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Information-Theoretic Approach to Financial Market Modeling

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ABSTRACT

The paper treats the financial market as a communication system, using four information-theoretic assumptions to derive an idealized model with only one parameter. State variables are scalar stationary diffusions. The model maximizes the surprisal of the market and minimizes the Kullback–Leibler divergence between the benchmark-neutral pricing measure and the real-world probability measure. The state variables, their sums, and the growth optimal portfolio of the stocks evolve as squared radial Ornstein-Uhlenbeck processes in respective activity times.

JEL Classification G10, G11

MSC(2010): 62P05, 60G35, 62P20

1 | Introduction

The Fundamental Theorem of Asset Pricing, derived by Delbaen and Schachermayer (1998), forms the basis of modern financial mathematics, as presented, for example, by Jarrow (2022). It states that the widely applied method of risk-neutral pricing is equivalent to the absence of free lunches with vanishing risk (FLVRs). Platen and Fergusson (2025) showed that the absence of FLVRs in real markets is not supported at a high significance level. This prompts reconsideration of financial market models to allow FLVRs and provide a sound, practical pricing method.

The benchmark approach provides a modeling framework where FLVRs can exist; see, for example, Platen (2006) and Platen and Heath (2006). Benchmark-neutral (BN) pricing, introduced in Platen (2025), uses the stocks' growth optimal portfolio (GOP) as numéraire and offers a practical, theory-based pricing approach. Developing related quantitative methods requires a financial market model that captures empirical data and enables precise contingent claim hedging.

The following key stylized empirical facts should be incorporated into any model aiming to accurately reflect financial markets:

1. Stock market index volatilities tend to follow processes with stationary probability densities, as demonstrated in Engle (1982).

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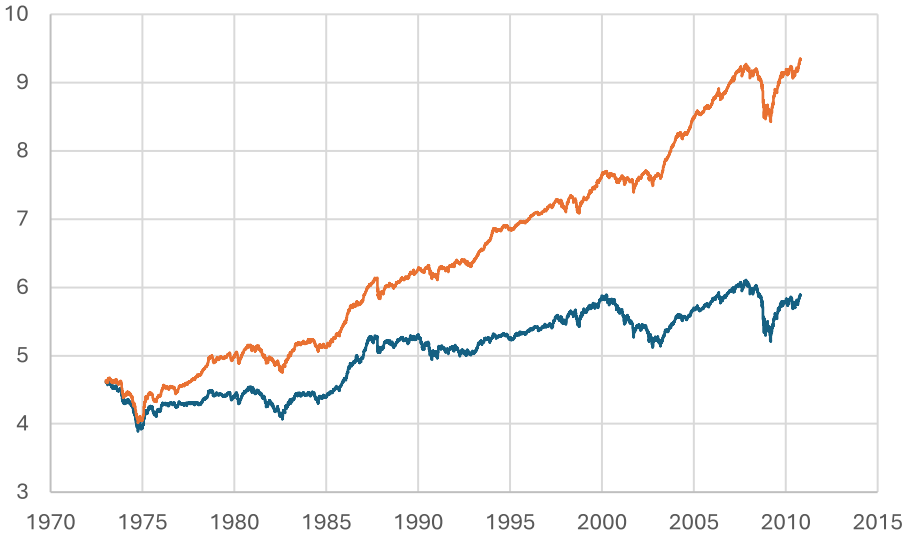


FIGURE 1 | Logarithms of US savings account denominated MSCI (blue) and EWI114 (red). [Color figure can be viewed at wileyonlinelibrary.com]

2. The log-return distributions of stock market indices closely approximate a Student-t distribution with four degrees of freedom across various statistical methodologies, as shown in Markowitz and Usmen (1996a), Markowitz and Usmen (1996b), Hurst and Platen (1997), and Fergusson and Platen (2006).
3. Stock market indices display the leverage effect: declines in the index are associated with increased volatility, and vice versa; see Black (1976b), Ait-Sahalia et al. (2013), and El Euch et al. (2018).
4. Volatility exhibits clustering and rough sample paths, including occasional spikes that differ from typical diffusion processes over calendar time, as discussed by Bayer et al. (2016) and El Euch et al. (2018).
5. Volatility comprises both rapidly and slowly changing components; see Fouque et al. (1999).
6. Log-returns of major indices exhibit scaling properties and self-similarity, as evidenced by Mandelbrot (2001) and Breymann et al. (2009).
7. Over extended periods, the logarithm of world stock market indices demonstrates approximately linear growth with mean-reverting characteristics, exemplified by the logarithm of the MSCI World total return stock index (MSCI) shown in Figure 1.

A " realistic" financial market model, as referenced in this paper, should provide robust explanations for these observed empirical phenomena.

This paper explores a novel interpretation of the financial market, viewing it as a *communication system*. By integrating information-theoretical principles in the modeling, an idealized market model is developed based on four fundamental assumptions. These assumptions serve as the foundation for a rigorous and structured approach to understanding market dynamics. To ensure analytical tractability, the state variables within the market are represented as scalar stationary diffusions. This choice enables a manageable yet robust framework for analyzing the evolution and behavior of these variables over time.

The existence of the GOP is assumed, providing a benchmark for portfolio performance. The GOP of a market is interchangeably called the Kelly portfolio, expected logarithmic utility-maximizing portfolio, or numéraire portfolio (NP); see, for example, Kelly (1956), Merton (1971), Long (1990), Becherer (2001), Platen (2006), Karatzas and Kardaras (2007), Hulley and Schweizer (2010), and MacLean et al. (2011). Kelly (1956) identified the GOP in gambling using Shannon's information theory, marking its discovery.

The expected information content, specifically the surprisal of the stationary densities of the state variables, is maximized. This approach seeks the most efficient encoding of market information with the release of the maximal possible expected information content through traded prices.

The BN pricing measure is in Platen (2025) proposed to replace the putative risk-neutral pricing measure. The Kullback–Leibler divergence between the BN pricing measure and the real-world probability measure is also minimized. This

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ensures that the pricing density remains as close as possible to observed market probabilities, maintaining both realism and theoretical soundness when pricing contingent claims.

The framework that emerges from these information-maximization criteria yields a distinct idealized financial market model, which is called *minimal market model* (MMM). It extends the similarly named model proposed in Platen (2001) for the dynamics of a GOP. The paper will show, under the MMM, the state variables, their aggregated sums, and the GOP of the stocks all evolve according to squared radial Ornstein-Uhlenbeck (SROU) processes as studied in Göing-Jaeschke and Yor (2003), each within their respective activity times. This result encapsulates the dynamic behavior of both individual and collective market components within an information-theoretic modeling context.

The properties of the proposed model will be derived endogenously. The model will be parsimonious with only one parameter and a minimum number of driving Brownian motions. It will facilitate the design of extremely accurate hedging methods. The modeling of the complex stochastic dynamical system represented by the financial market is significantly simplified by modeling its state variables, the *normalized factors*, as independent scalar diffusion processes with stationary probability densities.

Since stocks are securities that are driven by several sources of randomness, it is not ideal to employ these as the stochastic basis of a financial market model. Instead, the paper models stocks as portfolios of independent auxiliary securities, the *factors*, which form the theoretical stochastic basis of the market. A factor is assumed to represent a portfolio of stocks. It is modeled as the product of an exponential function of time and an independent state variable, the *normalized factor*. By construction, the volatilities of the normalized factors are stationary processes and equal to those of the respective factors. All securities are denominated in units of the savings account, which is added as a security to the factors to form the *extended market* so that the interest rate does not need to be modeled.

The extended market is assumed to permit continuous trading, instantaneous investing and borrowing, short sales with full use of proceeds, infinitely divisible securities, and no transaction costs. For the extended market, the modeling challenge reduces to the question, which independent stationary scalar diffusions should model the normalized factors. To answer this question, the paper optimizes suitable Lagrangians that capture mathematically the principle driving the market's dynamics, which is the maximization of expected information content. To capture this principle, the paper interprets the market as a communication system in the sense of Shannon (1948), where the expected information content is maximized, which is called the *surprisal*. This identifies the optimal stationary probability densities of the normalized factors.

The fact that information-theoretical concepts are particularly pertinent to the field of finance can be explained as follows: The trades reveal with their prices information. The average speed of the information flow is measured by the average trading intensity, which the paper models by the *market activity*. The integrated market activity is called the *market time*. At certain periods, the market moves rather fast and at other periods more slowly. One notices that the market activity represents a fast moving process because particular information can trigger enormous trading activity in a very short time. Important is the fact that the market participants observe the market evolution in calendar time. When observing the volatility of a stock market index in calendar time during a period of high market activity, one observes volatility spikes, which a diffusion that is evolving in calendar time cannot easily generate. The paper assumes that the market is moving in a common market time that moves faster at major market movements. However, the state variables, which are the normalized factors, move in market time as scalar diffusions. This separates the modeling of the fast moving market activity from that of the in market time as diffusions evolving normalized factors. The market activity or speed of the information flow can be measured by the average trading intensity. The modeling of the market activity is beyond the scope of the current paper and suggested to be modeled similarly as proposed in Platen and Rendek (2019). This leads to the first fundamental assumption of the paper:

A1: *The normalized factors evolve as independent, stationary scalar diffusions in market time, which is the integrated average trading intensity*.

The existence of the GOP can be interpreted as a *no-arbitrage condition* because Karatzas and Kardaras (2007) have shown that the existence of the GOP is equivalent to their *No Unbounded Profit with Bounded Risk* (NUPBR) condition. This no-arbitrage condition is weaker than the no FLVR condition of Delbaen and Schachermayer (1998). The assumption about the existence of the GOP is an extremely weak condition. When violated, the respective growth rate maximization would have no solution, and the candidate for the GOP would reach infinite values in finite time, which is not a financial market that the current paper aims to model. This leads to the second fundamental assumption of the paper:

A2 : *The market's GOP and a unique, strong solution of the system of stochastic differential equations (SDEs), characterizing the market's normalized factor dynamics, exist*.

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By maximizing the instantaneous growth rate of portfolios of factors when forming the GOP of factors, which is called the *benchmark*, a first Lagrangian comes into play. The Assumption **A2** requires the GOP of the entire market to exist, which is called NP and was named by Long (1990). By maximizing the growth rate of all admissible portfolios in the market, a second Lagrangian emerges in the modeling.

