Impact of Phase Noise in MIMO-OFDM Systems

Raghu Mysore Rao
Advanced Systems Technology Group, DSP Division,
XILINX INC, 2100 Logic Drive, San Jose, CA 95124
raghu.rao@xilinx.com

Abstract

As MIMO-OFDM systems gain in popularity and move towards commercialization, the impact of analog hardware impairments need to be addressed to ensure these systems achieve the significant capacity/throughput gains they are capable of. In this paper we discuss the impact of correlated and uncorrelated phase noise on MIMO-OFDM systems. Where transmit (or receive) antennas are not co-located the phase noise is uncorrelated. Even in systems where the antennas at the transmitter/receiver are co-located, the multiple RF chains could have dedicated PLLs which introduce uncorrelated phase noise. We compare the performance degradation due to phase noise and show the impact of correcting phase noise in both these situations.

Index Terms - Analog Impairments, Phase Noise, Lorenzian, OFDM, MIMO-OFDM.

1. Introduction

MIMO-OFDM is being considered for communication systems where high throughput and spectral efficiency are important factors. Theoretical capacity calculations show significant capacity/throughput gains from a MIMO-OFDM system. However, to measure the true performance of the system the impact of analog impairments needs to be considered. A spatial multiplexing MIMO-OFDM system transmits independent OFDM modulated data from multiple antennas simultaneously. At the receiver, after OFDM demodulation, MIMO decoding extracts the different transmitted data streams from each of the subcarriers, as long as the subcarriers are mutually orthogonal. If the subcarriers lose their orthogonality due to analog and RF impairments, the performance of the MIMO-OFDM system degrades dramatically.

Phase noise is caused by non-idealities in the local oscillators (LO) of the system causing the power spectral density (PSD) to exhibit skirts around the carrier frequency. The power spectrum of the noisy carrier turns out to be Lorenzian [1]. Phase noise in OFDM systems has been studied extensively in the literature [2]-[5]. In MIMO-OFDM systems, similar to OFDM systems, the interference due to phase noise can be separated into a common phase error (CPE) term and an inter-carrier interference (ICI) term [2]-[6][8]. The extent of CPE, which can be estimated and corrected, depends on a number of architecture and system level factors. As the number of subcarriers increases the CPE term decreases and the ICI term increases [3]-[5]. The CPE decreases as the number of antennas increases in a power constrained MIMO-OFDM system [2][6]. Similarly, when the phase noise is uncorrelated the amount of CPE decreases.

Correlated phase noise occurs when the RF chains share a common PLL for the LO signals. However, distributing high frequency LO signals requires expensive power splitters. Frequently, a low frequency clocking signal is distributed to the multiple RF chains which have dedicated PLLs to generate the LO signals. Also, many of the RF transceiver ICs in the market today, have built in PLLs. All of these cases, as well as when the multiple antennas of a MIMO system are not co-located result in uncorrelated phase noise.

Recently, the impact of phase noise on MIMO-OFDM systems has been studied in [6][8]-[10]. In [6][9], the statistics of phase noise interference are computed and phase noise estimation techniques are discussed in [8][9]. All of these assume correlated phase noise at the transmitters and at the receivers and simulations use correlated phase noise only at the receiver. In this paper we discuss MIMO-OFDM systems with uncorrelated phase noise which we believe is an equally common scenario in real-world systems and compare it with systems with correlated phase noise. More importantly, the issue of correlated v/s uncorrelated phase noise is a tradeoff that needs to be borne in mind during the system design phase. We show that in the case of uncorrelated phase noise at the transmitters, in a spatial multiplexing MIMO-OFDM system, the CPE needs to be estimated and corrected independently for the different data streams. In the case of correlated phase noise the CPE is common to the various data streams and the estimates improve due to diversity [8]. In general the amount of CPE, the correctable term, is much higher when the phase noise is correlated compared to the uncorrelated case.

