Differential Space Frequency Mapping Schemes and Norm Criterion

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Abstract

We proposed four types of differential modulation to map the unitary code into the OFDM signal. The time-varying channel model is established and the norm of detection metric is deduced. The norm is the inherent interference of the time-varying channel, so we can use it as criterion to evaluate the performance of our mapping schemes. The simulation results agree with the analytic conclusion.

1. Introduction

Since Hochwald and Sweldens[1], Hughes[2] have independently invented differential unitary space time code, it aroused huge enthusiasm on the design of differential modulation methods. No channel state information (CSI) is needed at the transmitter and the receiver side, so the demodulation is very simple. These works all concern the flat fading environment and low data rate case, assuming the channel is approximately constant for a coherence interval.

High data rate and highly mobility environment is always double selective, which is quite different from the basic assumption of original differential unitary space-time code. When data rate increases to some extend, if its order of magnitude is comparable with that of multi-path delay, the performance will degrade remarkably. Akatas [6] introduced semi-blind equalization to cope with it, however the constant module algorithm is too complicated and impractical in implementation.

Double selective fading, which means both time selective and frequency selective, impose hurdle on further improving data rate under highly mobility environment. Orthogonal frequency division multiplexing (OFDM) can effectively cope with the frequency selective, it use pilot signal to do channel estimation that can catch up with the channel variation. While the performance of pure OFDM is very poor, thus it needs interleaving and powerful forward error correction (FEC) code to get the time diversity.

For unitary space-time code, the code spread across the temporal dimension and spatial dimension. Space frequency code spread across the frequency dimension and spatial dimension. Then how to map the unitary code into the OFDM structure is the main issue of this paper. We propose four types of differential space frequency schemes and its performance is analyzed in detail. We also give some practical consideration on the mapping scheme, such as FFT size, spectral efficiency, and constellation structure.

2. Differential space frequency scheme

First let's review the classical form of differential unitary space-time code [2]. Multi-antenna communication system has M transmit and N receive antennas. Previous transmission matrix is $\mathbf{S}$, current modulation matrix is $\mathbf{V}$, both are chosen from a constellation matrix set G (its cardinality is g). Let the current transmission signal $\mathbf{X}$ be the product of $\mathbf{V}$ and $\mathbf{S}$, which is called the fundamental transmission equation. The received signal is $\mathbf{Y}$. The transmitted and received signals are related by

$$\begin{aligned} \mathbf{Y}_0 &= \sqrt{\rho}\mathbf{S}\mathbf{H}_0 + \mathbf{W}_0 \\ \mathbf{Y}_1 &= \sqrt{\rho}\mathbf{V}\mathbf{S}\mathbf{H}_1 + \mathbf{W}_1 \end{aligned} \qquad (1)$$

The subscript of matrix denotes the time sequence, 0 meaning previous block, and 1 meaning current block. Channel matrix $\mathbf{H} \in C^{M\times N}$, and the additive white Gaussian noise matrix $\mathbf{W} \in C^{M\times N}$. Both $\mathbf{H}$ and $\mathbf{W}$ have independent and indentical distribution entries (zero mean, unit variance, complex Gaussian). The transmission signal $\mathbf{S}$ is normalized to obey $E\left[\sum_{m=1}^{M}\left|S_{tm}\right|^2\right]=1$. Accordingly the variable ρ denotes the expected SNR at each receiver antenna.

Assuming the channel is nearly constant for 2M signal periods, i.e., $\mathbf{H}_0 = \mathbf{H}_1$. Thus get the fundamental differential receiver equation $\mathbf{Y}_1 = \mathbf{V}\mathbf{Y}_0 + \mathbf{W}_1 - \mathbf{V}\mathbf{W}_0$.

