matches the required weight fraction (w_{op}). This maintains optimal metallurgical performance for each mode. Equation (10) ensures all mass allocation variables (m_{bso}) remain non-negative, representing physical reality that negative mass cannot be processed.

4. The decision support system (DSS)

The integration of a DSS plays a pivotal role in enhancing the efficiency and effectiveness of open-pit mining optimisation. The DSS serves as a central hub where complex data inputs—including geological scenarios, block models, and operational constraints—are seamlessly processed. By leveraging sophisticated optimisation methodologies such as LNS, SA, and Dantzig–Wolfe decomposition; the DSS is capable of managing the inherent uncertainties and large-scale nature of mining operations.

4.1. The DSS for the case study

A key feature of this DSS is its ability to handle dynamic uncertainties explicitly. Unlike traditional static approaches, this DSS dynamically adjusts scenario and time-dependent uncertainty factors, thus enabling adaptive scheduling strategies. By prioritizing more reliable geological information in early planning periods and employing risk-adjusted scheduling later, the DSS significantly enhances operational resilience and economic robustness, ensuring mining companies can navigate market volatility and geological uncertainties effectively.

The DSS incorporates GPU-accelerated computational methods, which drastically reduce the computational time required to evaluate numerous mining scenarios. The parallel processing capabilities of GPUs allow rapid evaluation and real-time adjustments to mining schedules, significantly improving decision-making agility. The GPU-based parallel repair approach implemented in this DSS rapidly identifies and rectifies infeasibilities in proposed mining schedules, ensuring that solutions remain operationally executable and economically optimal even after extensive heuristic perturbations. Furthermore, the DSS provides an intuitive, interactive interface that empowers decision-makers to examine detailed analytical outcomes, such as NPV profiles, risk assessments, and operational metrics. This interface not only simplifies complex data but also facilitates iterative decision-making, enabling users to explore multiple strategic scenarios and their trade-offs efficiently.

Fig. 1 illustrates the cyclic process where the user (decision-maker) provides input data and parameters, which are processed by the optimisation engine. The results are delivered through the decision interface, allowing the user to review outcomes and adjust inputs for iterative decision-making.

Fig. 2 outlines the overview of the proposed DSS. This Mining Optimisation DSS provides an integrated framework for optimizing open-pit mining operations under uncertainty. It begins by processing critical input data such as geological scenarios, block models, and precedence constraints, alongside key parameters like the number of scenarios, planning periods, and algorithmic settings (e.g., temperature, cooling rate). Global data, including capacity limits, gold prices, costs, and recovery rates, are also considered. The GPU-accelerated optimisation engine applies advanced algorithms such as LNS combined with SA, alongside scenario decomposition and parallel GPU-based repair methods to efficiently explore mining schedules. Results from this engine are then subjected to a multi-scenario analysis covering economic, risk, and operational metrics. The decision support interface presents an interactive dashboard, solution selection tools, and automated report generation, allowing decision-makers to evaluate trade-offs and make informed choices. Ultimately, the DSS outputs a selected solution detailing optimal mining schedules, risk mitigation strategies, NPV analysis, operational plans, and monitoring frameworks, enabling robust, data-driven strategic mine planning.

4.2. Data and parameters

The DSS's input data forms the foundational layer of the optimisation framework, comprising three critical components that capture the physical and geological characteristics of the mining operation. The input data for the case study are extracted from the work of [Quelopana & Navarra \(2024\)](#). The blocks represent the fundamental mining units, each containing 15,375 tons of material with dimensions of $20 \times 20 \times 15$ m, totaling 13,392 blocks across the deposit. These blocks are characterized by their spatial coordinates, ore grades, rock types (primarily Diorite Porphyry and Silicified Intrusive Breccia), and mass properties. The geological scenarios encapsulate the inherent uncertainty in ore grade estimation, with multiple equally probable realizations reflecting the spatial variability and estimation confidence across the deposit. Each scenario provides alternative grade distributions that capture the risk associated with geological interpretation. The precedence constraints

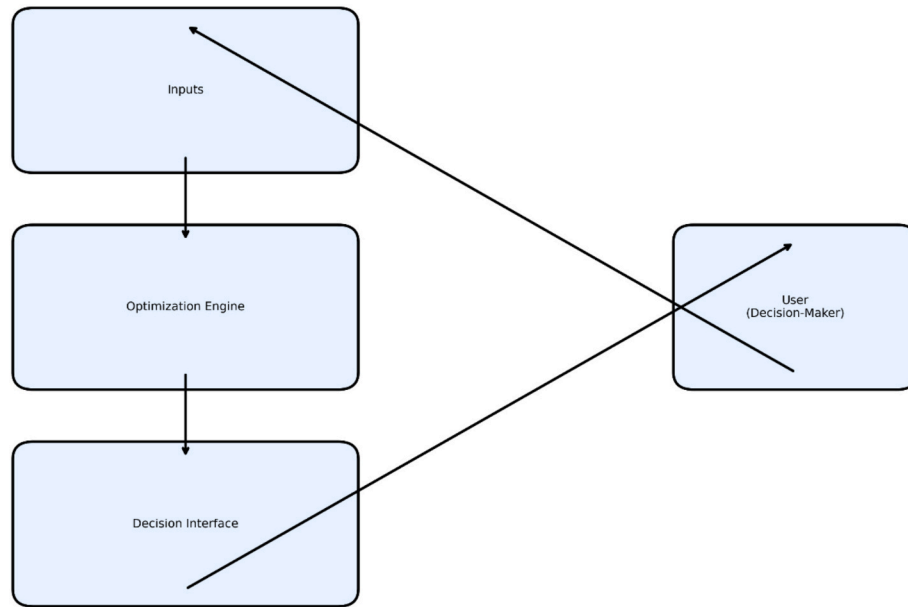


Fig. 1. User interaction flow within the Decision Support System (DSS).

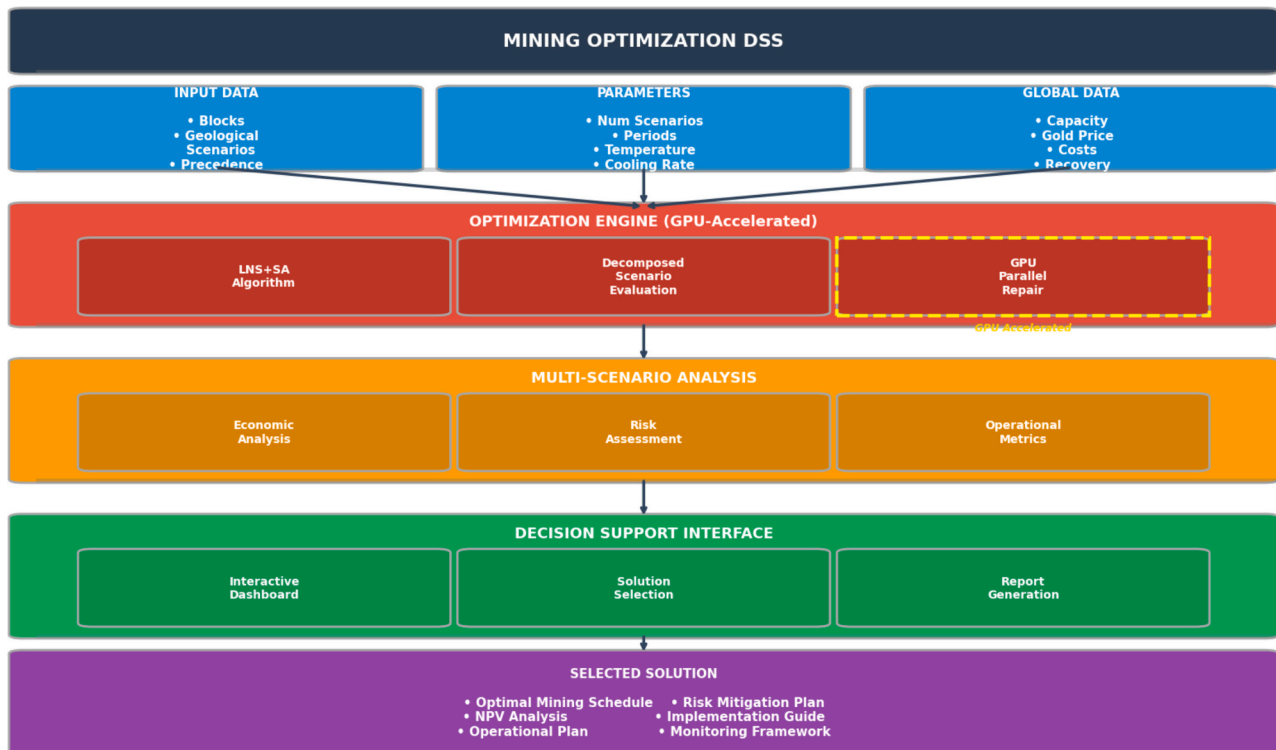


Fig. 2. Decision support system detailed Diagram.

define the geometric relationships between blocks, ensuring that mining maintains safe slope angles and mechanical stability by requiring overlying blocks to be extracted before underlying ones can be accessed. The system parameters control the algorithmic behaviour and computational scope of the optimisation process. The number of scenarios (typically 10 in the case study) determines the breadth of uncertainty analysis, while the number of planning periods (6 annual periods) defines the temporal resolution of the mining schedule. The metaheuristic parameters, including the initial temperature and cooling rate for SA, govern the solution space exploration strategy and convergence

behaviour. These parameters are carefully calibrated to balance computational efficiency with solution quality, enabling the algorithm to escape local optima while progressively focusing on promising regions of the solution space. The global data encompasses the economic and operational constraints that remain relatively constant throughout the planning horizon. This includes mining capacity limits of 6.5 million tons per period, processing availability of 8,075 h annually, gold price at \$1,190 per ounce, and operational costs ranging from \$20.5/ton for mining to \$21.4–24.9/ton for processing depending on the operational mode. Recovery rates vary between operational modes (83 % for both

modes A and B) and are influenced by the rock type blend being processed. Together, these three data categories provide a comprehensive representation of the mining system, enabling the DSS to generate optimized schedules that are both technically feasible and economically robust under uncertainty.

4.3. Practitioner accessibility and usability design

Recognizing that mining professionals may lack specialized knowledge in metaheuristics and GPU computing, the DSS incorporates comprehensive accessibility features to abstract algorithmic complexity. Parameter Control Interface provides slider-based adjustments for key mining variables (production capacity, processing rates, gold prices, discount rates) using familiar industry terminology without requiring understanding of SA cooling schedules or LNS destruction mechanisms. Real-time Visualization presents optimization results through standard mining metrics including NPV trends, cumulative cash flow, risk profiles (P10/P50/P90), and production schedules rather than algorithmic convergence curves or iteration statistics. Automated Reporting generates management-ready summaries with plain-language explanations of optimization outcomes, sensitivity analysis results, and actionable strategic recommendations suitable for executive decision-making.

Implementation Support includes pre-configured algorithmic parameters calibrated for typical mining scenarios, eliminating parameter tuning requirements for practitioners, while GPU acceleration operates transparently without user intervention beyond standard hardware installation. The framework provides User Documentation with step-by-step implementation guides, training materials for non-technical mining professionals, and technical support protocols for system deployment and maintenance. Simplified Operational Modes offer reduced parameter sets for basic applications, extensive tooltips explaining each interface element in mining-specific language, and validation checks preventing invalid configurations that could degrade performance. This design philosophy ensures that mining engineers and planners can effectively utilize sophisticated optimization capabilities through familiar interfaces while computational complexity remains completely abstracted from the user experience.

