Joint Power Control and Spreading Gain Optimisation for Minimising the Energy Consumption in CDMA based Ad hoc Networks

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Abstract

This paper addresses the impact of the spreading gain on the efficiency of mobile Ad Hoc networks using Code Division Multiple Access (CDMA) as a mode of transmission. In particular, we concentrate on the trade off between the increase in transmission data rate, hence decrease of transmission time, and the necessary power needed to reach acceptable levels of interference in the system. We formulate this trade off as an optimisation problem with associated constraints, where the cost function is the overall energy consumed in the network and the constraints are the acceptable signal to interference ratio for each node. The solution to this optimisation problem provide optimal values for both the transmission spreading gains and powers that need to be allocated to each node in order to minimise the total energy consumption of the network.

1. Introduction

CDMA is based on spread spectrum (SS) techniques, in which all users share the entire bandwidth and transmit at the same time. CDMA has been commonly used in second and third generation mobile communications and has recently been proposed in ad hoc networks. When perfect synchronisation is possible (as in the forward link of cellular networks) spreading sequences can be chosen to be perfectly orthogonal. In contrast, when synchronisation between all user transmissions in the network is not feasible, the baseband signals are spread using pseudo-random noise (PN) codes. This has been used in the reverse link of cellular systems and also proposed for ad hoc networks where synchronisation between all nodes is not feasible.

A key feature of CDMA systems which has been adopted for third generation systems is that by changing the spreading gain, one can also vary the rate of the data to be transmitted over the channel. More precisely, a decrease in the Spreading Gain allows for the initial stream of data to be transmitted at higher rates. While this is a desirable feature, especially for transmitting real time applications which requires high data rates and/or small latency, the drawback of reducing the spreading gain is that it requires increased transmission power, raising the level of interference created in the network.

In this paper, we explore the use of CDMA technology for ad hoc networks and propose a method for evaluating the optimum spreading gain to minimise the overall energy consumed in the network. The motivation behind choosing the energy consumption as criterion to be minimised is that it is one of the crucial issue of ad hoc and sensor networks due to limited battery life of the nodes. Note that the energy spent by the network depends on the transmission power used by every node as well as the time required to transmit the data. As mentioned previously, this transmission time depends on the data rate and therefore the CDMA spreading gain. We therefore claim that minimising the battery life can be achieved by appropriately choosing not only the transmitting power of every node but also the time it takes to transmit the data, hence the spreading gain.

Several CDMA based MAC protocols for MANETs have been proposed in the literature [3], [4], [5], [6]. None of these studies have discussed the choice of the spreading gain. Usually the previous studies have assigned an arbitrary value for the spreading gain in simulations and analytical studies. The only issue of concern is usually the distribution of the codes of a particular length. In [3], authors propose an algorithm for assigning codes in a dynamic, multihop wireless radio network. It is a distributed and asynchronous protocol. In [4] authors propose a code assignment algorithm based on the code reuse property. In [5] a slotted system is considered where a packet occupies a number of slots and presents two protocols which involves changing the spreading code of a transmission after an initial header is transmitted. However, the spreading gain value is not discussed specifically.

In [6] authors propose multiple access schemes, Distributed Reservation CDMA with Priority (DR_CDMA_P)
2. Problem statement

Power control [9],[10], [11], [12], [13] is an important issue to be solved in CDMA based ad hoc networks. Indeed, one can see that Multiple Access Interference (MAI) in ad hoc networks may be significant, since it is not feasible to use perfectly orthogonal codes in ad hoc networks due to difficulties in achieving perfect synchronisation among nodes. As a consequence, it is important to guarantee that spreading gain and transmitter powers for individual nodes are in an acceptable range to have the network functioning properly. Another important constraint in ad hoc networks is the limited energy availability in mobile nodes. As the data transmission is responsible for a significant portion of the total energy dissipation, it can be concluded that by minimising the total energy dissipation due to all packet transmissions we can extend the life-time of the network. In making this statement we have assumed that all nodes on average transmit similar number of packets and on average dissipate similar transmit energy. In this work we show that there exist a certain set of transmitted powers and a spreading gain that guarantees proper reception of all packets while minimising the total energy consumed by the network.

