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Review article

Advances in impact force identification: A comprehensive review of techniques and mathematical innovations

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ABSTRACT

This review provides a comprehensive and critical synthesis of state-of-the-art methodologies for impact force identification, a pivotal inverse problem in aerospace, automotive, civil infrastructure, and robotics systems. A systematic taxonomy is established to evaluate impact force reconstruction techniques, including deconvolution, subspace state-space formulations, and data-driven models, as well as localization strategies, such as triangulation, similarity-based matching, and optimization-based algorithms. The comparative analysis underscores the trade-offs between model-based approaches, which offer high computational efficiency in linear regimes, and machine learning methods, which demonstrate robustness in capturing nonlinear and high-dimensional system behaviors. The paper delves into recent mathematical advancements aimed at mitigating the inherent ill-posedness of inverse problems, emphasizing the roles of advanced regularization schemes, compressed sensing, and sparsity-promoting techniques. Notable emerging directions include hybrid physics-informed machine learning frameworks, domain adaptation and transfer learning to alleviate data dependency, and incremental learning paradigms suited for real-time deployment. Unresolved challenges are also identified, particularly in scenarios involving multiple concurrent impacts, sparse sensor networks, and online operation under dynamic environmental conditions. The review concludes by outlining future research trajectories to advance the accuracy, generalizability, and real-time feasibility of impact force identification methods.

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1. Introduction

Many engineering structures are frequently subjected to impact events throughout their service life, which can lead to progressive damage and, ultimately, structural failure. For instance, the compressive strength of composite laminates may decrease by 65 to 80 percent under impact forces, even when no visible external damage is observed [1]. Aircraft are particularly susceptible to various impact events, including damage caused by tire debris, hail, or bird strikes. Notably, bird strikes account for approximately 90 percent of reported incidents and can cause significant damage to forward-facing components of an aircraft [2], as shown in Fig. 1. Therefore, accurate identification of the impact location and magnitude is critical for effective structural health monitoring of aircraft.

Measurement of impact loads directly is often impractical or infeasible for several reasons: (i) installing sensors throughout a structure can be challenging and may alter the dynamic characteristics of the structure, (ii) capturing large impact forces over a short duration is difficult and may damage sensors over time, and (iii) positioning sensors in all desired locations, such as inaccessible areas between substructures in a large system to measure interaction forces, presents significant challenges [4,5]. As an alternative to direct measurement, inverse algorithms can be utilized to estimate impact forces based on available system dynamic responses. Typically, dynamic problems associated with impacts are classified into the following categories: (i) the forward problem, and (ii) the inverse problem, shown schematically in Fig. 2. In forward problems, the responses of a given system are found on the basis of given inputs. In inverse problems, system inputs are estimated from observed system dynamics and collected vibration responses. Additionally, determining the system model itself from known inputs and responses constitutes another type of inverse problem.

There are a noticeable number of studies in the literature that employ inverse algorithms for the reconstruction of impact forces, which can be categorized into two main classes: (i) model-based techniques [6–22], and (ii) machine/deep learning methods [23–29]. Most model-based approaches are only applicable to linear problems [30]. Generally speaking, machine learning methods are superior when (i) the underlying dynamics are inaccessible or too complex, and (ii) the identification problem is nonlinear [25,31]. On the other hand, machine and deep learning techniques have a significant drawback: their accuracy heavily depends on large amounts of training data. Consequently, the main advantages of model-based approaches and machine learning methods for solving inverse problems are, respectively, the reduced data and computational requirements for model-based methods, and the applicability of machine learning techniques to complex or inaccessible dynamics, see Fig. 3.

Structural integrity inspection can be performed quickly with non-destructive techniques [32] when the location of the impact is known beforehand [33]. When the impact location is not known, it can be potentially identified inversely based on available measurements and by employing impact localization techniques. There are different approaches for impact localization which are generally based on either (i) structural vibration, (ii) time differences and geometry, or (iii) machine and deep learning approaches. The structural vibration approach relies on the fact that the mode shape transformations are extremely sensitive to damage and hence can be employed to detect the impact force [34–36]. These methods require baseline health data, which is a significant limitation for use outside controlled settings. To overcome this issue, Gapped smoothing techniques in order to simulate the healthy structure [37,38] and the Wavelet Transform technique [39,40] have been proposed [41]. Moreover, this strategy is time-consuming for systems with finite degrees-of-freedom (DoF) and infeasible for distributed structures [16]. The time and geometry based approaches, that are based on positioning of impact signals and their time differences, are only applicable to isotropic materials as they assume that the wave propagation speed through the structure is uniform [42–45]. To remedy this issue, several studies have been conducted [46–53]. For anisotropic wave propagation, the localization problem can be defined as a nonlinear problem with geometrical constrains which can subsequently be solved by utilizing optimization algorithms [54,55]. The machine and deep learning approaches [24,56,57] rely on fitting or interpolating a known dataset with the impact data. The main limitation of these

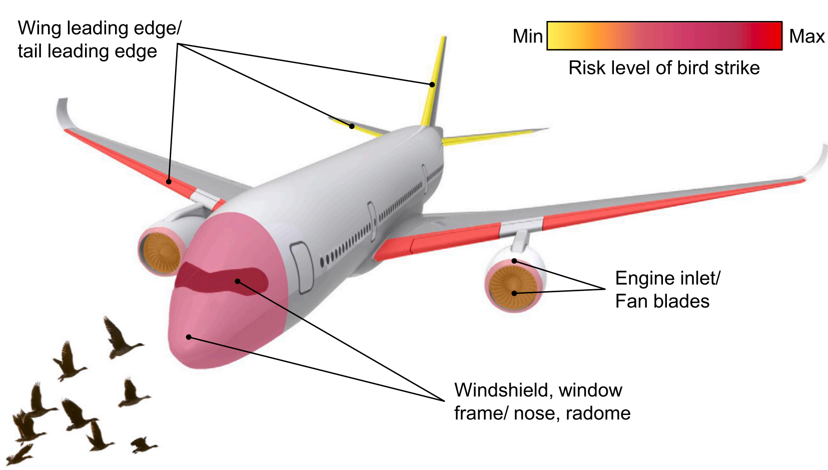

Fig. 1. Aircraft components most susceptible to bird strikes, highlighting areas prone to significant damage, such as the windshield, engines, and leading edges of wings [3].

Table 1
Illustrative summary of impact localization methods.

Impact localization approaches	Example	Shortcoming	Solution
Structural vibrations	(i) Time-reversal method, (ii) Normalized cross-correlation technique, (iii) Cosine similarity technique	(i) Need benchmark data of a healthy structure, (ii) Time consuming for finite DOF systems, (iii) Infeasible for continuous structures	Simulate the healthy structure using, e.g., (i) Gapped smoothing technique, (ii) Wavelet transform technique
Time difference and geometry	Triangulation method	Not applicable to anisotropic materials, and complicated structures	Express as a nonlinear problem with geometrical constraints, then solving it by, e.g., GA or PSO
Machine/deep learning	(i) Reference database, (ii) Neural Networks	Need large training dataset	Artificially create dataset

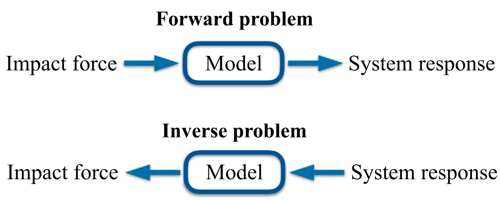

Fig. 2. Forward vs. inverse force problems.

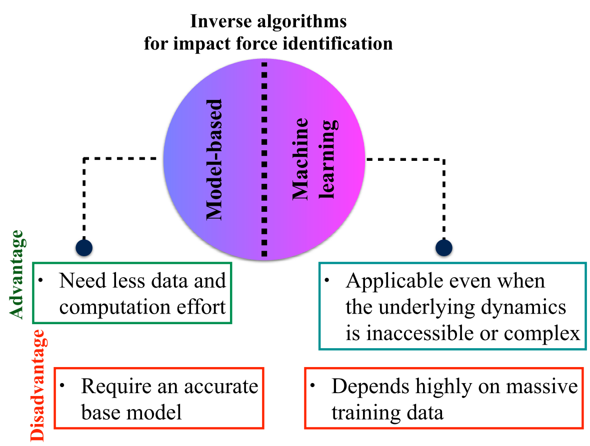

Fig. 3. Classification and comparison of different inverse algorithms for impact force reconstruction.

methods is their need for huge training data [44]. See Table 1 for an illustrative concise summary of impact localization methods.

An algebraic problem is considered well-posed if it satisfies the following conditions: (i) the solution is unique, (ii) a globally-defined solution exists for all reasonable data, and (iii) the solution continuously depends on the given data [58]. The force identification problem is typically ill-posed for two main reasons: (i) a high condition number of the transfer functions, making the problem sensitive to small perturbations such as measurement noise, and (ii) non-collocated excitation and observation points, leading to non-unique solutions due to time-delays and disturbances between input signal and measurement. The general approach to address this issue is to transform it into a well-posed problem by incorporating additional information about the desired solution [58,59]. Various approaches, typically referred to as filtering and regularization methods in this context, have been employed to address the ill-posedness of inverse reconstruction and localization problems.

This review paper investigates various methods for impact force reconstruction and localization, with an effort to classify the available approaches. It also introduces mathematical techniques to address the challenge of ill-posedness. The main contributions are twofold: first, it offers a comprehensive literature review of impact force identification; second, it provides a classification and comparison of the tools and techniques utilized.

The paper is organized as follows. Section 2 presents model-based impact force reconstruction methods. Next, the model-based impact localization techniques are presented in Section 3. A review of the application of machine learning and deep learning approaches in both the reconstruction and localization of the impact force is presented in Section 4. Section 5 introduces some relevant mathematical tools and techniques to deal with ill-posedness. Finally, concluding remarks are provided in Section 7.

2. Model-based impact force reconstruction

Model-based techniques utilize the collected vibration response along with an a priori structural model to reconstruct the unknown impact force. These approaches involve a model that describes the dynamic behavior of the structure under impact, which can be developed using various methods. [60]: (i) analytical solutions [61], (ii) numerical methods [8,62], or (iii) experiments (e.g., the transfer functions) [63,64].