The trades disclose price information. The stationary joint probability density of the normalized factors quantifies the probabilistic nature of the prices. When the surprisal, as defined in Kullback (1959), is maximized, the probability densities are so that all price-relevant expected information content that could be included by the buyer and seller in a trade has been included under the given constraints. This leads to the third Lagrangian and the third fundamental assumption of the paper:

A3: *The surprisal, which is the expected information content of the stationary joint probability density of the normalized factors, is maximized.*

The paper calls an extended market a *surprisal-maximized* market when the above three assumptions are satisfied. As an alternative and practicable pricing method to risk-neutral pricing, *BN pricing* has been proposed in Platen (2025), which employs as its numéraire the benchmark that equals the GOP of the factors. Buyers and sellers of contingent claims have an interest to minimize the increase of the expected information content caused by the pricing density they use. The derivative of the *Kullback–Leibler divergence*, as defined in Kullback (1959), of the BN pricing measure from the real-world probability measure quantifies the increase of expected information content as the result of BN pricing. Under the first three assumptions, the latter will be shown to be the only practicable pricing method that is applicable for all admissible contingent claims. This identifies the fourth Lagrangian and leads to the formulation of the fourth fundamental assumption of the paper:

A4 : *The Kullback–Leibler divergence of the BN pricing measure from the real-world probability measure is minimized.*

We call an extended market an *information-maximized* market when the four assumptions are satisfied by its dynamics. The paper will show that the above four assumptions lead to an idealized financial market model that is "realistic" in the sense that it generates the empirical facts previously listed. As indicated earlier, the derived idealized information-maximized financial market model is called MMM. The MMM has only one constant parameter when viewed in market time, which is the *net risk-adjusted return*. This parameter is not needed when applying BN pricing under the MMM. The observable market time is the only input that is required for the pricing and hedging of a wide range of contingent claims expressed in terms of the market time.

Under the BN pricing measure, the factors, sums of factors and the benchmark follow squared Bessel processes that evolve in respective observable activity times. Squared Bessel processes are special SROU processes, which were studied, for example, in Revuz and Yor (1999) and Göing-Jaeschke and Yor (2003). They are generalizations of the CIR process, which was popularized in finance in Cox et al. (1985). These processes are highly tractable with many explicit formulas for their functionals; see, for example, Baldeaux and Platen (2013).

The paper is organized as follows: Section 2 introduces the extended market of factors. Section 3 maximizes the surprisal of the stationary densities of the normalized factors. Section 4 minimizes the Kullback–Leibler divergence of the BN pricing measure from the real-world probability measure, which reveals the MMM.

Five appendices prove the theorems of the paper.

2 | Market of Factors

2.1 | Factors

Under Assumption **A1**, the *market time* τ_t evolves nondecreasing with respect to the calendar time $t \in [t_0, \infty)$. The modeling is performed on a filtered probability space $(\Omega, \mathcal{F}, \underline{\mathcal{F}}, P)$ in market time, satisfying the usual conditions; see, for example, Karatzas and Shreve (1998). The filtration $\underline{\mathcal{F}} = (\mathcal{F}_\tau)_{\tau \in [\tau_{t_0}, \infty)}$ models the evolution of all events relevant to the modeling.

The securities are denominated in units of the savings account $S^0_\tau \equiv 1$ and specify $n \in \{1, 2, \ldots\}$ stocks $A^1_\tau, \ldots, A^n_\tau$ as the risky primary security accounts. The stocks reinvest all dividends or other payments and expenses. The n stocks are assumed to represent self-financing portfolios of the n independent *factors* $S^1_\tau, \ldots, S^n_\tau$ at the market time $\tau \in [\tau_{t_0}, \infty)$. The k-th factor S^k_τ, $k \in \{1, \ldots, n\}$, is assumed to be driven by the k-th Brownian motion W^k_τ. The n independent driving Brownian motions $W^1_\tau, \ldots, W^n_\tau$ evolve in market time $\tau \in [\tau_{t_0}, \infty)$ under the real-world probability measure P, where the filtration generated by the Brownian motions is a subfiltration of the given filtration $\underline{\mathcal{F}}$.

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The factors are independent, nonnegative auxiliary securities that form the stochastic basis of the market. We assume that each factor can be formed as a self-financing portfolio of the stocks, that is, we have the SDE

$$\frac{dS_\tau^j}{S_\tau^j} = \sum_{k=1}^{n} p_\tau^{k,j} \frac{dA_\tau^k}{A_\tau^k} \tag{1}$$

with initial value $S_{\tau_{t_0}}^j > 0$, where the weights $p_\tau^{j,1}, \ldots, p_\tau^{j,n}$ form predictable, square integrable processes for $\tau \in [\tau_{t_0}, \infty)$ and $j \in \{1, \ldots, n\}$.

Under Assumption **A2**, the market formed by the n factors has a GOP, which we call the *benchmark* and denote by S_τ^* at the market time τ. Since the set of portfolios that can be formed by the factors and those that can be formed by the stocks are assumed to be the same, it follows by Theorem 3.1 in Filipović and Platen (2009) that the benchmark is also the GOP of the market formed by the stocks. The market formed by the factors has, by construction, no locally riskfree portfolio (LRP), which is a portfolio with zero volatility.

Theorem 3.1 in Filipović and Platen (2009) reveals the general structure of a financial market with continuous securities that has a GOP and no LRP, which determines the structure of the market of factors. The most striking structural property of this market is the existence of a unique *net risk-adjusted return* $\hat{\lambda}_\tau$, which emerges as the Lagrange multiplier of the growth rate maximization that identifies the benchmark S_τ^*. The net risk-adjusted return $\hat{\lambda}_\tau$ is assumed to represent an adapted, square integrable process.

We assume for $k \in \{1, \ldots, n\}$ the *k-th factor process* $S^k = \{S_\tau^k, \tau \in [\tau_{t_0}, \infty)\}$ to satisfy the SDE

$$\frac{dS_\tau^k}{S_\tau^k} = \hat{\lambda}_\tau d\tau + \beta_\tau^k (\beta_\tau^k \omega^k d\tau + dW_\tau^k) \tag{2}$$

for $\tau \in [\tau_{t_0}, \infty)$ with strictly positive $\mathcal{F}_{\tau_{t_0}}$-measurable, random *initial k-th factor value* $S_{\tau_{t_0}}^k > 0$. The value of a factor may approach zero, at which instance it is assumed to be instantaneously reflected. For each independent driving Brownian motion $W_\tau^k, k \in \{1, \ldots, n\}$, the respective *k-th factor volatility* is denoted by β_τ^k and the respective *risk premium parameter* by $\omega^k > 0$. The volatility with respect to market time is assumed to represent a flexible, continuous, strictly positive, adapted, square integrable process in market time. The expected rate of return $\hat{\lambda}_\tau + (\beta_\tau^k)^2 \omega^k$ is flexible because it is formed by the sum of the net risk-adjusted return and the product of the flexible k-th risk premium parameter and the squared k-th factor volatility.

2.2 | Benchmark

Let us denote by β_τ the diagonal matrix with the factor volatilities at its diagonal and zeros at all off-diagonal elements. Furthermore, we denote by $\boldsymbol{\omega} = (\omega^1, \ldots, \omega^n)^\top$ the vector of the risk premium parameters. This allows us to write the SDE for the vector of factors $\mathbf{S}_\tau = (S_\tau^1, \ldots, S_\tau^n)^\top$ in the form

$$\frac{d\mathbf{S}_\tau}{\mathbf{S}_\tau} = \hat{\lambda}_\tau \mathbf{1} d\tau + \beta_\tau (\beta_\tau \omega d\tau + d\mathbf{W}_\tau) \tag{3}$$

with the vector of strictly positive *initial values* $\mathbf{S}_{\tau_{t_0}}$ and the vector process $\mathbf{W} = \{\mathbf{W}_\tau = (W_\tau^1, \ldots, W_\tau^n)^\top, \tau \in [\tau_{t_0}, \infty)\}$ of the n independent driving Brownian motions. Here we write $\frac{d\mathbf{S}_\tau}{\mathbf{S}_\tau}$ for the n-vector of stochastic differentials $(\frac{dS_\tau^1}{S_\tau^1}, \ldots, \frac{dS_\tau^n}{S_\tau^n})^\top$. By application of the Itô formula one obtains for a self-financing portfolio S_τ^π of factors with weight vector $\pi_\tau = (\pi_\tau^1, \ldots, \pi_\tau^n)^\top$ the SDE

$$\frac{dS_\tau^\pi}{S_\tau^\pi} = \pi_\tau^\top \frac{d\mathbf{S}_\tau}{\mathbf{S}_\tau} \tag{4}$$

and the *instantaneous growth rate*

$$g_\tau^\pi = \hat{\lambda}_\tau + (\pi_\tau)^\top \beta_\tau \beta_\tau \left(\omega - \frac{1}{2} \pi_\tau \right) \tag{5}$$

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as the drift of the SDE of its logarithm for $\tau \in [\tau_{t_0}, \infty)$. The benchmark S^*_τ is the portfolio that maximizes this growth rate, which involves the first Lagrangian and is defined as follows:

Definition 1. The benchmark S^*_τ is the strictly positive portfolio of factors with maximum instantaneous growth rate g^π_τ and initial value $S^*_{\tau_{t_0}} > 0$, where its weight vector $\pi^*_\tau = (\pi^{*,1}_\tau, \ldots, \pi^{*,n}_\tau)^\top$ is a solution of the well-posed n-dimensional constrained quadratic maximization problem

$$\max \left\{ g^\pi_\tau | \pi_\tau \in \mathbf{R}^n, \pi^\top_\tau \mathbf{1} = 1 \right\}, \tag{6}$$

for all $\tau \in [\tau_{t_0}, \infty)$.

Note that the above portfolios of factors do not invest in the LRP, which is the savings account. The following properties of the benchmark are derived in Appendix A:

Theorem 1. *For a market of factors and $\tau \in [\tau_{t_0}, \infty)$, the sum of the risk premium parameters is conserved and equals the constant 1, that is,*

$$\omega^\top \mathbf{1} = 1. \tag{7}$$

When constructing the benchmark, the vector of weights assigned to the factors corresponds to the vector of risk premium parameters

$$\pi^*_\tau = \omega \tag{8}$$

and the benchmark satisfies the SDE

$$\frac{dS^*_\tau}{S^*_\tau} = \hat{\lambda}_\tau d\tau + (\beta_\tau \omega)^\top (\beta_\tau \omega d\tau + d\mathbf{W}_\tau) \tag{9}$$

for $\tau \in [\tau_{t_0}, \infty)$ with $S^*_{\tau_{t_0}} > 0$.

By Equation (7), the above theorem reveals the important property that the sum of the risk premium parameters is a conserved quantity and equals 1. For the case when the risk premium parameters would be predictable, square integrable processes, the respective proof of the above theorem does still work, the risk premium factors would still add up to one, and these would still represent the weights for the benchmark.

