SEQUENCE SYNCHRONIZATION IN A WIDEBAND CDMA SYSTEM

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ABSTRACT
A mathematical model of a synchronization block of a DS-CDMA system is developed and then investigated using a simulation. For this synchronization a pilot sequence has been used that can be a pseudorandom sequence or a chaotic sequence of a limited length. The theoretical model is tested by simulation. It was shown that the system can be synchronized using a pilot sequence represented by a periodically repeated chaotic sequence.

KEY WORDS
Chaos communication, CDMA system, synchronization, pilot synchronization

1. Introduction

In ordinary DS-CDMA communication systems all the users transmit their information using their own basis signals, which are orthogonal to each other. These basis signals can be pseudo random sequences with good orthogonal characteristics, Walsh functions [1], wavelets [2] or chaotic sequences [3]. The sequences allow spread spectrum characteristics of the transmitted signals and enhance the security of the communication system as well as jamming protection of the system.

In this paper, a DS-CDMA system, which uses chaotic sequences as the carriers of users' messages, is analyzed. The system is supposed to achieve the best possible masking of the transmitted information contents and the best possible protections of the information contents, which depends mostly on the characteristics of the chaotic signal generated for each user. The system is analyzed in the presence of white Gaussian noise for the case when the transmitter generates a multiuser signal that is composed of N single-user chaotic signals. The mathematical model of the system is presented in [3].

The basic problem in such systems is how to synchronize the chaotic sequence transmitted and its replica generated inside the receiver. Due to the lack of sufficiently robust synchronization schemes [4, 5, 6, 7], the topic of synchronization within chaos based DS-CDMA systems is still an active area of research. In this paper we have made a theoretical analysis of a synchronization system and conducted simulation to confirm that the synchronization can be achieved.

2. CDMA System Description

Due to its wideband nature, a signal mapped by chaotic basis functions is more resistant to multipath propagation. A system to be analyzed, shown in Fig. 1) is composed of 4 basic parts. First part is a transmitter, which includes spreaders, a low-pass FIR filter and a modulator that up-converts the composite CDMA signal of the carrier frequency ω_c. Second part is the receiver, which demodulates the incoming signal, filters high frequency component, dispreads the CDMA signal and makes decision related to the message bit transmitted. The third part is the communication channel, represented by additive white Gaussian noise. These three parts, excluding the carrier modulation, are theoretically analyzed and simulated in [3]. The fourth part, to be analyzed in this paper, is the synchronization block.

2.1 Theoretical Model of a Chaotic PSK system

The chaotic sequences are denoted by $\{x_t\}$, the encoded message by $\{\gamma_t\}$ and γ_i is the i^{th} transmitted bit. Thus, the transmitter signal can be expressed as

$$s_t(t)=\sum_{g=1}^{N}s_t^g(t)=\sum_{g=1}^{N}\gamma_i^g(t)x_t^g(t)=\sum_{g=1}^{N}\gamma_i^g x_t^g \text{ ,} \tag{2.1}$$

where the g-th user sequence is expressed as

$$\begin{aligned} s_t^g(t) &= \gamma_i^g(t)x_t^g(t) \\ &= \begin{cases} -1\cdot x_t^g(t) & \gamma_i^g(t)=-1,\ for\, t\in[(i-1)T_c, iT_c \\ +1\cdot x_t^g(t) & \gamma_i^g(t)=+1,\ for\, t\in[(i-1)T_c, iT_c \end{cases} \end{aligned} \tag{2.2}$$

and the pilot chaotic sequence is modulated by all ones sequence that gives the pilot sequence expressed as

$$s_t^0(t)=\gamma_i^0(t)x_t^0(t)=+1\cdot x_t^0(t),\ for\, t\in[(i-1)T_c,iT_c. \tag{2.3}$$

Unlike the users' chaotic sequences, which are theoretically non-periodic sequences to preserve the security in message transmission using chaotic phase shift keying as explained in [3], the pilot chaotic sequence has a finite duration and is periodically repeated. The communication system is designed in such a way that the spreading and transmission of message bits for each user starts at the beginning of this pilot sequence. Thus, if the received pilot sequence at the receiver is successfully synchronized with its replica locally generated at the receiver side, then the receiver will "know" where to expect the beginning of the user chaotic sequence in order to take out the user message bits.

