



Thermal performance investigation of blade-shaped kerosene-Ag nanofluid in a top wavy octagonal cavity using RSM and ANN

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Abstract

This study explores the mathematical modeling of magnetohydrodynamics (MHD) in a two-dimensional dynamic scenario involving free convection (FC) and heat transfer (HT) with entropy generation (E_{gen}) within a wavy octagonal cavity filled with blade-shaped kerosene-Ag nanofluids, where nanoparticles exhibit Brownian motion. The lower wall maintains uniform thermal boundary conditions (TBC), while the undulating wall remains at a cooler temperature. All other boundaries of the domain are assumed to be adiabatic. The finite element method, combined with the central composite design (CCD) based response surface method (RSM) and artificial neural networks (ANN), is used to solve nonlinear PDE problems. Various parameters are investigated, including Hartmann number ($0 \leq Ha \leq 100$), Rayleigh number $10^3 \leq Ra \leq 10^6$, nanoparticles volume concentration ($0 \leq \phi \leq 0.04$), nanoparticles diameter $10nm \leq d_p \leq 40nm$, and wave numbers ($1 \leq \lambda \leq 3$). Streamlines and isotherms contour plots are used to display the thermal and flow behavior under various conditions. Increasing Ha shows that higher magnetic field strengths tend to suppress convective flow, thereby reducing the overall HT rate, while buoyancy forces enhance thermal transport. A higher concentration of Ag nanoparticles improves thermal conductivity and HT rate, whereas larger nanoparticle sizes reduce HT. Notably, the domain with three waves demonstrates superior HT rates compared to those with one or two waves as ϕ increases. In addition, the statistical analysis testing for the response function indicates high R^2 values (95.07%), which suggests a well-suited technique for estimating the Nusselt number (average). ANN shows a good alternative for predicting response function analysis. Furthermore, this research highlights the considerable potential for advancing industrial and engineering applications by exploring fluid flow and HT in intricate domains, including wavy octagonal cavities with kerosene-Ag nanofluids.

Keywords Nanofluid · Wavy octagon enclosure · FEM/RSM/ANN · Nanoparticle size · Brownian effect

List of symbols

A	Dimensionless amplitude of wave
B_0	Magnitude of magnetic field [$\text{kg s}^{-2} \text{A}^{-1}$]
C_p	Specific heat at constant pressure [$\text{J kg}^{-1} \text{K}^{-1}$]
Ha	Hartmann number

g	Gravitational acceleration [m s^{-2}]
k	Thermal conductivity [$\text{W m}^{-1} \text{K}^{-1}$]
L	Length of cavity [m]
N	Dimensionless outward normal vector
Nu_L	Local Nusselt number
Nu_{avg}	Mean Nusselt number
M	Dimensionless outward normal vector
Pr	Prandtl number
P	Non-dimensional pressure
Ra	Rayleigh number
T	Temperature of fluid [K]
U	Non-dimensional velocity along x-axis
V	Dimensionless velocity along y-axis
X	Non-dimensional x-axis
Y	Non-dimensional y-axis
n	Shape of nanoparticles
d_p	Size of nanoparticles

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Greek symbols

α	Thermal diffusivity [$\text{m}^2 \text{s}^{-1}$]
β	Thermal expansion coefficient [K^{-1}]
θ	Non-dimensional temperature
Δ	Dependent dimensionless variable
μ	Dynamic viscosity [$\text{kg m}^{-1} \text{s}^{-1}$]
ϕ	Solid volume fraction of nanoparticles
τ	Dimensionless time
ν	Kinematic viscosity [$\text{m}^2 \text{s}^{-1}$]
σ	Electric conductivity
ρ	Density [kg m^{-3}]
λ	Non-dimensional period of wave

Subscript

c	Cool surface
bf	Base fluid
sp	Solid particle
h	Hot surface
nf	Nanofluid

1 Introduction

The study of fluid dynamics and heat transfer (HT) through free convection (FC) has garnered significant attention for its practical relevance in a variety of fields, including heat exchange in petroleum reservoirs, electrical systems, nuclear reactor cooling, and vehicle cooling systems (Zafar et al. 2023). Free convection, driven by buoyancy forces, plays a significant role in these applications, where effective heat transfer is crucial for operational efficiency and reliability. Different researchers (Saboj et al. 2023a; Akter et al. 2024; Hosseinzadeh et al. 2021; Roshani et al. 2024) frequently utilize nanofluids (NF) to augment the rate of HT. When introducing complex domains, multiple factors come into play, including the type of wave (e.g., sine function or cosine function) and the number of waves. Considering these factors, the subsequent literature reviews are outlined and analyzed.

Numerous studies have examined the impact of different cavity shapes and nanofluid properties on heat transfer rates. For instance, Zarei et al. (2022) investigated HT in a square cavity filled with NF and featuring sinusoidal wavy walls of varying amplitudes and wavelengths. Their findings revealed that as the nanofluid volume fraction (NVF) increased from 0 to 0.04, the Nusselt number (average) increased by approximately 2.5 times. Cho (2020) analyzed the combined effects of wavy surfaces and porous media on heat transfer and entropy generation in a Cu – H_2O

nanofluid-filled square cavity. The study highlighted that convection heat transfer was dominant, particularly at higher Darcy and Rayleigh numbers, with complex circulation patterns observed. Barnoonet al. (2019) explored the interplay between natural convection and thermal radiation in non-Newtonian nanofluids within a porous cavity. Their findings revealed that both the inclination of the cavity and the size of internal cylinders had a significant impact on thermal performance and fluid flow patterns. Two-dimensional nanofluid flow across a porous stretched sheet with nonlinear thermal radiation and slip effects was examined by Zeeshan et al. (2023), with an emphasis on how these factors affect energy and heat transfer properties. Shirvan et al. (2017) investigated the effects of wavy surface characteristics in an NF-filled square domain with cosine corrugations on natural convective HT, demonstrating improvements in the Nusselt number (average) with increased NVF. They observed that Nu_{avg} improved with increasing NVF.

Arasteh et al. (2020) investigated heat transfer enhancement in a counter-flow sinusoidal parallel-plate heat exchanger. This study focuses on the effects of incorporating porous media using metal foam in the channels' divergent sections. Islam et al. (2023) examined the effects of heat generation and absorption on natural convective heat transfer in a wavy triangular cavity filled with nanofluid. The results indicate that both heat generation and absorption significantly impact the convective heat transfer characteristics, with the wavy geometry playing a crucial role in enhancing thermal performance. Shekaramiz et al. (2021a) explored MHD nanofluid free convection in a wavy triangular cavity, considering periodic temperature boundary conditions and velocity slip mechanisms. Their findings revealed significant impacts of these factors on convective heat transfer rates. Tang et al. (2017) explored natural convection (NC) HT using double sinusoidal wavy walls in an NF-filled cavity, revealing periodic fluctuations in the local Nu along the outer wavy wall and the superiority of this configuration over single sinusoidal walls. Islam et al. (2020, 2024) also studied the impact of magnetic fields on the natural convection heat transport performance of nanofluids. This research focuses on how different orientations and strengths of inclined magnetic fields influence the heat transfer characteristics and fluid flow patterns in nanofluids.

The behavior of heat and mass transfer was found to be significantly influenced by chemical processes and non-linear thermal radiation when Hosseinzadeh et al. (2019) investigated their effects on Maxwell fluid flow across a convectively heated porous plate. A computational analysis on heat and fluid transfer in a cavity holding a ternary nanofluid (Cu - MgO - ZnO/water) under the influence of a magnetic field was carried out by Alipour et al. (2023a) with the goal of maximizing its thermal performance. Barman and Rao (2021) investigated the aspect ratio's effect on FC within a

wavy porous cavity with a partial heat source, demonstrating the enhancement of cooling efficiency with an increasing aspect ratio. By examining the optimization of hybrid nanoparticle fluid flow in a porous enclosure under the combined influence of thermal radiation and a magnetic field, Zabeti et al. (2024) highlighted the impact on heat transfer efficiency and hydrothermal performance. Rao and Barman (2021) studied FC within a wavy porous cavity with a partial heat source, highlighting the dependence of convection on porosity at high Ra and the intensified convection with increased surface roughness. Khalil et al. (2022) optimized magnetohydrodynamic FC within a trapezoidal cavity with a porous wavy bottom wall, achieving the highest HT rate at $Ha = 40$, $Ra = 2.5 \times 10^4$, and $T_c = 0^\circ\text{C}$ while identifying the lowest Nu_{avg} at $Ha = 0$, $Ra = 5 \times 10^2$, and $T_c = 25^\circ\text{C}$.

Nabwey et al. (2023) conducted a study on the dynamics of an inclined wavy porous cavity housing a square obstacle, illustrating the impact of radiative effects on the unsteady magnetohydrodynamics (MHD) natural convective flow of a hybrid NF. They observed that Nu_{avg} increased with the rise in the NVF in the cavity, while the isotherms experienced a slight reduction with higher Ha . Ali et al. (2021) analyzed the hydromagnetic NC flow of a hybrid NF within a wavy-walled cavity containing a heat-generating cylinder. Their results indicated that the presence of a heat-generating obstacle in a wavy cavity amplified the intensity of fluid motion compared to a smooth cavity without obstruction. Alsabery et al. (2020) investigated the average energy generation (E_{gen}) and flow characteristics of hybrid NFs within a partially divided wavy cavity including solid blocks their research revealed that employing hybrid NF enhanced Nu in comparison to conventional NF. Afsana et al. (2021) explored E_{gen} and MHD natural convection in a wavy cavity filled with non-Newtonian ferrofluid and observed a decrease in E_{gen} by 80% in the absence of a magnetic field and by 54% in its presence with the increase in the power-law index. Al-Kouz et al. (2021) analyzed heat transfer and entropy generation for water- $\text{Fe}_3\text{O}_4/\text{CNT}$ hybrid magnetic nanofluid flow in a trapezoidal wavy enclosure with porous media. Boulahia (2021) numerically investigated different E_{gen} and MHD fluid flow scenarios within an NF-filled corrugated cavity with a hot object in the middle. They generally observed an increase in HT rate and mean E_{gen} with higher corrugation numbers and Ra . By employing an Ag-MgO hybrid nanofluid to study conjugate natural convection in a corrugated porous enclosure, Al-Dulaimi et al. (2024) showed better heat transfer and flow circulation as a result of improved thermal conductivity.

Dogonchi et al. (2021) investigated the effects of FC and E_{gen} within a nano liquid-filled crown wavy cavity. Their findings showed that Da , Ra , and NVF all enhanced. Additionally, they observed that the geometry featuring a wavy base wall and a cylinder on the left side performs best in terms of HT. Shekaramiz et al. (2021b) studied MHD nanofluid

FC within a wavy triangular cavity, considering velocity slip mechanisms and periodic temperature boundary conditions. Their results showed that, with increasing Ha in high Ra conditions, E_{gen} initially increased and then stabilized. Cho et al. (2016) examined FC and E_{gen} in an inclined wavy-wall cavity filled with $\text{Al}_2\text{O}_3\text{-H}_2\text{O}$ NF. They found that, for a constant Ra and inclination angle, Nu in a regular cavity is lower compared to that in a wavy-surface cavity, while total E_{gen} is also lower. Saleem et al. (2023) investigated natural convective HT in an L-shaped cavity filled with NF and subjected to a magnetic field and time-periodic temperature boundary. They observed that Nu_{avg} increased linearly with increasing NVF for Ha values from 0 to 50, while it remained steady at $Ha = 75$ and 100. Sheremet et al. (2016) studied MHD natural convection in an NF-filled inclined wavy cavity with a corner heater. Their results revealed that both HT rate and convective flow rate decreased with increasing Ha .

