# The Peak Factor of a 16-QAM/OFDM System

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## Abstract

*The close relationship between the peak factor of a QPSK/OFDM system and the one from a 16-QAM/OFDM system is presented and analyzed. It is shown that a 16-QAM/OFDM system shows larger peaks and is more difficult to analyze unless use is made of some results that relate both schemes in a straightforward manner.*

Index Terms—Orthogonal Frequency Division Multiplexing, Peak to Average Power Ratio (PAR).

## I. INTRODUCTION

It is well known that one of the major drawbacks of Orthogonal Frequency Division Multiplexing (OFDM) is the very high peak-to-average power ratio (PAR) of any OFDM signal. This problem is more important when use is made of higher modulation schemes like 16-QAM. In the case of QPSK, the OFDM Peak Factor has been considered in the past [1].

As noted in [2], all the previous techniques attempting to reduce the PAR in an OFDM system use almost exclusively PSK signal constellations. However, QAM constellations are commonly used in OFDM systems.

Fortunately, it has been shown that a 16-QAM system can be described as a linear combination of two QPSK signals [2]. Using this and the analysis presented in [3] we can derive some results for the case of a 16-QAM/OFDM system. This motivates our work in this paper and in particular the analysis of the peak factor of a 16-QAM/OFDM system.

The results derived in this paper can also be extended in a straightforward manner to 64-QAM/OFDM or 256-QAM/OFDM systems but has not been considered here.

## II. THE 16-QAM/OFDM SYSTEM

Figure 1 below shows a block diagram of a typical OFDM system. In this system, the input bit stream arriving to the encoder is grouped into blocks of k bits and encoded into a particular constellation. Then a serial to parallel process is implemented to obtain the time-domain OFDM symbols. Each symbol consists of N modulated tones that can be represented by a vector $\underline{X}$ with elements $X_n$, $0 \leq n \leq N-1$. Next a sampled version of the time domain signal is generated via an IFFT that can be described by the vector $\underline{x}$ with elements $x_k$, $0 \leq k \leq N-1$.

It is important to observe, that these samples of the discrete time signal can exhibit large peaks. As mentioned before, the peaks are larger for 16-QAM/OFDM than for QPSK/OFDM systems. For the case of a QPSK/OFDM system, it is known that the highest peak is equal to the total number of subcarriers being used. In the case of QAM/OFDM systems it has only been mentioned that the highest peak can be greater than the number of subcarriers used in the particular system under consideration but specific values for the highest peak or peaks have not been provided. We show in this paper the value of these peaks for a 16-QAM/OFDM system only.

It is not difficult to derive the peak values of an OFDM system using higher QAM constellations. To verify these values all we need to do is to implement a simulation searching for the highest peaks. The problem associated with this is the time required for the program to give a result due to the fact that the probability of obtaining the highest peak decreases with increasing constellation sizes.

We can show in a graphical way that the peaks are larger for 16-QAM/OFDM than for a QPSK/OFDM system with the help of the plots shown in Fig. 2 and Fig. 3. In both cases, the number of carriers and center frequencies are the same.

From these plots it is clear that the PAPR is a more serious problem in 16-QAM/OFDM than in a QPSK/OFDM system.

In general, an OFDM symbol consists of N subcarriers separated by a frequency distance $\Delta f$. Accordingly, the total system bandwidth B is divided into N equidistant subchannels with all subcarriers being mutually orthogonal within a time interval T = $1/\Delta f$.

The OFDM signal in the time domain is given as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi\Delta ft} \qquad 0 \leq t \leq T \tag{1}$$

To obtain the discrete time representation, the signal $x(t)$ has to be sampled. The sampling time needs to be

$\Delta t = \frac{1}{B} = \frac{1}{N\Delta f}$ so that the signal can be completely determined by its samples. The sampled signal is given as

$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi kn/N} \qquad k = 0,1 \ldots, N-1 \tag{2}$$

Equation (2) represents an N point inverse discrete Fourier transform (IDFT) of the input data $X_n$. These input symbols $X_n$ represent digitally modulated binary data. The IDFT can be implemented using the inverse fast Fourier transform (IFFT) algorithm.

