

Research on Self-Calibration of HF Ground Wave Radar Antenna Arrays

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Abstract: Since the performance of high-resolution direction finding algorithm for HF Ground Wave Radar (GWR) is severely degraded by sensor phase and amplitude errors, the radar system's phase calibration is the prerequisite of keeping the radar working in order. According to the characteristic of HF GWR's sea echo, this paper, based on an arbitrary triangular array, presents that Space-Time DOA (direction of arrival) Matrix Method, which is used to estimate 2D DOA under ideal conditions, can be used to estimate planar wave's DOA and sensor phase and amplitude errors simultaneously so as to achieve self-calibration. Its validity is verified not only by computer simulation, but also by comparing treatment results of measured data before and after calibration with the GPS-measured result.

Key words: HF Ground Wave Radar, Space-Time DOA Matrix Method, Triangular Array, Self-Calibration

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1. Introduction

In fact, most of the recent high-resolution spatial spectrum estimation algorithms (such as MUSIC^[1], ESPRIT^[2], et al), which perform well, are based on the assumption that the array manifold is known exactly. However, in a practical system, these algorithms are severely degraded and even out of work^[3,4,5] due to unavoidable perturbations in array manifold. Calibration with auxiliary sources and self-calibration are the two normal ways of array calibration. Calibration with auxiliary sources is impractical because it requires the exact position information of the sources and costs very much. Self-calibration is the nowadays research focus, and many self-calibration algorithms have arose.

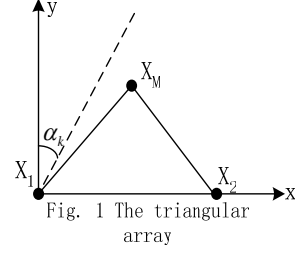
An HF Ground Wave Radar, working in shortwave band, usually consists of a large phase-controlled antenna array close to the seashore. It's difficult to calibrate this system's antenna array with auxiliary sources. So the problem becomes significant that how to calibrate the radar system' channels by making use of detected targets. As to HF GWR, it is unnecessary to estimate the elevation of the targets' echo that comes mainly from ocean's surface, so the HF Ground wave radar's antenna system is usually a uniform linear array (ULA). The two-dimension array constructed by adding new antennae to the ULA can estimate the 2D direction of arrival (DOA) of targets' echo. The two-dimension array will get redundant information when the DOA of echo is just one dimensional. Based on the premise that echo is one dimensional, our primary idea is to exploit the redundancy in order to achieve the HF Ground wave radar's channels' self-calibration.

In this paper, the shape of the array consisting of three whips is an arbitrary triangle. Space-Time DOA Matrix Method was initially devised to estimate 2D DOA based on precise knowledge of array manifold, but its formula under amplitude and phase errors can be deduced when echo is planar wave. DOA and phase perturbations can be estimated simultaneously through the redundancy generated by the 2D DOA technique applied to one dimension DOA estimation.

2、 Signal Model

Consider a triangular array with 3 sensors (X_1, X_2, X_M) as Fig.1 shows. X_1 lies on the grid

origin and X_2 lies on the X-axis whose distance from X_1 is d_x . X_M , an arbitrary point on the XY-plane, is d_y away from X-axis and r from the origin. Setting X_1 as the norm, each sensor's amplitude and phase error relative to X_1 can be written



as $G_1 \exp(j\varphi_1)$, $G_2 \exp(j\varphi_2)$ and $G_M \exp(j\varphi_M)$. Obviously, it is easy to get $G_1 = 1$ and $\varphi_1 = 0$ from the definition. Assume that there are d narrowband and mutually uncorrelated sources entered at wavelength λ and sufficiently far from the array to allow a planar wavefront. Additive noise is present at each sensor and is assumed to be zero mean, Gaussian process. The received signal of each sensor $x_i(t)$ is given by

$$x_i(t) = G_i \sum_{k=1}^d s_k(t) \cdot \exp[j \frac{2\pi}{\lambda} (i-1)d_x \sin \alpha_k] \cdot \exp[j\varphi_i] + n_i(t), \quad (i=1,2) \quad (1)$$

$$x_M(t) = G_M \sum_{k=1}^d s_k(t) \cdot \exp[j \frac{2\pi}{\lambda} d_p] \cdot \exp[j\varphi_M] + n_M(t) \quad (2)$$

where $s_k(t)$ denotes the signal emitted by the k -th source, α_k denotes the direction of arrival of the k -th source, $n_i(t)$ is additive noise with variance σ_i^2 and independent from sensor to sensor and $d_p = d_y \cos \alpha_k + \sqrt{r^2 - d_y^2} \sin \alpha_k$.

