Determinations of Stress Wave Velocity in a Timber Pole using Wavelet Transform

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Abstract

This paper presents an application of Wavelet Transform (WT) for determination of stress wave velocity for Non-destructive Testing of timber utility poles in service. For surface Non-destructive Testing (NDT), the hammer impact, which produces generally broadband frequency excitation, is used to generate stress wave. Moreover, due to practicality the impact location for field testing of a utility pole is on the side of the pole and 1.5 m above ground level. And the geometry of utility pole could not guarantee non-dispersive longitudinal wave. All of these issues have resulted in lack of accuracy and reliability of results from surface NDT in field testing. In recognition of such problem, this research explores methods to reliably calculate desired wave velocity by isolating wave mode and studying dispersive nature of utility pole. Fast Fourier Transform (FFT) is firstly conducted to determine the suitable frequency from a stress wave data. Then WT is applied on the wave data mentioned to perform time-frequency analysis. Velocity can be determined by time history data of desired frequency from WT results which will be compared with the available analytical solution for longitudinal wave velocity. The results of the investigation showed that wavelet transform analysis can be a reliable signal processing tool for non-destructive testing in terms of velocity determination, which in turn also helps to determine the embedded length of the timber pole.

Key words: non-destructive testing, timber pole, stress wave, wavelet transform

1. Introduction

Surface non-destructive testing (NDT) methods such as Sonic Echo, Bending Waves and Ultraseismic methods have been considered over the past decade to be simple and cost-effective tools for identifying the condition and underground depth of embedded structures, such as timber poles or piles in-service (Rix et al.(1), Davis(2), Lin et al.(3), Holt(4), Holt et al.(5)). Despite the wide spread use of these methods, the effectiveness and reliability of the methods on determination of embedded length and evaluation of underground conditions of poles, especially timber poles, are not addressed.

When it comes to field applications, these developed/to be developed NDTs face a significant challenge due to presence of uncertainties such as complex material properties (e.g. timber), environmental conditions, interaction of soil and structure, defects and deteriorations as well as coupled nature of unknown length and deteriorated condition. Moreover, due to the dispersive nature of the stress wave propagation, which is related to the types of the wave and geometry/boundary of the structure, multiple frequency components exist in the measured signals and each frequency component corresponds to an individual velocity. For NDT methods that are based on reflected wave signals, such as

Sonic echo and Impulse response methods, the degree of such complexity will be even much greater.

Although sonic echo and Impulse response have been used for many years for material and some structure testing, applications to timber poles are rarely seen (due to the aforementioned reasons). For NDT assessment of utility poles in service, the stress wave based techniques are still a preferable choice considering practical issues such as over-hanging electric cables, geotechnical environment and uncertainty of material properties of poles. Despite the advantages, propagation of stress wave in finite media (including soil) with multiple wave modes (due to the choice of the impact location) is very complex and challenging in nature. So understanding of the propagation of stress wave in timber utility poles in service is a key in development of suitable techniques for underground pole condition and embedment length determination. Such understanding will allow deployment of suitable advanced signal processing techniques to reveal hidden information related to condition and embedment length of poles from measured signals. Three main groups of signal processing techniques are often associated with wave based analysis: time domain analysis, frequency domain analysis and time-frequency analysis (such as Short Time Fourier Transform (STFT), Wigner–Ville Distribution (WVD) and Wavelet Transform (WT)).

