Phase Jitter Modeling for Uplink OFDMA

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Abstract

Multicarrier (MC) systems, such as OFDMA (Orthogonal Frequency Division Multiple Access), which are quite developed in the literature, were shown to be the adequate solution for multipath transmission. However, even with these systems, synchronization errors can't be avoided. In this contribution we study the sensitivity of uplink OFDMA system to the phase jitter for both models proposed : the gaussian and the rayleigh models. By investigating the performance degradation of the OFDMA system, we point out that, first, the degradation depends, not only, on the phase jitter variance, but also, on the jitter power density function, and then, the rayleigh model gives better performance regarding the SNR degradation.

1 Introduction

In this paper, we deal with MC systems and especially with OFDMA (Orthogonal Frequency Division Multiple Access) system. In fact, like all MC systems, this technique of modulation, offers high data rates and an immunity to channel dispersion. Besides that, OFDMA technique has received considerable attention since it is more robust against fading and interferences than single carrier technique.

In OFDMA system, the data streams, transmitted on the different carriers, belong to different users. In the case of downlink OFDMA, the signal transmitted to the different users are synchronized at the basestation. In addition, all transmitted carriers are upconverted with the same carrier oscillator so that all users have the same carrier phase error. However, in uplink OFDMA system, the transmitters are on different locations. Each transmitter generates his own carrier oscillator signal that has phases estimated by means of a Phase Locked Loop (PLL). At the basestation, the sum of the received signals is downconverted using a network synchronization signal. This implies that different users exhibit different synchronization errors. Hence, performance of uplink OFDMA system depends on the carrier phase jitter or carrier phase error.

This paper introduces new results for an uplink OFDMA by modeling the phase jitter with gaussian and rayleigh models. We will compare the effect of these models on system performance to the effect of the random process model proposed in [1], [3] and [4]. The outline of the paper is as follows : section 2 gives a description of the uplink OFDMA system in presence of phase jitter. Different interference terms are pointed out. Section 3 proposes gaussian and rayleigh models for phase jitter. It gives SNR degradation expressions for both models. Simulations and results are described in section 4.

2 Uplink OFDMA system description

We can distinguish mainly two levels of synchronization errors: carrier synchronization and timing synchronization. Several synchronization errors are studied in the literature. We restrict our analysis here on the carrier phase jitter.

MC systems were shown to be very sensitive to carrier phase jitter and depend only on the jitter variance [4]. OFDMA, one of these MC systems, has been proposed as an access technique for the return path in a CATV (Communication Area Television) network [2]. In OFDMA scenario, where the basesattion broadcasts a network synchronization signal, each user estimates his sinusoidal carrier from this received synchronization signal by means of Phase Locked Loop. And then the same synchronization signal is used for the downconversion at the receiver. Since the users in uplink OFDMA are in deferent locations, each transmitted signal is affected by a phase jitter. The conceptual block diagram of the uplink OFDMA is shown in figure 1. For each user, the data symbols to be transmitted to the basestation are organized into $(N_F + \nu)$ blocks where ν is the length of the cyclic prefix. Let $a_{i,n}$ denotes the n^{th} data symbol of the i^{th} block. Feeding these symbols to an inverse fast fourier transform (IFFT), we obtain the samples $s_{i,n,l}$, transmitted by user l, given by :

$$s_{i,n,l} = \frac{1}{\sqrt{N_F + \nu}} a_{i,n} e^{j2\pi \frac{nl}{N_F}} \tag{1}$$

The transmitted sequence $s_{i,n,l}$ applied to the transmit filter $p(t)$ yielding the signal $s_l(t)$ given by :

$$s_l(t) = \sum_{i=-\infty}^{+\infty} \sum_{n=-\nu}^{N_F-1} s_{i,n,l} p(t - (i(N_F + \nu) + n)T - \tau_{c,l}) \tag{2}$$

where 1/T is the symbol rate per carrier and $\tau_{c,l}$ is the time delay corresponding to the phase of the transmitter.

