Comparisons of Performance of Various Transmission Schemes of MIMO System Operating under Rician Channel Conditions

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Abstract

In this paper, we compare various transmission schemes, as described by the covariance matrix of transmitted signals, for a Multiple Input Multiple Output (MIMO) system operating under Rician channel conditions. It is shown that in order to obtain the best performance in terms of the maximum communication rate the receiver has to have full knowledge of the channel while the transmitter requires the knowledge of the Rice factor and the signal to noise ratio that are measured at the receiver. Based on this, a new transmission scheme is described. Analytical and simulation results are presented showing that the proposed transmission scheme outperforms the other schemes, which were reported earlier in the literature.

1. Introduction

Recent years have shown a strong research interest in multiple input multiple output (MIMO) wireless communications systems because they are capable of delivering much larger capacities than conventional single input single output (SISO) systems in a multipath rich Rayleigh propagation environment. In general, the MIMO system capacity (*the maximum communication rate*) depends on properties of the complex channel matrix, the covariance matrix of transmitted signals (signal transmission scheme) and the signal to noise ratio (SNR). It has been shown [1], [2] that the independent transmission scheme represented by a diagonal covariance matrix with equal coefficients realizes the maximum MIMO capacity in an ideal Rayleigh environment when the channel state information (CSI) is known to the receiver.

Real propagation environments deviate from Rayleigh conditions, as both line of sight (LOS) and non-line of sight (NLOS) signal components, are present at receiver. These environments can be modeled by the Rician channel. Determination of MIMO capacity and obtaining suitable transmission schemes under Rayleigh and Rician channel conditions has been the subject of many recent investigations [1]-[9]. It has been postulated that the information about the Rice factor k (the power ratio between LOS and NLOS components) at the transmitter can lead to an enhanced transmission scheme [3], [6]-[7]. In [3], the authors reported an upper bound for the MIMO capacity under an arbitrary Rician channel and in [6]-[7] they proposed a new transmission scheme applicable to arbitrary Rician conditions. By performing calculations, one can find a shortcoming in the formulas presented in [3] and [7], which occurs for low values of the Rice factor or SNR. The physical reason for this could be due to the fact that in practice the Rice k factor can not always be measured. For, example, this situation occurs when either power in LOS or NLOS components drop below the noise level. Therefore it is important to include this limitation in the theory presented in [3] and [7].

In our previous work [13], we have overcome the above mentioned limitations by completing derivations of expression for the upper bound capacity. The modified expression has been made valid for an arbitrary case of Rician channel. Also we have proposed a transmission scheme which achieves higher *communication rate* in comparison with the other transmission schemes, which have been described in the literature. The presented in [13] simulated and analytic curves for the MIMO capacity have shown very similar trends. However, no perfect match between the two sets of values has been reached. This can be explained by the fact that the simulated results concerned the average capacity while the analytic results were for upper bound capacity. In undertaken MIMO system simulations [13], the channel properties, as described by the channel matrix, are governed by random distributions of scattering objects. Some of these distributions create communication channels offering *communication rate* considerably lower than the upper bound.

In order to provide fair comparisons, the simulated results should focus on the scattering objects distributions leading to maximum instantaneous capacity. In this paper, we focus on this refined comparison. Under these new assumptions, we undertake investigations into different MIMO transmission schemes operating in a Rician channel and we investigate them for varying values of Rician k factor, SNR and number of antennas.

2. Upper bound capacity of MIMO system

The capacity (*the maximum communication rate*) of MIMO system with N_T antennas at the transmitter and N_R antennas at the receiver operating with an arbitrary signal transmission scheme is given as [7]:

$$C = \max_{tr\{\mathbf{Q}\} \le P; q_{ii} \ge 0} E_H \left\{ \log_2 \det(\mathbf{I}_{N_T} + \frac{1}{\sigma_n^2} \mathbf{Q}\mathbf{H}^\dagger \mathbf{H}) \right\} \quad (1)$$

where $E_H\{(\cdot)\}$ denotes the expectation value over channel matrix **H**, $\mathbf{Q}$=E$\{xx^\dagger\}$ is the covariance matrix of transmitted signals defining a transmission scheme, E$\{(\cdot)\}$ denotes the expectation value over time and $(\cdot)^\dagger$ is the conjugate and transpose operation of the matrix or so-called Hermitian operation. $\mathbf{I}_{N_T}$ is the $N_T \times N_T$ identity matrix. The transmitted signals are constrained by the average total power tr$\{\mathbf{Q}\}$< P , where tr$\{\mathbf{Q}\}$ is the sum of the diagonal elements of matrix **Q**. σ_n^2 is the noise power at receiving antennas. The **H** matrix coefficients are described by the Rice distribution with Rice factor $k = \mu^2 / 2\sigma^2$, where $\mu / \sqrt{2}$ is the mean value and σ^2 is the variance.