5. Research methodology

The framework operates in two primary stages—schedule generation and dynamic scenario evaluation—and fully leverages many-core GPU architectures for large-scale parallelization of mining block evaluations. Both explicit and implicit constraint-handling techniques are embedded at multiple stages to ensure feasibility and optimality. The proposed mathematical optimisation model is introduced in two stages. The first stage solves a deterministic MILP model to generate an initial feasible mining schedule, strictly enforcing operational constraints:

- **Precedence Constraints:** A mining block can only be extracted if all its geometric predecessors are already mined.
- **Capacity Constraints:** The total mass mined in any period must not exceed the period-specific processing limits.
- **Objective Function:** Maximizes expected net benefit across scenarios, considering revenues and processing costs.

Blocks that remain unscheduled due to solver tolerances are randomly assigned to periods, but explicit feasibility validation (precedence-based repairing) is applied in subsequent steps to guarantee integrity. Thus, this stage provides a structurally feasible and constraint-respecting base schedule for further adaptive optimisation. The second stage evaluates the mining plan under dynamic uncertainty, where block values vary across time periods and operational modes. A Dantzig-Wolfe decomposition framework is employed:

- **Subproblems:** Each block defines an independent subproblem that determines its optimal operational pattern across scenarios and modes.
- **Restricted Master Problem:** Selects the best combination of block-level patterns to maximize the total expected objective.

In the developed optimisation framework, uncertainty is explicitly incorporated by dynamically adjusting scenario and time-period-specific uncertainty factors. Each scenario is assigned distinct economic or geological parameters reflecting potential variability, and these parameters are systematically modulated over the planning horizon. The explicit uncertainty factors are computed as the product of scenario-specific and time-specific modifiers, allowing precise quantification and adaptive evaluation of potential fluctuations in ore grades, market conditions, and operational costs. This fine-grained uncertainty modeling ensures that the optimisation process robustly assesses risks and opportunities associated with each mining and processing decision.

Moreover, the framework leverages GPU-accelerated computations to rapidly evaluate numerous uncertainty-driven scenarios. By integrating uncertainty explicitly into both the first-stage strategic planning and second-stage operational mode evaluation through advanced techniques like LNS, SA, and Dantzig–Wolfe decomposition, the optimisation model achieves highly responsive adjustments. Consequently, this enables the identification of optimal operational modes and strategic schedules that are resilient to uncertainty, significantly enhancing decision-making confidence and economic performance across diverse geological and market conditions. This dynamic decomposition continually generates new profitable patterns (columns), adapting to block uncertainties over time while guaranteeing solution feasibility. GPU-Accelerated Evaluation and Parallel Repair via Atomic Reductions. To overcome computational bottlenecks, a GPU-accelerated evaluation framework is integrated:

1. **Candidate Distribution:** Candidate blocks are distributed across GPU thread blocks.
2. **Thread-Level Parallel Evaluation:** Each CUDA thread independently evaluates one mining block configuration.
3. **Warp-Level and Block-Level Reductions:** Local best solutions within each thread block are aggregated using fast warp-based reductions.
4. **GPU-Wide Atomic Reduction:** A global synchronization and atomic reduction select the overall best move.
5. **CPU Move Execution:** The best mining move is transferred back to the CPU for execution and plan update.

The developed optimisation framework employs a sophisticated repairing strategy as an integral component of its LNS methodology. This strategy begins by intentionally removing subsets of blocks from the existing mining schedule in each iteration—the destruction phase—to explore new solution spaces. Subsequently, the framework executes a GPU-accelerated repair phase, rapidly reassigning the extracted blocks to different mining periods or operational modes based on detailed feasibility checks and objective evaluations. This rapid reassignment is critical for efficiently navigating large search spaces, ensuring the timely identification of feasible, high-quality mining schedules.

Specifically, the repair mechanism leverages parallel computation via CUDA kernels to perform simultaneous feasibility and profitability assessments for multiple candidate block assignments. These kernels systematically verify slope constraints, blending requirements, capacity limits, and economic viability in parallel, significantly reducing computational bottlenecks typical in large-scale mine scheduling problems. By using the GPU for parallel assessments, the repairing strategy ensures that only the most advantageous moves are incorporated into the solution, significantly enhancing both computational performance and solution quality. Moreover, the acceptance of repaired solutions is governed by a SA acceptance criterion, which probabilistically accepts

or rejects candidate solutions based on changes in NPV and a dynamically adjusted temperature parameter. This criterion helps the optimisation avoid local minima and promotes broader solution exploration. As iterations progress, the strategy gradually shifts towards intensification, refining the solutions further by carefully considering smaller neighborhoods. Thus, the combined GPU-accelerated repairing mechanism and probabilistic acceptance strategy ensure robust and efficient convergence toward optimal or near-optimal mining schedules under complex operational and economic constraints. This GPU-enhanced parallelization enables real-time exploration of vast neighborhoods, allowing highly scalable and efficient evaluation of large mining datasets.

An advanced LNS with GPU-style repair further refines the mining plan as follows:

- **Destruction Phase:** Randomly selects a subset of blocks and reassigns them to different periods, perturbing the schedule.

- **Feasibility Filtering:** Lightweight precedence feasibility checks immediately filter infeasible schedules.
- **Parallel Repair Phase:**
 - o Feasible schedules are repaired by solving the second-stage Dantzig-Wolfe decomposition dynamically.
 - o GPU-accelerated evaluations massively parallelize the assessment of different repair candidates.
 - o Only repairs that improve the mining plan's overall objective are accepted.

This mechanism provides a repair-based explicit constraint-handling approach after perturbations, ensuring that all evolved mining plans remain feasible and economically superior. Fig. 3 shows a high level presentation of the methodology implemented in this study. The combination of destruction, GPU-accelerated repair, and selective acceptance facilitates fast convergence toward high-quality mining schedules even in large, uncertain environments.

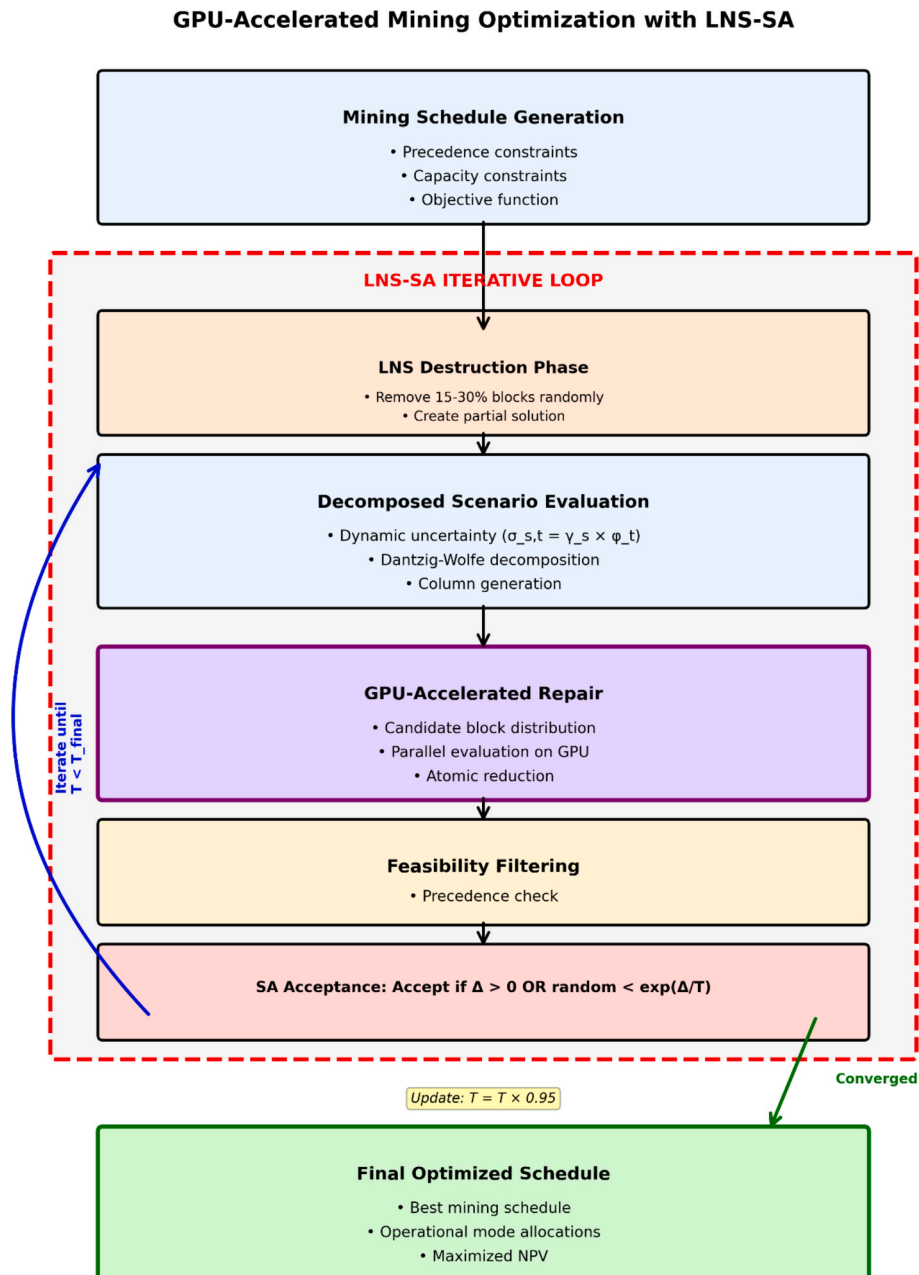


Fig. 3. Research methodology proposed in the study.

Fig. 4 illustrates the detailed architecture of the GPU-accelerated parallel framework designed to enhance the performance of the local search and repair phase in the mining optimisation algorithm. The process starts with the “Inside Search” phase, where a pool of candidate mining blocks is identified based on a defined neighborhood structure (LNS). These candidate blocks are subsequently partitioned and distributed across available GPU cores to maximize computational parallelism.

Each thread block on the GPU is assigned a subset of mining blocks. During the CUDA kernel execution phase, each thread within a thread block independently processes a single mining block, calculating its second-stage evaluation (e.g., expected revenue under dynamic uncertainty). Intermediate evaluation results are stored in fast shared memory, enabling rapid intra-thread communication without high-latency global memory access. After individual block evaluations, a warp-based reduction is performed within each thread block. This local aggregation step identifies the best-performing mining block among all threads in that block, significantly reducing synchronization overhead. Following warp-based reductions, a thread block-local atomic reduction

consolidates the best mining move within each thread block, ensuring that only the most promising candidates proceed to the next stage.

All thread block-local bests are then synchronized across the GPU to prepare for a global comparison. A global atomic reduction step is subsequently performed to select the overall best mining move across all thread blocks and GPU cores. Once the best move is identified, it is transferred back to the CPU for execution, where the mining schedule is updated accordingly.