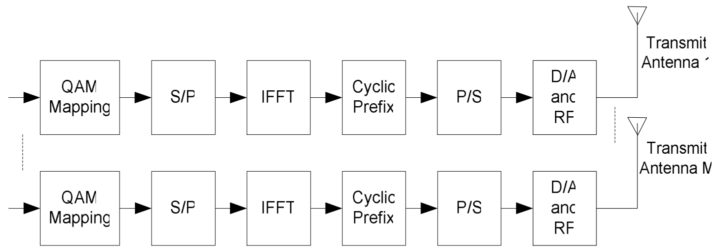


Figure 1 Transmitter of a MIMO-OFDM system

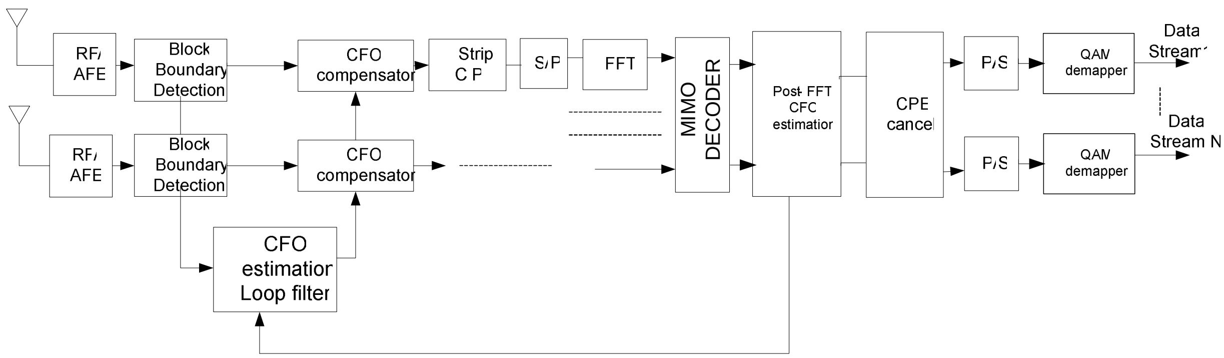


Figure 2 Receiver of a MIMO-OFDM system

The rest of the paper is organized as follows. In Section 2 the components of phase noise in a MIMO-OFDM system are derived for uncorrelated phase noise. Section 3 discusses simulation results and Section 4 concludes the paper.

2. MIMO-OFDM System With Phase Noise

The block diagram of the transmitter and receiver of a MIMO-OFDM system are shown in Figure 1 and Figure 2 respectively. When the LOs in the RF chain of each transmitter and each receiver share a common PLL the phase noise is correlated and uncorrelated when they have dedicated PLLs to generate their LO signals.

For the following discussion we use a MIMO-OFDM model identical to [8]. We have reproduced the model here for completeness with suitable modifications for uncorrelated phase noise. We consider a *M*x*N* MIMO-OFDM system with N_s subcarriers, where M is the number of transmit antennas and N is the number of receive antennas. The a^{th} transmit vector is $\hat{s}(a) = [s^T(0,a), s^T(1,a), ..., s^T(N_{s-1}, a)]^T$, where $s(n,a)$ denotes the MIMO transmit vector on the n^th^ subcarrier. This vector is transformed to the time domain and after adding a cyclic prefix (CP) and optional block shaping, is converted to the analog domain, upconverted to the carrier frequency and transmitted over the air. We assume a power constrained MIMO system and the total transmit power is equally distributed among the M transmit antennas. The baseband signal at the data converters is given by

$$\hat{m}(a) = \theta(F^{-1} \otimes I_M)\hat{s}(a) \quad (1)$$

where $\otimes$ represents the Kronecker product and F is the $N_s \times N_s$ Fourier matrix with the *(i,j)*th element being $\exp(-j2\pi \frac{ik}{N_s})$. F^{-1} represents the inverse Fourier transform. I_M is the $M \times M$ Identity matrix. θ is the matrix that adds the cyclic prefix. In the presence of phase noise at the transmitter RF frontend the signals are given by

$$\tilde{m}(a) = E_{Tx}\theta(F^{-1} \otimes I_M)\hat{s}(a) \quad (2)$$

$$E_{Tx} = diag[e_{Tx}(0,1),.,e_{Tx}(0,M),.,e_{Tx}(N_s + N_g, M)]$$

is the $(N_s + N_g)M \times (N_s + N_g)M$ phase noise matrix at the transmitter. $e_{Tx}(t,m) = \exp(j\theta(t,m))$ represents the phase distortion seen by the signal at antenna m at sample time t.

