A Lumpable Finite-State Markov Model for Channel Prediction and Resource Allocation in OFDMA Systems

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Abstract— **This paper presents novel closed-form solutions to the sub-channel and power allocation problems of an orthogonal frequency division multiple access (OFDMA) system. We model the Rayleigh fading channel as a finite-state Markov channel (FSMC) by partitioning the received signal-to-noise ratio (SNR) into several intervals. We use the sub-band formation and lumpability to reduce the size of channel state information (CSI) and to reliably predict the CSI with the corresponding state transition and steady-state probabilities. Simulation results show that the limited feedback scheme due to lumpable FSMC is not only experiencing less prediction error than the typical full feedback scheme but also achieving near-optimum capacity.**

I. Introduction

Orthogonal frequency division multiple access (OFDMA) is a multi-user OFDM in which each user is assigned a subset of sub-carriers for use, and each sub-carrier is assigned exclusively to one user. The entire bandwidth is shared by multiple users and this allows them to transmit simultaneously. Adaptive modulation and resource allocation are the common techniques for significantly improving spectral efficiency in an OFDMA system. With the knowledge of perfect channel state information (CSI) at both the transmitter and receiver, the Shannon's capacity of a fading channel can be achieved using optimal adaptation of control variables such as transmit power, data rate, coding rate or sub-carrier sharing factor [1]–[3]. Although the feedback of CSI helps to achieve higher spectral efficiency, a lot of resources is needed to convey the exact CSI of each sub-carrier from receiver to transmitter. In this paper, we propose to model the time-varying Rayleigh fading channel in an OFDMA system as a finite-state Markov channel (FSMC). By partitioning the received signal-to-noise ratio (SNR) into several intervals, the required feedback to the transmitter can be reduced to a quantized vector that carries the instantaneous states of the current channel. The corresponding state transition and steady-state probabilities are used to predict CSI for the next transmission cycle.

The study of FSMC was initiated by Gilbert [4] and Elliott [5] with a two-state Markov channel, which is inadequate to describe the channel quality that varies dramatically. Numerous researchers [6]–[8] utilized the idea of FSMC to partition the range of received SNR into a finite number of intervals, where each interval forms the state of the Markov chain. By modeling the fading channel of an OFDMA system as FSMC, its state transition matrix size grows exponentially as the number of sub-carriers increases. The computational load has been huge. To reduce the load, a two-step method is proposed in this paper to reduce the size of the feedback information. It involves the formation of sub-band and lumpable FSMC that reduces the expanded Markov channel to multiple smaller Markov channels while maintaining similar behaviour. By integrating the lumpable FSMC with a channel prediction scheme, we can predict the states of sub-bands ahead for each transmission. We then formulate capacity optimization problems in the basis of sub-channel and power allocation, and present the closed-form expressions. With these expressions, we investigate the reliability of the proposed channel prediction scheme with limited feedback based on the lumpable FSMC.

This paper is organized as follows. We formulate the optimal resource allocation problem in OFDMA system in Section II. The Markov model and the expanded Markov model are outlined in Section III, followed by the implementation of FSMC in capacity optimization problems with channel prediction in Section IV. Simulation results and conclusions are then presented in Sections V and VI respectively.

II. Optimal Resource Allocation in OFDMA

Consider an OFDMA system of K users and N sub-carriers with a time-varying, frequency-selective fading channel. Assuming that the sub-carrier separation is smaller than the coherent bandwidth, each sub-carrier can be considered as a flat fading sub-channel. Assume unity average transmission power, the downlink received signal is modeled as $\mathbf{Y} = \mathbf{HX} + \mathbf{n}$, where $\mathbf{H}$ is the channel matrix, $\mathbf{X}$ and $\mathbf{Y}$ are the transmitted and received signals, respectively, and $\mathbf{n}$ is the additive white Gaussian noise (AWGN). For an arbitrary user k, where $k \in [1, K]$, its channel vector, $\mathbf{h}_k = [h_{k,1}, \dots, h_{k,n}, \dots, h_{k,N}]$, is extracted from the channel matrix $\mathbf{H}$ and the SNR for the n^{th} sub-channel is expressed as $\gamma_{k,n} = |h_{k,n}|^2/\sigma_{k,n}^2$, where $\sigma_{k,n}^2$ is the noise variance of AWGN.