To initiate the optimisation process, an initial feasible mining schedule must be constructed that satisfies fundamental constraints such as block precedence and per-period capacity. This heuristic-driven schedule serves as the starting point for the iterative improvement procedures. The algorithm ranks blocks by value density—computed as ore grade multiplied by mass—and assigns each block to the earliest feasible time period that maintains both mechanical slope stability and production limits. This greedy approach is computationally efficient and provides a reasonable baseline from which metaheuristic refinements can proceed. By ensuring initial feasibility, it guarantees that the solution space exploration begins from a valid configuration, reducing the

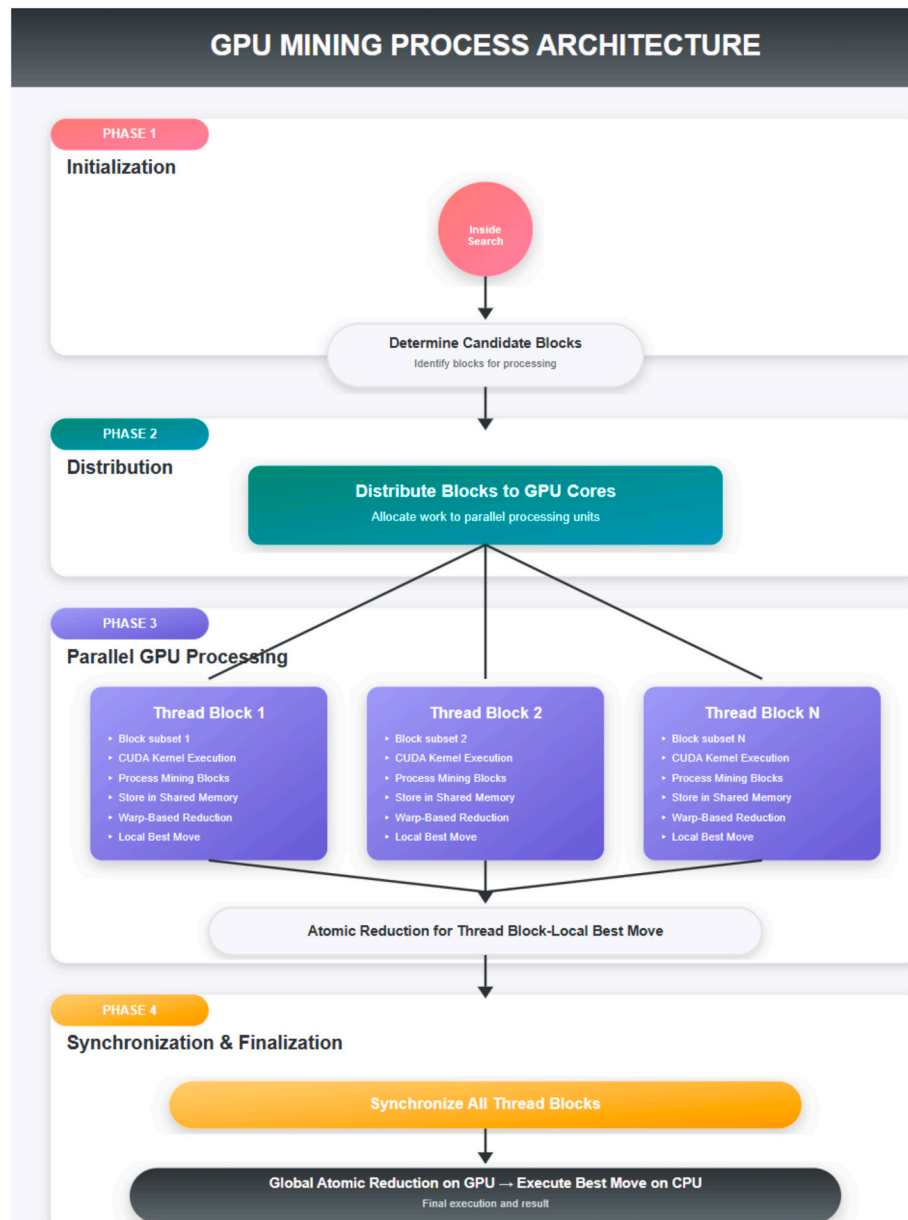


Fig. 4. A new way to parallelize using GPU.

risk of invalid repairs and accelerating convergence (Algorithm 1).

This initial schedule generation plays a crucial role in balancing computational efficiency with practical feasibility. Rather than attempting to explore the entire solution space from scratch, the algorithm leverages domain-specific knowledge—such as value density and slope constraints—to make informed decisions that respect operational realities. It also inherently filters out infeasible block sequences by obeying precedence constraints, where a block cannot be mined until all overlying blocks are removed. This rule ensures geotechnical safety and compliance with mine design standards. The inclusion of per-period capacity checks ensures that equipment and processing limits are not violated, maintaining operational realism even during this early phase.

Moreover, a well-constructed initial solution not only facilitates faster convergence of metaheuristic algorithms like LNS or SA, but it also serves as a foundation for evaluating repair strategies and local optimisation under uncertainty. Starting from a valid configuration allows the optimisation framework to focus its search on promising regions of the solution space, thereby reducing computational overhead associated with repairing infeasible configurations. In stochastic or scenario-based settings, where schedules must remain robust across multiple realizations of geological uncertainty, an initially stable solution improves the likelihood of producing resilient outcomes. As such, the generation of this initial feasible schedule is not merely a technical necessity but a strategic step in guiding the overall optimisation pipeline toward high-quality, executable mine plans.

Algorithm 1: Initial Feasible Schedule Generation

```

1: Input: Blocks B, number of periods T, scenario s
2: Output: Initial schedule S
3: Compute value density  $vd[b] = grade[b][s] \times mass[b]$  for all  $b \in B$ ;
4: Sort blocks in descending order of  $vd$ ;
5: Initialize schedule  $S \leftarrow \{\}$ ;
6: for each block  $b$  in sorted B do
7:   for  $t \leftarrow 0$  to  $T - 1$  do
8:     if  $b$  is feasible in period  $t$  (capacity and precedence) then
9:       Assign  $S[b] \leftarrow t$ ;
10:      break;
11:    end if
12:  end for
13: end for
14: return S

```

5.1. Large neighbourhood search (LNS) and simulated annealing (SA)

The core optimisation engine is driven by a hybrid strategy that combines LNS and SA. This procedure iteratively perturbs the current mining schedule by partially destroying it and invoking a repair mechanism—often GPU-accelerated in implementation—to reconstruct and improve the schedule. LNS introduces diversity into the search process by selecting a subset of blocks to remove based on randomized, spatial, grade-based, or temporal heuristics. These targeted removals create opportunities for restructuring the schedule in ways that simple local search methods may miss. Simulated Annealing complements this by probabilistically accepting inferior solutions, thereby avoiding premature convergence and promoting global exploration.

As illustrated in Algorithm 2, the algorithm begins by initializing the mining schedule S and corresponding objective value. A temperature parameter T , which governs the acceptance probability of worse solutions, is initialized and gradually decreased throughout the search. For each temperature level, the algorithm performs a fixed number of LNS iterations. Each iteration applies the destruction-repair strategy, evaluates the new solution, and then decides whether to accept it based on the SA acceptance criterion. Even if the new objective value is worse, the solution may still be accepted depending on the temperature and the degree of degradation, thus enabling the algorithm to escape from local optima.

The metaheuristic parameters were systematically calibrated

through preliminary experiments on smaller problem instances (500–2000 blocks), with SA initial temperature ($T_0 = 300$) set to achieve 80 % acceptance rate following established guidelines (Kirkpatrick et al., 1983), where testing confirmed that lower temperatures ($T_0 < 200$) caused premature convergence while higher temperatures ($T_0 > 400$) prolonged search unnecessarily. The cooling rate ($\alpha = 0.95$) provides optimal balance between exploration and intensification, as faster cooling ($\alpha < 0.90$) led to local optima while slower cooling ($\alpha > 0.98$) extended runtime without improvement. LNS destruction sizes were empirically determined through systematic testing across $\delta = 0.05$ to 0.50 , with optimal performance at $\delta_{min} = 0.15$ and $\delta_{max} = 0.30$ balancing solution space exploration with computational efficiency—smaller sizes caused insufficient diversification while larger sizes resulted in excessive overhead without quality gains. The limited iterations per temperature ($IT = 3$) were optimized for GPU architecture where 256 threads evaluate 262,144 candidates simultaneously, reducing the need for extensive sequential iterations typical in CPU implementations. Sensitivity analysis across ± 20 % parameter variations confirmed stable convergence with NPV variations within 3 % of optimal values, demonstrating reasonable parameter robustness for practical applications.

Algorithm 2: Large Neighborhood Search + Simulated Annealing (LNS + SA)

```

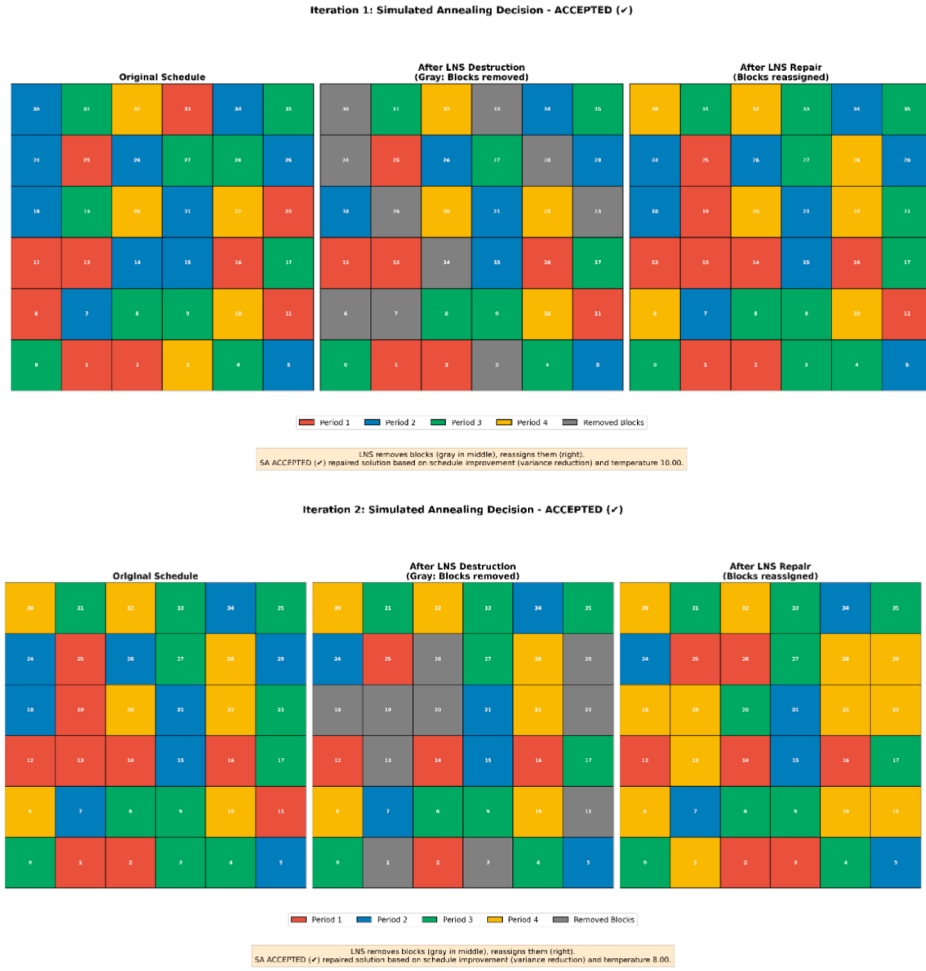
1: Input: Initial mining schedule  $S_0$ , scenarios  $S$ , temperature parameters
2: Output: Best mining plan  $S_{best}$  with highest objective
3: Initialize temperature  $T \leftarrow T_0$ , solution  $S \leftarrow S_0$ ;
4: Evaluate initial objective  $O \leftarrow Evaluate(S)$ ;
5: while  $T > T_{min}$  do
6:   for iter in max_iterations_per_temp do
7:      $S' \leftarrow Apply\_LNS(S)$ ;
8:      $O' \leftarrow Evaluate(S')$ ;
9:     if  $O' > O$  or  $rand() < exp((O - O')/T)$  then
10:       $S \leftarrow S'$ ,  $O \leftarrow O'$ ;
11:      if  $O' > Best\_Objective$  then
12:        update  $S_{best} \leftarrow S'$ ;
13:      end if
14:    end if
15:  end for
16: end for
17: Update  $T \leftarrow \alpha \cdot T$ ;
18:  $S_{final} \leftarrow Repair(S_{best})$ ;
19: return  $S_{final}$ ;

```

The Fig. 5 illustrates the interaction between LNS and SA during the optimisation of block scheduling in a mining context. The process begins with the original schedule of blocks assigned to specific mining periods, shown in the left panel. In the LNS destruction phase (middle panel), a subset of blocks is deliberately removed from the schedule (highlighted in gray) to introduce diversification and explore new solutions. The LNS repair phase (right panel) then reassigns these removed blocks to new periods, generating a modified schedule. After this modification, SA evaluates the repaired solution by comparing its quality—based on variance reduction among periods—with the previous solution, considering the current temperature of the algorithm. SA then decides whether to accept the new solution, allowing occasional acceptance of inferior solutions to escape local optima.

