The modified critical-state framework of a waste mixture concerning the addition of rubber

Yujie Qi[1,*] **and Buddhima Indraratna**[2]

[1] Senior Lecturer, Program Co-leader, Transport Research Centre (TRC), University of Technology Sydney (UTS), 15 Broadway, Ultimo NSW 2007, Australia; E-mail: yujie.qi@uts.edu.au
[2] Distinguished Professor, Director of Transport Research Centre (TRC), University of Technology Sydney (UTS), 15 Broadway, Ultimo NSW 2007, Australia; E-mail: buddhima.indraratna@uts.edu.au

Abstract The practical application of recycled and marginal materials such as scrap tyres and mining by-products in railways is becoming more prevalent. This paper investigates the fundamental stress-strain behaviour of a granular waste matrix (steel furnace slags blended with coal wash and recycled rubber granules) to serve as the railway subballast. The findings indicate that the incorporation of rubber has a substantial impact on the geotechnical properties of the waste mixtures, especially the dilatancy response and the ability to attain the critical state. For the mixtures with a higher amount of rubber (20-40%) that could not achieve the critical state, extrapolation was adopted to obtain the critical state parameters. A critical state surface was developed by capturing the effect of rubber inclusions. Moreover, a semi-empirical model was established to predict the dilatancy response of the waste composites by modifying the critical state parameters and incorporating the energy input.

Keywords: recycled rubber, critical state surface, dilatancy, railway subballast.

1 Introduction

The adoption of recycled materials in geotechnical engineering has increased significantly in response to global circular economy goals and sustainability initiatives. This trend is driven by the need to reduce waste, minimize environmental impact, and develop cost-effective, durable infrastructure. Many research has already proved that using recycled materials (e.g. mining rejects, waste tyres, demolished construction materials, recycled glass) is not just an alternative; it is becoming a mainstream solution for achieving sustainable, cost-effective, and resilient infrastructure (Arulrajah et al., 2014, Indraratna et al., 2025). For instance, the use of recycled rubber products, including recycled rubber geogrid, rubber shock-absorb mats, tyre cells, and recycled rubber granules in railway foundations, has been found to efficiently mitigate track deterioration due to their ability to absorb energy and increase ductility (Indraratna et al., 2021, Qi et al., 2024).

Among the recycled rubber materials, rubber granules or rubber crumbs from waste tyres have become prevalent in the railway ballast and subballast layers. Arachchige et al. (2021) developed a rubber intermixed ballast stratum (RIBS) to replace the bottom layer of ballast, and both large-scale laboratory and field tests have proved that RIBS could significantly reduce ballast breakage and the stress propagation to the track substructure (Indraratna et al., 2024). Recycled rubber crumbs (RC) have been mixed with coal wash (CW; coal rejects) with or without steel furnace slags (SFS; steel-making by-products) to serve as subballast materials (the layer between ballast and subgrade), which could serve as energy-absorbing reservoir to mitigate ballast breakage, track vibration and lateral movement (Hunt et al., 2022, Qi and Indraratna, 2022, Malisetty et al., 2022). However, rubber is a viscoelastic material which is different from traditional natural aggregates which are elasto-plastic. The incorporation of rubber particles changes the fundamental mechanisms of the mixtures; hence, the analysis of the rubber-included waste mixtures, is imperative for the design of future sustainable railway foundations.

This paper aims to look into the stress-strain behaviour of the mixtures of SFS-CW-RC with special relevance to the critical state and dilatancy responses. The consolidated drained triaxial test results from Qi et al. (2018) for the mixtures with the optimal SFS:CW ratio (7:3) and varying rubber contents 0-40% (by mass) under the effective confining pressure ($\sigma'_3 = 10, 40, and\ 70\ kPa$) are adopted in this paper.