A. Sub-channel Allocation

In a single-user environment, an optimal sub-channel allocation can be obtained by allowing transmission at any particular sub-channel which is experiencing the least fading. However, sub-channel allocation often poses a more difficult problem in a multi-user environment. One form of sub-channel allocation problem was solved in [1] of which the goal of the optimization is to minimize the transmission power for a given transmission rate. However, no closed-form solution was obtained. In order to find the optimal assignment, we modify the goal to maximize the achievable capacity. Note that, at this stage we have not considered power allocation yet. The primal objective of this problem can be written as Problem P1:

$$\begin{aligned} \max_{\beta_{k,n}} \quad & \sum_{k=1}^{K}\sum_{n=1}^{N} \frac{B}{N}\log_2\left(1+\beta_{k,n}\gamma_{k,n}\right) \\ \text{s.t.} \quad & \sum_{k=1}^{K}\beta_{k,n} \leq 1, \quad \sum_{k=1}^{K}\sum_{n=1}^{N}\beta_{k,n} \leq N, \end{aligned} \tag{1}$$

where $\gamma_{k,n}$ is the SNR of user k at sub-channel n and $\beta_{k,n}$ is defined as the sharing factor of sub-carrier n. With Lagrangian method and the property of duality [9], the optimal solution to Problem P1 is given as

$$\beta_{k,n}^{*} = \frac{\sum_{i=1}^{K}\frac{1}{\gamma_{i,n}}+1}{K} - \frac{1}{\gamma_{k,n}}, \tag{2}$$

in which the following Karush-Kuhn-Tucker (KKT) conditions can be satisfied:

$$\begin{aligned} \sum_{k=1}^{K}\left(\beta_{k,n}^{*}\right)-1 \quad &\leq \quad 0\ ; \\ \sum_{k=1}^{K}\sum_{n=1}^{N}\left(\beta_{k,n}^{*}\right)-N \quad &\leq \quad 0\ ; \\ \frac{\partial}{\partial\beta_{k,n}}L_P\left(\beta_{k,n}^{*},\lambda_n^{*},\mu^{*}\right) \quad &= \quad 0\ ; \\ \lambda_n^{*}\left[\sum_{k=1}^{K}\left(\beta_{k,n}^{*}\right)-1\right] \quad &= \quad 0\ . \end{aligned}$$

The solution presented in (2) is the optimal solution, however, $\beta_{k,n}$ takes values within the interval of $\left[-(1-\frac{2}{K}),1-\frac{2}{K}\right]$. In reality, sub-carrier sharing is only possible if extra coding scheme is implemented across sub-carriers. To sustain the fact that no sub-carrier sharing is allowed among users in each transmission, $\beta_{k,n}$ should strictly take the value of either 0 or 1 to indicate that the sub-carrier is not in used or in used, respectively. This particular problem involves some fairness issues because it is likely that some sub-channels may be equally good for multiple users at one time instant. The optimal solution in (2) would produce equally likely portion to the relevant users but our goal is to allocate that particular sub-channel to only one user out of all the contenders. Note that user prioritization or fairness is not the main emphasis of this paper, we will not deal with this further.

Proposition 1: For an arbitrary sub-carrier n, only one user is allowed for transmission at that sub-carrier. If more than one user is assigned to the same sharing factor for sub-carrier n, any user which enters the system earlier (user with lower notation value) will be favourable. It can be expressed as:

$$\beta_{k,n}^{*} = \begin{cases} 1, & \text{if } \beta_{k,n}^{*} > \beta_{i,n}^{*}, \forall i \neq k \text{ and } \beta_{k,n}^{*} = \beta_{j,n}^{*}, \forall j > k; \\ 0, & \text{otherwise.} \end{cases} \qquad \blacksquare$$

Hence, Proposition 1 presents the closed-form solution of Problem P1, which satisfies KKT conditions whilst meeting the requirement of only occupying transmission of one user at each sub-carrier.

B. Power Allocation

Based on Problem P1, we extend the problem to maximize the sum-capacity while maintaining the power distribution under a limited power budget. Similar problem with power budget of individual user is considered in [2]. Due to the implementation of sub-channel allocation scheme, it will only be fair if we consider the total power budget of the system. Thus, the problem is posed as a constrained optimization problem, defining by a primal objective as Problem P2:

$$\begin{aligned} \max_{P_{k,n}} \quad & \sum_{k=1}^{K}\sum_{n=1}^{N} \frac{B}{N}\log_2\left(1+P_{k,n}\beta_{k,n}\gamma_{k,n}\right) \\ \text{s.t.} \quad & \sum_{k=1}^{K}\sum_{n=1}^{N}\beta_{k,n}P_{k,n} \leq P^{\max},\ P_{k,n} \geq 0, \end{aligned} \tag{3}$$