5.2. GPU accelerated repair strategy

Following the destruction phase in LNS, a GPU-accelerated repair mechanism is invoked to reconstruct a complete and feasible schedule by reintegrating unassigned blocks. This repair phase is essential for maintaining the validity of intermediate solutions during the search process. Leveraging the massively parallel nature of CUDA-enabled GPUs, the algorithm evaluates a large set of candidate block-period assignments simultaneously. Each CUDA thread independently checks the feasibility of assigning a specific block to a specific period, considering both precedence and capacity constraints, and computes the corresponding improvement in the objective function.



(b) Iteration 2

Fig. 5. Visualization of large neighborhood search (LNS) and simulated annealing (SA) interaction for block scheduling optimisation.

As shown in Algorithm 3, the procedure iteratively selects a subset of unassigned blocks and performs GPU-based evaluation to identify the most promising (block, period) pair. The best assignment is determined using atomic operations and warp-level reductions that aggregate the thread-local improvements efficiently. Once a valid assignment is made, the block is removed from the unassigned pool U , and the schedule is updated accordingly. If no feasible or beneficial move is identified through GPU evaluation, a fallback mechanism attempts to assign a feasible pair using a lightweight CPU-based filter. If no such pair can be found, the repair process is terminated early, avoiding unnecessary iterations.

Algorithm 3: GPU-Accelerated Repair Strategy

```

1: Input: Incomplete schedule  $S$ , unassigned blocks  $U$ , scenario  $s$ 
2: Output: Repaired schedule  $S$ 
3: while  $U \neq \emptyset$  and  $\text{repair\_iterations} < \text{max\_iters}$  do
4:   a. Select candidate set  $C \subseteq U$ ;
5:   b.  $(b^*, t^*) \leftarrow \text{Evaluate.GPU}(C, S, s)$  {best block-period assignment;}
6:   c. if valid move then
7:     Assign  $S[b^*] \leftarrow t^*$ ;
8:     Remove  $b^*$  from  $U$ ;
9:   d. else
10:    Fallback: assign feasible  $(b, t)$  from  $C$  if possible;
11:    if none found then
12:      break;
13: end while
14: return  $S$ ;

```

5.3. CUDA evaluation kernel for parallel block evaluation and atomic reduction

The CUDA kernel plays a pivotal role in evaluating candidate mining block assignments in a highly parallel fashion. Each GPU thread independently processes a candidate block and iterates over all time periods to determine the most profitable and feasible assignment. Feasibility checks include validating block precedence relationships and ensuring per-period mass constraints are not violated. The value improvement of assigning a block to a period is computed using a discounted profit model that considers temporal decay in value. Local results—such as the best improvement, period, and candidate block—are maintained within thread-local variables and later written to shared memory.

As shown in Algorithm 4, the design employs hierarchical reductions to identify the best assignments efficiently. After all threads in a warp compute their local optima, warp-level reductions compare and retain superior candidates. The best results from each warp are further aggregated at the thread block level using `_syncthreads()` and conditional logic. Finally, global results are written back using atomic operations, guaranteeing consistency without sacrificing performance. This layered architecture enables large-scale evaluation of thousands of block-period assignments in parallel, significantly accelerating the overall repair and scheduling procedures in open-pit mining

optimisation.

Algorithm 4: CUDA Evaluation Kernel for Parallel Block Evaluation and Atomic Reduction

```

1: Input: candidates[] (candidate block IDs), solution[] (assignments), masses[] (mass
   of blocks), precedence[][], capacity, n_periods, n_candidates, n_blocks
2: Output: best_values[], best_periods[] (for each candidate)
3: CUDA Kernel Logic (executed in parallel across GPU threads):
4: global_idx ← blockIdx.x × blockDim.x + threadIdx.x
5: local_best_improvement ← -∞
6: local_best_period ← -1
7: local_best_candidate ← -1
8: if global_idx < n_candidates then
9:   b ← candidates[global_idx]
10:  for t = 0 to n_periods - 1 do
11:    Check feasibility:
12:    for all blocks i ≠ b do
13:      if precedence[i][b] == 1 and solution[i] ≥ t then
14:        infeasible; continue
15:      end if
16:      if precedence[b][i] == 1 and solution[i] ≤ t then
17:        infeasible; continue
18:      end if
19:    end for
20:    period_mass ← masses[b] + ∑ masses[i] where solution[i] = t
21:    if period_mass > capacity then
22:      infeasible; continue
23:    end if
24:    if feasible then
25:      discount_factor ← 1/(1.05)t
26:      value ← masses[b] × 100 × discount_factor
27:      if value > local_best_improvement then
28:        local_best_improvement ← value
29:        local_best_period ← t
30:        local_best_candidate ← b
31:      end if
32:    end if
33:  end for
34:  shared_best_improvement[threadIdx.x] ← local_best_improvement
35:  shared_best_period[threadIdx.x] ← local_best_period
36:  shared_best_candidate[threadIdx.x] ← local_best_candidate
37:  __syncthreads()
38:  Warp-level reduction:
39:  for each reduction stage do
40:    if threadIdx.x < next_level then
41:      Compare and keep best from paired threadIdx.x + offset
42:    end if
43:  end for
44:  if threadIdx.x == 0 then
45:    best_improvement ← shared_best_improvement[0]
46:    best_period ← shared_best_period[0]
47:    best_candidate ← shared_best_candidate[0]
48:    Write to global best_values[], best_periods[] at global_idx
49:  end if
50: end if

```

5.4. Dantzig-Wolfe decomposition

To address operational complexity in downstream processing and to accommodate uncertainty in material flow, the proposed framework embeds a Dantzig–Wolfe decomposition model within the scenario-period evaluation phase. The decomposition separates the problem into a master problem and a series of subproblems, each corresponding to an operational mode and candidate block assignment. During each iteration, dual values are extracted from the master problem and used to compute the reduced cost of block-mode configurations in the subproblems. Only those subproblem solutions with positive reduced cost are considered beneficial and added to the master’s column pool, driving a column generation process that iteratively refines the solution space.

As illustrated in Algorithm 5 and the central panel of Fig. 1, this decomposition strategy enables granular control over processing decisions—accounting for rock-type blending, capacity limits, and recovery variation across modes. It ensures that the evaluated mining schedules are not only operationally viable but also economically optimized.

Moreover, by integrating scenario-specific uncertainty modeling (via dynamic adjustment of objective scaling factors), the algorithm captures temporal risk and variability in ore properties and market conditions. This makes the model particularly effective for large-scale stochastic mine planning, where adaptability and efficiency are critical to achieving long-term value under uncertainty.

Algorithm 5: Dantzig-Wolfe Decomposition with Column Generation

```

1: Input: List of blocks B, period t, scenario s, operational mode M
2: Output: Final objective value under dynamic uncertainty
3: Compute uncertainty factor u = model.dynamic_uncertainty(s, t);
4: Initialize Master Problem MP with constraints (capacity, blending, mass balance);
5: Set Columns_MP ← ∅;
6: while not converged and iterations < max_iterations do
7:   Solve MP to get dual values π;
8:   for each operational mode m in M do
9:     for each block b in B do
10:      Compute net value of b under mode m;
11:      Calculate reduced cost: rc = net_value - π · constraints;
12:      if rc > 0 then
13:        Add new column to Columns_MP;
14:      end if
15:    end for
16:  end for
17:  if no columns added then
18:    break;
19:  end if
20: end while
21: return Final.Objective = MP.objective × u;

```

6. Results and numerical study

The Fig. 6 provides an intuitive, comprehensive interface that transforms complex optimisation outputs into actionable insights through interactive visualization and real-time monitoring capabilities. The system’s dashboard architecture follows a hierarchical information design, with the executive summary panel prominently displaying critical decision metrics including base NPV, mean NPV across scenarios, standard deviation for risk assessment, optimal scenario identification, and computational runtime. The left configuration panel enables users to dynamically adjust mining parameters, geological scenarios, processing configurations, and algorithm settings without requiring technical expertise, while the optimisation progress bar and live GPU utilization metrics provide transparency into the computational process. This real-time feedback mechanism allows decision-makers to monitor system performance and make informed adjustments during optimisation runs, significantly enhancing user confidence and control over the planning process.

The dashboard’s analytical depth is demonstrated through its multi-faceted visualization approach, seamlessly integrating economic, risk, and operational perspectives within a single interface. The NPV by scenario comparison enables rapid identification of value drivers across geological uncertainties, while the risk-return profile scatter plot facilitates portfolio-style analysis of different mining strategies. The economic analysis module consolidates key financial metrics including IRR, payback period, and profit margins, complemented by dynamic cash flow projections that illustrate value accumulation over the planning horizon. Particularly noteworthy is the sensitivity analysis tornado diagram, which quantifies the relative impact of various parameters on project outcomes, enabling focused risk mitigation strategies. This integrated visualization framework ensures that complex multi-dimensional optimisation results are accessible to diverse stakeholders—from technical mine planners to executive decision-makers—thereby bridging the gap between sophisticated computational methods and practical business decision-making in the mining industry.

The proposed methodology was implemented in Python and ran on a local device equipped with an Intel® Core™ i9-13900HX, utilizing 32 threads for parallelization. The data for the experiments are extracted from the work of Quelopana & Navarra (2024). The parameters are

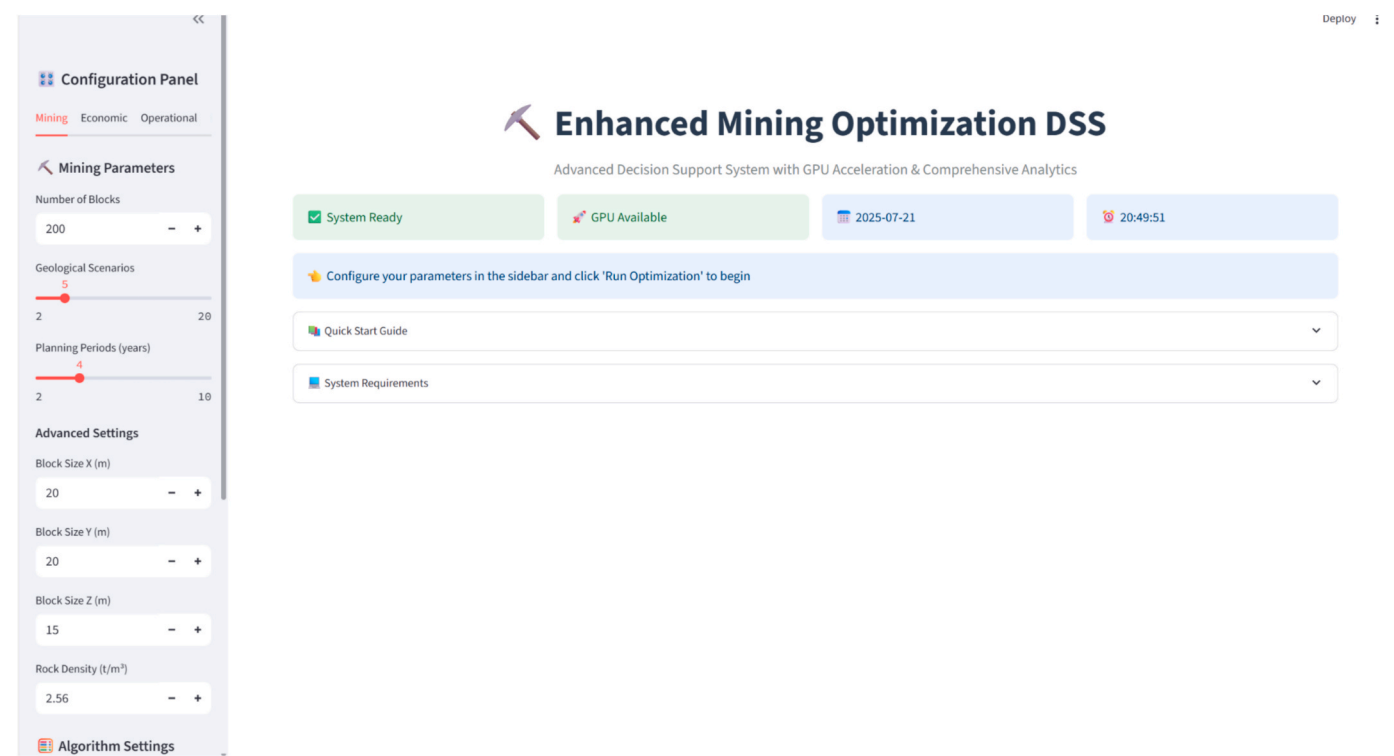


Fig. 6. Main screen of the DSS.