2 Stress-strain Responses and Critical State

As the amount of rubber increases in the SFS-CW-RC mixtures, the composites exhibit a reduced shear strength, with an increase in the axial strain at the peak deviator stress, and the strain softening response is weakened, as shown in Fig. 1 (a). This indicates a more ductile behaviour with the incorporation of rubber, which will benefit the railway foundation by preventing abrupt failure. Moreover, the volumetric strain of the mixtures becomes more contractive with the inclusion of rubber (Fig. 1b). This is because when preparing the testing samples, a higher rubber content induced a higher void ratio due to elasticity rebound, and this higher void was compressed when subjected to continuously loading. The high compressibility nature of rubber makes it difficult for the volumetric strain (ε_v) to reach a constant by the end of the test (25% axial strain) when the rubber content is ≥20%. Therefore, the critical state for the rubber-included composites (e.g. sand-rubber mixtures, CW+RC, and SFS-CW-RC mixtures) with a high rubber content usually cannot be achieved under laboratory conditions.

It has been found that SFS-CW-RC composites having 0% and 10% rubber amount (R_b) can attain a critical state at $\sigma'_3 = 10, 40, and\ 70\ kPa$, while the composites with $R_b \geq 20\%$ cannot attain a critical state within 25% axial stain but still show a trend to achieve beyond the laboratory strain limit. On this basis, the curve extrapolation was adopted in this research to obtain the critical state parameters for the waste composites with $R_b \geq 20\%$ (Fig. 1cd), inspired by Carrera et al. (2011).

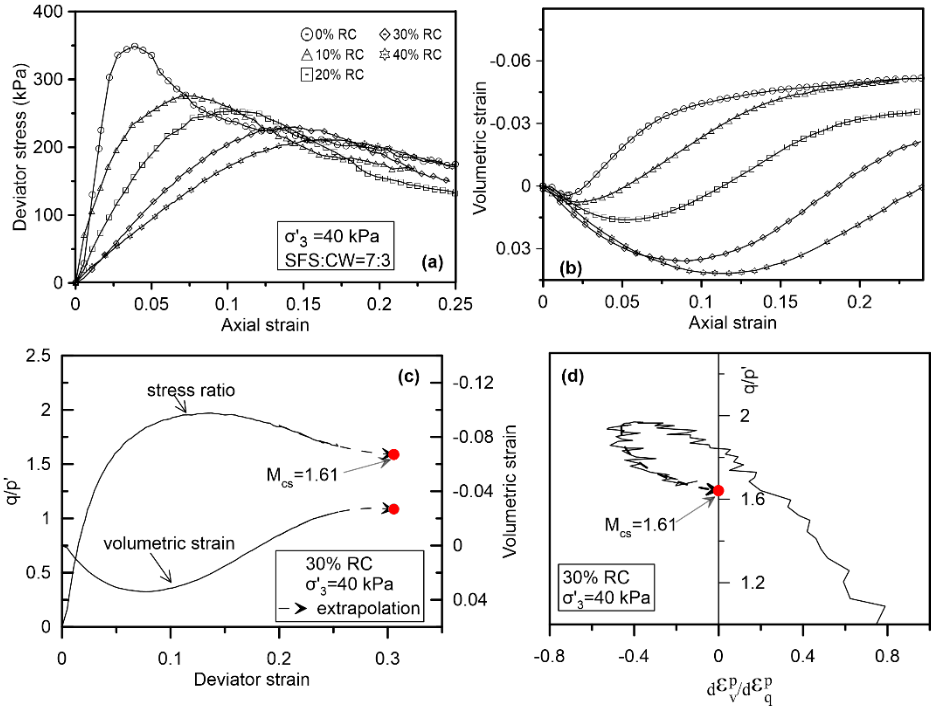

Fig. 1 Stress-strain behaviour under the confining pressure of 40 kPa (a) stress-strain response, (b) volumetric strain changes with axial strain; (c-d) obtain the critical state ratio via extrapolation (modified after Qi et al., 2018)