Maximum likelihood decoding of differential modulation is

$$\mathbf{V} = \arg \max_{l=0,\ldots,g-1} \left\|\mathbf{Y}_1 - \mathbf{V}_l\mathbf{Y}_0\right\|^2 \qquad (2)$$

When the duration of unitary code is comparable with multi-path delay, multi-path will cause superimposition of previous unitary code, a well-known phenomenon called inter-symbol interference (ISI). Assuming the number of multi-path is L, if omitting the impact of additive noise, and taking the extreme example M=N=1,

$$\begin{aligned} \mathbf{Y}(k) &= \sum_{l=0}^{L} \mathbf{X}(k-l)\mathbf{H}(l) \\ \mathbf{Y}(k+1) &= \sum_{l=0}^{L} \mathbf{X}(k+1-l)\mathbf{H}(l) \end{aligned} \tag{3}$$

Note only the k*th* modulated signal matrix has the relationship $\mathbf{X}(k+1) = \mathbf{V}\mathbf{X}(k)$. Then the detection metric become

$$\left\| \mathbf{Y}(k+1) - \mathbf{V}\mathbf{Y}(k) \right\| = \left\| \sum_{l=1}^{L} \left(\mathbf{X}(k+1-l) - \mathbf{V}\mathbf{X}(k-l) \right) \mathbf{H}(l) \right\| \tag{4}$$

This is the inherent interference that will cause the performance of unitary code degrade. When M>1, the expression is more complicated, and the result will become even worse. In our simulation, there is an error floor in BER curve no matter how bigness the SNR is.

The reason we choose OFDM as transmission method is based on the aforementioned consideration. All the signals are transmitted over frequency domain, thus avoiding the signal superimposition in time domain, for the apparent advantage of OFDM is ISI free. But the frequency response of channel is neither flat nor constant, which does not satisfy the basic assumption of unitary code. In next section this problem will be analyzed in detail.

OFDM decouple the frequency selective channel into a set of parallel flat channel, making the equalization very easy to implement. However, the channel estimation and equalization is unnecessary for differential modulation in frequency domain. The sampling period of OFDM signal is T_s , FFT size is N_0 , and the cyclic prefix (or guard interval) length is L_0 , so the total time of one OFDM symbol is $T_0 = (N_0 + L_0)T_s$. The guard interval must be larger than the channel length L, thus guaranteeing the ISI free.

The column vector of $\mathbf{X}$ is the signal transmitted over different antennas. There are totally M column vectors $\mathbf{X} = [\mathbf{X}_1,,,\mathbf{X}_M]$. We need to allocate the previous column vector $\mathbf{X}_i(k)$ and current column vector $\mathbf{X}_i(k+1)$ into the N_0 subcarriers of OFDM on the corresponding antenna. The unitary code of the i*th* antenna will be still transmitted over the same antenna, 1<i<M. So the frequency response during one OFDM symbol period is the nominal CSI, it's also time-dependent. The mapping strategy must follow the same principle as unitary code: the nominal channel of consecutive blocks should change as small as possible.

The differential relationship of $\mathbf{X}(k)$ and $\mathbf{X}(k+1)$ may be mapped over the frequency, or over the consecutive period of OFDM symbols. The mapping scheme can be classified into four types. Here we use the M=2 case as example.

$$\mathbf{X}(k) = \begin{bmatrix} a & c \\ b & d \end{bmatrix}, \ \ \mathbf{X}(k+1) = \begin{bmatrix} e & g \\ f & h \end{bmatrix}$$

Type1: transmission by frequency, differential by OFDM period

1 [ab...][ef...]
2 [cd...][gh...]

Type2: transmission by OFDM period, differential by OFDM period

1 [a...][b...][e...][f...]
2 [c...][d...][g...][h...]

Type3: transmission by OFDM period, differential by frequency

1 [ae...][bf...]
2 [cg...][dh...]

Type4: transmission by frequency, differential by frequency

1 [abef...]
2 [cdgh...]

The bracket means one OFDM symbol, and the ellipsis means other OFDM subcarriers that can be allocated according to the same rule. The performance of these four types depends on many parameters. The following sections give detail analysis.

3. Time varying model of differential modulation

It's Peel and Swindlehurst [5] that first studied the impact of continuously changing channel on the unitary space-time code. They proposed a Gauss-Markov model to characterize performance of differential modulation. This model only incorporates the Doppler frequency, not containing the effect of multi-path. Thus the new model of Double selective channel should be established.

