

DOA Estimation of a Wideband Signal Using a 2-D Array Antenna with Spatial Processing Capability

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Abstract

This paper describes investigations into Direction-Of-Arrival (DOA) estimation of a wideband signal by a two-dimensional array antenna, which employs only spatial signal processing for beam forming. The elements of this array are arranged in a horizontal rectangular lattice to steer a beam in azimuth over a wide frequency band. By applying the concept of interpolated array, a composite covariance matrix is produced. This composite covariance matrix is a simple addition of covariance matrices of narrowband virtual arrays, being stretched or compressed versions of a nominal array, all featuring the same radiation pattern. DOA is estimated by eigen-decomposition of the composite covariance matrix using the narrowband MUSIC algorithm. The performance of the proposed DOA estimation method is demonstrated by computer simulations. The obtained results indicate that the two-dimensional array provides better estimation of DOA than the one-dimensional one when the interpolated array technique in conjunction with the MUSIC algorithm is applied.

1. Introduction

It has been recently argued that the performance requirements of future wireless systems can not be met without the use of smart antennas [1]. It has been postulated [2] that the use of a smart antenna can increase the spectral density and the Signal-to-Noise Ratio (SNR) for both Wireless Fidelity (WiFi) and Worldwide Interoperability for Microwave Access (Wi-MAX). This opinion forms the ground for the claim that smart antenna techniques will play a significant role in the development of future wireless systems. Because of the demand for high speed data transmission it can be expected that the main focus will be on wideband smart antennas.

Wide band smart antennas have been the subject of investigations in [3]–[10]. In [3] and [4], space-time signal processing is investigated to achieve wideband beamforming. Time-Delay Lines (TDLs) are used in each array branch to equalize the signal delay across a wide

frequency band. In [5] and [6], the bank of frequency filters is employed to decompose the received wideband signals into non-overlapping narrowband signals so that a narrowband weighting scheme can be applied for beam formation. The works described in [7]–[9] aim at avoiding TDLs and frequency filters to perform wideband beamforming. To this purpose, a two-dimensional array antenna (instead of one-dimensional) arranged in a rectangular lattice is used to perform signal processing in both space and frequency domains. Amongst the three alternatives, the fully spatial method of wideband beamforming seems to be attractive because it eliminates the use of filters and TDLs. Furthermore, it requires only real-valued constant weights at individual antenna elements to achieve wideband beamforming. Such weights can be easily realized in practice, in an analog manner, using amplifiers or attenuators.

Designs of smart antennas based on the spatial signal processing concept [7]–[10] rely on the assumption that DOA of desired signal is known to the system, for example, using one of the already existing DOA methods. Unfortunately, many of them, such as the ones described in [11]–[16], are not suitable for the 2-D spatial beamformer. The reason is that they rely on the application of TDLs, which this type of beamformer avoids. However, the DOA estimation methods of wideband signals such as those described in [17]–[19] are worthwhile considering. The method described in [17] utilizes the covariance matrix of received signals by assuming that all sources have a flat power spectrum over the frequency band of interest. As this is not a realistic assumption in many commonly occurring situations, this approach is not pursued here. The method shown in [18] and [19] concerns a direction finding algorithm for multiple wideband signals received by one-dimensional array. It applies an interpolation technique to generate a set of virtual arrays, each for a different frequency band, having the same array manifold. The covariance matrices of these arrays are added up to produce a composite covariance matrix. DOA estimates are produced by eigen decomposition of this composite covariance matrix using the narrowband Multiple Signal Classification (MUSIC)

algorithm. This approach looks attractive from the point of view of applying it to the 2-D spatial beamformer described in [7]–[10]. However, it requires a suitable extension from the 1-D to 2-D array case.

In this paper, we modify the concept of the interpolated array technique for a 1-D array [18]–[19] and adapt it to the case of the 2-D spatial wideband beamformer described in [10]. The considerations commence with the 2-D array signatures. By applying the interpolated technique the composite covariance matrix is produced. Next, the MUSIC algorithm is used to the composite covariance matrix to estimate DOA of the desired wideband signal. The performance of the DOA estimation algorithm for the 2-D array is compared against the one for the 1-D array in [19].

