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Never say never: Optimal exclusion and reserve prices with expectations-based loss-averse buyers ☆

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We analyze reserve prices in auctions with independent private values when bidders are expectations-based loss averse. We find that the optimal public reserve price excludes fewer bidder types than under risk neutrality. Moreover, we show that public reserve prices are not optimal as the seller can earn a higher revenue with mechanisms that better leverage the "attachment effect". We discuss two such mechanisms: i) an auction with a secret and random reserve price, and ii) a mechanism where an auction with a public reserve price is followed by a negotiation if the reserve price is not met. Both of these mechanisms expose more bidder types to the attachment effect, thereby increasing bids and ultimately revenue.

1. Introduction

Reserve prices are a prevalent tool auctioneers use to raise their expected revenue. A reserve price acts as an additional bid placed by the auctioneer since, in order to win, a buyer must also outbid the reserve. Thus, a reserve price increases the competitiveness of an auction. Yet, this comes at a cost for the auctioneer because trade does not happen if no buyer bids at least the reserve price. Indeed, a reserve price excludes buyers with relatively low values from the auction and reduces the overall probability of trade. Seminal theoretical contributions by Myerson (1981) and Riley and Samuelson (1981) have characterized the revenue-maximizing reserve price as the solution to this trade-off between decreasing the probability of trade and amplifying competitive pressure. In particular, they show that with risk-neutral bidders having independent private values, the optimal reserve price coincides with the classical monopoly price; hence, it is (i) deterministic and public, (ii) always higher than the seller's own value, and (iii) under mild conditions on the distribution of bidders' values, independent of the number of bidders.

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However, these features are not always empirically observed. For instance, in real-world auctions sellers often use secret reserve prices. A prime example is that of real-estate auctions in the Australian state of Queensland, where prospective buyers are allowed to know whether the seller set a reserve price, but not its exact value.[1] Moreover, some studies show that in practice reserve prices are often significantly lower than what the classical models predict; see Paarsch (1997) and Haile and Tamer (2003). There is also evidence of reserve prices that vary with the number of bidders and auctions with no reserve price at all; see Davis et al. (2011) and Gonçalves (2013). Overall, this evidence suggests that sellers may face additional trade-offs not captured by the classical model with risk-neutral (or risk-averse) bidders.

In this paper, we analyze reserve prices in first-price auctions (FPA) and second-price auctions (SPA) where symmetric bidders have independent private values (IPV) and are expectations-based loss averse á la Kőszegi and Rabin (2006, 2007). We derive the revenue-maximizing reserve price for each format and highlight how loss aversion modifies the seller's trade-off between increasing competitive pressure and reducing the probability of trade. In particular, we show that loss aversion can rationalize reserve prices that (i) are secret, (ii) vary with the number of bidders, and (iii) are lower than what the theory predicts for risk-neutral and risk-averse bidders.

Section 2 introduces the auction environment and bidders' preferences, and describes the solution concept. Following Kőszegi and Rabin (2006), we posit that, in addition to classical material utility, a bidder also experiences "gain-loss utility" when comparing her material outcomes to a reference point equal to her expectations regarding those same outcomes, with losses being more painful than equal-size gains are pleasant; furthermore, in order to simplify the exposition, we abstract from loss aversion over money in our baseline model.

We apply the solution concept of "unacclimating personal equilibrium" (UPE) introduced by Kőszegi and Rabin (2007). According to this concept, bidders choose the strategy that maximizes their payoff keeping expectations fixed, and the distribution of outcomes so generated must coincide with the expectations; hence, when deviating from her equilibrium bid, a bidder holds her reference point fixed.[2] As there might be multiple UPEs, we assume bidders select their preferred personal equilibrium (PPE) — the one that maximizes their utility from an ex-ante perspective.

We begin our analysis in Section 3 by deriving the revenue-maximizing public reserve price in the FPA.[3] First, we show that the "attachment effect" (Kőszegi and Rabin, 2006) is the main driving force behind the bidding behavior of loss-averse buyers. In particular, the higher the probability with which a bidder, in equilibrium, expects to win the auction, the bigger the loss she endures if she ends up losing it. Hence, a bidder has an incentive to increase her bid, so as to win more often and avoid experiencing the loss. Thus, the attachment effect induces an upward pressure on the equilibrium bidding strategy, ensuring that a bidder's cost from an upward deviation is higher than the benefit. Importantly, the attachment effect only influences the incentives of a bidder who expects to win the auction with strictly positive probability — however small — and is thus exposed to potential losses in equilibrium; yet, it does not affect those bidders who abstain from the auction since they do not incur a loss when not winning it.

The fact that bidders who do not expect to win are not exposed to the attachment effect has several implications for the characterization of the revenue-maximizing public reserve price. First, it puts downward pressure on the optimal reserve price. To see why, take the revenue-maximizing one with risk-neutral bidders. In the risk-neutral benchmark, the seller has no incentive to lower this reserve price because it optimally balances exclusion and competitive pressure. However, with expectations-based loss-averse bidders, lowering the reserve price creates attachment for some bidders who were previously excluded. This, in turn, raises their willingness to pay, thereby increasing the competitive pressure on all bidders with higher types. Hence, with expectations-based loss aversion, a seller optimally excludes fewer types. This finding is especially relevant for those empirical studies that use the theoretical insights of Myerson (1981) and Riley and Samuelson (1981) to estimate the revenue-maximizing reserve price as the minimum bid that excludes all bidders with a "virtual value" lower than the seller's value. For instance, Paarsch (1997) and Haile and Tamer (2003) find that sellers exclude fewer types than their estimation predicts as revenue maximizing. Yet, our result implies that it is optimal to serve bidders with virtual values lower than the seller's own value because of the attachment effect. Risk aversion can rationalize such low reserve prices in the FPA (Hu et al., 2010), but not in the SPA.[4]

Furthermore, the optimal public reserve price varies with the number of bidders. Indeed, the more bidders are present, the less optimistic each of them is about her chances of winning, which reduces their attachment. Yet, this does not imply that the reserve price always increases in the number of bidders since, by raising the reserve price, the seller forgoes the attachment effect of the excluded types. However, with already many bidders participating, adding an extra one reduces this cost, leading to more exclusion.[5] With risk aversion, instead, the optimal reserve price (in the FPA) is always decreasing in the number of bidders; see Vasserman and Watt (2021).

[1] Secret reserve prices are also documented by Elyakime et al. (1994) and Li and Perrigne (2003) in timber auctions, by Ashenfelter (1989) in auctions for fine art and wine, and by Bajari and Hortaçsu (2003, 2004) and Hasker and Sickels (2010) in internet auctions.

[2] For other applications of UPE see, for instance, Heidhues and Kőszegi (2008, 2014), Karle and Peitz (2014, 2017), Karle and Möller (2020), Karle and Schumacher (2017), and Rosato (2016).

[3] As shown by Balzer and Rosato (2021), under UPE the FPA and SPA are revenue equivalent; therefore, our results on the reserve price for the FPA carry over to the SPA.

[4] Beyond risk aversion, several explanations for low reserve prices in both the SPA and FPA have been proposed. These include correlated or interdependent values (Levin and Smith, 1996; Quint, 2017; Hu et al., 2019), endogenous entry (McAfee, 1993; Levin and Smith, 1994; Peters and Severinov, 1997), bidders' selection neglect with privately-informed sellers (Jehiel and Lamy, 2015), seller's uncertainty over the distribution of bidders' values (Hernández-Chanto and Kim, 2024), level-k bidders (Crawford et al., 2009) and taste projection (Gagnon-Bartsch et al., 2021).

[5] Menicucci (2021) obtains a similar result in the classical IPV risk-neutral model when the bidders' virtual values are not monotone; in contrast, our result holds also for the regular case of increasing virtual values.

The fact that the attachment effect does not operate on those bidders excluded by a public reserve price suggests that a seller could raise an even higher revenue by exposing more bidders to this effect. In Section 4 we show that this intuition is correct. In particular, we characterize two tactics whereby a seller can expose almost all bidders to the attachment effect, resulting in a strictly larger revenue than an auction with a public reserve price.

In Subsection 4.1 we show that secret and random reserve prices are revenue superior to public and deterministic ones. To see why, notice that with a secret reserve price each bidder type expects to win the auction with strictly positive — albeit potentially arbitrarily small — probability. In such an auction, therefore, every bidder is exposed to potential losses and thus has an incentive to bid more aggressively in order to avoid them. Hence, by transforming the public reserve price into a secret one, the seller can ensure that every bidder experiences the attachment effect, which enhances revenue. By doing so, however, the seller also reduces the competitive pressure on the buyers' side, which could potentially harm revenue since those low-type bidders excluded under a public reserve would be participating now. Yet, the seller can choose a distribution for the (secret) reserve price that puts large probability mass on relatively high prices and arbitrarily small mass on low ones. Such a distribution ensures that, while the seller exposes every bidder type to the attachment effect, the competitive pressure is almost the same as under a public reserve price.

Thus, expectations-based loss aversion provides a novel rationale for secret *and* random reserve prices. This result is reminiscent of those in Heidhues and Kőszegi (2014) and Hancart (2024), who characterize the optimal pricing strategy for a monopolist selling to an expectations-based loss-averse buyer. In line with the findings of Azevedo and Gottlieb (2012), who showed that risk-neutral sellers benefit from offering gambles to consumers exhibiting prospect-theory preferences, these papers find that the monopolist benefits from using random prices. In particular, Heidhues and Kőszegi (2014) show that if the seller has sufficient commitment power, a stochastic pricing scheme featuring low, variable sale prices and a high, sticky regular price yields more revenue than posting a single price. Our secret and random reserve price scheme has similar features, but differs from their characterization since we consider an environment with multiple, privately-informed buyers. Nonetheless, we are able to draw a connection between the optimal reserve price and the optimal monopoly pricing scheme with expectations-based loss-averse buyers that is analogous to the well-known one for risk-neutral buyers.

In Subsection 4.2 we show that the seller can achieve a higher revenue than what is achievable with a public reserve price by employing a simple two-stage mechanism. In this mechanism, the seller first runs an auction with a public reserve price; then, if the reserve price is not met, with some probability the seller posts a price that would be accepted by some bidder types who did not bid in the auction. In this way, the seller exposes to the attachment effect also some bidder types who were excluded from the initial auction; this, in turn, pushes all the bidder types who participate in the auction to bid more aggressively, thereby increasing the overall revenue.

We analyze two extensions of our baseline model in Section 5. First, we allow for loss aversion also in the money dimension and show that our main result — that a profit-maximizing seller facing loss-averse bidders prefers secret and random reserve prices to deterministic ones — continues to apply. As in the baseline case, secret (and random) reserve prices allow the seller to expose all bidder types to potential losses, leading to more aggressive bidding. Moreover, we show that a seller who is confined to using deterministic reserve prices continues to exclude fewer buyer types compared to the risk-neutral benchmark if loss aversion in money is moderate relative to loss aversion in the good's dimension. Finally, we establish that random exclusion is a feature of any optimal static (potentially asymmetric) selling mechanism.

Section 6 concludes the paper by summarizing the results of our model and discussing some further implications. All proofs are relegated to Appendix A.

2. The model

In this section, we introduce the auction environment and bidders' preferences, and provide a formal definition of our solution concept (UPE) in the context of sealed-bid auctions.

2.1. Environment

A seller auctions off an item to $N \geq 2$ bidders via a sealed-bid auction. Each bidder $i \in \{1, 2, ..., N\}$ has a private value (or type) t_i independently drawn from the support $[\underline{t}, \overline{t}]$, with $\overline{t} > \underline{t} = 0$, according to the same cumulative distribution function F.[6] We assume that F is continuously differentiable, with strictly positive density f on its support. Further, we impose the standard assumption that F has a monotone hazard rate; i.e., $\frac{f(x)}{1-F(x)}$ is increasing for all $x \in [\underline{t}, \overline{t}]$. This, in turn, implies that bidders' "virtual values" are increasing; i.e., $V(t_i) \equiv t_i - \frac{1-F(t_i)}{f(t_i)}$ is increasing in t_i. The seller has a commonly-known value $t^S \in [0, \overline{t})$.

We consider two canonical selling mechanisms: the first-price sealed-bid auction (FPA) and the second-price sealed-bid auction (SPA).[7] We restrict attention to symmetric monotone equilibria in pure strategies; in such equilibria, the bidder with the highest type wins the auction, conditional on placing a bid above the reserve price. Hence, by focusing on symmetric equilibria, we concentrate on environments where the FPA and SPA are revenue equivalent and efficient conditional on trade. Let F_1 denote the cumulative

6 We normalize $\underline{t} = 0$ to simplify the exposition. Moreover, under this assumption, a seller facing risk-neutral bidders would always choose a non-trivial reserve price; i.e., there are no corner solutions.

7 Throughout the paper, we only consider symmetric (i.e., non discriminatory) auction mechanisms; for a recent analysis of asymmetric auctions with expectations-based loss-averse bidders see Muramoto and Sogo (2024).

distribution function of the highest order statistic among $N-1$ draws, and denote by f_1 its corresponding density. Finally, let r_{RN} denote the revenue-maximizing reserve price with risk-neutral bidders, and notice that $r_{RN} > 0$ since $\underline{t} = 0$.

2.2. Bidders' preferences and solution concept

Consider a bidder participating in either an FPA or an SPA; depending on her bid and her opponents' ones, she might either win the auction ($q = 1$) in which case she receives the item and pays a price $p \in \mathbb{R}_+$, or lose the auction ($q = 0$) in which case she does not obtain the good and pays nothing. We assume bidders have expectations-based reference-dependent preferences as formulated by Kőszegi and Rabin (2006, 2007). Accordingly, the utility of bidder i with type t_i has two components. First, her material utility is given by $q(t_i - p)$, with $q \in \{0, 1\}$. Second, the bidder also derives psychological utility from comparing the realized outcome to a reference outcome given by her recent expectations (probabilistic beliefs).[8] Hence, given an outcome (q, p) and a deterministic reference point $\tilde{q} \in \{0, 1\}$, a bidder's total utility is

$$U\left[(q,p)\,|\tilde{q}, t_i\right] = q(t_i - p) + \mu\left(qt_i - \tilde{q}t_i\right) \tag{1}$$

where

$$\mu(x) = \begin{cases} \eta x & \text{if } x \geq 0, \\ \eta\lambda x & \text{if } x < 0 \end{cases}$$

is *gain-loss utility*, with $\eta \geq 0$ and $\lambda > 1$. The parameter η captures the weight a bidder attaches to gain-loss utility while λ is the coefficient of loss aversion.[9] Notice that bidders' preferences reduce to the risk-neutral benchmark for $\eta = 0$.