Hatami and Safari (2016) explored the effects of NF-filled wavy-wall domains with a heated cylinder. In their study, the cylinder was movable in X and Y directions, which affected the temperatures, streamlines, and concentration contours of nanoparticles. Acharya (2022) investigated the hydrothermal behavior and entropy analysis of free convective MHD hybrid NF flow inside an octagonal cavity equipped with fins for thermal energy storage. The study concluded that entropy increased with NVF and Ra , while it decreased with magnetic number. Similar impacts were observed for heat transport, and higher fin length decreased entropy. Saha et al. (2024a) explored the dynamic MHD of TiO_2 -kerosene NF flow within a wavy octagonal cavity. Their findings revealed a decline in HT rate with increased Lorentz force, while buoyancy force led to its augmentation. Islam et al. (2022a) studied unsteady flow with magnetic field and uniform/non-uniform temperature in a half-moon cavity. Their findings showed that a non-uniform magnetic field results in a greater Nu_{avg} compared to a uniform field. Saha et al. (2023a) investigated the influence of heated fins on MHD free convection flow of NF in a wavy square cavity. They observed a 7.65% rise in HT rate with blade-shaped nanoparticles and a 2.86% increase with spherical-shaped nanoparticles. For additional recent literature, refer to Saboj et al. (2024), Saha et al. (2023b, 2024b, c), Ikram et al. (2021, 2023, 2024), Saboj et al. (2023b), Alipour et al. (2023b).

Islam and Nasrin (2022), Islam et al. (2022b, 2021), Islam (2024) studied unsteady convective heat transfer in a quarter-circular domain under MHD effects, considering the Brownian motion of nanoparticles. Using FEM and RSM, this study found that heat transfer improves with smaller particles, Brownian motion, and specific magnetic field conditions, with maximum heat transfer ($Nu = 24.045$). Optimizing a wavy trapezoidal porous cavity with a mixture of hybrid nanofluids (ethylene glycol Go - Al_2O_3) using the response surface approach is investigated by Alipour et al.

(2023b). Ziaur et al. (2024) analyzed mixed convection in a magnetically influenced square cavity with a star-shaped obstacle filled with hybrid nanofluid, showing that heat transfer increases with additional nanoparticles but decreases with Ha , with a notable 49.54% rise in heat transfer as ϕ increases from 1 to 10%. The high R^2 value (98.72%) suggests the model's reliability for estimating heat transfer rates (Nu). Bairagi et al. (2024) explored thermophysical effects in a cavity using Al_2O_3 nanofluid and found that as Ha increases, fluid velocity decreases while temperature and Nusselt number rise. The highest Nusselt number occurs when $Ha = 0$, while $Ha = 45$ results in the lowest. Their ANN model achieved 99% accuracy with a mean square error of 0.00069. Using FEM, RSM, and ANN methodologies to execute heat-line and sensitivity analyses, Uddin et al. (2025) carried out a comparative investigation on MHD nanofluid convection in a square chamber with a circular heater, augmenting our comprehension of heat transfer processes in such structures.

Despite the latest developments, there is still no comprehensive numerical framework that incorporates FEM, ANN, and RSM to investigate MHD free convection in wavy octagonal cavities with blade-shaped kerosene-Ag nanofluid. This study aims to address this research gap by examining unsteady MHD fluid flow and natural convective heat transfer in an octagonal cavity with wavy top wall configurations, using blade-shaped kerosene-Ag nanofluid. By varying the number of waves, this research seeks to offer new insights into optimizing cavity configurations for improved heat transfer performance, incorporating streamlines and isotherms, as well as to evaluate the impact of significant factors such as Ra , Ha , ϕ , n , and d_p on the average Nu . A CCD is used to organize the computational experiments, and ANOVA is employed to verify the model's correctness. The goal is to provide insights applicable to practical scenarios like heat exchangers and cooling systems, focusing on optimizing cavity configuration for maximum HT rates. This research stands out for its numerical integration of MHD effects and Brownian motion in this domain, as well as its exploration of different wave configurations in a wavy cavity. By varying the number of waves, this research aims to provide new insights into optimizing cavity configurations for enhanced heat transfer performance. By investigating these complex interactions, the study contributes to a deeper understanding of how geometric and material parameters can be tuned for efficient thermal management in engineering applications. The following are some key points emphasizing the study's novelty:

- With the FEM technique, the ANN and RSM are deployed to optimize HT with the generation of entropy in a wavy octagonal cavity under MHD influences.

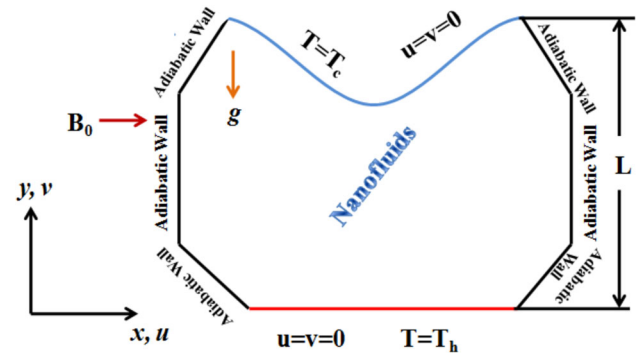


Fig. 1 Physical domain of our study

- Using CFD simulation data, an ANN is utilized to estimate HT with respect to average Nu , enabling accurate estimation without continuously resolving complex numerical models.
- A comprehensive study is carried out utilizing isotherms and streamlines to give physical insight into how various factors affect the flow in nanofluid.
- To ensure the reliability of optimization and predicted outcomes, the statistical validity of the RSM-based models is extensively investigated using ANOVA.
- The best parameter amalgamations for HT efficiency are offered by RSM, which improves the optimization of the response functions (average Nu).

2 Mathematical model and physical domain

This study develops a mathematical model to examine MHD flow and HT of NF in a wavy octagonal cavity. The flow is treated as incompressible, two-dimensional, and time-dependent. All walls, except the upper and lower boundaries, are assumed to be adiabatic. The lower wall is uniformly heated with a specified temperature boundary condition (TBC), while the upper wall is maintained at a lower temperature, creating a temperature gradient. Specifically, the wavy top wall is cooled to a temperature T_c , whereas the heated bottom wall is maintained at a higher temperature T_h . Figure 1 illustrates the geometry along with the x and y axes. Furthermore, the thermo-physical characteristics of different nanoparticles and base fluids are available in Islam et al. 2022a, and the relevant are listed in Table 1.

2.1 Dimensional analysis

Employing a non-dimensional analysis, the governing equations with boundary constraints are simplified by the following non-dimensional variables (Islam et al. 2022a):

Table 1 Thermo-physical properties of base fluid and nanoparticles (Saleem et al. 2023)

Base fluid/nanoparticles	c_p [J kg ⁻¹ K ⁻¹]	ρ [kg m ⁻³]	k [W m ⁻¹ K ⁻¹]	μ [kg m ⁻¹ s ⁻¹]	$\beta \times 10^{-5}$ [K ⁻¹]	σ [Sm ⁻¹]	Pr
Kerosene	2090	780	0.149	0.00164	99	6.0×10^{-10}	23
Ag	233	10,500	429	–	1.8	6.30×10^7	–

At bottom wall: $U = 0$, $V = 0$, $\theta = 1$.

$$\left. \begin{aligned} X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{uL}{\alpha_{bf}}, V = \frac{vL}{\alpha_{bf}}, \theta = \frac{T-T_c}{T_h-T_c}, P = \frac{\rho L^2}{\rho_{bf} \alpha_{bf}^2}, \\ \tau = \frac{t \alpha_{bf}}{L^2}, \lambda = \frac{\lambda_0}{L}, A = \frac{a}{L} \end{aligned} \right\} \quad (1)$$

2.2 Non-dimensional governing equations (Islam et al. 2022a)

$$\left. \begin{aligned} \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} &= 0 \\ \frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} &= -\frac{\rho_{bf}}{\rho_{nf}} \frac{\partial P}{\partial X} \\ &+ Pr \frac{\mu_{nf}}{\mu_{bf}} \frac{\rho_{bf}}{\rho_{nf}} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \\ \frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} &= -\frac{\rho_{bf}}{\rho_{nf}} \frac{\partial P}{\partial Y} \\ &+ Pr \frac{\mu_{nf}}{\mu_{bf}} \frac{\rho_{bf}}{\rho_{nf}} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{(\rho\beta)_{nf}}{\rho_{nf} \beta_{bf}} Ra Pr \theta \\ &- \frac{\rho_{bf}}{\rho_{nf}} \frac{\sigma_{nf}}{\sigma_{bf}} Ha^2 Pr V \\ \frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} &= \frac{\alpha_{nf}}{\alpha_{bf}} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \end{aligned} \right\} \quad (2)$$

The dimensionless numbers are defined as follows; Prandtl number is $Pr = \frac{\nu_{bf}}{\alpha_{bf}}$, Rayleigh number is, $Ra = \frac{g \beta_{bf} (T_h - T_c) L^3}{\mu_{bf} \alpha_{bf}}$, and $Ha = B_o L \sqrt{\sigma_{bf} / \mu_{bf}}$ is the Hartmann number.

2.3 Dimensionless initial and boundary conditions

For $\tau = 0$; entire domain:

$$U = 0, V = 0, \theta = 0, P = 0 \quad (3)$$

For $\tau > 0$: Top wall surface equation is: $Y = a \cos 2\pi \lambda X$ where $\lambda = 1, 2, 3$

At top wall: $U = 0$, $V = 0$, $\theta = 0$.

At vertical walls: $U = 0$, $V = 0$, $\frac{\partial \theta}{\partial X} = 0$.

At inclined walls: $U = 0$, $V = 0$, $\frac{\partial \theta}{\partial M} = 0$.

2.4 Nanofluid properties

The thermophysical properties of the nanofluid have an immense effect on improving its heat transmission performance. Viscosity, density, thermal diffusivity, heat capacitance, thermal expansion coefficient, electrical conductivity, and thermal conductivity are recognized as the thermal and physical traits of nanofluids that are considered as follows (Islam et al. 2022a):

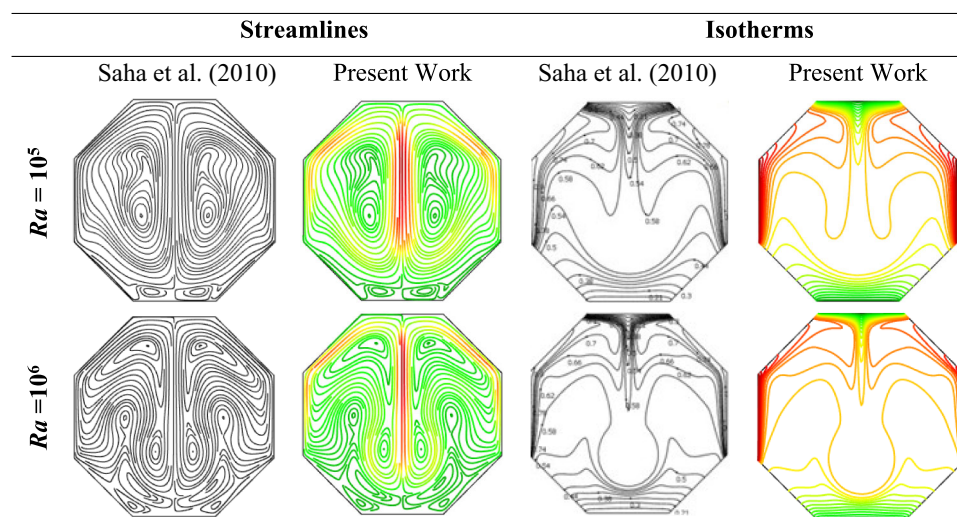
$$\begin{aligned} \mu_{nf} &= \mu_{bf} \frac{1}{(1 - \phi)^{2.5}} \\ \rho_{nf} &= (1 - \phi) \rho_{bf} + \phi \rho_{sp} \\ \alpha_{nf} &= \frac{k_{nf}}{(\rho c_p)_{nf}} \\ (\rho c_p)_{nf} &= (1 - \phi) (\rho c_p)_{bf} + \phi (\rho c_p)_{sp} \\ (\rho \beta)_{nf} &= (1 - \phi) (\rho \beta)_{bf} + \phi (\rho \beta)_{sp} \\ \sigma_{nf} &= \frac{\sigma_{sp} + 2\sigma_{bf} - 2(\sigma_{bf} - \sigma_{sp})\phi}{\sigma_{sp} + 2\sigma_{bf} + (\sigma_{bf} - \sigma_{sp})\phi} \sigma_{bf} \\ k_{nf} &= \frac{k_{sp} + (n - 1)k_{bf} - (n - 1)(k_{bf} - k_{sp})\phi}{k_{sp} + (n - 1)k_{bf} + (k_{bf} - k_{sp})\phi} k_{bf} \\ &+ \frac{\phi \rho_{sp} c_{p,sp}}{2} \sqrt{\frac{2K_B T_{ref}}{3\pi d_p \mu_{nf}}} \end{aligned} \quad (4)$$

