REDUCTION OF MULTIPLE ACCESS INTERFERENCE IN MC-CDMA IN THE PRESENCE OF RAYLEIGH FADING

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ABSTRACT
A new technique is proposed that offers significant reduction of multiple access interference (MAI) encountered in asynchronous multicarrier code division multiple access (MC-CDMA) detection. Using a dual frequency switching component at the transmitter enables the removal of a portion of unwanted interference signals from the decision variable. Bit-error rate (BER) comparisons are shown for the use of both Gold and Walsh-Hadamard (WH) spreading sequences. The proposed switching technique clearly outperforms the existing MC-CDMA technique, however, additional bandwidth is required.

1. INTRODUCTION
Multicarrier code division multiple access (MC-CDMA) [1] has recently received a great deal of attention, investigating its potential benefits over the direct-sequence CDMA (DS-CDMA) technique featuring predominantly in third generation (3G) cellular technologies such as UMTS, W-CDMA and cdma2000. Over asynchronous channels, where multiple access interference (MAI) is significantly degrading, the use of MC-CDMA has been demonstrated to outperform the single carrier DS-CDMA technique [2][3][4]. The multicarrier modulation format of the MC-CDMA technique allows the number of subcarriers to be designed such that each subchannel is subject to frequency nonselective fading [3] and furthermore, the insertion of a cyclic prefix has been shown to mitigate intersymbol interference in the presence of multipath [5]. MAI remains a destructive presence and a major limitation in CDMA detection and here we propose a dual frequency switching technique that reduces the amount of MAI incurred during asynchronous transmission.

The proposed scheme is named dual frequency MC-CDMA (DF/MC-CDMA) and involves periodic switching of the carrier frequency between two selected frequencies, which induces a Doppler separation between a portion of the unwanted MAI and the desired data. Such Doppler separation alleviates the decision variable of the MAI emanating from lagging data symbols in a symbol-by-symbol recovery method.

This paper is organized as follows: section 2 details the considered cellular scenario, specifying the asynchronous channel model. The signal models of the existing MC-CDMA and proposed DF/MC-CDMA systems are established in sections 3 and 4 respectively. In these sections the MAI is formulated and the partial reduction of MAI achieved by the DF/MC-CDMA technique is deduced in section 5. Section 6 defines the bit-error rate (BER) performance metric and simulation results are given in section 6 showing a comparison between the MC-CDMA and DF/MC-CDMA performances for the use of both Gold and Walsh Hadamard (WH) codes. Finally, section 7 concludes the paper.

2. SYSTEM DESCRIPTION
A single cell consisting of K users \((k = 1, \ldots, K)\) transmitting asynchronously over the uplink channel is considered. The \(ith\) data symbol of user \(k\) is denoted as \(b_{ki}\) for which all data is BPSK modulated and \(i.i.d\) such that \(Pr\{b_{ki} = -1\} = Pr\{b_{ki} = 1\} = 0.5\). The spreading factor of each code is given as \(N\), \(T_s\) and \(T_c\) where \(T_s\) and \(T_c\) denote the symbol and chip durations respectively. The data stream and assigned spreading code of the \(kth\) user are respectively expressed as

\[ b_k(t) = \sum_{i=-\infty}^{\infty} b_{ki} \cdot u_{T_s}(t - iT_s) \] (1)

and

\[ c_k(t) = \sum_{n=1}^{N} c_{kn} \cdot u_{T_c}(t - nT_s + T_c) \] (2)

where \(c_{kn}\) is the \(n\)th chip of the \(k\)th code and

\[ u_T(t) = \begin{cases} 1, & 0 \leq t < T \\ 0, & \text{elsewhere} \end{cases} \] (3)

is the rectangular pulse shaping waveform.
We consider an asynchronous slow varying Rayleigh fading uplink channel in which \( \tau_k \) denotes the timing offset of the \( k \)th user. We model the channel attenuation experienced by the \( n \)th subchannel of the \( k \)th user as a zero-mean Gaussian random variable \( \beta_k \exp(\jmath \varphi_k) \) for which \( \beta_k \) is Rayleigh distributed with \( E[\beta_k^2] = 1 \) and \( \varphi_k \) is equally distributed over the interval \([-\pi, \pi]\). Moreover, we consider slow varying Rayleigh fading and hence we assume the gain is constant over two symbol durations, \( 2T_s \).