The complex baseband impulse response of a mobile wireless channel [7] can be described by

$$h(t,\tau) = \sum_{k=0}^{L} a_k(t)\delta(\tau - \tau_k) \tag{5}$$

where τ_k is the delay of the k*th* path and $a_k(t)$ is the corresponding complex attenuation amplitude. The L+1 amplitudes are independent narrow-band complex Guassian processes, following the assumption: wide sense stationary (WSS), uncorrelated scatter (US). So the cross-correlation function of any two paths is zero. The self-correlation function of the k*th* path

$$r_k(\Delta t) = E\left\{ a_k(t+\Delta t) a_k^*(t) \right\} = \sigma_k R_{hh}(\Delta t) \tag{6}$$

where σ_k is the average power of the k*th* path. All the paths have the same normalized correlation function, which is the Jake's model for land mobile fading channel

[4]

$$R_{hh}(\Delta t) = J_0(2\pi f_d \Delta t) \quad (7)$$

$J_0(\cdot)$ is the zero order Bessel function of the first kind, and f_d is the maximum Doppler frequency in the mobile environment.

The frequency response of time-varying radio channel is

$$h(t,f) = \int_{-\infty}^{+\infty} h(t,\tau)e^{-j2\pi f\tau}d\tau = \sum_{k=0}^{L} a_k(t)e^{-j2\pi f\tau_k} \quad (8)$$

Its equivalent discrete expression for OFDM modulation is

$$h(n,m) = \sum_{K=0}^{L} a(nT_0,k)\exp\left(-j2\pi\tau_k \frac{m}{N_0 T_s}\right) \quad (9)$$

Here $h(n,m)$ denotes the frequency response of the n*th* OFDM symbol and m*th* subcarrier.

Since channel is time-varying, the equation (2) is no longer valid. Supposing during the transmission of signal $\mathbf{V}$ and $\mathbf{SV}$, the time-varying channel is indexed by time label, its sequence is $\mathbf{H}(1),,,\mathbf{H}(M),\mathbf{H}(M+1),,,\mathbf{H}(2M)$. Let $\mathbf{V}_i$ and $\mathbf{S}_i$ denote the row vector, for $1 \le i \le M$. If the additive noise is neglected, the detection metric become

$$\|\Omega\| \triangleq \|\mathbf{Y}_1 - \mathbf{V}\mathbf{Y}_0\| = \left\| \begin{bmatrix} \mathbf{V}_1\mathbf{SH}(M+1) \\ \vdots \\ \mathbf{V}_M\mathbf{SH}(2M) \end{bmatrix} - \mathbf{V}\begin{bmatrix} \mathbf{S}_1\mathbf{H}(1) \\ \vdots \\ \mathbf{S}_M\mathbf{H}(M) \end{bmatrix} \right\| \quad (10)$$

The m*th* row vector of detection matrix $\mathbf{\Omega}$ is

$$\mathbf{\Omega}_m = \mathbf{V}_m\mathbf{SH}(M+m) - \mathbf{V}_m \begin{bmatrix} \mathbf{S}_1\mathbf{H}(1) \\ \vdots \\ \mathbf{S}_M\mathbf{H}(M) \end{bmatrix} \quad (11)$$

Because the detection metric is measured by Frobenius norm, we consider only N=1 for simplification (even N>1 will follow the same step).

Assume the channel transfer function matrix $\mathbf{H}$ is spatially white, every entry is uncorrelated with each other,

$$E[\mathbf{H}_{pq}(\cdot)\mathbf{H}^*_{mn}(\cdot)] = \delta(p-m)\delta(q-n) \quad (12)$$

In our unitary code mapping scheme, $\mathbf{H}_{tn}(M+m)$ is corresponding to the p*th* subcarrier of c*th* OFDM symbol, and $\mathbf{H}_{tn}(k)$ is corresponding to the q*th* subcarrier of d*th* OFDM symbol.