2.3 | NP

The savings account denominated savings account $S^0_\tau \equiv 1$ equals the constant 1 for $\tau \in [\tau_{t_0}, \infty)$. Let us extend the market of factors by adding the savings account $S^0_\tau \equiv 1$ to the factors $\mathbf{S}_\tau = (S^1_\tau, \ldots, S^n_\tau)^\top$. In the *extended market* one can form self-financing portfolios by combining the $n+1$ securities $S^0_\tau, \ldots, S^n_\tau$. A positive self-financing portfolio S^π_τ in the extended market is described by its initial value $S^\pi_{\tau_{t_0}} > 0$ and its risky security weights $\pi_\tau = (\pi^1_\tau, \ldots, \pi^n_\tau)^\top$, where the savings account weight equals

$$\pi^0_\tau = 1 - \pi^\top_\tau \mathbf{1} \tag{10}$$

for $\tau \in [\tau_{t_0}, \infty)$. The portfolio S^π_τ satisfies the SDE

$$\frac{dS^\pi_\tau}{S^\pi_\tau} = \pi^\top_\tau \frac{d\mathbf{S}_\tau}{\mathbf{S}_\tau}, \tag{11}$$

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and its instantaneous growth rate, the drift of its logarithm, is of the form

$$g_\tau^\pi = \pi_\tau^\top \mathbf{1}\hat{\lambda}_\tau + (\pi_\tau)^\top \beta_\tau \beta_\tau (\omega - \frac{1}{2}\pi_\tau) \tag{12}$$

for $\tau \in [\tau_{t_0}, \infty)$.

Long (1990) introduced the NP as a general market notion for the GOP of the extended market, showing that risk-neutral prices can be derived using the NP as numeraire and the real-world probability measure P as pricing measure, which requires an equivalent risk-neutral measure. The benchmark approach emphasizes that only the existence of the NP is needed to compute minimal prices for replicable contingent claims. Karatzas and Kardaras (2007) showed that the existence of the NP is equivalent to their NUPBR condition. In analogy to Definition 1, and assuming **A 2**, the second Lagrangian is then relevant, leading to the following concept:

Definition 2. For the extended market, the NP S_τ^{**} is the positive portfolio of the savings account and the factors with maximum instantaneous growth rate $g_\tau^{\pi^{**}}$ and initial value $S_{\tau_{t_0}}^{**} > 0$, where its weight vector $(\pi_\tau^{**,0}, \pi_\tau^{**,1}, \ldots, \pi_\tau^{**,n})^\top = (\pi_\tau^{**,0}, \pi_\tau^{**\top})$ is a solution of the well-posed $(n+1)$-dimensional constrained quadratic maximization problem

$$\max\left\{g_\tau^\pi | (\pi_\tau^0, \pi_\tau^\top)^\top \in \mathbf{R}^{n+1}, \pi_\tau^0 + \pi_\tau^\top \mathbf{1} = 1\right\}, \tag{13}$$

for all $\tau \in [\tau_{t_0}, \infty)$.

Under the Assumption **A2**, the existence of the NP is secured and the following result derived in Appendix B:

Theorem 2. *For the given extended market, the NP S_τ^{**} satisfies the SDE*

$$\frac{dS_\tau^{**}}{S_\tau^{**}} = \theta_\tau^{**\top}(\theta_\tau^{**} d\tau + d\mathbf{W}_\tau) \tag{14}$$

with initial value $S_{\tau_{t_0}}^{**} > 0$, market price of risk vector

$$\theta_\tau^{**} = \hat{\lambda}_\tau \beta_\tau^{-1}\mathbf{1} + \beta_\tau \omega, \tag{15}$$

the NP weights

$$\pi_\tau^{**} = (\pi_\tau^{**,1}, \ldots, \pi_\tau^{**,n})^\top = \hat{\lambda}_\tau \beta_\tau^{-2}\mathbf{1} + \omega, \tag{16}$$

to be invested in the factors $S_\tau^1, \ldots, S_\tau^n$, and the weight

$$\pi_\tau^{**,0} = -\hat{\lambda}_\tau \mathbf{1}^\top \beta_\tau^{-2}\mathbf{1} \tag{17}$$

to be invested in the savings account S_τ^0 for $\tau \in [\tau_{t_0}, \infty)$.

The flexibility of factor volatility and risk premium parameters, along with Theorem 3.1 in Filipović and Platen (2009), ensures a broad set of continuous market dynamics can be described using the extended market model, including stocks formed as portfolios of factors. The theorem remains valid when risk premium parameters are predictable, square-integrable processes.

3 | Surprisal-Maximized Market

3.1 | Normalized Factors

By applying the Assumption **A2**, we characterize below the state variables, the normalized factors, as independent, stationary, scalar diffusion processes that evolve in respective activity times. We assume for $k \in \{1, \ldots, n\}$ the *k-th*

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normalized factor $Y^k_{\tau^k_\tau}$ to be given as the ratio

$$Y^k_{\tau^k_\tau} = \frac{S^k_\tau}{B_\tau e^{\tau^k_\tau}} \tag{18}$$

with *basis exponential*

$$B_\tau = \exp\left\{\int_{\tau_{t_0}}^{\tau} \hat{\lambda}_s ds\right\} \tag{19}$$

for $\tau \in [\tau_{t_0}, \infty)$, representing a scalar diffusion with stationary probability density p^k_∞. The stationary process $Y^k_{\tau^k_\tau}$ is assumed to evolve in the *k-th activity time*

$$\tau^k_\tau = \tau_{t_0} + (\tau - \tau_{\tau_{t_0}})a^k \tag{20}$$

for $\tau \in [\tau_{t_0}, \infty)$, where $a^k > 0$ denotes the strictly positive *k-th activity* and $\tau_{t_0} \in (-\infty, \infty)$ the *initial market time*. By Equation (18) it follows the *k-th initial factor value*

$$S^k_{\tau_{t_0}} = Y^k_{\tau_{t_0}} e^{\tau_{t_0}} > 0. \tag{21}$$

By application of the Itô formula it is straightforward to see that a normalized factor has the same volatility as the respective factor. For the stationary k-th normalized factor process $Y^k = \left\{Y^k_{\tau^k}, \tau^k \in [\tau_{t_0}, \infty)\right\}$, $k \in \{1, \ldots, n\}$, we denote by p^k_∞ its stationary probability density, where

$$\mathbf{E}^{p^k_\infty}\left(f(Y^k_{\tau^k_\tau})\right) = \int_0^\infty f(y) p^k_\infty(y) dy \tag{22}$$

denotes the average with respect to this density.

Four constraints for the k-th normalized factor dynamics emerge as follows:

First, the average level of a normalized factor is a crucial quantity because it determines the average level of the value of the factor at a given market time, which models an average price. Since there would exist some ambiguity concerning the average level of the k-th normalized factor, we parametrize the arithmetic average of Y^k, with a view towards the proof of Theorem 3, so that

$$4\omega^k = \mathbf{E}^{p^k_\infty}\left(Y^k_{\tau^k_\tau}\right) > 0 \tag{23}$$

for $\tau \in [\tau_{t_0}, \infty)$.

Second, under the benchmark approach, the logarithm of portfolios and, in particular, the logarithmic average of the portfolio with the largest instantaneous growth rate the GOP, are of central importance. Therefore, we parametrize the logarithmic average of Y^k by the flexible constant

$$\mathbf{E}^{p^k_\infty}(\ln(Y^k_{\tau^k_\tau})) = \zeta^k \in (-\infty, \infty) \tag{24}$$

for $\tau \in [\tau_{t_0}, \infty)$.

Third, by Theorem 1 and 7, we have the constraint

$$\sum_{k=1}^{n} \omega^k = 1. \tag{25}$$

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Fourth, with a view towards the proof of Theorem 5, we introduce for the activities the constraint

$$\sum_{k=1}^{n} \omega^k \sqrt{\frac{1}{a^k}} = 1. \tag{26}$$

It is our goal to specify the k-th normalized factor as a flexible, stationary scalar diffusion under the above described constraints. We achieve this by setting the k-th factor volatility equal to the function

$$\beta_\tau^k = \sqrt{\frac{4a^k}{\phi^k(Y^k_{\tau^k_\tau})}}, \tag{27}$$

where the *k-th volatility function* $\phi^k(.)$ is a flexible, infinitely often continuously differentiable, strictly positive function of the value of the k-th normalized factor. According to Assumption **A2**, when assuming stationary normalized factors, it is required that the NP S^{**}_τ exists and that the k-th normalized factor in the k-th activity time is modeled by a unique, independent scalar diffusion process; see, for example, Section 7.7 in Platen and Heath (2006). By the equations (18), (2), (20), and application of the Itô formula, the k-th normalized factor $Y^k_{\tau^k_\tau}$ satisfies the SDE

$$dY^k_{\tau^k_\tau} = Y^k_{\tau^k_\tau}\left(4\phi^k(Y^k_{\tau^k_\tau})^{-1}\omega^k - 1\right)a^k d\tau + Y^k_{\tau^k_\tau}\phi^k(Y^k_{\tau^k_\tau})^{-\frac{1}{2}}\sqrt{4a^k}dW^k_\tau \tag{28}$$

of a scalar diffusion with instantaneous reflection at zero for $\tau \in [\tau_{t_0}, \infty)$. In the above SDE, the diffusion coefficient of the k-th normalized factor is that of a flexible scalar diffusion process.

The drift coefficient is determined using the Itô formula. Consequently, this SDE encompasses a wide variety of scalar diffusion processes that evolve according to their activity times. For instance, assigning $\phi^k(y) = \bar{y}$ constant models factors with constant volatilities, as described by Black and Scholes (1973). If $\phi^k(y)$ is set as a power function, such as $\phi^k(y) = y^x$, the resulting dynamics exhibit constant elasticity of variance-type stochastic volatility, following the work of Cox (1975). Choosing $\phi^k(y) = y$ (the identity function) produces 3/2-type stochastic volatility models, as noted by Heston (1997) and Platen (1997). If $\phi^k(y) = y^{-1}$, the model coincides with the Heston (1993) volatility specification, while $\phi^k(y) = y^{-2}$ corresponds to the volatility of the Bachelier model (Bachelier (1900)). By allowing $\phi^k(.)$ to be any suitable general function of the normalized factor, one obtains a broad family of local volatility function models; see, for example, Breeden and Litzenberger (1978), Derman and Kani (1994), and Dupire (1994). Considered in calendar time, the class of stochastic volatility models becomes even broader, since market time remains highly adaptable and can be defined in numerous ways, with its derivative serving as a subordinator (see Bochner (1955), Clark (1973), and Hagan et al. (2002)). The Assumption **A1** allows sufficient flexibility to choose the market time as a nondecreasing stochastic process, potentially extending beyond the semimartingale framework.

