A Game Theoretical Approach for Resource Bargaining in Shared WCDMA Networks: Symmetric and Asymmetric Models

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***Abstract*— The high cost associated with the rollout of 3G services encourages operators to share network infrastructure. Network sharing poses a new challenge in devising fair, efficient and Pareto optimal resource allocation strategies to distribute system resources among users of different operators in the network. Cooperative game theory provides a framework for formulating such strategies. In this paper, we propose two models (i.e. symmetric and asymmetric) for cooperative resource bargaining in shared networks based on the concept of preference functions. The symmetric model assumes that all players have equal bargaining powers while in the asymmetric case, players are allowed to submit bids to the network operator to influence the final bargaining outcome. The bargaining solutions proposed vary according to a parameter β that considers the tradeoff between one's gain and the losses of others. The well-known Nash and Raiffa-Kalai-Smorodinsky solutions are special instances of the solutions proposed.**

I. Introduction

Network infrastructure sharing has become a popular strategy among operators in the rollout of 3G services, especially in the wake of substantial investments in licensing and slow 3G user growth. Operators are attracted to share network resources because of the lower capital expenditure (CAPEX) in infrastructure establishment and reduced operation expenditure (OPEX) in the long run. For example, a greenfield operator can save considerable costs by sharing its infrastructure with an incumbent operator. The acceleration of roll-out of 3G services, enabled by substantial cost savings, facilitates an earlier user acceptance of WCDMA and its related services. Besides, operators can increase coverage by sharing or having complementary, geographically separated sites, especially in low-density suburban and rural areas where it is more cost-effective to share.

Referring to Fig. 1, there are several sharing models available [1], [2]: site sharing; radio access network (RAN) sharing; RAN sharing with gateway core; and, complete sharing. Complete sharing can be further categorised into:

- *Mobile Virtual Network Operators (MVNOs)* – Operators share a full-scale 3G infrastructure (RAN, Core network and backbone). This approach is primarily used by MVNOs who do not have a 3G licence and have little infrastructure of its own except for their own home location register (HLR) and billing systems. At least one operator has a 3G licence.
- *Geographical network sharing* – Operators have complementary 3G infrastructure in different areas of a country and share them via national roaming to extend coverage.

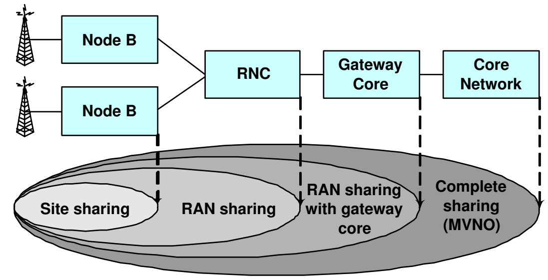

Fig. 1. Models of network sharing.

Network sharing among competing operators opens up a whole new range of research opportunities, especially in devising Radio Resource Management (RRM) strategies in a shared network. The notion of axiomatic bargaining in cooperative game theory provides a good analytical framework to derive a desirable operative point that is fair and Pareto optimal. Pareto optimality is a condition in which it is impossible to make any one party better off without making any other worse off. Cooperative game theory has been applied in a number of resource allocation problems. In [3], a game-theoretic framework based on Nash bargaining solution for bandwidth allocation for elastic traffic in broadband networks is considered. Similarly in [4], the authors demonstrate that a noncooperative game leads to a solution that is not Pareto optimal and in some cases "unfair". In order to achieve an optimal operating point, some arbitration, e.g. by the network operator, is required.

In this paper, we derive a new set of fair, efficient and Pareto optimal bargaining solutions for the resource allocation problem in a shared WCDMA network based on the concept of preference functions developed by Cao for a two-user problem in [5]. Some very well-known bargaining solutions are Nash [6], Raiffa-Kalai-Smorodinsky (Raiffa, hereafter) [7], [8], utilitarian and modified Thomson [5]. Preference functions quantify the tradeoff between one's gain and the losses of others using a weighting factor β and enable us to find a range of solutions on the Pareto optimal boundary with Nash, Raiffa and modified Thomson as special instances.

Our resource bargaining approach is different from conventional allocation schemes such as [9] and [10], which focus on maximising some throughput or social welfare objectives and ignore how much each user gains or loses compared to its requirements. Moreover, the concept of users' utility or preferences used in [10] is abstract and only known in some qualitative sense and such an approach cannot be used to provide concrete numerical answers [3]. Instead, we focus on the notion of fairness and resources are allocated according to the minimum and maximum requirements of the users.