Because in many situations expectations are stochastic, Kőszegi and Rabin (2006, 2007) allow for the reference point to be described by a distribution H over the possible values of $\tilde{q}$; then, fixing H, a bidder's total utility from the outcome (q, p) can be written as

$$U\left[(q,p)\,|H, t_i\right] = q(t_i - p) + \int_{\tilde{q}} \mu\left(qt_i - \tilde{q}t_i\right) dH(\tilde{q}).$$

In words, a bidder compares the realized outcome to all possible outcomes in the reference lottery, each one weighted by its respective probability.

A bidder learns her type before submitting a bid and, hence, maximizes her interim expected utility. If the distribution of the reference point is H and the distribution of consumption outcomes is $G = (G^g, G^m)$, the interim expected utility of a bidder with type t_i is

$$EU\left[G|H, t_i\right] = \int_{\{q,p\}} \int_{\{\tilde{q}\}} U\left[(q,p)\,|\tilde{q}, t_i\right] dH(\tilde{q})\, dG(q,p).$$

We assume that bidders first observe the details of the auction and the reserve price r (if public), and then form expectations about consumption outcomes based on their strategies. A strategy for bidder i is a function $\beta_i : [\underline{t}, \bar{t}] \to \mathbb{R}_+$. Fixing all other bidders' strategies, $\boldsymbol{\beta}_{-i}$, the bid of bidder i with type t_i, $\beta_i(t_i)$, induces a distribution over the set of final consumption outcomes. Let $\Gamma\left(\beta_i(t_i), \boldsymbol{\beta}_{-i}\right)$ denote this distribution. According to Kőszegi and Rabin (2007), when a decision is made shortly before outcomes realize, the reference point is fixed by past expectations; then, when the decision maker chooses the bid that maximizes her expected utility, she takes the reference point as given. Being fully rational, therefore, she can plan to submit a bid only if she is willing to follow it through, given the reference point determined by the expectation to do so. This is what Kőszegi and Rabin (2007) call unacclimating personal equilibrium (UPE):

Definition 1. A strategy profile $\boldsymbol{\beta}^*$ constitutes an Unacclimating Personal Equilibrium (UPE) if for all i and for all t_i:

$$EU\left[\Gamma\left(\beta_i^*(t_i), \boldsymbol{\beta}_{-i}^*\right)|\Gamma\left(\beta_i^*(t_i), \boldsymbol{\beta}_{-i}^*\right), t_i\right] \geq EU\left[\Gamma\left(b, \boldsymbol{\beta}_{-i}^*\right)|\Gamma\left(\beta_i^*(t_i), \boldsymbol{\beta}_{-i}^*\right), t_i\right]$$

for any $b \in \mathbb{R}_+$.

Thus, if a bidder deviates to a different bid, her reference point does not change. Notice that there might exist multiple UPEs; that is, multiple bids that the bidder is willing to follow through. In this case, following Kőszegi and Rabin (2006, 2007), we assume that

[8] Banerji and Gupta (2014), Rosato and Tymula (2019) and Eisenhuth and Grunewald (2020) provide experimental support for the Kőszegi and Rabin's model in the context of sealed-bid auctions.

[9] To clearly highlight the implications of the attachment effect on bidding incentives, we depart from the original formulation of Kőszegi and Rabin (2006, 2007) by considering buyers who are loss averse only with respect to their value for the item, but not with respect to the price they might pay; in other words, we assume buyers are risk neutral over money. As argued in Kőszegi and Rabin (2009), this assumption is reasonable if buyers' income is already subject to large background risk; relatedly, Novemsky and Kahneman (2005) propose that money given up in purchases is not generally subject to loss aversion. However, as we argue in Section 5, our results continue to hold when allowing for loss aversion over money.

the bidder selects the UPE that provides her with the highest expected utility. Hence, bidders play according to their (symmetric) Preferred Personal Equilibrium (PPE).[10]

Kőszegi and Rabin (2007) also propose an alternative solution concept, called choice-acclimating personal equilibrium (CPE), whereby agents choose the strategy that maximizes their expected payoff given that the strategy determines both the distribution of the reference point and the distribution of outcomes; in an auction setting, this entails that a bidder would not keep her reference point fixed when deviating from her equilibrium bid. We choose to focus on UPE rather than CPE for two reasons. First, from a theoretical point of view, optimal reserve prices under CPE preferences have already been studied by Eisenhuth (2019) for symmetric mechanisms and Muramoto and Sogo (2024) for (potentially) asymmetric ones. These authors show that under CPE the optimal reserve price is always public and deterministic. The reason is that with CPE bidders are able to commit to their strategy in advance; in turn, this makes bidders more risk averse under CPE than UPE, which reduces the seller's ability to manipulate their expectations by using random and secret reserve prices. Second, from a more applied point of view, we think that UPE is the more appropriate solution concept for describing the behavior of consumers in private-value auctions. Indeed, while institutional bidders are usually able to commit to their bids via, for instance, an agent who bids on their behalf, occasional bidders can often revise them just before the auction, at which point their reference point is sunk. Moreover, auctions with private values, where bidders do not have to extract information from their competitors' bids, share many similarities with the kind of purchasing decisions with which consumers are already familiar (e.g., bidding for airplane tickets or hotel accommodation on platforms such as Expedia and Priceline.com). For these purchasing decisions, UPE is the most common solution concept; see Heidhues and Kőszegi (2014), Rosato (2016), and Karle and Schumacher (2017).

Finally, we should make a remark about the terminology going forward. In the classical risk-neutral benchmark, a bidder's type coincides with her valuation or willingness to pay. However, with expectations-based loss aversion this is not the case anymore since, even fixing her type, a bidder's willingness to pay varies with her reference point and hence her expectations; therefore, bidders' valuations are endogenous to the mechanism. To avoid confusion, throughout the paper we use the term "value" when referring to a bidder's type and to properties of the type distribution; i.e., the t's. We instead use "willingness to pay" to refer to a bidder's overall valuation, which is shaped by the rules of the mechanism and how these affect her reference point.

3. Deterministic and public reserve price

This section characterizes the optimal public reserve price. Let $q(t)$ denote the probability with which, in equilibrium, a type-t bidder wins the auction. In the FPA, without a reserve price, the highest bidder wins the good and pays her bid. Hence, in a symmetric equilibrium, it holds that $q(t) = F_1(t)$ for all $t \in [\underline{t}, \bar{t}]$. However, with a (binding) reserve price r, there is a threshold type t_r such that $q(t) = 0$ for all $t \in [\underline{t}, t_r)$ and $q(t) = F_1(t)$ for $t \in [t_r, \bar{t}]$; that is, all bidders with types below t_r prefer not to participate in the auction.

Fix a symmetric and increasing bidding strategy, $\beta_I : [\underline{t}, \bar{t}] \mapsto \mathbb{R}_+$. Moreover, fix r and the implied t_r, to be determined shortly, and consider a type-t bidder who mimics a *larger* type $\tilde{t} > t$.[11] With a slight abuse of notation, denote her expected payoff by $EU(\tilde{t}, t)$; this is given by

$$EU(\tilde{t}, t) = q(\tilde{t})(t - \beta_I(\tilde{t})) - \eta\lambda(1 - q(\tilde{t}))q(t)t + \eta(1 - q(t))q(\tilde{t})t, \tag{2}$$

where $q(x) = F_1(x)$ if $x \geq t_r$ and $q(x) = 0$ otherwise.

The first term on the right-hand side of (2), $q(\tilde{t})(t - \beta_I(\tilde{t}))$, is the bidder's expected material payoff. The second and third terms represent the (expected) gains and losses for a bidder who planned to bid $\beta_I(t)$, hence expecting to win with probability $q(t)$, but then deviates and bids $\beta_I(\tilde{t})$, and thus wins with probability $q(\tilde{t})$. Whenever she loses, the bidder experiences a loss of $\eta\lambda q(t)t$ weighted by the probability of losing the auction, $1 - q(\tilde{t})$. Similarly, given that the bidder expected to lose the auction with probability $1 - q(t)$, if she ends up winning it she experiences a gain of $\eta(1 - q(t))t$ weighted by the probability with which that gain occurs, $q(\tilde{t})$.

Now consider a bidder with type $t \in [\underline{t}, t_r)$. In equilibrium, such a bidder does not want to mimic the threshold type t_r implying that

$$\begin{aligned} &EU(t_r, t) \leq EU(t, t) \Leftrightarrow \\ &F_1(t_r)(1 + \eta)t \leq F_1(t_r)r. \end{aligned} \tag{3}$$

To understand condition (3) note that, in equilibrium, bidders whose types are in $[\underline{t}, t_r)$ do not participate in the auction and thus expect to win the good with zero probability. If one of these bidders deviates and mimics type t_r, she then wins the auction with probability $F_1(t_r)$. Thus, her expected gains from deviating entail a material gain of $F_1(t_r)t$ and a psychological gain of $F_1(t_r)\eta t$, since she expected to lose the auction for sure; hence, the terms on the left-hand side of (3) represent the benefits from deviating and submitting a bid equal to the reserve price. The right-hand side of (3) captures the expected costs from such a deviation — the increase in the expected payment, $F_1(t_r)r$. Letting $t \to t_r$ from below, and noting that (3) holds with equality in the PPE, we obtain the following relationship between the reserve price and the threshold type:[12]

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[10] See also Heidhues and Kőszegi (2014), Rosato (2016), Freeman (2019), and Balzer and Rosato (2021).

[11] As shown by Balzer and Rosato (2021), in the PPE bidders' upward incentive constraints are the binding ones. In particular, for a fixed reserve price r, there is a continuum of symmetric UPEs and we focus on the one with the highest threshold type t_r; as the proof of Lemma 1 shows, this is every type's preferred symmetric UPE.

[12] Note that $r > t_r$ follows from our assumption of no loss aversion over money. See Section 5 for the case where the bidder is also loss averse with respect to money.

$$r = (1+\eta)t_r. \tag{4}$$

For a given type t', in the following we denote the solution to (4) by $r(t')$ (i.e., the reserve price assuring that $t_r = t'$). Using the relationship between the threshold type t_r and the reserve price, we then apply the standard logic from auction theory: equilibrium behavior shapes the bidding function up to a constant, which is pinned down by type t_r's expected payment, $F_1(t_r)r$. The next lemma formally states a type-t bidder's expected payment in the PPE for a given t_r.

Lemma 1. *Consider the PPE of an FPA with a deterministic reserve price. Let t_r be the lowest type that receives the good with strictly positive probability. The expected payment from a bidder with type $t \geq t_r$ is:*

$$F_1(t)\beta_I^*(t) = \int_{t_r}^{t} [1+\eta\lambda F_1(s) + \eta(1-F_1(s))] f_1(s) s\, ds + (1+\eta)F_1(t_r)t_r \tag{5}$$

and 0 for any $t < t_r$.

Notice that expressions (4) and (5), and hence $\beta_I^*(t)$, reduce to their well-known risk-neutral analogues if $\eta = 0$. For $\eta > 0$ and $t \geq t_r$, instead, $\beta_I^*(t)$ is strictly larger than its risk-neutral counterpart since buyers have an additional incentive to raise their bids in order to try to win more often and reduce their expected losses. This is the attachment effect: the larger the probability with which a bidder expects to win, the bigger the loss she feels if she loses, and hence the stronger her incentives to raise her bid. Thus, because of the attachment effect, bidders' willingness to pay endogenously depends on their expectations. In what follows, we will focus on the relationship between the attachment effect and the bidders' marginal willingness to pay, and its implications for the seller's choice of the optimal reserve price.

Consider a type-t bidder who expects to win the auction with probability q. How much does this bidder value an increase, $\Delta q > 0$, in her probability of winning? If her probability of winning increases by Δq, the bidder obtains the good more often; hence, she makes a material gain equal to $t\Delta q$. Moreover, the bidder's chances of enjoying a psychological gain increase, which she values at $\eta(1-q)\Delta q t$, given that she expected to lose with probability $1-q$. Similarly, by winning more often, the bidder's chances of experiencing a loss are also reduced; she values this reduction in the probability of making a loss at $\eta\lambda q\Delta q t$. Adding up all these terms, the bidder's marginal willingness to pay for such an increase in the probability of obtaining the good is equal to

$$\underbrace{t\Big(1+\eta\lambda q + \eta(1-q)\Big)}_{MWTP(t;q)}\Delta q. \tag{6}$$

It is easy to see that $MWTP(t;q)$ increases not only in the bidder's type t, but also in the probability q with which she already expects to win since $\lambda > 1$; in particular, a bidder who expects to never get the good (i.e., $q = 0$) is willing to pay less for a given Δq than a bidder who expects to obtain the good with strictly positive probability (i.e., $q > 0$). This is because only a bidder who rationally expects to win the good experiences a loss when not winning.

Consider now a bidder with a type equal to the threshold type, t_r. In equilibrium, as under risk neutrality, her expected payment is equal to the expectation of the willingness to pay of her strongest opponent; that is,

$$\begin{aligned} F_1(t_r)r &= \int_0^{t_r} MWTP(t_r;0) f_1(s)\, ds \\ &= F_1(t_r)(1+\eta)t_r. \end{aligned}$$

In other words, in equilibrium the reserve price makes the type *marginally below* the threshold type indifferent between participating by imitating the threshold type or abstaining, given that she expected not to participate.