In our case, each modulated symbol $X_n$ is chosen from a 16-QAM signal constellation that can be described by the following expression [4]:

$$s_{16-QAM}(t) = 2[A_n Cos\, 2\pi f_c t + B_n Sin 2\pi f_c t] + \\ + [C_n Cos 2\pi f_c t + D_n Sin 2\pi f_c t] \tag{3}$$

$\{A_n\},\{B_n\},\{C_n\},\{D_n\}$ are statistically independent binary sequences with elements from the set $\{-1,+1\}$.

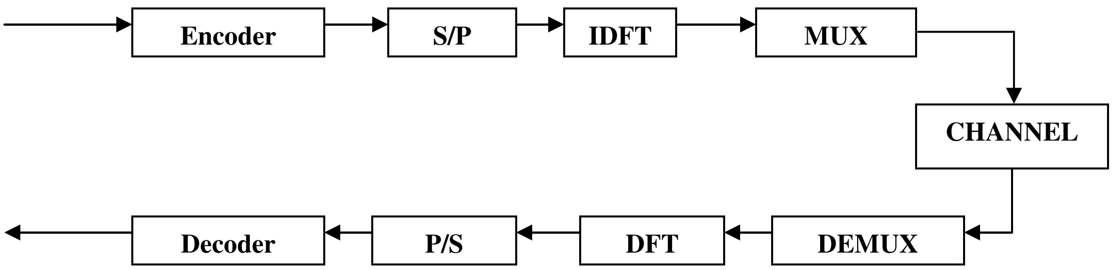


Fig. 1 Block diagram of an OFDM system

This expression results from the observation that a 16-QAM constellation can be realized as the sum of two QPSK signals as explained in [2].

Using $I_n = 2A_n + C_n$ and $Q_n = 2B_n + D_n$ we can simplify equation (3) and obtain:

$$s_{16-QAM}(t) = I_n Cos 2\pi f_c t + Q_n Sin 2\pi f_c t \tag{4}$$

Here $I_n, Q_n$ take elements from the set $\{\pm 1, \pm 3\}$ only.

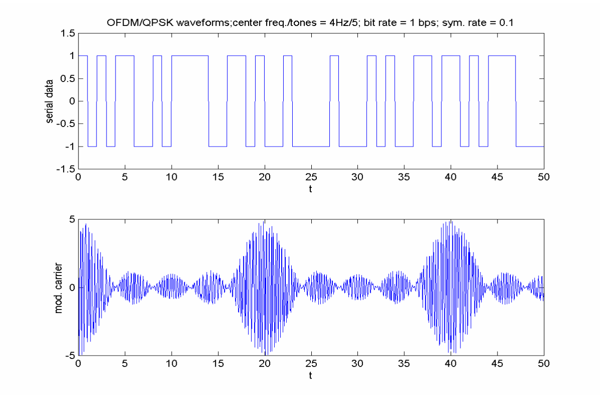


Fig. 2 QPSK/OFDM signal

Having introduced the relationship between QPSK and 16-QAM, we can proceed with the analysis of a 16-QAM/OFDM system in terms of QPSK/OFDM. To this end, following the notation of [3] we can write the following expression for the output waveform of a 16-QAM/OFDM modulation system:

$$v(t) = \frac{1}{\sqrt{10N}} \{\sum_{k=0}^{N-1} a_k Cos[2\pi(f_c + \frac{k}{T})t] \\ + b_k Sin[2\pi(f_c + \frac{k}{T})t]\} \tag{5}$$

Here, $a_k$ and $b_k$ are amplitude coefficients taking elements from the set $\{\pm 1, \pm 3\}$. The factor $\sqrt{10}$ is a normalization value used in connection with the 16-QAM constellation to make the expectation value equal to unity.