Definition 3. We call the above described market, formed by the savings account S^0_τ and the n factors $S^1_\tau, \ldots, S^n_\tau$ as primary security accounts, a *stationary market*, where the net risk-adjusted return $\hat{\lambda}_\tau$ represents an adapted, square integrable process and the normalized factors stationary, independent, scalar diffusions in market time.

This does not mean that in a stationary market the factors or resulting stocks have to be stationary processes. Only the volatilities of the normalized factors are modeled in market time as processes with stationary probability densities. The k-stationary probability density $p^k_\infty(y)$, $y \in (0, \infty)$, of the k-th normalized factor is the solution of a respective ordinary differential equation, the Fokker–Planck equation, as shown, for example, in Section 4.5 in Platen and Heath (2006).

3.2 | Surprisal Maximization

The identification of stationary dynamics in normalized factors depends on the principle driving financial market behavior. For a stationary market, this principle is formulated through a Lagrangian based on stationary probability densities, which must be maximized to find the optimal joint density. Shannon's information theory, as described in Shannon (1948), highlights information content as a key concept derived from event probabilities, with the *surprisal* representing the expected information content. This paper assumes in Assumption **A3** that market prices maximize the

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surprisal of the joint probability density of normalized factors, suggesting prices embed maximum beneficial information on average. Information content for a continuous variable is measured by the negative logarithm of its probability density, as described in Kullback (1959), and surprisal is the expected information content. Thus, rather than focusing on utilities, returns, or monetary measures, fundamental financial market quantities are in this paper interpreted as information-theoretic quantities. Following Shannon and Kullback, this leads to the proposed notion below.

Definition 4. For a stationary market with stationary joint density

$$p_\infty = \prod_{k=1}^{n} p_\infty^k \tag{29}$$

of the normalized factors, the *surprisal* $\mathcal{I}(p_\infty)$ of p_∞ is defined as its average information content, which is given by the sum of the integrals

$$\mathcal{I}(p_\infty) = -\sum_{k=1}^{n} \int_0^\infty p_\infty^k(y) \ln(p_\infty^k(y)) dy, \tag{30}$$

as long as, the above quantities exist.

The stationary market reaches its highest surprisal when, within the given specific parametrization and set of constraints, the expected information content of the stationary joint density of normalized factors is maximized. This process yields the third Lagrangian in our pursuit of an idealized market model. We now present the following kind of market:

Definition 5. A stationary market is said to be *surprisal-maximized* if the surprisal $\mathcal{I}(p_\infty)$ of the stationary joint density p_∞ of the normalized factors is maximized for the given parametrization and constraints.

The surprisal-maximized market can be seen as "efficient," though this differs from efficiency as defined by Fama (1970) and Grossman and Stiglitz (1980). Surprisal, mathematically equal to entropy, is important in the derivation of natural laws across disciplines (see Kosmann-Schwarzbach (2018)). The next theorem explains the results of maximizing the surprisal of p_∞, as per Assumption **A3**; its proof is provided in Appendix C.

Theorem 3. *For $k \in \{1, \dots, n\}$ and $\tau \in [\tau_{t_0}, \infty)$, the dynamics of the k-th normalized factor $Y^k_{\tau^k_\tau}$ of a surprisal-maximized market is that of a CIR process of dimension $\frac{4}{n}$, which evolves in the k-th activity time τ^k_τ as the unique strong solution of the SDE*

$$dY^k_{\tau^k_\tau} = \left(\frac{4}{n} - Y^k_{\tau^k_\tau}\right) a^k d\tau + \sqrt{Y^k_{\tau^k_\tau} 4a^k} dW^k_\tau, \tag{31}$$

with its initial value $Y^k_{\tau_{t_0}} = S^k_{\tau_{t_0}} e^{-\tau^1_{\tau_{t_0}}}$ distributed according to the surprisal-maximized stationary density p^k_∞. The latter is a gamma density with $\frac{4}{n}$ degrees of freedom and mean $\frac{4}{n}$.

As requested by Assumption **A3**, the k-th normalized factor is a stationary scalar diffusion. It generates a stationary volatility process in market time and has, by construction, the same volatility as the k-th factor. Therefore, the squared volatility

$$(\beta^k_\tau)^2 = \frac{4a^k}{Y^k_{\tau^k_\tau}} \tag{32}$$

of the k-th factor with respect to the market time τ is generated by the squared volatility of the k-th normalized factor. It follows by application of the Itô formula and using (27) the 3/2 *stochastic volatility dynamics* that are characterized by the

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SDE

$$d(\beta_\tau^k)^2 = \left(a^k + \left(1 - \frac{1}{n}\right)(\beta_\tau^k)^2\right)(\beta_\tau^k)^2 d\tau - \left((\beta_\tau^k)^2\right)^{\frac{3}{2}} dW_\tau^k \tag{33}$$

for $\tau \in [\tau_{t_0}, \infty)$, as long as $Y^k_{\tau^k_\tau} > 0$. A 3/2 volatility model was proposed in Platen (1997) as the squared volatility model of a GOP. The latter model became in Platen (2001) named MMM; see Hulley and Schweizer (2010). A 3/2-type volatility model was independently suggested in Heston (1997) for derivative pricing due to its excellent tractability. The current paper names the entire idealized financial market model MMM that results from applying all four assumptions.

By applying Equation (18), (2), and the Itô formula, one obtains the factor dynamics as follows:

Corollary 1. *For a surprisal-maximized market and $k \in \{1, \ldots, n\}$, the k-th factor can be expressed as the product*

$$S_\tau^k = Y^k_{\tau^k_\tau} B_\tau e^{\tau^k_\tau}, \tag{34}$$

which satisfies in the market time $\tau \in [\tau_{t_0}, \infty)$ the SDE

$$dS_\tau^k = \hat{\lambda}_\tau S_\tau^k d\tau + \frac{4}{n} B_\tau a^k e^{\tau^k_\tau} d\tau + \sqrt{S_\tau^k 4 B_\tau a^k e^{\tau^k_\tau}}\, dW_\tau^k \tag{35}$$

of a SROU process with dimension $\frac{4}{n}$ for $\tau \in [\tau_{t_0}, \infty)$.

3.3 | Benchmark Dynamics

Applying Theorem 1, we see that in a market that maximizes surprisal, the benchmark is created by assigning equal weights to all factors. Specifically, at any market time $\tau \in [\tau_{t_0}, \infty)$, the weight of the k-th factor is $\pi_\tau^{*,k} = \frac{1}{n}$. Since the volatility of a factor can become infinite if that factor reaches zero, we define $\mathcal{T} \subset [\tau_{t_0}, \infty)$ as the set of all market times when every one of the n factors remains strictly positive. The SDEs governing the SROU processes for both the factors and their normalized counterparts have unique strong solutions, and these solutions are still well-defined even at points when a factor hits zero. To prevent complications from infinite volatility, we focus only on market times contained in $\mathcal{T} \subset [\tau_{t_0}, \infty)$. The dynamics for the benchmark under these conditions are detailed in Appendix D.

Theorem 4. *Consider a surprisal-maximized market at a market time $\tau \in \mathcal{T}$. The benchmark S_τ^* represents the equal-weighted factor portfolio (FP) and satisfies the SDE*

$$dS_\tau^* = \hat{\lambda}_\tau S_\tau^* d\tau + S_\tau^* Z_\tau d\tau + S_\tau^* \sqrt{Z_\tau} dW_\tau^* = \hat{\lambda}_\tau S_\tau^* d\tau + 4e^{\tau_\tau^*} a_\tau^* d\tau + \sqrt{S_\tau^*} \sqrt{4e^{\tau_\tau^*} a_\tau^*}\, dW_\tau^*, \tag{36}$$

with initial value $S^*_{\tau_{t_0}} > 0$ and squared benchmark volatility

$$Z_\tau = \frac{1}{n^2} \sum_{k=1}^n \frac{4a^k}{Y^k_{\tau^k_\tau}}, \tag{37}$$

where W_τ^* is a Brownian motion with stochastic differential

$$dW_\tau^* = Z_\tau^{-\frac{1}{2}} \frac{1}{n} \sum_{k=1}^n \sqrt{\frac{4a^k}{Y^k_{\tau^k_\tau}}}\, dW_\tau^k \tag{38}$$

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and initial value $W^*_{\tau_{t_0}} = 0$. *The* normalized benchmark

$$Y^*_{\tau^*_\tau} = \frac{S^*_\tau}{e^{\tau^*_\tau}} \tag{39}$$

follows an SROU process of dimension four that is evolving in the benchmark time

$$\tau^*_\tau = \tau^*_{\tau_{t_0}} + \int_{\tau_{t_0}}^{\tau} a^*_s ds \tag{40}$$

and satisfies the SDE

$$dY^*_{\tau^*_\tau} = \hat{\lambda}_\tau Y^*_{\tau^*_\tau} d\tau + \left(4 - Y^*_{\tau^*_\tau}\right) a^*_\tau d\tau + 2\sqrt{Y^*_{\tau^*_\tau} a^*_\tau} dW^*_\tau \tag{41}$$

with initial value

$$Y^*_{\tau^*_{\tau_{t_0}}} = S^*_{\tau_{t_0}} e^{-\tau^*_{\tau_{t_0}}} \tag{42}$$

and benchmark activity

$$a^*_\tau = \frac{Z_\tau Y^*_{\tau^*_\tau}}{4}. \tag{43}$$

The normalized benchmark is an SROU process with dimension four in benchmark time τ^*_τ, always positive and uniquely solving the SDE (41). Its squared volatility Z_τ with respect to market time, as given in (37), becomes infinite when any normalized factor reaches zero, causing an upward jump in benchmark time. This highlights that benchmark volatility may spike at certain moments; however, this does not imply that the diffusion coefficient itself becomes infinite. It also suggests the market time-integrated growth rate of the benchmark can have upward jumps when a factor hits zero.

The benchmark is built using a dynamic asset allocation approach. In actual trading, reallocations can only occur at specific intervals since trades are discrete, and the SDEs representing both the factors and the benchmark are theoretical models that assume continuous trading. The continuous-time framework approximates what typically occurs when factor values become extremely small.