3、 Extended Spatial Time DOA Matrix Method

Original Space-Time DOA Matrix Method was devised to estimate 2D DOA of received signals under ideal conditions when X_M lies on Y-axis. Based on the signal Model described in section 2, Space-Time DOA Matrix Method can be rewritten as follows:

(1), (2) are written in a matrix form as:

$$X(t) = As(t) + n(t) \quad (3)$$

$$X = [x_1(t), x_2(t), x_M(t)]^T, n = [n_1(t), n_2(t), n_M(t)]^T, s = [s_1(t), \dots, s_d(t)]^T \quad (4)$$

$$A = [\mathbf{a}_1, \dots, \mathbf{a}_d], \mathbf{a}_k = [a_{1k}, a_{2k}, a_{Mk}] \quad (5)$$

$$a_{ik} = G_i \exp[j \frac{2\pi}{\lambda} (i-1)d_x \sin \alpha_k + j\varphi_i], \quad (i=1,2) \quad (6)$$

$$a_{Mk} = G_M \exp[j \frac{2\pi}{\lambda} d_p + j\varphi_M] \quad (7)$$

According to the definition of correlation, the correlation matrices of $x_M(t)$ and $x_2(t)$ with each sensor are

$$\begin{aligned} R_{x_i x_M}(\tau) &= E\left\{x_i\left(t + \frac{\tau}{2}\right)x_M^*\left(t - \frac{\tau}{2}\right)\right\} \\ &= \sum_{k=1}^d [R_{s_k s_k}(\tau) a_{Mk}^*] \cdot a_{ik}, (\tau \neq 0, i = 1, 2, 3) \end{aligned} \quad (8)$$

$$\begin{aligned} R_{x_i x_2}(\tau) &= E\left\{x_i\left(t + \frac{\tau}{2}\right)x_2^*\left(t - \frac{\tau}{2}\right)\right\} \\ &= \sum_{k=1}^d [R_{s_k s_k}(\tau) a_{2k}^*] \cdot a_{ik} \\ &= \sum_{k=1}^d [R_{s_k s_k}(\tau) a_{Mk}^*] \cdot a_{ik} \cdot \frac{a_{2k}^*}{a_{Mk}}, (\tau \neq 0, i = 1, 2, 3) \end{aligned} \quad (9)$$

Where $R_X(\tau) = [R_{x_1 x_M}(\tau), R_{x_2 x_M}(\tau), R_{x_M x_M}(\tau)]^T$, $R_Y(\tau) = [R_{x_1 x_2}(\tau), R_{x_2 x_2}(\tau), R_{x_M x_2}(\tau)]^T$,
 $R_S(\tau) = [R_{s_1 s_1}(\tau) \cdot a_{M1}^*, R_{s_1 s_1}(\tau) \cdot a_{M2}^*, \dots, R_{s_d s_d}(\tau) \cdot a_{Md}^*]^T$. Thus (8) and (9) can be rewritten as

$$R_X(\tau) = AR_S(\tau), R_Y(\tau) = A\Phi R_S(\tau) \quad (10)$$

where Φ is $D \times D$ dimensional diagonal matrix.

$$\begin{aligned} \Phi &= \text{diag}\left[\frac{a_{21}^*}{a_{M1}^*}, \dots, \frac{a_{2k}^*}{a_{Mk}^*}, \dots, \frac{a_{2d}^*}{a_{Md}^*}\right] \\ &= \frac{G_i}{G_M} \text{diag}\left[e^{j\frac{2\pi}{\lambda}(d_y \cos \alpha_1 - d_a \sin \alpha_1 + \varphi_M - \varphi_2)}, \dots, e^{j\frac{2\pi}{\lambda}(d_y \cos \alpha_1 - d_a \sin \alpha_1 + \varphi_M - \varphi_2)}\right] \end{aligned} \quad (11)$$

