New Detection Method for Vehicle Dominant Vibration Mode based on Energy Distribution

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Abstract
The main obstacle for commercialization of active suspensions is that it requires significant power to operate. In order to reduce the energy consumption, practical active suspension design focuses on primary control target identified from the general control object. This research proposes a novel method for onboard monitoring of the energy levels of vibration modes of a vehicular system, and identifying the dominant vibration mode, e.g., one of roll, pitch, bounce and articulation four modes, to be suppressed as the first priority. The method proposed for the identification of dominant vibration mode is so called mode energy method. This method can fast calculate mode energy from system state vector to indicate the intensity of a vibration mode. Simulation on a 4 degree of freedom half car model is performed to validate the method, under an impulse maneuver input. The system vibration mode energy distribution has been investigated for both undamped and damped cases.

Key words: vibration monitor, modal superposition, mode energy,

1. Introduction

At the centre of the debate about how we should control vehicle's motion and vibration is the question of how much energy and effort should be spend and make in controlling[1]. Having too few control targets in the system may leave the vehicle uncovered and vulnerable in some circumstances. Too many control targets, on the other hand, could result in huge energy consumption and the application becomes impractical. The main obstacle for commercialization of active suspensions is that it requires significant power to operate. In order to reduce the energy consumption, practical active suspension design focuses on primary control target identified from the general control object. The control target identification could base on frequency, such as low-bandwidth system[2; 3] , or depend on ride mode, such as primary ride mode control.

Compared with passive or semi-active suspensions, active suspensions are more flexible and effective to achieve integrated vehicle handling and safety control. Conventional active suspensions generally adopt independently controlled actuators to control vehicle motions that are in different frequency range and coupled together. This kind of unfocused control is too general to be efficient, and brings high design cost and energy consumption.

In order to reduce energy consumption, the more practical active suspensions focus on primary control target which is identified from the general control object. The identification could be based on frequency, such as low-bandwidth system, or depending on ride mode, such as primary ride mode control. This research proposes a novel active concept which is based on vehicle primary vibration modes, such as roll, pitch, bounce and articulation. The purpose is to decompose vehicle motions into uncoupled vehicle vibration modes, and identify the dominant mode and apply control accordingly.

From vehicle dynamics, the sprung mass dominated motions are characterized by their low frequencies and larger displacement, and are concerned with vehicle safety and handling performance, such as bounce, roll and pitch. Within vehicle body's large motions, roll and pitch are the most import motions that may results in serious consequences.

Vehicle rollovers are one type of the most dangerous and fatal accidents that have been frequently reported around the world in recent years. Four-wheel drive vehicles (4WDs), typically having higher mass centers, are particularly vulnerable to this type of accident, with over one third of 4WD fatalities involving rollover[4]. The events leading to vehicle rollovers are complex, with many factors influencing the vehicle motion. Suspension systems play a key role in reducing vehicle roll rates during extreme maneuvers, as such an advanced suspension can greatly reduce vehicular rollover propensity.

Pitch affects passenger ride comfort. Moreover, vehicle body's pitch motion can largely affect drivers' judgment in terms of speed estimation, even misleading the driver to make the wrong decisions under critical conditions. For example, the heavy pitch motion from braking during cornering may give a driver a false impression of understeering, and so induce oversteering, causing the vehicle to move out of control.

Bounce is well known for its contribution to ride comfort and handling performance. Bounce motion is the most intensively studied topics of vehicle active control. From literature, this motion is generally treated as a combined effect from two vibration modes: body dominated mode and wheel dominated mode. The body dominated mode is in low frequency range with large body displacement, and the wheel dominated mode is in higher frequency range, 10-15Hz, with large wheel displacements. Those two modes can be easily included in a two degree quarter car model, and many suspension designs take these two modes as control targets.