In WCDMA, the resource usage requirements of the users or players can be measured in terms of their uplink and downlink load factors. Based on these requirements, we first derive a class of parameterised resource bargaining solutions that varies according to β. Next, we explore the case where players are allowed to submit bids to the network operator in order to influence the bargaining outcome. This is achieved by relaxing the axiom of symmetry used in [5]. The class of asymmetric bargaining solutions will then vary with the bargaining powers of the players, in addition to their minimum and maximum resource usage requirements. Our focus is on networks that are completely-shared, where the operators and MVNOs share the core network, gateway core, RAN and sites, as in the case of MVNO and geographical network sharing.

There are very few existing published works that explore resource allocation strategies for operators in a shared network environment. A simple admission control strategy with non-preemptive priority queueing has been proposed in [11], which sets the call admission priority of an operator according the ratio of its pre-agreed guaranteed load and its current load. In [12], the authors discuss a framework to manage radio resources using Service Level Agreements (SLA) among a network operator and its MVNOs. The downside of this proposal is that this SLA needs to be repeatedly renegotiated when the users traffic characteristics evolve.

In section II, we present our system model and derive the uplink and downlink load factors. Then, we introduce our model of resource bargaining and derive the solutions in Sections III and IV respectively. Result analysis and conclusion are presented in Sections V and VI.

II. System Model

We consider a shared network with one operator and M MVNOs, denoted by $m \in \{1, \ldots, M\}$. Apart from serving its users, the operator sells unused resources to its MVNOs. These MVNOs do not own any resources and only have the ability to purchase them from the network operator and then resell them to their users. It is reasonable to assume that there is a pre-existing SLA between the operator and each MVNO to guarantee it at least $R_m^{\min}$ units of resources. We denote the number of users associated with the operator or any of the mth MVNO as N_0 and N_m respectively.

Assume that the services provided by the operator are elastic and defined by a range of transmission rates bounded by minimum and maximum $R^{\min}$ and $R^{\max}$. For example, the UMTS Adaptive Multi-Rates (AMR) codec offers transmission rates that vary between 4.75 and 12.2 kbps for conversational voice service [13]. The transmission rate can be dynamically adjusted every 20 ms. We assume that users can select their acceptable QoS level by setting their range of transmission rates. In order to allocate *resources* in a fair, efficient and Pareto optimal way, we first need to derive the meaning of a unit of resource. In a WCDMA network, resources can be expressed in terms of the uplink and downlink load factors of the users. The load factors are commonly used to make a semi-analytical prediction of the capacity of a WCDMA cell, without performing system-level simulations [13].

A. Uplink Load Factor

Consider a single WCDMA cell. In order for a signal to be received, the ratio of its received power to the sum of the background noise and interference must be greater than a given target. The target quality is translated to the following inequality that must be satisfied for each user $i = \{1, \ldots, N\}$ [14], [15]:

$$\frac{W}{\nu_i x_i} \frac{g_i p_i}{\sum_{j \neq i}^{N} g_j p_j + \sigma} \geq \left(\frac{E_b}{N_0}\right)_i, \tag{1}$$

where W is the WCDMA chip rate, ν_i is the activity factor, x_i is the allocated transmission rate, g_i is the path gain between the base station and user i, p_i is transmission power, σ is the background thermal noise, $I_i = \sum_{j \neq i}^{N} g_j p_j$ is interference received by the base station from all the other users within the same cell and $\left(\frac{E_b}{N_0}\right)_i$ is the target bit-energy-to-noise-density required to meet predefined bit error rate (BER). In the case multiple cells, the interference from other cells can be taken into account by using a coefficient f, i.e. $I_i = (1 + f) \sum_{j \neq i}^{N} g_j p_j$. Interference coefficient f typically has values between 0.1 and 0.6 [13].

Assuming perfect power control and solving the set of equations in (1), we obtain the following:

$$g_i p_i = \eta_i^{\mathrm{UL}} \left(\sum_j g_j p_j + \sigma\right) = \frac{\sigma \eta_i^{\mathrm{UL}}}{1 - \sum_{j=1}^{N} \eta_j^{\mathrm{UL}}}, \tag{2}$$

where the load factor η_i^{UL} and total interference I are respectively given by

$$\eta_i^{\mathrm{UL}} = \frac{1}{\frac{W}{\left(\frac{E_b}{N_0}\right)_i \nu_i x_i} + 1} \tag{3}$$

$$I = \sum_j g_j p_j = \frac{\sigma \sum_{j=1}^{N} \eta_j^{\mathrm{UL}}}{1 - \sum_{j=1}^{N} \eta_j^{\mathrm{UL}}}. \tag{4}$$