Let us generally explain how this received pilot is synchronized with a locally generated pilot at the receiver for the case when the *g*-th user is supposed to receive the message. The communication system, including the synchronization circuits, is shown in Fig. 1. Suppose the receiver pilot is $x_t^0(t-\tau)$, which is a time shifted replica of the transmitted pilot $x_t^0(t)$. The pilot multiplies the received signal as shown in Fig. 1. The calculated control variable *Z* is used to reduce the time delay of the receiver pilot using a feed-back loop as shown in Fig. 1. When this delay is reduced to zero, the pilot synchronization is achieved and a control signal $C_{initial}$ is generated at the output of the Pilot Synchronization block. This signal sets the *g*-th user Chaotic Sequence generator at the time instant where the transmitted sequence of the *g*-th user is supposed to start. Thus, when the transmitter starts to generate message bits for the *g*-th user, the *g*-th user chaotic sequence generator at the receiver side will be synchronized to the incoming sequence.

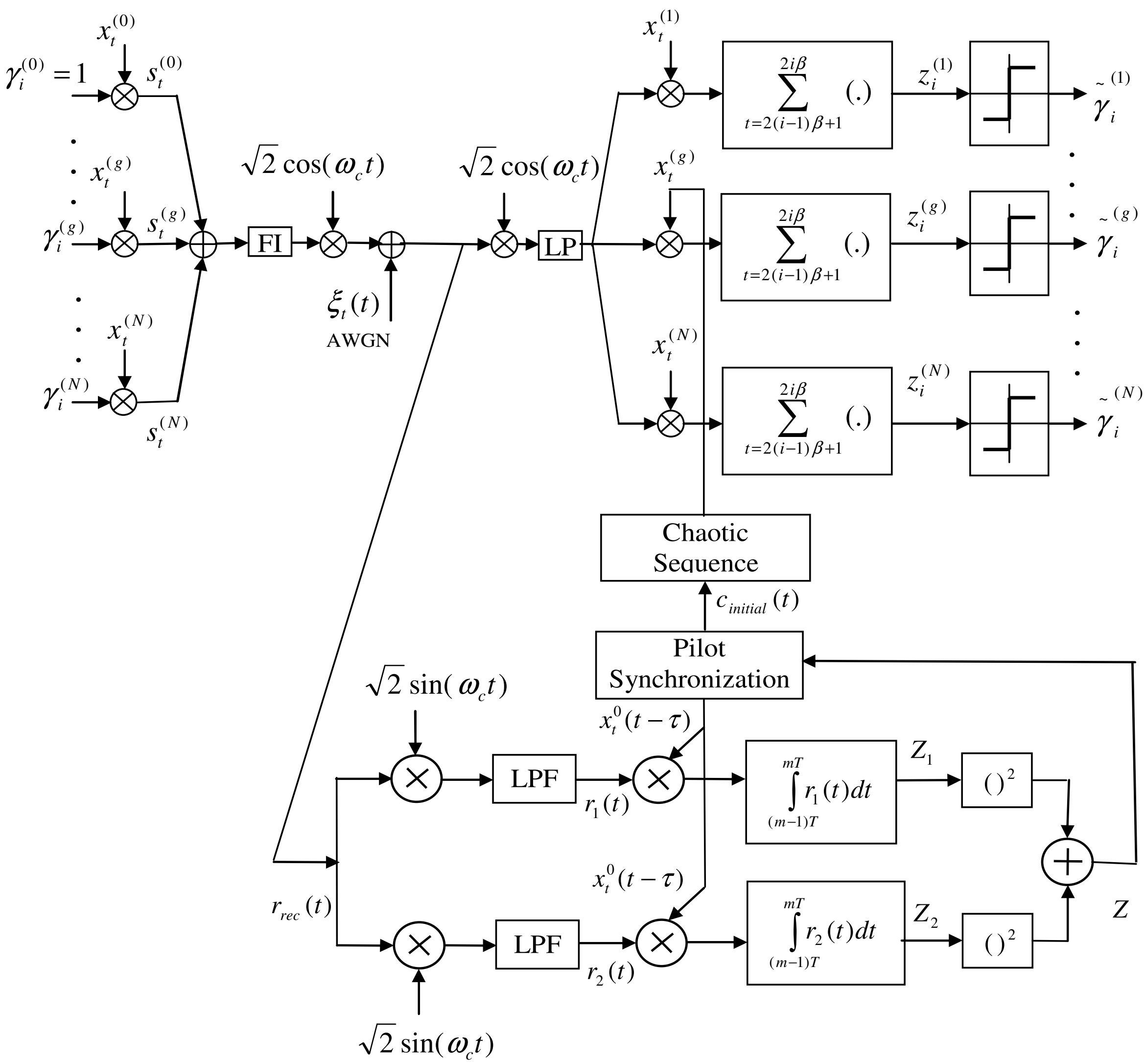

Figure 1 DS-CDMA CPSK system with a synchronization block

2.2 Theoretical Model of a Synchronization Block