Next, consider a type-t bidder with $t > t_r$. In equilibrium, $q(t) = F_1(t)$ and thus $\Delta q = f_1(t)$; hence, such a bidder's $MWTP(t;q)$ from mimicking a slightly larger type (and thus increasing her winning probability by $f_1(t)$) must equal the increase in the expected payment, $(F_1(t)\beta_I^*(t))'$, from such a deviation. That is, $F_1(t)\beta_I^*(t) = \int_{t_r}^{t} MWTP(s;F_1(s))f_1(s)ds + F_1(t_r)r$, i.e., (5).

The preceding discussion highlights how the attachment effect, which is revenue enhancing for the seller, has a bigger influence on the behavior of those bidder types who expect to win with strictly positive probability than on the behavior of the type marginally below the threshold type, whose bidding incentives determine the reserve price. This occurs because the seller can charge the threshold type a price capturing only the additional gain from winning experienced by a marginally smaller type, but not the benefit from avoiding losses. This asymmetry is crucial for the determination of the optimal reserve price under loss aversion r^* and, in turn, the degree of bidder exclusion, as the next result shows.

Proposition 1. *The optimal threshold type in the FPA, t_r^*, is smaller than that under risk neutrality, t^{RN}. Moreover, as λ approaches infinity, both t_r^* and r^* converge to zero, even if $t^S > 0$.*

Hence, with expectations-based loss-averse bidders, the seller optimally excludes fewer types than in the risk-neutral benchmark. To see the intuition, consider the seller's trade-off when setting the optimal threshold type; she chooses t_r in order to maximize her expected profit given by

$$N \int_{t_r}^{\bar{t}} F_1(s)\beta_I^*(s)f(s)ds + F(t_r)^N t^S, \tag{7}$$

where the integral term represents the expectation over buyers' expected payments, as given in equation (5), and the last term is the seller's payoff if no trade takes place. The derivative of the seller's profit with respect to t_r takes the following form:

$$N\left[(1+\eta)f(t_r)\left(\frac{1-F(t_r)}{f(t_r)} - t_r\right) + f(t_r)t^S - \eta(\lambda-1)(1-F(t_r))f_1(t_r)t_r\right]F_1(t_r). \tag{8}$$

Note first that, because this is a symmetric environment, all terms in the first-order condition are multiplied by the number of bidders, N. The first term is proportional to the (negative of the) virtual value of the threshold type, and it captures the standard trade-off between raising competitive pressure and risking not to sell the good at all. The second term captures the standard effect of the seller's opportunity cost of selling: reducing the probability of trade by marginally raising the threshold type is less costly the higher is the seller's own value for the good. Finally, the last term captures a novel trade-off due to expectations-based loss aversion. Indeed, using (6), it is easy to see that this term equals $(MWTP(t_r,0) - MWTP(t_r, F_1(t_r)))f_1(t_r)$; i.e., the difference between the threshold type's marginal willingness to pay when expecting to lose for sure, $MWTP(t_r,0)$, and her marginal willingness to pay when expecting to win with probability $F_1(t_r)$, $MWTP(t_r, F_1(t_r))$, multiplied by the change in the winning probability when marginally increasing the threshold type, $f_1(t_r)$. If the seller raises the threshold type, she transforms an interior type into the new threshold type, thus reducing this type's attachment effect.

Therefore, from the seller's point of view, the attachment effect creates an additional cost associated with raising the threshold type and thus with excluding more types. Moreover, this additional cost increases in the degree of bidders' loss aversion. Indeed, as λ approaches infinity, the optimal reserve price converges to zero even though the seller's value for the item is strictly positive. As a consequence, the optimal threshold type with loss aversion is lower than under risk neutrality, which in turn implies less bidder exclusion.

By relating the optimal threshold type under expectations-based loss aversion to its risk-neutral counterpart, Proposition 1 is relevant not only from a theoretical point of view but also from an applied one. Indeed, several empirical papers (e.g., Paarsch, 1997; Haile and Tamer, 2003) use bids submitted in actual auctions to estimate the distribution of bidders' values and, given these estimates, conclude that sellers in the field set reserve prices, and hence threshold types, that are lower than Myerson (1981)'s optimal one (i.e., the one that equates a bidder's "virtual value" with the seller's value), resulting in too little exclusion. However, Proposition 1 shows that, for a known (or separately estimated) distribution of values, such seller behavior is consistent with profit maximization if bidders are expectations-based loss averse.

The next proposition describes how the optimal threshold type, and hence the reserve price, vary with the number of bidders.

Proposition 2. *The probability of no trade and the reserve price depend on the number of bidders. For any $t^S \geq 0$, the no-trade probability and the optimal reserve price increase in N if and only if $ln(F(t_r^*)) < -1/(N-1)$. Moreover, if $t^S = 0$ (resp. $t^S > 0$), as $N \to \infty$, the no-trade probability converges to (resp. is strictly lower than) its risk-neutral counterpart.*

From Proposition 1, we already know that $t_r^* \leq t^{RN}$. Then, to see the intuition behind Proposition 2, suppose first that $t^S = 0$ and recall that the optimal threshold type in the risk-neutral benchmark is independent of N. With expectations-based loss aversion the optimal level of bidder exclusion depends also on the attachment to which the threshold type is exposed, as captured by the last term in expression (8). It is easy to see that this term depends on $f_1(t_r)$ and hence on N. In particular, when $f_1(t_r^*)$ decreases in N, which happens if and only if $ln(F(t_r^*)) < -1/(N-1)$, the seller's cost of raising the threshold type due to the forgone attachment effect decreases in the number of bidders; consequently, she excludes more types.

In line with this intuition, as $N \to \infty$ every bidder type except $\bar{t}$ expects to win the auction with (almost) zero probability (i.e., if N is sufficiently large $f_1(t_r^*)$ decreases in N and converges to zero); in turn, the attachment effect of any bidder type but the highest one becomes negligible. In this case, the seller does not affect any such type's expectations of winning when raising the reserve price, and thus sets the same threshold type as in the risk-neutral benchmark. However, the limit probability of no trade under loss aversion is lower than its risk-neutral counterpart if $t^S > 0$. The reason is that loss-averse buyers bid more aggressively than risk-neutral ones; i.e., the seller raises more revenue from a loss-averse buyer than from a risk-neutral buyer with the same type. Hence, the seller has a weaker incentive to exclude them.

Finally, note that with risk-averse bidders the optimal reserve price depends on the number of bidders in the FPA, but not in the SPA; see Hu et al. (2010) and Hu (2011). In contrast, all the results for loss-averse bidders derived in this section hold also for the SPA. Indeed, Balzer and Rosato (2021) show that both auction formats yield the same expected revenue to the seller. Moreover, since the threshold type t_r does not face any risk in her payment conditional on winning, the relationship between r and t_r satisfies (4) also in the SPA. Hence, the optimal reserve price and the optimal threshold type in the SPA coincide with those in the FPA.

4. Exposing more bidders to the attachment effect

As the previous section highlighted, the seller benefits from the attachment effect, as it pushes buyers to bid more aggressively; however, the excluded buyers are not exposed to this effect. In this section, we investigate tactics that expose more bidders to the attachment effect, thereby boosting the seller's revenue. We focus on two such tactics. The first one is a standard auction with a secret and random reserve-price regime. Heidhues and Kőszegi (2014) and Hancart (2024) show that for a monopolist facing a loss-averse buyer, committing to a stochastic pricing strategy yields a higher revenue than posting a single price, as in this way the seller ensures that the consumer is fully attached to the good. In our setting with multiple buyers, a public (and deterministic) reserve price corresponds to a single posted price whereas a secret and random reserve price corresponds to a stochastic pricing strategy.[13] The second tactic is an auction with a public reserve price followed by a take-it-or-leave-it (TIOLI) negotiation if the reserve price is not met.

4.1. Random and secret reserve prices

In what follows, we first assume that every bidder type is exposed to the attachment effect — even those excluded — and find the revenue-maximizing auction under this assumption. This exercise leads to a strict upper bound on the seller's revenue. We then relax the assumption that all bidders are exposed to the attachment effect, and show that an auction mechanism using secret and random reserve prices achieves a revenue arbitrarily close to this upper bound.

We begin by analyzing a symmetric pseudo auction where every bidder type is exposed to the attachment effect; that is, let $\widehat{MWTP}(t; F_1(t)) \equiv MWTP(t; q)|_{q=F_1(t)}$ (as defined in (6)), even for bidder types that are excluded in the actual auction, i.e., bidders for whom $q(t) = 0$. We then calculate the bidding function, $\hat{\beta}$, for this hypothetical situation. Given this pseudo bidding function, the seller maximizes her revenue by using a public reserve price $\hat{r}$ that, if $t^S = 0$, excludes the same set of types as the optimal auction under risk neutrality; this is because if buyers were to bid according to $\hat{\beta}$, the seller would not face the additional cost from the foregone attachment effect of the marginally excluded type.

Consider first a bidder with type $t_{\hat{r}}$ who in equilibrium bids $\hat{r}$; i.e., the threshold type. As under risk neutrality, in equilibrium her expected payment equals the expectation of the willingness to pay of her strongest opponent:

$$\begin{aligned} F_1(t_{\hat{r}})\hat{r} &= \int_0^{t_{\hat{r}}} \widehat{MWTP}(t_{\hat{r}}; F_1(s)) f_1(s) ds \\ &= [1 + \eta + \eta(\lambda - 1)F_1(t_{\hat{r}})/2] F_1(t_{\hat{r}}) t_{\hat{r}}. \end{aligned}$$

Take now a type-t bidder with $t > t_{\hat{r}}$. In this hypothetical equilibrium, $q(t) = F_1(t)$ and thus $\Delta q = f_1(t)$; hence, such a bidder's (pseudo) marginal willingness to pay to increase her winning probability by $f_1(t)$ must equal the increase in the expected payment, $(F_1(t)\hat{\beta}(t))'$, from such deviation. Thus,

$$F_1(t)\hat{\beta}(t) = \int_{t_{\hat{r}}}^{t} \widehat{MWTP}(s; F_1(s)) f_1(s) ds + F_1(t_{\hat{r}})\hat{r} \tag{9}$$

$$= \int_{t_{\hat{r}}}^{t} s\Big(1 + \eta\lambda F_1(s) + \eta(1 - F_1(s))\Big) f_1(s) ds + [(1 + \eta) + \eta(\lambda - 1)F_1(t_{\hat{r}})/2] F_1(t_{\hat{r}}) t_{\hat{r}},$$

for all $t \geq t_{\hat{r}}$. Given these expected payments, the seller chooses the threshold type to maximize her profit:

$$N \times \Big(\max_{t_{\hat{r}}} \int_{t_{\hat{r}}}^{\bar{t}} F_1(t)\hat{\beta}(t) f(t) dt \Big) + F^N(t_{\hat{r}}) t^S. \tag{10}$$

Notice that $F_1(t)\hat{\beta}(t)$ represents an upper bound on the expected payment of a type-t bidder since $\hat{\beta}(t)$ is the (pseudo) bidding function in an equilibrium where every bidder type is exposed to the attachment effect, even those who are excluded by the reserve price. In this hypothetical case, the revenue-maximizing threshold type is the same as in the risk-neutral benchmark since the attachment effect impacts these types as well. Hence, the solution to the above problem provides an upper bound on the seller's profit.

Lemma 2. *In the FPA, under PPE, a strict upper bound on the seller's payoff is given by the maximum value of* (10)*.*

Next, we show that the seller can garner a profit arbitrarily close to the maximum value of (10) by using secret and random reserve prices. Specifically, before buyers submit their bids, the seller publicly announces the distribution of the reserve price and commits to drawing a reserve price according to this distribution; then, after the buyers submit their bids, the seller reveals the realization

[13] Note that in Heidhues and Kőszegi (2014) a consumer knows the realized price at the time of purchase, whereas in our model buyers submit their bids before observing the realization of the stochastic reserve price. We think this timing is reasonable as long as bidders learn about the auction rules and environment — and thus form their reference points — only once they arrive at the auction. This is the reason why our mechanism features random and *secret* reserve prices; with our timing, if the seller made the realization of the reserve price public *before* buyers submitted their bids, then the setting would essentially become one with a deterministic reserve price.

of the reserve price. While such a selling mechanism undoubtedly requires some commitment on the part of the seller, there are some real-world examples where sellers seem to have such commitment power.[14] For instance, Li and Perrigne (2003) study timber auctions conducted by a French government agency who naturally has commitment power, and where the reserve price is revealed only after all bids are submitted.[15] The next proposition formally states the result.

Proposition 3. *In the FPA, under PPE, there exists a distribution of random and secret reserve prices that yields a revenue arbitrarily close to the upper bound characterized in Lemma 2. Hence, for every public reserve price, there exists a distribution of secret and random reserve prices that enhances the seller's payoff.*

To gain some intuition, consider an FPA with a secret reserve price drawn from a commonly known distribution over some interval $[\underline{r}, \overline{r}]$; then, in a symmetric equilibrium in increasing strategies, for each possible realization of the reserve price, $r \in [\underline{r}, \overline{r}]$, there exists a corresponding threshold type, $\tilde{t}_r$, whose bid coincides with r.[16] Hence, a bidder with a type below the highest such threshold type, $\tilde{t}_{\overline{r}}$, wins the good if both (i) her type is larger than that of all other bidders, and (ii) she bids higher than the secretly drawn reserve price. In what follows, it will prove convenient to directly work with the implied distribution of threshold types. That is, suppose that the seller first draws $\tilde{t}_r \in [\tilde{t}_{\underline{r}}, \tilde{t}_{\overline{r}}]$ according to some distribution F_0. Then, the seller computes the reserve price $r(\tilde{t}_r)$ that matches the equilibrium bid of a bidder with type $\tilde{t}_r$, who expects to win the good with probability $q(t) = F_1(\tilde{t}_r)F_0(\tilde{t}_r)$.[17]

Note that the above framework nests a public reserve price as a special case that occurs if all probability mass is on one threshold type, say, $\tilde{t}_r^p \in [\tilde{t}_{\underline{r}}, \tilde{t}_{\overline{r}}]$; that is, $F_0(\tilde{t}_r) = 0$ if $\tilde{t}_r < \tilde{t}_r^p$ and $F_0(\tilde{t}_r) = 1$ otherwise. However, as argued in Section 3, with such a discontinuous threshold-type distribution, the seller forgoes the attachment effect of the bidder with type $\tilde{t}_r^p$. Indeed, the implied winning probability, $q(t)$, jumps from 0 to $F_1(\tilde{t}_r^p)$ at the threshold type and, in a PPE, this type's bid is determined by the attachment level of a type that loses the auction for sure. More generally, consider an arbitrary threshold-type distribution that might have discontinuous jump points. At each such jump point, the corresponding threshold type's bid depends on the attachment level of the type immediately below. As the seller's revenue increases in the bidders' attachment level, the seller prefers continuous distributions that smooth out jump points (see the left panel of Fig. 1).