Brownian motion and its Gaussian transition probability density effectively describe the limiting behavior in continuous time for a sum of independently moving particles. This principle underlies models such as Bachelier's Bachelier (1900) approach for sums of independent capital units and the Black-Scholes model by Black and Scholes (1973), where it is applied to the logarithm of these sums. Empirical evidence, such as the Student-t log-return distribution observed in stock indices, indicates that these models do not sufficiently represent real market behavior. When analyzing the evolution of aggregated independent capital units, it becomes apparent that their trajectory is influenced by the exact count of units present. Within this framework, the SROU process and its noncentral chi-square transition probability density effectively model the limiting continuous-time dynamics of a system comprising independently moving particles that may generate new entities or cease to exist, as detailed in Feller (1971). This insight offers an alternative and intuitive perspective on the derived surprisal-maximized market dynamics, framing them as the aggregate activity of independent capital units that either proliferate or disappear autonomously.

4 | Information-Maximized Market

4.1 | BN Pricing

Hedging an inexpensive zero-coupon bond, which pays one unit of the savings account at maturity, necessitates trading both the numéraire and the savings account; for further details, refer to Fergusson and Platen (2023) and Barone-Adesi

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et al. (2024). This requirement implies that, in general, it is essential to be able to trade the numéraire used for pricing when hedging contingent claims. Subject to Assumption **A2**, real-world pricing may be employed with the NP S_τ^{**} as the numéraire and the real-world probability measure P as the pricing measure, as outlined in Platen (2006). Regarding the derived surprisal-maximized dynamics, risk-neutral pricing can result in prices that are considerably higher than necessary, as demonstrated by Platen (2025).

Within the current financial market, the NP represents a leveraged portfolio that takes a short position in the savings account. Since practical trading occurs at discrete intervals, a proxy for the NP, an extremely volatile portfolio, cannot be assured to remain strictly positive. Additionally, modeling the dynamics of the NP requires knowledge of the net risk-adjusted return $\hat{\lambda}_\tau$, a drift parameter process that is particularly challenging to estimate. To address these difficulties, Platen (2025) introduced the BN pricing method, which utilizes the benchmark S_τ^* as the numéraire and the corresponding BN pricing measure Q_{S^*} as the pricing measure. The BN pricing measure Q_{S^*} is specified by the *Radon-Nikodym derivative*

$$\Lambda_{Q_{S^*}}(\tau) = \frac{dQ_{S^*}}{dP}\Big|_{\mathcal{F}_\tau} = \frac{\frac{S_\tau^*}{S_\tau^{**}}}{\frac{S_{\tau_{t_0}}^*}{S_{\tau_{t_0}}^{**}}} \tag{44}$$

for $\tau \in [\tau_{t_0}, \infty)$; see Geman et al. (1995).

Let $L_Q^1(\mathcal{F}_{\tau_T})$ denote the set of Q-integrable, $\mathcal{F}_{\tau_T}$-measurable random variables in a filtered probability space $(\Omega, \mathcal{F}, \underline{\mathcal{F}}, Q)$. It is shown in Platen (2026) for a more general market model than the above derived surprisal-maximized market model that the BN pricing measure Q_{S^*} is an equivalent probability measure and BN pricing determines the minimal possible prices for replicable contingent claims. More precisely, one can conclude directly from Theorem 3.4 in Platen (2026) the following facts:

Corollary 2. *For a surprisal-maximized market with $n > 2$ and $\tau \in [\tau_{t_0}, \infty)$, where the absolute value of the net risk-adjusted return $\hat{\lambda}_\tau$ is constant and finite, the following statements hold:*

(i) The BN pricing measure Q_{S^*} is an equivalent probability measure.

(ii) For a replicable contingent claim H_{τ_T}, with stopping time τ_T that is bounded as a market time, where $\frac{H_{\tau_T}}{S_{\tau_T}^{**}} \in L_P^1(\mathcal{F}_{\tau_T})$, and $\frac{H_{\tau_T}}{S_{\tau_T}^*} \in L_{Q_{S^*}}^1(\mathcal{F}_{\tau_T})$, its minimal possible price H_τ at the market time $\tau \in [\tau_{t_0}, \tau_T]$ equals the BN price provided by the BN pricing formula

$$H_\tau = S_\tau^* \mathbf{E}^{Q_{S^*}}\left(\frac{H_{\tau_T}}{S_{\tau_T}^*}|\mathcal{F}_\tau\right). \tag{45}$$

(iii) For $k \in \{1, \dots, n\}$, the process $\bar{W}^k = \{\bar{W}_\tau^k, \tau \in [\tau_{t_0}, \infty)\}$, satisfying the SDE

$$d\bar{W}_\tau^k = \hat{\lambda}_\tau \sqrt{\frac{S_\tau^k}{4a_\tau^k B_\tau e^{\tau_\tau^k}}} d\tau + dW_\tau^k \tag{46}$$

for $\tau \in [\tau_{t_0}, \infty)$ with $\bar{W}_{\tau_{t_0}}^k = 0$, is under the BN pricing measure Q_{S^*} a Brownian motion in market time.

(iv) The benchmark satisfies the SDE

$$dS_\tau^* = Z_\tau S_\tau^* d\tau + \sqrt{Z_\tau} S_\tau^* d\bar{W}_\tau^* = 4e^{\tau_\tau^*} a_\tau^* d\tau + \sqrt{S_\tau^* 4e^{\tau_\tau^*} a_\tau^*} d\bar{W}_\tau^* \tag{47}$$

of a time-transformed squared Bessel process of dimension four, where $\bar{W}^* = \{\bar{W}_\tau^*, \tau \in [\tau_{t_0}, \infty)\}$ is under the BN pricing measure Q_{S^*} a Brownian motion in market time, satisfying the SDE

$$d\bar{W}_\tau^* = Z_\tau^{-\frac{1}{2}} \hat{\lambda}_\tau d\tau + dW_\tau^* \tag{48}$$

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for $\tau \in [\tau_{t_0}, \infty)$ *with* Z_τ *given in (37) and* $\bar{W}^*_{\tau_{t_0}} = 0$.

The BN pricing measure Q_{S^*} substitutes in the SDEs of the factors under the real-world probability measure the net risk-adjusted return $\hat{\lambda}_\tau$ by zero, and the processes $W^1_\tau, \ldots, W^n_\tau$ by respective processes $\bar{W}^1_\tau, \ldots, \bar{W}^n_\tau$, which are Brownian motions under the BN pricing measure. This means, the knowledge of the net risk-adjusted return $\hat{\lambda}_\tau$ is not needed for BN pricing and hedging. The dimensions of the SROU processes that form the normalized factors, factors, and the benchmark remain under the BN pricing measure Q_{S^*} the same as under the real-world probability measure P.

In a wide class of potential numéraires the benchmark has been shown in Platen (2025) to be the only portfolio that yields as numéraire an equivalent pricing measure. In Schmutz et al. (2025) BN pricing has been generalized for not fully replicable contingent claims by introducing the concept of BN risk-minimization.

4.2 | Information Minimization

We denote for the normalized factors the BN stationary joint density by the product

$$q_\infty = \prod_{k=1}^{n} q^k_\infty \tag{49}$$

and the real-world stationary joint density by

$$p_\infty = \prod_{k=1}^{n} p^k_\infty. \tag{50}$$

A trade at the market time $\tau \in [\tau_{t_0}, \infty)$ reveals information through its price. Trades are discrete in time and the trading intensities, which quantify the speed of the flow of price information, are modeled by the market activities in the surprisal-maximized market. By following Kullback (1959), we employ the following notion:

Definition 6. For a surprisal-maximized market with BN stationary joint density q_∞ and real-world stationary joint density p_∞ of the normalized factors at the market time $\tau \in [\tau_{t_0}, \infty)$, the *Kullback–Leibler divergence* $I(p_\infty, q_\infty)$ of q_∞ from p_∞ is defined as

$$I(p_\infty, q_\infty) = \frac{d}{d\tau} \mathbf{E}^{p_\infty}\Big(-\ln(\Lambda_{Q_{S^*}}(\tau))\Big). \tag{51}$$

The above Kullback–Leibler divergence measures the increase of expected information content when employing BN pricing. One notices by (51) and (44) that the Kullback–Leibler divergence equals the average growth rate of the NP S^{**}_τ denominated in units of the benchmark S^*_τ.

The Assumption **A4** postulates that the increase of expected information content from BN pricing is minimized in the market, which means that the Kullback–Leibler divergence $I(p_\infty, q_\infty)$ is minimized. This minimization involves our fourth Lagrangian required by the Noether method.

Definition 7. We call a surprisal-maximized market *information-minimized* when the above Kullback–Leibler divergence of q_∞ from p_∞ is minimized.

The results of the above minimization are described in the following theorem, where the proof is provided in Appendix E:

Theorem 5. *For an information-minimized market, it follows for* $k \in \{1, \ldots, n\}$ *that the k-th activity equals*

$$a^k = 1, \tag{52}$$

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and the net risk-adjusted return

$$\hat{\lambda}_\tau = \hat{\lambda} \tag{53}$$

equals a constant, where the divergence of q_∞ from p_∞ amounts to

$$I(p_\infty, q_\infty) = \frac{\hat{\lambda}^2}{2} \tag{54}$$

for $\tau \in [\tau_{t_0}, \infty)$.

For an information-minimized market the activities equal 1 and the net risk-adjusted return $\hat{\lambda}_\tau = \hat{\lambda}$ is a constant. As indicated earlier we name the following model:

Definition 8. The idealized financial market model that results from the application of the four assumptions **A1**, **A2**, **A3**, and **A4**, yielding an information-minimized market, is called MMM.

Under the MMM and for $k \in \{1, \ldots, n\}$, the k-th factor

$$S_\tau^k = Y_\tau^k B_\tau e^\tau, \tag{55}$$

has by (2), (18), and application of the Itô formula, the following properties:

Corollary 3. *For $k \in \{1, \ldots, n\}$, the k-th factor has under the MMM the dynamics of an SROU process with* dimension $d_k = \frac{4}{n}$. *It satisfies the SDE*

$$dS_\tau^k = \hat{\lambda} S_\tau^k d\tau + \frac{4}{n} B_\tau e^\tau d\tau + \sqrt{S_\tau^k}\sqrt{4B_\tau e^\tau} dW_\tau^k \tag{56}$$

for $\tau \in [\tau_{t_0}, \infty)$, with random initial value $S_{\tau_{t_0}}^k = Y_{\tau_{t_0}}^k e^{\tau_{t_0}}$, where $Y_{\tau_{t_0}}^k$ has the stationary probability density $p_{\tau_{t_0}}^k = p_\infty^k$ as its density.