Fourier transform (FT) is a classical signal processing tool that breaks down a signal into constituent sinusoids of different frequencies and transforms it from a time domain to frequency domain. As a result of such transformation, all time information is lost. To overcome this deficiency, Gabor introduced short time Fourier transform (STFT) with the windowing techniques, that is, by considering a section of the signal at a time. The STFT maps a signal into a 2-D function of time and frequency. However, the method suffers from the disadvantage that the time and frequency information obtained have limited precision due to the size of the window. A high resolution cannot be obtained simultaneously for both time and frequency domains and once the time window size is chosen, it remains fixed for all frequencies(6). Wigner-Ville distribution (WVD), on the other hand, has an excellent frequency resolution but suffers from the cross term. This limitation can be overcome by Choi-Williams distribution, although it decreases the time-frequency resolution(7). Different from these time-frequency methods, continuous wavelet transform (CWT) allows analysing signals for every frequency with a different window size. It, therefore, allows choice of long time intervals when more precise low-frequency information is needed, and shorter ones when high frequency information is desired. In recent years, time-frequency analysis has attracted a great deal of interest from researchers for data processing in stress wave based damage detection methods. Some researchers have also applied such techniques on traditional NDT methods such as sonic echo or impulse response method(8).

CWT displays all the frequency components and corresponding time histories presented in the signal, which can be used advantageously to determine the time history associated with particular frequency required for analysis, especially for stress wave generated by an impact that produces broadband frequency excitation. With a combination of FFT and DFT (Discrete Fourier Transform), it is possible to obtain the time history corresponding to the chosen frequency and, therefore, to enable the calculation of the velocity of the stress wave for desired wave mode.

2. Theoretical background

2.1 Non-destructive test (NDT):

Sonic Echo/Impulse Response (SE/IR) tests have been used to evaluate the integrity of the pole / pile condition and to determine the embedment length. The Sonic Echo methods require measurement of the travel time of stress waves echo (in time domain) and Impulse Response methods use spectral analysis (in frequency domain) for interpretation. These two methods are sometimes called Pile Integrity Testing methods (PIT). The Sonic Echo method is also known as Echo, Seismic, Sonic, Impulse Echo and Pulse Echo method. Other names for the Impulse Response methods include Sonic Transient Response, Transient Dynamic Response and Sonic.

In both (SE/IR) tests, the reflection of longitudinal compressive waves from the bottom of the tested structural element or from a discontinuity such as a crack or a soil intrusion is measured. The generated wave from an impulse hammer travels down a shaft or a pile until a change in acoustic impedance (a function of velocity, density, and changes in diameter) is encountered, at which point the wave reflects back and is recorded by a receiver placed next to the impact point.

Sonic Echo data are used to determine the length of the pile/pole based on the time separation between the first arrival and the first reflection events or between any two consecutive reflection events (Δt) according to the following equation:

$$D = V \times \frac{\Delta t}{2} \tag{1}$$

Where D is the reflector depth and V is the velocity of compressive waves.

A reflector can be the bottom of the pole or any discontinuity along the embedded part of the pile/pole. The Sonic Echo data can also be used to determine the existence of a bulb or a neck in a shaft or the end conditions of the shaft based on the polarity of the reflection events.

The Impulse Response data are also used to determine the depth of reflectors according to the following equation:

$$D = \frac{V}{2 \times \Delta f} \tag{2}$$

Where Δf is the distance between the two consecutive peaks in mobility graph (i.e. velocity versus frequency).

In both methods, it can be observed that, the accuracy to determine the length or the position of the damage solely depends on the identification of correct peak(s) of the stress wave in either time or frequency domain. Although longitudinal wave in pole/pile is often considered less dispersive in nature, whereas bending wave is highly dispersive, the assumption can be breached due to geometric aspect ratio of testing structures, which requires dispersive analysis of the wave data to extract desired wave modes for velocity calculation. It is also necessary to point out that due to practicality in the field testing, the impact imparted to generate stress wave on utility pole is on the side of poles near ground level which produces multiple wave types and multiple wave modes.

2.2 Stress wave velocities

Kolsky[9] stated that, waves propagating in infinite media have mainly two forms, i,e, dilatational wave and distortional wave. Dilatational waves cause a change in the volume of the medium in which it is propagating but no rotation; while distortional waves involve rotation but no volume changes. The velocities of these waves are merely a function of material properties of the medium. On the contrary, wave propagation in finite media is much more complicated in which dispersion and mode conversion becomes more prominent. However, Peterson[10] suggested that, under certain conditions, it is possible to simplify the case to produce a single mode, large diameter waveguides. However, it is not applicable for the in-service timber pole. Moreover, in utility pole testing, using hammer impact to generate waves is considered to be practical and simple which unfortunately results in a broadband excitation that generates multimode stress wave propagating through the media. At low wave frequency, there are few wave modes but they will be joined by higher wave modes once excitation frequency increases. The frequency, at which one particular mode starts generating, is called the cut off frequency of that mode. To analyse an

output signal, it is necessary to choose a particular frequency and to know their corresponding modes and velocities as well.