In fact, in the proof of Proposition 3 we show that the seller can achieve a revenue arbitrarily close to the maximum value of (10) by using a continuous distribution of threshold types. This distribution is such that every bidder type expects to win the auction with strictly positive probability, as tiny as that might be; that is, $\tilde{t}_{\underline{r}} = \underline{t}$. More precisely, the distribution is such that bidders with types strictly below $\tilde{t}_{\overline{r}}$ expect to win the auction with a small probability, and this probability steeply increases for types in a neighborhood below $\tilde{t}_{\overline{r}}$. Moreover, a revenue-maximizing seller chooses $\tilde{t}_{\overline{r}}$ such that the virtual value of the largest threshold type equals zero. One way for the seller to implement such a distribution of threshold types is to use the CDF $F_0(\tilde{t}_r) = (\frac{\tilde{t}_r}{\tilde{t}_{\overline{r}}})^K$, with $K \in \mathbb{R}_+$ and "large". The right panel of Fig. 1 depicts F_0 when $\tilde{t}_{\overline{r}} = 0.5$ and $K = 30$.

Therefore, with a secret and random reserve price, all bidders with a type below the largest threshold type $\tilde{t}_{\overline{r}}$ expect to win with strictly positive probability and are thus exposed to (potential) losses. In particular, the steep increase of $q(t)$ from (almost) zero to (almost) $F_1(\tilde{t}_{\overline{r}})$ in the neighborhood below $\tilde{t}_{\overline{r}}$ ensures that types slightly below $\tilde{t}_{\overline{r}}$ have an incentive to bid aggressively in order to reduce their potential losses.

In order to obtain the distribution of the secret reserve price, start by fixing F_0, the distribution of the threshold type $\tilde{t}_r$. Then, substitute the drawn $\tilde{t}_r$ into the equilibrium bidding function that applies without a reserve price, but where a type-t bidder wins the auction with probability $q(t) = F_0(t)F_1(t)$; that is, for $\tilde{t}_r \in [\underline{t}, \tilde{t}_{\overline{r}}]$, we have

$$\beta_I^*(\tilde{t}_r) = \frac{\int_{\underline{t}}^{\tilde{t}_r} [1 + \eta\lambda q(s) + \eta(1 - q(s)]q'(s)s\,ds}{q(\tilde{t}_r)}. \tag{11}$$

In equilibrium, bidders correctly anticipate that the seller implements reserve price $r = \beta_I^*(\tilde{t}_r)$ when drawing $\tilde{t}_r$ according to CDF $F_0(\tilde{t}_r)$, and their optimal response is given by the bidding function in (11), replacing $\tilde{t}_r$ with a buyer's type t.

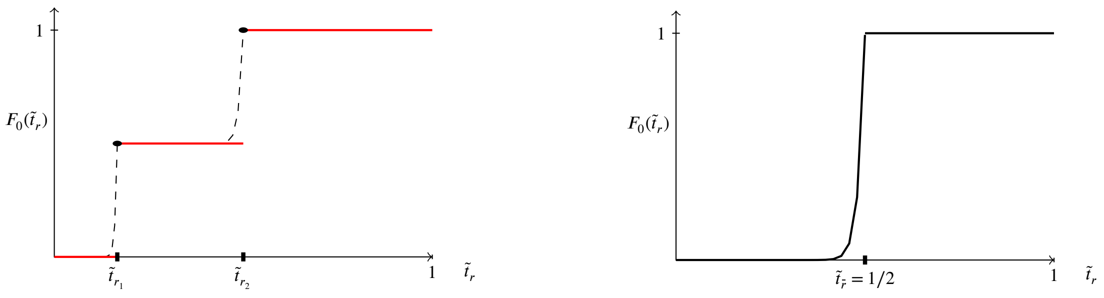
As argued by Bajari and Hortaçsu (2004), secret reserve prices are common in internet auctions. However, different from our result, these secret reserve prices are usually deterministic. Yet, from the bidders' perspective, the secret reserve price might appear as random if they do not precisely observe how the seller chooses it. Indeed, as shown in Fig. 1, the distribution of the secret reserve price has most of the mass on the upper bound of its support, and arbitrarily little mass everywhere else. While this characterization is admittedly extreme, it is in line with the real-world observation that hidden reserve prices can lead buyers to bid more aggressively. For instance, eBay suggests sellers to use a secret reserve price in order to "set a low starting price for your auction and boost interest

[14] The commitment power needed to implement a random reserve-price regime is similar to the commitment power that the monopolist in Heidhues and Kőszegi (2014) and Rosato (2016) needs to implement a stochastic pricing strategy. Hancart (2024) shows that such commitment power is essential in their setting.

[15] Li and Perrigne (2003) report that in such auctions interested bidders would first submit sealed bids; the auctioneer would then open the bids and announce the reserve price. The government agency did not use a fixed rule to determine the reserve price but instead considered various factors, such as market conditions and its own financial constraints. See also Andreyanov and Caoui (2022) for examples of sellers committing to a distribution of secret reserve prices.

[16] We start by positing the existence of an equilibrium such that for every r there exists a $\tilde{t}_r$ for which $\beta(\tilde{t}_r) = r$; then, using the implied properties of this equilibrium, we verify its existence by deriving a closed-form expression for the bidding function.

[17] The equilibrium bid depends on the distribution of the reserve prices itself; see equation (11) below.



(a) Arbitrary Distributions.

(b) Optimal Distribution for Unif[0, 1] and $t^S = 0$.

Fig. 1. The solid (red) lines in the left panel depict the distribution of threshold types under an (arbitrary) secret reserve price scheme with two reserve prices, r_1 and r_2. The dashed curve is a continuous approximation that leaves the seller with strictly larger revenue and uses infinitely many reserve prices. The right panel depicts the optimal CDF of the threshold types when bidders' types are distributed according to a Unif[0, 1]. The induced distribution of secret reserve prices, $\beta_I^*(\tilde{t}_r)$, can be obtained from (11). (For interpretation of the colors in the figure, the reader is referred to the web version of this article.)

in your item".[18] Similarly, Lucking-Reiley (2000) reports that most internet auctions employ no minimum bid but a secret reserve in order to generate interest and bidding momentum. Hence, consistent with our model, the rationale behind this practice seems to be the belief that if the reserve price is posted publicly, the likelihood that a bidder submits a bid above the reserve (i.e., participates in the auction) is smaller than when the reserve price is hidden.

Thus, expectations-based reference points offer a novel explanation for the use of secret reserve prices. Secret reserve prices can also be rationalized under risk aversion and in common-value auctions *if* the seller's value is privately known.[19] Furthermore, secret reserve prices can emerge with uninformed bidders who learn their value as the auction unfolds (Hossain, 2008), with an uninformed seller who uses the information in submitted bids to decide whether to trade (Andreyanov and Caoui, 2022), or with competing sellers if not all buyers correctly anticipate the distribution of reserve prices across sellers (Jehiel and Lamy, 2015). However, none of these models can predict secret and random reserve prices within the canonical IPV setting. Rosenkranz and Schmitz (2007) provided an earlier rationale for secret (but not random) reserve prices in auctions with independent private values; in their model, publicly announcing the reserve price plants a reference point in the bidders' minds, making it less attractive to win at a price higher than the reserve. While their explanation is also a reference-dependent one, it is based on loss aversion in money, whereas ours leverages the attachment effect.

Our final result in this section compares the degree of bidder exclusion under the optimal secret and random reserve-price scheme with that under risk neutrality.

Proposition 4. *Under the optimal secret and random reserve-price scheme, the following holds:*

1. *If $t^S = 0$, the probability of trade is arbitrarily close to that under risk neutrality.*
2. *If $t^S > 0$, the probability of trade*
   (a) *is strictly larger than that under risk neutrality;*
   (b) *converges to a limit value larger than that under risk neutrality as $N \to \infty$.*

The results of Proposition 4 and the intuition behind them are similar to those in Propositions 1 and 2 under a public and deterministic reserve price. Indeed, because with secret reserve prices the attachment effect does not affect the optimal no-trade probability anymore, differences with the risk-neutral benchmark are driven solely by the fact that, in contrast to the bidders, the seller is not loss averse. Therefore, the level of bidder exclusion, and hence inefficiency, is lower with loss-averse bidders than with risk-neutral ones. Finally, we re-emphasize that the analysis and the results in this section carry over to the SPA.

4.2. Auctions followed by negotiation

In this section, we show that the seller can achieve a larger revenue with an auction followed by a take-it-or-leave-it (TIOLI) negotiation than by holding a standard auction with a (revenue-maximizing) public reserve price and committing to not selling the good if the reserve price is not met — the latter being the optimal mechanism with risk-neutral bidders.

[18] On eBay the reserve price is secret unless the seller explicitly wants to share it with bidders; see https://www.ebay.com.au/help/selling/listings/selling-auctions/reserve-prices?id=4143.

[19] Li and Tan (2017) show that a seller with a privately known value may prefer a secret reserve price to a public one when facing risk-averse buyers. The reason is that, as the optimal reserve price depends on the seller's value, the fact that the seller is privately informed makes the reserve price random from the buyers' perspective. However, if the seller's value was commonly known, as in our model, a public reserve price would then be optimal. A similar argument holds in common-value auctions with risk-neutral bidders; see Vincent (1995).

Consider the following static mechanism. For fixed p and r in $\mathbb{R}_+$, with $p < r$, buyers submit a bid in $p \cup [r, \infty)$. The buyer who submits the highest bid above r wins the good and pays her bid. If no buyer submits a bid above r, with probability $v \in [0, 1]$ the seller allots the good randomly among those buyers that bid p (if any) and the randomly chosen winner pays the price p.[20]

Fixing the reserve price r and posted price p, there are two threshold types, t_r and t_p with $t_r > t_p$. The first one, t_r, is the bidder type that submits a bid exactly equal to the reserve price. The other threshold type, t_p, is the smallest type who is willing to buy the good at the posted price in case the reserve price is not met. In addition, let $q(t_p)$ be the (ex-ante) probability that type t_p receives the good and let $\alpha \in (0, 1)$ denote the solution to $\alpha F_1(t_r) = q(t_p)$; note that $q(t_p)$, and hence α, depend on v.[21]

Fixing p, α, and r, the following two conditions must hold:

$$(1+\eta)t_p = p,$$

$$t_r\Big((1-\alpha)(1+\eta) + \eta(\lambda-1)(1-\alpha)\alpha F_1(t_r)\Big) + (1+\eta)\alpha t_p = r.$$

The first condition, $(1+\eta)t_p = p$, is intuitive: in the PPE, the threshold type t_p is the lowest type for which not buying at the posted price is not a personal equilibrium. Given the first condition, the second one pins down the threshold type t_r as the smallest type whose downward adjacent neighbor is just indifferent between bidding the reserve price or bidding below the reserve price and buying the good at the posted price with probability $\alpha F_1(t_r)$, given she expected not to participate in the auction. Crucially, notice that, in contrast to a situation without the possibility of buying at the posted price, when not participating in the auction the threshold type t_r's adjacent neighbor still expects to obtain the good with strictly positive probability, $\alpha F_1(t_r)$. In turn, deviating and participating in the auction by submitting a bid equal to the reserve price becomes more attractive for this bidder type, as it reduces her potential losses.

Given these threshold types, equilibrium behavior is then straightforward. Buyers whose types are strictly below t_p abstain from the mechanism. Buyers with types in $[t_p, t_r)$ bid p, while those with types weakly higher than t_r bid $\beta(t) = \int_{t_r}^{t}(1+\eta+\eta(\lambda-1)F_1(s))sf_1(s)ds + F_1(t_r)r$.

As the next proposition shows, there exist values of v and p (with an implied α) such that, by exposing more bidder types to the attachment effect, the seller strictly benefits from using a post-auction TIOLI negotiation.

Proposition 5. *Consider either an FPA or an SPA with a public reserve price. Under PPE, for every such auction, there is an auction followed by a TIOLI negotiation that raises more revenue.*

Intuitively, by granting a probability of winning also to types below that of the marginal bidder, the seller attaches them to the good. This induces a competitive pressure, due to psychological motives, on the threshold type's bid: in equilibrium, this type has to raise her bid as otherwise marginally lower types would strictly prefer to imitate her. In other words, by not excluding a set of types below the threshold type t_r, the seller inflates their attachment, forcing the threshold type to bid more aggressively. However, there is also a cost for the seller since the larger the probability with which a type below the marginal bidder receives the good, the lower the competitive pressure, due to the material motives, on the threshold type. The seller optimally trades off these two effects — increasing the competitive pressure on the threshold type via the attachment effect vs. decreasing it by allowing some post-auction negotiation — by choosing a posted price p that only types that are very close to t_r would accept.

There are many real-world examples of auctions followed by some form of negotiation if the reserve price is not cleared. For instance, sellers may choose to negotiate with interested parties who did not meet the reserve; see Elyakime et al. (1994) on timber sales, Bulow and Klemperer (1996) on fine-art auctions, and Ashenfelter and Genesove (1992) and Ong (2006) on real-estate auctions.[22] Perhaps strikingly, Proposition 5 shows that the auctioneer benefits from engaging in some post-auction negotiation even though a TIOLI offer is not necessarily the optimal post-auction form of negotiation from the seller's perspective. Moreover, with risk-neutral bidders a seller would optimally commit to never negotiate with them after the auction if no one bids above the reserve.[23] In contrast, Proposition 5 shows that such negotiations are not necessarily a sign that a seller does not have the necessary commitment power; rather, the possibility of such post-auction negotiations can be beneficial for the seller if bidders are expectations-based loss averse.