Two main approaches have been proposed for model-based impact force reconstruction [65]: (i) the deconvolution method, and (ii) state estimation. In the deconvolution approach, the relationship between unknown forces and measured responses is first established, after which the reconstruction is formulated as a least squares optimization problem [66]. However, such inverse problems are typically ill-posed [6] making the least squares solution highly sensitive to measurement noise. To address this, several mathematical techniques are employed, which will be discussed in Section 5. Additionally, the deconvolution technique is not computationally efficient for large-scale structures or extended time periods. Two main approaches to remedy this issue are as follows: (i) employing dictionaries [7,14,65,67] in order to reduce the number of variables which, in turn, leads to the reduction of computation cost, and (ii) employing iterative regularization methods, such as Landweber method [68,69], Kaczmarz method [69–71], and Krylov Subspace methods [72], which avoid extensive regularization parameter selection. On the other hand, in the state estimation approach, impact forces are formulated as state variables. The superiority of this approach is that it can identify the external impact forces, the system state variables, and even the parameters of the system, simultaneously. Note that the state estimation approach is not applicable in the event of sudden changes as it assumes that the forces are time-invariant with stochastic perturbation noises [65]. Though, it is still more robust and computationally more efficient than the deconvolution approach. The above discussion is illustratively summarized in Fig. 4. In the following, several model-based methods to reconstruct impact forces are presented.

2.1. Deconvolution technique

The deconvolution technique is the most frequently employed strategy for impact force identification [73,74]. This method is known as a straightforward method to reconstruct impact forces [75]. However, it has two main limitations, namely, (i) it is only applicable to linear problems, i.e., it is limited to small deflections, and (ii) it is ill-posed, i.e., is sensitive to measurement noise [6]. To deal with the first limitation, a

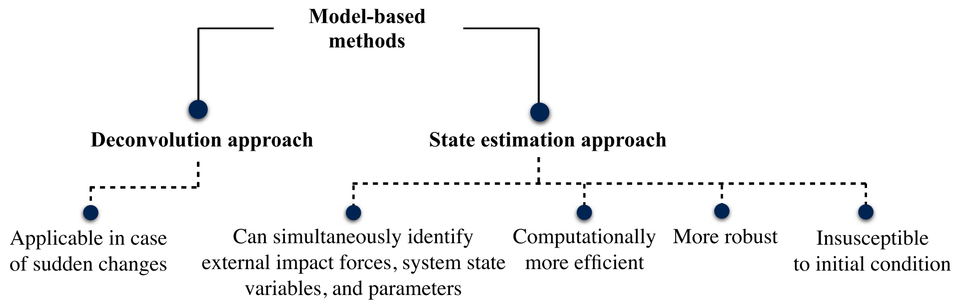

Fig. 4. The pros of each model-based approach for impact force reconstruction.

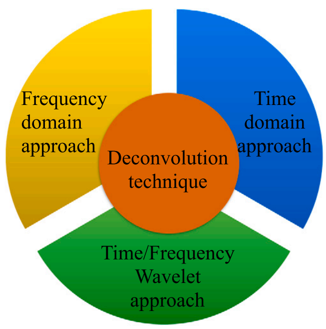

Fig. 5. Classification of approaches for the deconvolution method used for impact force identification.

generalized form of the convolution integral has been proposed in the literature [25], which expresses the nonlinear system as a series of infinite sums of convolutional integrals, named Volterra series [76,77]. To solve the second issue (ill-posedness), several mathematical techniques such as regularization methods are proposed in the literature, which will be reviewed in Section 5.

Three main approaches of the deconvolution method are (i) the time domain approach [15,16,60,78], (ii) the frequency domain approach [79,80], and (iii) the time/frequency domain wavelet technique [81–84], see Fig. 5. In general, frequency domain strategies require lower computational effort, nevertheless, (i) they are not applicable when time series data is accessible over a relatively short duration [85] and hence they are commonly infeasible for transient phenomena, and (ii) for some applications such as robotic surgery, a real time identification of the force is preferred. In particular, when using frequency domain methods, inverse Fourier Transform is required to obtain the forces in time domain. Those methods are more suitable for stable and stationary random forces.

2.1.1. Time domain deconvolution

Time domain deconvolution has been widely used in the literature for identifying impact forces [6,16,18,86–90]. The overall idea behind this method is as follows.

Consider an impact applied to a structure at a known location, with n sensors mounted on the structure to measure system responses, as illustrated in Fig. 6. The convolution integral, describing the relation between the response r and the impact force f, is as follows [18]:

$$r(y,t) = \int_0^t T_s(x,y,t-\zeta) f(x,\zeta) d\zeta, \tag{1}$$

where $T_s(x,y,t-\zeta)$, $s = 1,\ldots n$, is the transfer function between the impact force, applied at point x, and the sth sensor, located at point y, at time $t=\zeta$. Note that Eq. (1) does not account for the influence of the system's initial conditions. This is based on the assumption that the system remains at rest prior to the application of the impact force. Equation (1) can be discretized as follows:

$$\mathbf{r} = \mathbf{T}_s \mathbf{f}, \tag{2}$$

where $\mathbf{r} \in R^m$ is the measured response vector with m the number of samples, $\mathbf{f} \in R^m$ is the impact force vector (to be reconstructed), and $\mathbf{T}_s \in R^{m\times m}$ is the impulse response matrix, given by:

$$\mathbf{r} = \begin{bmatrix} r(\Delta t) \\ r(2\Delta t) \\ \vdots \\ r((m-1)\Delta t) \\ r(m\Delta t) \end{bmatrix}, \mathbf{f} = \begin{bmatrix} f(\Delta t) \\ f(2\Delta t) \\ \vdots \\ f((m-1)\Delta t) \\ f(m\Delta t) \end{bmatrix},$$
$$\mathbf{T}_s = \begin{bmatrix} T_s(\Delta t) & 0 & \ldots & 0 \\ T_s(2\Delta t) & T_s(\Delta t) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ T_s((m-1)\Delta t) & T_s((m-2)\Delta t) & \ldots & 0 \\ T_s(m\Delta t) & T_s((m-1)\Delta t) & \ldots & T_s(\Delta t) \end{bmatrix}, \tag{3}$$

with Δt the time interval. The solution of Eq. (2) can be regarded as the following least-squares problem:

$$min \|\mathbf{r} - \mathbf{T}_s \mathbf{f}\|_2^2, \tag{4}$$

where $\mathbf{T}_s$ can be obtained by using the following relation:

$$\mathbf{r} = \mathbf{F}\mathbf{t}_s, \tag{5}$$

where $\mathbf{F} \in R^{m\times m}$ is a reference impact force, $\mathbf{r} \in R^m$ is its corresponding measured response, and $\mathbf{t}_s \in R^m$ is the vector of transfer function, given by:

$$\mathbf{F} = \begin{bmatrix} f(\Delta t) & 0 & \ldots & 0 & 0 \\ f(2\Delta t) & f(\Delta t) & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ f((m-1)\Delta t) & f((m-2)\Delta t) & \ldots & f(\Delta t) & 0 \\ f(m\Delta t) & f((m-1)\Delta t) & \ldots & f(2\Delta t) & f(\Delta t) \end{bmatrix},$$
$$\mathbf{t}_s = \begin{bmatrix} T_s(\Delta t) \\ T_s(2\Delta t) \\ \vdots \\ T_s((m-1)\Delta t) \\ T_s(m\Delta t) \end{bmatrix}. \tag{6}$$

Again, the solution of Eq. (5) can be regarded as a least squares problem, as follows:

$$min \|\mathbf{r} - \mathbf{F}\mathbf{t}_s\|_2^2. \tag{7}$$

Note that while Eqs. (2)-(4) are similar to Eqs. (5)-(7), they describe two different problems. However, both problems represented by Eq. (4) and Eq. (7) are ill-posed, which can lead to instability in the solution. Mathematical methods to deal with ill-posedness are presented in Section 5.

2.1.2. Frequency domain approach

The frequency domain strategy relies on spectral analysis. Similar to the time domain deconvolution approach, the impact force is identified at each frequency by using the measured response vector and the

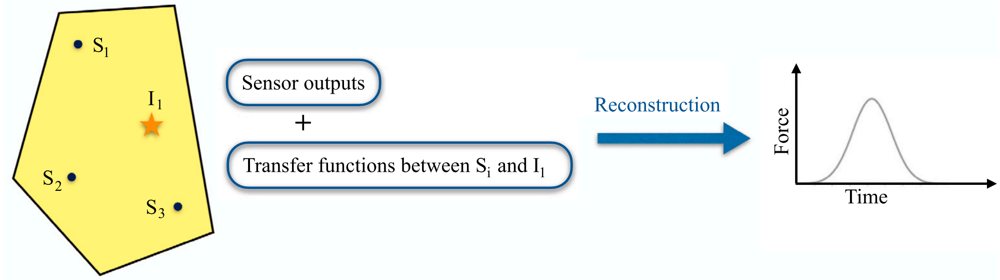

Fig. 6. Illustration of impact force reconstruction (the location of the impact force and the sensors are respectively shown by I_1 and S_i).

pseudo-inverse of the frequency response function matrix at that specific frequency [80], which is discussed below in detail.

The frequency domain deconvolution formulation is obtained by applying the Fourier Transform to both sides of Eq. (1) and rearranging as follows [79,80,91]:

$$\tilde{r}(y,\omega) = \tilde{T}_s(x,y,\omega)\tilde{f}(x,\omega), \tag{8}$$

where $\tilde{r}, \tilde{T}_s$, and $\tilde{f}$ are complex functions of ω that is the harmonic circular frequency. For a scenario that several responses are gauged at various locations, Eq. (8) can be expressed in the following matrix form:

$$\tilde{\mathbf{r}}(\omega)_{l\times 1} = \tilde{\mathbf{T}}_s(\omega)_{l\times q}\tilde{\mathbf{f}}(\omega)_{q\times 1}, \tag{9}$$

with $\tilde{\mathbf{T}}_s(\omega)$ the Frequency Response Function (FRF) matrix. In Eq. (9), l is the number of locations with mounted sensors, and q is the number of unknown forces. Pre-multiplying Eq. (9) by $\tilde{\mathbf{T}}_s^{-1}(\omega)$ can lead to the computation of $\tilde{\mathbf{f}}(\omega)$. This pre-multiplication can be done if, firstly, $\tilde{\mathbf{r}}(\omega)$ and $\tilde{\mathbf{T}}_s(\omega)$ are known, and secondly, $l = q$. However, using a least squares scheme with $l > q$, which involves employing additional measurement locations, leads to a more accurately identified force. Then, the formulation is as follows:

$$\tilde{\mathbf{f}}(\omega) = \tilde{\mathbf{T}}_s(\omega)^{+}\tilde{\mathbf{r}}(\omega), \tag{10}$$

where $\tilde{\mathbf{T}}_s(\omega)^{+}$ is the pseudo-inverse of $\tilde{\mathbf{T}}_s(\omega)$, which is a non-square matrix, defined by:

$$\tilde{\mathbf{T}}_s^{+} = (\tilde{\mathbf{T}}_s^{H}\tilde{\mathbf{T}}_s)^{-1}\tilde{\mathbf{T}}_s^{H}, \tag{11}$$

with the superscript H, the Hermitian transpose.