Craddock and Platen (2004) analyze Lie group symmetries for PDEs governing diffusion dynamics with square-root state variable diffusion coefficients. For the normalized factors in (56), Theorem 4.4.3 in Baldeaux and Platen (2013) identifies a specific Lie group symmetry aligning with these dynamics. This symmetry provides an explicit formula for the transition probability density, which is shown to be that of an SROU process (see Equation 5.1.2 in Baldeaux and Platen (2013)). Therefore, the transition probability density for normalized factor dynamics relates directly to a corresponding Lie group symmetry of its governing PDE.

Noether's Theorems in Noether (1918) predict that when a system of partial differential equations for state variables has a Lie group symmetry, a conservation law exists. In an information-maximized market, this conservation law appears as Equation (53), which means the net risk-adjusted return remains constant. This makes economic sense and enhances our understanding of how economies work: The parameter $\hat{\lambda}$, a Lagrange multiplier, can be seen as the average extra net return a firm needs above the interest rate, assuming its business activity is risk-free. If firms earn less than $\hat{\lambda}$, their operations would quickly become unprofitable. Conversely, if the average extra net return exceeds $\hat{\lambda}$, new competitors would enter the market, driving down returns until they settle at the natural constant $\hat{\lambda}$.

4.3 | Additivity Property of Sums of Independent Factors

The sum of independent squared Bessel processes evolving in the same time forms another squared Bessel process, as shown by Shiga and Watanabe (1973). This distinctive additivity arises from the specific partial differential equation governing the transition probability density of the squared Bessel process. These processes are particular types of SROU processes; see Revuz and Yor (1999). By applying Corollary 3, the following result is derived:

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Corollary 4. *For a set $\mathcal{A} \subseteq \{1, \ldots, n\}$ of indexes of independent factors, the respective* sum of independent factors

$$S_\tau^{\mathcal{A}} = \sum_{k \in \mathcal{A}} S_\tau^k \tag{57}$$

satisfies under the MMM the SDE

$$dS_\tau^{\mathcal{A}} = \hat{\lambda} S_\tau^{\mathcal{A}} d\tau + d_{\mathcal{A}} B_\tau e^\tau d\tau + \sqrt{S_\tau^{\mathcal{A}} 4 B_\tau e^\tau} dW_\tau^{\mathcal{A}} \tag{58}$$

of an SROU process with net risk-adjusted return $\hat{\lambda}$, dimension

$$d_{\mathcal{A}} = \sum_{k \in \mathcal{A}} \frac{4}{n}, \tag{59}$$

and initial value

$$S_{\tau_{t_0}}^{\mathcal{A}} = \sum_{k \in \mathcal{A}} S_{\tau_{t_0}}^k = \sum_{k \in \mathcal{A}} Y_{\tau_{t_0}}^k e^{\tau_{t_0}}, \tag{60}$$

where $W_\tau^{\mathcal{A}}$ is a Brownian motion with stochastic differential

$$dW_\tau^{\mathcal{A}} = \frac{1}{\sqrt{S_\tau^{\mathcal{A}}}} \sum_{k \in \mathcal{A}} \sqrt{S_\tau^k} dW_\tau^k \tag{61}$$

for $\tau \in [\tau_{t_0}, \infty)$ with initial value $W_{\tau_{t_0}}^{\mathcal{A}} = 0$. The respective normalized sum of factors

$$Y_{\tau_\tau^{\mathcal{A}}}^{\mathcal{A}} = \frac{S_\tau^{\mathcal{A}}}{B_\tau e^\tau} = \sum_{k \in \mathcal{A}} Y_{\tau_\tau^k}^k \tag{62}$$

satisfies the SDE

$$dY_\tau^{\mathcal{A}} = \left(d_{\mathcal{A}} - Y_\tau^{\mathcal{A}}\right) d\tau + \sqrt{4 Y_\tau^{\mathcal{A}}} dW_\tau^{\mathcal{A}} \tag{63}$$

and evolves as a CIR process of dimension $d_{\mathcal{A}}$ in the market time $\tau \in [\tau_{t_0}, \infty)$.

The result demonstrates that factor sums have a unique additive property under the MMM. The sum of all independent factors, called the FP S_τ^{FP}, is particularly important. Using Corollary 4, the Itô formula, and Equation (7), we conclude the following:

Corollary 5. *The FP, which is the sum of the factors,*

$$S_\tau^{FP} = \sum_{k=1}^n S_\tau^k, \tag{64}$$

follows under the MMM a time-transformed squared Bessel process of dimension four. The normalized FP

$$Y_{\tau_\tau^{FP}}^{FP} = \sum_{k=1}^n Y_{\tau_\tau^k}^k = \frac{S_\tau^{FP}}{B_\tau e^\tau} \tag{65}$$

represents a CIR process of dimension four in market time with average 4.

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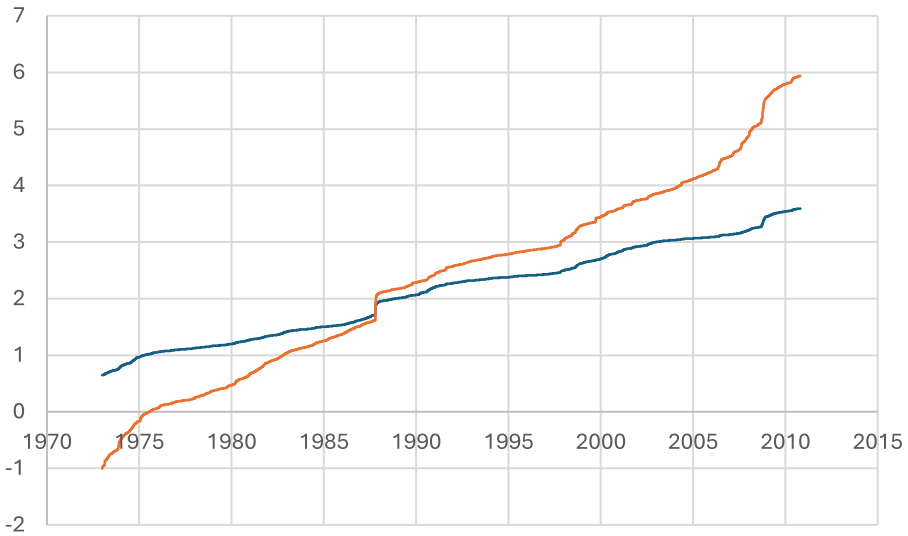


FIGURE 2 | Market time τ_t (blue) and benchmark time $\tau^*_{\tau_t}$ (red). [Color figure can be viewed at wileyonlinelibrary.com]

4.4 | Illustrations

For the SROU process S^{FP}_τ evolving in the market time τ_t one obtains via application of the Itô formula its activity time via the formula

$$\tau_t = \ln\left([\sqrt{S^{FP}_\cdot}]_{t_0,t} + e^{\tau_{t_0}} \right) \tag{66}$$

for $t \in [t_0, \infty)$; see Platen (2025). The quadratic variation $[\sqrt{S^{FP}_\cdot}]_{t_0,t}$ is observable. The only parameter needed to determine activity time is the initial activity time τ_{t_0}. For stock market indices, activity time typically evolves in an almost linear manner, making it easy to estimate the initial value through linear regression; see Platen (2025). This estimation only needs to be done once, after which the initial activity time becomes a known parameter. Once established, it remains unchanged, allowing for straightforward calculation of the current activity time when pricing contingent claims. By setting the maturity of a contingent claim as a fixed activity time value of the underlying security and finely discretizing calendar time, research by

Platen (2025) and Platen and Fergusson (2025) has shown that hedge errors for inexpensive zero-coupon bonds are extremely small. Only noticeable discretization errors arise when activity time moves faster than expected in calendar time, resulting in larger activity time steps. The outstanding hedges achieved are due to the observed activity time capturing all features of the security trajectory not explained by diffusion dynamics modeled in activity time. Most of these features can be accounted for using Assumption **A1**, as shown in Platen and Rendek (2019).

For illustration, consider the MSCI World Total Return stock index (MSCI) in US savings account denomination as proxy for the FP S^{FP}_τ and the EWI114 constructed in Platen and Rendek (2012) as proxy for the benchmark S^*_τ. The latter refers to an equally weighted portfolio of 114 global industry subsector indices, which can be considered as factors. Figure 1 shows the logarithms of both stock portfolios, revealing their roughly linear progression. Figure 2 displays the market time τ_t and the benchmark time $\tau^*_{\tau_t}$, both following nearly linear paths. As illustrated in Platen (2025) and Platen (2026), for a fixed maturity at the end of the dataset measured in benchmark time, an inexpensive zero-coupon bond, paying out one unit from the savings account, has been priced using BN pricing and hedging, with the EWI114 serving as a proxy. The resulting hedge error, shown in Figure 3, is less than 0.0001 units of the savings account at maturity, where one unit equals the payout. Notably, hedging inexpensive zero-coupon bonds with very long maturities turns out to be less costly than traditional risk-neutral pricing suggests, an insight highlighted by Platen (2026) and significant for pension and insurance industries.

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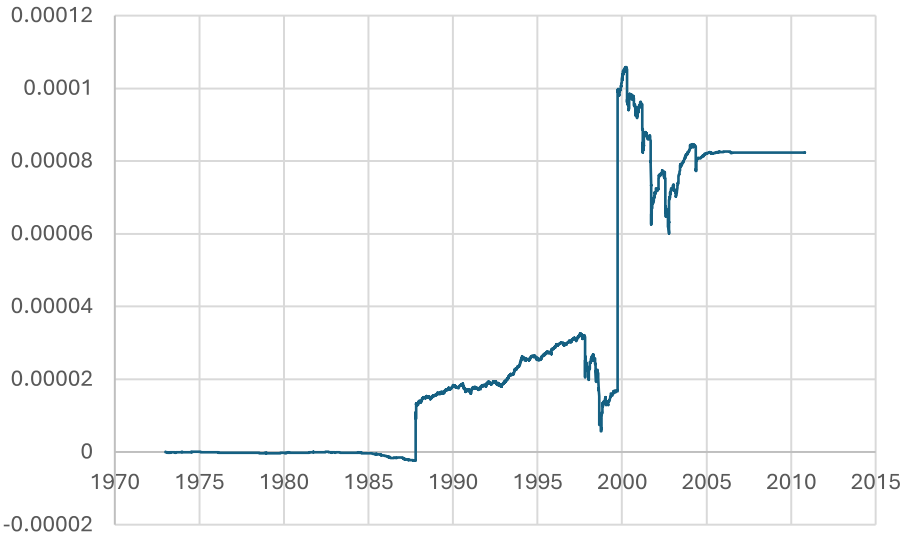


FIGURE 3 | Hedge error for inexpensive zero-coupon bond. [Color figure can be viewed at wileyonlinelibrary.com]

4.5 | Empirical Evidence

Let us compare predicted properties with empirical data from the real market, as outlined in the introduction: By design, stock market index volatilities are stationary processes; see Engle (1982). The Student-t density with four degrees of freedom emerges when estimating the log-return density of a security whose squared volatility follows an inverse gamma distribution with four degrees of freedom; see, for example, Hurst and Platen (1997) or Platen and Rendek (2008). When the MSCI is viewed as the FP, the MMM predicts its log-return density as a Student-t density with four degrees of freedom, which Fergusson and Platen (2006) found to have high significance among numerous competing models. Breymann et al. (2009) demonstrated that, across many observation frequencies, the hypothesis that MSCI log-returns follow a Student-t distribution with approximately four degrees of freedom cannot be easily dismissed when valued in major currencies. Similar findings were obtained for the S&P500 in Markowitz and Usmen (1996a) and in Hurst and Platen (1997) for stock indices of various countries.