Let us present a theoretical model of sequence synchronization based on Fig. 1. For the sake of the generality of explanation we assume that there is a phase difference φ between the carrier at the transmitter and the receiver side. For that reason the synchronization circuits are presented with two branches in Fig. 1. The received signal, assuming that a phase shift is φ, may be expressed as

$$
\begin{aligned}
r_{rec}(t) &= \left(\sum_{g=0}^{N} \gamma_i^g x_t^g\right)\sqrt{2}A\cos(\omega_c t+\varphi)+\xi_t(t) \\
&= \left(\sum_{g=0}^{N} \gamma_i^g x_t^g\right)\sqrt{2}A[\cos(\omega_c t)\cos(\varphi)+\sin(\omega_c t)\sin(\varphi)] \\
&\quad +\sqrt{2}\xi_t^I(t)\cos\omega_c t-\sqrt{2}\xi_t^Q(t)\sin\omega_c t
\end{aligned}
\tag{2.4}
$$

The first output is

$$
\begin{aligned}
r_1(t) &= r_{rec}(t)\sqrt{2}\cos\omega_c t \\
&= \left\{\begin{matrix}\left(\sum_{g=0}^{N} \gamma_i^g x_t^g\right)\sqrt{2}A[\cos(\omega_c t)\cos(\varphi)+\sin(\omega_c t)\sin(\varphi)] \\ +\sqrt{2}\xi_t^I(t)\cos\omega_c t-\sqrt{2}\xi_t^Q(t)\sin\omega_c t\end{matrix}\right\}\sqrt{2}\cos\omega_c t \\
&= \left(\sum_{g=0}^{N} \gamma_i^g x_t^g\right)A[1+\cos(2\omega_c t)]\cos(\varphi)+\left(\sum_{g=0}^{N} \gamma_i^g x_t^g\right)A\cos 2\omega_c t\sin(\varphi) \\
&\quad +\xi_t^I(t)[1+\cos(2\omega_c t)]-\xi_t^Q(t)\cos 2\omega_c t
\end{aligned}
$$

which is passed through the LP filter and gives

$$
\begin{aligned}
r_1(t) &= \left(\sum_{g=0}^{N} \gamma_i^g x_t^g\right)A\cos(\varphi)+\xi_t^I(t) \\
&= \gamma_t^0 x_t^0 A\cos(\varphi)+\left(\sum_{g=1}^{N} \gamma_i^g x_t^g\right)A\cos(\varphi)+\xi_t^I(t)
\end{aligned}
$$