The number of users that can be supported by the network is limited by maximum uplink system load factor allowed by the network, i.e.

$$\sum_{j=1}^{N} \eta_i^{\mathrm{UL}} \leq \bar{\eta}^{\mathrm{UL}} < 1. \tag{5}$$

When the total load factor approaches unity, the system reaches its pole capacity and the total interference increases to infinity in (4). If this constraint is violated, the target $\left(\frac{E_b}{N_0}\right)_i$ for all users will not be satisfied. We say that the uplink is interference-limited. Users cannot increase their power without bound because of the increased interference they caused to other users. The corresponding transmission rate allocated is

$$x_i = \frac{W}{\left(\frac{E_b}{N_0}\right)_i \nu_i \left(\frac{1}{\eta_i^{\mathrm{UL}}} - 1\right)}, \tag{6}$$

which increases according to the load factor allocated.

B. Downlink Load Factor

In the downlink, the target signal quality of user i is [15]

$$\frac{W}{\nu_i x_i}\frac{g_i p_i}{\theta_i g_i \sum_{j\neq i} p_j + \sigma} \geq \left(\frac{E_b}{N_0}\right)_i, \tag{7}$$

where $I_i = \theta_i g_i \sum_{j\neq i} p_j$ and θ_i is the orthogonality factor of the codes used in the downlink. Although WCDMA employs orthogonal codes, users will receive part of the base station signal due to multipath propagation. In the uplink, transmission is asynchronous and therefore the signals are not orthogonal. Typically, the orthogonality is between 0.4 and 0.9 in multipath channels [13]. The total transmission power in the downlink is limited by the maximum power that the base station can transmit, i.e. $\sum_{j=1}^{N} p_j \leq p^{\max}$. Assuming perfect power control, the transmission power to the ith user is

$$p_i \quad = \quad \frac{\sum_{j=1}^{N} p_j + \frac{\sigma}{g_i\theta_i}}{1 + \frac{W}{\theta_i\left(\frac{E_b}{N_0}\right)_i \nu_i x_i}}. \tag{8}$$

Using (8) and (7), the downlink load factor can be derived as follows

$$\eta_i^{\mathrm{DL}} = \frac{1 + \frac{\sigma}{g_i\theta_i p^{\max}}}{1 + \frac{W}{\theta_i\left(\frac{E_b}{N_0}\right)_i \nu_i x_i}}. \tag{9}$$

Unlike in the uplink, the downlink load factor depends on the orthogonality factor and path gain between the user and the base station. Similar to the uplink, the total downlink load factors must satisfy

$$\sum_{i=1}^{N} \eta_i^{\mathrm{DL}} \quad \leq \quad 1. \tag{10}$$

Using $\hat{\eta}_i^{\mathrm{DL}} = \frac{\eta_i^{\mathrm{DL}}}{1+\frac{\sigma}{g_i\theta_i p^{\max}}}$ to express (8) in terms of the downlink load factor, we have the following:

$$p_i \quad = \quad \hat{\eta}_i^{\mathrm{DL}}\left(\frac{\sum_{j=1}^{N} \hat{\eta}_j^{\mathrm{DL}} \frac{\sigma}{g_j\theta_j}}{1 - \sum_{j=1}^{N} \hat{\eta}_j^{\mathrm{DL}}} + \frac{\sigma}{g_i\theta_i}\right). \tag{11}$$

Given η_i^{DL}, the transmission rate allocated to the ith user is then expressed as

$$x_i = \frac{\frac{W}{\theta_i\left(\frac{E_b}{N_0}\right)_i \nu_i}}{\frac{1}{\eta_i^{\mathrm{DL}}}\left(1 + \frac{\sigma}{g_i\theta_i p^{\max}}\right) - 1}, \tag{12}$$

which increases as the allocated downlink load factor η_i^{DL} and path gain g_i increase.

III. Resource Bargaining Model

In the bargaining framework, the players or bargainers in our problem are the MVNOs and users of the operators. Therefore, there are a total number of $N = N_0 + M$ players in the network. Define $\mathcal{S}$ as the bargaining domain or the feasible set of all possible outcomes and $\mathcal{S}$ is assumed to be convex, closed and bounded sets of $\mathbb{R}^N$. The players compete for the use of resources and each player $i \in \{1, \ldots, N\}$ has a

- *utility function* $u_i = \eta_i$, which is represented by the allocated load factor. Any point $\mathbf{u} \in \mathcal{S}$ represents an outcome or solution of the game.
- *desired initial performance* $u_i^{\min}$, which is the minimal performance required by the user without any cooperation in order to enter the game. It is also known as the *disagreement point* or *threat point*. Players will not enter the game if it is not achievable.