Also, the local and average Nusselt numbers are defined as (Islam et al. 2022a):

$$\begin{aligned} Nu_L &= \frac{L q_w}{k_{bf} (T_h - T_c)}; \quad q_w = -k_{nf} \left(\frac{\partial T}{\partial y} \right)_{y=0}, \\ Nu_{avg} &= - \left(\frac{k_{nf}}{k_{bf}} \right) \int_0^1 \frac{\partial \theta}{\partial Y} dX \end{aligned} \quad (5)$$

Table 2 Variation of average E_{gen} and comparisons

Ra	Ikram et al. (2024)	Saboj et al. (2023b)	Islam et al. (2022a)	Present study
10^3	1.418	1.402	1.411	1.409
10^4	4.150	4.138	4.142	4.139
10^5	8.673	8.532	8.637	8.602

Fig. 2 The variation of streamlines and isotherms patterns for $Ra = 10^5$ and $Ra = 10^6$ of Saha et al. (2010), and the present study

Entropy generation due to heat transfer (E_{HT}) is defined by

$$E_{HT} = \frac{k_{nf}}{k_{bf}} \left[\left(\frac{\partial \theta}{\partial x} \right)^2 + \left(\frac{\partial \theta}{\partial y} \right)^2 \right] \quad (6)$$

3 Code validation and computational procedure

3.1 Code validation

In Table 2, values of average E_{gen} for different Ra values are presented, and compares the results with previous studies Ikram et al. (2024), Saboj et al. (2023b) and Islam et al. (2022a). It is seen that for $Ra = 10^3$, the maximum percentage error observed is 0.635%, similarly, for $Ra = 10^4$ showed a marginal difference of 0.265%, while for $Ra = 10^5$ presented a slightly higher deviation of 0.818%. A good agreement is seen with a maximum percentage error of less than 1%. To further validate the accuracy of the present numerical method, a comparison was performed with the results reported by Alnaqi et al. (2019), using streamline contours and isothermal lines at $Pr = 0.71$ as shown in Fig. 2. An

exceptional level of acceptance is shown by the result, indicating the accuracy and reliability of the present numerical code utilized in this experiment.

3.2 Computational procedure

We systematically explore the fluid dynamics within a complex octagonal domain using numerical methods. Utilizing the COMSOL Multiphysics software, we employ the finite element method (FEM) to solve a set of Eqs. (1–2) governing the system. Our focus lies on simulating blade-shaped kerosene-Ag nanofluids, accounting for the Brownian motion of nanoparticles. Employing a finely discretized triangular mesh, we analyze the velocity and temperature variations across both cooled and heated boundaries, as depicted in Fig. 3. The FEM-generated equations are efficiently solved using the stationary linear direct solver PARDISO, with convergence ensured by setting a relative tolerance of 10^3 . By reducing errors to a manageable level, this convergence criteria guarantees that the numerical solution is sufficiently correct.

This study optimizes thermal transport parameters using both RSM and ANN along with to the FEM CFD solver. The consequences of Ra , Ha and ϕ on the average Nu can be quickly and efficiently evaluated using RSM. This process is greatly enhanced by ANN, which captures complex nonlinear correlations and improves the optimization

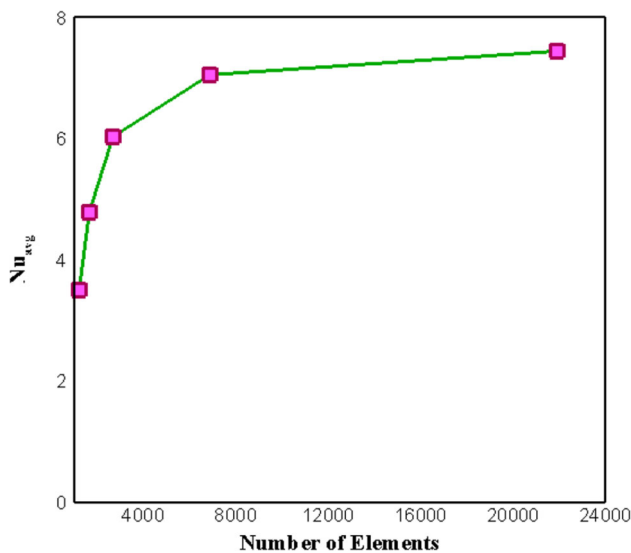


Fig. 3 Variation of Nu_{avg} for different number of elements when $Ra = 10^6$, $Ha = 100$, $\phi = 0.04$, $n = 8.6$, $d_p = 40$ nm and $\tau = 1$

results. When coupled, the two methods provide a complementary approach: RSM provides systematic sensitivity insights, while ANN enhances forecasts and model precision. Compared to employing solely traditional FEM solvers, this work combines RSM and ANN to optimize thermal system performance more fully and effectively. All RSM calculations were performed using Design-Expert software to simulate dependent variable responses.

3.3 Grid independence

To verify the adequacy of the mesh in the intricate octagonal cavity, a non-uniform mesh analysis was conducted for specific parameters: $Ra = 10^6$, $Ha = 100$, $\phi = 0.04$, $n = 8.6$, $d_p = 40$ nm and $\tau = 1$. Five non-uniform grid variations were examined, containing: 1202, 1638, 2652, 6841 and 21,867 elements. Numerical calculations of Nu_{avg} were conducted for different grid sizes, and the results are evaluated. As depicted in Fig. 3, this evaluation guides the refinement of the grid to enhance Nu_{avg} . Based on this comparative analysis, a grid containing 6841 elements were chosen for the simulations.

4 Results and discussion

In this section, findings are exhibited in terms of streamlines, isotherms, and Nu (average) for Ra varying from 10^3 to 10^6 , Ha varying from 0 to 100, and ϕ varying from 0 to 0.04. The particle diameter (d_p) varies from 10 to 40 nm and the number of waves (λ) ranges from 1 to 3. As shown in Figs. 4, 5, 6, 7, 8, 9, 10 and Table 3.

4.1 Effects of Rayleigh number

Figure 4 illustrates the influence of Ra on streamline contours and isothermal lines for kerosene-Ag nanofluid at various λ values ranging from 1 to 3, with uniform TBC under both unsteady state ($\tau = 0.1$) and steady-state ($\tau = 0.7$). This analysis was conducted with fixed parameters: $\phi = 0.04$, $Ha = 20$, and $n = 8.6$. Under unsteady conditions at $Ra = 10^5$, two oppositely directed symmetric circulating cells are observed within the cavity. As the solution progresses towards its steady state, a robust circulation pattern emerges. The size of the cells' cores varies slightly with different λ values; however, overall, λ does not induce significant changes. At higher Ra ($Ra = 10^6$), for $\lambda = 1$, the left cell is larger than the right cell in both steady and unsteady states. For $\lambda = 2$, two nearly symmetric circulating cells with opposite directions are observed under both conditions. In the steady state at $\lambda = 3$, the right-sided cell is larger than the left one, whereas under unsteady conditions, the cells are nearly symmetric.

These observations can be attributed to the thermal boundary conditions. Specifically, the heated wall causes the fluid adjacent to it to rise due to buoyancy forces, while the cooler kerosene-Ag nanofluid near the wavy wall moves downward towards the heated wall. This creates a symmetric flow pattern, characterized by the circulating fluid within the cavity. The Ra 's impact on flow and heat transfer is crucial as it signifies the balance between buoyant and viscous forces within the fluid. Higher Ra values indicate stronger buoyancy forces, which enhance convective currents and improve heat transfer efficiency. The detailed behavior of the flow, as depicted in the streamlines and isotherms, helps in understanding how the cavity geometry and Ra interact to optimize thermal management in complex domains.

In the case of isotherm contours at $Ra = 10^5$, regardless of the values λ , the isothermal lines exhibit a similar pattern under unsteady conditions. These lines form narrow bands starting from the bottom wall and gradually align parallel to the upper wall. This pattern persists in the steady state, where the isotherms curve from the upper-left corner of the cavity to the lower-right corner. At $Ra = 10^6$, the isothermal lines show significant distortion. For $\lambda = 1$, the distortion occurs on the right side of the domain, while for $\lambda = 2$, it is centralized. In the case of $\lambda = 3$, the distortion is centralized in the unsteady state and shifted to the left side in the steady state. These observations highlight the significant role of conduction in HT, particularly at lower buoyancy-driven parameters. As Ra increases, which indicates stronger buoyant forces, the intensity of fluid currents also increases, leading to enhanced convective forces. With rising Ra , the isothermal lines near the heated wall become increasingly distorted. This distortion reflects the transition from conduction-dominated heat transfer to convection-dominated heat transfer as Ra rises.