Let us define a neighborhood of $2M$ nodes, where, $\mathcal{M}_T \in \{1,2,\ldots,M\}$ are the transmitters and $\mathcal{M}_R \in \{1,2,\ldots,M\}$ are the receivers in the network. Furthermore, each transmitter $i \in \mathcal{M}_T$ want to simultaneously send packets to all the receivers $(1,\ldots,M) \in \mathcal{M}_R$. From the locations of the nodes and the propagation characteristics of the environment we can compute the channel gain matrix, $G$:

$$G = \begin{bmatrix}
G_{11} & G_{12} & G_{13} & \cdots & G_{1M} \\
G_{21} & G_{22} & G_{23} & \cdots & G_{2M} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
G_{M1} & G_{M2} & G_{M3} & \cdots & G_{MM}
\end{bmatrix}$$

where, $G_{ij} \geq 0$ is the instantaneous gain between the transmitter $i$ and the receiver $j$. Similarly we can also define $P$, the transmitter powers used by the $M$ transmitters, to transmit packets to all the $M$ receivers.

Let us consider an area of $250m \times 250m$, where all nodes are randomly distributed. At this stage, we also assume that all transmitters can directly communicate with any other receiver within this area. i.e., no multihop transmission is required in the network. Later we will explain how the solution to the optimisation problem can be extended to a multihop environment. The objective of this paper is to minimise the total energy consumed by the network by finding an optimum set of $P_{ij}$ and fixed spreading gain $N$, such that all packets are received properly.

Recalling that the total energy consumed by the network during a transmission is the product of the transmitter power and the length of the transmission. We can derive an expression for the total energy consumption as follows:

In spread spectrum systems, the spreading gain $N$, user data rate $R_{ij}$ and spread signal bandwidth $W$ can be expressed as:

$$N = \frac{W}{R_{ij}}$$

Our objective is to vary the user data rate of the users by varying the spreading gain of the system while keeping the spreaded signal bandwidth constant. Hence the time required to transmit a fixed amount of data, $T$ is proportional to the spreading gain $N$ of the system.

$$T \propto N$$

Therefore the total energy consumption of all the nodes in the network, $E$, is proportional to $N \sum_{i=1,j=1}^{M} P_{ij}$. Hence, the objective of the paper is to minimise $N \sum_{i=1,j=1}^{M} P_{ij}$ subject to the constraint that all packets are received properly.
3. Solution to the optimisation problem

To receive data properly, the Signal to Interference Ratio (SIR) at each receiver must be greater than a target $\beta$. The SIR at the receiver node $j$ when receiving data transmitted by transmitter node $i$ is given by:

$$\frac{P_{ij}G_{ij}}{\frac{1}{N} \left( \sum_{k=1, k \neq j}^M P_{ik}G_{ij} + \sum_{l=1, l \neq i}^M \sum_{k=1}^M P_{lk}G_{lj} \right) + \sigma^2} \geq \beta \quad (1)$$

Where $\sigma^2$ is the thermal noise power at the receiver. To reduce the dimensionality of the problem, without losing generality, we can assume that at any given time transmitter $i$ only intends to transmit to receiver $i$. In other words, all $P_{ij} = 0 \forall i \neq j$. Also substituting $P_i = P_{ii}$, we can simplify equation (1) to:

$$\frac{P_{ii}G_{ii}}{\frac{1}{N} \sum_{k=1, k \neq i}^M P_{ki}G_{ki} + \sigma^2} \geq \beta \quad (2)$$