2. Signal Model

The configuration of a wideband smart antenna that employs a fully spatial signal processing for beam forming [7]–[10] is shown in Fig. 1a. It is constituted by $N_1 \times N_2$ wideband antennas arranged in a rectangular lattice in which amplifiers or attenuators connected to individual array elements produce weighting coefficients. Then, the signal is combined by a summing network. From Fig. 1a, d_1 and d_2 represent array spacing in two orthogonal directions and are usually chosen as half-wavelength at the highest frequency of a given frequency band of operation. Antenna elements are denoted by indices m_1 and m_2 , where $-M_1 \leq m_1 \leq M_1$ and $-M_2 \leq m_2 \leq M_2$. The relation between N and M is $M_i = (N_i - 1)/2$.

In order to see the difference between this array antenna and the conventional 1-D wideband antenna with TDLs, the configuration of the latter is shown in Fig. 1(b). The two configurations have the same task of beam forming in azimuth (the x - y plane). In the 2-D array the function of TDLs is somehow performed by a rectangular structure of the 2-D array. As the signal is assumed to arrive from the direction not perpendicular to the array's plane, the 2-D array's elements receive the signal's replicas with different phases. This results in a set of signals, which can be used for processing both in frequency and angular domains.

In order to form a beam in a specific direction, the signal weights have to be properly chosen in the two types of arrays. The algorithms for generating these weights have already been given in [10] and [20] and are not repeated here. However, it can be noted that in both cases, these weights are real-valued.

Having explained the operation of the two types of wideband array antennas, we are ready to move to describing the DOA estimation algorithm based on the interpolation technique that was described in [18]–[19].

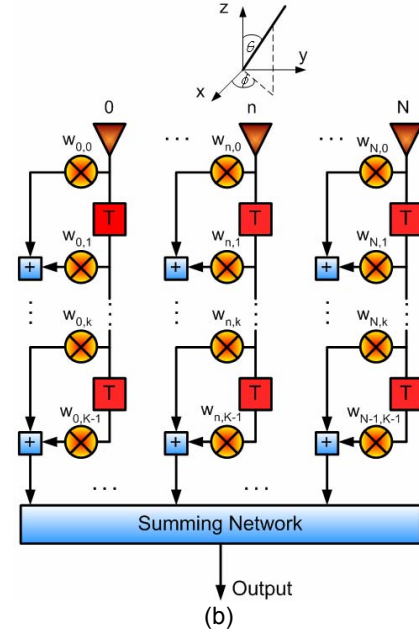
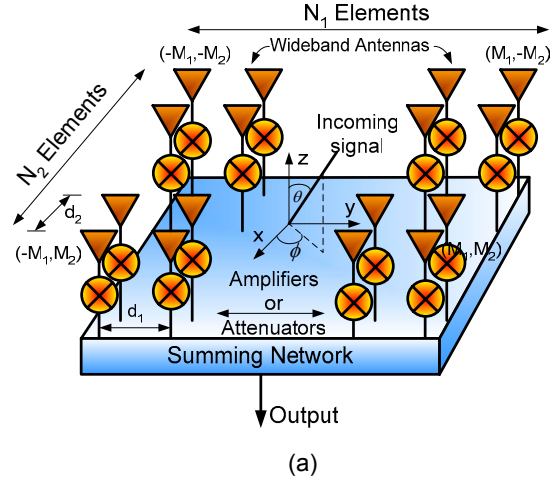


Fig. 1: (a) Configuration of fully spatial wideband smart antenna (b) Configuration of a conventional wideband smart antenna.