Selecting a particular frequency in analysis is also very important as every mode has a peak stress function corresponding to a certain frequency and between two modes the stress functions have compromised values, that is if the chosen frequency is related to the highest peak of a particular modal frequency, then the signal is non-dispersive at that point and the group velocity is close to the traditional bar velocity $C_L = (E/\rho)^{1/2}$, (E = modulus of elasticity, ρ = density). However, according to Puckett [11], the group velocity of a signal will be less than the traditional bar velocity and very dispersive, if the chosen frequency is in between the highest peak stress function of two consecutive modes.

Another important parameter is the length to radius ratio of the pole and if such ratio is very large, the medium can be considered as infinite. Thus, the group velocity of the modes approaches the velocity in an unbounded solid, dispersion becomes more negligible and a high degree of accuracy can be obtained [11]. Moreover, the number of lateral reflections decreases and as a result less transverse waves are generated and longitudinal waves become dominant. However, in the case of utility poles, such ratio is sufficiently large to consider them as infinite media; therefore, wave reflections on side/bottom boundary are expected. The assumptions such as wave propagation at traditional bar velocity and no dispersion are not necessarily true for wave propagation in utility poles.

Although timber pole should be considered as finite media, velocity of wave in finite media is related to velocity of dilatational wave and velocity of distortional wave in unbounded media as well as Rayleigh surface wave velocity. According to Puckett & Peterson [12], at low frequency, fewer modes are generated and the velocity approaches the traditional wave velocity or can be considered as one dimensional wave problem and at high frequency, more modes are generated and the velocity approaches the Rayleigh surface wave velocity. The equations governing these velocities are as follows:

$$C_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}} \tag{3}$$

$$C_2 = \sqrt{\frac{\mu}{\rho}} \tag{4}$$

$$C_L = \sqrt{\frac{E}{\rho}} \tag{5}$$

Where, C_1= velocity of dilatational wave in infinite media
C_2 = velocity of distortional wave in infinite media
C_L = traditional bar velocity
μ, λ = Lame's constant
ρ = density of the material
E = modulus of elasticity

$$\kappa_1^6 - 8\kappa_1^4 + (24 - 16\alpha_1^2)\kappa_1^2 + (16\alpha_1^2 - 16) = 0 \tag{6}$$

$$\kappa_1 = \frac{C_s}{C_2} \; and \; \alpha_1 = \frac{C_2}{C_1} \tag{7}$$

And C_s = velocity of Rayleigh surface wave.

The governing equation for C_1 and C_2 is given by Kolsky[10]

$$\rho \frac{\partial^2 \Delta}{\partial t^2} = (\lambda + 2\mu)\nabla^2 \Delta \qquad (8)$$

Where, Δ is the sum of three normal strains. From equation (8) using and irrotational condition, velocity C_1 and under no dilatation condition velocity C_2 can be defined. Lame's constant and shear modulus can be found from Poisson's ratio (ν) and modulus of elasticity by the following formulas:

$$\mu = \frac{E}{2(1+\nu)} \qquad (9) \qquad \lambda = \frac{\mu(E - 2\mu)}{(3\mu - E)} \qquad (10)$$

The properties of the timber poles in Table-1 are chosen from the data provided by the power distributing companies and the velocities are calculated from the above equations.