The bargaining problem and outcome can be defined as $(S, \mathbf{u}^{\min})$ and $F(S, \mathbf{u}^{\min}) \in \mathcal{S}$ respectively. Approaches to bargaining fall into two divisions: *strategic* and *axiomatic* bargaining. Strategic bargaining, such as the Rubinstein's model of bargaining [16], assumes that there is a bargaining process where the solution is achieved in a series of offers and counteroffers. The bargaining solution emerges as the equilibrium of a sequential game. The need for a bargaining process among the players is time-consuming and therefore unsuitable for WCDMA network with many users.

Axiomatic bargaining ignores the bargaining process and assumes some desirable properties about the outcome of the bargaining process and then identifies process rules or axioms that guarantee this outcome. The operator serves as the arbitrator in the cooperative resource bargaining game. Nash specifies four axioms, which impose properties that a bargaining solution should satisfy:

A1 *Invariance with respect to affine transformation*: If $\mathbf{u}^*$ is the solution to $(\mathcal{S}, \mathbf{u}^{\min})$ and y is any positive affine transformation, the solution to $(y(\mathcal{S}), y(\mathbf{u}^{\min}))$ is $y(\mathbf{u}^*)$.

A2 *Symmetry*: If the bargaining problem is symmetric, in the sense that (e.g. $N = 2$) $u_1^{\min} = u_2^{\min}$ and $(u_1, u_2) \in \mathcal{S} \Leftrightarrow (u_2, u_1) \in \mathcal{S}$, then $F_1(\mathcal{S}, \mathbf{u}^{\min}) = F_2(\mathcal{S}, \mathbf{u}^{\min})$. This means that two players with symmetric utilities get the same payoff.

A3 *Pareto Optimality*: The bargaining solution will be on the Pareto boundary. If $(\mathcal{S}, \mathbf{u}^{\min})$ is a bargaining problem and $\mathbf{u}, \mathbf{u}' \in \mathcal{S}$ and $u_j' > u_j$, $j = 1, \ldots, N$, then the outcome $F(\mathcal{S}, \mathbf{u}^{\min}) \neq \mathbf{u}$.

A4 *Independence of Irrelevant Alternatives*: If $(\mathcal{S}, \mathbf{u}^{\min})$ and $(\mathcal{S}', \mathbf{u}^{\min})$ are bargaining problems with $\mathcal{S} \subseteq \mathcal{S}'$ and $F(\mathcal{S}', \mathbf{u}^{\min}) \in \mathcal{S}$, then $F(\mathcal{S}, \mathbf{u}^{\min}) = F(\mathcal{S}', \mathbf{u}^{\min})$.

Axiom A4 received a number of criticisms. In particular, [7], [8] argued that one's gain should be proportional to its maximum gain but the Nash solution fails to satisfy this requirement. They retained A1-A3 and proposed a new axiom:

A5 *Monotonicity*: If $\mathcal{S} \subseteq \mathcal{S}'$ $(N = 2)$, $u_1(\mathcal{S}') = u_1(\mathcal{S})$ and $u_2(\mathcal{S}') \geq u_2(\mathcal{S})$, $F_2(\mathcal{S}', \mathbf{u}^{\min}) \geq F_2(\mathcal{S}, \mathbf{u}^{\min})$.

Cao in [5] explained that the Nash and Raiffa solutions represent different solution points on the Pareto boundary. There is no special reasons why they should be chosen and one might to choose another point on the boundary if one dislikes the properties of the Nash and Raiffa solutions. Bargaining solutions can be analysed using players' preference function. In the two-user case, with disagreement points $u_1^{\min} = u_2^{\min} =$

0, the players' preference functions are defined as

$$v_1 = u_1 + \beta(1-u_2) \quad (13)$$
$$v_2 = u_2 + \beta(1-u_1), \quad (14)$$

where $0 \leq u_1, u_2 \leq 1$ and β is a weighting factor that measures the trade-off between one's gain and another's loss. The bargaining outcome, $\mathbf{u}^*$ is the solution to $\mathbf{u}^* = \arg\max_{\mathbf{u}}(v_1 v_2)$. The special cases of $\beta = 0, 1, -1$ correspond to the Nash, Raiffa and modified Thomson solutions. The Nash solution only considers individual's gains and ignores how much other players may gain or lose. On the other hand, the Raiffa solution places the same weight on individual gain and other players' losses. The modified Thomson solution, also known as the relative utilitarian outcome, maximises the sum of all players' normalised utilities.