With the given measured responses and the FRF matrix, the unknown impact forces can be obtained by using Eq. (10). Note that this formulation needs the matrix inversion process to be as accurate as possible. With an ill-conditioned $\tilde{\mathbf{T}}_s$, the response errors will be amplified and can ultimately lead to unstable results. This clarifies the importance of employing regularization methods, which can be exploited if a numerical solution of Eq. (9) exists.

2.1.3. Wavelet deconvolution technique

The wavelet deconvolution technique can provide more accurate results for identifying impact forces, which have an inherently finite time duration, compared to the frequency domain deconvolution method [25]. This is because the Fourier Transform represents the impact force signal as a linear combination of sinusoidal functions, which are infinite in time [81]. Whereas, an arbitrary impact force signal can be represented as a linear combination of wavelets, which are finite in time, by taking the Wavelet Transform. Furthermore, this approach can help alleviating the ill-conditionedness of the transfer function matrix, as a dictionary to decompose the load into basic functions and coefficients.

Wavelets were first employed for impact force reconstruction only with time domain shifting [92], however, it is more effective when the basic wavelets are also scaled [81]. Efforts to modify and enhance the use of wavelets to identify impact forces have been published [93–95]. Considering both scaling and shifting of the wavelet for impact force reconstruction is studied in [82]. Controlling both shift and scale components provides a pseudo regularization and hence reduces the ill-posed nature of the problem [84]. More recently, in [96], the wavelet deconvolution method is exploited to reconstruct the exerted impact forces on a rectangular carbon fiber-epoxy honeycomb composite sandwich panel. Therein, the efficacy of different mother wavelets is also investigated. The wavelet deconvolution method is also verified experimentally on a polycarbonate plate in [84].

Using the discrete Wavelet Transform theory, the impact force $f(t)$ can be approximated by the following summation [84]:

$$f(\tau) = \sum_{m=m_0}^{M}\sum_{n=n_0}^{N_m} \tilde{f}^{d}_{m,n}\psi_{m,n}(\tau) + \sum_{n=n_0}^{N_M} \tilde{f}^{a}_{M,n}\varphi_{M,n}(\tau), \tag{12}$$

where m and n are integers and τ is the normalized time. In Eq. (12), $\tilde{f}^{d}_{m,n}$ and $\tilde{f}^{a}_{M,n}$ are, respectively, the expansion coefficients at scaling levels m and M with the superscripts d and a denoting the detail and approximation terms. The scaled and shifted wavelet function $\psi(\tau)$ and scaling function $\varphi(\tau)$ are, respectively, given by [82]:

$$\psi_{m,n}(\tau) = a_0^{-m/2}\psi\Big(\frac{\tau - nb_0a_0^{m}}{a_0^{m}}\Big), \tag{13a}$$

$$\varphi_{M,n}(\tau) = a_0^{-M/2}\varphi\Big(\frac{\tau - nb_0a_0^{M}}{a_0^{M}}\Big), \tag{13b}$$

with $a_0^{-m/2}$ and $a_0^{-M/2}$ the normalization parameters, a_0^{-m} and a_0^{-M} the scaling parameters, and $nb_0a_0^{m}$ and $nb_0a_0^{M}$ the shifting parameters. Consequently, the response function $r(\tau)$ can be obtained as follows based on the new formulation of impact force in Eq. (12):

$$r(\tau) = \sum_{m=m_0}^{M}\sum_{n=n_0}^{N_m} \tilde{f}^{d}_{m,n}\Psi_{m,n}(\tau) + \sum_{n=n_0}^{N_M} \tilde{f}^{a}_{M,n}\Phi_{M,n}(\tau), \tag{14}$$

where,

$$\Psi_{m,n}(\tau) = \int_0^{\tau} T_s(\tau-\zeta)\psi_{m,n}(\zeta)d\zeta, \tag{15a}$$

$$\Phi_{M,n}(\tau) = \int_0^{\tau} T_s(\tau-\zeta)\varphi_{M,n}(\zeta)d\zeta, \tag{15b}$$

can be regarded as the responses to the wavelet force. In Eq. (15), T_s is the impulse response function. Equations (12) and (14) can be discretized in the following form:

$$\{f\} = [\lambda]\{\tilde{f}\}, \tag{16a}$$

$$\{r\} = [\Lambda]\{\tilde{f}\}, \tag{16b}$$

where the elements of column vector $\tilde{f}$ are $\tilde{f}^{d}_{m,n}$ and $\tilde{f}^{a}_{M,n}$, the matrix $[\lambda]$ consists of $\psi_{m,n}(\tau)$ and $\varphi_{M,n}(\tau)$, and matrix $[\Lambda]$ composed of $\Psi_{m,n}(\tau)$ and $\Phi_{M,n}(\tau)$. Given Eq. (16a) and Eq. (16b), the impact force can be reconstructed as follows:

$$\{f\} = [\lambda][\Lambda]^{+}\{\tilde{r}\}, \tag{17}$$

with $[\Lambda]^{+}$ the Moore-Penrose generalized inverse of $[\Lambda]$. Note that $[\Lambda]$ can be constructed using a reference impact and a reference response with the same procedure of impact force reconstruction, see [82,84].

An adaptive wavelet-regularized time-domain deconvolution method has also been utilized for efficient impact force identification (IFI) [97]. Adaptive impact windows are generated using wavelet transform and multi-resolution analysis to reduce signal length, with wavelet bases filtered based on energy contributions for regularization. Validation through experiments with small-mass hammer and large-mass drop tower impacts demonstrated the method's computational efficiency and accuracy, particularly for large-mass impacts with significant local deformation.

2.2. State estimation-based approach

In this approach, force identification is considered as a state estimation problem, where the forces applied to the system are considered as the inputs. Then, various state estimation methods can be exploited to find the solution. The main advantages of the current approach, compared to deconvolution approach, are as follows [12,65]: (i) state variable formulation can be employed to simultaneously identify the external loads, the state variables, and even the parameters of the system, and (ii) these methods are computationally more efficient for large-scale structures. However, state variable formulation methods may lose their efficacy when the applied force is prone to abrupt changes as they assume that the forces are time-invariant within a time step. The accuracy of this approach can be improved by adjusting the time step sufficiently small. Nonetheless, this remedy results in heavy computation burden, and increases the ill-posedness [12]. Alternatively, the principle of momentum can be employed to transform the equation of motion, which is second-order, into a first-order momentum equation that can hone the rapid changes of the load [98]. Another approach presented in the literature to overcome the mentioned issue is utilizing the conventional implicit Newmark method [9], which is beneficial especially in the case of low sampling frequency. Still, very small time steps should be used with direct integration methods, including the Newmark method, in order to deal with discontinuous loadings [99]. In [12], an improved state-space method is proposed that considers the function interpolation for the external force. Therein, two interpolation functions are presented, (i) the linear interpolation, and (ii) the sigmoid curve interpolation, which both are shown effective in the case of long sampling time and/or low sampling frequency. Furthermore, an extended method based on dividing the time step and making the problem over-determined, is proposed which can also be applied to systems prone to high noise level.

The linear equation of motion of a structure is generally expressed by a continuous-time second-order differential equation [10]:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t), \tag{18}$$

where $\mathbf{u}(t) \in \mathbb{R}^n$, with n the number of DOF, is the vector of displacement and $\mathbf{f}(t) \in \mathbb{R}^n$ is the vector of excitation force. Herein, the matrices $\mathbf{M} \in \mathbb{R}^{n\times n}, \mathbf{C} \in \mathbb{R}^{n\times n}$ and $\mathbf{K} \in \mathbb{R}^{n\times n}$ are, respectively, the mass, damping and stiffness matrices. These coefficient matrices are positive-definite, symmetric, and real-valued.

The state-space formulation of the system dynamics in Eq. (18) is given by [10]:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{f}(t), \tag{19}$$

where the matrices $\mathbf{A}$ and $\mathbf{B}$ are functions of the coefficient matrices, $\mathbf{M}, \mathbf{C}$, and $\mathbf{K}$, with the state vector $\mathbf{x}(t)$ introduced as:

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{u}(t) \\ \dot{\mathbf{u}}(t) \end{bmatrix}. \tag{20}$$

In a general form, the measurement equation is defined as follows:

$$\mathbf{r}(t) = \begin{bmatrix} \mathbf{S}_d & 0 & 0 \\ 0 & \mathbf{S}_v & 0 \\ 0 & 0 & \mathbf{S}_a \end{bmatrix} \begin{bmatrix} \mathbf{u}(t) \\ \dot{\mathbf{u}}(t) \\ \ddot{\mathbf{u}}(t) \end{bmatrix}, \tag{21}$$

where $\mathbf{r}(t)$ is the measurement vector, and $\mathbf{S}_d, \mathbf{S}_v$ and $\mathbf{S}_a$ are the selection matrices. Equation (21) can also be reformulated into state-space form, as follows:

$$\mathbf{r}(t) = \mathbf{G}\mathbf{x}(t) + \mathbf{J}\mathbf{f}(t), \tag{22}$$

with the matrices $\mathbf{G}$ and $\mathbf{J}$ functions of the coefficient matrices and the selection matrices. Equations (19) and (22) form the full order state-space formulation which can be utilized for input and state estimation.