The obtained critical state ratios (M_{cs}) for SFS-CW-RC mixtures are not a constant value but change with R_b and σ'_3 (Qi et al., 2028). This is highly related to the increased energy-absorbing capacity due to rubber inclusions. To reflect this, an empirical relationship between M_{cs} and the total work input W_{total} was established (Qi et al., 2018) (Fig. 2a):

$$M_{cs}^{*} = a\left(\frac{W_{total}}{W_0}\right)^{b} \tag{1}$$

where M_{cs}^{*} is the modified critical state ratio capturing the energy absorption property; a and b are material constants, and their values are shown in Fig. 2 (a); W_0 is a unit pressure. W_{total} can be obtained by integrating the area of the stress-strain curve:

$$dW_{total} = p' d\varepsilon_v + q d\varepsilon_q \tag{2}$$

where p' is the effective mean stress, q is the deviator stress, ε_v is the volumetric strain, and ε_q is the shear strain. The critical state lines in the $e - \ln p'$ plane for SFS-CW-RC mixtures have been found rotating clockwise with the increase in R_b. Hence, a critical state surface (Fig. 2b) was identified with the critical state lines and R_b (Qi et al., 2021):

$$e_{cs}^{*} = (\Gamma_1 + \Gamma_2 R_b) + (\lambda_1 + \lambda_1 R_b) \ln p'_{cs} \tag{3}$$

where e_{cs}^{*} is the modified critical state void ratio; $\Gamma_{1,2}$ and $\lambda_{1,2}$ are regression parameters.

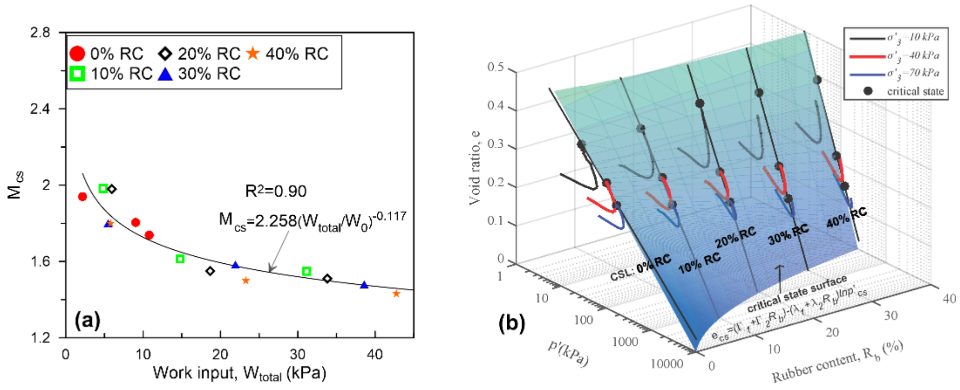

Fig. 2 (a) Empirical relationship for M^*_{cs}; (b) critical state surface of the void ratio (modified after Qi et al., 2019a)

3 Dilatancy

The incorporation of rubber crumbs in the waste composite causes a reduction in the dilatancy of the composites under static loading (Fig. 3). The soil dilatancy is usually related to the state and soil density, while for SFS-CW-RC composites, it is also influenced by the rubber content. The dilatancy equation can be expressed as below, inspired by Li and Dafalias (2000),:

$$d=\frac{d\varepsilon_v^p}{d\varepsilon_q^p}=d_0(e^{m\psi}-\frac{\eta}{M^*_{cs}}) \tag{4}$$

where d_0 and m are material constants; η refers to the stress ratio; $\psi=e-e^*_{cs}$ is the state parameter, which is the difference between the current and the critical void ratio under the same pressure. Through the modified critical state parameters e^*_{cs} in ψ and M^*_{cs}, the dilatancy model incorporates the influence of the rubber content in the waste composite. The parameter d_0 and m can be obtained via the peak deviator stress state and the phase transforming state (PTS):

$$d_0=\frac{d_{peak}}{(e^{m\psi_{peak}}-(\eta_{peak}/M^*_{cs}))} \tag{5}$$

$$m=\frac{1}{\psi_{PTS}}\ln(\frac{\eta_{PTS}}{M^*_{cs}}) \tag{6}$$

Fig. 3 shows the predicted and the measured dilatancy for SFS-CW-RC composites having various rubber contents under different effective confining pressures. The predicted dilatancy matches the test results reasonably well, indicating that the dilatancy model (Eq. 4) successfully incorporates the effect of rubber inclusion.