Articulation, also called warp, is a motion that only diagonally-opposed wheels move in phase, relative to vehicle body[5]. This is a non-planar mode, allowing the vehicle to travel on spatial surfaces, important to maintain a good road surface contact [6]. Not much research has been done on articulation, so its characteristic and effects are remain unclear. However, from literature, articulation is generally reported to be associated with road holding, braking/traction, roll and yaw stability.

Dominant vibration mode means at a point in time there is one vibration mode that contributes to vehicle body vibration most. This dominant vibration mode has the highest level of energy relative to other vibration modes. Since vehicle is a fairly damped system, with a group of close frequencies with unknown force input, conventional vibration monitoring method can hardly be implemented into real time control. How to uncouple different vehicle vibration modes? What is the scientific basis for dominant vibration mode identification? And how can it be done in real time? This paper intends to propose some practical solutions to these questions.

2. Mode Energy Method

The method proposed for the identification of dominant vibration mode is mode energy method. It requires system state vector to be measured in real time. The estimation of vehicle state vector using sensing signal has been done in earlier 2011 by the research team from Cambridge University[7]. Here we assume the vehicle state vector is known, and treat it as the input to mode energy method.

Mode energy method uses mode superposition to uncouple each component of primary modes and calculate the potential energy and kinetic energy in each mode. Finally the sum of the system potential energy and kinetic energy in a mode, called mode energy, is calculated. The mode energy level indicates the intensity of a certain vibration mode. After normalization of the mode energy, the energy contribution ratios of individual vibration mode are compared to determine which mode is of active control priority.

A 4 degree of freedom bilateral symmetric half car model is employed to illustrate the method. From the new perspective of vibration mode energy, findings from the investigation of a vehicular system under impulse are presented.

The model is a 4 DOF linear damped limped-mass system with 4 vibration modes. The parameters are estimated from a Ford Territory and presented in Table 1[8] .

Table 1. Model parameters and system state

Notation	Description	Units	Values
I	Sprung mass inertia	kgm^2	390
m_s	Sprung mass	kg	900
m_u	Unsprung mass	kg	40
k_s	Suspension stiffness	N/m	44e3
k_t	Tire stiffness	N/m	2.7e5
c_s	Damping coefficient	N/m^2	2000
CG	Height of Centre of Gravity	m	0.637
l	Distance from suspension coil to vehicle centreline	m	0.575
z_s	Displacement of sprung mass in Z direction	m	------
θ	Roll angle of sprung mass	rad	------
z_{u1}	Displacement of left unsprung mass	m	------
z_{u2}	Displacement of right unsprung mass	m	------

The vehicle model consists of sprung and unsprung masses, which are connected by springs and dampers, shown in fig. 1. The derived model has good accuracy and matches the vehicle experimental data in both roll and bounce modes [8].

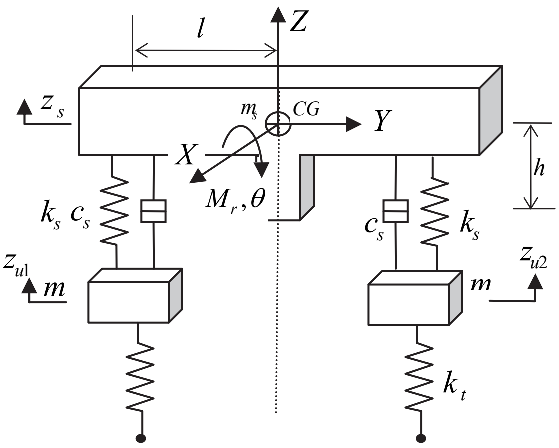

Fig. 1 A 4-DOF vehicle model

In fig. 1, point CG is the centre of the gravity for the sprung mass and also the rolling centre for simplicity. The differential equations for the heave and roll movements of the vehicle body, and the heave movements of the two wheels are given as follows.