By assuming K radiating sources in azimuth direction (ϕ), being in the plane of the array, the signal at the output of each array element is given by

$$x_{m_1, m_2}(t) = \sum_{k=1}^K s_k(t - \psi_{k, m_1, m_2}) + q_{m_1, m_2}(t) \quad (1)$$

where S_k is the radiated signal and q_{m_1, m_2} is the noise signal. The parameter ψ_{k, m_1, m_2} stands for the delay associated with the signal propagation time from k^{th} source to the $(m_1, m_2)^{\text{th}}$ element of array. This parameter contains the information about location of sources as shown in (2).

$$\psi_{k,m_1,m_2} = (2\pi f/c)(d_1 m_1 \sin \phi_k + d_2 m_2 \cos \phi_k) \quad (2)$$

where ϕ_k is DOA from k^{th} source in azimuth. The time-domain signal given in (1) can be transformed to frequency-domain as shown in (3).

$$X_{m_1,m_2}(f_i) = \frac{1}{N} \sum_{n=0}^{N-1} x_{m_1,m_2}(n\Delta t) e^{-j2\pi f_i n\Delta t} \quad (3)$$

where $\Delta t = 1/f_s$, $\Delta f\Delta t = 1/N$, f_s is sampling frequency and N is the number of frequency bins. Using (3), (1) can be rewritten in terms of frequency, as shown in (4)

$$X_{m_1,m_2}(f_i) = \sum_{k=1}^K e^{j(2\pi f_i/c)(d_1 m_1 \sin \phi_k + d_2 m_2 \cos \phi_k)} S_k(f_i) + Q_{m_1,m_2}(f_i) \quad (4)$$

where $S_k(f_i)$ and $Q_{m_1,m_2}(f_i)$ are the Fourier coefficients of $s_k(t)$ and $Q_{m_1,m_2}(t)$ at the frequency bin f_i respectively. Then, (4) can be simplified to the equation as given in (5)

$$\mathbf{X}(f_i) = \mathbf{A}(f_i)\mathbf{S}(f_i) + \mathbf{Q}(f_i) \quad (5)$$

where

$$\begin{aligned} \mathbf{X}(f_i) &= [\mathbf{X}_{-M_2}(f_i) \quad \mathbf{X}_{-M_2+1}(f_i) \quad \cdots \quad \mathbf{X}_{M_2}(f_i)]^T \\ \mathbf{X}_{m_2}(f_i) &= [\mathbf{X}_{-M_1,m_2}(f_i) \quad \mathbf{X}_{-M_1+1,m_2}(f_i) \quad \cdots \quad \mathbf{X}_{M_1,m_2}(f_i)]^T \\ \mathbf{S}(f_i) &= [\mathbf{s}_1(f_i) \quad \mathbf{s}_2(f_i) \quad \cdots \quad \mathbf{s}_K(f_i)]^T \\ \mathbf{Q}(f_i) &= [\mathbf{Q}_{-M_2}(f_i) \quad \mathbf{Q}_{-M_2+1}(f_i) \quad \cdots \quad \mathbf{Q}_{M_2}(f_i)]^T \\ \mathbf{Q}_{m_2}(f_i) &= [\mathbf{Q}_{-M_1,m_2}(f_i) \quad \mathbf{Q}_{-M_1+1,m_2}(f_i) \quad \cdots \quad \mathbf{Q}_{M_1,m_2}(f_i)]^T \\ \mathbf{A}(f_i) &= [\mathbf{a}(f_i, \phi_1) \quad \mathbf{a}(f_i, \phi_2) \quad \cdots \quad \mathbf{a}(f_i, \phi_K)] \\ \mathbf{a}(f_i, \phi_k) &= [\mathbf{a}_{-M_2}(f_i, \phi_k) \quad \mathbf{a}_{-M_2+1}(f_i, \phi_k) \quad \cdots \quad \mathbf{a}_{M_2}(f_i, \phi_k)]^T \\ \mathbf{a}_{m_2}(f_i, \phi_k) &= [e^{j\psi_k(-M_1,m_2)} \quad e^{j\psi_k(-M_1+1,m_2)} \quad \cdots \\ &\quad e^{j\psi_k(M_1,m_2)}]^T \end{aligned}$$