Let $\Phi(m,k) \triangleq E\left[\mathbf{H}_{tn}(M+m)\mathbf{H}^*_{tn}(k)\right]$, then

$$\Phi(m,k) = E\left\{\left[\sum_{i=0}^{L} a(cT_0,i)\exp(-j2\pi\tau_i \frac{p}{N_0T_s})\right] \times \left[\sum_{i=0}^{L} a(dT_0,i)\exp(-j2\pi\tau_i \frac{q}{N_0T_s})\right]^*\right\}$$

From the equation (6), we can obtain

$$\Phi(m,k) = \sum_{i=1}^{L}\sigma_i^2 J_0\left(2\pi f_d(c-d)T_0\right)\exp(-j2\pi\tau_i \frac{p-q}{N_0T_s}) \quad (13)$$

Let $\Theta(k,m) \triangleq E\left[\mathbf{H}_{tn}(k)\mathbf{H}^*_{tn}(M+m)\right]$, with relationship of the zero order Bessel function of the first kind: $J_0(x) = J_0(-x)$, it can be easily get relationship $\Theta(k,m) = [\Phi(m,k)]^*$.

The norm of $\mathbf{\Omega}$ reflects the impact of time-varying channel. It resembles the inherent additive noise that cannot be removed (not necessarily white Guassian). The mean of the norm of $\mathbf{\Omega}_m$ is

$$E[\mathbf{\Omega}_m\mathbf{\Omega}^*_m] = 2\mathbf{V}_m\mathbf{SS}^H\mathbf{V}_m^H - \mathbf{V}_m\mathbf{SS}^H Diag[\Phi(m,1),...,\Phi(m,M)]\mathbf{V}_m^H - \mathbf{V}_m Diag[\Theta(1,m),...,\Theta(M,m)]\mathbf{SS}^H\mathbf{V}_m^H$$

Because $\mathbf{S}$ is unitary matrix, $\mathbf{SS}^H = \mathbf{I}_M$.

Let $\mathrm{var}(m,k) \triangleq 2\,\mathrm{Re}\left[\Phi(m,k)\right]$.

From the above equations, the mean norm of $\mathbf{\Omega}$ is

$$E\left[\|\mathbf{\Omega}\|^2\right] = \sum_m\sum_k \left|\mathbf{V}_{m,k}\right|^2 [2 - \mathrm{var}(m,k)] \quad (14)$$

The equation (14) is the basis for further analysis, and we will use it as norm criterion to measure the performance of our mapping schemes. It depends on the design of unitary code constellation and channel realization. The reason we don't continue to use the method in paper [6] is that it's too complex to get the exact pair-wise error probability. Norm criterion is enough to give a baseline judgement.

4. Analysis of differential scheme

In the following expression, $1 \le m \le M,\ 1 \le k \le M$, all the types are listed below.

For type 1,

$$\mathbf{H}_{tn}(M+m) = \sum_{i=0}^{L} a((c+1)T_0,i)\exp(-j2\pi\tau_i \frac{r+m}{N_0T_s})$$

$$\mathbf{H}_{tn}(k) = \sum_{i=0}^{L} a(cT_0,i)\exp(-j2\pi\tau_i \frac{r+k}{N_0T_s})$$

where c is the time index of previous block, and r is the beginning subcarrier index of previous block. Thus

$$\mathrm{var}(m,k) = 2\sum_{i=1}^{L}\sigma_i^2 J_0\left(2\pi f_d T_0\right)\cos(2\pi\tau_i \frac{m-k}{N_0T_s}) \quad (15)$$

For type 2,

$$\mathbf{H}_{tn}(M+m) = \sum_{i=0}^{L} a((c+M+m)T_0,i)\exp(-j2\pi\tau_i \frac{r}{N_0T_s})$$

$$\mathbf{H}_{tn}(k) = \sum_{i=0}^{L} a((c+k)T_0,i)\exp(-j2\pi\tau_i \frac{r}{N_0T_s})$$

where c is the beginning time index of previous block, r is the subcarrier index of both blocks. Thus

$$\mathrm{var}(m,k) = 2\sum_{i=1}^{L}\sigma_i^2 J_0\left(2\pi f_d(M+m-k)T_0\right) \quad (16)$$