$$m_s \ddot{z}_s = -k_s(2z_s - z_{u1} - z_{u2}) - c_s(2\dot{z}_s - \dot{z}_{u1} - \dot{z}_{u2}) \quad (1)$$

$$I\ddot{\theta} = lk_s(z_{u1} - z_{u2} - 2\theta l) + lc_s(\dot{z}_{u1} - \dot{z}_{u2} - 2\dot{\theta} l) + M_r \quad (2)$$

$$m_u\ddot{z}_{u2} = -k_t z_{u2} + k_s(z_s - \theta l - z_{u2}) + c_s(\dot{z}_s - \dot{\theta} l - \dot{z}_{u2}) \quad (3)$$

$$m_u\ddot{z}_{u1} = -k_t z_{u1} + k_s(z_s + \theta l - z_{u1}) + c_s(\dot{z}_s + \dot{\theta} l - \dot{z}_{u1}) \quad (4)$$

Based on the differential equations, we obtain the following state equation for the half-car system:

$$M\ddot{Z} + C\dot{Z} + KZ = F \quad (5)$$

where $Z = [\theta \quad z_s \quad z_{u1} \quad z_{u2}]^T$ (6)

$$M = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & m_s & 0 & 0 \\ 0 & 0 & m_u & 0 \\ 0 & 0 & 0 & m_u \end{bmatrix}, \quad C = \begin{bmatrix} 2l^2c_s & 0 & -lc_s & lc_s \\ 0 & 2c_s & -c_s & -c_s \\ -lc_s & -c_s & c_s & 0 \\ lc_s & -c_s & 0 & c_s \end{bmatrix} \quad (7,8)$$

$$K = \begin{bmatrix} 2l^2k_s & 0 & -lk_s & lk_s \\ 0 & 2k_s & -k_s & -k_s \\ -lk_s & -k_s & k_s + k_t & 0 \\ lk_s & -k_s & 0 & k_s + k_t \end{bmatrix} \quad (9)$$

If define $X = [Z \quad \dot{Z}]^T$, then equation 5 can be written in the state space force while considering external force vector F.

$$\dot{X} = AX + F$$

Where $A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$ (10)

The system characteristic matrix is defined as A, 4×4. We know that for a 4 DOF non-symmetric real matrix eigenvalue problem it should have 4 pairs of complex conjugate as eigenvalue and 4 pairs of corresponding complex conjugate eigenvectors, expressed as following[9] .

$$\overline{\Omega} = diag[\Lambda \quad \Lambda^*], 8\times 8 \quad (11)$$

$$U = \begin{bmatrix} \Psi & \Psi^* \\ \Psi\Lambda & \Psi^*\Lambda^* \end{bmatrix}, \ 8\times 8 \quad (12)$$

Note, * represents the conjugated counterpart of a complex number, vector or matrix. Λ and Ψ are further specified as following:

$$[\Lambda] = diag[\lambda_1 \quad \lambda_2 \quad \dots \quad \lambda_n], \ 4\times 4 \quad (13)$$

$$[\Psi] = [\phi_1 \quad \phi_2 \quad \dots \quad \phi_n], \ 4\times 4 \quad (14)$$

Using Λ and Ψ to form modal transition matrix Γ, 8×4:

$$\Gamma = [\Psi \quad \Psi\Lambda]^T \quad (15)$$

In equation 16, the measured or estimated system state vector X, 8×1, can be represented as a superposition of its normal modes plus noise/error vector ε, 8×1. Noise/error vector ε refers to the noise/error from three aspects: noise from sensor and related process; the error generated from state estimation; the error from the linearization of modeling.

$$X = \Gamma q + \varepsilon \quad (16)$$

modal amplitude vector q, 4×1 from state vector by taking advantage of least square method. Weighting matrix $[W]$ is employed to optimize the performance of least square method minimizes error ε .

$$q = \left(\Gamma^T[W]\Gamma\right)^{-1}\Gamma^T[W]X \quad (17)$$

The obtained modal amplitude vector q, in equation 18, has the following form. Each element in vector q is a complex number, which contains information of the amplitude and phase of its according vibration mode at this time point.