[20] One could also interpret this mechanism as having two stages: first, the seller runs a standard auction with a public reserve price r; then, if the reserve price is not met, with probability v the seller posts a price $p < r$ at which any buyer can buy the good. Under this interpretation, the seller must commit to both v and p in the first stage. Indeed, if buyers update their reference points, the information that the good was not sold in the first stage would make them more attached and thus willing to accept an even larger price in the second stage; see also Balzer et al. (2022). We abstract from this dynamic formulation for notational convenience.

[21] Consider a bidder with type $t \in [t_p, t_r)$. If she accepts the posted price, the bidder receives the good with probability $1/\#$ where $\#$ is the number of buyers with types in $[t_p, t_r)$. Conditional on the event that no other buyer bids above the reserve (which happens with probability $F_1(t_r)$), $\#$ is a random variable that follows a binomial distribution with success probability $1 - F(t_p)/F(t_r)$. Let $Pr(\#; N-1)$ be the probability that exactly $\#$ out of $N-1$ bidders have a type in $[t_p, t_r)$. Then, $q(t_p) = vF_1(t_r)\sum_{\#=0}^{N-1} Pr(\#; N-1)/(\#+1) = vF_1(t_r)\sum_{v=0}^{N-1}\binom{N-1}{\#}(1-F(t_p)/F(t_r))^{\#}(F(t_p)/F(t_r))^{N-1-\#}/(\#+1) = vF_1(t_r)\frac{1-(F_1(t_p)/F_1(t_r))^N}{N(1-F_1(t_p)/F_1(t_r))}$ and thus $\alpha = v\frac{1-(F_1(t_p)/F_1(t_r))^N}{N(1-F_1(t_p)/F_1(t_r))}$.

[22] While not exactly the same, eBay's "Second Chance Offer", which allows the seller to make an offer to the highest bidder below the (secret) reserve, is in a similar spirit.

[23] There is a substantial theoretical literature focusing on (optimal) selling mechanisms for auctioneers lacking power to commit not to trade; see, for instance, McAfee and Vincent (1997) on price-posting sellers, and Skreta (2015) and Liu et al. (2019) on sellers having access to more general mechanisms. All these contributions show that the seller is hurt by being unable to commit to not selling the good if her initial mechanism does not allocate it.

5. Extensions and robustness

In this section, we show that our main results continue to hold when bidders are loss averse also over money. Moreover, we highlight that random exclusion, as achieved for instance via secret reserve prices, improves not only upon the revenue of a symmetric first-price auction but also upon every static, deterministic (and potentially asymmetric) selling mechanism.

5.1. Loss aversion over money

We now introduce loss aversion over money. Abusing notation slightly, we denote by $\lambda^k > 1$, for $k \in \{m, g\}$, the bidders' coefficient of loss aversion in the monetary domain ($k = m$) and in the good domain ($k = g$). Thus, a bidder assesses gains and losses separately over each dimension of consumption utility. For ease of notation, we assume that a bidder's weight on gain-loss utility, $\eta > 0$, is the same in both dimensions.

In the following, we show that in the FPA the seller still prefers random reserve prices to deterministic ones. Consider first a type-t bidder, with $t \geq t_r$, who expects to win the good with probability $q(t) > 0$ but mimics type $\tilde{t} > t$; her expected payoff is

$$EU(\tilde{t}, t) = q(\tilde{t})(t - \beta_I(\tilde{t})) - \eta\lambda^g(1 - q(\tilde{t}))q(t)t + \eta q(\tilde{t})(1 - q(t))t$$
$$-\eta\lambda^m(1 - q(t))q(\tilde{t})\beta_I(\tilde{t}) + \eta q(t)(1 - q(\tilde{t}))\beta_I(t) - \eta\lambda^m q(\tilde{t})q(t)(\beta_I(\tilde{t}) - \beta(t)). \tag{12}$$

Compared to Section 2, the second line of (12) is new and it captures the bidder's additional psychological gains and losses from the uncertainty in the monetary payment. In particular, the first term in the second line of (12) describes the situation where the bidder expected to lose the auction (and thus pay nothing) with probability $1 - q(t)$, but instead wins it (with probability $q(\tilde{t})$) and experiences a loss proportional to her bid, $\beta_I(\tilde{t})$. The expected loss is thus equal to $-\eta\lambda^m(1 - q(t))q(\tilde{t})\beta_I(\tilde{t})$. The second term is the respective gain when not winning the auction. Finally, the last term captures the additional loss a bidder experiences when deviating and winning with a larger than expected bid; i.e., $\beta_I(\tilde{t}) > \beta_I(t)$.

Next, consider a selling mechanism with a deterministic reserve price r, and a bidder with type $t \in [\underline{t}, t_r)$. Such a bidder does not want to mimic the threshold type t_r and therefore

$$EU(t_r, t) \leq EU(t, t) \Leftrightarrow$$
$$F_1(t_r)(1 + \eta)t \leq F_1(t_r)r(1 + \eta\lambda^m). \tag{13}$$

Condition (13) differs from its counterpart without loss aversion over money (see (3)) by the additional sensation of loss arising from the monetary payment; i.e., $\eta\lambda^m F_1(t_r)r$. Letting $t \to t_r$ from below, and making (13) hold with equality, we obtain the following relationship between the reserve price and the threshold type:

$$r = \frac{1 + \eta}{1 + \eta\lambda^m} t_r. \tag{14}$$

Note that, for a fixed threshold type t_r, stronger loss aversion over money (i.e., a larger λ^m) implies that the corresponding reserve price, the threshold type's payment conditional on winning, decreases. The reason is what Kőszegi and Rabin (2006) call the "comparison effect": because in the PPE the marginally excluded bidder does not expect to pay anything, deviating and bidding the reserve price would entail a loss in the money dimension, which in turn lowers the bidder's willingness to pay.

As for the case with loss aversion only in the good dimension, because of the attachment effect, the willingness to pay of the threshold type (and thus that of any higher type) increases the more optimistic she is about her chances of winning the auction. Moreover, a higher chance of winning the auction also implies a higher probability of making a strictly positive payment, thereby weakening the comparison effect. Hence, by inducing more bidder types to expect to win the auction with strictly positive probability, a random and secret reserve price can increase the attachment effect while simultaneously reducing the comparison effect. Therefore, as the following proposition formally states, random and secret reserve prices continue to raise more revenue than deterministic ones.

Proposition 6. *Consider the revenue raised in the FPA with a deterministic reserve price r. For any η and λ^k with $k \in \{m, g\}$, there exists a distribution of random and secret reserve prices such that the FPA generates a strictly higher expected revenue and almost the same level of exclusion.*

Our final result in this subsection concerns how loss aversion over money affects the probability of trade under deterministic reserve prices. As equation (14) reveals, loss aversion over money reduces the payment of the threshold type; i.e., the reserve price. Consequently, the seller's marginal cost from raising the threshold type is lower compared to the case of $\lambda^m = 0$. Thus, loss aversion over money puts upward pressure on the revenue-maximizing threshold type. However, we know from Section 3 that loss aversion in the good dimension has the opposite effect; and the latter effect might outweigh the former, as the following proposition shows.

Proposition 7. *For fixed λ^g, there exists a $\bar{\lambda}^m > 1$ such that if $1 < \lambda^m < \bar{\lambda}^m$, the optimal deterministic reserve price under loss aversion generates less exclusion than its risk-neutral counterpart.*

Therefore, if loss aversion in the money domain is not too strong, it continues to be optimal for the seller to exclude fewer bidder types compared to the risk-neutral benchmark. In Online Appendix B, we provide three examples highlighting how, for the case where $\lambda^g = \lambda^m \equiv \lambda > 1$, loss aversion can lead to either less or more exclusion compared to the risk-neutral benchmark.

5.2. Arbitrary mechanisms

Our last extension highlights that the optimality of random reserve prices extends to any static selling mechanism featuring exclusion.

Consider any static (possibly asymmetric) selling mechanism.[24] First, bidders learn the rules of the selling mechanism and form their reference points. Then, bidders simultaneously make their reports to the mechanism designer. Finally, depending on the submitted reports and the mechanism's rules, a final outcome $(q_i, p_i)_{i=1,\dots,N}$ realizes, where $q_i \in [0,1]$ denotes the allocation probability of bidder i, and $p_i \in \mathbb{R}$ is her payment.

With a slight abuse of notation, let $q_i(t_i, t_{-i})$ and $p_i(t_i, t_{-i})$ denote the equilibrium outcomes for bidder i if she behaves according to her equilibrium strategy, her opponents behave according to their equilibrium strategies, and the realized type profile is (t_i, t_{-i}). Consider type t_i and define the joint CDF of her opponents' types as $\Phi_{-i}(t_{-i}) \equiv \prod_{j \neq i} F(t_j)$. Then, let $q_i(t_i) \equiv \int_{t_{-i}} q_i(t_i, t_{-i}) d\Phi_{-i}(t_{-i})$ and $P_i(t_{-i}) \equiv \int_{t_{-i}} p_i(t_i, t_{-i}) d\Phi_{-i}(t_{-i})$.

Definition 2. We say that a mechanism is deterministic if for all i, t_i and t_{-i} such that $t_i \neq t_{-i}$, it holds that $q_i(t_i, t_{-i}) \in \{0, 1\}$.

Notice that Definition 2 rules out random reserve prices, whereby even knowing all submitted bids (i.e., the realized type profile) the final allocation is not determined yet without knowing the realization of the reserve price. Also, define $t^i_{\underline{r}}$ as $t^i_{\underline{r}} \equiv inf\{t_i : q_i(t_i) > 0\}$; this is the lowest of bidder i types that obtains the good with strictly positive probability. Note that, if a deterministic mechanism features exclusion, it holds that $t^i_{\underline{r}} > \underline{t}$. Take now any mechanism that implements an allocation rule q_i with exclusion. In the following we describe a mechanism that uses random exclusion and provides the seller with a strictly higher revenue.

Fixing $q_i(t_i)$, consider the following allocation rule of a mechanism with random exclusion:[25]

$$\hat{q}_i(t_i; K) = \begin{cases} q_i(t_i) & \textit{if } t_i > t^i_{\underline{r}}, \\ (\frac{t_i}{t^i_{\underline{r}}})^K F_1(t_i) & \textit{if } t_i \leq t^i_{\underline{r}}, \end{cases}$$

where $K \in \mathbb{R}_+$. In words, according to the new allocation rule $\hat{q}_i$, types that were not excluded before receive the good with the same probability as in the original selling mechanism. However, types that were excluded in the original selling mechanism, now receive the good with probability $(\frac{t_i}{t^i_{\underline{r}}})^K F_1(t_i)$. Then, we have the following result.

Proposition 8. *Take the allocation rule q_i of any static selling mechanism with deterministic exclusion and assume that q_i is continuous for all $t_i > t^i_{\underline{r}}$ with at most countably many non-differentiable points. There is no static mechanism implementing the same q_i for all $t_i > t^i_{\underline{r}}$ that yields the seller a larger revenue than the one implementing $\hat{q}_i(\cdot; K)$ with $K \to \infty$.*

The intuition behind Proposition 8 is the same as the one provided in Section 4.1: by being opaque about which types are excluded, the seller can expose every bidder to the attachment effect, thereby increasing revenue. Notice that we require $q_i(\cdot)$ to be continuous for all $t > t^i_{\underline{r}}$ merely for technical convenience. Discontinuous jumps in $q_i(\cdot)$ at $t \in (t^i_{\underline{r}}, \bar{t})$ can be smoothed out in the same revenue-enhancing way as the jump at $t^i_{\underline{r}}$.[26] The following then is a straightforward corollary of Proposition 8.

Corollary 1. *Consider the class of static and symmetric selling mechanisms with differentiable allocation rule for all non-excluded types. The FPA with a random reserve price is an optimal selling mechanism.*

Finally, recall that the FPA and SPA are revenue equivalent under UPE; hence, the result in the above corollary readily extends to the SPA.

[24] As Gershkov et al. (2022), we focus on static mechanisms to prevent the seller from exploiting the potential dynamic inconsistency in the bidders' preferences. Indeed, as shown in Balzer et al. (2022), dynamic mechanisms such as Dutch auctions outperform their static counterparts when bidders are expectations-based loss averse.

[25] Note that the associated (expected) payments are pinned down in the PPE by incentive compatibility. See Lemma 3 in the proof of Proposition 8 for more details.

[26] In addition, limiting attention to allocation rules with at most countably many non-differentiable points assures that bidders' direct utilities are differentiable in their types almost everywhere. In turn, the envelope theorem pins down the expected payments almost everywhere on $[t^i_{\underline{r}}, \bar{t}]$.

6. Conclusion

This paper contributes to the recent and growing literature on the market implications of expectations-based loss aversion. Indeed, over the last decade, the model of expectations-based loss aversion developed by Kőszegi and Rabin (2006, 2007, 2009) has found many fruitful applications in several areas of economics, including firms' pricing and advertising strategies (Heidhues and Kőszegi, 2008, 2014; Rosato, 2016; Karle and Peitz, 2014, 2017; Karle and Schumacher, 2017), incentive provision (Herweg et al., 2010; Daido and Murooka, 2016; Macera, 2018), bargaining (Rosato, 2017; Herweg et al., 2018; Benkert, 2025), labor supply (Crawford and Meng, 2011), school choice (Dreyfuss et al., 2022; Meisner and von Wangenheim, 2023), asset pricing (Pagel, 2016, 2018; Meng and Weng, 2018), and life-cycle consumption (Pagel, 2017). In particular, there have been several studies on the implications of expectations-based loss aversion in auctions; see Lange and Ratan (2010), Eisenhuth (2019), and Balzer and Rosato (2021) on static auctions, and von Wangenheim (2021), Balzer et al. (2022), and Rosato (2023) on dynamic ones.