For type 3,

$$\mathbf{H}_{tn}(M+m)=\sum_{i=0}^{L} a((c+m)T_0,i)\exp(-j2\pi\tau_i \frac{r+1}{N_0T_s})$$

$$\mathbf{H}_{tn}(k)=\sum_{i=0}^{L} a((c+k)T_0,i)\exp(-j2\pi\tau_i \frac{r}{N_0T_s})$$

where c is the beginning time index of both blocks, r is the subcarrier index of previous block. Thus

$$\mathrm{var}(m,k)=2\sum_{i=1}^{L}\sigma_i^2 J_0\left(2\pi f_d (m-k)T_0\right)\cos(2\pi\tau_i \frac{1}{N_0T_s}) \quad (17)$$

For type 4,

$$\mathbf{H}_{tn}(M+m)=\sum_{i=0}^{L} a(cT_0,i)\exp(-j2\pi\tau_i \frac{r+M+m}{N_0T_s})$$

$$\mathbf{H}_{tn}(k)=\sum_{i=0}^{L} a(cT_0,i)\exp(-j2\pi\tau_i \frac{r+k}{N_0T_s})$$

where c is the time index of both blocks, r is the subcarrier index of previous block. Thus

$$\mathrm{var}(m,k)=2\sum_{i=1}^{L}\sigma_i^2 \cos(2\pi\tau_i \frac{M+m-k}{N_0T_s}) \qquad (18)$$

We now make the following discussion, maximizing the variance $\mathrm{var}(m,k)$ is our comparison criterion.

1) FFT size

Because $T_0=(N_0+L_0)T_s$, once the sample period T_s is fixed (which is related to the specific data rate), the variance mainly depends on FFT size N_0. In our interesting scope, Bessel function $J_0(\cdot)$ is a decreasing function with N_0, while $\cos(\cdot)$ is an increasing function with N_0. For type 1 and 3, maximizing the variance is a hard work, as it's not easy to get an explicit expression for N_0. The optimization can be done by trial of a series of value. For type 2, less N_0 is good. But there is some requirement for N_0 in ISI free rule, i.e., $N_0 > L_0 > L$. For type 4, the bigger N_0 is the preferable choice. However when considering modulation latency and hardware implementation, N_0 cannot go up unbounded. So the suitable FFT size is related to the concrete scenario.

2) Diversity order

The available diversity order of multi-path MIMO channel is MNL, but for MIMO OFDM the diversity order become MN. In our mapping scheme, the unitary code is transmitted across M subcarriers of OFDM. Firstly, these nominal channels are correlated; Secondly, the unitary code requires nominal channel keep constant, or else additive interference may arise. Therefore no diversity is provided by our scheme except spatial diversity, i.e., MN.

3) Constellation structure

Sholkrollahi, etc [8] elaborate the method to design high-rate constellations with full diversity. They give an exhaustive classification on the unitary matrices. Here we only concentrate on the diagonal structure and non-diagonal structure. By our mapping scheme, $\mathrm{var}(m,m) > \mathrm{var}(m,k)$, the norm of diagonal constellation is larger than that of non-diagonal. Therefore we can expect the performance of diagonal constellation is better. In paper [8], cyclic group u=(1,1) and quaternion group Q_1 have the same diversity product. However, the simulation shows the BER performance curves of both are almost identical. This is the drawback of norm criterion, i.e., different constellation structure cannot be compared directly.

5) Signal in first OFDM symbol

This problem only relates with type 1 and 2. If we put all the identity matrices in the first OFDM, then there exist repeated periodic pattern, such as [100100…]. After the IFFT, only the energy in time index $t=mN_0/M,\ m=0,\dots,M-1$ is nonzero, others are all zero. The peak to average ratio (PAR) of OFDM will be very high, especially the value in time index 0, which should be avoided by the engineering implementation. So the signal in first OFDM symbol shouldn't be periodic signals, and identity matrix is not imperative. We can use a series of known unitary signals as initial matrices.

6) Robustness to multi-path realization

Note in expression (16), when the average power of multi-path are normalized to be $\sum_{i=1}^{L}\sigma_i^2=1$, and the variance becomes $\mathrm{var}(m,k)=2J_0\left(2\pi f_d(M+m-k)T_0\right)$. It only relates with the Doppler frequency, and the multi-path delay profile is no longer important. So the type 2 shows its robustness to frequency selective channel.