Although physical phenomena are continuous in time, the experimentally measured response data are discrete in time. The discretized form of Eq. (19) with sampling frequency $1/\Delta t$ is given by [11]:

$$\mathbf{x}_{k+1} = \mathbf{A}_d\mathbf{x}_k + \mathbf{B}_d\mathbf{f}_k,\ k = 0, 1, 2, ..., N, \tag{23}$$

with N the number of samples, and the discrete time instants are defined as $t_k = k\Delta t$. Herein, the matrices $\mathbf{A}_d$ and $\mathbf{B}_d$ are defined as follows:

$$\mathbf{A}_d = exp(\mathbf{A}\Delta t), \tag{24a}$$

$$\mathbf{B}_d = \mathbf{A}^{-1}(\mathbf{A}_d - \mathbf{I})\mathbf{B}. \tag{24b}$$

Note that to arrive at Eq. (23), the force has been assumed constant in the integration time step. Moreover, the discretized system output is given by:

$$\mathbf{r}(k) = \mathbf{G}_d\mathbf{x}(k) + \mathbf{J}_d\mathbf{f}(k), \tag{25}$$

with $G_d = G$ and $J_d = J$.

2.2.1. Markov parameters precise computation

The impact force can be identified based on the impulse response of the system, i.e., the system Markov parameters [11]. Suppose a unit impulse load is applied to the system at time $t = 0$ (i.e., $\delta_0 = 1$) and the initial conditions of the system are zero. Then, given Eq. (23) and Eq. (25), the impulse responses at various time points are given by:

$$\mathbf{H}_0 = \mathbf{G}_d\mathbf{x}(0) + \mathbf{J}_d\delta_0 = \mathbf{J}_d,$$

$$\mathbf{x}(1) = \mathbf{A}_d\mathbf{x}(0) + \mathbf{B}_d\delta_0 = \mathbf{B}_d,$$

$$\mathbf{H}_1 = \mathbf{G}_d\mathbf{x}(1) + \mathbf{J}_d\delta_1 = \mathbf{G}_d\mathbf{B}_d,$$

$$\mathbf{x}(2) = \mathbf{A}_d\mathbf{x}(1) + \mathbf{B}_d\delta_1 = \mathbf{A}_d\mathbf{B}_d,$$

$$\mathbf{H}_2 = \mathbf{G}_d\mathbf{x}(2) + \mathbf{J}_d\delta_2 = \mathbf{G}_d\mathbf{A}_d\mathbf{B}_d,$$

$$\mathbf{x}(3) = \mathbf{A}_d\mathbf{x}(2) + \mathbf{B}_d\delta_2 = \mathbf{A}_d^2\mathbf{B}_d,$$

$$\mathbf{H}_3 = \mathbf{G}_d\mathbf{x}(3) + \mathbf{J}_d\delta_3 = \mathbf{G}_d\mathbf{A}_d^2\mathbf{B}_d,$$

$$\vdots$$

$$\mathbf{H}_i = \mathbf{G}_d\mathbf{A}_d^{i-1}\mathbf{B}_d. \tag{26}$$

Therefore, the response can be obtained by the convolution of the impulse response and the input force as follows:

$$\mathbf{r}(k) = \sum_{i=0}^{k}\mathbf{H}_i\mathbf{f}(k-i), \tag{27}$$

which is known as Markov parameter representation of a structural system with the matrices $\mathbf{H}_i$ called forward Markov parameters. The forward Markov parameters are obtained based on system dynamic properties, and represent the response of a discrete system subjected to a unit impulse. These parameters can be calculated either analytically or experimentally, where the system output is measured for a known input.

In the case of inverse problem, i.e., estimating the input force based on given forward Markov parameters and responses of the system, the convolution relation is as follows:

$$\mathbf{f}(k) = \sum_{i=0}^{k} \mathbf{h}_i \mathbf{r}(k-i), \tag{28}$$

where the matrices $\mathbf{h}_i$ are the inverse Markov parameters, given by (see [11]):

$$\mathbf{h}_0 = (\mathbf{H}_0^T \mathbf{H}_0)^{-1} \mathbf{H}_0^T,$$
$$\mathbf{h}_k = -\mathbf{h}_0 \sum_{i=1}^{k} \mathbf{H}_i \mathbf{h}_{k-i}. \tag{29}$$

Although determining the inverse Markov parameters from forward Markov parameters needs intensive computation, it is to be performed only once for a specific system. Therefore, highly precise force identification can be performed without much computing cost. Nevertheless, the force identification problem is still ill-posed as a result of the noise in the measurements and the inversion process [11].

2.2.2. Kalman filter approach

In the process of force identification by inverse approaches, two kinds of uncertainty that are (i) modeling uncertainties, and (ii) uncertainty in the measured responses are present. Deterministic-stochastic methods are employed to take these uncertainties into account, which are usually represented in state-space. These methods presume that both the measurements and the state variables are subjected to noise, while the unknown force is treated as a deterministic quantity. Most of stochastic (or deterministic-stochastic) force identification methods rely on the Kalman filter [100].

In Kalman filter methods, the model error and the measurement noise are considered as stochastic processes in state-space. The external force and structural responses are then considered as unknowns [8,101–103]. The external force is estimated in a separate process as the state estimation [104], which may result in biased identification errors [13]. Generally, the Kalman filter has several advantages over deterministic methods. This includes (i) noise intervention cancellation, (ii) robustness, and (iii) online monitoring [105]. In the following, several Kalman filter approaches employed in the literature are briefly introduced:

- **Augmented Kalman filter:** An augmented Kalman filter technique is developed in [8] for force identification, where noise is considered as a stochastic process. In [8], unknown forces are included in the state vector. This augmented vector is then estimated by employing the standard Kalman filter. This deterministic-stochastic method is presented in time domain and demonstrates insensitivity to modeling and measurement errors [105].
- **Dual Kalman filter:** To estimate unknown input and system states, a dual implementation of the Kalman filter is proposed in [10]. This method, which is compatible with linear systems, is numerically applicable even in case of un-observability and rank deficiency of the augmented problem formulation.
- **Sparse Kalman filter:** This method is a time domain recursive strategy which facilitates simultaneous localization and reconstruction of unknown forces [17]. Its main strength is that it can detect large number of forces, applied on potential locations, with employing a relatively fewer sensors and in less time, compared to conventional methods. Furthermore, the time delay between measurement and the corresponding input estimation is considered. On the other hand, the main defect of this method is the increase in the computational burden.
- **Unscented Kalman filter:** Like Sparse Kalman filter method, this method can also localize and reconstruct the input simultaneously [13,75], while it can be employed for nonlinear structures.
- **Extended Kalman filter:** This method is employed for nonlinear structures in order to identify the parameters and estimate the external forces [106–108]. It is applicable on linear systems or structures with slight nonlinear property. Nevertheless, the Unscented Kalman filter leads to higher identification accuracy for nonlinear systems [109] compared to Extended Kalman filter.

Wrapping up, among model-based impact force identification methods, those based on the state-space formulation have been shown to outperform the methods based on the deconvolution approach. Still, the application of mentioned methods in complicated structures needs to be investigated in more detail. Additionally, the performance of each approach for real-time identification is still inconclusive in the literature. It is also unclear to the authors which approach performs better in the cases of (i) multiple impact occurrence, and (ii) limited instrumentation.

3. Model-based impact localization

The force reconstruction methods, presented in Section 2, assume that the impact location is known a priori. In the current section, some mostly-used model-based localization techniques are reviewed with two different approaches, (i) based on time difference and geometry, and (ii) based on structural vibrations.

3.1. Methods based on time difference and geometry

These methods seek the impact location based on propagating elastic waves resulting from the impact and the corresponding Time of Arrival (ToA) at a specific sensor location [55]. Hence, these methods are normally only applicable to isotropic materials through which the wave propagation speed is uniform. To make them applicable for anisotropic materials, some modifications have been proposed in the literature relying on optimization algorithms such as Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) [54,55]. In the following, the idea behind this approach is presented through the Triangulation method.

One of the most popularly employed wave-based localization strategies is the Triangulation method, which has been presented in time domain and frequency domain [33]. Fig. 7 illustrates the idea of the triangulation method on a rectangle plate with 4 sensors mounted at its corners [44]. First, the distance between each sensor and the impact location is estimated. Then, a circle is drawn around each sensor with a radius equal to the related estimated distance of the impact location. Subsequently, the common area of these circles is defined as the impact region. Mathematically, one could first search for the midpoint of the intersection line between every two adjacent circles, shown by P_1, P_2, P_3, and P_4 in Fig. 7, and then determine the estimated impact location as the intersection point of lines $P_1 - P_2$ and $P_3 - P_4$. The distance between the impact point and each sensor location can be estimated from the magnitudes of sensor signals, see [44,45], or the wave propagation speeds, see [42,43,55]. Furthermore, several pieces of research have been conducted in the literature to calculate ToA, which is needed for both signal magnitude and wave propagation approaches [52,110–113]. Among which, (i) Threshold method, (ii) Correlation method, and (iii) Likelihood algorithm are compared in [43] and a detailed discussion on the pros and cons of each method is presented.

Early triangulation techniques [114] were initially designed for isotropic materials, relying on the assumption that wave propagation speed remains uniform throughout the structure. However, recent advancements have extended these methods to accommodate anisotropic materials [46–53]. In [51], a hybrid two-step approach was introduced for acoustic source prediction in anisotropic plates. The first step simplified the problem by assuming that waves travel along straight paths from the acoustic source to the sensor, enabling initial localization. This preliminary estimate was then refined in the second step through an optimization process. Experimental validation confirmed that this second step consistently adjusted the estimates to more accurately align with the actual source location.

Still, this method loses its efficiency for complex structures as wave propagation highly relies on the structure geometry and properties

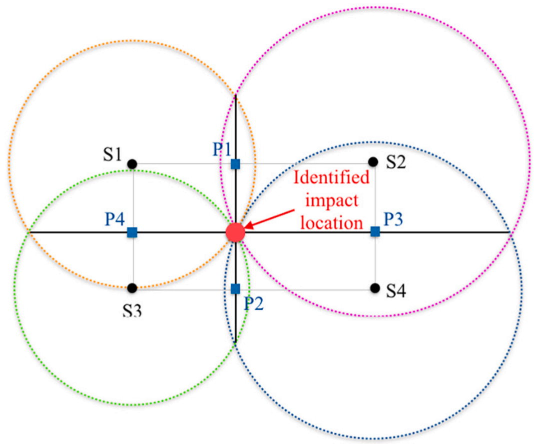

Fig. 7. Triangulation method for localizing impact location.

[115]. More advanced signal processing methods have been also exploited for impact localization on complex structures, e.g., similarity searching strategies [16,96,116–118] that will be discussed in the following sections.

3.2. Methods based on structural vibrations

These methods employ the mode shape transformations of a structure before and after an impact event. The main shortcoming of these methods is that they rely on a benchmark data of the intact structure. Additionally, they are generally computationally inefficient, especially for large or distributed structures [16]. The methods based on structural vibrations are classified into two groups: (i) similarity searching strategies, and (ii) optimization-based techniques.