Because the pilot is always one, i.e., $\gamma_i^0(t)=+1$, and having in mind notation in (1), we may have

$$
r_1(t)=x_t^0(t)A\cos(\varphi)+\xi_1(t), \tag{2.5}
$$

where the noise term includes the AWGN and the inter-user interference. In analog way we can find the second output

$$
r_2(t)=x_t^0(t)A\sin(\varphi)+\xi_2(t) \tag{2.6}
$$

These outputs are multiplied by an m-bit reference sequence containing M chips, which is a time shifted version of the received pilot sequence, and then integrated which results in

$$
\begin{aligned}
Z_1 &= \int_{t=(m-1)T_c}^{t=mT=mMT_c} r_1(t)x_t^0(t-\tau)dt=\int_{t=(m-1)T}^{t=mT}\left[Ax_t^0(t)\cos(\varphi)+\xi_1(t)\right]x_t^0(t-\tau)dt \\
&= A\cos(\varphi)\int_{t=(m-1)T}^{mT} x_t^0(t)x_t^0(t-\tau)dt+\int_{t=(m-1)T}^{mT}\xi_1(t)x_t^0(t-\tau)dt \\
&= A\cos(\varphi)TR(\tau)+N_1
\end{aligned}
$$

The correlation procedure takes place at the receiver as shown in Fig. 1. The first M chips of the pilot sequence are aligned with the reference sequence, multiplied and then summed to obtain the first correlation value. If this value is less than the threshold value this procedure is repeated for the next chip intervals until the correlation value calculated is greater than the threshold value, when we assume that the synchronization is acquired. Because the amplitude of the pilot is $A=\sqrt{2E_c/T_c}$ and the number of chips in the interval T is $M = T/T_c$, we may have

$$
Z_1=T\sqrt{\frac{2E_c}{T_c}}\cos(\varphi)R(\tau)+N_1 \tag{2.7}
$$

And, derived in the same manner,

$$
Z_2=T\sqrt{\frac{2E_c}{T_c}}\sin(\varphi)R(\tau)+N_2 \tag{2.8}
$$

where N_1 and N_2 are zero-mean Gaussian random with the variance $N_0^{'}T/2$, expressed in these forms

$$
N_1 \Rightarrow G\left(0,N_0^{'}T/2\right) \quad \text{and} \quad N_2 \Rightarrow G\left(0,N_0^{'}T/2\right). \tag{2.9}
$$

Therefore, the decision variables Z_1 and Z_2 are also Gaussian random variables expressed as

$$
\begin{aligned}
Z_1 &\Rightarrow G\left(T\sqrt{\frac{2E_c}{T_c}}\cos(\varphi)R(\tau),N_0^{'}T/2\right) \\
&= \sqrt{N_0^{'}T/2}\cdot G\left(2\sqrt{\frac{TE_c}{T_c N_0^{'}}}\cos(\varphi)R(\tau),1\right) \\
&= \sqrt{N_0^{'}T/2}\cdot G\left(2\sqrt{\frac{ME_c}{N_0^{'}}}\cos(\varphi)R(\tau),1\right)
\end{aligned}
\tag{2.10}
$$

And

$$
Z_2 \Rightarrow \sqrt{N_0^{'}T/2}\cdot G\left(2\sqrt{\frac{ME_c}{N_0^{'}}}\sin(\varphi)R(\tau),1\right), \tag{2.11}
$$

because $M = T/T_c$. The decision variable, which controls the correlation procedure, and consequently the synchronization procedure, is $Z=Z_1^2+Z_2^2$. The new variable Z is $\sigma^2 = N_0^{'}T/2$ times a noncentral chi-squared random variable with two degrees of freedom. The noncentrality parameter is

$$\lambda = \left[2\sqrt{\frac{ME_c}{N_0^{'}}}\sin(\varphi)R(\tau)\right]^2 + \left[2\sqrt{\frac{ME_c}{N_0^{'}}}\cos(\varphi)R(\tau)\right]^2 \\ = 4\frac{ME_c}{N_0^{'}}R^2(\tau) = 4MR^2(\tau)\cdot\frac{E_c}{N_0^{'}} \quad (2.12)$$