For the multi-player case, we define the ith player's preference function with $u_i^{\min}$ and maximum utility $u_i^{\max}$ as follows:

$$v_i(\beta) = u_i - u_i^{\min} + \frac{\beta}{N-1}(\sum_{j\neq i} u_j^{\max} - u_j). \quad (15)$$

where $\beta = 0, 1, -(N-1)$ corresponds to the Nash, Raiffa and utilitarian solutions respectively. Our definition does not require u_i to be normalised by its maximum value since $u_i^{\max}$ is included and is general enough to include the special case of normalised utility in [5] and [17]. The bargaining outcome $\mathbf{u}^*(\beta)$ is the solution to

$$\mathbf{u}^*(\beta) = \arg\max \prod_{i=1}^{N} v_i(\beta). \quad (16)$$

The solution depends on β and we call this the symmetric *parameterised* solution of the bargaining problem.

IV. Resource Bargaining in WCDMA

In our WCDMA resource bargaining problem, $\eta_i^{\min}$ and $\eta_i^{\max}$ correspond to the minimum and maximum acceptable load factors based on the player's requirement for the minimum and maximum transmission rates $x_i^{\min}$ and $x_i^{\max}$, as defined in (3) and (9). If the player is a user of the operator, $R^{\min} \leq x_i^{\min}, x_i^{\max} \leq R^{\max}$, $i \in \{1, \ldots, N\}$. For the mth MVNO, its guaranteed minimum resource allocation is $R_m^{\min}$, which can be in terms of the transmission rate or load factor. Its maximum requirement will be in terms of the maximum requirements of all of its N_m users.

A. Symmetric Bargaining

We first derive the symmetric bargaining problem with $u_i = \eta_i$ where all players are assumed to have equal bargaining power. The symmetric resource bargaining problem is defined as (P1):

$$\max_{\boldsymbol{\eta}} \quad \prod_{i=1}^{N}\left(\eta_i - \eta_i^{\min} + \frac{\beta}{N-1}\sum_{j\neq i}(\eta_j^{\max} - \eta_j)\right)$$
$$\text{s.t.} \quad \eta_i \geq \eta_i^{\min}, \eta_i \leq \eta_i^{\max}, \sum_{i=1}^{N}\eta_i \leq T. \quad (17)$$

Referring to (5) and (10), the resource constraint parameter T corresponds to $\bar{\eta}^{\mathrm{UL}}$ and 1 respectively for the uplink and downlink. Note that a similar formulation has been considered in [3] but the authors only focus on the Nash solution, i.e. $\beta = 0$. We are interested in deriving a range of bargaining solutions, parameterised by β, on the Pareto boundary.

Proposition 1: Under the assumption of $\eta_i^{\min} \leq \eta_i \leq \eta_i^{\max}$, $\sum_{i=1}^{N}\eta_i \leq T$ and $\sum_{i=1}^{N}\eta_i^{\min} < T$, the symmetric bargaining solution, parameterised by weighting factor β, $-(N-1) < \beta \leq 1$, of the problem (P1) is given by

$$\eta_i(\beta) = \min\{\tilde{\eta}_i(\beta), \eta_i^{\max}\}, \quad (18)$$

$$\text{where} \quad \tilde{\eta}_i(\beta) = \frac{T}{N} + \frac{(N-1)(N\eta_i^{\min} - \sum_{j=1}^{N}\eta_j^{\min})}{N(N-1+\beta)} + \frac{\beta(N\eta_i^{\max} - \sum_{j=1}^{N}\eta_j^{\max})}{N(N-1+\beta)}. \quad (19)$$

Proof: Let bargaining domain $\mathcal{S}$ be a nonempty, convex and compact set. Taking the logarithm of (P1), the Lagrangian equation of the problem is given as

$$\mathcal{L}(\boldsymbol{\eta}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \gamma) = \sum_{i=1}^{N}\ln\left(\eta_i - \eta_i^{\min} + \frac{\beta}{N-1}\sum_{j\neq i}(\eta_j^{\max} - \eta_j)\right) - \sum_{i=1}^{N}\lambda_i(\eta_i^{\min} - \eta_i) - \sum_{i=1}^{N}\mu_i(\eta_i - \eta_i^{\max}) - \gamma(\sum_{i=1}^{N}\eta_i - T).$$