where $P^{\max}$ is the total power budget and $\beta_{k,n}$ is the sharing factor of sub-carrier n which only takes the value of 0 or 1. With Lagrangian method and the property of duality [9], the closed-form solution to Problem P2 is

$$P_{k,n}^{*} = \frac{P^{\max}+\sum_{i=1}^{K}\sum_{j=1,\beta_{i,j}=1}^{N}\frac{1}{\gamma_{i,j}}}{N} - \frac{1}{\beta_{k,n}\gamma_{k,n}}, \tag{4}$$

in which the following KKT conditions can be satisfied:

$$\begin{aligned} \sum_{k=1}^{K}\sum_{n=1}^{N}\left(\beta_{k,n}P_{k,n}^{*}\right)-P^{\max} \quad &\leq \quad 0\ ; \\ \lambda^{*} \quad &\geq \quad 0\ ; \\ \lambda^{*}\left[\sum_{k=1}^{K}\sum_{n=1}^{N}\left(\beta_{k,n}P_{k,n}^{*}\right)-P^{\max}\right] \quad &= \quad 0\ ; \\ \frac{\partial}{\partial P_{k,n}}L\left(P_{k,n}^{*},\lambda^{*}\right) \quad &= \quad 0\ . \end{aligned}$$

The solution presented in (4) is a sub-optimal solution to Problem P2 since the positivity constraint of $P_{k,n}^{*}$ may be violated under certain extreme circumstances. Thus, we need to assign zero power for all violated $P_{k,n}^{*}$ and adjust the remaining terms so that the total power budget remains unchanged.

Proposition 2: For an arbitrary sub-carrier n, the corresponding transmission power for user k of $P_{k,n}^{*}$ that violates the positivity constraint is suppressed to 0; whilst other non-negative counterparts are reallocated. It can be expressed as:

$$P_{k,n}^{*} = \begin{cases} \dfrac{P_{k,n}^{*}P^{\max}}{\sum_{i=1}^{K}\sum_{j=1,P_{i,j}^{*}>0}^{N}P_{i,j}^{*}}, & \text{if } \mathrm{P_{k,n}^{*}} > 0 \text{ and } \beta_{\mathrm{k,n}}^{*} = 1; \\ 0, & \text{otherwise.} \end{cases} \qquad \blacksquare$$

Hence, Proposition 2 presents the closed-form solution of Problem P2, which satisfies KKT conditions whilst not violating the positivity constraint.

III. Finite-State Markov Channel

A. Markov Model

Let $\mathcal{S} = \{s_1, s_2, \ldots, s_M\}$ denote a set of M states and $\{S_t\}$, $t = 0, 1, \ldots$, be a constant Markov process, which has stationary transitions [6]. The illustration of the M-state Markov chain is shown in Fig. 1. Let π_i be the steady-state probability and a_{ij} the state transition probability, which are given as

$$\pi_i = \Pr(S_t = s_i), \tag{5}$$

$$a_{i,j} = \begin{cases} \Pr(S_{t+1} = s_j \mid S_t = s_i), & \text{for } |i-j| \leq 1; \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

respectively, for all $i, j \in \{0, 1, \ldots, M-1\}$. With (5), we can define an $(M \times M)$ transition matrix, $\mathbf{A}$ and an $(M \times 1)$ steady-state probability vector, $\boldsymbol{\pi}$, with the properties that the sum of the elements on each row of $\mathbf{A}$ equals to 1, i.e. $\sum_{j=0}^{M-1} a_{i,j} = 1$, and the sum of all the elements in $\boldsymbol{\pi}$ equals to 1, i.e. $\sum_{i=0}^{M-1} \pi_i = 1$. In a typical multipath propagation channel, the received signal envelope can be modeled using a Rayleigh distribution. Let γ denotes the received SNR which is proportional to the square of the signal envelope. One can show that γ is exponentially distributed [10] with its probability density function as

$$p(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right), \tag{7}$$

where $\bar{\gamma}$ is both the mean and standard deviation of γ. The expected number of times per second (also known as *level crossing rate*) the received SNR γ passes downward across a quantized level γ_m is given by

$$N_m = \sqrt{\frac{2\pi\gamma_m}{\bar{\gamma}}} f_d \exp\left(-\frac{\gamma_m}{\bar{\gamma}}\right), \tag{8}$$

where f_d is the maximum Doppler frequency, which is defined as $f_d = v f_c / c$, for v/c is the ratio of moving speed of mobile terminal to speed of light and f_c is the carrier frequency. With the thresholds of the quantized SNR levels define as $0 = \gamma_0 < \gamma_1 < \ldots < \gamma_{M-1} = \infty$, the Rayleigh fading channel is said to be in state s_m if the received SNR is within the interval of $[\gamma_m, \gamma_{m+1})$. With the exponential distributed SNR, the steady-state probability for each state is given as

$$\pi_m = \int_{\gamma_m}^{\gamma_{m+1}} \frac{1}{\bar{\gamma}} \exp\left(-\frac{u}{\bar{\gamma}}\right). \tag{9}$$