Respectively, when $\tau = T_s, 2T_s, \dots, NT_s$, we can get N groups of $R_X(\tau)$, $R_Y(\tau)$ and construct the following matrices.

$$X = [R_X(T_s), R_X(2T_s), \dots, R_X(NT_s)] \quad (12)$$

$$Y = [R_Y(T_s), R_Y(2T_s), \dots, R_Y(NT_s)] \quad (13)$$

From (10), we can get

$$X = AS, \quad Y = A\Phi S \quad (14)$$

where $S = [R_S(T_s), R_S(2T_s), \dots, R_S(NT_s)]$.

The spatial time DOA matrix is constructed as $R_{TS} = Y \cdot [X]^-$ (15), where $[X]^-$ denotes the pseudo inverse $[X]^- = X^H (XX^H)^{-1}$.

It is known from [9] that if both A and B are full rank, if all the diagonal elements of Φ are

different, D nonzero eigenvalues of R_{TS} are equal to D diagonal elements of Φ and moreover these eigenvalues' corresponding eigenvectors are equal to the corresponding signal steering vectors.

$$R_{TS}A = A\Phi \quad (16)$$

So we eigen-decompose R_{TS} to obtain A and Φ . For A is $3 \times d$ dimensional, the biggest number of DOA that can be estimated is 2. Through k -th ($0 < k < 3$) source's steering vector \mathbf{a}_k ,

it is easy to get it is easy to get $a_{2k} = G_2 e^{j\frac{2\pi}{\lambda}d_x \sin \alpha_k + j\varphi_2}$ and $a_{Mk} = G_M e^{j\frac{2\pi}{\lambda}d_p + j\varphi_M}$ whose

corresponding eigenvalue is $\Phi(k, k) = \frac{G_2}{G_M} e^{j\frac{2\pi}{\lambda}(d_y \cos \alpha_k - d_a \sin \alpha_k + \varphi_M - \varphi_2)}$.

4. The algorithm

In summary, the steps of the algorithm are:

Step1: estimate the number of sources in the received signal;

Step2: construct spatial time DOA matrix R_{TS} from (8), (9), (12), (13);

Step3: eigen-decompose R_{TS} ;

As to the k -th source, we can get 3 equations:

$$a_{2k} = e^{j\frac{2\pi}{\lambda}d_x \sin \alpha_k + j\varphi_2} \quad (17)$$

$$a_{Mk} = e^{j\frac{2\pi}{\lambda}d_y \cos \alpha_k + j\frac{2\pi}{\lambda}\sqrt{r^2 - d_y^2} \sin \alpha_k + j\varphi_M} \quad (18)$$

$$\Phi(k, k) = e^{j\frac{2\pi}{\lambda}(d_y \cos \alpha_k - d_a \sin \alpha_k + \varphi_M - \varphi_2)} \quad (19)$$

Step4: form simultaneous equations to solve DOA, amplitude and phase errors.

Amplitude errors can be figured out directly from (17), (18).

$$G_2 = abs(a_{2k}) \quad (20)$$

$$G_M = abs(a_{Mk}) \quad (21)$$

As to phase errors, from (17), (18) and (19) we can obtain:

$$angle(a_{2k}) = \frac{2\pi}{\lambda}d_x \sin \alpha_k + \varphi_2 \quad (22)$$

$$angle(a_{Mk}) = \frac{2\pi}{\lambda}(d_y \cos \alpha_k + \sqrt{r^2 - d_y^2} \sin \alpha_k) + \varphi_M \quad (23)$$

$$angle(\Phi(k, k)) = \frac{2\pi}{\lambda}(d_y \cos \alpha_k - d_a \sin \alpha_k) + \varphi_M - \varphi_2 \quad (24)$$

Since (24) is just a linear combination of (23) and (24), there are actually only two equations about

three unknowns $\alpha_k, \varphi_2, \varphi_M$. But when there are two sources in the received signal, we can get four equations about $\alpha_1, \alpha_2, \varphi_2$ and φ_M .