Table- 1: Properties of timber and stress wave velocities

E (Pa)	ρ (kg/m³)	ν	μ (Pa)	λ (Pa)	C_1 (m/s)	C_2 (m/s)	C_L (m/s)	C_S (m/s)
14x10⁹	750	0.3	5.3x10⁹	8.15x10⁹	5,021	2,878	4,321	2,500

2.3 Pochhammer-Chree equation

The Pochhammer-Chree equation for wave propagation in an infinite cylinder is of primary interest for many wave solutions. The equation is derived for an infinite cylinder in a three dimensional space. But if the length to radius ratio is high, then it can be considered as a one dimensional wave and can be determined by traditional bar velocity. The Pochhammer-Chree equation is given as follows [13]:

$$\frac{2\alpha}{r}(\beta^2 + k^2)J_1(\alpha r)J_1(\beta r) - (\beta^2 - k^2)^2 J_0(\alpha r)J_1(\beta r) - 4k^2\alpha\beta J_1(\alpha r)J_0(\beta r) = 0 \qquad (11)$$

Where, $\alpha = (\omega^2/C_1^2)\text{-}k^2$
ω = frequency
c_p = phase velocity
$\beta = (\omega^2/C_2^2)\text{-}k^2$
r = radius of the cylinder
J_1 and J_0 = Bessel function of the first kind of order zero and one

The root of the equation is wavenumber (k), which is related to the phase velocity of a signal. The relation is,

$$k = \frac{\omega}{c_p} \qquad (12)$$

For a timber utility pole, this equation can be solved for given dimensions. After substituting all the values in equation (11), the Bessel function can be expanded as follows

$$J_0(\alpha r) = 1 - \frac{1}{4}(\alpha r)^2 + \frac{1}{64}(\alpha r)^4 - \cdots \qquad (13)$$

$$J_1(\beta r) = \frac{1}{2}(\beta r) - \frac{1}{16}(\beta r)^3 + \cdots \; etc \qquad (14)$$

By considering only the first term of the Bessel expansion and substituting the values in equation (11), two roots or two values of wavenumber are found. These two roots are constant and lead to two velocities with values of 4,320 m/s and 2,682 m/s. The former is very close to the traditional bar velocity and the latter is close to the shear wave velocity in infinite media, i.e. presenting the non-dispersive wave for which the velocity is unique and depends on material property. The accuracy of the results will increase by considering more terms from the expanded series of the Bessel function. If the first two terms is considered, then the roots could be complex (damping/evanescent mode), imaginary (non-propagating mode) and real (propagating mode). So dispersion occurs and the velocity is closed to the traditional bar velocity at low frequency. Figure 1 shows the graph of considering first two terms of Bessel's expansion and can be seen that longitudinal wave is less dispersive in nature.

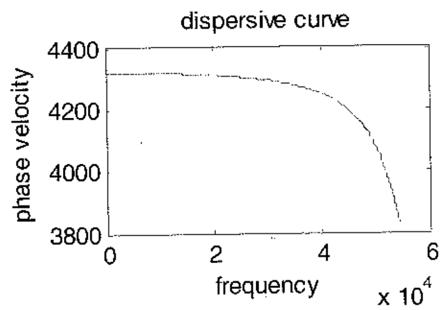


Fig. 1 Dispersive curve (considering first two terms of Bessel expansion)

2.4 Phase velocity due to broadband excitation

To determine the phase velocity, it is necessary to know the frequency related wavenumber and this can be done by applying Fast Fourier Transform (FFT) to a spatial data. Moreover, wavenumber is also related to the wave mode. So in a particular frequency, more than one wavenumber may be present. To determine the wavenumber, location vs axial stress graph is generated. In the case of a broadband signal, it is also important to determine which wavenumber is related to a particular frequency of wave modes. The procedure of determining the wavenumbers for a broadband signal is shown in Figure 2.

In this paper, axial stresses at both ends are considered to determine the presence of modes. From the results of the analysis, dispersive curve is generated and the presence of modes at different frequencies is determined by this procedure and verified by analysing the same signal using wavelet transform.

2.5 Continuous Wavelet Transforms (CWT)

The continuous wavelet transform (CWT) can compute each single spectral component with any desired width of the window, which is probably the most significant characteristic of the wavelet transform.