The proposed dilatancy model for rubber-waste mixtures is applicable for rubber-included mixtures that cannot reach a critical state due to the compressibility of rubber. The rubber material in the mixtures is limited only to granulated rubber, as larger rubber particles (e.g. rubber chips) may keep deforming without conforming to a critical state.

As this model is focused on static loading, cyclic loading conditions will be considered for future research.

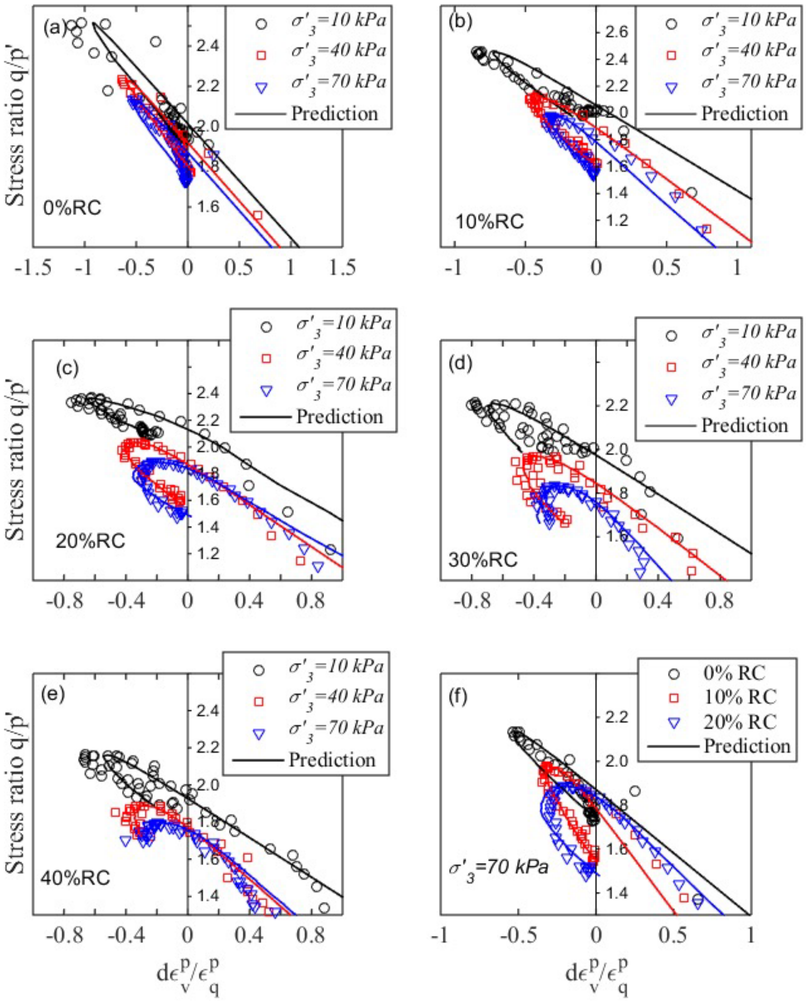

Fig. 3 Laboratory-obtained and model-predicted dilatancy of SFS-CW-RC mixtures

4 Conclusions

This paper investigated the influence of adding rubber on the stress-strain behaviour, critical state and dilatancy of the SFS-CW-RC composites under static loading. It revealed that the incorporation of rubber increased the compressibility and improved the ductility of the composites, reduced the overall shear strength, and made it harder for the mixtures to reach the critical state when $R_b \geq 20\%$. Curve extrapolation was adopted to obtain the critical state parameters, and a dilatancy model was developed, capturing the influence of rubber contents via the modified critical state parameters with R_b.

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