The transition probabilities $a_{m,m+1}$ and $a_{m,m-1}$ can be approximated by

$$a_{m,m+1} \approx \frac{N_{m+1} T}{\pi_m}, \quad m = 0, 1, \ldots, M-2 \tag{10}$$

$$a_{m,m-1} \approx \frac{N_m T}{\pi_m}, \quad m = 1, 2, \ldots, M-1 \tag{11}$$

where T represents the time interval for each transmission over the channel, i.e. symbol duration. Other transition probabilities are given by

$$a_{m,m} = \begin{cases} 1 - a_{m,m+1}, & \text{for } m = 0; \\ 1 - a_{m,m-1}, & \text{for } m = M-1; \\ 1 - a_{m,m-1} - a_{m,m+1}, & \text{otherwise.} \end{cases}$$

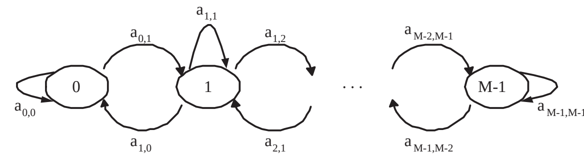

Fig. 1. Illustration of *M*-state Markov chain.

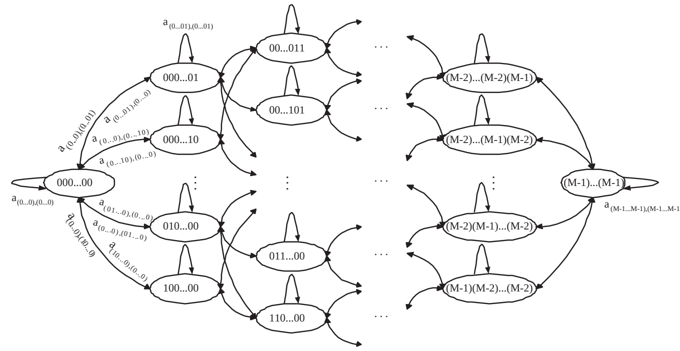

Fig. 2. Illustration of M^N-state Markov chain where each state comprises a sequence of N components.

B. Expanded Markov Channel

For an arbitrary user k of an OFDMA system, the SNR at sub-channel n is expressed as $\gamma_{k.n} = |h_{k,n}|^2 / \sigma_{k,n}^2$ and modeled as an M-state Markov chain where $\gamma_{k,n} \in \{0, 1, \ldots, M-1\}$ indicates the quantized SNR level. Assume that at each time instant, only one sub-channel shall vary no more than one quantized level at either direction. By expanded process, each state is formed with all N consecutive conditions in the channel matrix. An M^N-state Markov channel is formed and shown in Fig. 2. Now consider the Markov chain with states $\mathbf{s}_0 = (00\ldots0), \mathbf{s}_1 = (0\ldots01), \ldots, \mathbf{s}_{M^N-1} = (M-1\ldots M-1)$, the steady-state probability, $\pi_{\mathbf{s}_i}$, and the state transition probability, $a_{\mathbf{s}_i,\mathbf{s}_j}$, are defined as

$$\pi_{\mathbf{s}_i} = \Pr(S_t = \mathbf{s}_i), \tag{12}$$

$$a_{\mathbf{s}_i,\mathbf{s}_j} = \begin{cases} \Pr(S_{t+1} = \mathbf{s}_j \,|\, S_t = \mathbf{s}_i), & \text{for } |\mathbf{s}_i - \mathbf{s}_j| \leq 1; \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

The transition from state $\mathbf{s}_i$ to state $\mathbf{s}_j$, which originally occurs in N successive steps with the original chain, is restricted to one step transition such that only one sub-channel is susceptible to a state transition.

In any practical system, the SNR level can be quantized to at least 2 levels whereas the number of sub-channels of an OFDMA system can go up to 512. Simply relating these figures to M and N, the matrix dimension can approach to $2^{512} \approx 1.34 \times 10^{154}$, which is too big to be practical. We define a two-step method to reduce the size of the matrix in order to realize the channel prediction at transmitter.