Thus, the probability density function of Z is

$$p_Z(z) = \begin{cases} \frac{1}{2\sigma^2}e^{-\frac{1}{2}(\lambda+z/\sigma^2)}I_0\left(\sqrt{\frac{\lambda z}{\sigma^2}}\right) & z>0 \\ 0 & otherwise \end{cases}. \quad (2.13)$$

2.3 Decision Procedure

Suppose the decision is made on the basis of two hypothesis tests: H_1 is the hypothesis that the received and locally generated sequences are aligned within one chip, and H_0 is the hypothesis the received sequence is not aligned with the local one, which can be expressed formally as

$$H_0 : |\tau| > T_c \Rightarrow R(\tau) \cong 0, N_0^{'} > N_0 \text{ , and}$$
$$H_1 : |\tau| \leq T_c \Rightarrow R(\tau) > 0, N_0^{'} \cong N_0. \quad (2.14)$$

The probability density functions, conditioned on these hypotheses, are

$$p_Z(z|H_0) = \frac{1}{2\sigma^2}e^{-\frac{1}{2}(z/\sigma^2)} \quad (2.15)$$

And

$$p_Z(z|H_1) = \frac{1}{2\sigma^2}e^{-\frac{1}{2}(\lambda+z/\sigma^2)}I_0\left(\sqrt{\frac{\lambda z}{\sigma^2}}\right). \quad (2.16)$$

because the noncentrality parameter is zero in the first case and different from zero in the last case. Thus, the single attempt (m = 1) false alarm probability is

$$p_F(m=1) = p(Z > z_T | H_0) = \int_{z_T}^{\infty}\frac{1}{2\sigma^2}e^{-\frac{1}{2}(z/\sigma^2)}dz = e^{-\frac{1}{2}(z/\sigma^2)} \quad (2.17)$$

and the threshold value is

$$z_T = -2\sigma^2\ln p_F(m=1) = -N_0^{'}T\ln p_F(m=1) \quad (2.18)$$

For this threshold value, the single-run acquisition detection probability is

$$p_D(m=1) = p(Z > z_T | H_1) = \int_{z_T}^{\infty}\frac{1}{2\sigma^2}e^{-\frac{1}{2}(\lambda+z/\sigma^2)}I_0\left(\sqrt{\frac{\lambda z}{\sigma^2}}\right)dz = e^{-\frac{1}{2}(z/\sigma^2)}$$

Transforming z to be $z = x^2\sigma^2$ we may have

$$p_D(m=1) = p(Z > z_T | H_1) = \int_{z_T}^{\infty}\frac{1}{2\sigma^2}e^{-\frac{1}{2}(\lambda+z/\sigma^2)}I_0\left(\sqrt{\frac{\lambda z}{\sigma^2}}\right)dz \\ = \int_{\sqrt{z_T/\sigma^2}}^{\infty}x\cdot e^{-\frac{1}{2}(\lambda+x^2)}I_0\left(\sqrt{\lambda}x\right)dx.$$

This integral is the Marcum's Q-function, therefore we may have

$$p_D(m=1) = Q_M\left(\sqrt{\lambda}, \sqrt{z_T/\sigma^2}\right) \quad (2.19)$$

For the hypothesis H_1 we have that $R(\tau) > 0,\ N_0^{'} \cong N_0$ and $\sigma^2 = N_0T/2$, which gives the expression for λ in this form

$$\lambda = 4MR^2(\tau)\cdot\frac{E_c}{N_0} \quad (2.20)$$

Thus, inserting this expression and also the expression for the threshold value (2.18) into (2.19), we may have

$$p_D(m=1) = Q_M\left(2R(\tau)\sqrt{M\cdot\frac{E_c}{N_0}}, \sqrt{-2\frac{N_0^{'}}{N_0}\ln p_F(m=1)}\right) \quad (2.21)$$