The timing offset of each user is assumed to be distributed over one symbol duration, \( T_s \), with equal probability. Timing offsets are quantized to integer multiples of the chip duration, \( T_c \), making \( \tau_k \in [0, (N-1)T_c] \). Note that timing offsets \( \tau_k \) account for the propagation delay and the timing misalignments amongst users. All timing offsets are made with respect to the reference user, denoted as user \( x \), for which \( \tau_x = 0 \) and for simplicity purposes, all interferers are temporally offset such that \( \tau_k \geq 0 \), \( \forall k \). This offset condition of the asynchronous channel is depicted in Fig. 1.

We assume perfect power control at the base station and the signal power of each user is normalized to unity in both systems. Moreover, the assumption of perfect carrier and symbol recovery is made for both the MC-CDMA and DF/MC-CDMA systems.

### 3. EXISTING MC-CDMA DETECTION

The transmitted MC-CDMA signal of the \( k \)th user, ignoring any random phase offset, is expressed as

\[
s_k(t) = \sum_{n=1}^{N} b_k(t) c_k \exp(\jmath \omega_n t)
\]  

where \( b_k(t) \) is defined in (1) and \( \omega_n = \omega_c + \frac{2\pi n}{T_s} \) is the \( n \)th subcarrier frequency, neighbouring subcarriers being orthogonal and separated by \( \Delta \omega = \frac{2\pi}{T_s} \). The MC-CDMA received signal at the base station is written as [4]

\[
r(t) = \sum_{k=1}^{K} \sum_{n=1}^{N} \beta_k b_k(t - \tau_k) c_k \exp(\jmath \omega_n t - \phi_k + \varphi_k) \]

where \( \phi_k = \omega_c \tau_k \) and \( \varphi_k \) is the phase offset experienced over the \( n \)th subcarrier of user \( k \). Following subcarrier demodulation and despreading, the matched filter output of the MC-CDMA system is given as [6]

\[
y_{x_k}^{mc} = \frac{1}{T_s} \int_{T_s}^{(n+1)T_s} r(t) \sum_{n'=-1}^{N} \alpha_{x,n'} c_{x,n'} \exp(-j \omega_{n'} t + \varphi_{x,n'}) \ dt
\]

where \( \alpha_{x,n'} \) is dependent on the combining strategy used. Applying equal gain combining (EGC) [7], making \( \alpha_{x,n'} = 1 \), the matched filter output in (6) becomes [6]

\[
y_{x_k}^{mc} = \frac{1}{T_s} \sum_{k=1}^{K} \sum_{n=1}^{N} \beta_k c_k \sum_{n'=1}^{N} \exp(-j \omega_{n'} t + \varphi_{x,n'}) \int_{T_s}^{(n+1)T_s} \ dt
\]

where \( \xi_{k,n'} = -\phi_k + \varphi_k - \varphi_{x,n'} \). Due to the symbol boundary misalignments caused by \( \tau_k \), the matched filter output is conditioned on \( b_{k,i} \) as well as the previous lagging symbol \( b_{k,(i-1)} \) of each interfering signal, which is expressed as [4]

\[
y_{x_k}^{mc} = \frac{1}{T_s} \sum_{k=1}^{K} \sum_{n=1}^{N} \beta_k c_k \sum_{n'=1}^{N} \exp(-j \omega_{n'} t + \varphi_{x,n'}) \int_{T_s}^{(n+1)T_s} \ dt
\]

This expression in (9) is simplified as

\[
y_{x_k}^{mc} = \frac{1}{T_s} \sum_{k=1}^{K} \sum_{n=1}^{N} \beta_k c_k V_{k_x}(\tau_k) + b_{k,i} \tilde{V}_{k_x}(\tau_k)
\]

where the partial spectral correlation functions [8], \( V_{k_x}(\tau_k) \) and \( \tilde{V}_{k_x}(\tau_k) \), are given by

\[
V_{k_x}(\tau_k) = \sum_{n=1}^{N} \sum_{n'=1}^{N} \beta_k c_k \exp(j \xi_{k,n'}) \int_{0}^{T_s} \exp(j \omega_n - \omega_{n'}) t \ dt
\]

\[
\tilde{V}_{k_x}(\tau_k) = \sum_{n=1}^{N} \sum_{n'=1}^{N} \beta_k c_k \exp(j \xi_{k,n'}) \int_{T_s}^{T_{s}(n+1)} \exp(j \omega_n - \omega_{n'}) t \ dt
\]
\[ \hat{V}_{kz}(\tau_k) = \sum_{n=1}^{N} \sum_{n'=1}^{N} \beta_{kn} c_{kn} c_{zn'} \exp(j \xi_{kn'}) \int_{\tau_n}^{T_s} \exp(j(\omega_n - \omega_{n'}) t) \, dt \]  