In practice, one can obtain the frequency response (5) from a number of snapshots (samples) in time domain. Assuming N_s is the number of snapshots, and the number of frequency samples (L) is equal to $N_s - N + 1$, the covariance matrix of each frequency bin can be evaluated by averaging over L times as shown in (6)

$$\begin{aligned} \mathbf{R}(f_i) &= E\{\mathbf{x}(f)\mathbf{x}^\dagger(f)\} \\ &= \mathbf{A}(f_i)\mathbf{R}_S(f_i)\mathbf{A}^\dagger(f_i) + \mathbf{R}_Q(f_i) \end{aligned} \quad (6)$$

where $\mathbf{R}_S(f_i) = \frac{1}{L} \sum_{l=1}^L \mathbf{S}(f_i)\mathbf{S}^\dagger(f_i)$

and

$$\mathbf{R}_Q(f_i) = \frac{1}{L} \sum_{l=1}^L \mathbf{Q}(f_i)\mathbf{Q}^\dagger(f_i)$$

Note that \dagger stands for the conjugate and transpose operation (Hermitian operation). From (6), it can be noticed that the covariance matrix has the structure similar to the narrow band case [21]. This analogy enables the use of an efficient subspace search technique like the one offered by the MUSIC algorithm. Note that before implementing the MUSIC algorithm, N sets of covariance matrix from N frequency bins has to be considered. In order to tackle this problem, the concept of interpolated array [19] is adapted. It allows for transferring the steering vector of any frequency bin into the specific frequency's steering vector which is usually chosen at the center frequency of the band of operation.

3. Interpolated array technique

The concept of interpolated array for estimating DOA of wideband signals has been described in [18]–[19]. In this concept, a wide frequency band is divided into multiple narrow bands with center frequencies f_i . In each band the signal is assumed to be received by a virtual array, which is “stretched” or “compressed” version of nominal array. The stretch/compress factor is such that all of the virtual arrays have the same response (radiation pattern). Because the virtual arrays have the same response (at their operating frequencies) it is possible to combine the covariance matrices for the different frequencies by simple addition. The outputs from virtual arrays are obtained via interpolation technique from the real one-dimensional array. The interpolation coefficients are selected so as to minimize the interpolation error for a signal arriving from a given sector (a range of bearing angles), at a particular frequency. The size of the sector is chosen to give good estimates of the virtual array outputs. The design of interpolator is done once and off-line.

By assuming a constant matrix \mathbf{B}_i , the relation between the steering vector of frequency bin f_i and the center frequency f_c for any angle of arrival ϕ within the limited sector $[\phi_1, \phi_2]$ can be shown in (7)

$$\mathbf{a}(f_c, \phi) = \mathbf{B}_i \mathbf{a}(f_i, \phi) \quad (7)$$

The steering matrix $\mathbf{A}_B(f_i)$ which is needed to determine \mathbf{B}_i appeared in (7) is formed by a set of steering vectors $\mathbf{a}(f_i, \phi)$ cooperating with the set of angles within the sector as given in (8)

$$\mathbf{A}_B(f_i) = [\mathbf{a}(f_i, \phi_1) \quad \mathbf{a}(f_i, \phi_1 + \Delta\phi) \quad \cdots \quad \mathbf{a}(f_i, \phi_2)] \quad (8)$$

The interpolated matrix \mathbf{B}_i is then given by (9).

$$\mathbf{B}_i = \mathbf{A}_B(f_c) \mathbf{A}_B^\dagger(f_i) [\mathbf{A}_B(f_i) \mathbf{A}_B^\dagger(f_i)]^{-1} \quad (9)$$

The interpolation accuracy has been described in [19] and therefore is not discussed here.