By rearranging:

$$\phi \beta \sum_{k=1, k \neq i}^M P_{ki}G_{ki} - P_{ii}G_{ii} + \sigma^2 \beta \leq 0$$

$$g_i(P, \phi) \leq 0 \quad (3)$$

Where,

$$g_i(P, \phi) = \phi \beta \sum_{k=1, k \neq i}^M P_{ki}G_{ki} - P_{ii}G_{ii} + \sigma^2 \beta$$

and

$$\phi = 1/N$$

Furthermore, all the transmitted powers and the spreading gain of the system must be strictly positive. Hence,

$$P_i > 0 \forall i \in \mathcal{M}_T$$

$$N > 0$$

$$-\frac{1}{\phi} < 0$$

Now we can define the optimisation problem:

minimise $F(P, \phi) = \frac{1}{\phi} \sum_{i=1}^M P_i$ \quad (4)

subject to

$$g_i(P, \phi) \leq 0 \forall i \in \mathcal{M}_T \quad (5)$$

$$-P_i < 0 \forall i \in \mathcal{M}_T \quad (6)$$

$$-\frac{1}{\phi} < 0$$ \quad (7)

This is an optimisation problem with inequality constraints. Therefore, we form the Lagrangian of the system by adding non-negative multipliers as follows:

$$L(P, \phi, \Lambda, \Theta, \gamma) = F(P, \phi) + \sum_{i=1}^M \lambda_i g_i(P, \phi)$$

$$+ \sum_{i=1}^M \theta_i (-P_i) + \gamma \left( -\frac{1}{\phi} \right) \quad (8)$$

Where $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_M\}$, $\Theta = \{\theta_1, \theta_2, \ldots, \theta_M\}$ and $\gamma$ are all non-negative values.

As $\theta_i \to \infty$, the term $\theta_i (-P_i)$ becomes an arbitrary large negative value, making $L(P, \phi, \Lambda, \Theta, \gamma)$ arbitrary small. Hence to minimise $L$, all $\theta_i$ must be zero [14]. Similarly, from inequality (7), we can deduce $\gamma = 0$. Now equation (8) can be rewritten as:

$$L(P, \phi, \Lambda) = F(P, \phi) + \sum_{i=1}^M \lambda_i g_i(P, \phi) \quad (9)$$

Hence, The necessary conditions for optimality are:

$$\frac{dL}{d\phi} = 0$$ \quad (10)

$$\frac{dL}{dP_i} = 0 \forall i \in \mathcal{M}_T$$ \quad (11)

$$\lambda_i g_i(P, \phi) = 0 \forall i \in \mathcal{M}_T$$ \quad (12)

$$\lambda_i \geq 0 \forall i \in \mathcal{M}_T$$ \quad (13)

From equation (10):

$$\frac{dL}{d\phi} = -\frac{1}{\phi^2} \sum_{m=1}^M P_m + \beta \sum_{m=1}^M \lambda_m \sum_{k=1, k \neq m}^M P_k G_{km} = 0$$
From equation (11):

\[
\frac{dL}{dP_i} = \frac{1}{\phi} + \phi \beta \sum_{k=1, k \neq i}^{M} \lambda_k G_{ki} - \lambda_i G_{ii} = 0
\]  

(14)

By rearranging terms:

\[
\lambda_i = \frac{\frac{1}{\phi} + \beta \phi \sum_{k=1, k \neq i}^{M} \lambda_k G_{ki}}{G_{ii}}
\]  

(15)

As \( \phi > 0 \), \( \beta > 0 \), \( G_{ki} > 0 \ \forall \ (k, i) \in (\mathcal{M}_T, \mathcal{M}_R) \) and \( \lambda_k \geq 0 \ \forall \ k \in \mathcal{M}_T \), \( \lambda_i \ \forall \ i \in \mathcal{M}_T \) must be strictly positive \( [14] \). Hence at optimality:

\[
g_i(P, \phi) = 0
\]  

(16)

This implies that at optimality, the constraints in (5) should be satisfied as equalities. Hence, the optimisation problem can be re-written as:

\[
\text{minimise } F(P, \phi) = \frac{1}{\phi} \sum_{i=1}^{M} P_i
\]

subject to

\[
-P_i < 0 \ \forall \ i \in \mathcal{M}_T
\]

\[
g_i(P, \phi) = 0 \ \forall \ i \in \mathcal{M}_T
\]

\[
-\frac{1}{\phi} < 0
\]