$$q = [q_1 \quad q_2 \quad \dots \quad q_n]^T \quad (18)$$

System's state can be represented as a combination of each vibration mode. Let's define the contribution of the ith vibration mode $(i = 1,2,3,4)$ to system state is $[Z_i \quad \dot{Z}_i]$, so system state $[Z \quad \dot{Z}]$ can be represented by each vibration mode contribution in the following two equations:

$$Z = \sum_{i=1}^{4} Z_i \qquad \dot{Z} = \sum_{i=1}^{4} \dot{Z}_i \tag{19}$$

$$Z_i = real(q_i \phi_i) \text{ and } \dot{Z}_i = real(q_i \phi_i \lambda_i) \tag{20}$$

Let's look at one vibration mode, e.g., the ith vibration mode $(i = 1,2,3,4)$. Now we are able to use the modal amplitude q_i to calculate energy level in this vibration mode. The kinetic energy ek_i and potential energy ep_i stored in the ith mode are derived, the sum of which, e_i, is the energy level in the ith vibration mode.

$$ek_i = \frac{1}{2} M \dot{Z}_i^{\,2} \tag{21}$$

$$ep_i = \frac{1}{2} H[\mathrm{T}\,\Omega\; Z_i]^2 \tag{22}$$

$$e_i = ek_i + ep_i \tag{23}$$

where M is mass and H is stiffness matrix for energy calculation. H is different from stiffness coefficient matrix K in equation 9. T and Ω are transition matrices used to convert the state vector into system displacement in its force filed (stiffness).

$$H = diag[k_s \quad k_s \quad k_t \quad k_t], \tag{24}$$

Where $\mathrm{T} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, and $\Omega = \begin{bmatrix} l & 1 & 0 & 0 \\ -l & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (25,26)

The sum of the energy from all modes $(i = 1,2,3,4)$ is written as $E = \sum_{i=1}^{4} e_i$. (27)

The vibration mode contribution ratio can be written as $\eta_i = \dfrac{e_i}{E}$. (28)

Please note η_i is a function of time, and will be calculated at each time point. System 4 vibration modes are briefly presented in the following Table 2.

Table 2. System Vibration Modes

Mode Number	Description	Frequency(Hz)	Damping ratio
1	Wheel dominant roll mode	12.8379	0.3589
2	Vehicle body dominant roll mode	1.4078	0.2195
3	Wheel dominant bounce mode	12.7841	0.3614
4	Vehicle body dominant bounce mode	1.2687	0.1972

3. Simulation

A moment impulse has been applied to the CG of the 4 DOF vehicle model to simulate a quick maneuver to stimulate a particular vibration mode, i.e., body dominant roll mode. The simulation based analysis is presented for undamped system and damped system, shown in fig.2 and fig. 3 respectively. Since the lateral acceleration is usually taken to measure the severer of vehicle maneuver, here the equivalent vehicle lateral acceleration is presented rather than the moment itself.

Fig.2, illustrates system vibration energy distribution of an undamped system under impulse excitation. The damping coefficient has been set to 0. The results include 6 graphs:

the lateral acceleration to vehicle, the total vehicle vibration energy, and the sub-energy in each individual vibration mode, namely, body dominated roll mode, wheel dominated roll mode, body dominated bounce mode and wheel dominated bounce mode. The red line represents the total dynamic energy that exists in the system. The blue line represents potential energy and the green line refers to kinetic energy. Since no damping, the system's total energy should be constant after excitation which occurred at 1s and last for 0.1s, while in each vibration mode the energy continuously transferring between kinetic energy and potential energy. Note that energy does not transfer from one vibration mode to other since vibration modes are orthogonal. From fig.2, body dominant roll mode has the highest energy level over other 3 vibration modes, thus easily we can see body dominant roll mode is the dominant vibration mode.