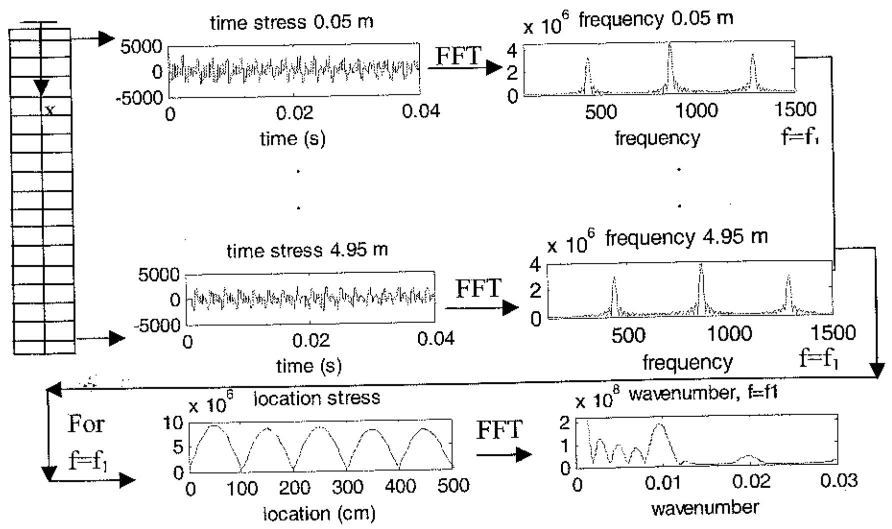


Fig. 2 Graphical representation of getting wave number for broadband signal

$$CWT_x^{\psi}(\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t)\psi^*\left(\frac{t-\tau}{s}\right) dt \tag{15}$$

As seen in the above equation, the transformed signal is a function of two variables, τ and ψ, the translation and scale parameters, respectively. ψ is the transforming function, and it is called the mother wavelet . There are many types of mother wavelets, such as Beta wavelet, Hermitian wavelet, Hermitian hat wavelet, Mexican hat wavelet, Shannon wavelet, Morlet wavelet, etc. Choosing the mother wavelet depends on the problems needed to be dealt with.

3. Numerical modeling

Wave propagation in a timber pole is complicated due to its nature (the properties, tapered shape etc.). Therefore, numerical study will begin with considering only the timber pole without soil to gain a basic understanding of wave propagation in timber itself. Numerical modelling is accomplished by Finite Element (FEM) analysis using ANSYS under free end boundary conditions which can be realised by suspending the pole with two pinned rigid links during testing. Impact location is vital for the generation of various waves (longitudinal, bending etc). In the numerical modeling, impact is imparted at the centre of the cross section at top of the pole, producing mainly longitudinal waves. A 5 m pole with a diameter of 300 mm at the bottom and tapered to 260 mm to the top is considered. This configuration is the same as the timber pole experimentally tested in the laboratory. Figure 3 shows the mesh set up for the numerical simulation of the timber pole. Transient analysis (i.e. time history analysis) is conducted on the numerical model and recorded impulse loading from the hammer test during the experimental test was used as the loading input.

3.1 Choosing the adequate frequency

Figure 4 shows the time history and frequency contents of the loading where the input frequency of 0 to 3,000 Hz is considered. To obtain a needed frequency resolution and wave number, zero padding technique is used. Figure 5 shows the temporal and spectral signal at a location of 3 m from the top (impact location) of the pole. It is clearly seen that there are mainly four dominating frequencies present in the signal, i.e. 437 Hz, 868 Hz, 1,288 Hz and 1,720 Hz. Near both ends of the pole, some additional frequencies exist such as, 2,156 Hz, 2,552 Hz (Figure 6) and the stress value at the end of the pole is very small compared to the stresses at the mid position.