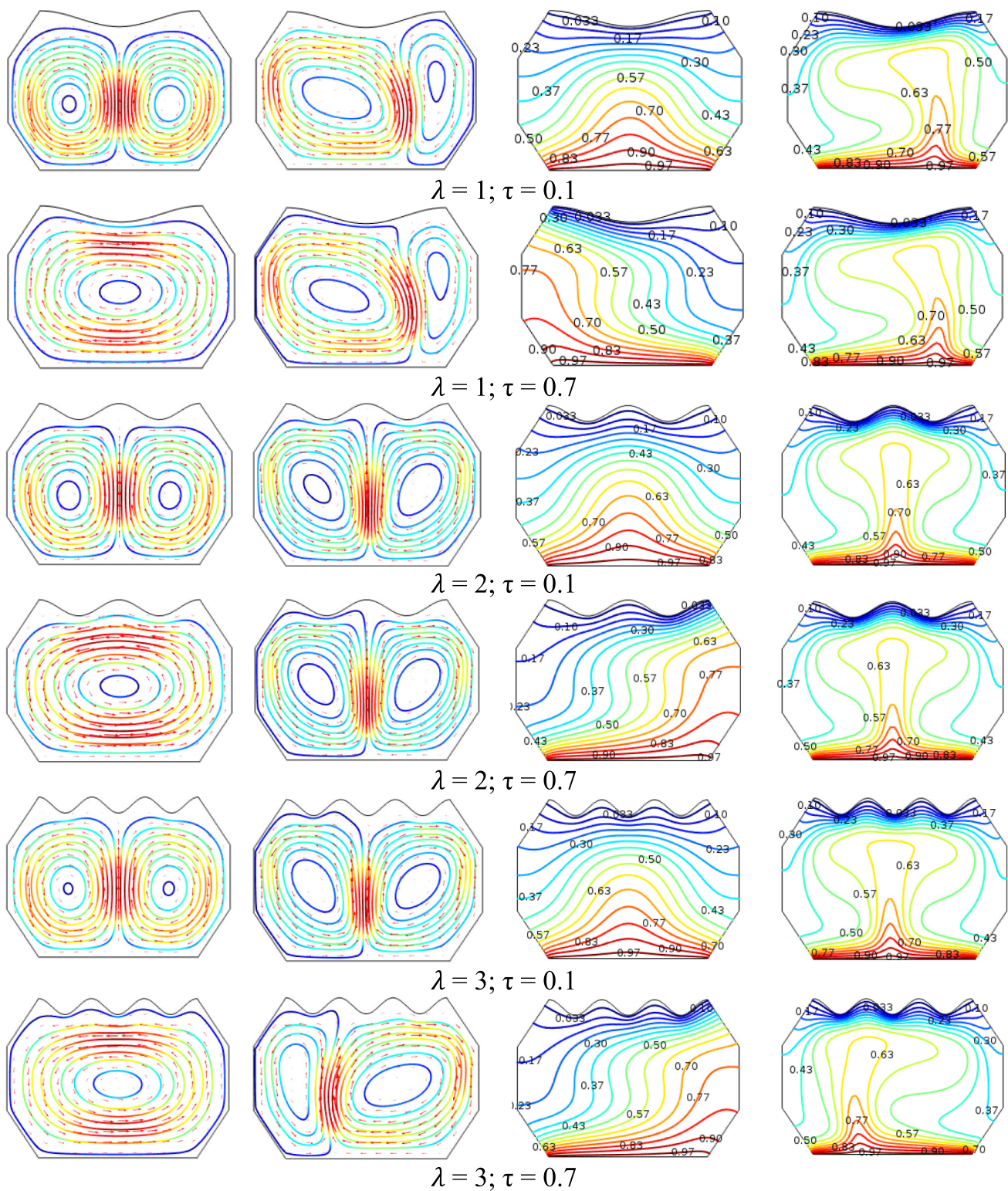
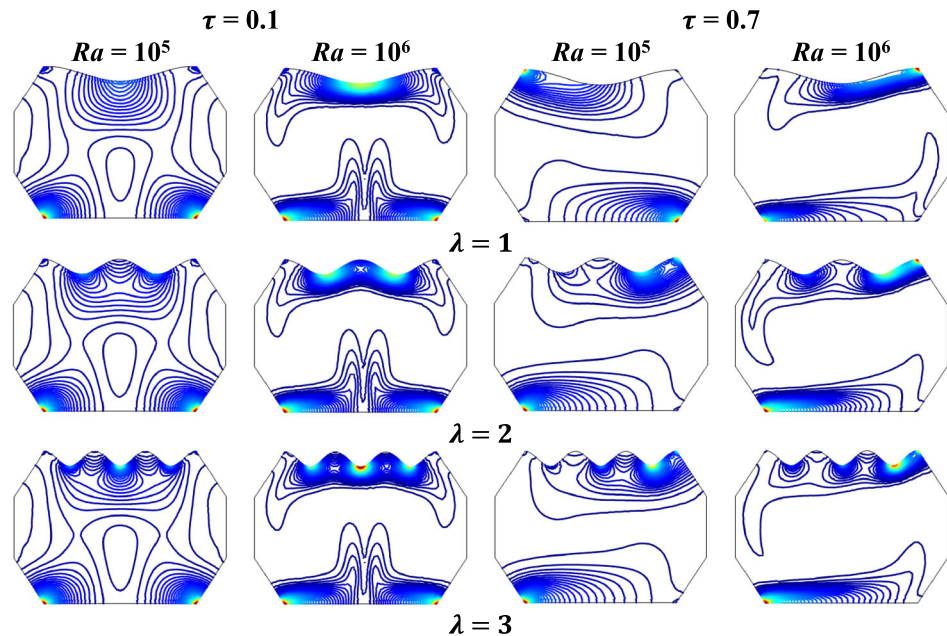


Fig. 4 Variation of streamlines and isotherms for different Ra for $\phi = 0.04$, $Ha = 20$, $n = 8.6$, and $d_p = 10$ nm

Fig. 5 Entropy generation due to heat transfer (E_{HT}) for different λ and Ra when $Ha = 20$, $\phi = 0.04$, $n = 8.6$, $d_p = 10$ nm



The formation of pronounced convection cells and the distortion of isotherms signify that convection becomes the dominant mode of heat transfer within the cavity as buoyancy forces increase. At lower Ra values, the gradual alignment of isotherms suggests that conduction plays a more significant role when buoyant forces are weaker, whereas, at higher Ra values, convection begins to dominate the thermal behavior within the cavity.

Figure 5 demonstrates the entropy generation due to heat transfer (E_{HT}) across different Ra . Generally, E_{HT} increases significantly as Ra rises. This is because a higher Ra indicates stronger NC, which leads to enhanced HT rates. The elevated HT and associated fluid motion result in greater thermal gradients and shear stresses, both of which contribute to an increase in E_{HT} . When a magnetic field is present, higher Ra also leads to stronger current flows, further amplifying E_{HT} . This is due to the interaction between the thermal and electrical fields, commonly referred to as magnetohydrodynamic (MHD) effects. For a fixed Ra , increasing the undulation number results in irregular variations in E_{HT} . These variations are linked to changes in the flow pattern, which may lead to localized regions of enhanced viscous dissipation and altered current dynamics. This shift in flow structure, combined with the complex interactions between thermal and electrical fields, further contributes to total E_{gen} .

4.2 Effects of Hartmann number

Figure 6 illustrates the streamlines and isotherms influenced by the Ha in a kerosene-Ag NF, considering various values of λ (ranging from 1 to 3), with uniform TBC under both

unsteady states ($\tau = 0.1$) and steady-state ($\tau = 0.7$) conditions. This analysis is performed with fixed parameters: $\phi = 0.04$, $Ra = 10^5$, and $n = 8.6$. In the absence of a magnetic field ($Ha = 0$), the streamline patterns exhibit two symmetric circulation cells: one clockwise and other counterclockwise, within the enclosure across all λ values in both steady and unsteady states. However, at $\lambda = 3$, a slight asymmetry is noticed, with the right-sided cells appearing slightly larger than the left-sided ones under both conditions. This asymmetry is attributed to the wavy nature of the cavity walls, which influences the overall flow dynamics. When the magnetic field is strongest ($Ha = 100$), regardless of λ , four smaller cells are discernible within the cavity, with streamlines clustering near the bottom cells. Although the overall circulation pattern remains similar, subtle variations in cell size and configuration are noted. Specifically, for $\lambda = 2$, additional cells are observed near the upper boundary under unsteady conditions, while one extra cell appears in the same region under steady conditions. These changes highlight the effect of the magnetic field on the flow structure.

The impact of the applied magnetic field strength is evident in the flow behavior, as stronger fields result in a noticeable reduction in flow intensity. This phenomenon is due to the Lorentz force generated by the magnetic field, which opposes fluid motion and consequently weakens the circulation patterns within the cavity. As Ha increases, the Lorentz force impedes the fluid's velocity, resulting in smaller, more numerous cells. The isothermal lines exhibit a banded pattern predominantly concentrated in the central region of the cavity when no magnetic field is applied ($Ha = 0$). The isotherms gradually form an inverted “jump” shape

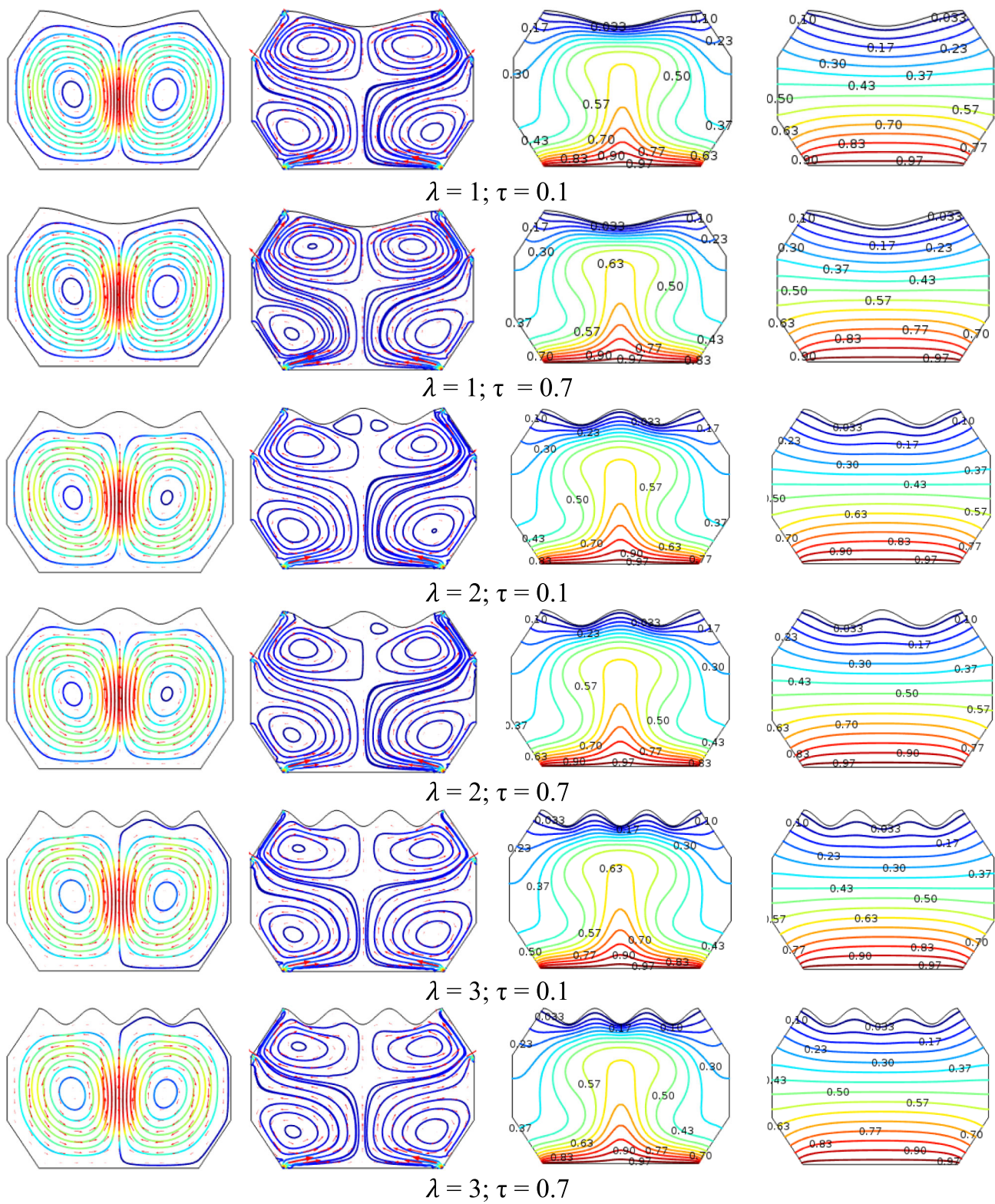


Fig. 6 Variation of streamlines and Isotherms for different Ha for $\phi = 0.04$, $Ra = 10^5$, $n = 8.6$, and $d_p = 10$ nm