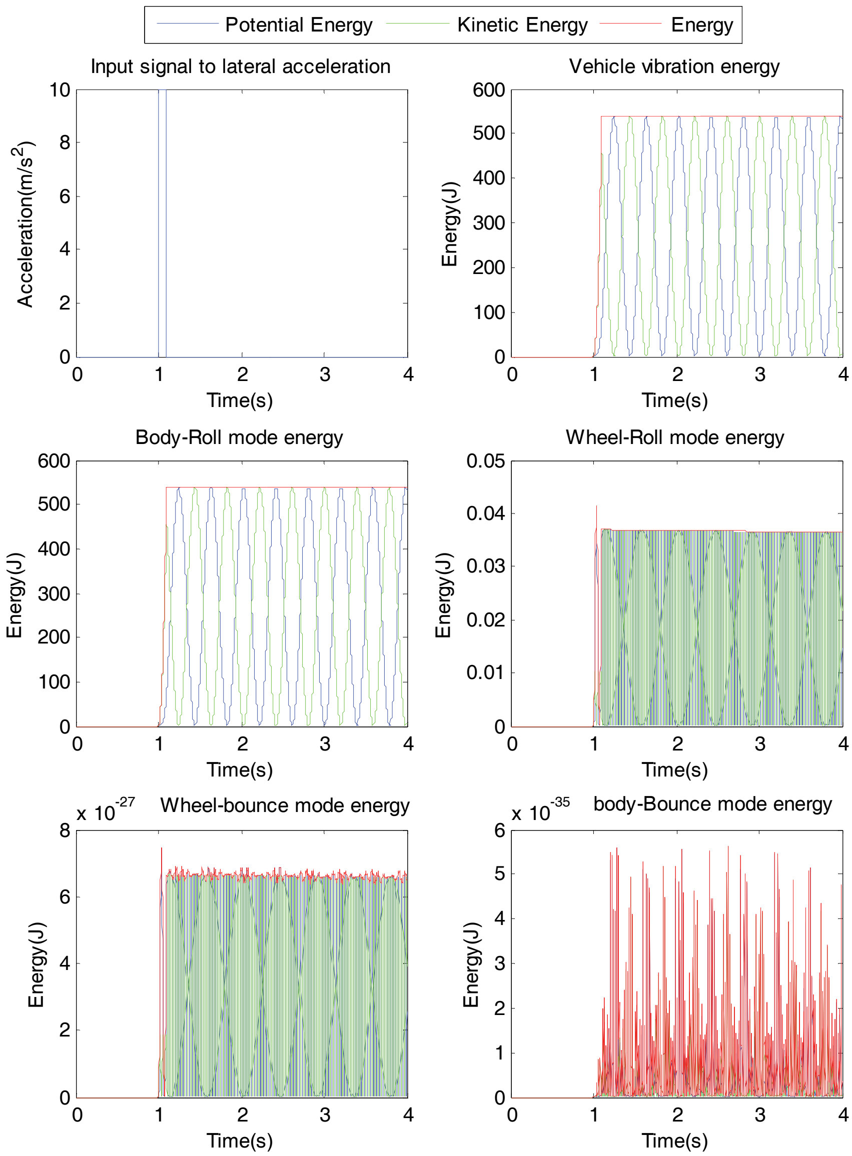

Fig. 2 Mode energy analysis of an undamped system under impulse

Damped system impulse mode energy response is shown in fig.3. The damping coefficient c_s is set to 2000 N/m^2. The system is fairly damped. System's total energy, as well as each individual mode energy decays over time due to damping. Because the applied viscous damper is only effective when there is a relative speed, kinetic energy decays according to this damping effects while it is converting into/out from potential energy. Over the testing period, the body dominant roll mode is also the dominant vibration mode.