The output signals of the dispersive channel are summed and disturbed by additive white gaussian noise $w_{LP}(t)$. the resulting signal is affected by the carrier phase error $\phi_l(t)$ given by:

$$\phi_{\mathbf{l}}(t) = \theta_{c,l}(t) - \theta_r \tag{3}$$

where θ_r is the phase carrier oscillator of the basestation and $\theta_{c,l}(t)$ is the transmitter carrier phase.

At the basestation, the signal is applied to the receiver filter and sampled to give the $v_{i,k}$ samples.

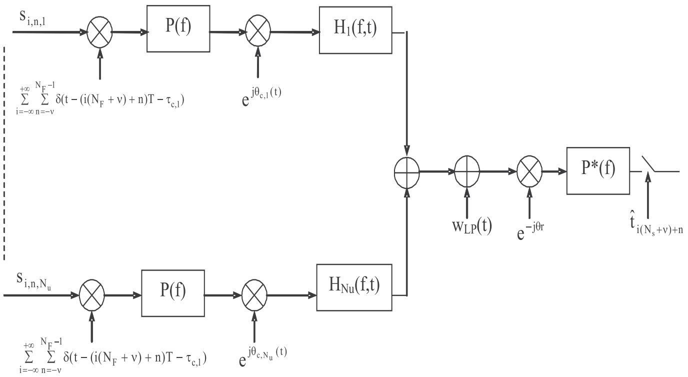

Figure 1: The conceptual block diagram of an uplink OFDMA

Keeping the N_F samples outside the cyclic prefix, the output samples are fed to the FFT and then to an equalizer with coefficients $g_{i,l}$. As a result, the samples $z_{i,l}$ are given by :

$$z_{i,l} = \sqrt{\frac{N_F}{N_F + \nu}} \sum_{l'=0}^{N_u-1} \sum_{i'=-\infty}^{+\infty} a_{i,i',l,l'} I_{i,i',l,l'} + w_{i,l} \tag{4}$$

where $w_{i,l}$ is the term referred to the AWGN (Additional White Gaussian Noise) and $I_{i,i',l,l'}$ is given by :

$$I_{i,i',l,l'} = \frac{1}{N_F} g_{i,l} \delta_{i,i'} \sum_{k=0}^{N_F-1} e^{-j2\pi \frac{k(l-l')}{N_F}} e^{-j\phi_l(t)} \tag{5}$$

The quantity $I_{i,i',l,l'}$ denotes the contribution of the symbol $a_{i',l'}$ on the l^{th} FFT output during the i^{th} block. The samples $z_{i,l}$ can be decomposed into four contributions : an average useful component $I_{i,i,l,l}$, a zero mean useful component $I_{i,i,l,l} - E(I_{i,i,l,l})$ or self interference, an intersymbol interference ($i' \neq i$ and $l' = l$) and an intercarrier or interuser interference ($l' \neq l$).

In order to evaluate the system degradation of the uplink OFDMA system described above, we will analyze in the next section the signal to noise ratio (SNR) using different interference powers.

3 Performance degradation

we investigate here the SNR degradation of the uplink OFDMA system when the carrier phase jitter is present. To clearly isolate the effect of synchronization phase error, we consider the case of an ideal channel.
The SNR is defined as the ratio of the power of the average useful component P_U to the sum of the powers of the self interference P_{SI}, the intersymbol interference P_{ISI}, the intercarrier (or interuser) interference P_{IUI} and the noise AWGN. This yiels :

$$SNR(\phi_l) = \frac{\frac{N_F}{N_F+\nu} E_{s,l} P_{u,l}}{N_0 + \frac{N_F}{N_F+\nu} E_{s,l}(P_{SI,l} + P_{ISI,l} + P_{IUI,l})} \tag{6}$$

Where
$E_{s,l}$ is the energy per symbol on the l^{th} carrier
N_0 is the variance of the AWGN given by $N_0 = E(|w_{i,l}|^2)$
and
$P_{u,l} = |E(I_{i,i,l,l})|^2$
$P_{SI,l} = E(|I_{i,i,l,l} - E(I_{i,i,l,l})|^2)$
$P_{ISI,l} = \sum_{i'=-\infty \ et \ i' \neq i}^{+\infty} E(|I_{i,i',l,l}|^2)$
$P_{IUI} = \sum_{l'=0 \ et \ l' \neq l}^{N_u - 1} \frac{E_{s,l'}}{E_{s,l}} E(|I_{i,i,l,l'}|^2)$