The necessary and sufficient Karush-Kuhn-Tucker conditions for optimality for $i \in \{1, \ldots, N\}$ are

$$f(\eta_i) - \frac{\beta}{N-1}\sum_{j\neq i}^{N} f(\eta_j) = \lambda_i - \mu_i + \gamma \quad (20)$$
$$\gamma(\sum_{i=1}^{N}\eta_i - T) = 0 \quad (21)$$

$$\text{using} \quad f(\eta_i) = \frac{1}{\eta_i - \eta_i^{\min} + \frac{\beta}{N-1}\sum_{j\neq i}(\eta_j^{\max} - \eta_j)}. \quad (22)$$

When constraints $(\sum_{i=1}^{N}\eta_i - T)$ is inactive and $(\eta_i^{\min} - \eta_i)$ and $(\eta_i - \eta_i^{\max})$ are inactive for all $i \in \{1, \ldots, N\}$, $\lambda_i = \mu_i = 0$ and $\gamma \geq 0$. Solving these equations, we have $f(\eta_i) = f(\eta_j)$ for all $j \neq i$, $i, j \in [1, N]$ or

$$\eta_i = \eta_j + \frac{(N-1)(\eta_i^{\min} - \eta_j^{\min}) + \beta(\eta_i^{\max} - \eta_j^{\max})}{N-1+\beta}. \quad (23)$$

Using (23) and condition $\sum_{j=1}^{N}\eta_j = T$, the solution to the bargaining problem (P1) can be derived accordingly. ■

Proposition 2: When players take into account the utility loss of other players in their preference function by setting weighting factor $0 < \beta \leq 1$, the absolute gap between the new outcome and the Nash solution increases by up to

$$\Delta_i = \frac{(N-1)(\eta_i^{\max} - \eta_i^{\min}) - (\sum_{j\neq i}^{N}\eta_j^{\max} - \eta_j^{\min})}{N^2} \quad (24)$$

when $\beta = 1$. The utility, measured in terms of the load factor, of ith player using the Raiffa solution ($\beta = 1$) is more than the Nash solution if $\eta_i^{\max} - \eta_i^{\min} > \sum_{j\neq i}^{N} \frac{\eta_j^{\max}-\eta_j^{\min}}{N-1}$.

Proof: From Proposition 1, the Nash ($\beta = 0$) and Raiffa ($\beta = 1$) solutions are respectively given by $\eta_i(\beta) = \min\{\tilde{\eta}_i(\beta), \eta_i^{\max}\}$ and

$$\tilde{\eta}_i(0) = \eta_i^{\min} + \frac{T - \sum_{j=1}^{N} \eta_j^{\min}}{N} \quad (25)$$

$$\tilde{\eta}_i(1) = \tilde{\eta}_i(0) + \Delta_i \quad (26)$$

The second part of the proposition is obvious. ■

The Nash solution in (25) is the tangent point of the hyperbola $\prod_{i=1}^{N}(\eta_i - \eta_i^{\min}) =$ constant and only takes into account the individual's gain $\eta_i - \eta_i^{\min}$. This solution is known as the *split-the-difference* rule and coincides with the two-user bargaining outcome derived in [16]. On the other hand, the Raiffa solution in (26) places the same importance on one's gain and the losses of others. On the other hand, as β approaches $-(N-1)$, more weight is placed on other players' gain. When $\beta = -(N-1)$, the problem maximises the sum of utilities of all players. However, there is no trivial solution for this problem as $(N-1+\beta)$ approaches 0 in (1) when β approaches $-(N-1)$. When $-(N-1) < \beta < 0$, the weight on other players' utility is less than 1.

B. Asymmetric Bargaining

Imposing the Axiom of symmetry A2 in (P1) assumes that all players have equal bargaining skills. In practice, the bargaining outcome may be influences by other variables such as the tactics employed by the bargainers, the negotiation procedure and the information structure [16]. In our asymmetric resource bargaining model, we allow the final outcome to be influenced by the price paid by all players. Suppose that each player $i \in \{1, \ldots, N\}$ can submit a bid $\tau_i \in \mathbb{R}$ to the network operator, which is also the arbitrator. We then define the asymmetric resource bargaining problem (P2) as follows:

$$\max_{\boldsymbol{\eta}} \quad \prod_{i=1}^{N}\left(\eta_i - \eta_i^{\min} + \frac{\beta}{N-1}\sum_{j\neq i}^{N}(\eta_j^{\max} - \eta_j)\right)^{\tau_i}$$
$$\text{s.t.} \quad \eta_i \geq \eta_i^{\min}, \eta_i \leq \eta_i^{\max}, \sum_{i=1}^{N}\eta_i \leq T. \quad (27)$$