As seen in (7), different steering vectors of each frequency bin f_i can be transformed into the steering vector at the center frequency of the band. By using this transformation, the composite covariance matrix \mathbf{R} is obtained as given in (10)

$$\mathbf{R} = \sum_{i=1}^N \mathbf{B}_i \mathbf{R}(f_i) \mathbf{B}_i^\dagger = \mathbf{A}(f_c) \bar{\mathbf{R}}_S \mathbf{A}^\dagger(f_c) + \bar{\mathbf{R}}_Q \quad (10)$$

where $\bar{\mathbf{R}}_S = \sum_{i=1}^N \mathbf{R}_S(f_i)$ and $\bar{\mathbf{R}}_Q = \sum_{i=1}^N \mathbf{B}_i \mathbf{R}_Q(f_i) \mathbf{B}_i^\dagger$

4. MUSIC algorithm

The MUSIC algorithm is considered to be an effective algorithm to estimate DOA of multiple narrowband signals in a wireless system. It has gained a considerable attention in the last decade. This is because of its advantages in terms of hardware implementation as well as due to its simplicity against other well-known DOA techniques [21]. Steps required in the MUSIC algorithm can be summarized as follows [21]:

1. Perform eigen decomposition on \mathbf{R} from (10)

$$\mathbf{R} \mathbf{V} = \mathbf{\Lambda} \mathbf{V} \quad (11)$$

where $\mathbf{V} = [\mathbf{v}_0 \quad \mathbf{v}_1 \quad \dots \quad \mathbf{v}_{N_1 N_2 - 1}]$
 $\mathbf{\Lambda} = \text{diag}(\lambda_0, \lambda_1, \dots, \lambda_{N_1 N_2 - 1})$
 $\lambda_0 \geq \lambda_1 \geq \dots \geq \lambda_{N_1 N_2 - 1}$

2. Assuming K sources, the covariance matrix \mathbf{C} is given in (12).

$$\mathbf{C} = \mathbf{V}_n \mathbf{V}_n^\dagger \quad (12)$$

where $\mathbf{V}_n = [\mathbf{v}_K \quad \mathbf{v}_{K+1} \quad \dots \quad \mathbf{v}_{N_1 N_2 - 1}]$

3. The MUSIC spectrum is obtained using (13)

$$P_{MUSIC}^{-1}(\phi) = \frac{\mathbf{a}^\dagger(f_c, \phi) \mathbf{a}(f_c, \phi)}{\mathbf{a}^\dagger(f_c, \phi) \mathbf{C} \mathbf{a}(f_c, \phi)} \quad (13)$$

Here, this algorithm is applied to DOA estimation of multiple wideband signals when the interpolation array algorithm, described in the previous section of this paper,

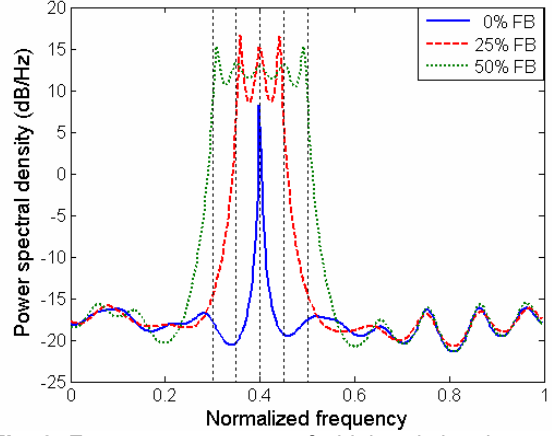


Fig. 2: Frequency spectrum of wideband signals

is applied to convert the wideband DOA estimation problem to the narrowband DOA estimation problem.

5. Simulation results

The proposed wideband DOA estimation method for the 2-D spatial beamformer of [10] was assessed via MATLAB[®] simulations. In particular, its performance was compared with the 1-D case, described in [18], [19].