The signals are transmitted over a multi-path Rayleigh fading channel and are received by multiple receive antennas. The time domain channel matrix C is block-circulant and is defined as

$$C = \begin{bmatrix} c(0) & 0 & & c(L) & \cdots & c(1) \\ c(1) & c(0) & & & & \\ c(2) & c(1) & c(0) & & & c(L) \\ \vdots & & & \ddots & & \\ & & & & & \\ 0 & & c(L) & \cdots & & c(0) \end{bmatrix} \quad (3)$$

where the length of the channel is $L+1$, and

$$c(n) = \begin{bmatrix} c_{11}(n) & \cdots & c_{1M}(n) \\ \vdots & \ddots & \\ c_{N1}(n) & & c_{NM}(n) \end{bmatrix} \quad (4)$$

and $c_{nm}(n)$ represents the complex amplitude of the n^{th} channel tap from transmit antenna *m*, to receive antenna *n*. We assume that the cyclic prefix is longer than the length of the channel to avoid ISI.

The signals at the receiver baseband, impacted by uncorrelated phase noise during the down-conversion process, after stripping the cyclic prefix and transforming to the frequency domain can be represented as

$$\hat{y}(a) = (F \otimes I_N)\theta^{-1}E_{Rx}(a)[C\tilde{m}(a) + \hat{v}(a)] \quad (5)$$

$$\hat{y}(a) = G_{Rx}HG_{Tx}\hat{s}(a) + \hat{n}(a) \quad (6)$$

E_{Rx} is defined similar to E_{Tx} and represents phase noise at the receiver. θ^{-1} represents the matrix that strips the cyclic prefix. It is shown in [8] that in the absence of phase noise $\theta^{-1}C\theta$ can be diagonalized by the DFT and IDFT operations and therefore the subcarriers are orthogonal. However in the presence of phase noise $\theta^{-1}E_{Rx}CE_{Tx}\theta$ is no longer block circulant and therefore cannot be diagonalized by the FFT and IFFT operations. This indicates ICI due to phase noise will occur. G_{Tx} and G_{Rx} represent the convolution matricies in the frequency domain of the phase noise processes. The phase noise process in the frequency domain is given by

$$g(a,k) = \frac{1}{\sqrt{N_s}} \sum_{n=0}^{N_s-1} e^{j\theta(a,n+N_g)} e^{\frac{-j2\pi nk}{N_s}} \quad (7)$$

At this point, we can also examine the components of the received signal vector $\hat{y}(a)$ in detail. The received signal at receiver *n* and subcarrier *k'* is given by (we ignore *a* for simplicity),

$$\hat{y}_n(k') = \frac{1}{N_s} \sum_{l=0}^{N_s-1} \sum_{k=0}^{N_s-1} \tilde{g}_{Rx,n}(k') [H(k)\tilde{s}(k) e^{j\frac{2\pi l(k-k')}{N_s}}] + \hat{n}(k') \quad (8)$$

$$\tilde{s}(k) = \left[s_0(k)\tilde{g}_{Tx,m}(k'), .., s_m(k)\tilde{g}_{Tx,m}(k') \right]^T$$

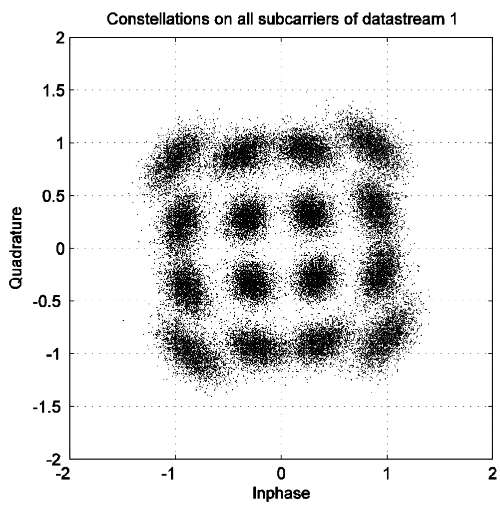
The MIMO channel matrix $H(k)$ on each subcarrier is a 1-tap Rayleigh fading channel. We see that on each subcarrier the transmit vector is rotated by the phase noise at the transmitter and the phase noise at the receiver. The phase noise terms $\tilde{g}_{Tx}(k)$ and $\tilde{g}_{Rx}(k)$ represent in the frequency domain the rotation on subcarrier *k* due to the phase noise at the transmitter and receiver respectively. Grouping the phase noise terms and invoking the small signal approximation for phase noise $\exp(j\phi) \approx 1 + j\phi$, we get