3.2.1. Similarity searching methods

Mathematically, a similarity searching method can be utilized to evaluate the similarity level between two time series. These methods are employed to detect the most similar time series. The similarity between the time series is quantified as a similarity metric. The similarity searching methods are utilized to localize the impact force by comparing the measured impact signal with pre-obtained reference data. The similarity metric will be higher for those training points which are located closer to the actual impact force than other training points, which, in turn, determines the impact location. In the following, several methods based on similarity searching approach are presented, namely, (i) Time reversal, (ii) Normalized cross-correlation, (iii) Cosine similarity, and (iv) Indexing strategy using Wavelet Transform.

- **Time reversal method:**

The Time Reversal method consists of two steps: (i) forward propagation step, and (ii) backward propagation step [119]. In the first step, the following information is acquired and stored: (1) the time history of low-velocity impacts which are performed at m excitation points by a hand-held modal hammer, and (2) the responses of the structure, which are measured by n sensors mounted on the structure. This step gives an $n \times m$ matrix of stored responses, shown by ($G\mathbf{r}_m$). Suppose an impact with an unknown location is performed on the structure. The relating responses form an $n \times 1$ matrix ($G\mathbf{r}_{m0}$). In the second step, a correlation between ($G\mathbf{r}_{m0}$) and ($G\mathbf{r}_m$) is performed, which leads to $n \times m$ functions R_{TR}, named “time reversal operators”.

Let us define $E_{Gr_{m0}}$ as the geometric mean of the energy of the unknown impact response, and E_{Gr_m} as the m energies of the impact responses, stored in the first step. Then, for the responses measured by a specific sensor, the moduli of the $1 \times m$ calculated R_{TR_s} are normalized with $E_{Gr_{m0}}$ and E_{Gr_m}, respectively. To evaluate the similarity between two signals, the correlation coefficient c_{TR} is employed, defined as [120,121]:

$$c_{TR} = max\left(\frac{|R_{TR}|}{\sqrt{E_{Gr_m} E_{Gr_{m0}}}}\right). \tag{30}$$

When the signals are similar, $0 < c_{TR} < 1$ is close to 1, which happens at the true impact location. For each excitation point, n correlation coefficients are obtained, while the mean correlation coefficient is considered. And for each cell, a mean value of the coefficients related to its four corners is calculated, called a global correlation coefficient $c_{TR-GLOBAL}$. The cell with the maximum $c_{TR-GLOBAL}$ can then be chosen as the impact cell, which can be localized by a center-of-gravity method [122]:

$$x_I = \frac{\sum_{i=1}^{4} x_i c_{TR_i}}{\sum_{i=1}^{4} c_{TR_i}},$$
$$y_I = \frac{\sum_{i=1}^{4} y_i c_{TR_i}}{\sum_{i=1}^{4} c_{TR_i}}, \tag{31}$$

with c_{TR_i} the averaged correlation coefficient related to the i^{th} node, and impact source coordinates x_I and y_I, where x_i and y_i are the coordinates of the i^{th} node of the impact cell.

- **Normalized cross-correlation technique:**

The normalized cross-correlation technique is another impact localization method which relies on the similarity searching between a reference database and the measured signals of the impact with unknown location [117,123]. The cross-correlation between signals f and g with τ time lag is given by:

$$(f * g)(\tau) = \int_{-\infty}^{+\infty} f(t)g(t+\tau)dt, \tag{32}$$

where $*$ is the cross-correlation operator. Consequently, the normalized cross-correlation is as follows:

$$\left(\frac{f}{F} * \frac{g}{G}\right)(\tau), \tag{33}$$

with normalizing constants F and H that correspond to signals f and g, respectively, given by:

$$F = \int_{-\infty}^{+\infty} |f(t)|dt, \tag{34a}$$

$$G = \int_{-\infty}^{+\infty} |g(t)|dt. \tag{34b}$$

The normalized cross-correlation in Eq. (33) is ideally 1 when two signals f and g are similar and the time-lag τ is zero.

Fig. 8 illustrates the concept of the cross-correlation method, which localizes an impact force by employing the normalized cross-correlation between the measured impact signals excited in the structure and a reference database. Suppose n sensors are mounted on the structure, hence, for each training point, (n) datasets are available. Similarly, when an impact occurs on the structure, n signals are recorded by sensors. Next, for all the training points, the normalized cross-correlation of each reference data is compared with the acquired signal. The point that corresponds to the higher value of normalized cross-correlation is identified as the impact location [117,123].

- **Cosine similarity technique:**

Cosine similarity method exploits the cosine of the angle between two non-zero signals to evaluate the error between them, as shown in

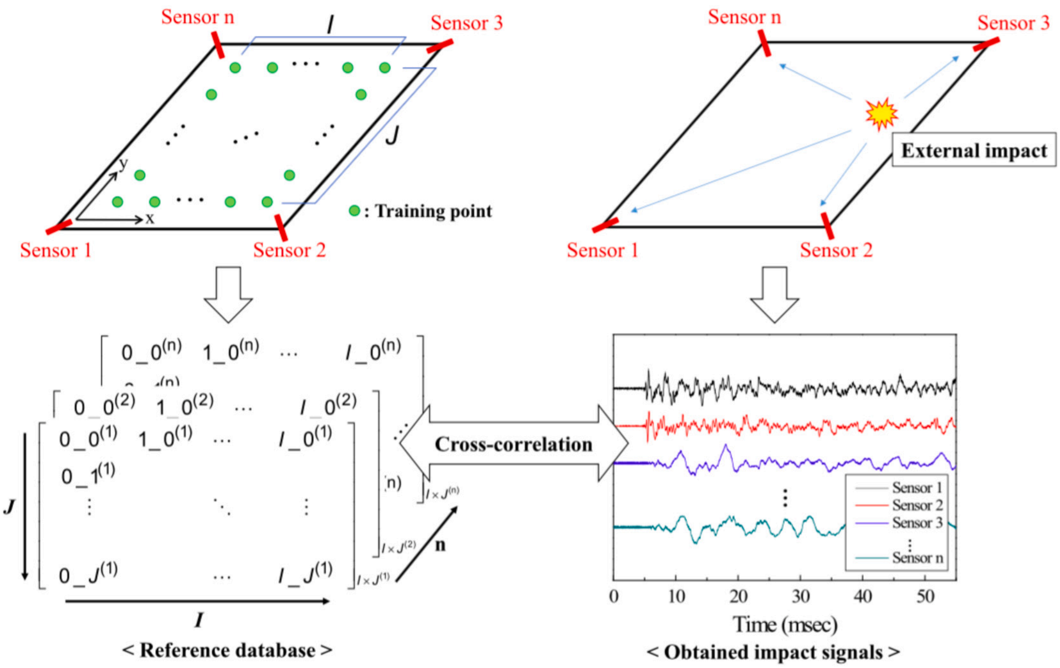

Fig. 8. General idea of the normalized cross-correlation technique for impact localization [117].

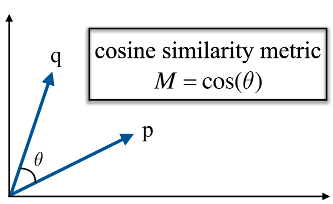

Fig. 9. Searching for the similarity between two vectors by Cosine similarity technique, employed for impact force localization.

Fig. 9, while it neglects the signals magnitude. The cosine similarity of vectors $p = (p_1, p_2, ..., p_n)$ and $q = (q_1, q_2, ..., q_n)$ is evaluated by [16,118]:

$$M(p,q) = \frac{p.q}{\|p\|\|q\|} = \frac{\sum_{i=1}^{n} p_i q_i}{\sqrt{\sum_{i=1}^{n} p_i^2}\sqrt{\sum_{i=1}^{n} q_i^2}}. \tag{35}$$

Two vectors with the same orientation will result in the cosine similarity M equal to 1, whereas perpendicular vectors give the value $M = 0$. To localize an impact force, the reconstructed forces at each potential impact location are compared with a half-sine vector. The idea behind this is that normally an impact force signal is similar to a half-sine function, even in the case of damage [63]. In [118], the following half-sine signal is introduced:

$$p = \begin{cases} sin(\frac{\pi}{a}t) & 0 < t < a \\ 0 & a < t < T \end{cases}, \tag{36}$$

with a the scaling parameter and T the time window. The definition of the similarity index δ is then given by:

$$\delta = max\|M\|. \tag{37}$$

The impact location can be then identified as the value of δ is maximum at the true impact location among all the other possible locations.

- **Indexing strategy using Wavelet Transform:**

Continuous Wavelet Transform has been employed for the localization of damage in beam structures with single damage [124–130] or multiple damages [131,132]. The same idea and formulation can be employed for impact localization as well. An indexing strategy can be used for each reconstructed force signal based on the determined wavelet coefficients [90,133]. The true impact location will be then identified as it has the highest index.

This mathematical transformation, continuous Wavelet Transform, facilitates the similarity measurement between a signal $f(t)$ and an analyzing function $\psi(t)$ [41,128]. The wavelet coefficient W, which is a measure of similarity between the signal and the mother wavelet, is given by [96]:

$$W(a,b) = \frac{1}{\sqrt{a}} \int f(t)\psi(\frac{t-b}{a})dt, \tag{38}$$

where real parameters $a > 0$ and b are, respectively, the scale and translation parameters. The larger the value of W is, the finer the correlation between the signal and the wavelet is, at that particular location. When the mother wavelet and the true impact force have similar shapes, the wavelet coefficient will be larger for the true impact force than the one for the false reconstructed forces. Consequently, in [90], a similarity index is proposed as:

$$\delta = \sum_a \sum_b |W(a,b)|, \tag{39}$$

where δ is the force localization index, which will be maximum for the true impact force.

3.2.2. Optimization-based methods

In model-based localization methods, the difference between the computed analytical data and the measured vibration data at all sensors is minimized for all possible impact locations. The location and the time history of the impact force can be obtained when the difference is least. Different formulations have been employed in the literature as an objective function, e.g.,

$$E = \sum_{i=1}^{m} ||\mathbf{T}_s^{(i)}\mathbf{f} - \mathbf{r}^{(i)}||, \tag{40}$$

where $\mathbf{r}^{(i)}$ is the response of ith measurement point, $\mathbf{f}$ is the impact force, $\mathbf{T}_s^{(i)}$ is the transfer function of ith measurement point and m denotes the number of possible impact locations. The optimization-based techniques normally seek the minimum value of the objective function (e.g., function E in Eq. (40)).