Since for an SROU process the volatility is proportional to the inverse of its square root, the leverage effect, observed by Black (1976a), is endogenously built into the MMM dynamics of stock indices and several other securities. The 3/2 stochastic volatility model for stock indices of the MMM explains the clustering and heteroscedasticity of observed volatilities. Moreover, the rough volatility paths observed, for example, in Bayer et al. (2016), can be explained, as is shown in Platen and Rendek (2019), due to the fact that the market activity evolves in market time but is viewed in calendar time, which generates volatility spikes. The paper Platen and Rendek (2019) demonstrates that the market activity under the MMM is modeling the fast moving component of the stock market index volatility, whereas the inverse of the square root of the normalized stock market index represents its much slower moving component. This aligns with the findings in Fouque et al. (1999). The logarithm of a stock market index is under the MMM, by construction, approximately linearly evolving in a mean-reverting manner, as visible in Figure 1 for the MSCI. As shown in Platen and Fergusson (2025) and Figure 3, the MMM delivered extremely accurate hedges for thousands of inexpensive extreme maturity bonds.

Under the MMM, benchmark dynamics follow an SROU process with dimension four in benchmark time. Platen and Rendek (2012) constructed the EWI114 as a proxy for the world stock market benchmark, which shows strong average growth compared to the MSCI, as shown in Figure 1. Similarly, Platen and Rendek (2008) introduced the EWI104; its log-return density is shown to align with a Student-t distribution with four degrees of freedom, consistent with MMM predictions.

Goard and Mazur (2013) found that the 3/2-volatility model of the MMM outperforms other stochastic volatility models when applied to VIX derivatives. Mandelbrot (2001) observed self-similarity in stock indices, which the MMM also predicts through its modeling of sums of independent factors as squared Bessel processes; see Revuz and Yor (1999). Thus, the MMM accounts for key empirical facts and can be considered to represent a "realistic" financial market model.

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Conclusion

Based on four assumptions, a parsimonious, idealized information-minimized financial market model has been derived. The stock market index and the GOP of the stocks are under this model and under the BN pricing measure squared Bessel processes that evolve in respective activity times. No parameters are required for the accurate BN pricing and hedging of a wide range of contingent claims. Only the observable market time and the observable benchmark value enter the respective formulas for prices and hedges. The findings indicate that the financial market could be viewed as a communication system, and concepts from information theory are particularly pertinent to the field of finance. Further research could revisit long standing questions in finance from the perspective of this paper and may lead to interesting answers.

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Data Availability Statement

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

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Appendix A

Proof of Theorem 1

Proof. By applying Theorem 3.1 in Filipović and Platen (2009) for $\tau \in [\tau_{t_0}, \infty)$, we obtain from (2) with the SDE 3.10 in Theorem 3.1 in Filipović and Platen (2009) for the portfolio S^{π}_{τ} with weight vector $\pi_\tau = (\pi^1_\tau, \ldots, \pi^n_\tau)^\top$ for the holdings in the factors $\mathbf{S}_\tau$ the SDE

$$\frac{dS^{\pi}_{\tau}}{S^{\pi}_{\tau}} = \pi^\top_\tau \frac{d\mathbf{S}_\tau}{\mathbf{S}_\tau} = \hat{\lambda}_\tau d\tau + \sum_{k=1}^{n} \pi^k_\tau \beta^k_\tau (\beta^k_\tau \omega^k d\tau + dW^k_\tau). \tag{A.1}$$

By the matrix equation (3.5) in Theorem 3.1 in Filipović and Platen (2009) it follows for the k-th factor-GOP weight

$$(\beta^k_\tau)^2 \pi^{*,k}_\tau + \hat{\lambda}_\tau = \hat{\lambda}_\tau + (\beta^k_\tau)^2 \omega^k, \tag{A.2}$$

which yields

$$\pi^{*,k}_\tau = \omega^k \tag{A.3}$$

for $k \in \{1, \ldots, n\}$ and $\tau \in [\tau_{t_0}, \infty)$. By applying Equation (3.8) in Filipović and Platen (2009) we obtain the k-th benchmark volatility

$$\pi^{*,k}_\tau \beta^k_\tau = \omega^k \beta^k_\tau \tag{A.4}$$

for $k \in \{1, \ldots, n\}$ and $\tau \in [\tau_{t_0}, \infty)$. Furthermore, we obtain by Equation (3.4) in Theorem 3.1 in Filipović and Platen (2009) and (A.3) the sum

$$\sum_{k=1}^{n} \omega^k = \sum_{k=1}^{n} \pi^{*,k}_\tau = 1, \tag{A.5}$$

which proves Equation (7). By (A.3) one obtains the Equation (8) and the SDE (9), which completes the proof of Theorem 1. □

Appendix B

Proof of Theorem 2

Proof. Since the savings account $S^0_\tau \equiv 1$ is a traded security in the extended market, it follows from Theorem 3.1 in Filipović and Platen (2009) that the risk-adjusted return of this market is zero. For a savings account denominated portfolio S^{π}_{τ}, which invests with the weight π^0_τ in the savings account S^0_τ and with the weight π^k_τ in the k-th factor S^k_τ, $k \in \{1, \ldots, n\}$, it follows by Equation (3.8) and Equation (3.11)

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in Theorem 3.1 in Filipović and Platen (2009) that this portfolio equals the NP S_τ^{**} when it satisfies the SDE

$$\frac{dS_\tau^{**}}{S_\tau^{**}} = \sum_{k=1}^{n} \theta_\tau^{**,k} \left(\theta_\tau^{**,k} d\tau + dW_\tau^k \right) \tag{B.1}$$

with

$$\theta_\tau^{**,k} = \pi_\tau^{**,k} \beta_\tau^k, \tag{B.2}$$

which proves Equation (14). By Equation (3.8) in Theorem 3.1 in Filipović and Platen (2009) we have the k-th market price of risk and by the matrix equation (3.5) in Theorem 3.1 in Filipović and Platen (2009) the equation

$$(\beta_\tau^k)^2 \pi_\tau^{**,k} = \hat{\lambda}_t + (\beta_\tau^k)^2 \omega^k, \tag{B.3}$$

which is solved by the optimal k-th NP weight

$$\pi_\tau^{**,k} = \frac{\hat{\lambda}_\tau}{(\beta_\tau^k)^2} + \omega^k \tag{B.4}$$

and yields the k-th market price of risk

$$\theta_\tau^{**,k} = \frac{\hat{\lambda}_\tau}{\beta_\tau^k} + \omega^k \beta_\tau^k \tag{B.5}$$

for $k \in \{1, \dots, n\}$ and $\tau \in [\tau_{t_0}, \infty)$. We obtain by (B.4) and (7) the NP weight

$$\pi_\tau^{**,0} = 1 - \sum_{k=1}^{n} \pi_\tau^{**,k} = 1 - \hat{\lambda}_\tau \sum_{k=1}^{n} (\beta_\tau^k)^{-2} - 1 = -\hat{\lambda}_\tau^* \sum_{k=1}^{n} (\beta_\tau^k)^{-2} \tag{B.6}$$

to be invested in the savings account S_τ^0. This completes the proof of Theorem 2. □

Appendix C
Proof of Theorem 3

Proof. We perform the maximization of the surprisal

$$\mathcal{I}(p_\infty) = \sum_{k=1}^{n} \mathcal{I}(p_\infty^k), \tag{C.1}$$

of the stationary joint density p_∞ in several steps.

1. At first, we derive the stationary densities of the normalized factors. For $k \in \{1, \dots, n\}$, the stationary density p_∞^k of the k-th normalized factor, when it is evolving in the market time τ, is by the SDE (28) the solution of the stationary Fokker–Planck equation

$$\frac{d}{dy} \left(p_\infty^k(y) y ((\phi^k(y))^{-1} 4\omega^k - 1) a^k \right) - \frac{1}{2} \frac{d^2}{dy^2} \left(p_\infty^k(y) y^2 \frac{4a^k}{\phi^k(y)} \right) = 0, \tag{C.2}$$

as described, for example, in Chapter 4 in Platen and Heath (2006), which is a second-order ODE. Its solution is given by the formula

$$p_\infty^k(y) = \frac{C_k \phi^k(y)}{4y^2} \exp\left\{ 2 \int_1^y \frac{\omega^k - \frac{\phi^k(u)}{4}}{u} du \right\} \tag{C.3}$$

for $y \in (0, \infty)$ and some constant $C_k > 0$. The latter ensures that p_∞^k is a probability density.