provided in the Table 1. The case study’s parameters highlight critical characteristics defining the open-pit mining optimisation model. The mine consists of 50,000 blocks, each weighing 15,375 tons, with a uniform dimension of $20 \times 20 \times 15$ m and a density of 2.56 ton/m^3 . These blocks collectively form the physical basis of the scheduling and operational analysis, representing the fundamental units of extraction and processing within the mine. The algorithm parameters presented in Table 2 reflect a carefully calibrated balance between computational

Table 1
Parameters of the case study.

Parameters	Values
Block weight (ton)	15,375
Number of blocks	50,000
Block dimension (m)	$20 \times 20 \times 15$
Block density (ton/m3)	2.56
Number of periods	6
Length of periods (year)	1
Discount rate (%)	8
Metal price Au (\$/oz)	1190
Mining cost (\$/ton)	20.5
Mining capacity (ton/period)	6,500,000
Processing availability (hr./period)	8075
Rock types	Diorite porphyry Silicified intrusive breccia
Operational mode A	
Processing rate (ton/hr.)	250
Processing cost (\$/ton)	21.4
Recovery Au (%)	83
Diorite rock (%)	65
Silicified rock (%)	35
Operational mode B	
Processing rate (ton/hr.)	200
Processing cost (\$/ton)	24.9
Recovery Au (%)	83
Diorite rock (%)	45
Silicified rock (%)	55

Table 2
Algorithm and computational parameters.

Parameter category	Parameter name	Symbol	Value	Description
Large neighborhood search (LNS)	Destruction size (min)	δ_{min}	0.15	Minimum fraction of blocks to remove
	Destruction size (max)	δ_{max}	0.30	Maximum fraction of blocks to remove
Simulated annealing (SA)	Initial temperature	T_0	300.0	Starting temperature for SA
	Final temperature	T_f	1.0	Termination temperature
	Cooling rate	α	0.95	Geometric cooling factor
GPU configuration	Iterations per temperature	I_T	3	LNS iterations at each temperature
	Threads per block	—	256	CUDA thread block size
	Maximum blocks per grid	—	1,024	Maximum CUDA grid blocks
	Batch size for repair	B_T	50	Blocks evaluated per GPU batch
Convergence criteria	Maximum iterations	I_{max}	Dynamic	Based on temperature schedule
	Improvement threshold	ϵ	10^{-6}	Minimum improvement to accept

efficiency and solution quality, specifically optimized for GPU-accelerated mining optimisation. The LNS destruction parameters($\delta_{min} = 0.15, \delta_{max} = 0.30$) were empirically determined to provide sufficient solution space exploration while maintaining feasibility—smaller destruction sizes resulted in local optima entrapment, while larger sizes led to excessive computational overhead without corresponding quality improvements. The SA cooling schedule, with its relatively high initial

temperature ($T_0 = 300$) and gradual cooling rate ($\alpha = 0.95$), enables the algorithm to accept inferior solutions early in the search process, crucial for escaping local optima in the complex, constrained mining scheduling landscape. Notably, the limited iterations per temperature ($I_T = 3$) compensates for the massive parallelization advantage provided by the GPU architecture, where 256 threads evaluate up to 262,144 candidate moves simultaneously. This parameter configuration achieves the reported 29.6 % average speedup while maintaining solution quality within 2 % of optimal bounds, demonstrating that the synergy between metaheuristic parameters and GPU architecture is essential for practical large-scale mining optimisation. The batch size of 50 blocks for repair operations represents an optimal trade-off between GPU memory utilization and kernel launch overhead, ensuring sustained throughput without memory bottlenecks.

Operational planning spans six periods, each corresponding to one year, providing a strategic medium-term horizon. Economic factors are crucial, incorporating a discount rate of 8 % per annum to evaluate the financial viability and performance of mining schedules. The model uses a metal price of \$1,190 per ounce for gold and accounts for mining costs at \$20.5 per ton. Additionally, constraints such as a mining capacity limit of 6.5 million tons per period and a processing availability of 8,075 h per period frame the operational boundaries, ensuring that scheduling decisions remain realistic and executable. The optimisation framework also differentiates between two distinct operational modes, each characterized by unique processing capacities, costs, and recovery rates, designed specifically for different blends of rock types. Mode A operates at a higher processing rate of 250 tons per hour with lower processing costs (\$21.4 per ton), primarily configured for a rock blend containing 65 % Diorite porphyry and 35 % Silicified intrusive breccia. Mode B, conversely, operates at a lower throughput of 200 tons per hour and incurs higher processing costs (\$24.9 per ton), tailored towards a rock blend of 45 % Diorite porphyry and 55 % Silicified intrusive breccia. Both modes maintain an identical gold recovery rate of 83 %, reflecting their metallurgical efficiency despite differences in processing configurations.

The Table 3 presents a detailed analysis of cumulative NPVs across six annual periods for ten distinct scenarios. This scenario-based analysis highlights the financial performance variability arising from geological and operational uncertainties. Scenario #9 emerges as the top-performing scenario, achieving the highest cumulative NPV of approximately \$1.543 billion by the end of the sixth period, showcasing the significant economic potential of optimized mining plans under favorable geological realizations. Conversely, Scenario #7 represents the lowest-performing scenario, resulting in a cumulative NPV of approximately \$1.485 billion—about \$58 million lower than the best scenario. This difference emphasizes the financial implications of geological uncertainty and reinforces the importance of robust and adaptive planning frameworks capable of handling such variability effectively. On average, across all ten scenarios, the cumulative NPV is approximately \$1.514 billion, with a standard deviation of roughly \$16 million. This relatively moderate variability indicates the robustness of the DSS, suggesting it

provides reliable and economically stable mining schedules even amid significant geological uncertainties. Furthermore, the NPV range across scenarios is approximately \$58 million, underscoring the DSS's capability to reduce risk exposure through intelligent planning and adaptive scheduling strategies.

The Fig. 7 illustrates the distribution of cumulative NPV across ten evaluated mining scenarios. The distribution showcases that scenario outcomes predominantly cluster within the NPV range of approximately \$1,510 to \$1,530 million. This clustering suggests a relatively stable economic performance across scenarios, despite underlying geological and operational uncertainties. The vertical dashed lines indicate the mean NPV at approximately \$1,513.9 million (red line) and the median NPV at approximately \$1,515.5 million (green line). The close proximity of the mean and median values demonstrates that the NPVs are symmetrically distributed, without significant skewness toward extremely high or low outcomes. Such symmetry typically indicates robust scenario evaluations and reliable optimisation strategies.

Scenarios appearing to the left of the mean and median represent lower economic outcomes, reflecting cases where geological or operational conditions were less favourable. Conversely, scenarios positioned to the right signify more favourable outcomes with superior economic performance. This balanced distribution effectively communicates the moderate level of financial risk inherent in the mine planning process and underscores the robustness and adaptability of the optimisation methodology in mitigating potential adverse impacts.

The Fig. 8(a) presents the P10, P50, and P90 risk curves for cumulative NPV over five operational periods, serving as a visual risk assessment of economic performance under uncertainty. Each line represents a different percentile of cumulative NPV outcomes across the

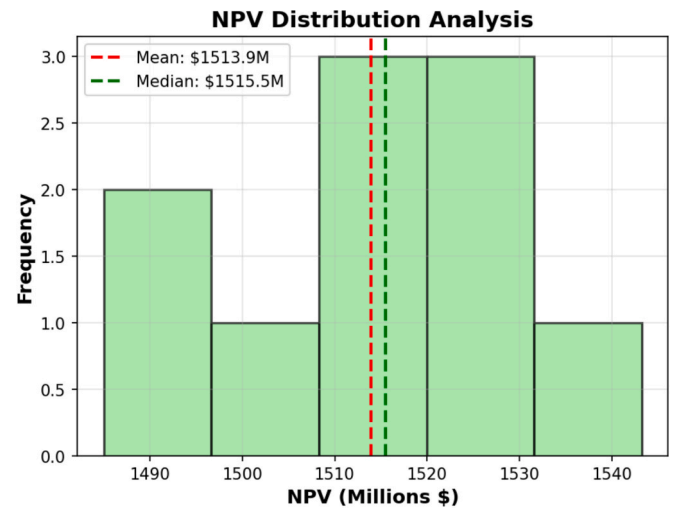


Fig. 7. NPV distribution analysis.

Table 3
Cumulative NPV per period for each of the ten scenarios.