$$\text{angle}(a_{21}) = \frac{2\pi}{\lambda} d_x \sin \alpha_1 + \varphi_2 \quad (25)$$

$$\text{angle}(a_{M1}) = \frac{2\pi}{\lambda} (d_y \cos \alpha_1 + \sqrt{r^2 - d_y^2} \sin \alpha_1) + \varphi_M \quad (26)$$

$$\text{angle}(a_{22}) = \frac{2\pi}{\lambda} d_x \sin \alpha_2 + \varphi_2 \quad (27)$$

$$\text{angle}(a_{M2}) = \frac{2\pi}{\lambda} (d_y \cos \alpha_2 + \sqrt{r^2 - d_y^2} \sin \alpha_2) + \varphi_M \quad (28)$$

The DOA of arrival and phase errors can be obtained through solving the equations. If there is only one source in the received signal, four equations like (25) ~ (28) can also be constructed by combining this group of signal with another group of received signal. Thus the DOA and phase errors can be figured out.

From the previous demonstration, it can be concluded that DOA, amplitude and phase errors can be estimated simultaneously by redundancy generated by applying Spatial Time DOA Matrix Method to one dimension DOA estimation.

5. Computer Simulation

Assuming there are two arrays, one is a right triangle whose $d_x = 13.3m$ and $r = d_y = 8m$; the other is a cute one whose $d_x = 13.3m, d_y = 8m$ and $r = 15.52m$. Simulated sea echo arrive at the arrays from arbitrary direction within $-90^\circ \sim 90^\circ$. Set the amplitude and phase errors of x_2 and x_M relative to x_M as $G_2 = 1.2, G_M = 0.8, \varphi_2 = 80^\circ, \varphi_M = 110^\circ$. Repeat the experiment 20 times respectively when the signal to noise rates (SNR) are 5dB, 10dB and 15dB. Under each SNR, the mean results of the amplitude errors are 1.2 and 0.8, and the variance of each result is smaller than 10^{-4} . It verifies the above algorithm can estimate amplitude errors precisely. The results of phase errors estimation are in the following table.

Array Manifold	SNR	φ_2		φ_M	
		Mean	Variance	Mean	Variance
The cute triangle	5dB	81.76	245.46	110.72	22.56
	10dB	81.57	55.30	110.45	5.80
	15dB	80.02	18.39	110.32	2.11
The right triangle	5dB	81.62	237.82	110.69	22.45
	10dB	81.44	56.50	110.32	5.95
	15dB	80.36	17.35	110.12	2.05

It can be concluded from the table that when the SNR is low, a single result of the phase errors may be far from the value previously set because of the noise and the error introduced by process of solving the equations. However, the mean of the results is very close to the set value even under

low SNR conditions.

6. Measured Data

Select the data collected at 11:20 on 26 June, 2004 by OSMAR2000 of Wuhan University. The shapes of the two arrays consisting of three whips are a right triangle and an obtuse one (showed in Fig.2).

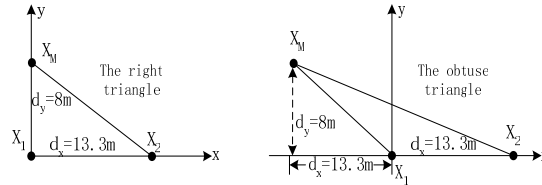


Fig. 2 The sketch map of arrays

Estimate the DOA of the boat's echo by MUSIC. The following table compares treatment results of measured data before and after calibration with the boat's azimuth measured by GPS.

Array Manifold	GPS Data	Result before calibration	Result after calibration
Right triangle	-24.72°	-12.62°	-24.93°
Obtuse triangle		-13.76°	-25.21°

This table verifies that the performance of MUSIC has been improved remarkably.

7. Conclusion

It can be concluded both from the computer simulation and measured data that this algorithm can calibrate the HF Ground wave radar system's triangular antenna array well so as to greatly improve the performance of MUSIC. Though the calibrating results of phase errors wave around the true errors when the SNR is low, the mean of the results is very close to the true value. All these verify that this algorithm is effective and valid.

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