An expression for the upper bound of the capacity was derived in [7, (15)] and is rewritten here as (2):

$$C \le \max_{tr\{\mathbf{Q}\} \le P; q_{ii} \ge 0} \log_2 \det\left(\mathbf{I}_{N_T} + \frac{1}{\sigma_n^2} \mathbf{U}^T \mathbf{Q}\mathbf{U}\mathbf{D} \right)$$
$$= \max_{tr\{\mathbf{Q}\} \le P; q_{ii} \ge 0} \log_2 \det\left(\mathbf{I}_{N_T} + \frac{1}{\sigma_n^2} \widetilde{\mathbf{Q}}\mathbf{D} \right) \quad (2)$$

In (2) the complex matrix **D** is a $N_T \times N_T$ diagonal matrix, which is obtained from the singular value decomposition (SVD) with respect to $E_H\{\mathbf{H}^\dagger \mathbf{H}\} = \mathbf{U}\mathbf{D}\mathbf{U}^T$. **U** is the composition matrix having orthogonal eigenvectors corresponding to **D** where $(\cdot)^T$ is the transpose operation of the matrix.

In order to overcome the shortcomings mentioned in [13] we introduce a new parameterε, as given by (3):

$$\varepsilon = \min\left\{ \frac{k(1+k)N_T}{\rho(1+N_T k)N_R}, 1 \right\} \quad (3)$$

where ρ is SNR, $\rho = \frac{P}{\sigma_n^2}$.

In order to have practical meaning, values of ε are limited to the range $0 \le \varepsilon \le 1$. This restriction is obtained from (17) in [3] that the diagonal elements ($\widetilde{q}_{ii}$) of matrix $\widetilde{\mathbf{Q}}$ have to be non-negative. In turn, the condition $0 \le \varepsilon \le 1$ restricts the allowable values of k as a function of ρ because of (3). The limitation that ε should not exceed 1 can be easily interpreted for the case of N_R=N_T =1. Assuming N_R=N_T =1 one finds from (3) that when ε = 1 the Rice factor becomes equal to SNR (k=ρ). For large k value, $\rho \approx P_{LOS}/P_{Noise} \approx k = P_{LOS}/P_{NLOS}$ and therefore k=ρ becomes the largest value which can be measured at the transmitter. Larger values of k can not be measured because P_{NLOS} drops below the noise level. This limitation can now be included in the theoretical expressions presented in [3] or [7].

As the result of the introduced limitation, the expression for the upper bound capacity for a MIMO system operating under Rician fading environment is given as (4).

$$C_{UB} = \begin{cases} \log_2\left(1 + k(N_T - 1) + \frac{k}{\varepsilon}\right) + \\ \quad (N_T - 1)\log_2\left(1 + \frac{(1-\varepsilon)\rho N_R}{(1+k)N_T}\right) & 0 \le \varepsilon < 1 \\ \log_2\left(1 + 0.5\left(\begin{array}{l} \rho N_T N_R - N_T \\ + \sqrt{(\rho N_T N_R - N_T)^2 + 4\rho N_T N_R} \end{array} \right)\right) & \varepsilon = 1 \end{cases} \quad (4)$$

3. New transmission scheme

In [6]-[7], the authors proposed a transmission scheme, as given by (5).

$$\mathbf{Q} = \mathbf{Q}^{\mathbf{AV}} = \frac{P}{N_T(1+k)}\left(\mathbf{I}_{N_T} + k\mathbf{\Psi}_{N_T}\right) \quad (5)$$

where $\mathbf{\Psi}_{N_T}$ is the $N_T \times N_T$ matrix of all ones, and compared it with so-called independent and dependent transmission schemes.