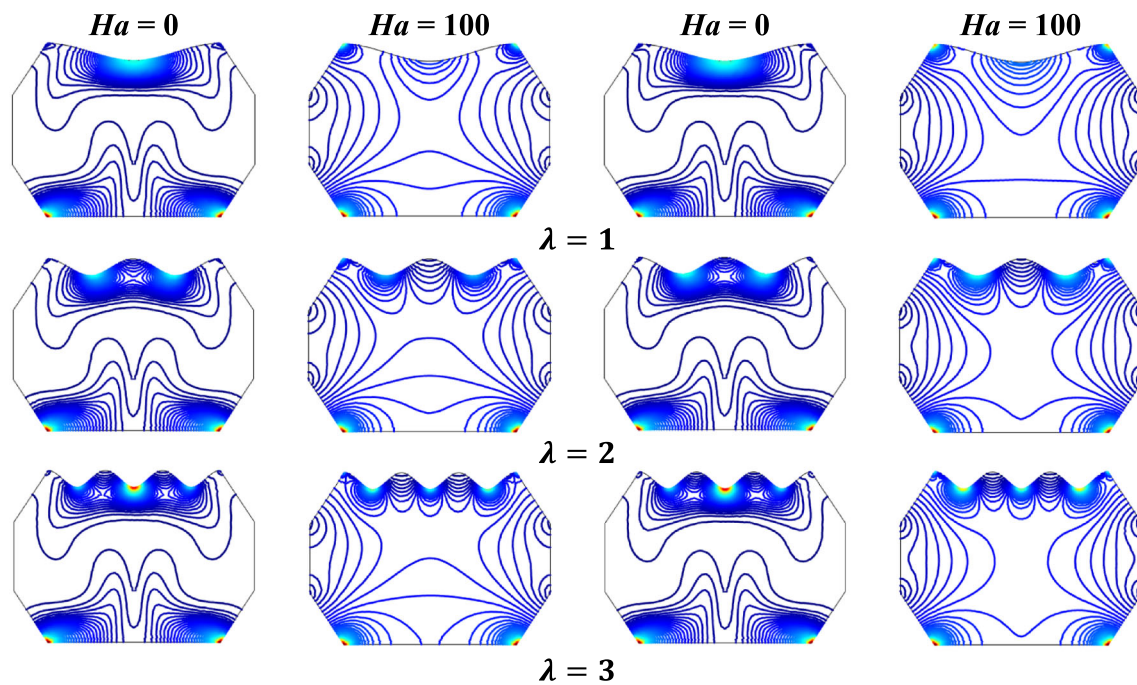


Fig. 7 Entropy generation due to heat transfer (E_{HT}) for different λ and Ha when $Ra = 10^5$, $\phi = 0.04$, $n = 8.6$, $d_p = 10$ nm

towards the center of the bottom wall before aligning parallel to the upper wall, a behavior consistently observed in both steady and unsteady states. This pattern reflects the balance between thermal conduction and natural convection. With a higher magnetic field strength ($Ha = 100$), the isothermal lines become uniformly parallel across all λ values. The parallel alignment of the isotherms indicates that convection is suppressed and heat transfer is increasingly dominated by conduction. As the Lorentz force intensifies, the isothermal lines shift progressively away from the heated bottom wall, indicating a reduction in the temperature gradient within the cavity. This shift reflects the magnetic field's influence on diminishing convective heat transfer and altering the thermal distribution in the cavity.

Figure 7 represents the relationship between E_{HT} and the Ha for different undulation numbers when $Ra = 10^5$, $\phi = 0.04$, $n = 8.6$, $d_p = 10$ nm. In general, the application of a magnetic field, represented by Ha , tends to reduce the E_{HT} . As Ha increases, the magnetic field imposes greater resistance to buoyancy-driven fluid motion, resulting in a reduction in the E_{HT} . This reduction in the E_{HT} can be attributed to the suppression of natural convection HT. As the magnetic field strengthens, the Lorentz force exert a damping effect on fluid movement, thereby reduces the convective HT. The diminished convective currents lead to lower temperature differentials within the fluid, reducing the E_{HT} . At a constant Ha , the E_{HT} exhibits a noticeable increase with rising undulation numbers. This behavior can be linked to the changes in the flow structure caused by the geometric

undulations, which introduce more complex fluid motion and potentially higher localized thermal gradients. These undulations, by altering the path of the flow, can lead to increased viscous dissipation and enhanced interactions between the thermal and magnetic fields, contributing to the overall E_{gen} . Additionally, the interactions between the magnetic field and the undulating surfaces can modify the distribution of Lorentz forces, further complicating the E_{HT} patterns.

4.3 Effects of nanoparticles volume fraction

Figure 8 illustrates the impact of the nanoparticle volume fraction (ϕ), ranging from 2 to 4%, on both streamlines and isotherms in a kerosene-Ag NF. The analysis considers various numbers of waves (λ) from 1 to 3, with uniform temperature boundary conditions (TBC) under both unsteady state ($\tau = 0.1$) and steady-state ($\tau = 0.7$), with fixed parameters $Ha = 20$, $Ra = 10^5$, and $n = 8.6$. At a lower nanoparticle volume fraction ($\phi = 2\%$), distinct symmetric circulations in opposite directions are observed within the enclosure for all λ values in both steady and unsteady states. These symmetric circulations are indicative of balanced buoyancy-driven convection within the fluid. In contrast, when ϕ is increased to 4%, the flow patterns exhibit notable changes. Under unsteady conditions, symmetric circulations in opposite directions persist across all λ values, reflecting the enhanced convective activity due to the higher volume fraction. However, under steady conditions, a single dominant circulation pattern emerges throughout the domain for any λ .

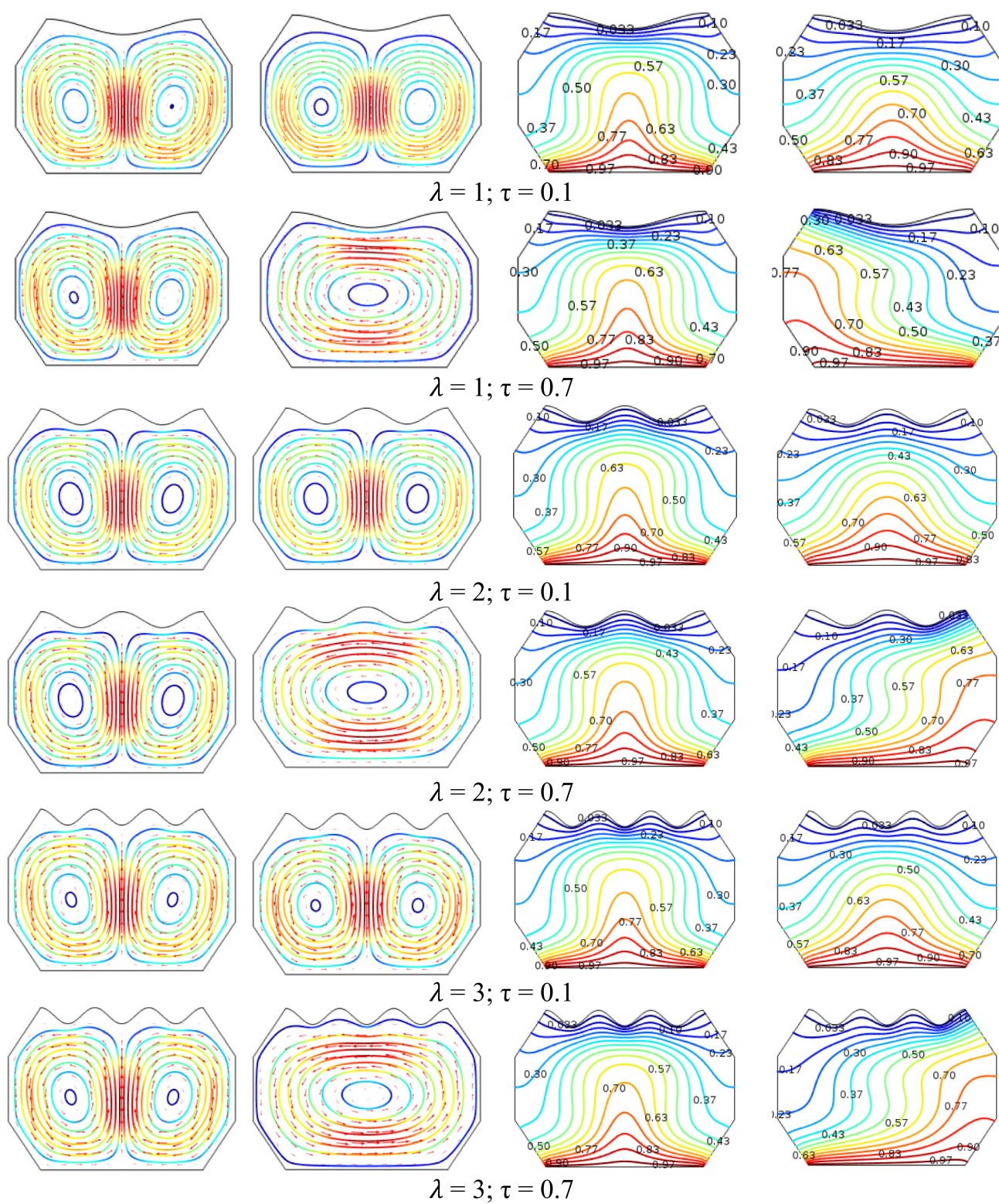


Fig. 8 Variation of streamlines and isotherms for different ϕ for $Ra = 10^5$, $n = 8.6$, $Ha = 20$, and $d_p = 10$ nm

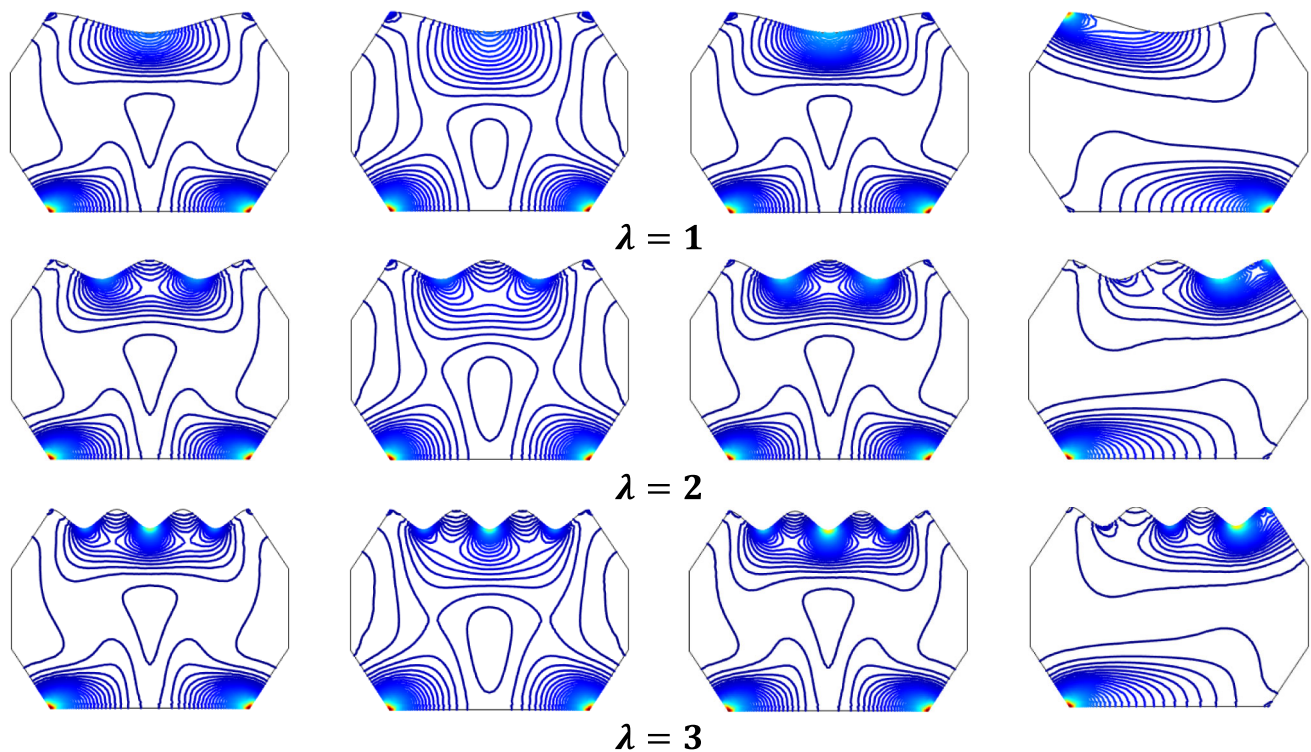


Fig. 9 Entropy generation due to heat transfer (E_{HT}) for different λ and ϕ when $Ra = 10^5$, $Ha = 20$, $n = 8.6$, $d_p = 10$ nm

This shift is due to the increased mass and viscosity of the fluid caused by the higher nanoparticle concentration, which amplifies the fluid's inertia and slightly reduce the overall flow velocity. Additionally, the increased viscosity reduces the effectiveness of fluid mixing and particle interactions within the cavity.