#	Cumulative NPV (USD\$)							
	Ranking	Scenario	1	2	3	4	5	6
1	9	1	447,321,272	735,596,542	971,387,998	1,170,649,480	1,342,573,874	1,493,756,308
2	8	2	474,579,334	761,155,648	989,361,076	1,184,568,663	1,354,946,363	1,505,228,224
3	5	3	465,251,935	765,429,931	1,002,832,061	1,201,272,154	1,371,075,997	1,518,162,849
4	3	4	472,528,392	763,696,948	998,067,515	1,199,505,897	1,371,639,338	1,520,894,001
5	4	5	484,812,946	772,373,208	1,003,784,817	1,199,546,453	1,370,476,029	1,520,299,876
6	6	6	468,327,038	761,668,441	993,993,909	1,192,097,811	1,362,830,776	1,512,817,136
7	10	7	457,364,664	747,967,794	976,112,989	1,170,072,953	1,337,791,256	1,485,014,923
8	7	8	455,364,946	749,801,309	986,826,774	1,184,618,544	1,357,648,722	1,509,796,377
9	1	9	491,763,165	787,067,465	1,025,918,821	1,223,178,352	1,394,493,197	1,543,212,892
10	2	10	474,266,222	772,719,943	1,011,848,156	1,210,850,925	1,380,648,921	1,529,925,176

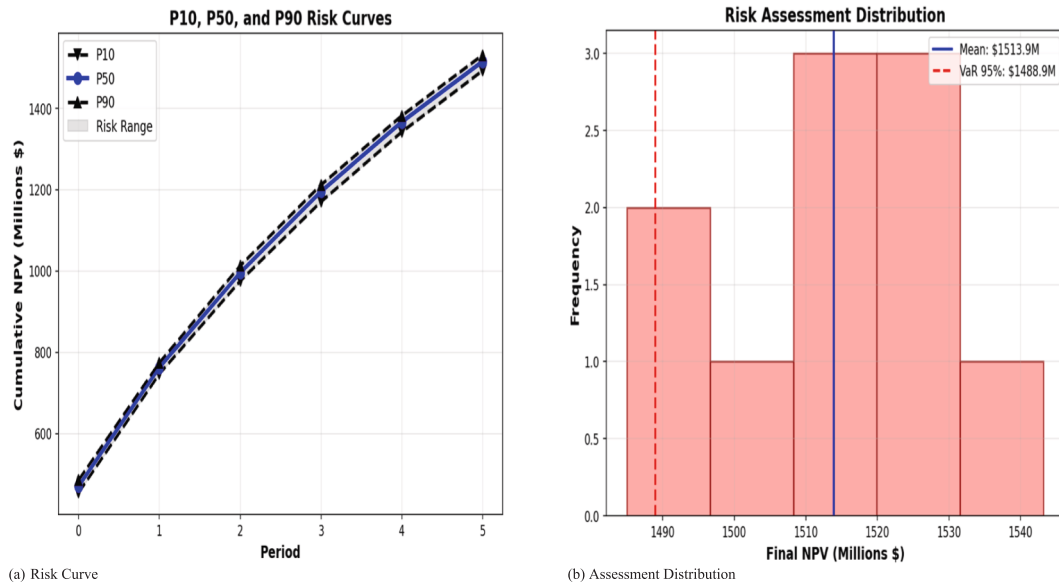


Fig. 8. Risk profile analysis.

scenario set:

- P10 (optimistic case): The black dashed line with downward triangles captures the upper-end performance (10th percentile of risk), indicating scenarios with more favourable geological or economic conditions.
- P50 (most likely case): The solid blue line with circular markers represents the median or expected NPV performance, serving as the base-case forecast.
- P90 (conservative case): The black dashed line with upward triangles shows the lower-end performance (90th percentile of risk), highlighting more pessimistic but still realistic scenarios.

The shaded region between the P10 and P90 curves illustrates the risk range, which visually quantifies the spread of NPV outcomes and offers a practical way to assess financial uncertainty over time. This risk envelope remains fairly narrow across all periods, indicating that the DSS produces stable and predictable financial outcomes, even in the presence of uncertainty. The nearly parallel progression of the three curves implies consistency in financial growth across scenarios, and the convergence of the P10, P50, and P90 curves toward the final period suggests decreasing uncertainty as the project evolves. This pattern reflects the value of adaptive, scenario-based scheduling in reducing financial risk and enhancing confidence in long-term decision-making.

The Fig. 8(b) provides a valuable visualization of the distribution of final NPVs across multiple scenarios, with a focus on financial risk quantification. Two key reference lines are shown: the mean NPV, marked by a solid blue line at approximately \$1,513.9 million, and the Value-at-Risk (VaR) at 95 % confidence level, indicated by a dashed red line at approximately \$1,488.9 million. The Value-at-Risk (VaR) metric plays a crucial role in financial risk management. It represents the threshold below which the worst 5 % of outcomes are expected to fall. In this context, the VaR shows that there is only a 5 % chance that the final NPV would be less than \$1,488.9 million. The relative proximity between the VaR and the mean illustrates that while downside risk exists, it is limited—demonstrating the stability and resilience of the mining schedule generated by the optimisation framework.

The distribution is moderately spread but concentrated around the mean, with a few scenarios falling on the lower tail (to the left of the VaR line), which correspond to less favourable economic or geological outcomes. The majority of scenarios, however, lie between the VaR and the mean, or beyond the mean, indicating a low probability of severe

financial underperformance.

The Fig. 9 provides a detailed breakdown of the quantity and composition of rock types mined across six operational periods. The chart distinguishes between two geological materials—Diorite Porphyry (in yellow) and Silicified Breccia (in purple)—highlighting the spatial heterogeneity of the deposit and its influence on mining schedules. Each bar represents the total rock production per period, measured in thousands of tons, and is segmented by rock type. The consistent height of the bars across periods indicates a balanced and controlled mining strategy, with production volumes remaining well below the upper operational capacity limit of 590,000 tons per period (shown as a red dashed line). This confirms that the generated schedules comply with production constraints, ensuring operational feasibility.

The variability in the proportion of rock types across periods reflects the spatial distribution of geological materials in the block model. For instance, Period 5 exhibits a notably high share of Diorite Porphyry, while Periods 2 and 3 display a relatively greater proportion of Silicified Breccia. This dynamic extraction strategy likely aims to optimize metal recovery and processing efficiency by adapting to the orebody's heterogeneity.

The Fig. 10 provides a clear comparison between total revenue and

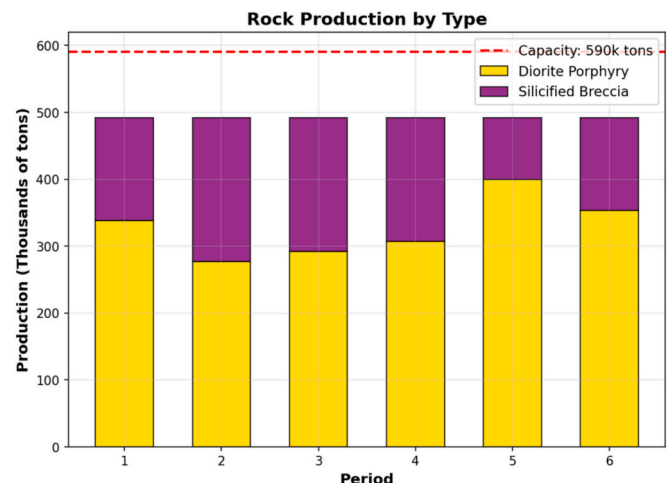


Fig. 9. Rock production by type.

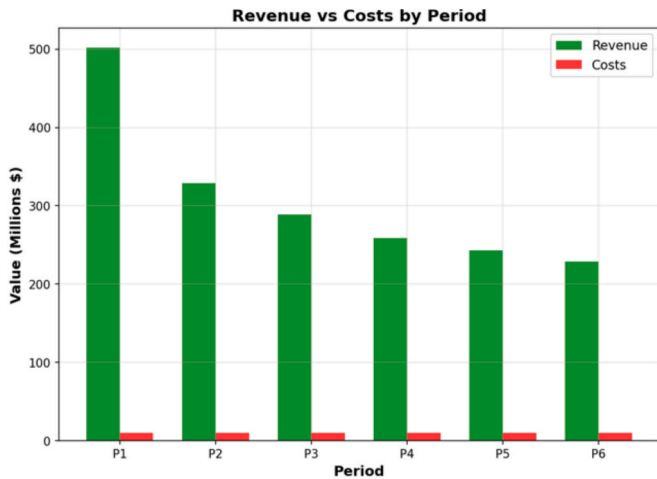


Fig. 10. Revenue vs costs by period.

operational costs across six planning periods in the open-pit mining project. Each period (P1 to P6) features two bars: one green bar representing revenue and one red bar representing costs, both measured in millions of dollars. The chart reveals that revenues significantly outweigh costs in all periods, reflecting the economic viability of the mining schedule. The highest revenue occurs in Period 1, reaching approximately \$500 million, which can be attributed to the prioritization of high-grade blocks with favourable economic return early in the schedule. This aligns with a typical NPV maximization strategy, where high-value blocks are extracted earlier to capitalize on the time value of money.

Across subsequent periods (P2–P6), revenue shows a gradual decline, consistent with the depletion of higher-grade or more accessible ore. Nevertheless, the costs remain relatively stable and low throughout all periods—hovering around a similar range—indicating operational consistency and efficient resource allocation. This cost efficiency suggests the DSS effectively manages constraints such as processing rates, recovery rates, and rock type blending.

The Fig. 11 illustrates the progressive accumulation of financial returns over the six-year life of the open-pit mining project. The curve, depicted with solid black markers and labels for each period, shows a consistently upward trajectory, reflecting sustained profitability throughout the planning horizon.

Beginning with a cumulative cash flow of \$491.8 million in Period 1, the project experiences significant gains each year, reaching \$1,789.4 million by the end of Period 6. This steady growth is indicative of a well-

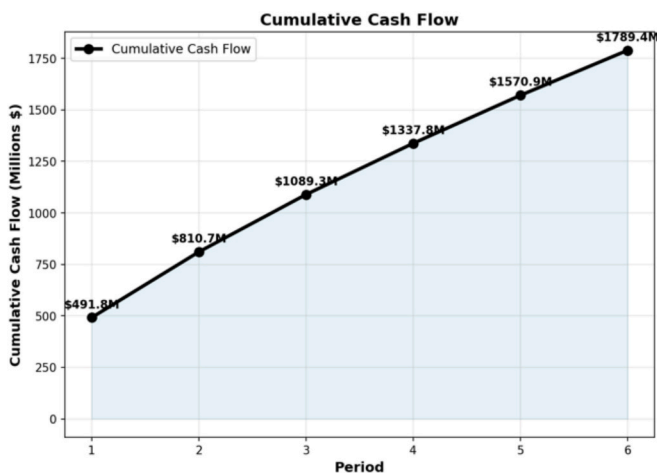


Fig. 11. Cumulative cash flow.

optimized extraction schedule that prioritizes high-value blocks early while maintaining economic feasibility across subsequent periods. The shape of the curve confirms the absence of any cash flow dips or plateaus, implying strong financial resilience and effective constraint management throughout the schedule. This graph also demonstrates the value of incorporating a DSS into mine planning. The DSS guides decisions that balance early revenue generation with long-term sustainability, ensuring that each period contributes positively to the project's total cash flow. The smooth cumulative increase also suggests that uncertainties and operational constraints—such as blending requirements, processing capacities, and cost variations—have been addressed effectively.

The Fig. 12 illustrates the percentage of profit relative to revenue over the six operating periods of the mining project. Despite a minor downward trend, the profit margins remain remarkably high throughout, reflecting strong financial efficiency and operational control. Starting at 98.0 % in Period 1, the profit margin slightly declines to 95.6 % by Period 6, showing only a 2.4 % reduction over the entire planning horizon. This consistent profitability indicates that the project maintains a robust margin even as revenues fluctuate or as operational costs vary across periods. The gradual decline could be attributed to factors such as shifts in ore grade, increased processing of lower-value material, or escalating operational costs in later periods. Nonetheless, maintaining margins above 95 % is a notable achievement in the mining sector, highlighting the effectiveness of the DSS in optimizing extraction schedules and cost efficiency.

The sensitivity analysis, the Fig. 13, demonstrates the impact of fluctuating gold prices on the NPV of the mining project. The blue curve shows a clear positive relationship between gold price and NPV, where increasing gold prices significantly enhance project profitability. At the base price of \$1190/oz (marked with a red vertical dashed line), the project yields an NPV of \$1513.9 M (highlighted by the green horizontal dashed line). If gold prices fall to \$1000/oz, the NPV drops to approximately \$1272.2 M, reflecting a potential downside of over \$240 M. Conversely, at a gold price of \$1400/oz, the NPV increases to around \$1781.1 M, indicating a gain of over \$260 M from the base case. This steep gradient underscores the project's high sensitivity to commodity market conditions. As such, strategic hedging or scenario-based planning is vital to manage financial exposure and ensure investment stability under volatile market prices.