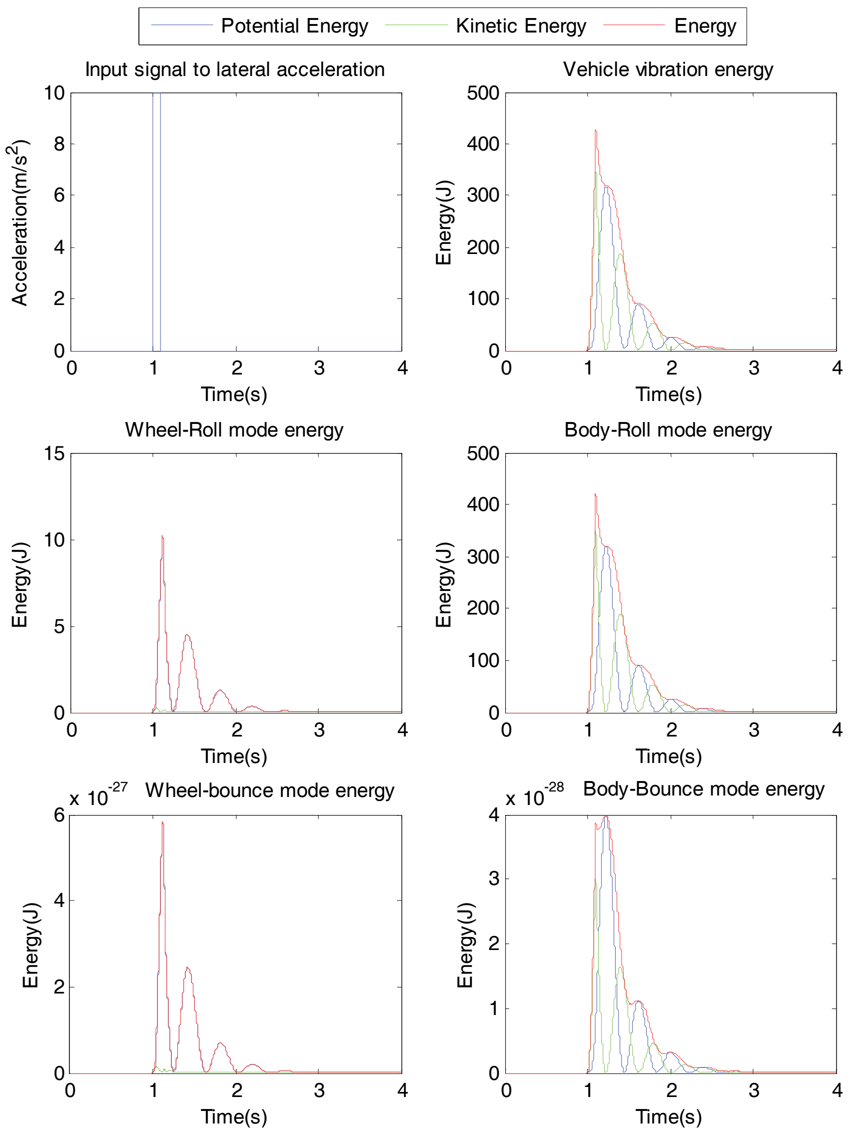

Fig. 3 Mode energy analysis of an damped system under impulse

Damped system mode energy distribution under road input is shown in fig.4. The input single to two wheels are shown in the first graph. To highlight the energy contribution ratio, $\eta_i \ (i=1,2,3,4)$ are plotted along the time in fig.5. We can see during the road bump period, from 1s to 1.5s, wheel dominant roll mode is the main contribution to vehicle vibration and motion; afterwards, vehicle body dominant roll mode becomes the main contribution.