The decision variable in (10) can finally be expressed as

\[ y_{x_i}^{mc} = \beta_x b_{x_i} \sum_{n=1}^{N} c^2_n + \sum_{k \neq x} M_{kz_i}^{mc} \]  

where

\[ M_{kz_i}^{mc} = \frac{1}{T_s} \left[ b_{k(i-1)} \hat{V}_{kz}(\tau_k) + b_{ki} \hat{V}_{kz}(\tau_k) \right] \]  

is the MAI incurred by user \( x \) from user \( k \) in the recovery of \( b_{x_i} \).

4. PROPOSED DF/MC-CDMA DETECTION

Excluding channel fading, the distinguishing attribute separating the desired information signal from the multiple interferences signals is the absence of timing misalignment between the desired symbols and the detection interval \( i T_s \leq t \leq (i + 1) T_s \). To exploit this condition, a controlled frequency separation is inserted between consecutive symbols (the MAI incurred by user \( x \) from user \( k \) in the recovery of \( b_{x_i} \)).

The time-variant switching pattern, denoted by \( \omega_d(t) \), of period \( T_s \) is represented by

\[ \omega_d(t) = \omega_d^i, \quad i T_s \leq t \leq (i + 1) T_s \]  

where \( \omega_d^i \in \{ \omega_d^1, \omega_d^2 \} \) denotes the switching frequency that alternates every \( T_s \) such that \( \omega_d^i \neq \omega_d^{(i+1)} \). Applying this switching component to the previously defined MC-CDMA format yields the transmitted DF/MC-CDMA signal of the \( k \)th user (depicted in Fig. 2), expressed as

\[ s_{k}^{df}(t) = \sum_{n=1}^{N} b_k(t) c_{kn} \exp(j(\omega_n + \omega_d(t))t) \]  

which is of similar format to frequency hopping modulated signals as in [9]. Choosing the switching period as \( T_s \) enables one frequency shift every symbol duration making \( \omega_d^i \), the switching frequency constant over the \( i \)th symbol and all frequency switching is assumed time-synchronous with the symbol boundaries.

\[ r(t) = \sum_{k=1}^{K} \sum_{n=1}^{N} \beta_{kn} b_k(t-\tau_k) c_{kn} \exp(j(\omega_n + \omega_d(t-\tau_k)) \{ t-\tau_k \} + \varphi_{kn}) \]  

For the detection of \( b_{x_i} \), the signal received at the BS is first deswitched by the appropriate frequency, \( \omega_d^x \), corresponding to the detection interval \( i T_s \leq t \leq (i + 1) T_s \), by \( \exp(-j\omega_d^x t) \). Subcarrier demodulation and despreading follows through the application of \( c_{zn'} \exp(-j\omega_{zn'}) \) for \( n' = 1, \ldots, N \). At this stage of the detection process we are dealing with baseband signal components (following the deswitching and subcarrier demodulation operations). Here a low-pass filter is applied to retain the signal components associated with \( \omega_d^i \) and remove those associated with \( \omega_d^{(i+1)} \). The output of the matched filter is then expressed as

\[ y_{x_i}^{df} = \frac{1}{T_s} \int_{i T_s}^{(i+1)T_s} r(t) \sum_{n'=1}^{N} \alpha_{zn'} c_{zn'} \exp(-j(\omega_{zn'} + \omega_d^x) t + \varphi_{zn'}) \, dt \]  

Employing EGC such that \( \alpha_{zn'} = 1 \) and substituting the received signal in (17) into (18) yields

\[ y_{x_i}^{df} = \frac{1}{T_s} \int_{i T_s}^{(i+1)T_s} \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{n'=1}^{N} \beta_{kn} b_k(t-\tau_k) c_{kn} c_{zn'} \exp(j(\omega_n - \omega_{zn'} + \omega_d(t-\tau_k) - \omega_d^x) \{ t-\tau_k \} + \varphi_{kn} - \varphi_{zn'}) \, dt \]  

