STUDY OF SPREAD CODES WITH BLOCK SPREAD OFDM

Ibrahim S. Raad, Xiaojing Huang, Darryn Lowe

School of Electrical, Computer and Telecommunications Engineering
University of Wollongong,
Wollongong, N.S.W Australia
ibrahim@uow.edu.au

ABSTRACT

This paper presents the study undertaken with block spread OFDM and compares three spreading matrices. The matrices include the Hadamard, Rotated Hadamard and Mutually Orthogonal Complementary Sets of Sequences (MOCSS). The study is carried out for block lengths of $M = 2$, $M = 4$ and $M = 8$ and it shows that all the spreading matrices show improvement and a better performance over the conventional OFDM over frequency selective channel as expected. As the size of M increased the spreading matrices which have better orthogonal qualities show greater improvement. [1]

Key Words-*OFDM, Spreading Matrices, Block Spread-OFDM, Golay-Paired Hadamard Matrix.*

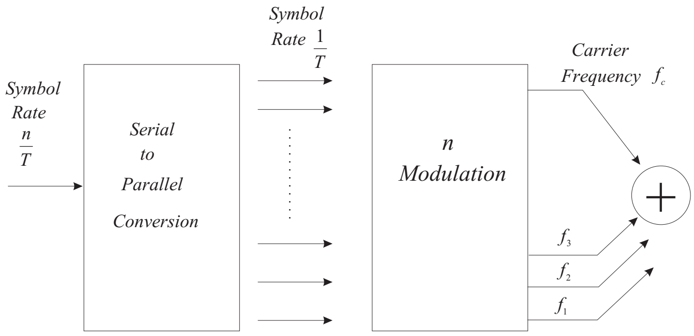

Fig. 1. Transmitter for multi-carrier modulation [1].

1. INTRODUCTION

A method used to implement mutually orthogonal signals is called Orthogonal Frequency Division Multiplexing (OFDM) and this is done by setting up multiple carriers at a suitable frequency separation and modulating each symbol stream separately [1]. By increasing the number of carriers the data rate per carrier can be reduced for a given transmission.

The symbol streams do not interfere with each other because of the carriers being mutually orthogonal. It is possible to mitigate fading through suitable interleaving and coding. One method of ensuring the signals are independent of each other is to select the frequency separation between each signal in a manner which will achieve orthogonality over a symbol interval. This can be seen in Figure 1. OFDM's operation can also be seen in Figure 2.

While OFDM will combat the effect of multipath transmission, other methods need to be used to mitigate the effect of fading and two are mentioned above. Another way of achieving this is called Diversity Transmission. Diversity transmission can be used to reduce or remove the effect of fading by the transmitted signal power being "split between two or more subchannels that fade independently of each other, then the degradation will most likely not be severe in all subchannels for a given binary digit" [2]. Then when all the outputs of these subchannels are recombined in the proper way the performance achieved will be better than the single transmission. There are a number of ways to achieve this diversity and the main methods include "transmission over spatially different times (space diversity), at different paths (time diversity) or with different carrier frequencies (frequency diversity)" [2].

Block Spread OFDM (BSOFDM) has been used to achieve frequency diversity and in frequency selective channels has shown significant improvement over conventional OFDM. This is done by dividing the N subcarriers into M sized blocks and spreading them by multiplying these blocks by spreading codes such as the Hadamard matrix.

This paper studies Block Spread OFDM (BSOFDM) with three different types of spreading codes in frequency selective environment. They include the following,

1. Hadamard matrix
2. Rotated Hadamard and
3. Mutually Orthogonal Complementary Sets of Sequences(MOCSS).

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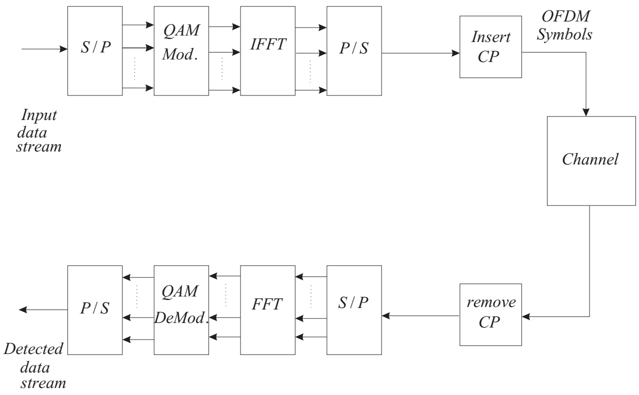

Fig. 2. A simplified block diagram of an OFDM system [3].