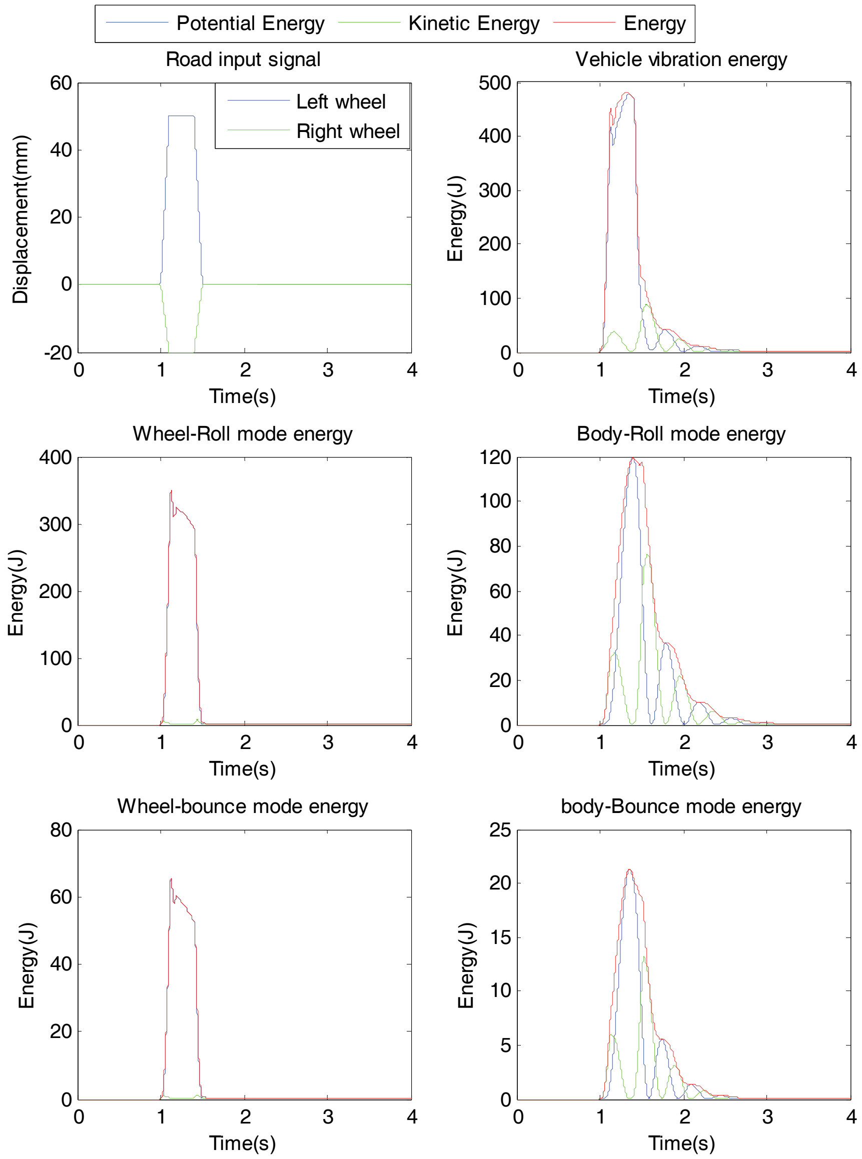

Fig. 4 Mode energy analysis of an damped system under road input

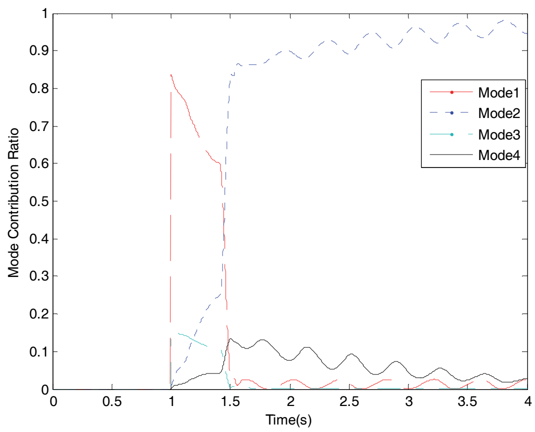

Fig. 5. Mode contribution ratio

4. Conclusion

Different from commonly used vibration energy concept in frequency domain, the proposed mode energy method is in time domain to ensure the fast response. Comparing to position and motion monitoring which relies on displacement and velocity, energy method is much steadier and more meaningful for vibration control. The energy continuously transfers between potential energy and kinetic energy, and the appearance of this energy transformation is vibration. Neither displacement nor velocity alone can give an accurate estimation of the vibration intensity at a point in time. The mode energy method provides a convenience for us to onboard detect the energetic level of each vehicle vibration mode, and provides a scientific base for the identification of the dominant vibration mode.

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