Equation (5) can be further simplified to yield:

$$v(t) = \frac{1}{\sqrt{10N}} \left| \sum_{k=0}^{N-1} (a_k + b_k) \exp(\frac{j2\pi kt}{T}) \right| Cos(2\pi f_c t + \phi(t)) \tag{6}$$

From equation (6), the peak envelope power (PEP) can be calculated as [5]:

$$PEP = \frac{\max_{\langle t \rangle} \{ | \sum_{k=0}^{N-1} (a_k + b_k) \exp(\frac{j2\pi kt}{T}) |^2 \}}{20N} \tag{7}$$

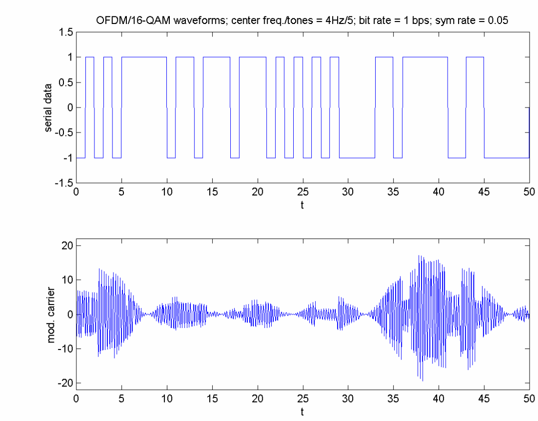


Fig. 3 16-QAM/OFDM signal

If all the carriers have the same phase at some particular time, the result of the sum becomes $36N^2$ giving a PEP of $1.8N$ meaning that the peak power can be $1.8N$ times the average power.

This result is in agreement with the explanation provided in [6]. There, it is mentioned that for an OFDM system using N subcarriers modulated by phase-shift keying (PSK), the theoretical upper bound of the PAR is N but it can be higher than N if a multilevel constellation like QAM is used.

## III. Simulation Results

To verify the validity of the expression for the PEP of a 16-QAM/OFDM modulation system obtained in the preceding section, we implemented a simulation for the case of 5, 10, and 15 subcarriers using a rectangular 16-point QAM signal constellation.

We denote the complex-valued signal points corresponding to the information symbols on the N subchannels by $X_n$, $n = 0,1,\ldots,N-1$. Hence, these information symbols $X_n$ represent the values of the discrete Fourier transform (DFT) of the OFDM signal, where the modulation on each subcarrier is 16-QAM and the symbol duration T = 100 seconds.

The information symbols $X_n$ are selected pseudorandomly to allow for different phases in the subcarriers. In this way we were able to obtain the highest envelope peak, that is, the highest PEP.

Figure 4 below shows the result of the simulation for the case of 5 subcarriers.

From this figure we observe that the highest peak has a value of 20.3102 W compared with the value of 29.0851 W obtained using the expression from the preceding section.

It is important to observe that Fig. 4 results after several runs of the program searching for the highest peak. In all cases the peaks never exceeded the predicted value of 1.8N times the average power.

Another important consideration is that with increasing number of subcarriers, it takes more time for the program to obtain the highest peak or peaks. This is to be expected because it is more difficult to obtain the same phase in all the subcarriers being used at one specific point in time. The situation just described becomes worst when higher QAM constellations like 64-QAM or 256-QAM are used in conjunction with the OFDM system. Again, this is to be expected.

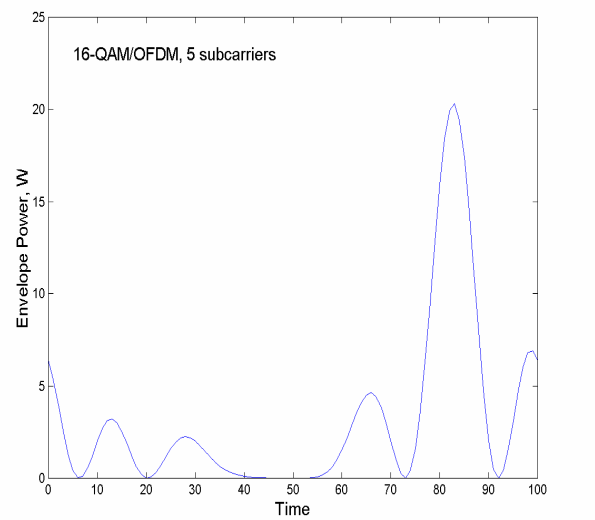


Fig. 4 16-QAM/OFDM System, 5 subcarriers, Pav = 3.2317 W

For the case of 10 subcarriers, Fig. 5 shows a peak of 77.9388 W compared with 84.0812 W from the expression of section II.

Finally, Fig. 6 shows the results for the case of 15 subcarriers. The highest peak has a value of 139.1858 W compared with 185.4378 W predicted by the expression from section II.