In the absence of the phase jitter, the SNR resulting is :

$$SNR(0) = \frac{N_F}{N_F + \nu} \frac{E_{s,l}}{N_0} \tag{7}$$

When phase jitter is present, the SNR is reduced, as compered to the case of the absence of synchronization error phase. The degradation of the SNR due to the phase jitter is given by :

$$\deg_l = -10 \log \frac{SNR(\phi_l)}{SNR(0)} \tag{8}$$

Without loss of generality, by replacing the SNR terms, the equation (8) can be simplified as follows :

$$\deg_l = -10 \log \frac{P_{u,l}}{1 + SNR(0)(P_{SI,l} + P_{ISI,l} + P_{IUI,l})} \tag{9}$$

Mathematically, each average term can be computed using the quantity $E(e^{j\phi})$. For small phase jitter, and when the jitter is assumed to be a stationary process, $\phi_l(t)$ can be reduced to $\phi_l(t) = \phi$.
At this point we have a relationship :

$$E(e^{j\phi}) = \int_0^{2\pi} p(\phi) e^{j\phi} d\phi \tag{10}$$

where $p(\phi)$ is the jitter probability law.
In [1], ϕ was modeled as a random process which is slowly varying. There was shown that the degradation of the uplink OFDMA system depends only on the jitter variance.
In this paper, we suppose that the phase jitter can follow a gaussian model or a rayleigh model. If the phase jitter ϕ has a gaussian probability law with variance σ_ϕ^2, then $p(\phi)$ is :

$$p(\phi) = \frac{1}{\sigma_\phi \sqrt{2\pi}} \exp(-\frac{\phi_l^2}{2\sigma_\phi^2}) \tag{11}$$

If we, now, simplify equation (10), then this quantity becomes:

$$E(e^{j\phi}) = \frac{e^{-\frac{\sigma_\phi^2}{2}}}{\sqrt{\pi}} \int_{-j\frac{\sigma_\phi}{\sqrt{2}}}^{\frac{\pi\sqrt{2}}{\sigma_\phi} - j\frac{\sigma_\phi}{\sqrt{2}}} e^{-x^2} dx \tag{12}$$

There is no classical method to calculate the expression above. Hence we can hardly found the exact value of the integral in equation (12). An approximate method of calculation is been used, based on trapezoids surface approximation. Let A be the approximate value of the integral. Thus, the degradation expression (9)is reduced to :

$$\deg = -10 \log \frac{|A|^2 \frac{e^{-\sigma_\phi^2}}{\pi}}{1 + SNR(0)[2 - \frac{e^{-\sigma_\phi^2}}{\pi} |A|^2]} \tag{13}$$

Looking at the expression (13), we see that, with small jitter, performance degradation of the uplink OFDMA system depends only on the phase jitter variance and is the same for all users.
Another approach can be presented to model the phase jitter. By choosing the rayleigh law instead of the gaussian law, identical theory results can be found. The probability of the rayleigh law is given by :

$$p(\phi_l) = \frac{\phi_l}{\sigma_\phi^2} \exp(-\frac{\phi_l^2}{2\sigma_\phi^2}) \tag{14}$$

As a consequence the quantity $E(e^{j\phi})$, denoted B, can be written :

$$E(e^{j\phi}) = B = \int_0^{2\pi} \frac{\phi_l}{\sigma_\phi^2} e^{-\frac{\phi_l^2}{2\sigma_\phi^2}+j\phi_l} d\phi \qquad (15)$$

Similarly, using an approximate method to calculate the integral quantity, we found the same degradation expression given in (13) function of B instead of A. This is means that the performance degradation depends on the phase jitter probability law.