Along the 5 m surface of the timber pole, a total of 100 nodal locations (in 50 mm intervals) are chosen for calculation of temporal and spectral signals and up to six

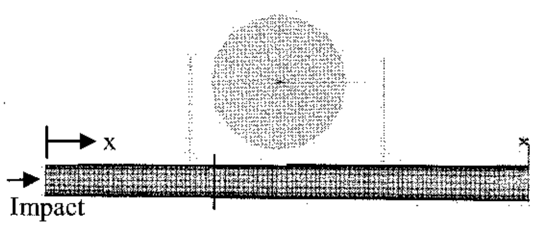


Fig. 3 Numerical modeling of the timber pole (cross section and front elevation)

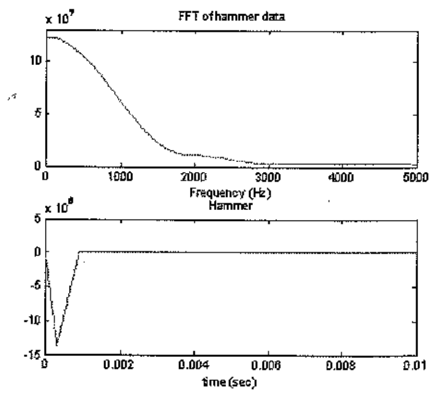


Fig. 4 Temporal & spectral signal of hammer

frequencies are chosen to determine the wavenumbers and number of modes corresponding to the frequency. Figure 7 shows wave numbers and number of modes at frequencies of 437 Hz, 1,288 Hz, 1,720 Hz and 2,156 Hz. It is clear from the wave number graph that, the first

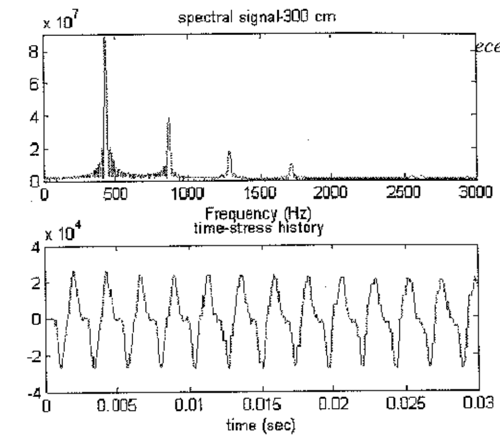


Fig. 5 Temporal & spectral signal at 3 m

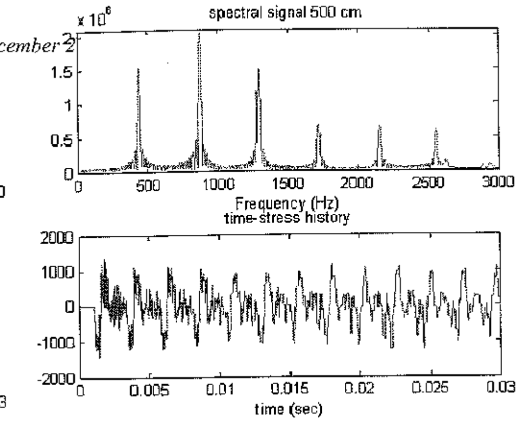


Fig. 6 Temporal & spectral signal at 5 m

mode is present at the frequency of 437 Hz and the second to forth modes are present at 1,288, 1,720 and 2,156 Hz, respectively.

3.2 Velocity of longitudinal wave velocity

In relation to practical application, it is noted that, rather than stresses, accelerations are often captured in testing and displacement is usually obtained in numerical analysis. Therefore, it needs to be verified that the same process can be conducted using continuous wavelet transform (CWT) on time acceleration/velocity/displacement data. In this paper time-velocity graph is used for both numerical and experimental cases. Velocity data can be obtained by differentiating displacement data for numerical case and integrating