$$\begin{aligned} \hat{y}_n(k') &= H(k')\hat{s}(k') + \\ & j \sum_{k=k',m=1}^{M} (\phi_{Tx,m} + \phi_{Rx,n}) h_{nm}(k') s_m(k') \\ & \frac{j}{N_s} \sum_{l=0}^{N_s-1} \sum_{k=0,k\neq k'}^{N_s-1} \sum_{m=1}^{M} (\phi_{Tx,m} + \phi_{Rx,n}) h_{nm}(k) s_m(k) e^{j\frac{2\pi l(k-k')}{N_s}} \\ & + \hat{n}(k') \end{aligned} \quad (9)$$

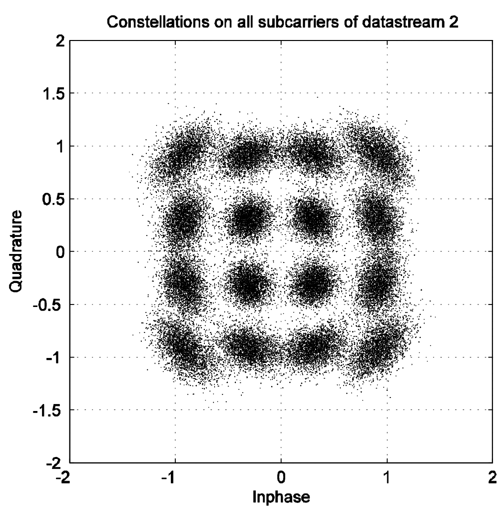
$\phi_{Tx,m}$ and $\phi_{Rx,n}$ represent the combined interference on each subcarrier after the small signal approximation, due to the phase noise at the transmitter and receiver respectively. We also separate the CPE and the ICI terms by setting $k = k'$ to generate the CPE term and $k \neq k'$ to generate the ICI term. We see that although the common phase error is common to all the subcarriers of a data stream, the interference from the phase noise at the transmitter causes the rotation to be independent on each of the data streams. This is shown in Figure 3. When the phase noise is correlated at the transmitters, we see that the phase noise interference is common across all the datastreams and can be written as $j\left(\phi_{Tx} + \phi_{Rx,n}\right)H(k')\hat{s}(k')$.

MIMO decoding involves cancelling the effects of the channel and estimating each of the transmitted data streams. When a weight matrix *W* (either an MMSE solution or a zero-forcing solution) is applied to the received vector $\hat{y}(k')$ on each subcarrier we see that the original data stream is recovered, but the constellations suffer a rotation due to CPE and are degraded due to the ICI term of phase noise. We also note that the CPE term is scaled by the transmit signal on each antenna similar to the SISO-OFDM case. This results in the CPE term decreasing as the number of antennas increases for a power constrained MIMO system [6].

The CPE term can be estimated and cancelled using continuous pilots. In the case of correlated phase noise, a joint estimate of the CPE across all the data streams can be used to correct CPE as shown in [9]. There is some gain due to diversity in this case [9]. However, when the phase noise is uncorrelated at the transmitters, the estimation needs to be per data stream. We also note that the presence of the ICI term from the phase noise components on the other data streams increases the phase noise distortion in the case of uncorrelated phase noise. Simulation results in the next section bear this out.



(a)



(b)

Figure 3 (a) Data stream 1 (b) Data stream 2 suffer independent rotations due to uncorrelated phase noise at the transmitters.