To avoid confusion in referring to various transmission schemes, as described by relevant covariance matrices of transmitted signals, we use capital letters A, B, C, and D to name them. Throughout this paper, the independent transmission scheme which corresponds to pure NLOS (k=0) is named as scheme A and given in (6). The dependent transmission scheme which corresponds to pure LOS (k=∞) is named as scheme B and given in (7). The transmission scheme in (5) is named here as scheme C. It can be noticed that scheme C can be viewed as a weighted average of scheme A and B.

$$\mathbf{Q} = \mathbf{Q}^0 = \frac{P}{N_T}\mathbf{I}_{N_T} \quad (6)$$

$$\mathbf{Q} = \mathbf{Q}^\infty = \frac{P}{N_T}\mathbf{\Psi}_{N_T} \quad (7)$$

The new transmission scheme which can realize the upper capacity bound for the case of an arbitrary Rice fading channel is given by **Q= Q^NEW^**.

This scheme assumes that the receiver has the full knowledge of channel matrix **H** and other statistical signal parameters while transmitter knows only the Rice factor k and SNR.

The solution of $\mathbf{Q}^{\mathrm{NEW}}$ has been shown in [13]. Here, we describe an alternative derivation, which leads to the same solution, as presented in [13]. By using SVD technique, we can find the relationship of composition matrix **U,** $N_T \times N_T$ matrix, as shown in (8)

$$E_H\{\mathbf{H}^\dagger\mathbf{H}\} = \mathbf{UDU}^T \tag{8.1}$$

$$E_H\{\mathbf{H}^\dagger\mathbf{H}\} = \left(\frac{N_R}{1+k}\right)\begin{bmatrix} 1+k & k & \cdots & k \\ k & \ddots & & \vdots \\ \vdots & & 1+k & k \\ k & \cdots & k & 1+k \end{bmatrix} \tag{8.2}$$

$$\mathbf{UDU}^T = \left(\frac{N_R}{1+k}\right)\mathbf{U}\begin{bmatrix} 1+N_T k & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & & 1 \end{bmatrix}\mathbf{U}^T \tag{8.3}$$

Next, we use an analogous approach to find the solution by performing matrix **U** in (8) to matrix $\widetilde{\mathbf{Q}}$ in [3, (17)]. The resulting covariance matrix is given as shown in (9).

$$\mathbf{Q}^{\mathrm{NEW}} = \mathbf{U}\widetilde{\mathbf{Q}}\mathbf{U}^T \tag{9.1}$$

$$\mathbf{U}\widetilde{\mathbf{Q}}\mathbf{U}^T = \left(\frac{P}{N_T}\right)\mathbf{U}\begin{bmatrix} (1+\varepsilon(N_T-1)) & 0 & \cdots & 0 \\ 0 & (1-\varepsilon) & & \vdots \\ \vdots & & \ddots & \\ 0 & & \cdots & (1-\varepsilon) \end{bmatrix}\mathbf{U}^T \tag{9.2}$$

$$\mathbf{Q}^{\mathrm{NEW}} = \left(\frac{P}{N_T}\right)\begin{bmatrix} 1 & \varepsilon & \cdots & \varepsilon \\ \varepsilon & \ddots & & \vdots \\ \vdots & & 1 & \varepsilon \\ \varepsilon & \cdots & \varepsilon & 1 \end{bmatrix} \tag{9.3}$$

From (9) the covariance matrix $\mathbf{Q}^{\mathrm{NEW}}$ for the new transmission scheme can be deduced and is expressed in (10) which is the same expression in [13]. The associated transmission scheme for (10) is named here as scheme D.

$$\mathbf{Q} = \mathbf{Q}^{\mathrm{NEW}} = \frac{P}{N_T}\left((1-\varepsilon)\mathbf{I}_{N_T} + \varepsilon\mathbf{\Psi}_{N_T}\right) \tag{10}$$

The expression (10) indicates the desired statistical properties of data transmitted by different antenna elements to reach the maximum communicate rate. From (10) it can be seen that the diagonal elements of $\mathbf{Q}^{\mathrm{NEW}}$ are equal to ρ/N_T meaning that equal power has to be transmitted by individual antennas. The other elements apart from diagonal elements are equal to $\rho\varepsilon/N_T$, where ε can be interpreted as the cross correlation coefficient between signals transmitted by different antennas.