5. Simulation results

We set up two kind of propagation scenario to test the OFDM scheme. Scenario A is an artificial environment, which is consisting of two equal amplitude echoes, and the second echo delay is 1us. Scenario B is COST 207 model, the typical urban (TU) model, its coefficients are listed in table.1.

TABLE 1
COST 207 TU MODEL

tapes	1	2	3	4	5	6
Delay (us)	0	0.2	0.6	1.6	2.4	5
Power (dB)	-3.0	0	-2	-6	-8	-10

The spectrum shapes of all tapes follow the classical Jakes' Doppler spectrum [4]. In this paper, we choose $f_d = 75\mathrm{Hz}$. The overall power of all tapes is normalized to 0dB.

OFDM is also simulated for comparison. To make the data rate equal, the digital modulation of OFDM is QPSK, and the sampling period keeps the same. The signal in odd subcarriers is used as pilot signal to do channel estimation, the signal in even subcarriers carry the information bits. The CSI of even number is obtained by interpolation the two nearby CSI of odd number, thus achieving the approximated CSI. We assume the perfect receiver structure, neither carrier frequency offset nor timing offset exist.

The transmit antenna number M=4, and the receive antenna number N=1. Fig.1~4 only consider diagonal constellation, as Hochwald [2] designed, cyclic group u=(1,3,5,7), R=1. For fig.1 and 2, the sampling period is 10^{-6} s, so the corresponding data rate is 1M bits/s. For fig.3 and 4, the sampling period is 10^{-7} s with data rate 10M bits/s. The FFT size is 256, and the guard interval is 24, which satisfy the ISI free requirement for most cases.

Here we calculate the norm in equation (14) for fig.1~4. Because it's diagonal constellation, the computation is very simple. Results are listed below.

TABLE 2
CALCULATED NORM

	Type 1	Type 2	Type 3	Type 4
Fig.1	0.0348	0.5475	0.0012	0.0193
Fig.2	0.0348	0.5475	0.0039	0.0623
Fig.3	3.4820e-004	0.0056	0.1199	1.7777
Fig.4	3.4820e-004	0.0056	0.3639	2.2039

Definitely the smaller the norm is, the better the BER performance. The simulation results tally with the calculated norm perfectly. Taking Fig.1 as the example: from the table the ascending sequence of four types is 3,4,1,2, and the performance curves of fig.1 also rank in the same sequence. Compared with the curve of OFDM, most types show excellent performance owing to the diversity gain.

From Fig.1 and 2, type 2 encounters the error floor. Because the differential matrix span 8 OFDM symbols, the time-varying channel may cause the inherent interference just as the equation (14) imply. The performance of type 3 is much better owing to its differential matrix only spanning 4 OFDM symbols.

However in Fig.3 and 4, where the data rate go up to ten times, the performance of type 2 show its superiority. Because one OFDM symbol period is much shorter, the channel frequency response of each subcarrier across consecutive symbol is nearly static. Thus transmission and differentiation across OFDM period is a better choice.