3. Simulation results

Simulations results were compared to analyze the impact of correlated and uncorrelated phase noise in MIMO-OFDM systems. Phase noise was modelled as a $1/f$ process using the Kasdin model [7] (also used in popular tools such as Simulink/Matlab). For the comparison between correlated and uncorrelated phase noise a 64 subcarrier 2x2 system with an MMSE MIMO decoder was used. 6 pilots were used on each of the transmit data streams for estimating phase noise. Identical phase noise was added at the transmit and receive chains. In one case the phase noise at the transmitter RF chains was uncorrelated and in the other it was correlated. Similarly at the receivers. Phase noise of -90dBc/Hz at 100KHz offset from the carrier and -100dBc/Hz at 100KHz were simulated. Figure 4 plots the cumulative distribution function (CDF) of post-processing SNR for -100dBc/Hz case. The post-processing SNR is defined as the SNR at the output of the MIMO decoder. It can be clearly seen that the case of correlated phase noise shows better performance. The extent of improvement with CPE cancellation is also higher with correlated phase noise. Figure 5 shows the same data for the -90dBc/Hz case. Figure 6 shows the impact of using distributed and dedicated pilots for estimating and correcting uncorrelated phase noise. It can be observed that dedicated pilots perform better when the phase noise is uncorrelated. We also simulate the impact of adding additional antennas in the presence of phase noise. These results are plotted in Figure 7 for correlated phase noise. It can be seen that as the number of antennas increases the amount of CPE decreases and therefore the performance gain with CPE cancellation also decreases. This is because the CPE term is scaled by the transmit data and when the transmit power is distributed (equally) across all the antennas the CPE term also decreases, as antennas are added.

4. Conclusions

In MIMO-OFDM systems the interference due to phase noise can be separated into the common phase error (CPE) and the random or ICI term similar to SISO-OFDM systems. The CPE can be estimated using pilots and corrected. In a power constrained MIMO system the CPE decreases as more antennas are added. An important system level tradeoff is the issue of correlated v/s uncorrelated phase noise. We showed that CPE in the presence of correlated phase noise can be corrected to a larger extent than CPE when the phase noise is uncorrelated at the various transmit/receive RF chains. We also showed that in the case of uncorrelated phase noise estimating phase noise with pilots dedicated to each data stream performs better than a joint estimate across all the data streams.

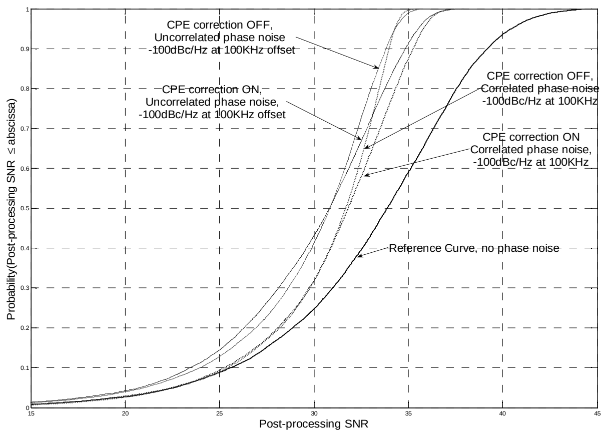


Figure 4 Correlated and uncorrelated phase noise of -100dBc/Hz, with and without CPE correction

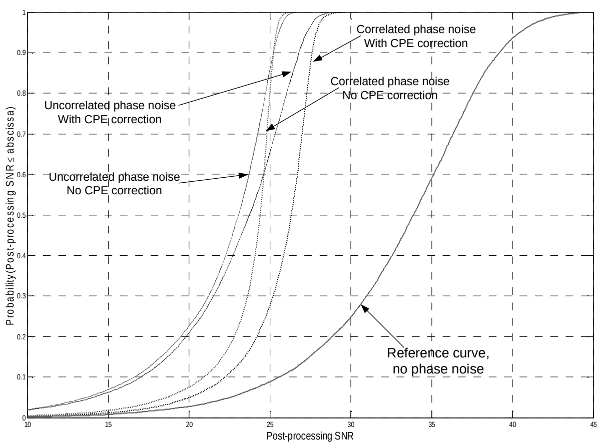


Figure 5 Correlated and uncorrelated phase noise of -90dBc/Hz, with and without CPE correction