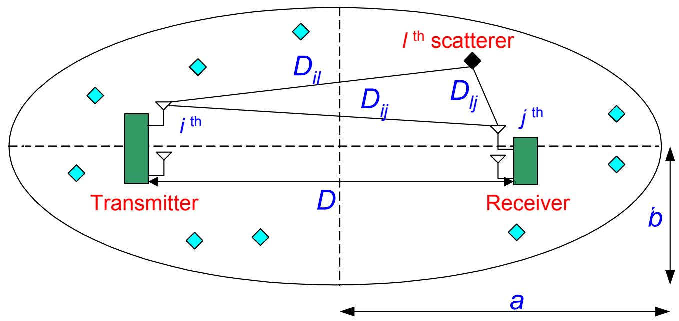

Figure 1. The scattering model of an indoor MIMO system.

4. Results and discussions

In order to assess the performance of the four transmission schemes (A, B, C and D) we analytically calculate the capacity bounds, as well as we calculate capacity via simulations assuming arbitrary Rician conditions of a channel. The modified single bounce scattering model as described in [10]-[11] is used to generate the channel matrix **H** coefficients. In order to obtain results relevant to indoor wireless communications standards, an indoor scattering model, as shown in Figure 1 is assumed. Note that other region shapes can also be easily included. The operating frequency and root mean square delay spread are assumed to be f = 5 GHz and τ_{RMS}=100ns respectively, which are typical values for WLAN standards such IEEE 802.11a and HIPERLAN/2 [12]. Following this initial assumption, the distance between transmitter and receiver D is set equal to 300λ (or 18m). Using τ_{RMS} and f, the ellipse major axis parameter is found to be a = 250λ (or 15m) [9]. Finally, the ellipse minor axis is calculated as $b=\sqrt{a^2-(D/2)^2}$ = 200λ (or 12m). The chosen dimensions describe an exhibition hall or a large office building. The number of scattering objects randomly distributed inside the area is 100. The inter-element spacing in antenna arrays is 0.5 λ for transmitter and receiver. 1,000 random channel matrices **H** are produced via Monte Carlo simulations, which are performed in MATLAB. For the assumed geometry, uniform distribution of scattering objects is assumed. For each distribution an instantaneous capacity is calculated for four transmission schemes according to (5), (6), (7) and (10). From this set, a maximum and average capacity values, and other statistical quantities are determined.

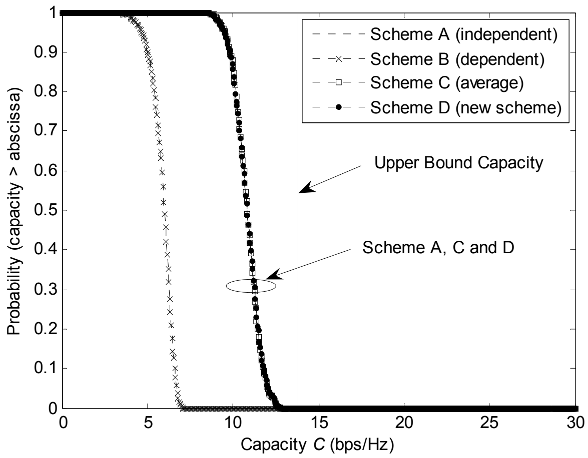

Figure 2. Complementary cumulative distribution function of capacity for 4x4 MIMO system, k = -10dB, ρ =10 dB.

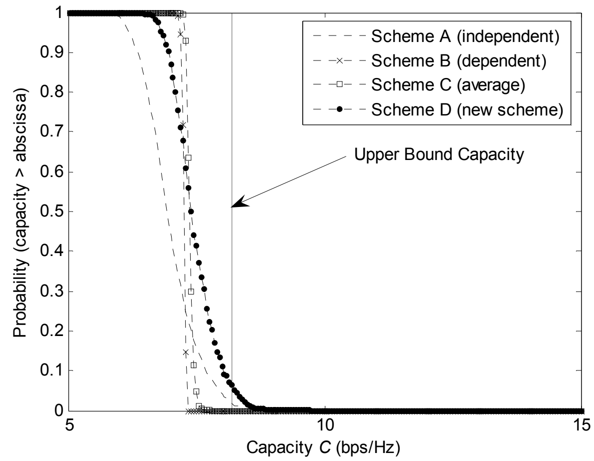

Figure 3. Complementary cumulative distribution function of capacity for 4x4 MIMO system, k = 10dB, ρ =10 dB