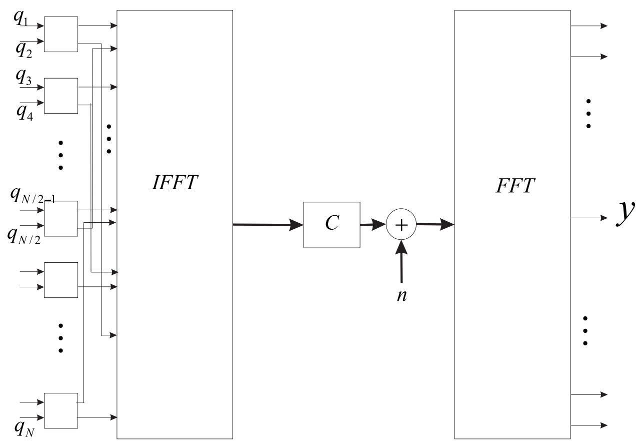

Fig. 3. Block diagram representation of the BOFDM channel for a block length of two [4].

The remainder of the paper is setup in the following format. Section Two discusses BSOFDM briefly. Section three presents the three types of spreading codes that will be studied in this paper. This is followed by a description of the system used in this study in Section Four. Section Five presents the results and finally Section Six gives the conclusion.

2. BLOCK SPREAD OFDM

In [4] a study into an optimal block spreading code for OFDM is presented, where the main idea of BSOFDM is to "split the full set of subcarriers into smaller blocks and spread the data symbols across these blocks via unitary spreading matrices in order to gain multipath diversity across each block at the receiver". They found that the spreading code presented was optimal for the quadrature amplitude modulation (QAM), binary phase shift keying (BPSK) and quadrature-phase shift keying (QPSK) modulation.

The output of the receiver's FFT processor is

$$y \quad = \quad Cq + n \tag{1}$$

where y is the FFT output, $q \in A^N$ is the vector of transmitted symbols, each drawn from an alphabet A, C is a diagonal matrix of complex normal fading coefficients, and n is a zero mean complex normal random vector. Equalization of the received data is done through multiplication by C^{-1} and then "quantized independently on each subcarrier to form the soft or hard decision $\hat{q}$ which may be further processed if the data bits are coded" [4]. There is no loss in performance when the detection is performed independently on each carrier due to the noise been independent and identically distributed with fading been diagonal [4].

The block spreading matrices are used to introduce dependence among the subcarriers. N subcarriers are split into $\frac{N}{2}$ for blocks of size 2. Then each of the blocks are multiplied by a 2×2 unitary matrix U_2. "The resulting length 2 output vectors are then interleaved to separate the entries in each block as far as possible across the frequency band so that they will encounter independent fading channels" [4]. The transmitter's IFFT has the interleaved data passed through it and this data is sent across the frequency selective channel. The data is passed through an FFT processor at the receiver and deinterleaved before using block by block processing. The BOFDM channel model is shown in Figure 3.

In this study three more spreading matrices are studied and compared in frequency selective environment using block sizes of $M = 2$, $M = 4$ and $M = 8$. These spreading matrices are used with BPSK modulation.

3. BLOCK SPREADING MATRICES

3.1. Hadamard Matrix

A square matrix of $N \times N$ consisting of two elements $+1$ and -1 is known as the Hadamard matrix and meets the following conditions,

$$H_N H_N^T \quad = \quad H_N^T H_N \tag{2}$$

$$\quad = \quad N I_N \tag{3}$$

where H_N^T is the transpose of H_N and I_N is the identity matrix of order N [7]. Equation 2 means that if any row or column taken they are orthogonal. Another example of Hadamard matrix is the Sylvester Hadamard matrix and can be generated using the following method,

$$H_{2N} = \begin{bmatrix} H_N & \tilde{H_N} \\ H_N & -H_N \end{bmatrix}$$

The following equation is used for the setup of the rotated Hadamard matrix.

$$U = \frac{1}{\sqrt{2}} \times H_{m\times m} \times diag(exp^{(\frac{j\times\pi\times m)}{rotatedcount}}) \quad (4)$$

where the $H_{m\times m}$ is the Hadamard matrix. The $rotated-count$ is the value of the constellation which rotates back onto it self. $m = 0 < m < M - 1$ where M is the block size.