While the prior literature has mostly abstracted from considering reserve prices, the focus of our paper is on how expectations-based loss aversion affects the optimal reserve price, and the resulting level of bidder exclusion, in the FPA and SPA.[27] Our analysis reveals that loss aversion delivers new implications for the design of optimal auctions that are likely to be of interest for both theorists and practitioners. In particular, we find that random and secret reserve prices outperform deterministic and public ones in both the FPA and SPA. Indeed, by using random and secret reserve prices, the seller introduces a small risk that exposes all bidders to the attachment effect, which in turn leads them to bid more aggressively. This result establishes a tight link between the optimal level of exclusion in an auction and the optimal monopoly pricing scheme with loss-averse consumers derived by Heidhues and Kőszegi (2014) and Hancart (2024), that is analogous to the well-known one under risk neutrality. More generally, our result implies that with loss-averse agents mechanisms that level the playing field by giving every player a chance to win might be better suited to generate competitive pressure than mechanisms with steeper incentives (e.g., winner-take-all contests).

If the seller is forced to use a deterministic and public reserve price, we find that its optimal level depends on the number of bidders in the auction and is typically lower than the optimal reserve price under risk neutrality. Moreover, we show that sellers can raise even more revenue by committing to engage in some post-auction haggling if the reserve price is not met, as this also exposes more bidders to the attachment effect; this is in stark contrast to the case of risk-neutral (or risk-averse) buyers, where post-auction negotiations are never optimal. Hence, expectations-based loss aversion rationalizes several features of reserve prices observed in real-world auctions which are hard to reconcile with the classical risk-neutral and risk-averse frameworks.

CRediT authorship contribution statement

Benjamin Balzer: Project administration, Methodology, Investigation, Formal analysis, Conceptualization. **Antonio Rosato:** Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

Proof of Lemma 1

Proof. Let $P(t) \equiv F_1(t)\beta_I^*(t)$ be the expected payment from a bidder with type $t \geq t_r$. We know from Balzer and Rosato (2021) that in the symmetric equilibrium of the FPA, a bidder's first-order condition satisfies

$$\Big((1+\eta\lambda F_1(t)+\eta[1-F_1(t)])f_1(t)t\Big) = P'(t). \tag{15}$$

Solving the differential equation in (15) yields

$$P(t) = \int_{t_r}^{t} (1+\eta\lambda F_1(s)+\eta[1-F_1(s)])f_1(s)s\,ds + C,$$

where C is a constant. Since $P(t) = F_1(t)\beta_I^*(t)$, the constant satisfies $C = P(t_r) = F_1(t_r)\beta_I^*(t_r) = F_1(t_r)r$.

Note that $P(t_r) = F_1(t_r)r$ is not uniquely determined in a symmetric UPE. Indeed, fixing r, any t_r satisfying $(1+\eta)t_r \leq r \leq (1+\eta+\eta(\lambda-1)F_1(t_r))t_r$ can be supported as UPE with threshold type t_r. The leftmost inequality denotes the indifference condition,

[27] Three notable exceptions are Rosenkranz and Schmitz (2007), Eisenhuth (2019) and Muramoto and Sogo (2024). Using a different solution concept, Choice-acclimating personal equilibrium (CPE), Eisenhuth (2019) shows that the optimal reserve price in the all-pay auction implies the same no-trade probability as under risk neutrality, while Muramoto and Sogo (2024) focus on asymmetric auction design, allowing for bidder-specific reserve prices. In Rosenkranz and Schmitz (2007), differently from our setting, bidders are loss averse only with respect to their monetary payment and use the (public) reserve price as a reference point.

between bidding the reserve price or not, when the threshold type expects not to participate in the auction. In contrast, the rightmost inequality denotes the same indifference condition when the threshold type expects to bid the reserve price.

In the following we show that, for a fixed r, every bidder type prefers the equilibrium satisfying $(1+\eta)t_r = r$ to any other symmetric UPE. Fixing r, define $\underline{t}_r$ as solution to $\underline{t}_r = \frac{r}{1+\eta+\,\eta(\lambda-1)F_1(\underline{t}_r)}$, and note that this is the lowest threshold that can be supported as UPE threshold. Moreover, let $t_r^0 \equiv \frac{r}{1+\eta}$ and $t_r^1 \in [\underline{t}_r, t_r^0)$.

We want to show that switching from the equilibrium with t_r^1 (and equilibrium bid β^1 and $P^1(t) = F_1(t)\beta^1(t)$) to that with t_r^0 (and respective bid β^0 and $P^0(t) = F_1(t)\beta^0(t)$), would increase every bidder type's expected utility. Indeed, consider any $t \in [t_r^1, t_r^0)$. Such a bidder submits a bid larger than r and, therefore, attains a negative payoff:

$$EU(t) = F_1(t)t(1-\eta(\lambda-1)(1-F_1(t))) - F_1(t)\beta^1(t) < 0,$$

which is satisfied for any $t \in [t_r^1, t_r^0)$ since

$$F_1(t)t(1-\eta(\lambda-1)(1-F_1(t))) - F_1(t)\beta^1(t) < F_1(t)t(1+\eta) - F_1(t)\beta^1(t) < F_1(t)t(1+\eta) - F_1(t)r,$$

where the last inequality comes from the fact that $\beta^1(t) = \int_{t_r^1}^t \beta^{1'}(s)ds + r > r$. Moreover, the last inequality is negative if and only if $t < \frac{r}{1+\eta} = t_r^0$. Thus, when moving from t_r^1 to t_r^0, we exclude types with negative expected utility.

Moreover, consider any type above t_r^0. In both equilibria, types above t_r^0 receive the good with the same probability. Thus, they prefer the equilibrium with the lower bid. Using that $P^1(t_r^0) = F_1(t_r^0)\beta^1(t_r^0) > F_1(t_r^0)\beta^0(t_r^0) = F_1(t_r^0)r = P^0(t_r^0)$ the result follows, as this implies that $P^1(t) = \int_{t_r^0}^t (1+\eta\lambda F_1(s) + \eta[1-F_1(s)])f_1(s)sds + P^1(t_r^0) > \int_{t_r^0}^t (1+\eta\lambda F_1(s)+\eta[1-F_1(s)])f_1(s)sds + P^0(t_r^0) = P^0(t)$ for any $t \geq t_r^0$. □

Proof of Proposition 1

Proof. Fix t_r with corresponding r (see equation (4)). The seller's objective is:

$$N \times \Big(\int_{t_r}^{\bar{t}} \Big[\int_{t_r}^{t} \{f_1(s)s(1+F_1(s)\lambda\eta + [1-F_1(s)]\eta)\}ds + F_1(t_r)t_r(1+\eta)\Big] f(t)dt + \frac{F(t_r)^N}{N} t^S \Big). \tag{16}$$

Notice first that the t_r which maximizes (16) is either $t_r = \underline{t}$, $t_r = \bar{t}$ or an interior solution. In the first case, the derivative of (16) is negative at $t_r = \underline{t}$, in the second-case it is positive at $t_r = \bar{t}$, while in the third case the derivative is zero at an interior solution and negative for a slightly larger t_r. Moreover, define

$$\tilde{V}(t_r) \equiv \frac{1}{f(t_r)} \int_{t_r}^{\bar{t}} \Big(1 - \frac{\eta(\lambda-1)}{1+\eta} f_1(t_r)t_r\Big) f(t)dt. \tag{17}$$

It is straightforward to show that the derivative of (16) with respect to t_r is increasing in t_r if and only if

$$\tilde{V}(t_r) + \frac{t^S}{1+\eta} - t_r \geq 0. \tag{18}$$

Note that equation (18) rules out the potential solution $t_r = \bar{t}$ as $\tilde{V}(\bar{t}) = 0$ and $\bar{t} > t^S$. Using expression (18), we see that the seller's revenue increases in t_r as long as

$$V(t_r) - t^S \leq -t^S \frac{\eta}{1+\eta} - \frac{\eta(\lambda-1)}{1+\eta} \frac{[1-F(t_r)]}{f(t_r)} f_1(t_r)t_r, \tag{19}$$

where $V(t_r) = t_r - [1-F(t_r)]/f(t_r)$ is the 'virtual value'. If $t_r = \underline{t} = 0$, the left-hand side is lower than the right-hand side.

The right-hand side of equation (19) is negative. This implies that at the optimal threshold type, t_r^*, (no matter whether it is at the lower bound or in the interior) we have $V(t_r^*) - t^S < 0$. Since V is an increasing function, it must be that $t_r^* < t^{RN}$ where the optimal threshold is interior, i.e., $V(t^{RN}) - t^S = 0$ (because $\underline{t} = 0$).

Finally, notice that for λ sufficiently large, we have that $t_r^* \to \underline{t} = 0$ as, for any $t_r > 0$, the right-hand side of (19) becomes arbitrarily negative when λ becoming arbitrarily large. Obviously, the optimal reserve price, r^*, is $r^* = (1+\eta)t_r^*$ and converges to zero if t_r^* does. □

Proof of Proposition 2

Proof. If, for some N, $t_r^* = 0$, then it trivially follows that the optimal threshold type is weakly increasing in the number of bidders. Thus, without of loss of generality, suppose that the optimal threshold type is in the interior. Then it follows from the proof of Proposition 1 that condition (19) holds with equality; hence, we have

$$-V(t_r^*) + \left(\frac{1}{1+\eta}\right) t^S - \frac{\eta(\lambda-1)}{1+\eta} \frac{[1-F(t_r^*)]}{f(t_r^*)} f_1(t_r^*) t_r^* = 0. \tag{20}$$

Moreover, note that the derivative of the left-hand side with respect to t_r, say $LHS_{t_r}(t_r^*)$, is negative at the optimal t_r^*, as otherwise increasing t_r at t_r^* would increase the seller's profit. Applying the implicit function theorem to (20) we have

$$\frac{dt_r^*}{dN} = -\frac{LHS_N(t_r^*, N)}{LHS_{t_r}(t_r^*)}, \tag{21}$$

where the term $LHS_N(t_r^*, N)$ denotes the derivative of the left-hand side of (20) with respect to N, evaluated at $t_r = t_r^*$. Hence, it follows that $\frac{dt_r^*}{dN} > 0$ if and only if $LHS_N(t_r^*, N) > 0$. Note that $LHS_N(t_r^*, N) = -\frac{\eta(\lambda-1)}{1+\eta} \frac{[1-F(t_r^*)]t_r^*}{f(t_r^*)} \frac{df_1(t_r^*)}{dN}$. We thus need $\frac{df_1(t_r^*)}{dN} = \frac{d((N-1)F^{N-2}(t_r^*)f(t_r^*))}{dN} < 0$, or, equivalently, $\frac{dln(f_1(t_r^*))}{dN} = 1/(N-1) + ln(F(t_r^*)) < 0$, which is the condition in the proposition. □

Proof of Lemma 2

Proof. We prove the proposition in two steps. First, we present a function that bounds the seller's expected-revenue function (with the reserve price being the argument) from above. Second, we find the public reserve price that maximizes that upper bound.

Step 1: Bounding the Seller's Revenue. Recall that the seller's revenue under a deterministic reserve price is given by

$$N \times \int_{t_r}^{\bar{t}} \Big(\int_{t_r}^{t} \{F_1'(s)s(1+F_1(s)\lambda\eta + [1-F_1(s)]\eta)\}ds + F_1(t_r)t_r(1+\eta) \Big) f(t)dt.$$

We replace $F_1(t_r)t_r(1+\eta)$ with an upper bound, say, $\hat{P}(t_{\hat{r}})$, where

$$\hat{P}(t_{\hat{r}}) = (1+\eta)t_{\hat{r}}F_1(t_{\hat{r}}) + \eta(\lambda-1)t_{\hat{r}}F_1(t_{\hat{r}})^2/2. \tag{22}$$

Further define $h(s) \equiv F_1'(s)sv(s)$ with $v(s) \equiv 1+\eta+\eta(\lambda-1)F_1(s)$, then the expected revenue's upper bound, $\hat{R}(t_{\hat{r}})$, (divided by N) is

$$\hat{R}(t_{\hat{r}}) = \int_{t_{\hat{r}}}^{\bar{t}} \Big(\int_{t_{\hat{r}}}^{t} h(s)ds + \hat{P}(t_{\hat{r}}) \Big) f(t)dt. \tag{23}$$

Step 2: Maximizing the upper Bound.
The derivative of $t^S F(t_{\hat{r}})^N/N + \hat{R}(t_{\hat{r}})$ with respect to $t_{\hat{r}}$ is

$$\begin{aligned} & t^S f(t_{\hat{r}})F_1(t_{\hat{r}}) - f(t_{\hat{r}})\hat{P}(t_{\hat{r}}) + \Big(-h(t_{\hat{r}}) + \hat{P}'(t_{\hat{r}}) \Big)(1-F(t_{\hat{r}})) \\ &= t^S f(t_{\hat{r}})F_1(t_{\hat{r}}) - f(t_{\hat{r}})c\hat{P}(t_{\hat{r}}) + \frac{\hat{P}(t_{\hat{r}})}{t_{\hat{r}}}(1-F(t_{\hat{r}})) \\ &= t^S f(t_{\hat{r}})F_1(t_{\hat{r}}) - \frac{\hat{P}(t_{\hat{r}})}{t_{\hat{r}}}\Big(f(t_{\hat{r}})t_{\hat{r}} - [1-F(t_{\hat{r}})] \Big), \end{aligned} \tag{24}$$

where we used that

$$\hat{P}'(t_{\hat{r}}) = \frac{\hat{P}(t_{\hat{r}})}{t_{\hat{r}}} + (1+\eta)F_1'(t_{\hat{r}})t_{\hat{r}} + \eta(\lambda-1)F_1(t_{\hat{r}})f_1(t_{\hat{r}})t_{\hat{r}}.$$

Recall that $V(t) = t - [1-F(t)]/f(t)$. Then, condition (24) reveals that the optimal threshold type of the upper bound, say $t_{\hat{r}}^*$, satisfies

$$V(t_{\hat{r}}^*) = \left[\frac{F_1(t_{\hat{r}}^*)t_{\hat{r}}^*}{\hat{P}(t_{\hat{r}}^*)}\right] t^S. \quad \square \tag{25}$$

Proof of Proposition 3

Proof. We show that the seller can achieve the maximized upper bound with the following secret and random reserve-price scheme. For given $K \in \mathbb{R}^+$, introduce a random variable, $\tilde{T}_r$, with realization $\tilde{t}_r \in [\underline{t}, \tilde{t}_{\overline{r}}]$, with $\tilde{t}_{\overline{r}} = t_{\hat{r}}^*$ (as defined in the proof of Lemma 2), drawn according to CDF $F_0(\tilde{t}_r) = (\frac{\tilde{t}_r}{\tilde{t}_{\overline{r}}})^K$. Moreover, for $q(t) = F_0(t)F_1(t)$ consider

$$\beta_I^*(\tilde{t}_r) = \Big(\int_{\underline{t}}^{\tilde{t}_r} (1+\eta\lambda q(s) + \eta[1-q(s)])q'(s)sds + \Big)/q(\tilde{t}_r). \tag{26}$$

If the secret reserve price follows the random function $\beta_I^* \circ \tilde{T}_r : [\underline{t}, \tilde{t}_{\bar{r}}] \mapsto \Delta([\beta_I^*(\underline{t}), \beta_I^*(\tilde{t}_{\bar{r}})])$, then, in a symmetric equilibrium in increasing strategies, type-t bidder expects to win the auction with probability $q(t) = F_0(t)F_1(t)$, which by Lemma 1 reinforces (26) as the equilibrium bidding strategy (when replacing $F_1(t)$ with $q(t)$ and replacing the support of (26) with $[\underline{t}, \bar{t}]$). Define $h_q(s) \equiv q'(s)sv(s)$ with $v_q(s) \equiv 1 + \eta + \eta(\lambda - 1)q(s)$. Then, replacing F_1 with q it is straightforward to observe that the expected payment, $P(t)$, satisfies

$$P(t) = \int_{\underline{t}}^{t} h_q(s)ds + P(\underline{t}), \tag{27}$$

for some constant $P(\underline{t})$ (see Lemma 1). Obviously, $P(\underline{t}) = \beta_I^*(\underline{t})q(\underline{t}) = 0$, as $q(\underline{t}) = 0$.