4 Results and interpretations

Figure 2 shows simulation result for the gaussian model with different SNR(0) values. In this figure, it is pointed out that the uplink OFDMA degradation is not only closely related to the jitter phase variance but also increases with increasing SNR(0).
Additional result is given in figure 3. In fact, in this figure, we compare the performance degradation of the gaussian model to the random model which has no specifications on the law followed by the phase jitter. The gaussian model performs better than the random model since it gives minimum degradation. In other words, OFDMA system is more stable and performing with a known behavior of the phase jitter.
Figure 4 focuses on comparison of three models for the phase jitter : gaussian model, rayleigh model and random model. It is shown that better uplink OFDMA performance is given with rayleigh model. This is can be justified by the fact that in mobile communication, we deal with multipath channel. This channel is in the most cases a rayleigh channel. The jitter phase is one of the synchronization errors that may occur due to the dispersive channel. Thus, it is natural that the rayleigh model performed better than the other models.

5 Conclusion

In this contribution we study the sensitivity of the uplink OFDMA system, in mobile communication, to the phase jitter. This jitter caused by no synchronization between transmitter and receiver, is modeled here by gaussian and rayleigh models. For this aim, we study the performance degradation of OFDMA system. We point out that, first the degradation depends not only on phase jitter variance but also on the jitter power density function, and then the rayleigh model gives better performance regarding the SNR degradation.

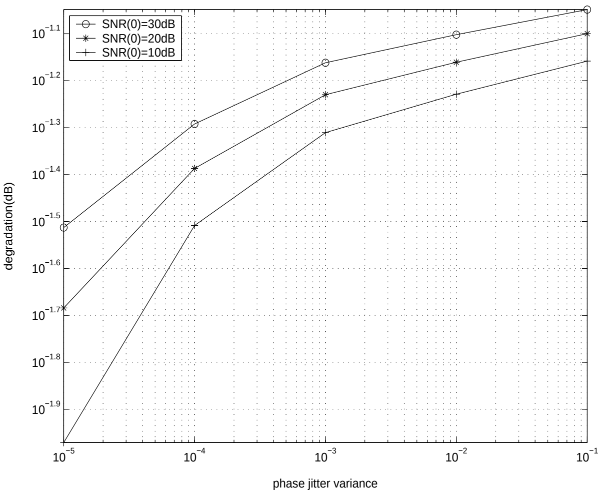

Figure 2: Simulation result for the gaussian model with different SNR(0) values

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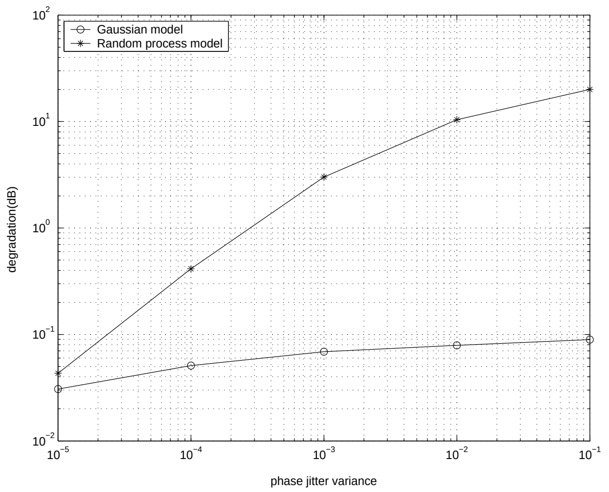

Figure 3: Performance degradation for the gaussian and the random models

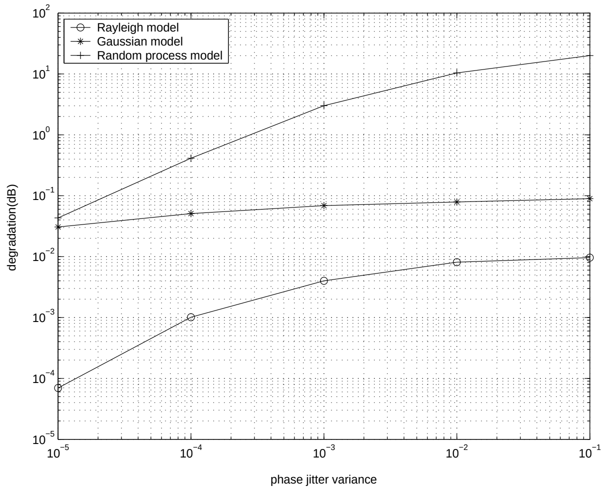

Figure 4: Comparison of three models for the jitter phase: gaussian model, rayleigh model and random model

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