Examining the isotherm patterns, for $\phi = 2\%$, the isothermal lines initially form narrow bands in the middle of the cavity, regardless of λ . As the fluid warms and mixes, these bands gradually widen and take on an inverted jump-like shape before aligning parallel to each other towards the top wall. This pattern reflects a transitional thermal distribution influenced by both conduction and convection. For $\phi = 4\%$, the isothermal lines show different behavior. Under unsteady conditions, the isothermal lines exhibit pronounced curvature near the middle of the domain, particularly near the lower wall, and become nearly parallel as they approach the top wall. This curvature indicates a more complex thermal gradient due to the increased fluid viscosity and slower convective mixing. Under steady conditions, the curvature of the isothermal lines is dependent on λ : for $\lambda = 1$, the lines curve from the upper-left corner to the lower-right corner of the enclosure; whereas for $\lambda = 2$ and 3, the curvature extends from the upper right corner to the lower left corner. These changes in curvature highlight the impact of nanoparticle volume fraction

on the thermal distribution and flow patterns, demonstrating how increased particle concentration alters both the flow dynamics and heat transfer characteristics within the cavity.

Figure 9 illustrates the relationship between E_{HT} and various values of ϕ for different numbers of undulations. The presence of nanoparticles in the fluid alters both the thermal conductivity and viscosity, which directly influence HT and fluid motion, leading to corresponding changes in E_{HT} . As ϕ increases, the enhanced effective thermal conductivity of the nanofluid typically improves HT rates. This increase in HT can result in higher temperature gradients, which contribute to greater E_{HT} .

Table 3 illustrates Nu_{avg} at the heated bottom wall for various values of ϕ and λ , respectively, at fixed $Ha = 50, 75$, and 100, where $n = 8.6$, $d_p = 10$ nm, and $\tau = 1$, across different Ra ranging from 10^3 to 10^6 . Despite Ha remaining constant, the HT increases as Ra , ϕ , and λ increase. This suggests that Ra , ϕ , and λ have a significant impact on HT even when Ha is unchanged. This is because a higher Ra indicates stronger buoyancy forces, which enhance convective heat transfer. This increase emphasizes the dominant role of natural convection in the system. Increasing the nanoparticle volume fraction (ϕ) enhances the effective thermal conductivity of the nanofluid, thereby improving heat transfer efficiency. The wavy geometry affects the flow patterns and thermal boundary layers, with different λ values altering the heat transfer

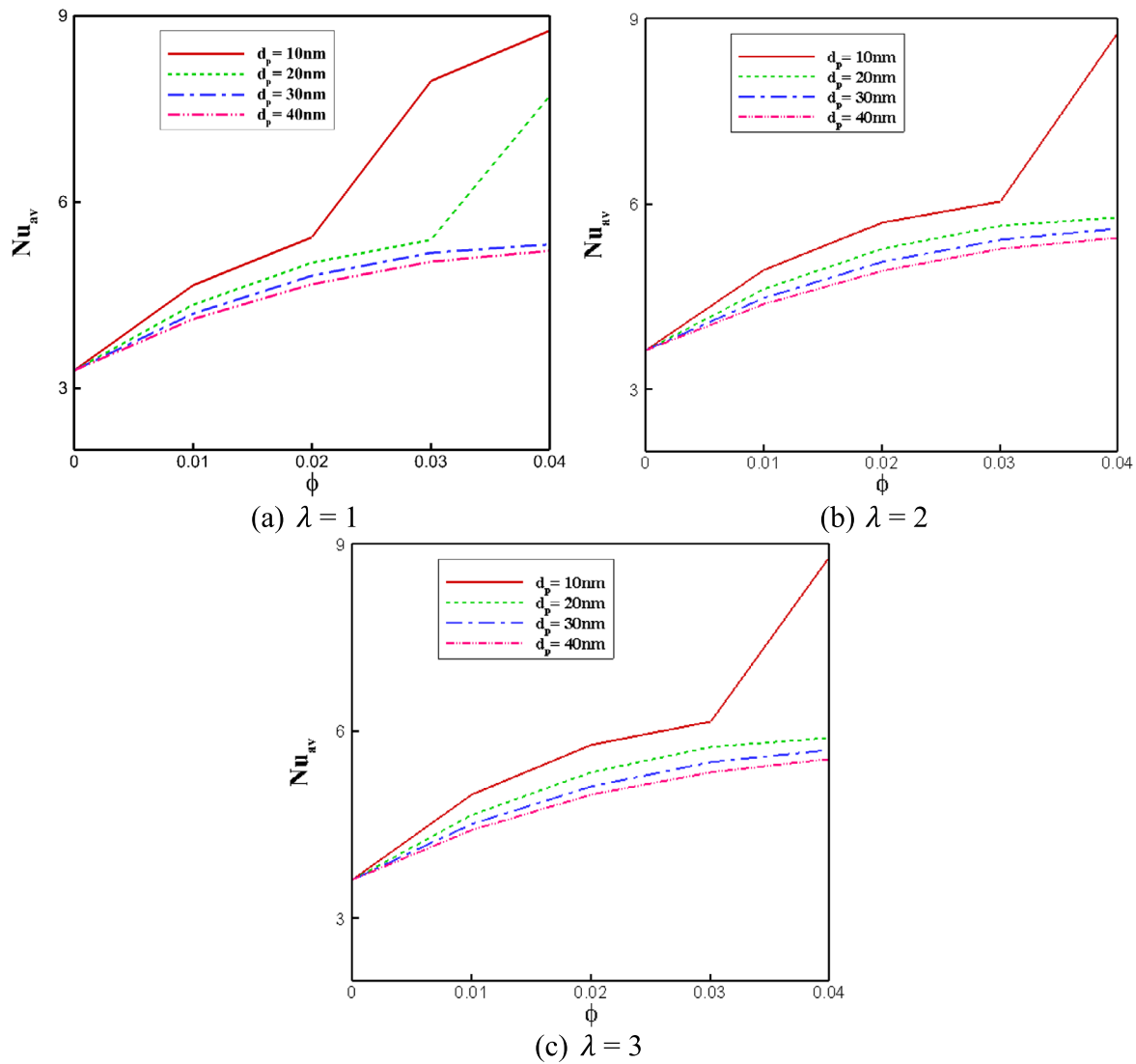


Fig. 10 Variation of Nu_{avg} with different ϕ and d_p

characteristics by impacting the fluid circulation and mixing within the cavity.

The table indicates that for $Ra = 10^6$, a noticeable shift occurs in the results between $\lambda = 1$ and $\lambda = 3$. Specifically, Nu_{avg} is highest for $\lambda = 1$ and lowest for $\lambda = 3$ when $Ra = 10^6$. This suggests that at very high Ra , the geometry's effect on HT becomes more pronounced, possibly due to the enhanced convective flows that dominate the heat transfer process. Summarizing the three tables, $\lambda = 3$ generally results in the highest heat transfer rate for most conditions. However, when the Ha is set at 75 and 100 and Ra is at $Ra = 10^6$, the situation changes: for these specific conditions $\lambda = 1$ yields the highest Nusselt number (average). This change suggests that under strong magnetic fields and high buoyancy-driven conditions, simpler wave structures ($\lambda = 1$) facilitate more efficient convective processes, possibly

due to reduced flow resistance and better alignment with the dominant flow structures in the presence of strong magnetic damping.

Figure 10 illustrates the variation of Nu_{avg} with different values of ϕ and d_p for three cases: (a) $\lambda = 1$, (b) $\lambda = 2$, and (c) $\lambda = 3$, under the conditions $Ra = 10^5$, $n = 8.6$, $Ha = 20$, and $\tau = 1$. When $\lambda = 1$, for $d_p = 10$ nm and 20 nm, the enhancement of Nu_{avg} shows irregular behavior with increasing ϕ . Specifically, between $\phi = 0.02$ and 0.03, the rate of increase is notably higher for $d_p = 10$ nm. For $d_p = 20$ nm, the increase follows a regular pattern up to $\phi = 0.03$, resembling a parabolic shape, with a subsequent acceleration in the rate of increase. Conversely, for $d_p = 30$ nm and $d_p = 40$ nm, the enhancement of Nu_{avg} follows a consistent pattern with increasing ϕ , displaying parabolic behavior. Similarly, when $\lambda = 2$, irregularities in the enhancement of Nu_{avg} are

Table 3 Variation of Nu_{avg} for the variety of ϕ and λ where, $n = 8.6$, $d_p = 10$ nm, and $\tau = 1$

ϕ	$Ra = 10^3, Ha = 50$			$Ra = 10^3, Ha = 75$			$Ra = 10^3, Ha = 100$		
	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$
0	1.24913	1.27268	1.27742	1.24902	1.27243	1.27461	1.17227	1.18822	1.21461
0.01	2.10642	2.13152	2.15261	2.10623	2.13076	2.15181	2.03731	2.06356	2.09616
0.02	2.95223	2.98152	3.00985	2.94596	2.94273	2.96108	2.88532	2.92144	2.96020
0.03	3.77493	3.80248	3.80716	3.76931	3.79131	3.80633	3.71525	3.76115	3.80627
0.04	4.58128	4.58455	4.63559	4.57624	4.58348	4.63482	4.52769	4.58299	4.63444

ϕ	$Ra = 10^4, Ha = 50$			$Ra = 10^4, Ha = 75$			$Ra = 10^4, Ha = 100$		
	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$
0	1.25718	1.27226	1.28378	1.25715	1.26456	1.27728	1.17426	1.26433	1.27716
0.01	2.11327	2.13866	2.15818	2.11323	2.13194	2.15244	2.03874	2.13174	2.15231
0.02	2.95226	2.98778	3.01502	2.94605	2.98188	3.01012	2.88639	2.98169	3.00998
0.03	3.77495	3.81511	3.85073	3.76943	3.81494	3.85062	3.71630	3.8148	3.85052
0.04	4.58129	4.63169	4.67466	4.57638	4.63156	4.67456	4.52820	4.63141	4.67443

ϕ	$Ra = 10^5, Ha = 50$			$Ra = 10^5, Ha = 75$			$Ra = 10^5, Ha = 100$		
	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$
0	2.22646	1.45445	1.50681	1.25719	1.27229	1.28381	1.24935	1.27223	1.28380
0.01	2.73307	2.92213	2.96094	2.11329	2.13870	2.15821	2.10575	2.13868	2.1582
0.02	2.95228	2.99509	3.11829	2.94629	2.98781	3.01504	2.88839	2.9878	3.01504
0.03	3.77496	3.82037	3.85513	3.76957	3.82035	3.85512	3.71649	3.82034	3.85506
0.04	4.58130	4.63642	4.67858	4.57650	4.63641	4.67857	4.52883	4.63639	4.67856

ϕ	$Ra = 10^6, Ha = 50$			$Ra = 10^6, Ha = 75$			$Ra = 10^6, Ha = 100$		
	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$
0	6.10677	5.80811	5.88464	4.46134	4.36903	4.79143	4.29004	3.13652	3.01631
0.01	7.79279	7.57941	8.02084	6.04819	5.83331	5.47113	4.76848	3.38115	3.34296
0.02	8.25938	9.29082	8.50239	6.95736	5.87245	6.01111	7.16749	3.71822	3.68347
0.03	9.85341	10.47125	10.12028	9.33321	6.32226	7.78684	7.80149	7.69204	7.78384
0.04	10.78022	11.21441	11.06311	9.85474	6.42086	8.02334	8.04662	7.93343	8.02085

observed only for $d_p = 10$ nm with increasing ϕ , with an intensified enhancement beyond $\phi = 0.03$. For other values of d_p , the trends exhibit parabolic shapes. A similar behavior is observed for $\lambda = 3$. In summary, Nu_{avg} decreases with increasing d_p , but increases with the enhancement of ϕ .