Throughout the simulations, for the 2-D configuration of Fig.1a, the 4x4-antenna array [10] was considered. In this array antenna elements are spaced half-wavelength at the highest frequency ($f_s/2$). For the 4x1 wideband beamformer, a 4x1 linear array with uniform spacing of half-wavelength at the highest frequency ($f_s/2$) was assumed. In the two cases, the frequency was normalized by setting f_s equal to 1. The center frequency was set at 0.4 and the number of frequency bins N was assumed to be 100. The simulated results were obtained using 400 snapshots. In order to take into account the random nature of noise, the results were evaluated over 50 experiments. The sector used in the interpolated technique was assumed to be from 0° to 60° with the step size of 1° . The wideband signal extending from f_l to f_h having the Fractional Bandwidth (FB) defined as

$$FB = \frac{(f_h - f_l)}{f_c} \times 100\% \quad (14)$$

was created assuming frequency bins of equal amplitude and phase shift. Examples of spectra of wideband signals with different values of FB are shown in Fig. 2.

Simulation results concerning various aspects of DOA estimation are presented in Fig.3 to 6. They are divided into four issues: DOA detection, effect of fractional bandwidth, effect of SNR, and effect of angle separation.

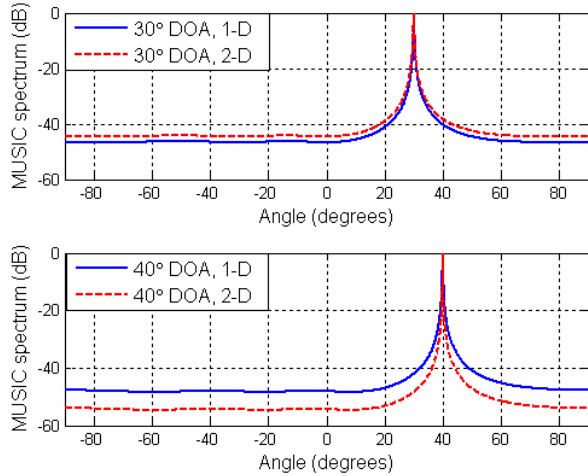


Fig. 3: MUSIC spectrum, SNR = 10dB, FB =50%, interpolated sector $[0^\circ \ 60^\circ]$, $N=100$, $N_s=400$, 50 experiments.

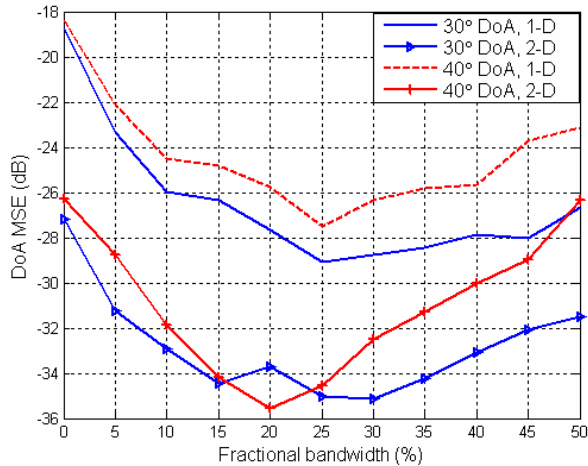


Fig. 4: DOA MSE (dB) versus fractional bandwidth (%), SNR = 10dB, $\phi = 30^\circ$ and 40° , interpolated sector $[0^\circ \ 60^\circ]$, $N=100$, $N_s=400$, 50 experiments.

Here we provide detailed comments with respect to each of the four issues.

5.1. DOA detection

Fig 3 shows the results for DOA using 2-D array and 1-D array when the desired signal comes from the angles of 30° and 40° off boresight direction. The plot indicates that the proposed method using 2-D array provides DOA estimation as good as the original 1-D array described in [18] and [19].

5.2 Effect of fractional bandwidth

Here the Mean Square Error (MSE) of DOA is used to assess the performance of the DOA detecting algorithm. According to Fig. 4, the use of the 2-D array provides less error in DOA estimation compared with 1-D array.