Conventional optimization methods are mostly gradient-based, which may lose their efficiency in case of highly nonlinear objective functions. More recently, artificial intelligent optimization algorithms are employed for impact localization, e.g., GA [62,134], and PSO [135]. Compared to GA, PSO is simpler and in many cases more efficient [136]. Additionally, it requires fewer function evaluations. Whereas, it can lead to premature convergence, like many other optimization algorithms, for which some enhancements are proposed in the literature [137,138].

Model based impact force localization techniques have been reviewed above. These methods have shown their efficacy in localizing impact forces applied on simple structures that undergo one impact force only at a time. The literature should still be enriched for more complex structures, potentially under multiple concurrent impacts, with limited number of sensors (as few as possible). Moreover, as far as the authors know, the compatibility of these approaches for real-time impact force localization has not been investigated. Investigation of the sensitivity of different localization strategies to modeling uncertainties is another potential future research.

4. Machine and deep learning approach

Machine Learning (ML) techniques have emerged as viable candidates for impact force reconstruction and localization in recent years [25–28,30,60,139–145]. In Structural Health Monitoring (SHM), these techniques are employed in order to obtain models which map the input pattern to the output target. However, typical ML techniques are restricted in processing raw form of huge amount of measured data [146]. Hence, engineering knowledge is essential for the extraction of features from raw data. The conundrum is that there is no guarantee that a set of features suitable for a specific structure can be used for other structures as well. To circumvent this conundrum, Deep Learning (DL) techniques have been explored in recent years, which facilitate using the data in raw form. Using DL methods, features are discovered in an intelligent way from even high-dimensional data. Though, the key challenge in many SHM applications is that the impact identification should be fulfilled in an unsupervised manner, since the data for different impact scenarios are often not available [146].

Artificial Neural Network (ANN) is an advanced ML algorithm which underpins most DL models, and can be employed to model complex nonlinear input-output relations [147,148]. ANNs also rely on data to perform supervised learning like other ML techniques. However, as the weight information is specified, it can be employed for online identification like adaptive impact absorption systems [149]. In [25], ANN is exploited to identify the impact forces that result in large deflections in a composite stiffened panel. Therein, two networks are introduced, one for impact force reconstruction and the other for peak prediction. The exciting point of the proposed ANNs is that they do not need the location of the impact a priori. Another interesting idea employed in this research to improve the predictions is classifying the impacts into (i) large mass and (ii) small mass, for which separate networks are trained. Still, not many applications of ANNs are available for impact force identification of nonlinear structures due to the following reasons [30]:

1. Most used ANNs have Multi-Layer Perceptron (MLP) architecture and the input of MLP is two dimensional, i.e., data scale and feature, while force reconstruction is three dimensional, i.e., data scale, feature, and time.
2. Human domain knowledge is needed for feature extraction.
3. It is essential to have dynamic responses from numerous sensors placed optimally like model-based approaches.

Deep Recurrent Neural Network (RNN) is another DL approach employed for impact force identification [30]. The deep RNN gives the impact history by using the raw response through a model trained by back-propagation through time algorithm. Compared to ANNs, RNNs establish more connections in the hidden layer, so that the hidden layer receives the hidden cells of the previous states as well as the original data. Like many other identification techniques, the deep RNN method suffers from instability due to data contamination. For this, data regularization can be a solution [150].

Generally, the main advantages of DL methods are as follows:

1. DL methods are able to deal with noisy data which is common in the real application,
2. DL methods can automatically extract the features and make them faster versus methods that rely on hand-craft.

The required training data can be constructed either synthetically [151–155] or experimentally (civil infrastructure [156], hydro-junction infrastructure [157], operating Vestas V27 wind turbine [155], PVC sandwich plate [158], bridge [159]) which makes the universal comparison difficult on one hand while showing how the DL method can be useful in real settings. These research papers mainly used DL as a tool without much contribution to the DL method itself.

Table 2 provides a summary of how Machine Learning models have been applied for impact force identification. The table highlights that contributions from the development of new ML models are limited, with ANNs being the most commonly used models. Algorithms such as Genetic Algorithms (GA), Wavelet Transform, and Neuro-Fuzzy models have been employed in combination with ANNs. The table also indicates that comparisons with baseline models are not comprehensive. The studies considered various scenarios, including composite plates, rotating multi-damage cantilever rotors, Perspex plate structures, reinforced concrete beams, curved composite plates, and aluminum plate structures.

In anisotropic structures, wave propagation exhibits strong direction-dependent behavior, requiring deep learning models to explicitly incorporate anisotropic effects for accurate impact localization and force identification. A hybrid approach integrates physics-based principles with data-driven learning to effectively account for material anisotropy, enhancing model robustness and accuracy. A hybrid strategy takes into account anisotropy using:

1. Orientation-aware input features: traditional deep learning models often rely solely on raw sensor signals, but for anisotropic materials, additional contextual information is essential. By incorporating material orientation angles at each impact location into the input vectors, the model can better capture directional variations in wave propagation, leading to improved localization accuracy.
2. Physics-based data augmentation: to address the challenge of limited experimental data, training datasets are augmented using transformed stiffness matrices derived from classical laminate theory. These matrices simulate variations in fiber orientation, allowing the model to generalize across different anisotropic configurations. This augmentation ensures that the network learns meaningful physical relationships rather than relying on purely empirical patterns.
3. Attention mechanisms for path weighting: in anisotropic materials, wave propagation efficiency varies based on direction. To account for this, neural attention layers are introduced, enabling the model to dynamically assign higher weights to sensor signals traveling along optimal paths. This mechanism improves the model's ability to distinguish meaningful signals from noise and enhances its robustness against directional dispersion effects.

By integrating these strategies, the hybrid framework effectively combines domain-specific knowledge with deep learning capabilities, significantly improving performance in anisotropic systems. Such physics-informed architectures pave the way for more reliable impact localization and force reconstruction in complex composite structures, ensuring greater adaptability in real-world applications.

Table 2
A list of papers using machine learning models for impact force identification.

Ref.	Main aim	Technical model	Conclusion
[160]	Impact identification on an aluminum plate	Support Vector Machine (SVM)	SVM accuracy is better than ANN
[161]	Simultaneous force localization and force history reconstruction	Distance-assisted Graph Neural Network (DAGNN)	The accuracy of DAGNN is higher than a conventional GNN model
[162]	Impact localization and force reconstruction of impacts applied to composite panel structures	Artificial neural networks (ANN)	The impact identification procedure is much faster than that of the traditional model-based techniques
[163]	Impact identification on composite stiffened panels	Artificial neural networks (ANN)	The performance of three ANNs is better than a single ANN trained
[164]	Prediction of the acceleration, deflection, and strain responses of on a smart reinforced concrete beam	Wavelet-based Time delayed Adaptive Neuro-Fuzzy Inference System (W-TANFIS)	W-TANFIS model has better performance over the ANFIS
[165]	Impact load identification and localization on real engineering structures	(i) Gradient Boosting Decision Tree (GBDT) model based on ensemble learning, (ii) Convolutional Neural Network (CNN) model and (iii) Bidirectional Long Short-Term Memory (BLSTM) model based on deep learning	The methods can accurately identify impact loads and its location
[166]	Impact force identification through establishing inverse mapping relationships between structural vibration responses and impact forces	Gated Temporal Convolutional Network (GTCN) method	The GTCN performs better than Temporal Convolutional Network (TCN) and Convolutional Neural Network (CNN)
[167]	Impact identification on Perspex plate structure	Artificial neural networks (ANN), Multilayer Perceptron	Impact localization with MAT feature yields the highest accuracy
[168]	Classification of pathological gait patterns using 3D Ground Reaction Force (GRFs) data	Nearest Neighbor Classifier (NNC) and Artificial Neural Networks (ANN)	The optimal feature set of six features enhances the accuracy up to 95 percent
[169]	Predicting and analyzing highly nonlinear behavior of integrated structure-control systems subjected to high impact loading	Time-delayed Adaptive Neuro-Fuzzy Inference System (TANFIS)	TANFIS modeling framework is an effective way to capture nonlinear behavior of integrated structure-MR damper systems under high impact loading
[140]	Impact identification on a Perspex plate structure	Radial Basis Function Network (RBFN)	The performance of the RBFN surpasses that of the conventional Multilayer Perceptron (MLP) by significantly reducing errors
[170]	Impact force reconstruction on composite structures to evaluate the health status of the structure	Artificial Neural networks (ANN), Genetic Algorithm (GA)	ANN+GA reconstructed method outperforms ANN.

Some state-of-the-art DL methods which can be used in the impact force reconstruction and localization are as follows:

1. **Incremental learning**: In this paradigm, the new data/feature is used for modifying the trained DL method without doing the learning from scratch. This makes the DL more applicable in the real setting when we expect the flow of data and emerging of new factors. By applying incremental learning models, the DL based impact reconstruction/localization can be embedded in the tools for the real application.
2. **Explainable DL**: In 2016, the European Parliament and Council of the European Union passed the rules namely, “The General Data Protection Regulation”. This requires organizations that use AI to tell their user what information they hold about them and how it is being used. This means for the construction industry, they should be able to explain how DL based impact reconstruction/localization is working. This also makes their solution and service more attractive and useful. Explainable DL can also lead to better accuracy when the feedback of the knowledge workers is obtained.
3. **Human-AI interaction**: The setting of the right interaction between AI solutions and its user is becoming important these days due to the wide adoption and utilization of AI-powered products across various ranges of industries.