Finally, we want to show that for $K \to \infty$ the seller's payoff converges to the maximized upper bound (23). Indeed, first note that

$$P(t) = \int_{\tilde{t}_{\bar{r}}}^{t} h_q(s)ds + \underbrace{\int_{\underline{t}}^{\tilde{t}_{\bar{r}}} h_q(s)ds}_{P(\tilde{t}_{\bar{r}})}. \tag{28}$$

Observe next that, for any $t \geq \tilde{t}_{\bar{r}} = t^*_{\hat{r}}$, we have $q(t) = F_1(t)$, implying $h_q(t) = h(t)$. Thus, the expectation of the first integral of $P(t)$ stated in (28) is equal to $\hat{R}^m(t^*_{\hat{r}}) - \hat{P}^m(t^*_{\hat{r}})(1 - F(t^*_{\hat{r}}))$ (see (23)).

It thus remains to show that $P(\tilde{t}_{\bar{r}})$ (stated in the under-bracket of (28)) converges to $\hat{P}^m(t^*_{\hat{r}})$ with $K \to \infty$. Applying partial integration reveals that

$$P(\tilde{t}_{\bar{r}}) = (1+\eta)q(\tilde{t}_{\bar{r}})\tilde{t}_{\bar{r}} + \eta(\lambda - 1)(\frac{q(\tilde{t}_{\bar{r}})^2}{2}\tilde{t}_{\bar{r}} - \int_{\underline{t}}^{\tilde{t}_{\bar{r}}} \frac{q(s)^2}{2} ds).$$

We now use that $q(t) \to 0$ if $t < \tilde{t}_{\bar{r}} = t^*_{\hat{r}}$. Thus, $P(\tilde{t}_{\bar{r}}) \to \hat{P}^m(t^*_{\hat{r}})$ and the claim follows. □

Proof of Proposition 4

Proof. Consider equation (25) from Step 2 in Lemma 2's proof. Assume that $t^S = 0$. The optimal threshold $t^*_{\hat{r}}$, satisfies $V(t^*_{\hat{r}}) = 0$, and thus the threshold coincides with the risk-neutral one. Now assume that $t^S > 0$. The optimal threshold then satisfies $V(t^*_{\hat{r}}) = t^S / \left(1 + \eta + F_1(t^*_{\hat{r}})\eta(\lambda - 1)/2\right) < t^S$. Since V is increasing in t, $t^*_{\hat{r}}$ is strictly smaller than the risk-neutral threshold type. Moreover, it is easy to see that $V(t^*_{\hat{r}})$ converges to $t^S/(1+\eta)$ for $N \to \infty$ and is thus lower than the risk-neutral optimal threshold. □

Proof of Proposition 5

Proof. Take any auction with public reserve price, where the seller commits not to sell the good in case no bidder meets the reserve. The corresponding threshold type is t_r. In the following, we show that there exists an auction followed by TIOLI negotiations that yields the seller larger revenues. For fixed t_r (as in the auction without TIOLI negotiations) and t_p, this latter mechanism is characterized by an interval from $[t_p, t_r)$. Types in that interval receive the good with constant probability $\alpha F_1(t_r)$, where $\alpha < 1$ and pay price $(1+\eta)t_p$ if they get the good. In the following we derive the bid function of t_r, that is, the reserve price, and show that there exist feasible choices of t_p (i.e., p) and v such that the seller raises more revenue than without having TIOLI negotiations.

Note that $F_1(t_r)r$ is determined by $\sup\{t | t < t_r\}$'s incentive constraints. We have

$$\begin{aligned}
&F_1(t_r)t_r - \eta\lambda\alpha F_1(t_r)(1 - F_1(t_r))t_r + \eta F_1(t_r)(1 - \alpha F_1(t_r))t_r - F_1(t_r)r \\
&\leq \alpha F_1(t_r)t_r - \eta\lambda\alpha F_1(t_r)(1 - \alpha F_1(t_r))t_r + \eta\alpha F_1(t_r)(1 - \alpha F_1(t_r))t_r - \alpha F_1(t_r)(1+\eta)t_p \\
&\Leftrightarrow (1-\alpha)F_1(t_r)t_r + \eta\lambda(1-\alpha)\alpha F_1(t_r)^2 t_r + \eta(1-\alpha)F_1(t_r)t_r + (1+\eta)\alpha F_1(t_r)t_p \leq F_1(t_r)r.
\end{aligned}$$

Dividing by $F_1(t_r)$ we have

$$\begin{aligned}
&t_r\Big((1-\alpha) + \eta\lambda(1-\alpha)\alpha F_1(t_r) + \eta(1-\alpha)\Big) + (1+\eta)\alpha t_p \leq r \\
&\Leftrightarrow t_r\Big((1-\alpha)(1+\eta) + \eta(\lambda-1)(1-\alpha)\alpha F_1(t_r)\Big) + (1+\eta)\alpha t_p \leq r.
\end{aligned}$$

In equilibrium, the above condition holds with equality and pins down the reserve price. Suppose the seller chooses $t_p \to t_r$ and $\alpha = 1/2$ (recall from footnote 21 that $\alpha = v\frac{1-(F_1(t_p)/F_1(t_r))^N}{N(1-F_1(t_p)/F_1(t_r))}$. Thus, $\alpha \to v$ if $t_p \to t_r$, and thus there exists a feasible choice of $v \in$

(0, 1) implementing $\alpha = 1/2$), in which case the reserve price becomes $t_r\Big(1+\eta+\eta(\lambda-1)F_1(t_r)/4\Big)$ which is strictly larger than the counterpart under no commitment $t_r\Big(1+\eta\Big)$. □

Proof of Proposition 6

Proof. The proof generalizes (and mirrors) that of Lemma 2 to a setting where the bidders are loss averse both in the monetary and in the good domain. By a slight abuse of notation, let $\Lambda^k \equiv \eta(\lambda^k - 1)$ for $k \in \{m, g\}$.

Fix a deterministic reserve price. We first present a function that bounds the seller's expected-revenue function (with the reserve price being the argument) from above. Second, we show that there exist secret and random reserve prices that lead to revenues arbitrarily close to the upper bound (and thus to strictly larger expected revenues compared to the auction with deterministic reserve) and imply almost the same level of exclusion as the deterministic counterpart.

Step 1: Bounding the Seller's Revenue. It is straightforward to extend Balzer and Rosato (2021) to allow for a public reserve price (in the spirit of Lemma 1). Doing so, the seller's revenue under a deterministic reserve price is given by

$$\int_{t_r}^{\bar{t}} \frac{\int_{t_r}^{t}\{f_1(s)s(1+\eta\lambda^g F_1(s)+\eta[1-F_1(s)])\}e^{\frac{\Lambda^m[F_1(t)-F_1(s)]}{1+\eta\lambda^m}}ds}{1+\eta\lambda^m} + \frac{(1+\eta)F_1(t_r)t_r}{1+\lambda^m\eta}e^{\frac{\Lambda^m[F_1(t)-F_1(t_r)]}{1+\eta\lambda^m}} f(t)dt.$$

We bound the seller's revenue from above and then show that secret and random reserve prices yield a revenue that is arbitrary close to that bound, $\bar{P}^m(t_r)$, where $\bar{P}^m(t_r)$ is defined as

$$\bar{P}^m(t_r) \equiv (1+\eta)t_r\frac{e^{\frac{\Lambda^m}{c}F_1(t_r)}-1}{\Lambda^m} + \frac{c^m\Lambda^g}{\Lambda^m}t_r\frac{e^{\frac{\Lambda^m}{c^m}F_1(t_r)}-1-\frac{\Lambda^m}{c^m}F_1(t_r)}{\Lambda^m}, \tag{29}$$

and $c^m \equiv 1+\eta\lambda^m$.[28] Further recall that $h(s) = f_1(s)sv(s)$ with $v(s) = 1+\eta+\Lambda^g F_1(s)$, then the expected revenue's upper bound, $\bar{R}^m(t_r)$ (times c^m), is

$$c^m\bar{R}^m(t_r) = \int_{t_r}^{\bar{t}}\Big(e^{\frac{\Lambda^m}{c^m}F_1(t)}\int_{t_r}^{t}h(s)e^{-\frac{\Lambda^m}{c^m}F_1(s)}ds + e^{\frac{\Lambda^m[F_1(t)-F_1(t_r)]}{c^m}}c^m\bar{P}^m(t_r)\Big)f(t)dt. \tag{30}$$

Step 2: Upper bound is the Supremum when using Secret Reserve Prices. Next, we need to show that the seller can achieve the upper bound with the following secret and random reserve prices. Let t_r be the implied threshold type in the auction with deterministic reserve. For given $K \in \mathbb{R}^+$, introduce a random variable, T_S, with realization $t_S \in [\underline{t}, t_r]$, drawn according to cdf $F_S(t_S) = (\frac{t_S}{t_r})^K$. Moreover, for $q(t) = F_S(t)F_1(t)$ consider

$$\hat{\beta}_I^*(t_S) = \Big(\int_{\underline{t}}^{t_S}\frac{1+\eta\lambda^g q(s)+\eta[1-q(s)]}{1+\eta\lambda^m}e^{\frac{\Lambda^m[q(t_S)-q(s)]}{1+\eta\lambda^m}}q'(s)sds + \Big)/q(t_S). \tag{31}$$

If the secret reserve price follows the random function $\hat{\beta}_I^* \circ T_S : [\underline{t}, t_r] \mapsto \Delta([\hat{\beta}_I^*(\underline{t}), \hat{\beta}_I^*(t_r)])$, then, in a symmetric equilibrium in increasing strategies, type-t bidder expects to win the auction with probability $q(t) = F_S(t)F_1(t)$. Define $h_q(s) \equiv q'(s)sv(s)$ with $v_q(s) \equiv 1+\eta+\Lambda^g q(s)$. Then, replacing F_1 with q it is straightforward to observe that the expected payment, $P(t)$, satisfies

$$c^m P(t) = e^{\frac{\Lambda^m}{c^m}q(t)}\int_{\underline{t}}^{t}h_q(s)e^{-\frac{\Lambda^m}{c^m}q(s)}ds + cP(\underline{t}), \tag{32}$$

for some constant $P(\underline{t})$ (see Lemma 1). Obviously, $P(\underline{t}) = \beta_I^*(\underline{t})q(\underline{t}) = 0$, as $q(\underline{t}) = 0$.

Finally, we want to show that for $K \to \infty$ the seller's payoff converges to the upper bound (30). Begin by noting that

$$\begin{aligned} c^m P(t) &= e^{\frac{\Lambda^m}{c^m}q(t)}\Big(\int_{t_r}^{t}h_q(s)e^{-\frac{\Lambda^m}{c^m}q(s)}ds + \int_{\underline{t}}^{t_r}h_q(s)e^{-\frac{\Lambda^m}{c^m}q(s)}ds\Big) \\ &= e^{\frac{\Lambda^m}{c^m}q(t)}\int_{t_r}^{t}h_q(s)e^{-\frac{\Lambda^m}{c^m}q(s)}ds + e^{\frac{\Lambda^m[(q(t)-q(t_r)]}{c^m}}\underbrace{e^{\frac{\Lambda^m}{c^m}q(t_r)}\int_{\underline{t}}^{t_r}h_q(s)e^{-\frac{\Lambda^m}{c^m}q(s)}ds}_{\equiv c^m\hat{P}^m(t_r)}. \end{aligned} \tag{33}$$

[28] Noting that $e^{\frac{\Lambda^m}{c^m}F_1(t)} > \frac{\Lambda^m}{c^m}F_1(t_r)+1$ proves that we bounded the revenue from above.

Next, observe that for any $t \geq t_r$, we have $q(t) = F_1(t)$, implying $h_q(t) = h(t)$. Hence, the expectation of the first term of (33) is equal to $c^m \bar{R}^m(t_r) - \int_{t_r}^{\bar{t}} e^{\frac{\Lambda^m[F_1(t)-F_1(t_r)]}{c^m}} c^m \bar{P}^m(t_r) f(t)dt$ (see (30)). It thus remains to show that $\hat{P}^m(t_r)$ (defined in (33)) converges to $\bar{P}^m(t_r)$ as $K \to \infty$.