3.2. Mutually Orthogonal Complementary Sets of Sequences

In 1961 Marcel J. E. Golay introduced the concept of Complementary Series in [5]. It was argued that a "set of complementary series is defined as a pair of equally long, finite sequences of two kinds of elements which have the property that the number of pairs of like elements with anyone given separation in one series is equal to the number of pairs unlike elements with the same given separation in the other series" [5].

This work was continued in 1972 by C. C. Tseng and C. L. Liu. They argued that for a set of equally long finite sequences to be complementary set of sequences if the sum of autocorrelation functions of sequences in that set is zero except for a shift term [6]. Also if the sum of the cross correlation functions of the corresponding sequences in these two sets is zero, it is said that these two sequences are complementary.

"Complementary sets of sequences are said to be mutually orthogonal complementary sets if any two of them are mates to each other" [6]. Finally, in 2002 X. Huang and Y. Li in [7] presented a family of complete complementary sets of sequences with closed form expression. An orthogonal Golay paired matrix called Golay-paired Hadamard matrix is derived and the general procedure for construction of these spreading matrix is given and their scalability is proven. The following will explain this procedure and these matrices will be used in this study and comparison.

The Golay paired Hadamard matrix of order $N = 2^n$ and its equivalent form have simple closed-form expressions for any element h_N(i,j) and $\tilde{h}_N$(i,j) at the i^{th} row and the j^{th} column of H_N. If one lets i_r and j_r, where $r = 0, 1, \cdots, n-1$, denote the r^{th} digit $(0 or 1)$ in the radix-2 representations of the integer i and j, that is,

$$i \equiv (i_{n-1}, i_{n-2}, \cdots, i_0)_2 = \sum_{r=0}^{n-1} 2^r i_r \quad (5)$$

$$j \equiv (j_{n-1}, j_{n-2}, \cdots, j_0)_2 = \sum_{r=0}^{n-1} 2^r j_r \quad (6)$$

Then,

$$h_N(i,j) \equiv (-1)^{\sum_{r=0}^{n-2}(j_{r+1}\oplus i_r)j_r + i_{n-1}j_{n-1}} \quad (7)$$

$$\tilde{h}_N(i,j) \equiv (-1)^{\sum_{r=0}^{n-2}(j_{r+1}\oplus i_r)j_r + \tilde{i}_{n-1}j_{n-1}} \quad (8)$$

where the $\oplus$ denotes the modulo-2 sum and $\tilde{i}_{n-1}$ denotes the binary inverse of i_{n-1}.

These complementary sequences can be used to construct orthogonal sequences sets which provide additional advantages over conventional orthogonal sequences such as the Walsh-Hadamard and offshoots.

4. DESCRIPTION OF SYSTEM

The data is transmitted and modulated using the Binary Phase Shift Keying modulation. After this the spread matrix is set using the parameters of the transmitted data. The first of these spreading matrices was the rotated Hadamard matrix. The second spreading method used was a normal Hadamard matrix.

The following matrix is the Hadamard 8×8 used for $M = 8$. The $+$ indicates a 1 and the $-$ indicates a -1.

$$Hadamard = \begin{bmatrix} + & + & + & + & + & + & + & + \\ + & - & + & - & + & - & + & - \\ + & + & - & - & + & + & - & - \\ + & - & - & + & + & - & - & + \\ + & + & + & + & - & - & - & - \\ + & - & + & - & - & + & - & + \\ + & + & - & - & - & - & + & + \\ + & - & - & + & - & + & + & - \end{bmatrix}$$

The third and final spreading matrix used is the Mutually Orthogonal Complementary Sets of Sequences (MOCSS), this set up is the same as that of the Hadamard. The following is an 8×8 Mutually Orthogonal Complementary Sets of Sequences matrix (MOCSS).