The upper bound on the acquisition detection probability is obtained in the case when system is in synchronization, i.e., when this condition is fulfilled $R(\tau) \cong R(0) = 1,\ N_0^{'} \cong N_0$, according to the expression

$$p_D(m=1) \leq Q_M\left(2\sqrt{M\cdot\frac{E_c}{N_0}}, \sqrt{-2\ln p_F(m=1)}\right) \\ \approx Q\left(\sqrt{-2\ln p_F(m=1)} - 2\sqrt{M\cdot\frac{E_c}{N_0}}\right) \quad (2.22)$$

Where Q is a Gaussian Q-function that accurately approximates the Marcum's Q-function Q_M. For a particular value of the signal-to-noise ratio we may plot the graphs of Q_M function for M as a parameter. Thus, for a worst expected signal-to-noise ratio value we can determine the value of the number of chips in the reference sequence M at the receiver side that allows us to synchronize the receiver with the desired value of both the acquisition detection probability and the false alarm probability.

The pilot sequence can also be a pseudo random sequence in the case when it is orthogonal to the all chaotic sequences used for spreading users' information bits. In that case the synchronization procedure will be practically similar to the procedure in classical CDMA systems like systems that use Walsh functions as orthogonal sequences [1].

3. Simulations and Analysis

A way of obtaining the empirical expressions for the probability of false alarm and the probability of detection is now presented. Assume that, at certain noise power level in the system, the output of the acquisition circuit is as given in Fig. 2.

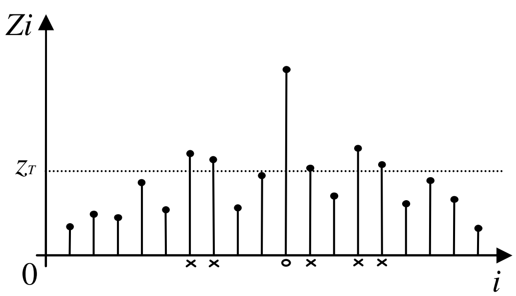

Figure 2 Discrete values of the decision variable

Then, for this noise power level the decision variable Zi will exceed the threshold value z_T six times. However in only one of those six times the incoming signal $r(t)$ and the basis function, $x_p(t-\tau)$, at the receiver will be in synchronism. Let the circle in Fig. 2 correspond to the case when the two are indeed in synchronism, and the crosses represent cases when they are not in synchronism but the threshold is exceeded.

Let us apply Baye's rule, expressed as

$$P_r(A \mid B) = \frac{P_r(A \cap B)}{P_r(B)} \tag{3.1}$$

Which states that the probability of event A given that event B has occurred is equal to the probability of both A and B occurring divided by the a priori probability of event B. The equation (2.17), which represents the probability of false alarm, can therefore also be written in the following form

$$\begin{aligned} p_F(m=1) &= P_r(Z_m > z_T \mid H_0) \\ &= \frac{P_r(Z_m > z_T \cap H_0)}{P_r(H_0)} \end{aligned} \tag{3.2}$$

The expression for the numerator term $P_r(Z_m > z_T \cap H_0)$ of equation (3.2) may be expressed in this form

$$P_r(Z_m > \beta_T \cap H_0) = \lim_{S\to\infty}\left(\frac{k}{S}\right), \tag{3.3}$$

In equation 3.3 k represents the number of crosses in Figure 2, that is, the number of times that the decision variables Zm exceeds the threshold value z_T, leading to the wrong decision. S represents the total number of discrete values of the decision variables Zm.

The expression for the denominator term $P_r(H_0)$ of equation 3.2 is given by equation 3.4.

$$P_r(H_0) = \lim_{S\to\infty}\left(\frac{S-1}{S}\right) \tag{3.4}$$

Substituting (3.3) and (3.4) into (3.2) equation 3.5 is obtained.

$$p_F(m=1) = \lim_{S\to\infty}\left(\frac{k}{S-1}\right) \tag{3.5}$$

Saying that S is unlimited implies that the number of synchronization bits is unlimited. In order to obtain an accurate result, when S is limited, the experiment must be run a number of times, that is, a large number of synchronization bits (periods) must be processed. Processing a large number, i, of bits, while keeping S limited, permit an accurate estimation of the probabilities, for a limited size of the synchronization bit. Running the experiment i number of times, that is, processing i number of bits, leads to the expression for the probability of false alarm expressed as

$$p_F(m=1) = \lim_{i\to\infty}\sum_{n=1}^{i}\left(\frac{k_n}{i(S_n-1)}\right) \tag{3.6}$$