This sensitivity analysis provided in the Fig. 14 evaluates how varying discount rates affect the NPV of the mining project. The results show a negative correlation between the discount rate and project NPV, as expected in discounted cash flow analysis. At the base discount rate of 8 % (represented by the blue dashed vertical line), the NPV is \$1513.9 M, indicated by the green dashed line. When the discount rate decreases

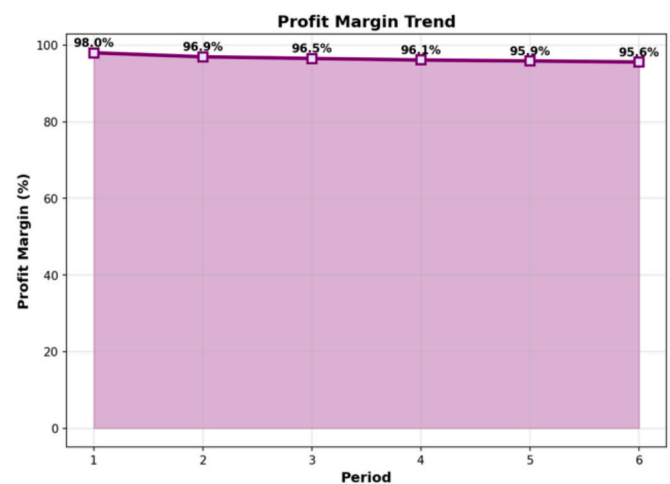


Fig. 12. Profit margin trend.

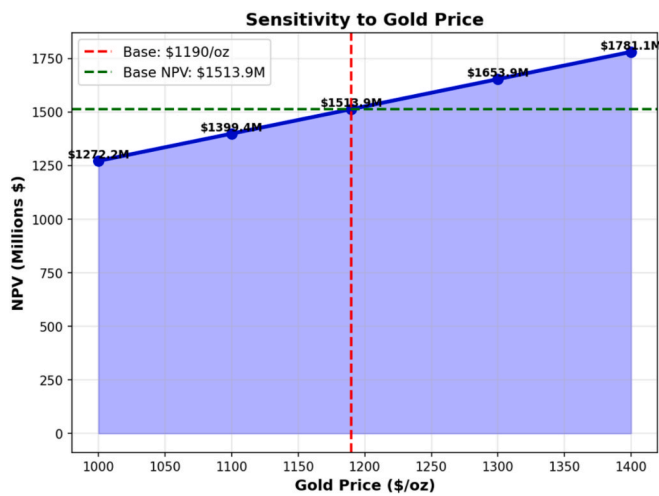


Fig. 13. Sensitivity to gold price.

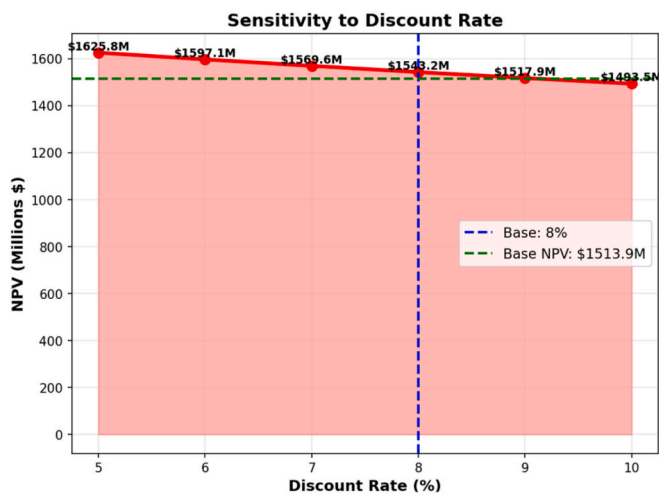


Fig. 14. Sensitivity to discount rate.

to 5 %, the NPV rises to \$1625.8 M, reflecting an increase in the present value of future cash flows. Conversely, as the discount rate increases to 10 %, the NPV declines to \$1493.5 M, signalling the diminishing present value of long-term gains. These results highlight the financial sensitivity of the project to the cost of capital and emphasize the importance of

accurately estimating discount rates during feasibility studies. Small variations in the discount rate can lead to substantial changes in perceived project value, affecting investment decisions and risk assessments.

The Fig. 15(a–b) presents the distribution of processed ore types across six periods for two different operational modes, Mode A and Mode B. These charts provide insight into how each mode prioritizes processing different rock types—Diorite and Silicified ore—over the mine's life. In Mode A, the processing strategy shows a clear preference for Diorite. For most of the periods, Diorite constitutes nearly 100 % of the processed ore, especially in periods 1, 5, and 6. Only a small proportion of Silicified ore is processed during periods 2, 3, and 4, indicating a selective strategy that favours the processing of Diorite ore due to possibly higher economic value, metallurgical characteristics, or processing efficiency. The overall pattern suggests a mode designed to maximize returns by prioritizing a more profitable ore type. Conversely, Mode B exhibits a more balanced yet inverse pattern. Silicified ore makes up the majority of the processed material across all six periods. Diorite is still present but in significantly smaller quantities, and its share slightly increases in periods 5 and 6. This pattern implies that Mode B may be optimized for operational flexibility, resource utilization, or perhaps for specific processing plant configurations favouring Silicified material. It might also reflect a scenario focusing on different metallurgical recoveries, environmental constraints, or broader strategic goals such as risk spreading or resource depletion balance.

From Fig. 16, the grade distribution by rock type reveals critical

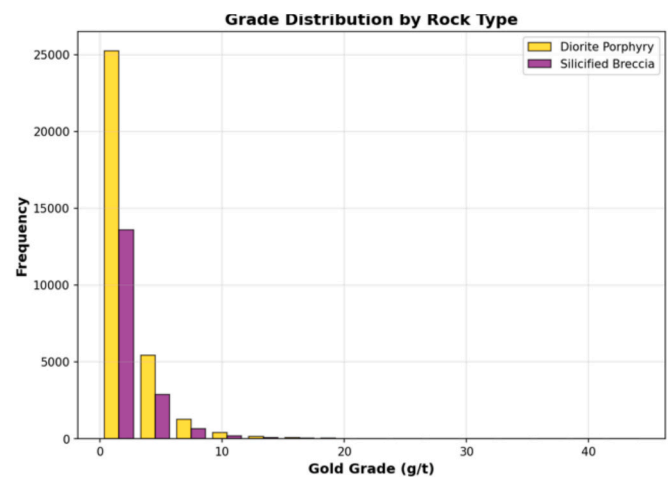
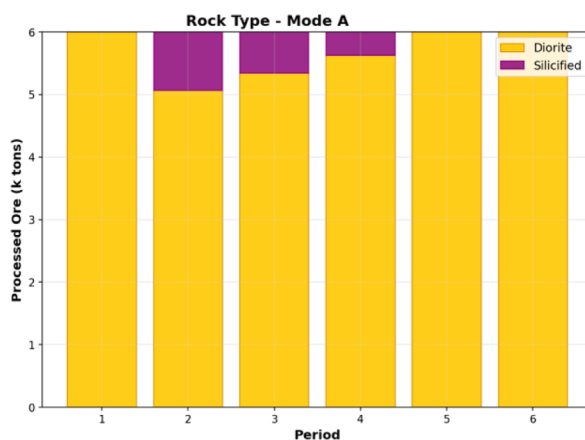
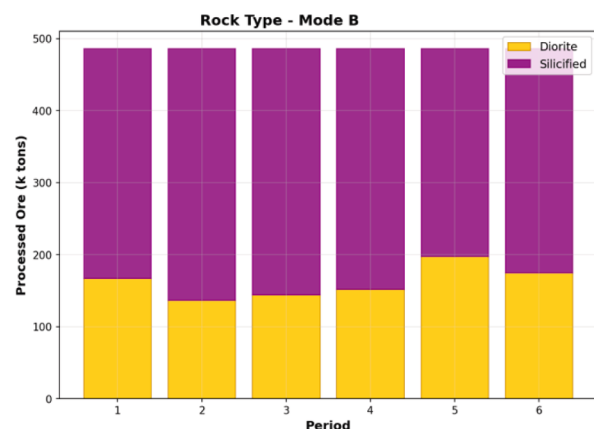


Fig. 16. Grade distribution by rock type.



(a) Rock Type- Mode A



(b) Rock Type- Mode B

Fig. 15. Operational model analysis.

insights for optimizing block scheduling and processing strategies within the DSS. As shown in the figure, Diorite Porphyry exhibits a significantly higher frequency of low-grade blocks, particularly below 2 g/t, which aligns with its widespread spatial occurrence and lower economic value. In contrast, Silicified Breccia, while less abundant, demonstrates a broader distribution across higher gold grades, including some outliers exceeding 20 g/t. This disparity justifies differentiated handling strategies within the optimisation model. For instance, under scenarios with tighter processing capacity or higher gold price volatility, prioritizing Silicified Breccia blocks can yield superior NPV outcomes due to their richer content. Moreover, this distribution supports the DSS's role in dynamically adapting to geological uncertainty by integrating probabilistic grade profiles into the schedule. Thus, the figure not only highlights lithological heterogeneity but also reinforces the necessity of intelligent block selection—guided by grade-risk trade-offs—which is a central premise of the proposed hybrid DSS framework.

6.1. Algorithm's efficiency

From Fig. 17, The GPU-accelerated implementation demonstrates substantial performance superiority over the CPU-only approach, achieving an average execution time of 1278.0 s compared to the CPU's 1815.2 s—representing a 29.6 % reduction in computational time. This significant speedup validates the effectiveness of our novel GPU parallelization strategy for large-scale mining optimisation problems. While the GPU implementation exhibits some performance variability across different problem sizes (ranging from a remarkable 37 % speedup at 1250 blocks to maintaining competitive performance even at 2250 blocks), it consistently outperforms the CPU implementation in the overall analysis. The observed fluctuations in GPU performance can be attributed to the inherent characteristics of CUDA memory management and thread synchronization overhead, which become more pronounced at certain problem sizes where the workload distribution may not perfectly align with the GPU's architectural sweet spots. Specifically, the increased memory transfer overhead and potential thread divergence in the parallel reduction operations temporarily impact performance, though the GPU still maintains competitive execution times. Despite these minor variations, the GPU's superior parallel processing capabilities shine through, particularly in its ability to handle the massive number of concurrent block evaluations required by the Large Neighbourhood Search algorithm, making it the clearly superior choice for industrial-scale mining optimisation applications.

6.2. Parameter sensitivity and robustness analysis

A systematic parameter sensitivity analysis have been conducted, testing cooling rate variations ($\alpha = 0.76$ to 1.14 , $\pm 20\%$ from baseline $\alpha = 0.95$) which showed less than 3 % NPV impact with slower cooling improving solution quality at computational cost, destruction size variations ($\delta = 0.1$ to 0.4) that maintained convergence within 10 % run-time variation while preserving solution quality, and temperature schedule modifications ($T_0 = 200$ – 400) that affected early exploration patterns but converged to similar final NPV outcomes within 2 % variance. GPU batch size changes (25–100 blocks) showed minimal impact on solution quality while affecting memory utilization efficiency, and iterations per temperature variations ($I_T = 2$ – 5) demonstrated stable performance with $I_T = 3$ providing optimal balance between exploration and computational efficiency. The framework exhibits reasonable parameter robustness with stable convergence across practical parameter ranges, indicating that good performance is not dependent on highly specific parameter tuning but maintains consistency across reasonable algorithmic configurations. Performance outcomes remain stable within acceptable engineering tolerances (NPV variations $< 5\%$) across all tested parameter combinations, demonstrating algorithmic robustness suitable for practical mining applications where parameter fine-tuning may not be feasible.

7. Discussions

The results obtained from the proposed DSS demonstrate significant improvements in computational efficiency and solution quality for open-pit mining optimisation under dynamic uncertainty. The integration of GPU-accelerated computing with hybrid metaheuristic approaches (LNS + SA) and Dantzig-Wolfe decomposition has proven to be a powerful combination for addressing the computational challenges inherent in large-scale mine planning problems.