It can be concluded that Nu_{avg} decreases with increasing d_p but increases with the enhancement of ϕ . This behavior can be attributed to the following physical effects: as ϕ increases, the effective thermal conductivity of the nanofluid improves, which enhances convective heat transfer. The increase in nanoparticle concentration boosts the fluid's heat transport capacity due to more effective energy exchange facilitated by the particles. Smaller particles have a higher

surface area-to-volume ratio, which enhances interactions between the nanoparticles and the base fluid, leading to improved heat transfer. Conversely, larger particles reduce the overall surface area available for heat exchange, diminishing the heat transfer enhancement. Moreover, the wavy geometry can alter flow patterns and the distribution of temperature gradients within the cavity. Higher values of λ can disrupt the symmetry and flow uniformity, potentially impacting the convection currents and overall heat transfer rate. However, λ itself does not strongly alter the general trend of heat transfer, which is more influenced by ϕ and d_p .

4.4 Response surface analysis (RSM)

This technique is a highly effective tool for evaluating the effects of multiple variables simultaneously with minimal resource usage. By utilizing quantitative data and a well-structured experimental design, RSM allows for efficient analysis. This approach is particularly valuable when the response variables are influenced by several interacting factors. The methodology also facilitates an exploration of the interrelationships between these parameters. Among the various RSM models available, the second-order quadratic RSM model, which includes linear, squared, and interaction terms, is widely considered sufficient for accurate response estimation (Islam et al. 2021; Islam 2024; Uddin et al. 2025).

The mathematical relationship between the independent variables and the response variable is expressed by the following second-order polynomial equation:

$$y = a_0 + \sum_{i=1}^3 a_i w_i + \sum_{i=1}^3 a_{ii} w_i^2 + \sum_{i=1}^3 \sum_{j=i+1}^3 a_{ij} w_i w_j \quad (6)$$

where, y represents the output function, a_0 is the intercept, a_i and a_{ii} , are the regression coefficients for the linear and quadratic terms of the i th factor, and a_{ij} represents the coefficients of the interaction terms between the i th and j th factors.

In this study, Ra , Ha , and ϕ are used as independent variables (input factors), while the dependent response function (y) is represented by the Nu (average). The goal is to maximize the dependent variable (Nu) in order to establish a robust relationship between the input factors and the response function. A second-order polynomial model, along with the CCD, is utilized for the RSM analysis in this study. The ranges for the input factors are as follows: $10^4 \leq Ra \leq 10^6$, $0 \leq Ha \leq 100$, and $0 \leq \phi \leq 0.04$. The study considers three levels for each factor, represented by -1 , 0 , and $+1$, indicating the lowest, medium, and highest values, respectively. In a CCD framework, the design has a total of 20 experimental runs with 14 non-centre points and 6 centre points.

A comprehensive overview of the coded (categorized) levels of the input variables utilized in the CCD is given in Table 4. The simulation configurations for the real data based on the CCD methodology are described in Table 5. The quadratic polynomial regression models are statistically analyzed using specialized analytical software, which included estimating coefficients and evaluating the impacts of different parameters using coded values. The coefficient of determination and optimal resource utilization were used to evaluate the goodness-of-fit for the experimental data. To ascertain whether or not the model fit is satisfactory, a 95% confidence level comparison of the derived p -values.

The model's statistical analysis reveals a high level of significance and reliability, with an R^2 of 0.9507, an Adjusted

R^2 of 0.9064, and an Adequate Precision of 19.407, indicating a strong signal-to-noise ratio. However, the predicted R^2 of 0.5647 is notably lower than the Adjusted R^2 , suggesting potential issues with the model, such as block effects or data irregularities, which may require model refinement or data adjustments. Significant factors in the model include Ra , Ha , ϕ , $Ra \times Ha$, and $Ha \times Ha$, based on p -values less than 0.05. The F -value of 21.44 further confirms the model's statistical significance, with a very low chance (0.01%) that the result is due to noise. In Table 6, ANOVA analysis shows that the model's total variability is well-explained, and the high R^2 of 95.07% for the response (Nu) demonstrates that the model effectively predicts the behavior of the system.

Based on the results of the RSM analysis, the following model has been developed to explore the relationship between the response variable and the input factors:

$$\begin{aligned} Nu_{avg} = & 2.84819 + 0.000011Ra - 0.117494Ha \\ & + 170.55865\phi - 5.89574 \\ & \times 10^{-8} Ra * Ha + 0.000637Ha^2 \end{aligned} \quad (7)$$

Figures 11, 12, 13 present response surface contour plots generated through the application of RSM. These visualizations are designed to analyze the effects of key factors namely Ra , Ha , and ϕ on the response variable, the Nu (average). The plots are provided in both two-dimensional (2D) and three-dimensional (3D) formats to offer a comprehensive view of how these factors influence the heat transfer performance.

Figure 11 focuses on the influence of Ra and Ha on Nu (average), with ϕ held constant at 0.04. The 2D contour plot in Fig. 11(a) reveals that as Ra increases, there is a corresponding rise in the Nu (average), indicating enhanced heat transfer. This pattern highlights the strong positive correlation between Ra and Nu (average), as Ra directly influences buoyancy-driven convection.

The 2D desirability plot in Fig. 11(b) provides a visual representation of the optimal regions for Ra and Ha , indicating the combination of these factors that maximizes the desirability (0.998) of the response (Nu). This plot helps in identifying the most favorable operating conditions for maximizing heat transfer. In addition, the 3D surface plot in Fig. 11(c) offers a more detailed, three-dimensional perspective on the combined effects of Ra and Ha on Nu (average). The 3D plot allows for better visualization of how changes in both parameters simultaneously affect the response. The trends observed in both the 2D and 3D plots are consistent, demonstrating that Ra has a significant positive effect on Nu (average), while Ha exhibits a more complex interaction, potentially acting as a suppressor of convection at higher values.

Table 4 Coded values and design factors for the CCD

Parameters	Ranges	Coded values		
		− 1 (low)	0 (Medium)	+ 1 (High)
Ra	$10^4 \leq Ra \leq 10^6$	1.00×10^4	5.05×10^5	1.00×10^6
Ha	$0 \leq Ha \leq 100$	0	50	100
ϕ	$0 \leq \phi \leq 0.04$	0	0.02	0.04

Table 5 CCD-based RSM experimental design

Run	Ra	Ha	ϕ	Average Nu	RSM prediction	ANN prediction
1	505,000	50	0.02	6.56638	6.04349	6.56638
2	505,000	50	0.02	6.56638	6.04349	6.56638
3	10,000	0	0.04	4.67377	4.78054	4.67377
4	505,000	50	0.02	6.56638	6.04349	6.56638
5	1,000,000	100	0.04	8.02085	9.3954	8.02085
6	1,000,000	100	0	3.01631	2.57305	3.01631
7	505,000	0	0.02	12.3230	11.8144	12.3230
8	505,000	50	0.02	6.56638	6.04349	6.56638
9	1,000,000	0	0.04	15.0547	20.6705	15.0547
10	505,000	50	0.02	6.56638	6.04349	6.56638
11	505,000	50	0	4.58320	2.63232	4.58320
12	10,000	50	0.02	3.01502	2.05769	3.01502
13	10,000	100	0	1.27716	2.48017	1.27716
14	505,000	50	0.02	6.56638	6.04349	6.56638
15	1,000,000	50	0.02	8.50239	10.0292	8.50239
16	10,000	0	0	2.59867	2.95819	2.59867
17	505,000	100	0.02	4.44849	3.45761	4.44849
18	505,000	50	0.04	6.71412	9.45466	6.71412
19	10,000	100	0.04	4.67465	4.34218	4.67465
20	1,000,000	0	0	8.97660	13.84820	8.97660

Table 6 Analysis of variance (ANOVA) regarding Nu (average)

Source	Sum of squares	df	Mean square	F value	p value	Comment
Model	190.70	9	21.19	21.44	< 0.0001	Significant
Ra	74.70	1	74.70	75.59	< 0.0001	
Ha	49.24	1	49.24	49.83	< 0.0001	
ϕ	34.92	1	34.92	35.33	0.0001	
$Ra \times Ha$	17.03	1	17.03	17.24	0.0020	Insignificant
$Ra \times \phi$	3.93	1	3.93	3.98	0.0740	
$Ha \times \phi$	0.0077	1	0.0077	0.0078	0.9312	
$Ra \times Ra$	2.94	1	2.94	2.98	0.1151	
$Ha \times Ha$	6.97	1	6.97	7.06	0.0240	
$\phi \times \phi$	3.60	1	3.60	3.65	0.0853	
Residual	9.88	10	0.9882			
Lack of fit	9.88	5	1.98			
Pure error	0.0000	5	0.0000			
Cor total	200.58	19				