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2. Under the constraints (23) and (24), we maximize according to (30) the surprisal

$$\mathcal{I}(p^k_\infty) = -\int_0^\infty \ln(p^k_\infty(y))p^k_\infty(y)dy \tag{C.4}$$

of the stationary probability density p^k_∞ of the k-th independent normalized factor $Y^k_{\tau^k_\cdot}$, $k \in \{1,\dots,n\}$. According to the formula (30), we maximize the Lagrangian

$$\begin{aligned}\mathcal{L}(p^k_\infty,\lambda_0,\lambda_1,\lambda_2) = &-\int_0^\infty \ln(p^k_\infty(y))p^k_\infty(y)dy + \lambda_0\left(\int_0^\infty p^k_\infty(y)dy - 1\right)\\ &+\lambda_1\left(\int_0^\infty y p^k_\infty(y)dy - 4\omega^k\right) + \lambda_2\left(\int_0^\infty \ln(y)p^k_\infty(y)dy - \zeta^k\right),\end{aligned} \tag{C.5}$$

where $\lambda_0,\lambda_1,\lambda_2$ are Lagrange multipliers. $\mathcal{L}(p^k_\infty,\lambda_0,\lambda_1,\lambda_2)$ is maximized when its Fréchet derivative $\delta\mathcal{L}(p^k_\infty,\lambda_0,\lambda_1,\lambda_2)$, i.e., the first variation of $\mathcal{L}(p^k_\infty,\lambda_0,\lambda_1,\lambda_2)$ with respect to admissible variations of p^k_∞, becomes zero. This implies for the surprisal-maximized stationary density p^k_∞ the equation

$$\delta\mathcal{L}(p^k_\infty,\lambda_0,\lambda_1,\lambda_2) = \int_0^\infty \left(-\ln(p^k_\infty(y)) + \lambda_0 + \lambda_1 y + \lambda_2\ln(y)\right)\delta p^k_\infty(y)dy = 0. \tag{C.6}$$

The solution of the above first-order condition is the gamma density

$$p^k_\infty(y) = \exp\{\lambda_0 + \lambda_1 y + \lambda_2\ln(y)\} \tag{C.7}$$

for $y \in (0,\infty)$ with the constraint

$$\int_0^\infty \exp\{\lambda_0 + \lambda_1 y + \lambda_2\ln(y)\}dy = 1, \tag{C.8}$$

and the Lagrange multipliers $\lambda_0,\lambda_1,\lambda_2$. It has $2(\lambda_2+1)$ degrees of freedom and as parameters and constraints the averages

$$\mathbf{E}^{p^k_\infty}(Y^k_\cdot) = \frac{\lambda_2+1}{-\lambda_1} = 4\omega^k \tag{C.9}$$

and

$$\mathbf{E}^{p^k_\infty}(\ln(Y^k_\cdot)) = \zeta^k.$$

3. The SDE for the k-th normalized factor is given by (28). Consequently, the stationary density $p^k_\infty(y)$ of the k-th normalized factor satisfies the Fokker–Planck equation with the drift and diffusion coefficient functions of the SDE (28). This yields the stationary density $p^k_\infty(y)$ in the form (C.3). The latter must equal the above-identified gamma density. By setting both expressions for the stationary density equal, respective conditions for the function $\phi^k(y)$ emerge.

The Weierstrass Approximation Theorem states that a continuous function can be approximated on a bounded interval by a polynomial. When using a polynomial for characterizing $\phi^k(y)$ and searching for a match of the stationary density (C.3) with the gamma density (C.7), then one finds by comparing the coefficients of the possible polynomials that only the polynomial

$$\phi^k(y) = y \tag{C.10}$$

provides such a match. This yields for the k-th normalized factor process $Y^k_\cdot$ the stationary density

$$p^k_\infty(y) = \frac{C_k y}{4y^2}\exp\left\{2\int_1^y \frac{\omega^k - \frac{u}{4}}{u}du\right\} = \frac{y^{2\omega^k-1}}{2^{2\omega^k}\Gamma(2\omega^k)}\exp\left\{-\frac{y}{2}\right\} \tag{C.11}$$

for $y > 0$. The above density is the gamma density with $d_k = 4\omega^k$ degrees of freedom and mean $4\omega^k$. We assumed the logarithmic average of the stationary density to equal a constant ζ^k, which emerges as

$$\zeta^k = \mathbf{E}^{p^k_\infty}(\ln(Y^k_\cdot)) = \ln(2) + \psi(2\omega^k), \tag{C.12}$$

where the function $\psi(x)$ is the digamma function; see, for example, Abramowitz and Stegun (1972) or Johnson et al. (1995).

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4. The resulting CIR process is a stationary process where its initial value $Y^k_{\tau_{t_0}} = S^k_{\tau_{t_0}}$ is distributed according to its stationary density. This yields with

$$\sqrt{\mathcal{I}} = \sum_{k=1}^{n} \frac{1}{n} \sqrt{\mathcal{I}(p^k_\infty)} \tag{C.13}$$

the surprisal of the stationary joint density

$$\begin{aligned} \mathcal{I}(p_\infty) &= n \sum_{k=1}^{n} \frac{1}{n} \mathcal{I}(p^k_\infty) = n \sum_{k=1}^{n} \frac{1}{n} \left(\sqrt{\mathcal{I}(p^k_\infty)} \right)^2 \\ &= n \left((\sqrt{\mathcal{I}})^2 + \sum_{k=1}^{n} \frac{1}{n} \left(\sqrt{\mathcal{I}(p^k_\infty)} - \sqrt{\mathcal{I}} \right)^2 \right). \end{aligned} \tag{C.14}$$

The latter is minimized for

$$\sqrt{\mathcal{I}(p^k_\infty)} = \sqrt{\mathcal{I}}, \tag{C.15}$$

which means by (7) that

$$\omega^k = \frac{1}{n}. \tag{C.16}$$

According to Göing-Jaeschke and Yor (2003), the k-th normalized factor is a CIR process and, more generally, an SROU process that represents a unique strong solution of the SDE (31), which completes the proof of Theorem 3. □

Appendix D
Proof of Theorem 4

Proof. 1. Let us consider a market time $\tau \in \mathcal{T}$ when no normalized factor equals zero. By employing Theorem 1 with (9) and (27), it follows for the benchmark the SDE

$$\frac{dS^*_\tau}{S^*_\tau} = \hat{\lambda}_\tau d\tau + \sum_{k=1}^{n} \frac{1}{n} \sqrt{\frac{4a^k}{Y^k_{\tau^k_\tau}}} \left(\frac{1}{n} \sqrt{\frac{4a^k}{Y^k_{\tau^k_\tau}}} d\tau + dW^k_\tau \right). \tag{D.1}$$

By using (37), one can rewrite this SDE in the form

$$dS^*_\tau = \hat{\lambda}_\tau S^*_\tau d\tau + Z_\tau S^*_\tau d\tau + \sqrt{Z_\tau} S^*_\tau dW^*_\tau, \tag{D.2}$$

where the Brownian motion W^*_τ has the stochastic differential given in (38) with initial value $W^*_{\tau_{t_0}} = 0$. This proves the SDE (36) together with (37) and (38).

2. Similarly as in (18), one can introduce the normalized benchmark $Y^*_{\tau^*_\tau}$ via the formula (39). By application of the Itô formula, Equations (39) and (37) it follows that $Y^*_{\tau^*_\tau}$ satisfies the SDE

$$\begin{aligned} \frac{dY^*_{\tau^*_\tau}}{Y^*_{\tau^*_\tau}} &= \hat{\lambda}_\tau d\tau + \left(Z_\tau - a^*_\tau \right) d\tau + \sqrt{Z_\tau} dW^*_\tau \\ &= \hat{\lambda}_\tau d\tau + \left(\frac{4}{Y^*_{\tau^*_\tau}} - 1 \right) a^*_\tau d\tau + \sqrt{\frac{4a^*_\tau}{Y^*_{\tau^*_\tau}}} dW^*_\tau \end{aligned} \tag{D.3}$$

and is forming an SROU process of dimension four satisfying the SDE (41) that evolves in the benchmark time

$$\tau^*_\tau = \tau^*_{\tau_{t_0}} + \int_{\tau_{t_0}}^{\tau} a^*_s ds \tag{D.4}$$

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with the benchmark activity

$$a_\tau^* = \frac{1}{4} Z_\tau Y^*_{\tau_\tau^*}. \tag{D.5}$$

This proves the remaining statements of Theorem 4. □

Appendix E

Proof of Theorem 5

Proof. For a surprisal-maximized market one can write the Radon-Nikodym derivative $\Lambda_{Q_{S^*}}(\tau)$, given in (44), of the BN pricing measure Q_{S^*} as the product

$$\Lambda_{Q_{S^*}}(\tau) = \prod_{k=1}^{n} \Lambda^k_{Q_{S^*}}(\tau), \tag{E.1}$$

of the independent martingales

$$\Lambda^k_{Q_{S^*}}(\tau) = \exp\left\{ -\frac{1}{2} \int_{\tau_{t_0}}^{\tau} \frac{\hat{\lambda}^2 Y^k_{\tau_s^k}}{4a^k} ds - \int_{\tau_{t_0}}^{\tau} \sqrt{\frac{\hat{\lambda}^2 Y^k_{\tau_s^k}}{4a^k}} dW_s^k \right\} \tag{E.2}$$

for $k \in \{1, \ldots, n\}$ and $\tau \in [\tau_{t_0}, \infty)$. According to Equations (51), (E.2), and (23) we have to minimize the Kullback–Leibler divergence

$$I(p_\infty, q_\infty) = \frac{1}{2} \sum_{k=1}^{n} \mathbf{E}^{p_\infty} \left(\frac{\hat{\lambda}_\tau^2 Y^k_{\tau_\tau^k}}{4a^k} \right) = \frac{1}{2n} \sum_{k=1}^{n} \frac{\hat{\lambda}_\tau^2}{a^k} \to \min \tag{E.3}$$

of q_∞ from p_∞.

In (26), the average of the inverse of the square root of the activity in market time is set to

$$\sum_{k=1}^{n} \frac{1}{n} \sqrt{\frac{1}{a^k}} = 1. \tag{E.4}$$

This allows us to write the Kullback–Leibler divergence in the form

$$\begin{aligned} I(p_\infty, q_\infty) = \frac{\hat{\lambda}_\tau^2}{2n} \sum_{k=1}^{n} \frac{1}{a^k} &= \frac{\hat{\lambda}_\tau^2}{2} \left(\left(\sum_{k=1}^{n} \frac{1}{n} \sqrt{\frac{1}{a^k}} \right)^2 + \sum_{k=1}^{n} \frac{1}{n} \left(\sqrt{\frac{1}{a^k}} - 1 \right)^2 \right) \\ &= \frac{\hat{\lambda}_\tau^2}{2} \left(1 + \sum_{k=1}^{n} \frac{1}{n} \left(\sqrt{\frac{1}{a^k}} - 1 \right)^2 \right). \end{aligned}$$

This expression is minimized with respect to the activities if all activities are equal with value

$$a^k = 1, \tag{E.5}$$

which proves (52) and yields

$$I(p_\infty, q_\infty) = \frac{\hat{\lambda}_\tau^2}{2} \tag{E.6}$$

for $\tau \in [\tau_{t_0}, \infty)$.

Since the stationary probability densities p_∞ and q_∞ do not change over market time, we obtain the net risk-adjusted return

$$\hat{\lambda}_\tau = \sqrt{2I(p_\infty, q_\infty)} = \hat{\lambda} \tag{E.7}$$

as a constant. This proves (54) and, therefore, Theorem 5. □

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