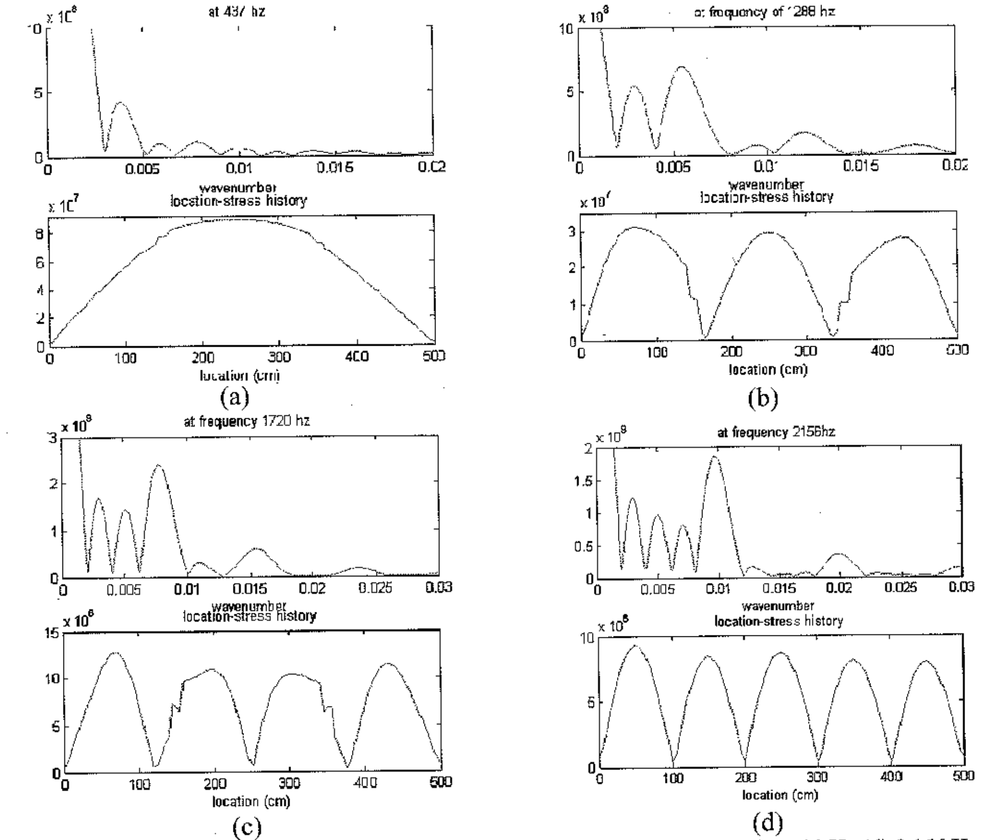
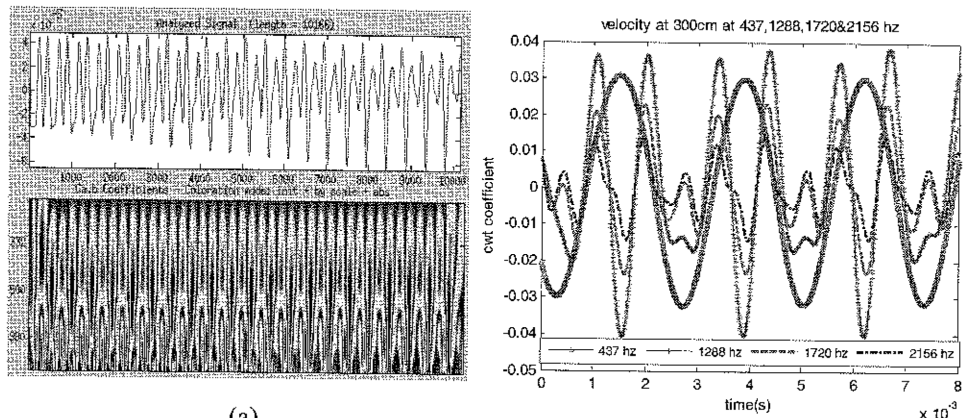


Fig. 7 Wavenumber and modes at the frequency of (a) 437 Hz (b) 1,288 Hz (c) 1,720 Hz (d) 2,156 Hz

acceleration data (with baseline correction) for experimental case. From the CWT, four frequencies are then selected from FFT results as aforementioned to determine the presence of modes and the results will be used for comparison with those from stress data.