Proposition 3: Under the assumption of $\eta_i^{\min} \leq \eta_i^{\text{AS}} \leq \eta_i^{\max}$, $\sum_{i=1}^{N}\eta_i \leq T$ and $\sum_{i=1}^{N}\eta_i^{\min} < T$ the asymmetric bargaining solution, parameterised by weighting factor β, $-(N-1) < \beta \leq 1$, of the problem (P2) is given by

$$\eta_i^{\text{AS}}(\beta) = \min\{\tilde{\eta}_i^{\text{AS}}(\beta), \eta_i^{\max}\}, \quad (28)$$

$$\text{where} \quad \tilde{\eta}_i^{\text{AS}}(\beta) = \hat{\tau}_i T + \frac{\beta(1-N\hat{\tau}_i)T}{N-1+\beta} + \frac{(N-1)(\eta_i^{\min} - \hat{\tau}_i\sum_{j=1}^{N}\eta_j^{\min})}{N-1+\beta} + \frac{\beta[\eta_i^{\max} + ((N-1)\hat{\tau}_i - 1)\sum_{j=1}^{N}\eta_j^{\max}]}{N-1+\beta}. \quad (29)$$

$\hat{\tau}_i = \frac{\tau_i}{\sum_{j=1}^{N}\tau_j}$ can be interpreted as the *bargaining power* of the ith player and the sum of all bargaining powers is equal to one. When the bids submitted by all players are the same, the asymmetric solution (28) is the same as the symmetric solution derived in (18).

Proof: The derivation is similar to the one in the previous section and will therefore be omitted. It is easy to see that the symmetric solution (18) is a special instance of the asymmetric solution. The second part of the proof can be obtained using $\hat{\tau}_i = \tau_i = \frac{1}{N}$. ■

Proposition 4: Similar to Proposition 2, when players take into account the utility loss of other players, i.e. $0 < \beta \leq 1$, the absolute gap between the new outcome and the Nash solution increases by up to

$$\Delta_i^{\text{AS}} = \frac{(1-N\hat{\tau}_i)T - \eta_i^{\min} + \hat{\tau}_i\sum_{j=1}^{N}\eta_j^{\min}}{N} + \frac{\hat{\tau}_i(N-1)\eta_i^{\max} + [(N-1)\hat{\tau}_i - 1]\sum_{j\neq 1}^{N}\eta_j^{\max}}{N} \quad (30)$$

when $\beta = 1$. When the Raiffa solution is used, the utility of the ith player will only be greater than the utility derive from the Nash solution when $\Delta_i^{\text{AS}} > 0$.

Proof: Using Proposition 3, the Nash and Raiffa solutions are respectively given by

$$\tilde{\eta}_i^{\text{AS}}(0) = \eta_i^{\min} + \hat{\tau}_i(T - \sum_{j=1}^{N}\eta_j^{\min}) \quad (31)$$

$$\tilde{\eta}_i^{\text{AS}}(1) = \tilde{\eta}_i^{\text{AS}}(0) + \Delta_i^{\text{AS}}. ■ \quad (32)$$

The asymmetric Nash and Raiffa solutions derived in (31) and (32) exhibit the same properties as the symmetric solutions in the previous section. The Nash solution only varies according to the minimum load factor requirements and bargaining powers of all players. The asymmetric Raiffa solution takes into account the maximum load factor requirements as well.

Proposition 5: When the bargaining power of the ith player varies by δ_i, the new load factor $\tilde{\eta}_i^{\text{AS}}$ can be written in terms of the previous load factor $\tilde{\eta}_i^{\text{AS,old}}$ as

$$\tilde{\eta}_i^{\text{AS}}(\beta) = (1 + K_i(\beta))\tilde{\eta}_i^{\text{AS,old}}(\beta) \quad (33)$$

where

$$K_i(\beta) = N\delta_i[1 - \frac{(N-1)x_i^{\min} + \beta(T - \sum_{j\neq i}^{N}x_j^{\max})}{\eta_i^{\text{AS,old}}(N-1+\beta)}].$$

Note that $\delta_i = \hat{\tau}_i - \hat{\tau}_i^{\text{old}}(\beta)$. When the bargaining power increases, i.e $\delta_i > 0$ and $K_i(\beta) > 0$, $\eta_i^{\text{AS}}(\beta) > \eta_i^{\text{AS,old}}(\beta)$. Similarly, using (18), $\eta_i^{\text{AS}}(\beta) > \eta_i(\beta)$ when $\hat{\tau}_i > \frac{1}{N}$. ■

Proof: Omitted.