4.1. The effect of Rician *k* factor

Figure 2, 3 and 4 shows the complementary cumulative distribution function of capacity for the four transmission schemes for a 4x4 MIMO system when ρ = 10 dB and k = -10dB, 10dB and 20dB respectively. The upper bound capacity is calculated by applying expression (4). It can be noticed that when k is very small the capacity of the weighted average (scheme C) and optimal (scheme D) schemes converge to the capacity of independent scheme (scheme A). A similar convergence to the capacity of dependent scheme is observed for weighted average (scheme C) and optimal (scheme D) schemes when k is very large. Scheme A fails to achieve high capacity for large values of k. Similarly, scheme B underperforms for low values of k. Scheme C avoids this failure and works almost optimally in the two extreme cases. However, it considerably deflects from the upper bound in an intermediate range of k values.

As can be seen from Figure 2, 3 and 4, the simulated capacity values vary below the upper bound limit. Among four schemes, the new scheme (scheme D) offers the highest capacity compared with the other schemes. It can be noticed that scheme D can offer the most probability to reach the upper limit for any particular Rician k factor.

4.2. The effect of signal to noise ratio

Figure 5 shows the results for capacity of a 4x4 MIMO system as a function of the SNR when the Rice factor value is fixed and equal to k = 10 dB. In general, the capacity increases when the SNR increases for all the four schemes, except the slope of increase is different is each

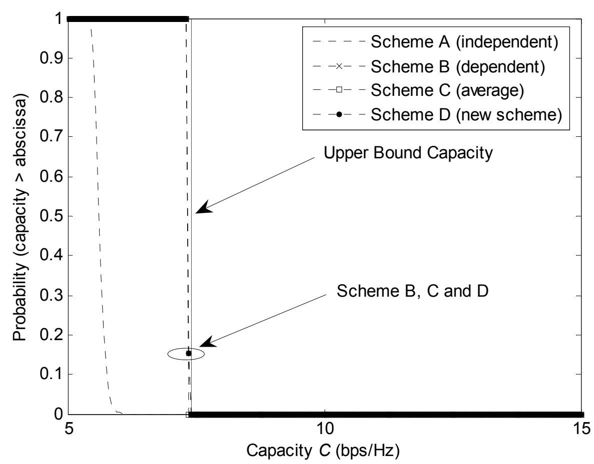

Figure 4. Complementary cumulative distribution function of capacity for 4x4 MIMO system, k = 20dB, ρ =10 dB

case. It can be observed that the optimal scheme (D) provides the highest capacity for any particular SNR.

Figure 6 shows the corresponding values for capacity obtained from Monte Carlo simulations. As noticed from Figure 2, 3 and 4, the average capacity is always lower than the upper bound capacity. In order to fairly compare the simulated results with the upper bound limit, the maximum instantaneous capacity value is selected from 1000 realizations. This value can be regarded as the simulated upper bound capacity value. The result is shown in Figure 6. As observed in Figure 6, the simulated upper bound capacity shows not only the same trend as the upper bound capacity in Figure 5 but the agreement is also achieved in terms of numerical values.

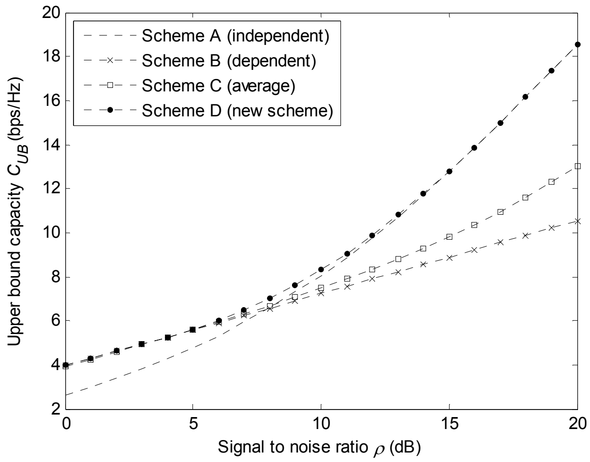

Figure 5. Upper bound capacity C_{UB} (bps/Hz) vs. signal to noise ratio ρ (dB) for 4x4 MIMO system, k =10 dB.