There are other state-of-the-art ML methods to deal with the data scarcity issue which are discussed in the next section:

4.1. Machine learning data requirements

One of the most significant challenges in applying ML to impact force identification is the limited availability of labeled data, particularly for real-world structural systems where generating large-scale experimental datasets is time-consuming, costly, or infeasible. Furthermore, the reliance on benchmark, i.e., healthy-state data presents a major obstacle in deploying ML methods for impact force identification in real-world settings. Recently, several approaches have emerged to address these limitations through the integration of physical knowledge and data-efficient learning frameworks:

1. **Synthetic data generation**: Physics-based simulations, such as Finite Element Models (FEM), can generate synthetic datasets that mimic real structural responses. These datasets are often used to pretrain models before fine-tuning on limited real measurements. Studies like [183–186] have demonstrated this simulation-to-reality transfer using hybrid FEM–ML pipelines, achieving good generalization across scenarios.
2. **Data augmentation**: Inspired by techniques in computer vision, data augmentation introduces variability by transforming existing data. For vibration signals, this can include time-warping, noise injection, or domain-specific transformations like frequency shifts. These techniques help improve generalization and reduce overfitting when only a few real samples are available.
3. **Transfer learning**: Transfer learning has gained significant prominence in the domains of Machine Learning and Deep Learning, owing to various challenges, including (i) concept drifts that lead to evolving data distributions, and (ii) limited access to training data,

often due to costly, time consuming, or hazardous data collection procedures. Impact force reconstruction training data collection face similar problems due to slow and costly process. As a best practice usually, the data collect in a region (domain target) and using transfer learning method can be used for the monitoring of other regions (target domains) [171–174]. These studies employ synthetic impact datasets generated via finite element simulations to pretrain ML models, which are then fine-tuned using a limited amount of experimental data. This approach allows for reliable force identification with minimal physical testing, reducing the dependency on benchmark datasets. This technique significantly lowers data requirements while maintaining accuracy. Thus, development of transfer learning techniques to support stakeholders in enhancing the impact force reconstruction task proves highly advantageous. Having said that, considering the high stakes nature of the impact force reconstruction task, the development of transfer learning methods is crucial.
4. **Self-supervised and unsupervised learning**: Unsupervised learning methods such as autoencoder-based anomaly detection can identify deviations from normal structural behavior without requiring labeled healthy-state data. These models learn compact latent representations from operational signals and flag anomalies, including impacts, based on reconstruction errors. As demonstrated in [146], this enables autonomous detection without needing predefined baseline responses.
5. **Zero-shot and Few-Shot Learning (FSL)**: The zero-shot and few-shot deep-based learning methods are able to classify new classes. Examination of these advanced techniques for the impact reconstruction/localization can be a new direction which increases the applicability of these methods, especially for cases in which safety is an important element for the stakeholders. This is particularly useful when labeled impact events are rare. Recent work [175] has shown that embedding physical laws, such as wave propagation dynamics, within model architectures enables accurate force localization and reconstruction even in cases where the impact lies outside the training distribution. This physics-informed machine learning paradigm combines the generalizability of physical models with the adaptability of data-driven methods, offering a major advancement over conventional data-intensive techniques.

These data-efficient learning techniques are becoming essential tools for extending machine learning methods to field-deployable structural health monitoring systems. They also represent a shift toward physics-informed, data-efficient, simulation-augmented, and transfer-aware ML paradigms, which aim to mitigate the need for large-scale data collection and are more practical for in-service structures lacking benchmark datasets or undergoing dynamic operational conditions.

5. Mathematical techniques to deal with ill-posedness

Given the ill-posed nature of inverse problems in impact force identification, noise sensitivity poses a significant challenge. Recent studies have introduced a range of stabilization techniques aimed at enhancing robustness against measurement noise. These techniques can be broadly categorized into sensor optimization, direct regularization schemes, including probabilistic (Bayesian) frameworks, and iterative regularization methods, each contributing to more accurate and stable force reconstructions.

Both the deconvolution technique and the state-space approach ultimately lead to the same type of algebraic problem (that is in the form of $y = Ax$). Although the notation differs in Section 2.1 and Section 2.2, one notation will be used in this section for clarity, i.e., $\mathbf{r} = \mathbf{Hf}$, which describes the forward problem. Consequently, the inverse problem would be formulated as $\mathbf{f} = \mathbf{H}^{-1}\mathbf{r}$. This problem is said well-posed, if its solution is stable as well as unique, otherwise, it is called ill-posed.

5.1. Posedness investigation

Singular Value Decomposition (SVD) is an efficient tool to investigate the problem posedness [176–178]. Applying SVD, the matrix $\mathbf{H}$ is decomposed as follows [100]:

$$\mathbf{H} = \mathbf{U}\Sigma\mathbf{V}^T, \tag{41}$$

where the diagonal matrix Σ consists of the singular values of $\mathbf{H}$. $\mathbf{U}$ and $\mathbf{V}$ are the left and right vectors of singular values, consisting of the eigenvectors of $\mathbf{HH}^T$ and $\mathbf{H}^T\mathbf{H}$, respectively. In other words, $\mathbf{U}$ and $\mathbf{V}$ can be considered the mode shapes of the response and the input, respectively. Eigenvalues of the matrices $\mathbf{HH}^T$ and $\mathbf{H}^T\mathbf{H}$ are identical, defined as σ_i^2. To investigate the posedness, the condition number can be employed, which is as follows for matrix $\mathbf{H}$:

$$cond(\mathbf{H}) = \frac{\sigma_1}{\sigma_m}, \tag{42}$$

with m the dimension of $\mathbf{r}$, the output. The condition number shows how sensitive the output is to small changes, errors, or noise in input [179,180]. Condition number of one states perfect orthogonality between columns of $\mathbf{H}$. Higher numbers of the condition number state weaker orthogonality. The condition number will be infinity if at least two columns are linearly dependent. Normally, a matrix is known as ill-conditioned when its condition number is greater than 1×10^3 [100]. In such cases, it is impossible to invert the matrix $\mathbf{H}$, hence, the problem $\mathbf{r} = \mathbf{Hf}$ can not be straightly inverted to find $\mathbf{f}$.

5.2. The least squares solution

The inverse problem can be solved with the Least Squares (LS) solution, where the following objective function is minimized [100]:

$$J = \|\mathbf{Hf} - \mathbf{r}\|_2, \tag{43}$$

which results in:

$$\mathbf{f}_{LS} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{r}. \tag{44}$$

This can be written in terms of the SVD, given by:

$$\mathbf{f}_{svd} = \mathbf{V}\Sigma^{-1}\Sigma^T\mathbf{r}, \tag{45}$$

or rewritten as follows:

$$\mathbf{f}_{svd} = \sum_{i=1}^{q} \frac{u_i\mathbf{r}}{\sigma_i} v_i, \tag{46}$$

with $q = rank(\mathbf{H})$. For very small singular values σ_i, even small perturbations in $\mathbf{r}$ lead to large changes in $\mathbf{f}_{svd}$. Due to this sensitivity, the least squares solution is not robust. In some cases, simply filtering the high-frequency contents in the response signal or adding more sensors to make the transfer function over-determined can eliminate the ill-posedness [75]. While regularization techniques can be principally employed to eliminate this amplification by regularizing the matrix $\mathbf{H}$ [65,181].

5.3. Sensor placement and measurement optimization

It has been shown that increasing the number of sensors can be one of the effective ways to avoid ill-posedness, while it is important to note that the number of the sensors and their positions are of significance and have a high impact on the result of the identification [6,18,88,182–184]. When these factors (number and positions of sensors) are not taken consciously, the identification is unreliable even if more sensors are mounted on the structure than the number of forces (over-determined case) [182]. In [184], the coherence-based placement strategies—specifically positioning sensors near anti-nodal regions of dominant vibration modes—is proposed to reduce the condition number of the system matrix. This reduction directly improves numerical stability and minimizes the amplification of noise during inversion. For more info on optimal sensor placement, refer to the review paper [185].

5.4. Regularization methods

In regularization methods, the matrix **H** is regularized such that the influence of some small singular values is decreased, which will, in turn, improve the problem posedness, however, as a result, the solution found is an approximation of the actual solution.

The literature on regularization approaches can be divided into two groups: (i) Direct approach, and (ii) Iterative approach. In the direct approach, the matrix **H** is decomposed by the SVD technique and the modes that are affected by noise are removed. The performance of these methods depends on the selection of the regularization parameter. On the other hand, methods based on iteration do not require the SVD which is computationally costly when the dimension of the matrix **H** is large. In this approach, the number of iterations is similar to the regularization parameter in the direct approach in terms of responsibility for the solution convergence and accuracy. In the following, some of the direct and iterative regularization methods are reviewed.

5.4.1. Direct approach for regularization

The most commonly employed regularization technique for impact force identification is the Tikhonov method [186–188]. This method balances the objective function and exploits a smoothness condition for the solution to moderate the ill-conditioning [30]. There are a number of methods for the parameter selection for the Tikhonov regularization technique, e.g., the L-curve method [189], the S-curve method [190], and the Generalized Cross Validation (GCV) method [191], which are based on the SVD of the coefficient matrix [75]. These methods cancel the part of the matrix relating to the very small singular values, and use the new reformulated matrix to solve the inverse problem. The main shortcomings of the Tikhonov regularization method are as follows: (i) it is computationally ineffective for large-size inverse problems as it takes too long to be of practical use and may even fail in computation [192], and (ii) it is only applicable in case of smooth excitation signals or fields [193]. One of the remedies proposed in the literature to deal with the second issue is employing LASSO regularization, which is also known as l_1-regularization [193]. This method develops the sparsity of the regularized solutions, whilst it still keeps the inverse problem convex [194–196].

The Truncated Singular Value Decomposition (TSVD) regularization method was introduced in [197,198] for linear ill-posed problems in impact force identification. TSVD has been also used for force identification in frequency domain [199]. For a comparison between simplified Generalized Singular Value Decomposition (GSVD) and TSVD with the Tikhonov technique, in time and frequency domain, refer to [6,200,201]. Therein, it is concluded that TSVD is useful when the high-frequency contents are responsible for the solution instability, while Tikhonov acts globally, and impacts the whole signal. Hence, roughly speaking, Tikhonov has shown to outperform TSVD in terms of identification accuracy [6,201]. In [202], the Truncated GSVD is combined with Tikhonov method in order to reach higher accuracy with less memory requirement and less computational time for large-scale ill-posed problems.

Lately, Bayesian regularization, also referred as the augmented Tikhonov regularization [203,204], has been employed for impact force identification, which lies within the Tikhonov framework with l_2-norm features [60]. In this method, the forces are considered as real random vectors with generalized Gaussian distribution [193,205–208]. Compared to the traditional Tikhonov method, the regularization parameters of this method are chosen adaptively and the noise level is detected from the measurements [192,209,210]. A Bayesian inference framework is formulated in [60] that not only reconstructs impact forces but also estimates the statistical characteristics of the noise. Their method reduced noise-induced reconstruction errors compared to conventional approaches, particularly when prior information about the system or loading is available. The power of the Bayesian regularization is that when it is used for impact force reconstruction, (i) the impact shape, (ii) the impact duration, (iii) the impact peak, and (iv) the impact energy can all be identified [75]. Moreover, this technique gives a probabilistic description of the impact force, enabling uncertainty analysis [211]. On the other hand, its main drawback is that the reconstructed impact forces are fluctuated with negative values while the impact force is intrinsically non-negative [212]. The application of Bayesian regularization in SHM and its potential in relaxing the ill-conditioning of the inverse problems are studied in [192,209,210].