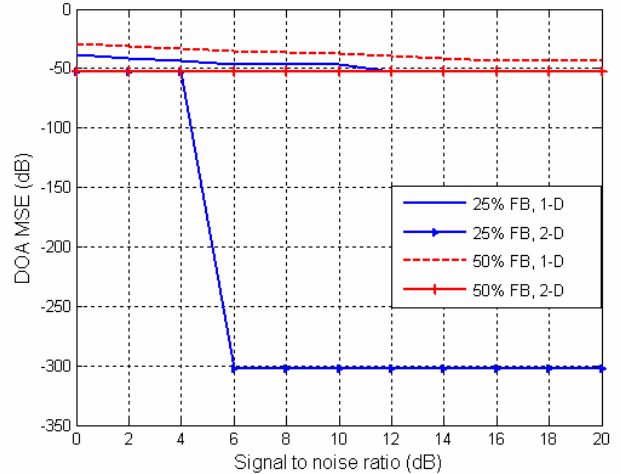


Fig. 5: DOA MSE (dB) versus signal-to-noise ratio (dB) for, $\phi = 30^\circ$, FB = 25% and 50%, interpolated sector $[0^\circ \ 60^\circ]$, $N=100$, $N_s=400$, 50 experiments.

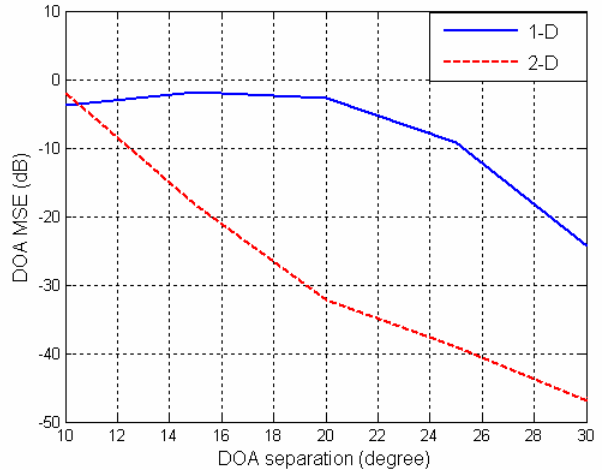


Fig. 6: DOA MSE (dB) versus DOA separation (degree) for FB = 50%, SNR = 10dB, $\phi = 0$, interpolated sector $[0^\circ \ 60^\circ]$, $N=100$, $N_s=400$, 50 experiments.

The reason for this improvement can be due to the fact that the 2-D array uses more antenna elements to perform DOA estimation.

5.3 Effect of Signal-to-Noise Ratio

Fig. 5 shows the error in DOA estimation (MSE) as a function of SNR. As can be seen in Fig.5, for FB of 50% the use of 2-D array provides less error than 1-D array. This is especially true for low values of SNR. The advantage of the 2-D array becomes more pronounced for the signals having FB=25%.

5.4 Effect of angle separation

In this simulation, two correlated and coherent signals were generated with different DOA. Fig. 6 shows an error in DOA detection when the angle separation between two

sources is varied. It can be seen that the 2-D array and the 1-D array offer nearly the same capability of detecting signal direction when the separation is small i.e. 10° . As the separation angle increases, the 2-D array shows lower MSE compared with the 1-D array.

By summarizing the findings obtained with respect to the four investigated issues 5.1–5.4, it can be stated that DOA estimation method of a wideband signal based on the interpolated technique produces superior results when the 2-D antenna array of [10] instead of the 1-D array [18], [19] is used.

6. Conclusion

In this paper, a method of DOA estimation of a wideband signal based on the interpolated array technique in conjunction with the MUSIC algorithm has been investigated. The performance of this method has been assessed for two- and one-dimensional wideband arrays. MATLAB simulations have been used as the assessment platform. The simulation results have shown that the 2-D spatial beamformer introduced in [7]–[10] offers better quality DOA estimation of a wideband signal than its 1-D counterpart part described in [18], [19].

The presented DOA algorithm completes the design of a wideband smart antenna with only spatial signal processing. By combining the work presented in [10] and the one described in this paper, the smart antenna designer is able to estimate DOA of a wideband signal arriving from any direction in azimuth and generate a desired wideband radiation pattern by using the two-dimensional array introduced in [10].

7. Acknowledgment

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8. References

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