V. Numerical Analysis and Discussions

In order to achieve the resource allocation outcome in Proposition 1, the arbitrator, i.e. the operator requires the knowledge of each player's $x_i^{\min}$ and $x_i^{\max}$. This is not difficult to achieve in real implementation because users can select their acceptable transmission rate range at the beginning of a call or even change it during. Moreover, for each service, the operator can set up several classes with different varying guaranteed quality for its users to select from. Using (3) and (9), the operator can calculate the minimum and maximum load factor requirements of its users for both uplink and downlink respectively. For the MVNOs, $\eta_i^{\min}$ is specified in their SLA with the operator and $\eta_i^{\max}$ is a function of the total maximum load factor requirements of the users supported by them. The MVNOs can in turn redistribute the resources allocated in a similar manner using (18).

The Nash and Raiffa solutions derived in Propositions 1 and 2 satisfy different axioms and are both on the Pareto optimal boundary. The Nash solution maximises the Nash product, i.e. the product of the gain of all players. The Raiffa solution also considers the size of the bargaining domain of each player, i.e. how much other players give up in addition to one's gain. To illustrate this, we consider the following simple game with $N = 2$. Player 1 is a user and Player 2 is an MVNO, which has two users with the same maximum load factor 0.6. Suppose that $T = 1$ and the minimum and maximum requirements of the players are $\eta^{\min} = (0.1, 0.2)$, $\eta^{\max} = (0.5, 1.2)$.

The bargaining solutions, parameterised by $\beta \in [0, 1]$, are depicted in Fig. 2 in solid lines. The Nash and Raiffa solutions are given by $\boldsymbol{\eta}(0) = (0.45, 0.55)$ and $\boldsymbol{\eta}(1) = (0.3, 0.7)$. As β increases from 0 to 1, the load factor allocated to Player 2, which has a higher maximum rate requirement, increases. Both solutions are Pareto optimal. However, by axiom A5, the Raiffa solution is at the point where each player's gain is proportional to its maximum gain and therefore "fairer" to player 2. Also, Player 1 is able to increase her load factor by increasing her bid, τ_1, to the operator. In that case, Player 2's bargaining power $\hat{\tau}_2$ decreases. The asymmetric solutions for the case of $\hat{\boldsymbol{\tau}} = (0.6, 0.4)$ are also depicted in Fig. 2 in dashed lines. Note that $\hat{\boldsymbol{\tau}} = (0.5, 0.5)$ in the symmetric model. By Proposition 5, the new Nash and Raiffa solutions are given by $\boldsymbol{\eta}^{\mathrm{AS}}(0) = (0.50, 0.48)$ and $\boldsymbol{\eta}^{\mathrm{AS}}(1) = (0.37, 0.63)$. Given the $\boldsymbol{\eta}$, the uplink or downlink transmission rate of the players can respectively be determined using (6) and (12).

VI. Conclusion

We have derived the symmetric and asymmetric resource bargaining solutions for a WCDMA network. Although our model is of a shared network, the results can be applied to other networks with similar resource allocation problem. Unlike conventional schemes that only aim to maximise some system objectives, the allocation approaches that we derived are Pareto optimal and fair according to the minimum and maximum requirements of each player. In the asymmetric model, the players, i.e. the users of the network operator and the MVNOs, can affect the bargaining outcome by submitting a bid to the network arbitrator. The solutions derived are parameterised by β, which quantifies the preference for one's gain and the losses of others. As β approaches 1, more weight is placed on how much other players give up and the bargaining solution favours the player with higher maximum requirement. When the solutions are all Pareto optimal, the selection of β is arbitrary.

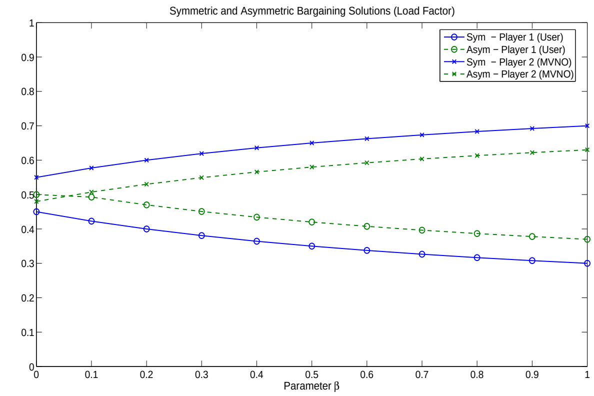

Fig. 2. Symmetric and asymmetric bargaining solutions with $\hat{\boldsymbol{\tau}} = (0.5, 0.5)$ and $\hat{\boldsymbol{\tau}} = (0.6, 0.4)$ respectively.

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