Another regularization method which has been shown to be efficient for impact force identification problems is the sparse regularization [67]. This method is based on the sparsity exploration of the forces in time or space domain and hence can be effectively employed for the identification of forces with sparse structure [65], namely, (i) impact forces (sparsity in time domain) [15,213], (ii) concentrated forces (sparsity in space domain) [214,215], and (iii) forces with sparse presentation, for example by employing Fourier series or wavelets [14,67]. However, sparse regularization can be highly sensitive to noise in the data, which may lead to inaccurate or unstable estimates, especially in the presence of significant noise. Furthermore, finding the optimal sparse solution often requires solving complex optimization problems, which can be computationally intensive and time-consuming, particularly for large datasets or high-dimensional problems. Moreover, the performance of sparse regularization depends heavily on the choice of the regularization parameter. Improper selection can lead to either overfitting (if too small) or underfitting (if too large), requiring careful tuning and validation.

Recent advancements have explored combining classical Tikhonov regularization with sparsity-promoting l_1-norm constraints. For instance, a hybrid regularization framework is proposed in [19] that balances smoothness and sparsity in the reconstructed forces. These hybrid methods are particularly effective in capturing localized or transient forces while suppressing high-frequency noise.

5.4.2. Iterative regularization

Iterative solvers have gained attention for their robustness in noisy environments. Unlike direct methods, these algorithms update the solution iteratively using randomly selected measurement equations, allowing for better error averaging and improved convergence under high noise conditions. There are various iterative regularization methods in the literature, such as Levenberg–Marquardt regularization method [216], the Landweber method [68,188,217,218], the Kaczmarz method [70,71], and Krylov subspace methods [72], which include Conjugate Gradients [219,220], Least Square QR (LSQR) [221], and LSMR [222]. The main advantages of iterative regularization methods are that they do not need extensive regularization parameter selection and are therefore suitable for large-scale problems [72], and that they generally produce more reliable and accurate solutions. Krylov subspace methods have become more popular recently compared to Landweber and Kaczmarz methods because of their faster convergence [223]. However, the Krylov subspace method may lead to poor solution accuracy without proper stopping criteria due to more semi-convergence behavior [224]. On the other hand, the Landweber method is still better in terms of simplicity and stability which makes it more applicable in some real-world situations [225,226]. Adaptive Landweber method [227] has been also developed to speed up the convergence of this method. But its performance is not enhanced when there is no suitable preset parameter. Generally, the weaknesses of iterative approaches can be listed as follows:

- (i) Iterative methods can sometimes converge to local minima rather than the global minimum, especially if the initial guess is not close to the optimal solution. This can result in suboptimal performance.

Table 3
A list of regularization methods.

Regularization method		Strength	Weakness
Direct approach	Tikhonov regularization	Simplicity	(i) Computationally ineffective for large-size inverse problems (ii) Not applicable in case of nonsmooth excitation signals
	Truncated Singular Value Decomposition (TSVD)	Effective when the solution instability is caused by high-frequency contents	Less identification accuracy
	Bayesian regularization	(i) Adaptive choice of the regularization parameters (ii) Detection of the noise level from the measurements (iii) Identification of the impact shape, duration, peak, and energy (iv) Probabilistic description of the impact force	Fluctuated reconstructed impact forces with negative values
	Sparse regularization	Effective for the identification of forces with sparse structure, e.g., impact force	(i) Sensitivity to noise (ii) Computational complexity (iii) Dependence on the choice of the regularization parameter
Iterative approach	Levenberg–Marquardt Landweber Kaczmarz Krylov subspace methods	(i) Suitable for large-scale inverse problems (ii) More reliable and accurate solutions compared to direct approach	(i) Possibility of suboptimal performance (ii) Computational cost (iii) Sensitivity to parameters (iv) Slow convergence (v) Dependence on initial guess

- (ii) Iterative methods can be computationally expensive, particularly for large-scale problems, as they require multiple iterations to converge. The computational cost increases with the number of iterations and the complexity of each iteration.
- (iii) These methods are sensitive to the choice of algorithm-specific parameters, such as step size in Landweber or damping factors in Levenberg–Marquardt. Improper tuning of these parameters can adversely affect the convergence rate and solution quality.
- (iv) In some cases, especially with ill-conditioned problems, iterative methods can exhibit slow convergence, requiring a large number of iterations to achieve a satisfactory solution.
- (v) The effectiveness of iterative approaches often depends on the quality of the initial guess. Poor initial guesses can lead to slow convergence or failure to converge.

Recently, a probabilistic regularized load reconstruction method was proposed, incorporating an iterative strategy to address uncertainty [228]. This method accounts for uncertainty factors in both regularization parameter selection and the reconstruction process. Compared to traditional methods, it provides more accurate and robust results, with the added benefit of quantifying the effect of uncertainty within the framework of probability theory.

A summary of the above discussion on direct and iterative regularization methods is provided in Table 3. Also, refer to [181] for a critical survey on modern nonlinear regularization techniques.

6. Applicability and real-time feasibility

Real-time identification and reconstruction of impact forces are essential for applications such as structural health monitoring and active control systems. Various methods have demonstrated real-time capability, balancing latency, computational load, and practical feasibility to different degrees.

Piezoelectric sensor-based methods utilize the direct measurement of stress waves induced by impact forces. These sensors detect transient stress wave signals with minimal delay, enabling impact force characterization in real time. Supported by finite element modeling for calibration, these approaches combine low latency and moderate computational effort, making them highly applicable for continuous monitoring and active control scenarios that demand rapid and precise impact detection [229].

The dynamic reduced dictionary approach leverages the sparsity of impact forces in both temporal and spatial domains, enabling efficient and rapid identification by focusing computations only on sparse features of the signal. This significantly reduces the computational load while maintaining detection accuracy, making it well-suited for real-time applications [230].

Data-physics hybrid-driven deep learning methods integrate physics-based models with data-driven deep learning architectures, specifically time-reversed LSTMs, to resolve the inverse problem of impact force identification. This approach effectively mitigates errors resulting from limited experimental data and inaccuracies in transfer matrices, achieving low latency with moderate computational demand suitable for active control system integration [231].

The Weighted Reference Database Method (WRDM) constructs a sparse reference database with bicubic interpolation to increase the density of impact reference points. It further employs a cosine distance variant for weighted localization, balancing localization accuracy and computational efficiency. This method achieves high impact localization accuracy with real-time capability. The use of a compact, interpolated database reduces computational cost and improves feasibility for deployment in active control and monitoring [232].

Challenges in data-driven impact localization methods arise when localizing impacts outside the training coverage area, particularly in anisotropic composite structures. These challenges have been addressed in [175] by incorporating the physical dispersion relations of impact-induced flexural waves, derived from first-order shear deformation and classical laminate theories, into the localization process. The wave velocity profile was explicitly formulated and optimized with respect to structural stiffness and wave frequency using gradient-based techniques. Experimental validation on composite panels demonstrated the efficiency and accuracy of this hybrid physics-based and data-driven method, requiring minimal training data and successfully localizing impacts, even outside the training coverage area. Additionally, the methods discussed in [233] further enhance impact localization by addressing uncertainties and improving group velocity profile estimation in composite structures, thereby expanding the applicability of the approach.

While computational efficiency is a key consideration, it is important to note that systematic runtime benchmarks are seldom reported in the literature. Performance metrics such as latency and throughput vary widely depending on factors such as problem size, noise levels, algorithmic formulation, and hardware configuration. If reported at all, such metrics are typically context-specific and not generalizable. For this reason, we have opted not to include detailed performance comparisons, as doing so without consistent data could be misleading.

Methods differ in their trade-offs among latency, computational load, and localization accuracy. Dynamic reduced dictionaries and deep learning hybrids emphasize efficient computation and accuracy, suitable for real-time control. Hybrid methods integrate recognition and optimization to speed processing while maintaining precision. WRDMs exploit sparse data structures with interpolation to enhance speed and accuracy balance, and piezoelectric sensor-based approaches provide rapid, direct force sensing essential for active monitoring. Selecting a method depends on system requirements related to computational resources, latency tolerance, and accuracy needs. Continued research is warranted to further optimize these techniques and develop hybrid strategies that integrate their strengths for robust real-time impact identification.

7. Conclusions

In many real-world applications, the impact forces acting on structures cannot be directly measured. In such cases, impact force reconstruction and localization techniques are employed. Different reconstruction as well as localization methods are reviewed in this paper, giving an overview of the relevant literature. Herein, both model-based and machine/deep learning approaches are presented. Available methods are categorized and some are described in detail to provide an insight about the characteristic of each category. Furthermore, the strengths and drawbacks of each method are discussed which can help researchers to choose the right approach based on their problem characteristics. As these inverse problems are ill-posed, some applicable mathematical tools are introduced, which can be exploited to relax the ill-conditioning. Lastly, some applications of force identification approaches are presented.

A solid basis has been established for impact force reconstruction and localization in the literature. Still, there is room to improve the applicability and efficiency of these techniques for real-world applications. Future works could, firstly, focus on the real-time suitability of available impact localization and reconstruction methods which is of the utmost importance particularly when active control needs to be performed. Secondly, the available techniques can be strengthened to work efficiently in case of limited instrumentation, since it might be impossible to install the desired number of sensors in a structure. Thirdly, the identification of multiple concurrent impact forces is another issue worthy of in-depth investigation. And fourthly, the employment of machine learning approaches for this research area is still in its early stages. In the current paper, some state-of-the-art deep learning methods were presented along with how they can be exploited for impact force identification.

CRediT authorship contribution statement

Hamed Kalhori: Writing – review & editing, Writing – original draft, Supervision, Resources, Project administration, Investigation, Funding acquisition. **Shabnam Tashakori:** Writing – original draft, Validation, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Ben Halkon:** Writing – review & editing, Supervision, Funding acquisition. **Mehrisadat Makki Alamdari:** Writing – review & editing, Supervision. **Bing Li:** Writing – review & editing, Visualization, Supervision. **Morteza Saberi:** Writing – original draft, Investigation, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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