C. Formation of Sub-bands

For an OFDMA system with N correlated sub-channels, we propose to gather a small fixed number of, say b, sub-channels into one sub-band. The original N sub-channels model with M^N-state Markov channel can then be transformed into N/b parallel sub-bands, each modeled as an M^b-state Markov channel with $(M^b \times M^b)$ transition matrices, $\mathbf{A}_s^{(z)}$, where $z \in \{1, 2, \ldots, N/b\}$. Since it becomes a parallel problem, the notation of $\mathbf{A}_s$ will be used throughout this paper for simplicity purpose. The M^b-state Markov chain can be illustrated similar to Fig. 2 except that each state only comprises a sequence of b components.

The choice of b is decided by the frequency correlation between sub-channels. Assume that B_c is the coherent bandwidth and B is the signal bandwidth. For a severe frequency-selective fading environment where $B_c \ll B$, all sub-channels are non-correlated, which leads to $b = 1$ because each sub-channel can be treated as an independent channel. As the correlation between sub-channels increases, the choice of b falls within the range of $1 < b < N$. When all the sub-channels are fully correlated, the best choice of b should be N because all sub-channels are now identical and they can be treated as a single channel, i.e. flat fading environment with $B_c \geq B$. Since we consider the channel as time-varying and frequency-selective with partially correlated sub-channels, the appropriate choice of b must be more than 1, much less than N and in the power of 2 (in order to satisfy $N/b \in \mathbb{Z}$).

D. Lumpability

The concept of lumpability of a Markov chain has been previously discussed in [11], [12]. In essence, the property of lumpability means that there is a partition of aggregated states of a Markov chain and yet the behavior of the Markov chain remains in a similar manner as far as the state dynamics and observation statistics are concerned. We first observe the pattern of an M^b-state Markov chain. Then, we present the concept of lumpability to form an aggregated state $\mathbf{L}$ from multiple atomic states of an FSMC and obtain its eventual transition matrix.

Definition 1: Consider an M^b-state Markov chain with states $\mathbf{s}_i = (s_{i1} s_{i2} \cdots s_{ib})$ where each of the sub-states, $s_{ik} \in \{0, 1, \ldots, M-1\}$ for all $i = 0, 1, \ldots, M^b - 1$ and $k = 1, 2, \ldots, b$. All M^b states can then be divided into $Q = [(M-1)b + 1]$ lumpable partitions, of which the q^{th} partition is defined as

$$\mathbf{L}_q = \left\{ \mathbf{s}_i \left| \sum_{k=1}^{b} s_{ik} = q, \forall i = 0, 1, \ldots, M^b - 1 \right. \right\}, \quad (14)$$

where $q = 0, 1, \ldots, Q-1$.

Definition 2: For the partitions, $\mathbf{L} = \{\mathbf{L}_0, \mathbf{L}_1, \ldots, \mathbf{L}_{Q-1}\}$, assume that the chain before lumping has $R = M^b$ states and after lumping has $Q = [(M-1)b + 1]$ states. Let $\mathbf{U}$ be the $(Q \times R)$ matrix whose i^{th} row is the probability vector having equal components for states in $\mathbf{L}_i$ and 0 for the remaining states. Let $\mathbf{V}$ be the $(R \times Q)$ matrix with the j^{th} column is a vector with 1's in the components corresponding to states in $\mathbf{L}_j$ and 0's otherwise. Given that the transition matrix of the M^b-state Markov chain is $\mathbf{A}_s$, the lumped transition matrix [11] is defined as

$$\mathbf{A}_l = \mathbf{U}\mathbf{A}_s\mathbf{V}, \quad (15)$$

where $\mathbf{A}_l = [a_{\mathbf{L}_x, \mathbf{L}_y}]$, for $x, y = 0, 1, \ldots, Q-1$.

Definition 3: Given that the steady-state vector of the M^b-state Markov chain is $\boldsymbol{\pi}_s$, the lumped steady-state probability vector is defined as

$$\boldsymbol{\pi}_l = \boldsymbol{\pi}_s \mathbf{V}, \quad (16)$$

where $\boldsymbol{\pi}_l = [\pi_{\mathbf{L}_q}]$, for $q = 0, 1, \ldots, Q-1$.

With Definitions 1-3, an M^b-state Markov channel is reduced to a smaller size by lumping some states among the M^b states to form a new $[(M-1)b + 1]$-state Markov channel.