The Lagrangian of the system:

\[
L(P, \phi, \Lambda) = F(P, \phi) + \sum_{i=1}^{M} \lambda_i g_i(P, \phi)
\]

Using equations (10), (11) and (16) we get,

\[
-\frac{1}{\phi^2} \sum_{m=1}^{M} P_m + \beta \sum_{m=1}^{M} \sum_{k=1, k \neq m}^{M} P_k G_{km} = 0
\]

\[
\frac{1}{\phi} + \phi \beta \sum_{k=1, k \neq i}^{M} \lambda_k G_{ki} - \lambda_i G_{ii} = 0 \ \forall \ i \in \mathcal{M}_T
\]

\[
\phi \beta \sum_{k=1, k \neq i}^{M} P_k G_{ki} - P_i G_{ii} + \sigma^2 \beta = 0 \ \forall \ i \in \mathcal{M}_T
\]

This results in \( 2M + 1 \) equations with \( 2M + 1 \) unknowns, which we can solve for \( P \) and \( N \). The solution provides the optimum spreading gain and the transmitter powers for all transmitters.

4. Simulation setup

We considered a neighborhood of \( 2M \) nodes, with \( M \) transmitters and \( M \) receivers placed across a square area of length 250 meters. The location of the mobile nodes are selected randomly within this area, where each node can directly communicate with any other nodes in the neighborhood. The simulation parameters and their values are summarised in table 1.

![Table I: Common parameters and their values](image)

The free space propagation model \( [15] \) is used to estimate the received signal strength assuming the transmitter and receiver have a clear, unobstruct line-of-sight path between them. The channel gain between transmitter \( i \) to receiver \( j \) can be calculated as,

\[
G_{ij} = \frac{K}{d^2}
\]

![Fig. 1. Simulation Scenarios for 3, 5, 10 and 25 Nodes](image)
where, $K$ is a constant based on free space propagation environment and $d$ is the distance between transmitter and receiver.

The network scenarios used in the simulations are shown in Figure 1. We started with a network of 3 communicating pairs and without changing the position of these nodes, repeatedly added more communicating pairs to increase the network size. In Figure 1, ◊ represents position of the transmitters and receivers associated with 3 nodes scenario, ◊, □, □ together represents 5 nodes scenario, ◊, □, ◊ together represents 10 nodes scenario and ◊, □, ◊, □ together represents 25 nodes scenario respectively. Arrowheads point to the receiver for a transmitterreceiver pair.

In the simulations, The transmitting powers of all $M$ nodes were estimated for a given spreading gain $N$ using the set of $M$ equations based on the receiver SIR as follows:

$$\frac{1}{N} \sum_{k=1,k\neq i}^{M} P_k G_{kj} + \sigma^2 = \beta \forall i \in \mathcal{M}_T$$

5. Simulation results

The simulation results presented in this section illustrate the potentials of our proposed method and also allow us to understand several consequences of varying the CDMA spreading gain in ad hoc networks.

The performance of the proposed method is demonstrated in Figure 2. It shows that for a network with 3, 5, 10, and 25 nodes, we can find a spreading gain and set of transmitter powers, which minimises the total energy consumed by the whole network during a particular transmission. This spreading gain is of course the result of the optimisation problem developed in this paper and the results suggest that it can show great variations, depending on the number of nodes within neighborhood as well as the position of the transmitters and receivers.