$$MOCSS = \begin{bmatrix} + & + & + & - & + & + & - & + \\ + & - & + & + & + & - & - & - \\ + & + & - & + & + & + & + & - \\ + & - & - & - & + & - & + & + \\ + & + & + & - & - & - & + & - \\ + & - & + & + & - & + & + & + \\ + & + & - & + & - & - & - & + \\ + & - & - & - & - & + & - & - \end{bmatrix}$$

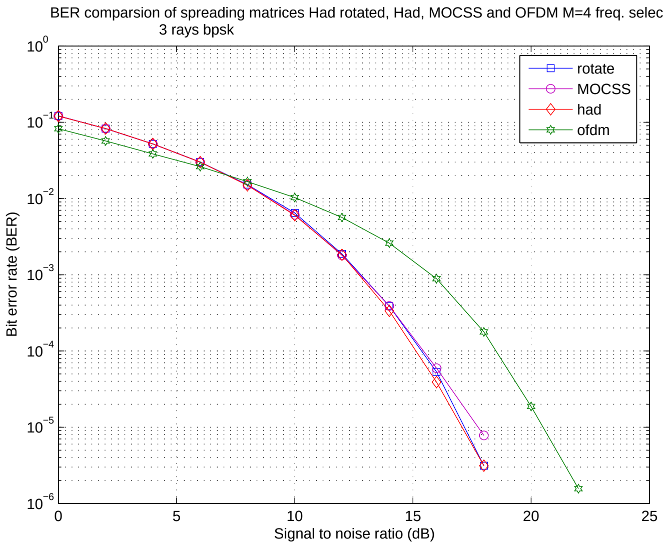

Fig. 4. Bit Error Rate versus SNR comparing the three orthogonal spreading matrices using BPSK with M=4.

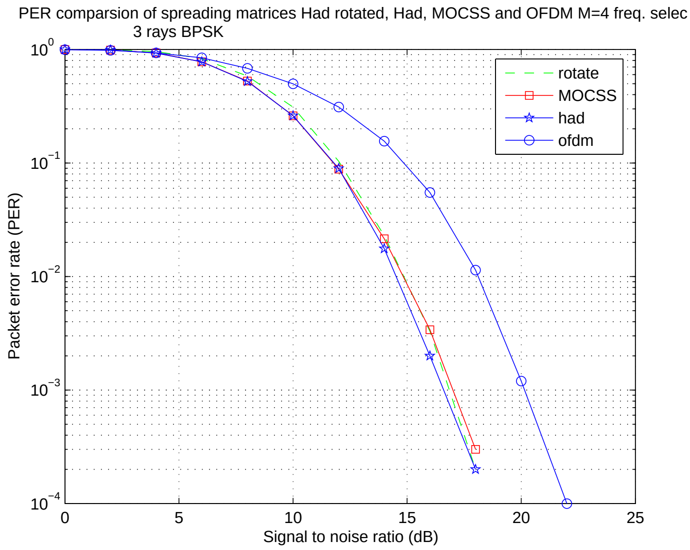

Fig. 5. Packet Error Rate versus SNR comparing the three orthogonal spreading matrices using BPSK with M=4.

Then each of the three spreading matrices are multiplied by the M-sized blocks. These spread blocks are interleaved using a random interleaver and passed through the IFFT block. Cyclic Prefix is added and then these are corrupted using frequency selective channel and then adding white gaussian noise.

At the receiver end the CP is removed and the data is sent through the FFT block. The symbols are deinterleaved and de-spread by using the inverse of the spreading matrices and passed through and are demodulated.

5. RESULTS

The results shown depict the simulations comparing the three spreading codes studied in this paper. The block sizes used with BSOFDM include $M = 2$, $M = 4$ and $M = 8$. The modulation used with these simulations was BPSK and using 3-path Frequency selective channel. The number of subcarriers used is $N = 64$ and the number of packets sent is 10000. As can be seen comparing the Figures below, all the spreading matrices improve on the conventional OFDM as expected. Comparing the three spreading matrices, it is observed that with smaller or shorter block sizes such as $M = 2$ and $M = 4$, depicted in Figures 4 and 5, the Hadamard and the Rotated Hadamard showed better performance than the MOCSS. As the size of the block was increased to $M = 8$, depicted in Figures 6 and 7, the MOCSS showed better performance due to its superior orthogonal properties.