IV. Resource Allocation with Predicted Channel

To utilize the proposal of FSMC, the term $\gamma_{k,n}$ in (1) of Problem P1 and (3) of Problem P2 can be replaced by the predicted channel state. Assume that a receiver can estimate the received channel conditions perfectly and the received SNR is used to estimate the average SNR for the time-varying channel. Suppose that the OFDMA system has K users with N sub-carriers and b sub-channels are accommodated into one sub-band. An M^b-state Markov channel can be reduced to $[(M-1)b+1]$-state Markov channel, and hence it reduces the size of the feedback information. Let $\mathbf{S}_t = \mathbf{s}_i$ be the current state of channel condition where $\mathbf{s}_i$ belongs to the partition $\mathbf{L}_q$. At the next transmission time frame, the channel condition is predicted to be $\mathbf{S}_{t+1} = \mathbf{s}_j$, where $\mathbf{s}_j \subset \mathbf{L}_r$, with a probability of

$$\begin{aligned} \Pr(\mathbf{S}_{t+1} = \mathbf{s}_j) &= \Pr(\tilde{\mathbf{S}}_{t+1} = \mathbf{L}_r) \\ &= \Pr(\tilde{\mathbf{S}}_{t+1} = \mathbf{L}_r | \tilde{\mathbf{S}}_t = \mathbf{L}_q) \cdot \Pr(\tilde{\mathbf{S}}_t = \mathbf{L}_q) \\ &= a_{\mathbf{L}_q, \mathbf{L}_r} \pi_{\mathbf{L}_q}. \end{aligned} \quad (17)$$

Since the reduced model of $[(M-1)b + 1]$-state Markov channel is a typical birth-death process, there are not more than three situations at the next transmission time frame, i.e. (i) transit to a set of lumpable states of higher order, (ii) transit to a set of lumpable states of lower order, and (iii) remain in the same set of lumpable states. Among these possible options, the partition $\mathbf{L}_u$ with the highest probability, such that

$$\Pr(\tilde{\mathbf{S}}_{t+1} = \mathbf{L}_u) > \Pr(\tilde{\mathbf{S}}_{t+1} = \mathbf{L}_r), \qquad \forall r \neq u,$$

is the predicted partition that might consist of more than one atomic state. The receiver is not able to retrieve each sub-channel condition of any sub-band. However, the receiver can quantized the sub-band condition in terms of a finite integer within the range of 0 to $(M-1)b$, where 0 indicates all sub-channels within the sub-band are experiencing worst fading and $(M-1)b$ indicates otherwise. In other words, the receiver can feedback less information with N/b finite integers, equivalent to $\frac{N}{b} \log_2 [(M-1)b + 1]$ bits compared to a full feedback scenario with N finite integers, equivalent to $N\log_2(M)$ bits.

For the case of limited feedback with lumpable states, all sub-channel conditions $\gamma_{k,1}, \gamma_{k,2}, \ldots, \gamma_{k,N}$ are grouped into N/b sub-bands. The normalized sub-band condition of user k at sub-band z is represented by the index of its predicted state, $\mathbf{L}_u^{(k,z)}$, which is expressed as

$$\hat{\gamma}_{k,z} = \frac{u^{(k,z)}+1}{(M-1)b+1}\,, \tag{18}$$

where $u^{(k,z)} \in \{0, 1, \ldots, (M-1)b\}$ is the index of the predicted state. Note that the shift of +1 is introduced mainly for avoiding division of zero. The definition of sub-carrier sharing factor, $\beta_{k,n}$, shall now be redefined as sub-band sharing factor, $\beta_{k,z}$. Problem P1 and Problem P2 need to be redefined and the corresponding optimal solutions $\beta^*_{k,z}$ and $P^*_{k,z}$, respectively, can be obtained as

$$\beta^*_{k,z} = \frac{\sum_{i=1}^{K}\frac{1}{\hat{\gamma}_{i,z}}+1}{K} - \frac{1}{\hat{\gamma}_{k,z}}\,, \tag{19}$$

$$P^*_{k,z} = \frac{P^{\max}+\sum_{i=1}^{K}\sum_{j=1,\beta_{i,j}=1}^{N/b}\frac{1}{\hat{\gamma}_{i,j}}}{N/b} - \frac{1}{\beta_{k,z}\hat{\gamma}_{k,z}}. \tag{20}$$

Hence the closed-form expressions in Propositions 1 and 2 are rewritten as

$$\beta^*_{k,z} = \begin{cases} 1, & \text{if } \beta^*_{k,z} > \beta^*_{i,z}, \forall i \neq k,\ \beta^*_{k,z} = \beta^*_{j,z}, \forall j > k; \\ 0, & \text{otherwise.} \end{cases}$$

$$P^*_{k,z} = \begin{cases} \dfrac{P^*_{k,z}P^{\max}}{\sum_{i=1}^{K}\sum_{j=1,P^*_{i,j}>0}^{N/b}P^*_{i,j}}, & \text{if } P^*_{k,z} > 0, \beta^*_{k,z} = 1; \\ 0, & \text{otherwise.} \end{cases}$$