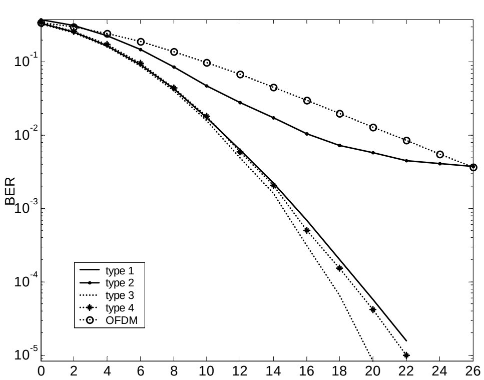

Fig.1. Two-ray of equal power, Ts=1e-6, BER vs. SNR

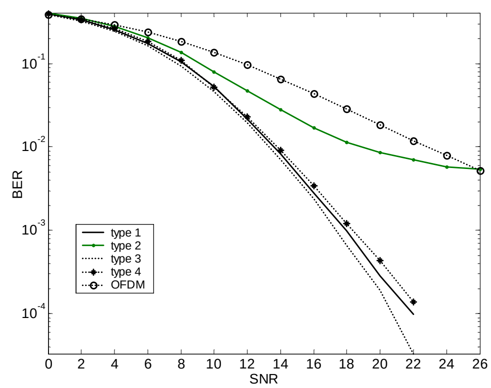

Fig.2. COST 207 TU, Ts=1e-6, BER vs. SNR

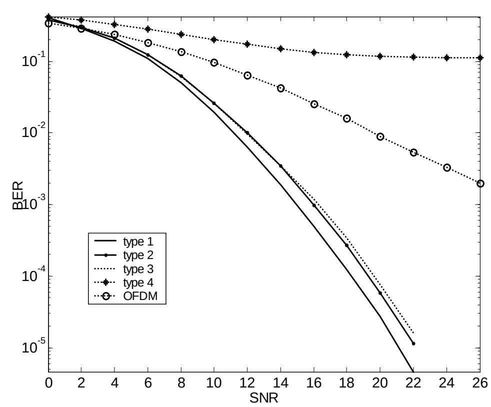

Fig.3. Two-ray of equal power, Ts=1e-7, BER vs. SNR

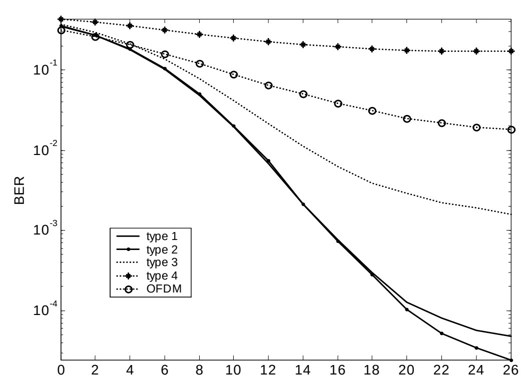

Fig.4. COST 207 TU, Ts=1e-7, BER vs. SNR

In Fig.4, the error floor happens for all the types.

Because the guard interval is only 2.4us, which is smaller than the maximum delay of COST 207 model, the ISI free rule is violated. When SNR is greater than 20dB, the ISI became the main trouble. Although the norm of type 1 is much smaller than that of type 2, its performance is a little worse than type 2. ISI cause the superimposing part of each OFDM symbols longer than the guard interval, and type 1 span four signals across four subcarriers, so it's more sensitive to the ISI. Type 4 span eight signals across eight subcarriers, surely its performance is rather awful.

FFT size also has impact on the performance. Fig.5 show the result of different FFT size, including 64/70,128/140,256/280, 512/560. The sampling period is 10^{-6} s, use scenario B. The mapping scheme is type 1. The norm is calculated in table 3. Here again we found the BER curve agree with the norm.

TABLE 3
CALCULATED NORM

FFT SIZE	64	128	256	512
Fig.5	0.0022	0.0087	0.0348	0.1387

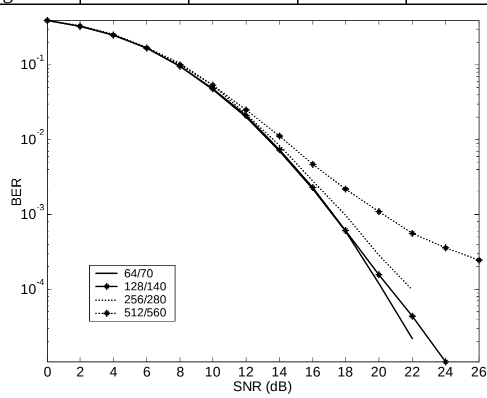

Fig.5. COST 207 TU, Ts=1e-6, BER vs. SNR, for different FFT size

6. Conclusion

Now we make some conclusion on the proposed four types: Type 1 is nearly excellent under all situations. Type 2 is best suited for the very high rate, promising for the rate greater than 10M bits/s. Type 3 and 4 are limited to low data rate, lower than 1M bits/s.

The norm criterion is a valuable tool to analyze the mapping schemes. It can only evaluate the same kind of unitary code, and different unitary code cannot be compared directly.

7. References

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