In all three cases, the number of program iterations was set to 100. If we increase the number of iterations, a better approximation is obtained.

We considered only 5, 10 and 15 subcarriers in our simulation because as the number of these subcarriers increases, the probability of the occurrence of the highest peak becomes negligible [6].

In our simulation the differences between simulated and calculated results are 30%, 7%, and 25% respectively. The main reasons for this is the random nature of the peak occurrence and the number of iterations needed to reduce the gap between simulated and calculated results.

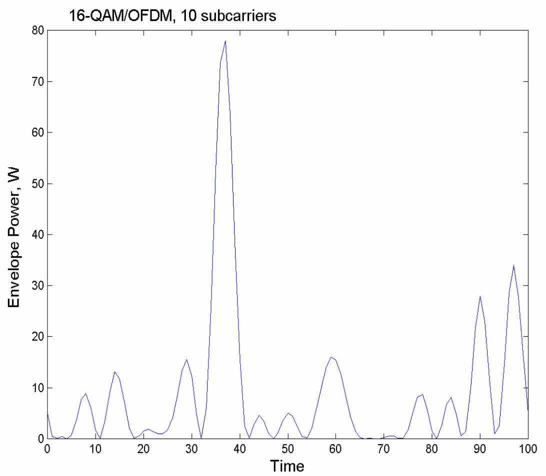


Fig. 5 16-QAM/OFDM System, 10 subcarriers, Pav = 4.6712 W

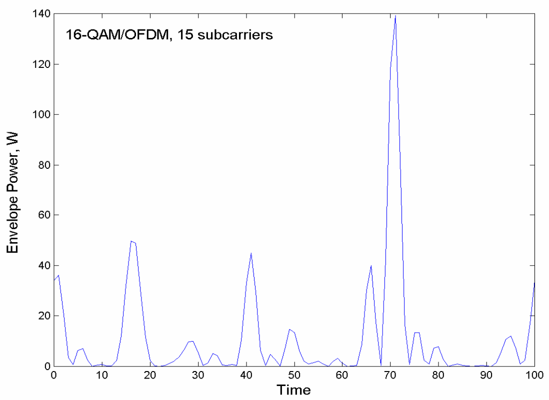


Fig. 6 16-QAM/OFDM System, 15 subcarriers, Pav = 6.8681 W

## IV. Conclusions

Many applications of OFDM use PSK signal constellations but at the same time16-QAM/OFDM is regularly used in multicarrier communications. It is well known that the PEP of a QPSK/OFDM system is N times the average power of the transmitted signal. It has also been mentioned [6] that the PEP in the case of an OFDM system using QAM constellations can be higher than N, but no specific simulation results have been provided. An analytical expression for the PEP is given in [7]. In this paper we have derived an expression for the PEP of a 16-QAM/OFDM system and verified it by simulation.

### References

[1] P. W. J. Van Eetvelt, S. J. Shepherd, S. K. Barton, "Distribution of peak factor in QPSK multi-carrier modulation",*Journal of Wireless Personal Commun.*, vol. 2, pp. 87-96, Nov. 1995.

[2] C. Rossing, V. Tarokh, "A construction of OFDM 16-QAM Sequences Having Low Peak Powers", *IEEE Trans. on Inform. Theory*, vol. 47, no. 5, July 2001.

[3] S. Shepherd, J. Orriss, S. Barton, "Asymptotic Limits in peak Envelope Power Reduction by Redundant Coding in Orthogonal Frequency-Division Multiplex Modulation", *IEEE Trans. on Comm.*, vol. 46, no. 1, January 1998.

[4] J. Proakis, "Digital Communications", Fourth Edition, McGraw-Hill 2001.

[5] L. W. Couch, "Digital and Analog Communication Systems", Sixth Edition, Prentice Hall 2001.

[6] H. Ochiai, H. Imai, "On the Distribution of the Peak-to-Average Power Ratio in OFDM Signals", *IEEE Trans. on Comm.*, vol. 49, no. 2, February 2001.

[7] Z. Wang, X. Ma, G. B. Giannakis, "OFDM or Single-Carrier Block Transmissions?", *IEEE Trans. on Comm.*, vol. 52, no. 3, March 2004.