V. Simulation and Discussion

In the simulation, the system is configured to have 512 sub-carriers and a received bandwidth of 5.12MHz, such that the sub-carrier spacing is determined as $\Delta f = 10$kHz. The carrier frequency is taken as $f_c = 2.4$GHz. Assume that the cyclic prefix is 11μs, thus symbol period is defined as $T = \frac{1}{\Delta f} + 11 = 111\mu$s. Throughout this section, the size of sub-band is $b = 4$.

By varying the vehicular speed of mobile users from 10km/h to 100km/h, the channel varies from slow ($f_dT = 0.0025$) to moderate ($f_dT = 0.01$), and to fast ($f_dT = 0.025$) fading channels. The average prediction errors of M^b-state Markov channels for $M = 2, 3, 4$ are illustrated in Fig. 3. As f_dT increases, it is shown that the case with limited feedback experiences up to 3dB lesser in error than the conventional prediction with full feedback. Since FSMC is an equivalent quantization of the real channel, it is understandable that the larger the size of M, the more accurate the real channel is represented by the M-state Markov channel. In the contrary, the smaller the size of M, the less room for error. Hence, it explains the phenomena of higher M experiences slightly higher prediction error.

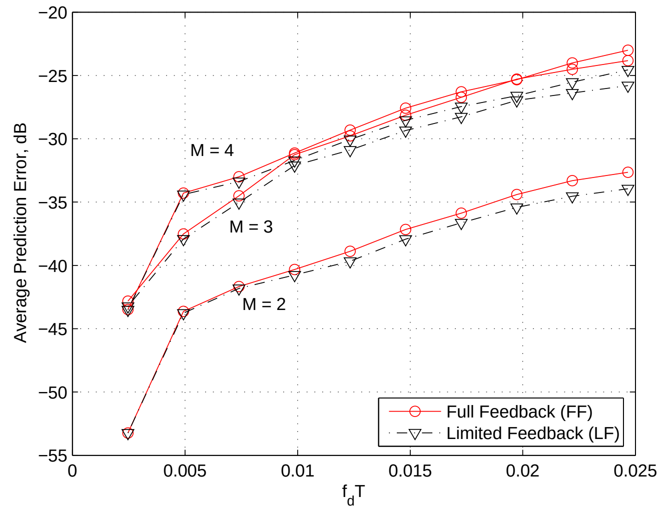

Fig. 3. Prediction error of M^b-state Markov channels for channel variation in terms of f_dT, where the size of sub-band is taken as $b = 4$.

In another simulation, the system is accommodated with 4 users, each with the same configuration as the previous simulation. Given that the system is constrained by a total power budget, each user is allocated with a number of sub-channels and distributed with some power for optimum capacity. With different feedback types, the achievable capacity is computed for slow ($f_dT = 0.0025$), moderate ($f_dT = 0.01$) and fast ($f_dT = 0.025$) fading channels:

- Optimum - Perfect feedback (detailed and exact channel condition).
- Full feedback (FF)- Full feedback of quantized channel condition.
- Limited feedback (LF) - Reduced feedback of quantized channel condition) with predicted partitions of lumpable states.
- Convention (Con) - No feedback and each user is pre-assigned to use a regular set of sub-carriers and average transmission power.

By varying the transmitted SNR, the ratio of the achievable capacities with limited feedback, full feedback and conventional schemes with respect to the optimum achievable capacity of M^b-state Markov channels for $M = 2, 3, 4$ are illustrated in Fig. 4, Fig. 5 and Fig. 6, respectively. These figures show that the conventional schemes often act as the lower benchmark for slow, moderate and fast fading channels. Since slow fading channel experiences less average prediction error, followed by moderate and fast fading channels, it is expected that the system is capable of achieving higher capacity at slow fading channel, followed by moderate and fast fading channels. The limited feedback scheme outperforms the full feedback schemes by being able to achieve higher capacity among slow, moderate and fast fading channels. This is within our expectation because we have shown that the average prediction error for limited feedback scheme is always