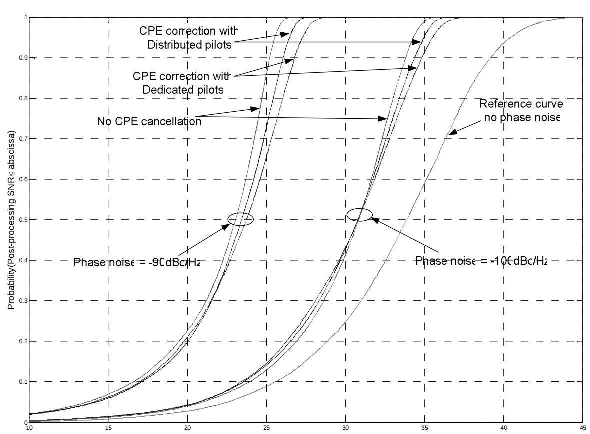


Figure 6 Phase noise cancellation with distributed and dedicated pilots

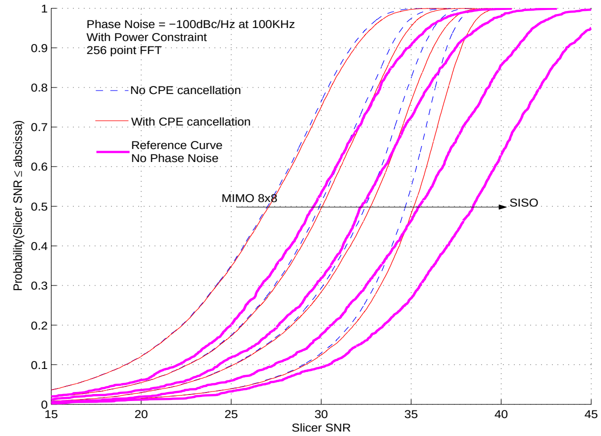


Figure 7 CDF of post-processing (slicer) SNR of a power constrained MIMO-OFDM system.

5. References

[1] T. H. Lee, A. Hajimiri, "Oscillator Phase Noise: A Tutorial", IEEE Journal of Solid-State Circuits, vol. 35, no. 3, march 2000.

[2] S. Lang, R. M. Rao, B. Daneshrad, "Development of a Software Configurable Broadband Wireless Communication Platform", IST Mobile Comm Summit, June 27-30, Lyon, France.

[3] M. S. El-Tanany, Y. Wu, L. Hazy, "Analytical Modeling and Simulation of Phase Noise Interference in OFDM-Based Digital Television Terrestrial Broadcasting Systems", IEEE Trans. on Broadcasting, vol. 47, no. 1, March 2001.

[4] A. G. Armada, "Understanding the Effects of Phase Noise in Orthogonal Frequency Division Multiplexing (OFDM)", IEEE Trans on Broadcasting, June 2001.

[5] P. Robertson, S. Kaiser, "Analysis of the Effects of Phase Noise in Orthogonal Frequency Division Multiplex (OFDM) Systems", ICC 1995, vol 3, pp 1652-1657, Seattle, USA.

[6] R. Rao, 'Performance analysis of MIMO-OFDM Systems", PhD Dissertation, UCLA 2004.

[7] N. J. Kasdin, "Discrete Simulation of Colored Noise and Stochastic Processes and $1/f^{\alpha}$ Power Law Noise Generation", Proc. of the IEEE, vol 83, no 5, pp 802-827, May 1995.

[8] T. C. W. Schenk, P. Mattheijssen, "Analysis of the Influence of Phase Noise in MIMO-OFDM based WLAN Systems", Proc. IEEE Symposium on Comms and Vehicular Tech., Eindhoven 2003.

[9] T. C. W. Schenk, X. –J. Tao, P. F. M. Smulders, E. R. Fledderus, "Influence and Suppression of Phase Noise in Multi-Antenna OFDM", VTC'04 – Fall, Sept 2004.

[10] T. C. W. Schenk, X.-J. Tao, P.F.M. Smulders, E. R. Fledderus, "On the Influence of Phase Noise Induced ICI in MIMO OFDM Systems", IEEE Commun. Letters, Feb. 2005.