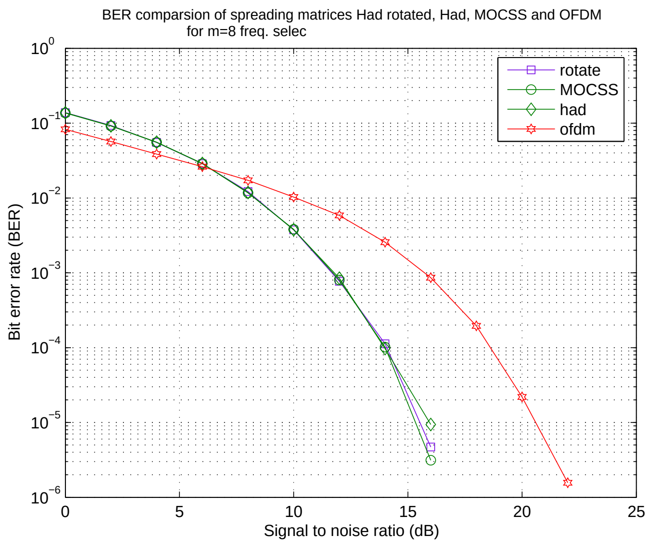

Fig. 6. Bit Error Rate versus SNR comparing the three orthogonal spreading matrices using QPSK modulation with M=8.

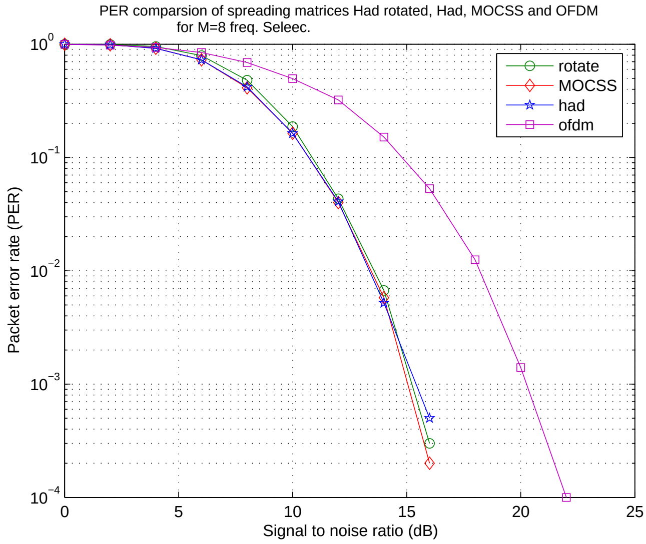

Fig. 7. Packet Error Rate versus SNR comparing the three orthogonal spreading matrices using BPSK with M=8.

6. CONCLUSION

In conclusion to this study it can be said that block spreading of conventional OFDM will in no doubt improve on the system performance. This study considered three types of spreading matrices which include Hadamard, rotated Hadamard and MOCSS. While all improved significantly on conventional OFDM as expected, the Hadamard and the Rotated Hadamard showed better performance than the MOCSS with the shorter block sizes. As the block size used became larger the MOCSS showed better performance than the two mentioned.

7. REFERENCES

[1] T. Öberg, "Modulation, Detection and Coding", 2001, John Wiley and Sons Ltd, West Sussex England.

[2] Rodger Ziemer and William Tranter, "Principles of Communications - Systems Modulation and Noise", 2002, fifth edition, John Wiley and Sons Ltd, NJ.

[3] G. Parsaee and A. Yarali, "OFDMA for the 4th generation cellular networks", Canadian Conference on Electrical and Computer Engineering, volume 4, 2004, 2325 - 2330, May.

[4] Michael L. McCloud, "Optimal Binary Spreading for Block OFDM on Multipath Fading Channels", WCNC / IEEE Communications Society, volume 2, 2004, 965-970, March.

[5] Marcel J. E. Golay, "Complementary Series", Information Theory, IEEE Transactions on Volume 7, Issue 2, April 1961 Page(s)82 - 87.

[6] Chin-Chong Tseng; C. Liu, "Complementary sets of sequences", Information Theory, IEEE Transactions on Volume 18, Issue 5, September 1972 Page(s)644 - 652.

[7] X. Huang and Y. Li, "Scalable complete complementary sets of sequences", Global Telecommunications Conference, 2002. GLOBECOM '02. IEEE, vol 2, 2002, 1056 - 1060, November.