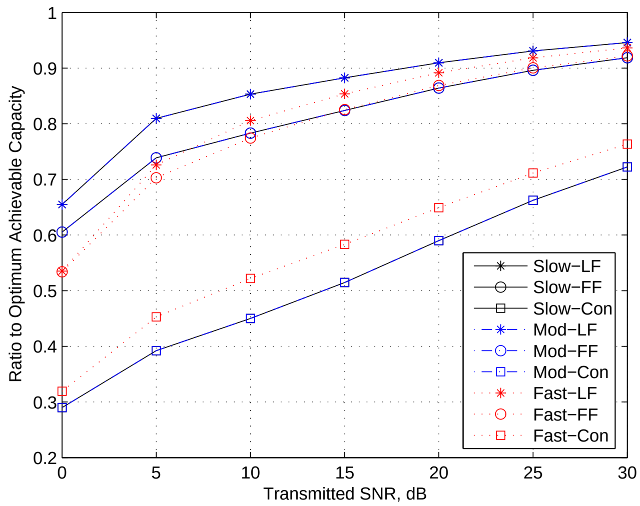

Fig. 4. Ratio of achievable capacities with limited feedback (LF), full feedback (FF) and conventional (Con) schemes with respect to optimum capacity, for slow, moderate and fast fading of 2^b-state Markov channels.

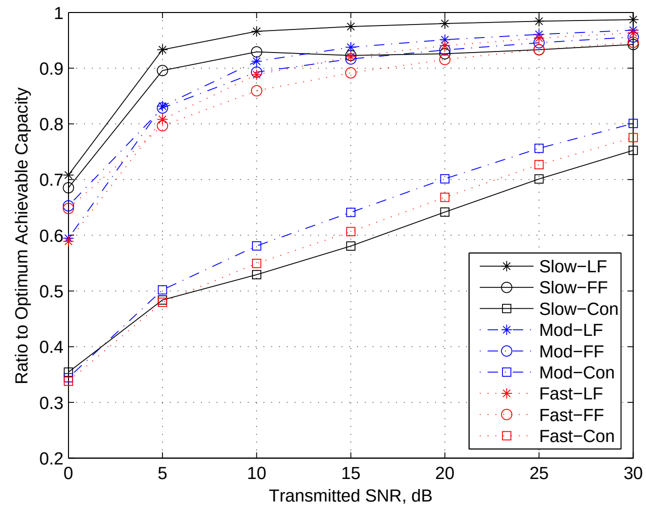

Fig. 6. Ratio of achievable capacities with limited feedback (LF), full feedback (FF) and conventional (Con) schemes with respect to optimum capacity, for slow, moderate and fast fading of 4^b-state Markov channels.

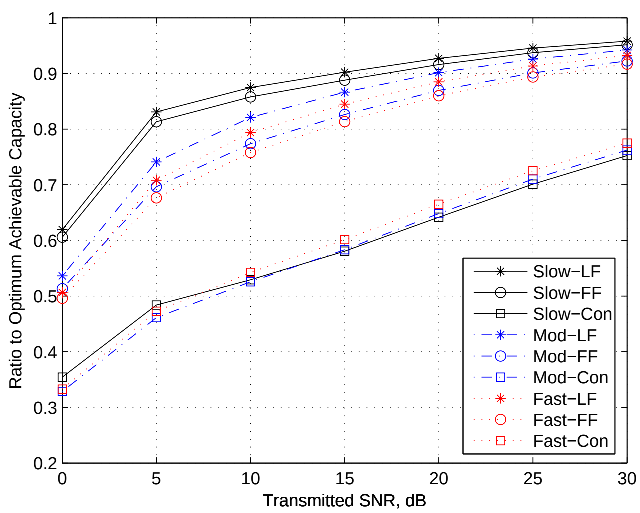

Fig. 5. Ratio of achievable capacities with limited feedback (LF), full feedback (FF) and conventional (Con) schemes with respect to optimum capacity, for slow, moderate and fast fading of 3^b-state Markov channels.

marginal better than full feedback due to less room for error. Hence, it is reasonable that limited feedback scheme is able to exploit the benefit of sub-channel (sub-band) and power allocation schemes slightly better than full feedback scheme.

VI. Conclusion

This paper addresses the problem of huge consumption of resources to feedback exact CSI of multi-carrier in OFDMA system. Conventionally, CSI is constructed with detailed current channel conditions in the form of amplitude or SNR. In this paper, the Rayleigh fading channel of an OFDMA system is modeled as an FSMC by partitioning the received SNR into several quantized levels. With the aid of sub-band formation and lumpability, the size of feedback information is reduced from $N\log_2(M)$ bits to $\frac{N}{b}\cdot\log_2[(M-1)b+1]$ bits, where N, M and b are number of sub-carriers, number of states and size of sub-band, respectively. Simulation results show that the integration of channel prediction with sub-channel (sub-band) and power allocation schemes is able to achieve near-optimum capacity.

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