According to the results shown in Figure 2, minimum spreading gain required for 3 and 5 nodes are very low, which is a positive outcome since it means that data can be transmitted at very high data rate without interfering with each other. On the other hand, as the network size increases to 10, spreading gain that leads to minimum energy also increases. The fact that the value of the optimum spreading gain is increasing with the number of nodes can be explained as follows:

As seen in Figure 1 for 3 and 5 nodes scenario, all receivers are relatively far away from the transmitters. as a result, it is possible to transmit at high data rate without creating too much interference on the other nodes in the network. As the network size increases to 10, receiver $R_3$ becomes very close to transmitter $T_1$ and transmitter $T_2$, increasing the interference generated at receiver $R_3$. In fact, an increase in the number of nodes leads to increases interference at the receiver as the distance between the receiver and any transmitters decreases. This requires spreading gain to be increased for the network to function properly. As the network grows to 25 nodes the amount of the interference at some receivers increases dramatically, which results in a very high spreading gain. It can be seen that this high spreading gain (more than 500) would not be a practical solution for a CDMA system. This observation motivates extension of our method by allowing the spreading gains to be different for each node, instead of a fixed spreading gain for all nodes. With this flexibility in the spreading gain, we will be able to better control the amount of interference created by each node so as to accommodate the non uniform topology of ad hoc networks.

Several sets of simulations were carried out for different node distributions. All the simulation result sets follow similar results as the one shown in Figure 2.

Furthermore, it was verified that for each simulation scenario the minimum energy spreading gain obtained as the minima of curves in Figure 2 always corresponds to the solution of the optimisation problem we presented in section 3. Therefore, we can conclude that using the method presented in section 3, ad hoc nodes can accurately calculate the spreading gain required for minimum energy, given that the channel gains are known between all the communication pairs.
5.1 Implication on Multihop Networks

In the above simulation study we have assumed a fully connected network, that is single hop network. In this section we will discuss the strategy to extend this approach to multihop scenario.

Using the transmitted power, the interference threshold and the propagation loss model we can calculate the transmission range of each transmitter as follows:

\[ d < \sqrt{\frac{\text{TransmittedPower} \times K}{\text{InterferenceThreshold}}} \]

where, \( K \) is a constant based on free space propagation environment and Interference threshold is the minimum received power required by the receiver to receive transmission properly.

The transmission distance for Transmitter \( T4 \) according to Figure 1 is 221m. The results shows that transmission range for transmitter \( T4 \) does not reach to receiver \( R1, R2 \) and \( R3 \). As a result transmitter \( T4 \) does not produce any interference to these receivers. This indicates that transmission powers could have been calculated locally for transmitter \( T1, T2 \) and \( T3 \) ignoring transmitter \( T4 \).

This leads to breaking the fully-connected network into a number of fully-connected sub networks. In terms of the optimisation problem the global optimisation problem can now be replaced with a number of local optimisation problems, that should be numerically less intensive than the global problem. We can use this approach to solve multihop network problems. After identifying suitable local neighborhoods the optimum transmission power or rate constraints for these neighborhoods can be estimated. A point of interest here would be the behavior of nodes that fall into more than one neighborhood. We will address this issue in future studies.

6. Conclusions

The aim of this work was to show that the energy consumption of CDMA based ad hoc networks can be minimised by dynamically controlling the spreading gain and transmitter powers of the communicating nodes at a given time. We have developed the relevant optimisation problem and provided a solution for the problem. Furthermore, using an extensive simulation study we have shown that the solution we developed always minimises the energy consumed by the networks for a given transmission.

Our proposed solution assumes that each node in the network has global knowledge of the system. This may not be feasible in ad hoc networks with a large number of nodes in a neighbourhood. In practice we can assume that each node will have channel gain information about a subset of nodes in the vicinity of the node. We have shown in subsection 5.1 that, in fact not all nodes contribute to interference at a given node. Hence, it is important to develop an approximate solution by solving a sub-optimisation problem at each neighbourhood taking into account the information available at each node. We can also introduce the multi-hop nature of the problem by solving the optimisation problem for localised neighbourhoods. With this scheme, each node will estimate the optimum spreading gain the transmitter power required to transmit to the next hop node.

Furthermore, we also intend to develop a more general optimisation problem where different communicating node pairs can use different spreading gains.

References