Figure 8 shows the coefficient plot of the signal at these four frequencies obtained by using CWT analysis of time-velocity data at the location of 300 cm from the top. It can be seen clearly that the increase of frequency resulted in the signal with more peaks. In other words, it has demonstrated the presence of multi-modes in high frequency. Moreover, Figure 9 shows the dispersive curve from the broadband input excitation of the numerical results. In terms of the input frequencies, the main frequency band for this case lies between 400 - 2,600 Hz at which the dispersion curve is drawn. At high frequency, the velocity of each mode approaches the Rayleigh surface wave velocity and the result matches with the known phenomenon[9]. Analytically, Rayleigh surface wave velocity should be 2,500 m/s in this case, whereas Figure 10 shows the value as 2,200 m/s. From this curve, the cut-off frequency of second mode was found as 1,300 Hz. By knowing the cut-off frequency of the second mode, it is possible to choose the frequency below that cut-off frequency from CWT and produce correct velocity from single wave mode.

Figure 10 shows the time lag of a time-velocity data between two locations (200 cm and 300 cm from top) at frequency of 990 Hz. At this frequency, mainly one mode is generated and the measured velocity from this time lag is found to be 4,400 m/s which also matches with Figure 9. So, the time-stress data is also matched with time-velocity data. From Figure 10 it can also be observed that the sensor 1 (200cm from the impact point) receives impact wave earlier than sensor 2 (300cm from the impact point) but inversed in receiving the reflect wave from the bottom end. It demonstrates that this selected frequency has isolated a single mode wave for which echo time can be obtained more reliably.



(a) (b)

Fig. 8 (a) Time signal & CWT of signal (b) Coefficient plot at frequency 437, 1,288, 1,720 & 2,156 Hz

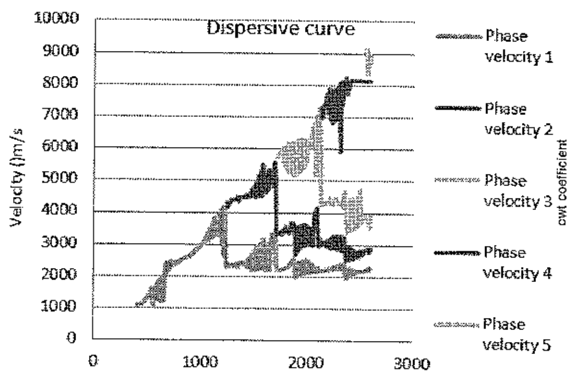


Fig. 9: Dispersive curve of longitudinal wave

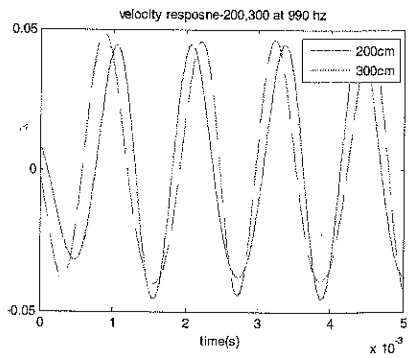


Fig. 10: Time lag between sensor at 200 & 300 cm at 990 Hz

4. Conclusion

In this paper, the method of FFT and DFT was used to find out various modes in a longitudinally dominated wave signal. Dispersive curve is also generated by CWT for impact load to find out velocities at different modes present in the measured wave signal. Continuous Wavelet Transform (CWT) technique has been applied for the determination of wave mode of timber poles through numerical investigations. The results have demonstrated that CWT is an effective tool for processing stress wave signals for identifying reflective waves, especially under more complicated situations such as impact at middle of specimens or presence of damage (under investigation). For these cases, traditional time domain analyses cannot provide satisfactory results due to multiple wave modes and broadband frequency excitation. This paper mainly focuses on the determination of stress wave velocity. Further investigation is needed to gain full understanding on effects of the geotechnical conditions and uncertainties of field testing to determine the embedded length and damage in the timber pole.

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