Also equation 3.7 which represents the probability of detection can be written as equation 3.7.

$$p_D(m=1) = P_r(Z_m > z_T \mid H_1) = \frac{P_r(Z_m > z_T \cap H_1)}{P_r(H_1)} \tag{3.7}$$

The expression for the numerator term $P_r(Z_m > z_T \cap H_1)$ of equation 3.7 has only two outcomes, depending on whether the threshold z_T has been exceeded or not. These two outcomes are given by

$$P_r(Z_m > z_T \cap H_1) = \begin{cases} \lim\limits_{S\to\infty}\left(\frac{1}{S}\right) = \frac{1}{S} & Z_m > z_T \\ \lim\limits_{S\to\infty}\left(\frac{0}{S}\right) = 0 & otherwise \end{cases} \tag{3.8}$$

The expression for the denominator term $P_r(H_1)$ of (3.7) is given by

$$P_r(H_1) = \lim_{S\to\infty}\left(\frac{1}{S}\right) \tag{3.9}$$

Substituting (3.8) and (3.9) into (3.7) we may have.

$$p_D(m=1) = \frac{P_r(Z_m > z_T \cap H_1)}{P_r(H_1)} = \begin{cases} \dfrac{\lim\limits_{S\to\infty}(1/S)}{\lim\limits_{S\to\infty}(1/S)} = 1 & Z_m > z_T \\ \dfrac{0}{\lim\limits_{S\to\infty}(1/S)} = 0 & otherwise \end{cases} \tag{3.10}$$

From (3.10) it is clear that to obtain the expression for the probability of detection one must run the experiment over more than a single synchronization bit, regardless of the length of the synchronization bit, that is, the synchronization bit period. Running the experiment i number of times leads to the expression for the probability of detection given by

$$p_D(m=1) = \lim_{i\to\infty}\sum_{n=1}^{i}\left(\frac{p_D(m=1)_n}{i}\right) \tag{3.11}$$

where $p_D(m=1)_n \in \{0,1\}$.

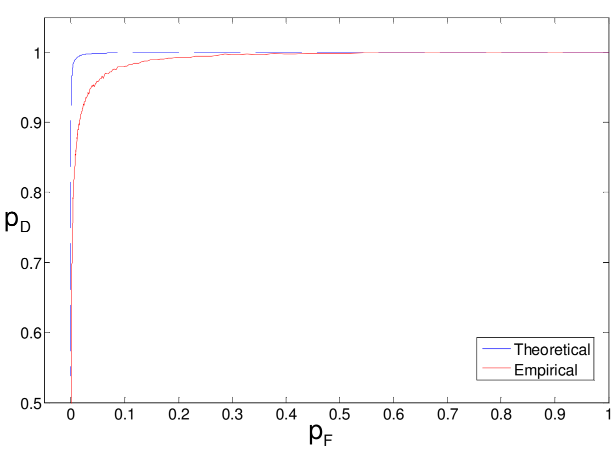

Figure 3 Theoretical and empirical curves at *Ec/No* = -15dB

Equations (3.6) and (3.11) have been evaluated for large i and M = 224. The results of simulation are shown in Fig. 3 where the dependence of p_D versus p_F is plotted. The solid curve is obtained by simulation and the dashed curve is obtained from the theory using equation (2.22). The theoretical result is somewhat better than the empirical one, as demonstrated by Fig. 3.

4. Conclusions

It was shown, by a theoretical analysis and simulation that a CDMA system, which is based on application of the chaotic sequences as carriers of the user information, can be synchronized using a pilot sequence. This pilot can be a repeated version of a chaotic sequence or a pseudo random sequence that is orthogonal to the chaotic sequences used in the system.

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