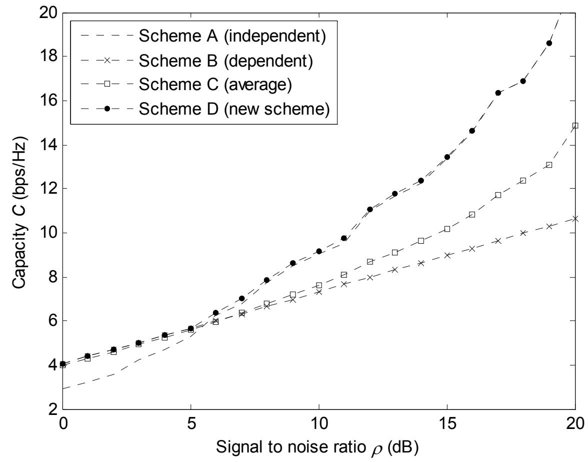

Figure 6. Simulated capacity C (bps/Hz) vs. signal to noise ρ (dB) for 4x4 MIMO system, k =10 dB.

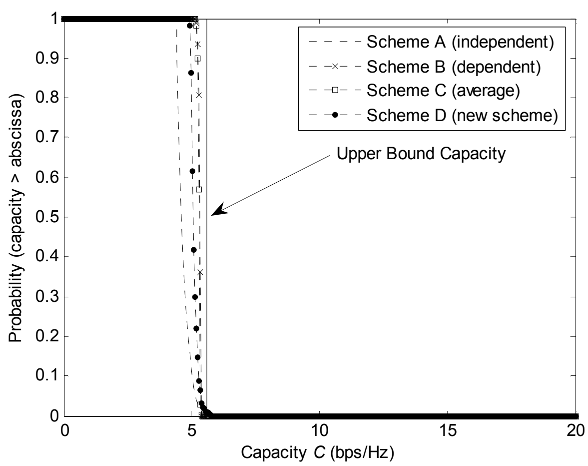

Figure 7. Complementary cumulative distribution function of capacity for 2x2 MIMO system, k = 10dB, ρ =10 dB.

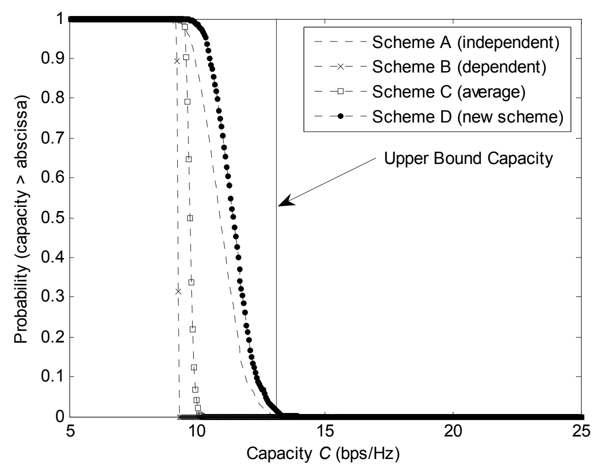

Figure 8. Complementary cumulative distribution function of capacity for 8x8 MIMO system, k = 10dB, ρ =10 dB

4.3. The effect of transmit/receive antennas

Figure 7 and 8 present the complementary cumulative distribution function of capacity for 2x2 and 8x8 MIMO systems, respectively. In general, the capacity increases when the number of antennas increases for all the four schemes. For 2x2 system (Figure 7), there is a small difference between each scheme but for 4x4 (Figure 3) and 8x8 (Figure 8) systems the highest capacity can be achieved by the new transmission scheme (scheme D). This trend is apparent when a larger number of antennas is used.

In all of the presented results, the newly proposed transmission scheme outperforms the three remaining schemes. This can be explained by the fact that a feedback loop between the receiver and the transmitter has to carry information about SNR in addition to Rice factor k.

5. Conclusion

In this paper we have investigated various transmission schemes, as described by the covariance matrix of transmitted signals, for MIMO system operating in a Rician fading environment. A new transmission scheme has been described and compared with the schemes proposed by other researchers. We have investigated the validity of the new scheme for varying values of Rician k factor, SNR and number of antennas by performing Monte Carlo simulations. The simulated results have shown superiority of the new scheme over the other schemes reported in the literature.

6. References

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