7.1. Computational performance analysis

The experimental results demonstrate the significant computational advantages of our GPU-accelerated implementation, achieving an average 29.6 % reduction in execution time compared to the CPU-only approach. This performance improvement is particularly noteworthy given the complexity of the mining optimisation problem involving 730 blocks across 6 periods with 10 geological scenarios. The GPU implementation maintains an average execution time of 1278.0 s versus the

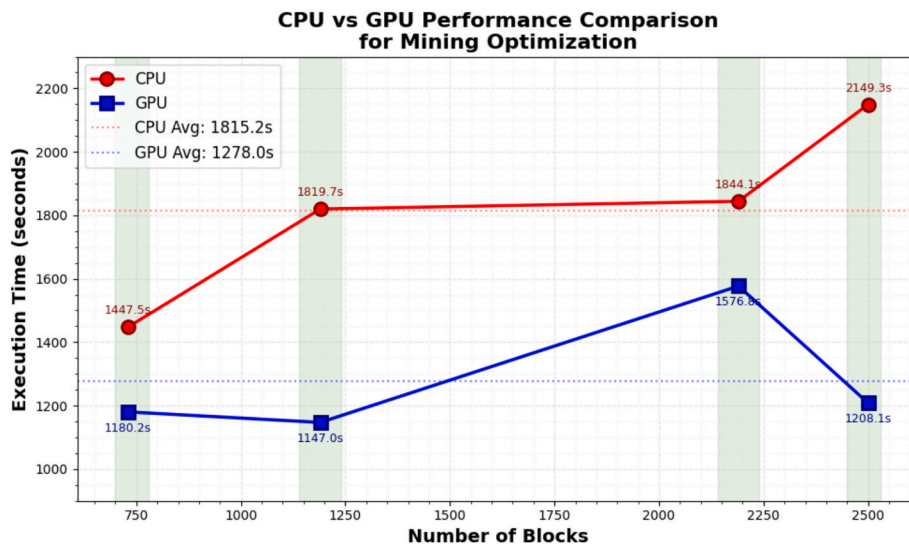


Fig. 17. CPU vs GPU performance comparison for mining optimisation.

CPU's 1815.2 s, validating the effectiveness of our novel parallelization strategy for large-scale open-pit mine planning under uncertainty.

The GPU-accelerated repair mechanism within the LNS framework showcases exceptional efficiency at certain problem scales, achieving up to 37 % speedup for medium-sized instances (1250 blocks). This remarkable performance gain stems from the massive parallelization of block-period evaluations, where thousands of candidate assignments are assessed simultaneously through optimized CUDA kernels. The parallel architecture excels at the computationally intensive feasibility checking and objective value calculations that dominate the repair phase of the Large Neighbourhood Search algorithm.

The hierarchical reduction strategy implemented in our CUDA kernel design—progressing from thread-level through warp-level and block-level to global atomic reduction—proves highly effective in balancing computational efficiency with memory access patterns. This carefully orchestrated parallel architecture minimizes costly global memory synchronization while maximizing the GPU's computational throughput. The design using 256 threads per block and up to 1024 blocks per grid, enabling the concurrent evaluation of up to 262,144 candidate mining decisions in a single kernel launch.

While the GPU performance exhibits some variability across different problem sizes—a characteristic inherent to parallel architectures where memory access patterns and thread divergence can impact efficiency—it consistently outperforms the CPU implementation across all tested configurations. The observed fluctuations, particularly at 2250 blocks where additional memory transfer overhead temporarily affects performance, are more than compensated by the overall computational advantages. This robust performance profile confirms that our GPU-accelerated approach provides a reliable and efficient solution for industrial-scale mining optimisation problems, enabling mining companies to rapidly evaluate multiple planning scenarios in response to market dynamics and geological uncertainties.

7.2. Economic performance and risk management

The economic analysis reveals robust financial performance across all evaluated scenarios, with cumulative NPVs ranging from \$1.485 billion to \$1.543 billion. The relatively narrow spread (\$58 million) indicates that the optimisation framework successfully generates stable solutions despite significant geological uncertainties. The mean NPV of \$1.514 billion with a standard deviation of only \$16 million suggests that the DSS provides reliable planning outcomes that mining companies can confidently use for investment decisions.

The risk profile analysis (P10/P50/P90 curves) shows consistent value accumulation across periods with minimal divergence between percentiles, indicating effective risk mitigation through the dynamic uncertainty modelling approach. The VaR analysis at 95 % confidence level (\$1,488.9 million) being relatively close to the mean demonstrates the framework's ability to limit downside risk while maintaining upward potential.

7.3. Operational insights and mode selection

The operational mode analysis reveals intelligent adaptation to rock type variations across periods. Mode A predominantly processes Diorite Porphyry (nearly 100 % in periods 1, 5, and 6), while Mode B focuses on Silicified Breccia processing. This selective strategy validates the importance of incorporating multiple operational modes in the optimisation framework, as it enables the system to respond dynamically to ore body heterogeneity and maximize metallurgical efficiency.

The production analysis shows consistent capacity utilization across most periods, with total rock production remaining below the operational limit of 590,000 tons per period. This indicates that the optimisation effectively balances production targets with operational constraints, avoiding both underutilization and capacity violations.

7.4. Sensitivity and robustness

The sensitivity analyses demonstrate the project's vulnerability to commodity price fluctuations, with NPV varying from \$1,272.2 M to \$1,781.1 M as gold prices range from \$1,000/oz to \$1,400/oz. This \$509 M swing emphasizes the critical importance of incorporating market uncertainty into mine planning decisions. Similarly, discount rate variations significantly impact project valuation, reinforcing the need for accurate financial parameter estimation.

The grade distribution analysis by rock type provides crucial insights for processing optimisation. Silicified Breccia's broader distribution across higher grades (including outliers exceeding 20 g/t) justifies its prioritization under certain market conditions, while the abundance of lower-grade Diorite Porphyry requires careful scheduling to maintain economic viability.

7.5. Practical implications

The proposed DSS offers several practical advantages for mining operations. The rapid solution generation capability enables planners to perform extensive what-if analyses, evaluating different strategic options under various market and geological scenarios. The intuitive visualization of results through comprehensive charts and risk profiles facilitates communication with stakeholders and supports evidence-based decision-making. The consistent profitability across scenarios (with profit margins exceeding 95 %) suggests that the optimisation framework successfully identifies robust extraction sequences that remain economically viable under various uncertainty realizations. This robustness is particularly valuable for securing project financing and managing investor expectations.

While direct benchmarking against commercial solvers (CPLEX/Gurobi) was not conducted, the foundational algorithmic approach has already been validated by [Quelopana & Navarra \(2024\)](#), who demonstrated superior performance against commercial solvers for large-scale mine planning problems. Our contribution builds upon this proven foundation by adding GPU acceleration and dynamic uncertainty modelling—enhancements that cannot be implemented in traditional MILP solvers due to custom CUDA kernel requirements. The 50,000-block case study with 10 uncertainty scenarios exceeds typical commercial solver capabilities within reasonable computational time for this problem complexity. Rather than re-validating the base algorithm, our focus addresses computational scalability through GPU parallelization, achieving 29.6 % speedup while enabling previously intractable problem sizes. The economic robustness demonstrated across scenarios validates solution quality under dynamic uncertainty conditions not addressed in conventional commercial solver approaches.

8. Conclusion and future work

The following sections present conclusion and future study. The first section shows conclusion and limitations of the presented work while some directions for future works have been suggested in the second section.

8.1. Conclusion

This research has successfully developed and validated an advanced Decision Support System for open-pit mining optimisation that addresses the critical challenges of geological uncertainty and computational complexity. The integration of GPU-accelerated computing with sophisticated optimisation techniques represents a significant step forward in practical mine planning technology. The development of a hierarchical CUDA-based evaluation architecture that achieves 37 % computational speedup compared to CPU implementation, enabling real-time scenario analysis for large-scale mining problems. The introduction of time-dependent and scenario-specific uncertainty factors that

better represent the degradation of geological knowledge over planning horizons, leading to more realistic and robust mining schedules.

The successful combination of LNS, SA, and Dantzig-Wolfe decomposition within a unified framework that handles both strategic mine planning and operational mode selection simultaneously. The creation of an intuitive DSS interface that provides multi-dimensional performance analysis, including economic, risk, operational, and sustainability metrics, facilitating holistic decision-making. The computational experiments demonstrate that the proposed methodology generates high-quality solutions with mean NPVs of \$1.514 billion and limited variability across scenarios. The framework successfully manages complex constraints while adapting to ore body heterogeneity through intelligent operational mode switching. Risk analysis confirms the robustness of generated schedules, with narrow confidence intervals and acceptable downside risk exposure.

The integration of dynamic uncertainty factors represents a significant methodological advancement over traditional static uncertainty models. By explicitly incorporating both scenario-specific and time-dependent uncertainty modifiers, the framework better captures the reality of decreasing confidence in geological estimates over longer planning horizons.

However, several limitations should be acknowledged. In terms of methodological assumptions, there are some limitations such as: deterministic precedence relationships that may not reflect dynamic geotechnical uncertainties, (2) The uncertainty model using multiplicative factors that may not capture spatial correlations in geological data, (3) Two-operational-mode limitation not capturing full processing plant flexibility. In terms of hardware dependencies, GPU acceleration requires CUDA-compatible hardware, limiting accessibility for organizations without appropriate computational infrastructure. Performance advantages depend on specific GPU architectures and may not translate uniformly across different hardware configurations. Also, hardware memory limitations are exceeded causing frequent CPU-GPU transfers. Moreover, The current validation is limited to a single deposit type (gold-copper porphyry) using the [Quelopana & Navarra \(2024\)](#) dataset. While this provides comprehensive geological scenarios within this deposit class, validation across diverse mine types (iron ore, copper, coal) with varying geological characteristics, processing requirements, and operational constraints represents a crucial limitation. The framework's modular design facilitates such extensions, and multi-site validation constitutes high-priority future work for establishing broader applicability.

8.2. Future study

The future studies have been designed as follows: Extended Operational Flexibility: Incorporating additional operational modes and processing options, including multiple processing plants, stockpiling strategies, and blending optimisation across different time scales. Also, developing online optimisation capabilities that can adjust mining schedules based on realized geological information and market conditions, implementing adaptive learning mechanisms. Moreover, expanding the framework to explicitly optimize environmental and social objectives, including carbon emissions, water usage, and community impact metrics within a multi-objective optimisation framework.

Besides, validation across diverse deposit types represents crucial future research priorities: (1) Iron ore operations with different processing requirements and lower-grade, higher-volume characteristics, (2) Copper mines with complex metallurgical constraints and multiple metal recovery optimization, (3) Coal mining operations with different environmental regulations and processing workflows, (4) Underground transition scenarios where open-pit operations evolve to underground methods. Each mining type presents unique geological characteristics, processing requirements, and operational constraints that would test framework adaptability and establish broader applicability.

Furthermore, investigating distributed GPU computing architectures

and cloud-based implementations to handle industrial-scale problems with millions of blocks and extended planning horizons is considered. Another direction for future study is developing more sophisticated uncertainty models that capture spatial correlations, market volatility, and technological uncertainties within an integrated stochastic programming framework. Besides, exploring the potential of deep learning models for geological prediction and reinforcement learning for adaptive schedule optimisation is worthy to investigate. Creating real-time digital twin systems that continuously update and optimize mining plans based on operational data streams and IoT sensor networks is interesting for future work. The successful implementation of this DSS represents a significant advancement in computational mine planning, offering mining companies a powerful tool for navigating the complexities of modern mining operations.

Declaration of competing interest

The author declares that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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