Applying partial integration reveals that

$$\hat{P}^m(t_r) = -\frac{e^{\frac{\Lambda^m}{c^m}q(t_r)}}{\Lambda^m}\Big(e^{-\frac{\Lambda^m}{c^m}q(t_r)}t_r v_q(t_r) - e^{-\frac{\Lambda^m}{c^m}q(\underline{t})}\underline{t}v_q(\underline{t})) - \int_{\underline{t}}^{t_r} e^{-\frac{\Lambda^m}{c^m}q(s)}v_q(s)ds +$$

$$\frac{c^m}{\Lambda^m}\Lambda^g(e^{-\frac{\Lambda^m}{c^m}q(t_r)}t_r - e^{-\frac{\Lambda^m}{c^m}q(\underline{t})}\underline{t}) - \frac{c^m}{\Lambda^m}\Lambda^g\int_{\underline{t}}^{t_r} e^{-\frac{\Lambda^m}{c^m}q(s)}ds\Big)$$

$$= \frac{\int_{\underline{t}}^{t_r} e^{\frac{\Lambda^m[q(t_r)-q(s)]}{c^m}}v_q(s)ds}{\Lambda^m} + \frac{\frac{c^m}{\Lambda^m}\Lambda^g\int_{\underline{t}}^{t_r} e^{\frac{\Lambda^m[q(t_r)-q(s)]}{c^m}}ds}{\Lambda^m}$$

$$-\frac{t_r v_q(t_r) - e^{-\frac{\Lambda^m[q(t_r)-q(\underline{t})]}{c^m}}\underline{t}v_q(\underline{t})}{\Lambda^m} - \frac{t_r - e^{-\frac{\Lambda^m[q(t_r)-q(\underline{t})]}{c^m}}\underline{t}}{\Lambda^m}\frac{c^m}{\Lambda^m}\Lambda^g.$$

We now use that $q(t) \to 0$ if $t < t_r$. For such a t, we have that $v_q(t) \to 1+\eta$ and $\hat{P}^m(t_r)$ becomes

$$\hat{P}^m(t_r) \to \frac{\int_{\underline{t}}^{t_r} e^{\frac{\Lambda^m}{c^m}q(t_r)}(1+\eta)ds}{\Lambda^m} + \frac{\frac{c^m}{\Lambda^m}\Lambda^g\int_{\underline{t}}^{t_r} e^{\frac{\Lambda^m}{c^m}q(t_r)}ds}{\Lambda^m} - \frac{(t_r-\underline{t})(1+\eta)}{\Lambda^m}(1+\frac{c^m}{\Lambda^m}\Lambda^g) - \frac{t_r}{\Lambda^m}\Lambda^g q(t_r)$$

$$= (1+\eta)t_r\frac{e^{\frac{\Lambda^m}{c^m}q(t_r)}-1}{\Lambda^m} + \frac{c^m\Lambda^g}{\Lambda^m}\frac{t_r(e^{\frac{\Lambda^m}{c^m}q(t_r)}-1) - t_r\frac{\Lambda^m}{c^m}q(t_r)}{\Lambda^m}.$$

Thus $\hat{P}^m(t_r) \to \bar{P}^m(t_r)$ and the claim follows. □

Proof of Proposition 7

Proof. Recall that $\Lambda^k = \eta(\lambda^k - 1)$ for $k \in \{m, g\}$. Then, fix t_r with corresponding r (see equation (14)). It is straightforward to extend Balzer and Rosato (2021) to allow for a public reserve price (in the spirit of Lemma 1), in which case the seller's payoff, normalized by N, is:

$$\int_{t_r}^{\bar{t}}\Big(\frac{\int_{t_r}^{t}\{f_1(s)s(1+\eta\lambda^g F_1(s)+\eta[1-F_1(s)])\}e^{\frac{\Lambda^m[F_1(t)-F_1(s)]}{1+\eta\lambda^m}}}{1+\eta\lambda^m}ds$$
$$+\frac{(1+\eta)F_1(t_r)t_r}{1+\eta\lambda^m}e^{\frac{\Lambda^m[F_1(t)-F_1(t_r)]}{1+\eta\lambda^m}}\Big)f(t)dt + \frac{F(t_r)^N}{N}t^S. \tag{34}$$

Notice first that the t_r which maximizes (34) is either $t_r = \underline{t}$, $t_r = \bar{t}$ or an interior solution. In the first case, the derivative of (34) is negative at $t_r = \underline{t}$, in the second-case it is positive at $t_r = \bar{t}$, while in the last case the derivative is zero at an interior solution and negative for a slightly larger t_r. Moreover, define

$$\tilde{V}^m(t_r) \equiv \frac{1}{f(t_r)}\int_{t_r}^{\bar{t}}\Big(1 - \{\frac{\Lambda^g}{1+\eta} + \frac{\Lambda^m}{1+\eta\lambda^m}\}f_1(t_r)t_r\Big)e^{\frac{\Lambda^m[F_1(t)-F_1(t_r)]}{1+\eta\lambda^m}}f(t)dt. \tag{35}$$

It is easy to show that the derivative of (34) with respect to t_r is increasing in t_r if and only if

$$\tilde{V}^m(t_r) + t^S\frac{1+\eta\lambda^m}{1+\eta} - t_r \geq 0. \tag{36}$$

Note that, under risk neutrality, the seller's optimal threshold type, t^{RN}, satisfies $V(t^{RN}) = t_S$. We want to establish that we can find λ^g and $\bar{\lambda}^m$ so that (36) is strictly negative at the risk-neutral threshold t^{RN}. Indeed, if $\lambda^m = 1$, then $\frac{\Lambda^m}{1+\eta\lambda^m} = 0$ and the condition in equation (36) (evaluated at t^{RN}) becomes:

$$-V(t_r) - \frac{1}{f(t_r)}\int_{t_r}^{\bar{t}}\{\frac{\Lambda^g}{1+\eta}\}f_1(t_r)t_r f(t)dt + t^S \geq 0.$$

At $t_r = t^{RN}$, $V(t^{RN}) = t^S$, and therefore the left-hand side of this condition is strictly negative, implying that the seller's payoff is strictly decreasing at $t_r = t^{RN}$. Moreover, as V is increasing in t_r, the left-hand side of the condition is strictly negative for any $t_r \geq t^{RN}$, implying that the seller's payoff is strictly decreasing for any $t_r \geq t^{RN}$. Therefore, the optimal threshold is below the risk-neutral one.

Finally notice that, as the left-hand side of condition (36) is continuous in $\frac{\Lambda^m}{1+\eta\lambda^m}$, which itself is continuous in λ^m, there exists $\bar{\lambda}^m > 1$ so that, for any $\lambda^m < \bar{\lambda}^m$, the left-hand side of (36) is strictly negative at any $t_r \geq t^{RN}$ for any $\lambda^m < \bar{\lambda}^m$, implying a lower threshold than under risk neutrality. □

Proof of Proposition 8

Proof. To prove Proposition 8, we first employ standard arguments from mechanism design to express the expected payment from bidder i to the seller, resulting from the PPE equilibrium play of a selling mechanism in which bidder i wins the good with probability q_i.

By a slight abuse of notation, let $\Lambda \equiv \eta(\lambda - 1) > 0$.

Lemma 3. *Consider any static (stochastic or deterministic) selling mechanism in which bidder t_i expects to win the good with probability $q_i(t_i)$ in the PPE. Further assume that $q_i(\cdot)$ is continuous with at most countably many non-differentiable points on $[t^i_{\underline{r}}, \bar{t}]$. The expected transfer of bidder i to the seller is given by*

$$\int_{\underline{t}}^{\bar{t}} P_i(t_i) f(t_i) dt_i = \int_{t_{\underline{r}^i}}^{\bar{t}} q_i(t)(1 + \eta + \Lambda/2 q_i(t_i)) V(t_i) f(t_i) dt_i - (1 - F(t^i_{\underline{r}}))\Lambda/2 q_i(t^i_{\underline{r}}) t^i_{\underline{r}}. \tag{37}$$

Proof. Consider any of bidder-i's types that does not expect to win the good with positive probability in equilibrium, i.e., $t < t^i_{\underline{r}}$. When submitting a bid, this type does not prefer to imitate the equilibrium action of a slightly larger type. Thus,

$$\begin{aligned} & q_i(t^i_{\underline{r}})t - P_i(t^i_{\underline{r}}) + \eta q_i(t^i_{\underline{r}})t \geq 0 \\ & \quad \Leftrightarrow P_i(t^i_{\underline{r}}) \leq q_i(t^i_{\underline{r}}) t(1+\eta), \end{aligned} \tag{38}$$

and with equality for $t \to t^i_{\underline{r}}$.

Now consider any larger type, $t \geq t^i_{\underline{r}}$. In equilibrium this type does not prefer to take the action of any other type $\hat{t}$, implying that $EU(t,t) \leq E(\hat{t}, t)$. Thus, by the envelope theorem it holds that for any t for which $q_i(t)$ is differentiable (see Milgrom and Segal (2002))

$$EU(t,t) = \int_{t^i_{\underline{r}}}^{t} q_i(s)\{1 - \eta\lambda(1 - q_i(s)) + \eta(1 - q_i(s))\} ds - \int_{t^i_{\underline{r}}}^{t} q_i'(s)\{\eta\lambda(1 - q_i(s))s + \eta q_i(s) s\} ds + EU(t^i_{\underline{r}}, t^i_{\underline{r}}), \tag{39}$$

where $EU(t^i_{\underline{r}}, t^i_{\underline{r}}) = -\eta\lambda q(t^i_{\underline{r}}) t^i_{\underline{r}}$ by (38). Using (39), bidder i's type t's expected equilibrium payment is

$$\begin{aligned} P_i(t) = {} & q_i(t)t(1 - \Lambda) + \Lambda q_i(t)^2 t - \int_{t^i_{\underline{r}}}^{t} q_i(s)\{1 - \eta\lambda(1 - q_i(s)) + \eta(1 - q_i(s))\} ds \\ & + \int_{t^i_{\underline{r}}}^{t} q_i'(s)\{\eta\lambda(1 - q_i(s))s + \eta q_i(s) s\} ds + \eta\lambda q(t^i_{\underline{r}}) t^i_{\underline{r}}. \end{aligned}$$

Applying partial integration to the last integral and re-arranging, this reads as

$$\begin{aligned} P_i(t) = {} & q_i(t)t(1+\eta) + \Lambda/2 q_i(t)^2 t - \Lambda/2 q_i(t^i_{\underline{r}})^2 t^i_{\underline{r}} \\ & - \int_{t^i_{\underline{r}}}^{t} q_i(s)\{1+\eta\} ds - \Lambda/2 \int_{t^i_{\underline{r}}}^{t} q_i(s)^2 ds. \end{aligned} \tag{40}$$

Next, consider a bidder with type t' where $q_i(t')$ is non-differentiable. Since q_i is continuous, incentive compatibility implies that $P_i(t') = \lim_{t \to t'} P_i(t)$ (i.e., (40)).

Applying partial integration to $\int_{t^i_{\underline{r}}}^{t} P_i(t) f(t) dt$ then yields the desired result. □

Proposition 8 then evolves around the following idea. Whenever bidder i is excluded deterministically if her type falls below $t^i_{\underline{r}}$, we can find a stochastic mechanism, $\hat{q}(\cdot; K)$, implementing the same q_i in the limit as $K \to \infty$, and yielding an expected payment equal to

$$\int_{\underline{t}}^{\bar{t}} \tilde{P}_i(t_i) f(t_i) dt_i = \int_{t^i_{\underline{r}}}^{\bar{t}} q_i(t_i)(1 + \eta + \Lambda/2 q_i(t_i)) V(t_i) f(t_i) dt_i. \tag{41}$$

Lemma 3 then implies that, for fixed q_i, expression (41) is an upper bound on the principal's payment generated by any static (random or deterministic) mechanism, as every such mechanism implies a negative constant of $-(1 - F(t^i_{\underline{r}}))\Lambda/2 q_i(t^i_{\underline{r}}) t^i_{\underline{r}}$. Moreover, the upper bound is strict if there is exclusion, i.e., $t^i_{\underline{r}} > \underline{t}$.

Take any stochastic or deterministic selling mechanism and take $t^i_{\underline{r}}$ and q_i as given. Consider the following stochastic selling mechanism: Fix $K \in \mathbb{R}_+$ and choose allocation probability $\hat{q}_i(t) = q_i(t)$ if $t \geq t^i_{\underline{r}}$ and $\hat{q}_i(t) = (\frac{t}{t^i_{\underline{r}}})^K F_1(t)$ if $t < t^i_{\underline{r}}$. For any $K < \infty$, and any $t \in (\underline{t}, t^i_{\underline{r}}]$ we have that $F_1(t) > \hat{q}(t) > 0$. The first inequality implies that $\hat{q}$ is resource feasible. The second inequality implies that, according to Lemma 3, the selling mechanism implements an expected payment equals

$$\int_{\underline{t}}^{\bar{t}} \tilde{P}_i(t_i) f(t_i) dt_i = \int_{\underline{t}}^{\bar{t}} \hat{q}_i(t_i)(1 + \eta + \Lambda/2 \hat{q}_i(t_i)) V(t_i) f(t_i) dt_i.$$

As this expression is continuous in K, it follows that

$$\int_{\underline{t}}^{\bar{t}} \tilde{P}_i(t_i) f(t_i) dt_i \underbrace{\rightarrow}_{K \to \infty} \int_{t^i_{\underline{r}}}^{\bar{t}} q_i(t_i)(1 + \eta + \Lambda/2 q_i(t_i)) V(t_i) f(t_i) dt_i \geq \int_{\underline{t}}^{\bar{t}} P_i(t_i) f(t_i) dt_i,$$

where the inequality is strict if and only if $t^i_{\underline{r}} > \underline{t}$.

Finally, we notice that equation (37) in Lemma 3 immediately implies that there is no static mechanism implementing allocation rule q and yielding a strictly larger revenue than $\int_{t^i_{\underline{r}}}^{\bar{t}} q_i(t_i)(1 + \eta + \Lambda/2 q_i(t_i)) V(t_i) f(t_i) dt_i$. $\square$

Appendix B. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jet.2025.106045.

Data availability

No data was used for the research described in the article.

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