Fig. 2. DF/MC-CDMA Transmission Signal \( s_{k}^{df}(t) \)
Expanding this expression into distinct symbol components yields

\[
\begin{align*}
&= \frac{1}{T_s} \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{n'=1}^{N} \beta_k c_k c_{xn'} \\
&\quad \cdot \int_{iT_s + \tau_k}^{(i+1)T_s} \exp \left( j \left( \omega_n - \omega_{n'} + \nu_i \right) t \right. \\
&\quad \left. + \Psi_k(c_{kn'}) \right) dt \\
&+ \frac{b_k}{T_s} \sum_{i=1}^{K} \sum_{n=1}^{N} \sum_{n'=1}^{N} \beta_k \int_{iT_s + \tau_k}^{(i+1)T_s} \exp \left( j \left( \omega_n - \omega_{n'} + \nu_i \right) t + \Psi_k(c_{kn'}) \right) dt
\end{align*}
\]

(20)

where \( \Psi_k(c_{kn'}) = -\{ \omega_n + \omega_{n'} \} \tau_k + \phi_k n - \phi_k n' + \nu_i = \omega_d(i-1) - \omega_d(i) \). Separating the expression in (20) into the desired information and MAI components gives, leaves

\[
y^{(i)}_{\bar{x}} = b_i \sum_{n=1}^{N} \beta_{kn} c_n^2 + \sum_{k \neq i} \tilde{M}^d_{kzi}
\]

(21)

which comprises of the desired information component and a sum of MAI components from the \( K - 1 \) interfering users. The MAI contribution from a single interferer consists of two components such that

\[
\tilde{M}^d_{kzi} = \tilde{M}^d_{kzi} + \tilde{M}^d_{kzi}
\]

(22)

where \( \tilde{M}^d_{kzi} \) and \( \tilde{M}^d_{kzi} \) are the MAI components over the temporal intervals \( iT_s \leq t \leq iT_s + \tau_k \) and \( iT_s + \tau_k \leq t \leq (i + 1)T_s \) respectively. The first MAI component is expressed as

\[
\tilde{M}^d_{kzi} = \frac{1}{T_s} \sum_{n=1}^{N} \sum_{n'=1}^{N} \beta_k c_k c_{xn'} \\
\cdot \exp \left( j \left( \omega_n - \omega_{n'} + \nu_i \right) t + \Psi_k(c_{kn'}) \right) dt
\]

(23)

and represents the MAI incurred over the interval \( iT_s \leq t \leq iT_s + \tau_k \). It is seen that all interfering signal components are successfully removed by the low-pass filter due to the presence of \( \nu_i \) over the interval \( iT_s \leq t \leq iT_s + \tau_k \) (23). It can therefore be written that

\[
\tilde{M}^d_{kzi} = 0 \quad \forall k \neq x \quad \text{iff} \quad \nu_i \geq \frac{N}{T_s}
\]

(24)

and it follows that no MAI is incurred over this portion of the symbol detection interval.

The second MAI component (representing MAI incurred over the detection interval \( iT_s + \tau_k \leq t \leq (i + 1)T_s \)) is not a function of the Doppler offset \( \nu_i \) and is expressed as

\[
\tilde{M}^d_{kzi} = \frac{1}{T_s} \sum_{n=1}^{N} \sum_{n'=1}^{N} \beta_k c_k c_{xn'} \\
\cdot \exp \left( j \left( \omega_n - \omega_{n'} + \nu_i \right) t + \Psi_k(c_{kn'}) \right) dt
\]

(25)

or alternatively as

\[
\tilde{M}^d_{kzi} = \frac{b_k}{T_s} \tilde{V}_{kz} (\tau_k) \exp \left( -j \omega_d \tau_k \right)
\]

(26)

where \( \tilde{V}_{kz} (\tau_k) \) was defined in (12).

\section{5. PERFORMANCE}

To assess the benefits of the proposed DF technique it is desirable to compare the BER performance against that of the existing DS-CDMA technique. With all MAI being zero-mean, the total MAI power inflicted by each interferer during detection is \( \tilde{M}^d_{kzi} \). For a given offset, the MAI power can take different values depending on \( b_k(i-1) \) and \( b_k \) and therefore we consider the conditional MAI power in our system performance analysis which is the conditional expectation of the MAI power for a given offset value, defined as

\[
\sigma_z^2 (\xi) = \sum_{k \neq z} E \left[ \tilde{M}^d_{kzi} \left( \beta_k, b_k, b_k(i-1), \tau_k \right) | \tau_k \right]
\]

(27)

where \( E[a|b] \) denotes the expectation of \( a \) given \( b \). \( \beta_k = [ \beta_k \ldots \beta_k ] \) and \( \xi \) denotes the vector of offsets defining the asynchronous channel for which \( \tau_k = 0 \). In the absence of additive noise, the signal-to-interference ratio (SIR) is given by

\[
\gamma = \left( \frac{\beta_z}{\sigma_z} \right)^2
\]

(28)

and the resulting probability of error for BPSK modulated data is given by [10]

\[
P_e = Q \left( \sqrt{\gamma} \right) = Q \left( \frac{\beta_z}{\sigma_z} \right)
\]

(29)

where

\[
Q (x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{t^2}{2} \right) dt
\]

(30)

is the complementary Gaussian cumulative distribution function.

\section{6. SIMULATIONS}

Monte Carlo simulations were performed to verify the MAI reduction capability of the proposed DF/MC-CDMA technique. The MAI reduction achieved by the proposed method can be experienced using any set of spreading sequences, however, only the results for the use of Gold and WH codes are presented here. Both the MC-CDMA and DF/MC-CDMA systems considered were simulated, running 10,000 realizations and randomly generating a different offset vector \( \xi \) in each realization. To ensure a fair performance comparison, the same channel and offset vector were applied in each system. The bandwidth expansion in the proposed DF/MC-CDMA
is minimized by taking the separation between the dual frequencies as $N/T_s$.

To demonstrate the reduction of MAI incurred by the reference user, the observed total MAI power, $\sigma_z^2$, was recorded for each realization and the corresponding histograms are displayed in Figs.3 and 4 for a $K = 64$ loaded system using Gold codes of $N = 63$ and WH codes of $N = 64$ respectively. These histograms are successfully fitted by the Nakagami-m distribution [11] as shown by the smooth curves in Figs.3 and 4; the details of this fitting form the topic of a previous study [12]. It can be seen that these histograms follow the Nakagami-m power distribution given by [12]

$$p(\sigma_z^2) = \left(\frac{m}{\zeta}\right)^m \frac{(\sigma_z^2)^{m-1}}{\Gamma(m)} \exp\left(-\frac{m\sigma_z^2}{\zeta}\right) \quad (31)$$

where $m = E[\sigma_z^2]^2 / \text{var}(\sigma_z^2)$, $\zeta = E[\sigma_z^2]$ and $\Gamma(m)$ is the Gamma function given by

$$\Gamma(m) = \int_0^\infty x^{m-1} \exp(-x) \, dx \quad (32)$$

As is evident from these histograms, the DF/MC-CDMA scheme offers significant MAI reduction and the range of total MAI power values is considerably lower than those of the MC-CDMA scheme.

Figure 5 shows the BER performance for an increasing number of interferers, of both systems for the use of Gold codes of $N = 63$ while Fig. 6 illustrates the corresponding BER performance using WH codes of $N = 64$. It can clearly be observed in both figures that the proposed dual frequency technique offers an improved BER performance as a
result of the partial MAI reduction. The major drawback of this reduction technique however, is the bandwidth increase associated with the addition of the dual frequency switching component. From a design point of view, a decision on whether additional bandwidth is expendable to achieve more attractive BERs needs to be made when considering the proposed dual frequency system described here; with $\nu = N/T_s$ the bandwidth is increased to approximately twice that of the MC-CDMA system.

7. CONCLUSION

A method for the partial reduction of MAI incurred by an MC-CDMA system over an asynchronous Rayleigh fading channel has been proposed. Applying a periodic switching component to the MC-CDMA technique shifts a portion of the unwanted interference signals to an alternative frequency band. The proposed DF/MC-CDMA technique offers an improved BER performance over its MC-CDMA counterpart, however, this improvement comes at the cost of bandwidth sacrifice.

8. REFERENCES