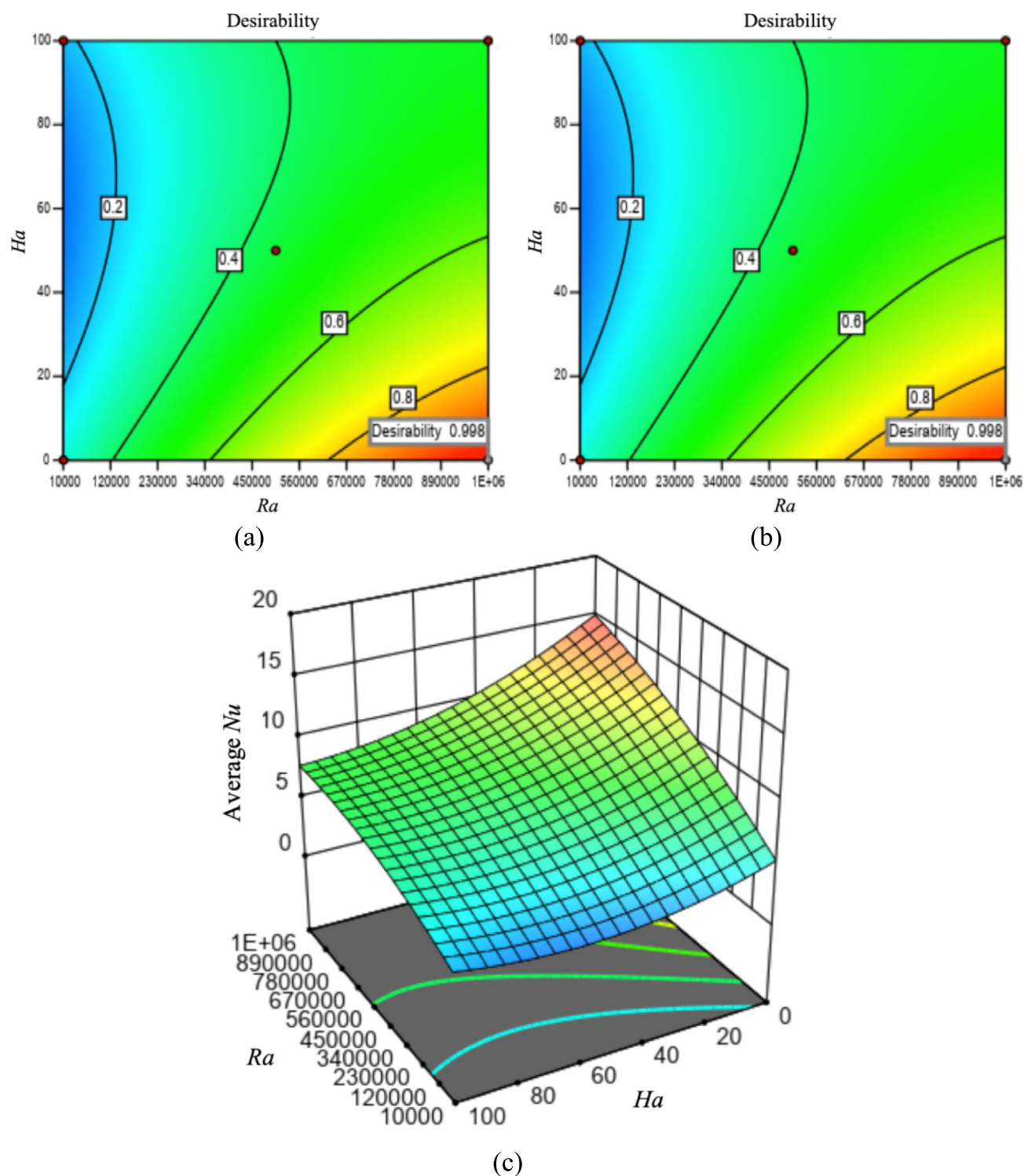


Fig. 11 Variations in the Nu (average) for different values of Ra and Ha , with $\phi = 0.04$ held constant: (a) 2D contour plot, (b) 2D desirability plot, and (c) 3D plot

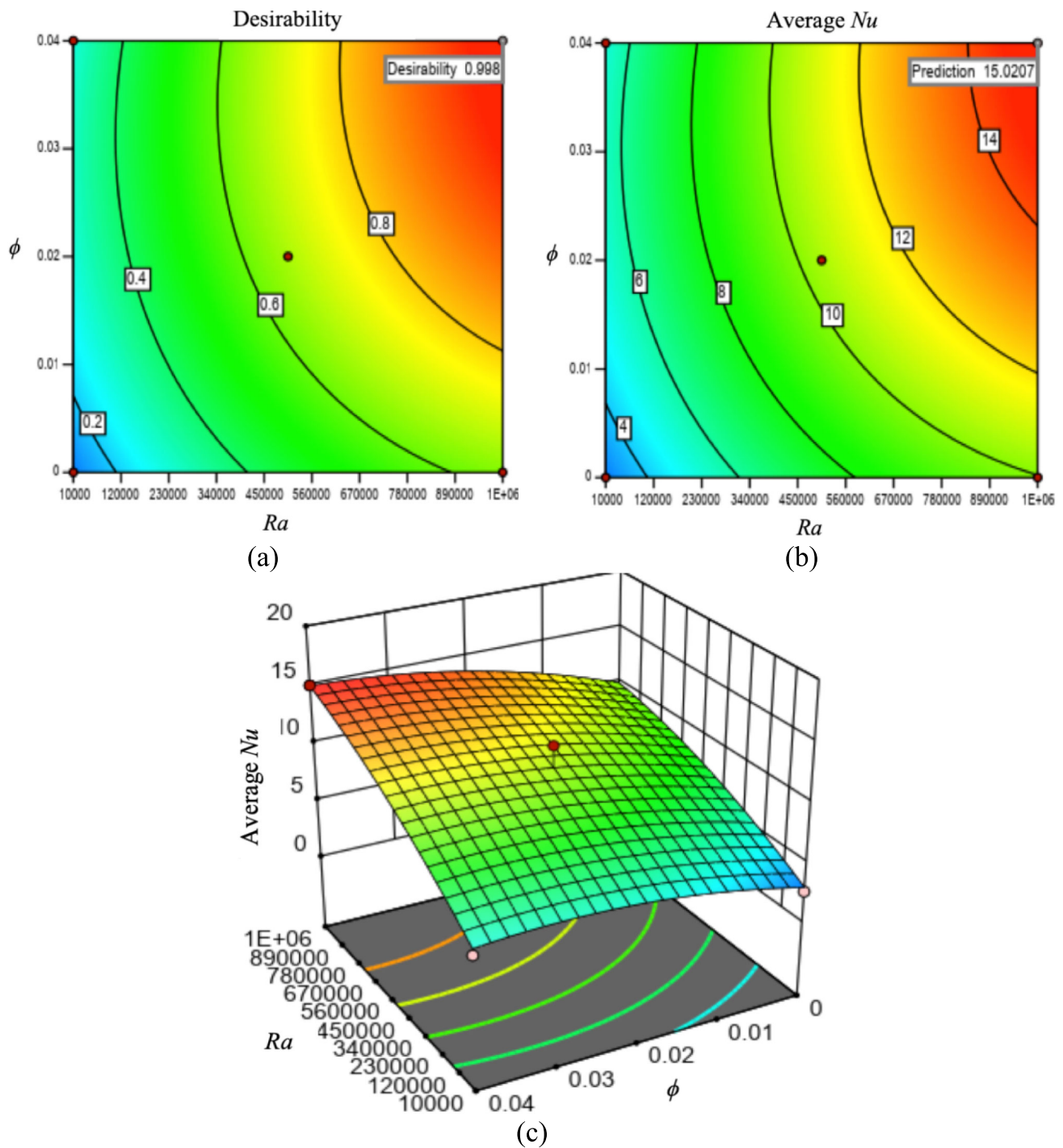


Fig. 12 Variations in the Nu (average) for different values of Ra and ϕ at $Ha = 20$: (a) 2D contour, (b) 2D desirability plot, (b) 3D plot

Figure 12(a) presents a 2D contour plot showing the effect of Ra and nanoparticle volume fraction (ϕ) on the Nu (average). As both Ra and ϕ increase, there is a corresponding rise in Nu (average), signifying enhanced heat transfer. The highest heat transfer rate is observed at $Ra = 10^6$ and $\phi = 0.04$, where buoyancy-driven convection and the improved thermal conductivity of the nanoparticle-enhanced fluid maximize

the heat transfer. The increase in Ra promotes stronger convection currents, while higher ϕ boosts thermal conductivity, facilitating more efficient energy transport within the fluid. The 2D desirability plot in Fig. 12(b) highlights the regions where the combinations of Ra and ϕ yield the most favorable outcomes for optimizing Nu (average). The desirability contours provide insight into the optimal balance between

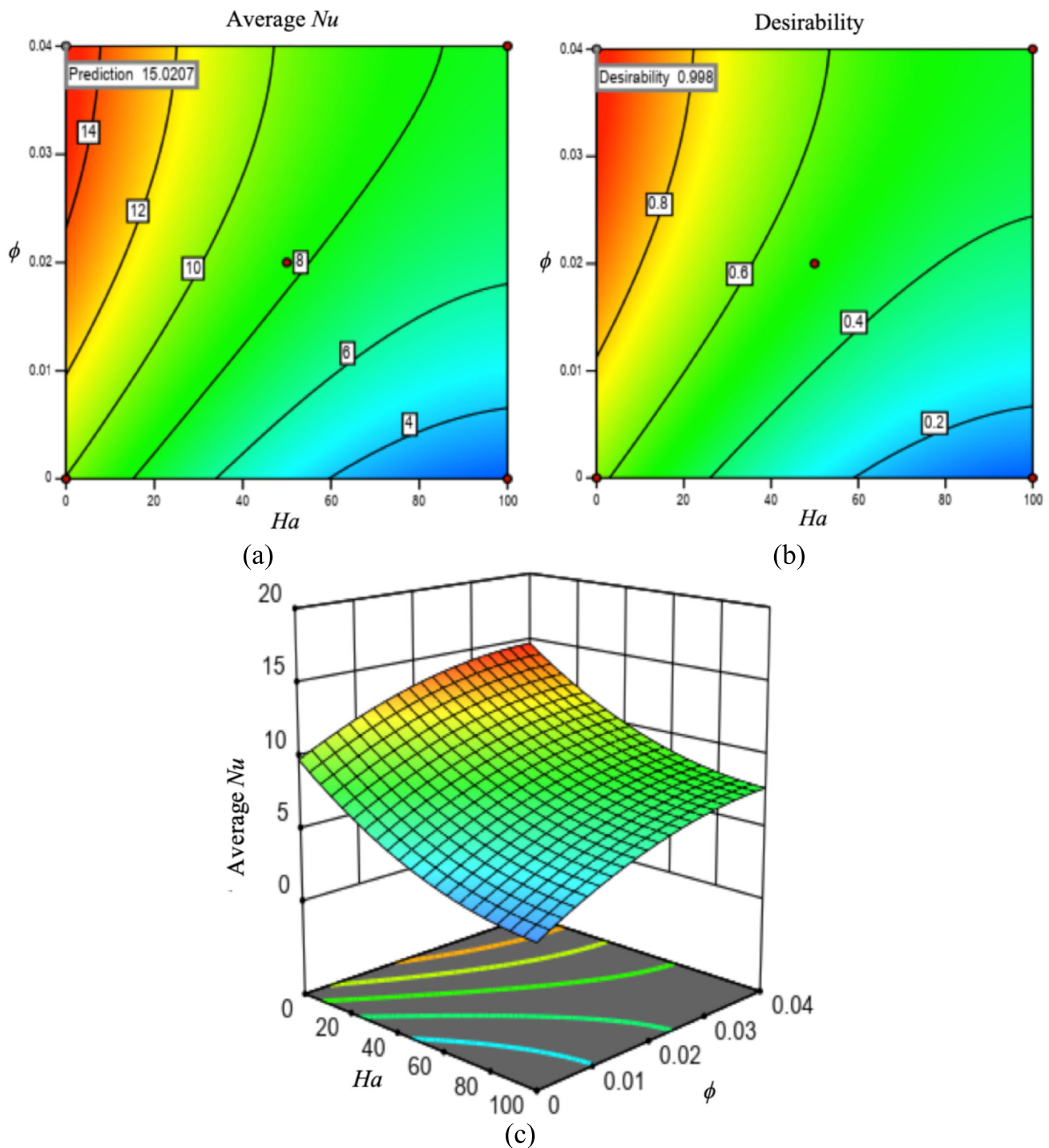


Fig. 13 Variations in the Nu (average) for different values of Ha and ϕ at $Ra = 10^5$: (a) 2D contour, (b) 2D desirability plot, (c) 3D plot

convection (Ra) and nanoparticle influence (ϕ) to achieve the best heat transfer results. Furthermore, Fig. 12(c) represents a 3D surface plot, offering a clear visualization of the combined impact of Ra and ϕ on Nu (average). This plot demonstrates how increasing both Ra and ϕ leads to improved heat transfer performance, particularly when ϕ enhances the thermal

properties of the fluid, amplifying the convective heat transfer driven by Ra .

In Fig. 13(a), the 2D contour plot illustrates the relationship between the Ha and ϕ on Nu (average). The results show that Nu (average) is highest when $Ha = 0$ and $\phi = 0.04$, as

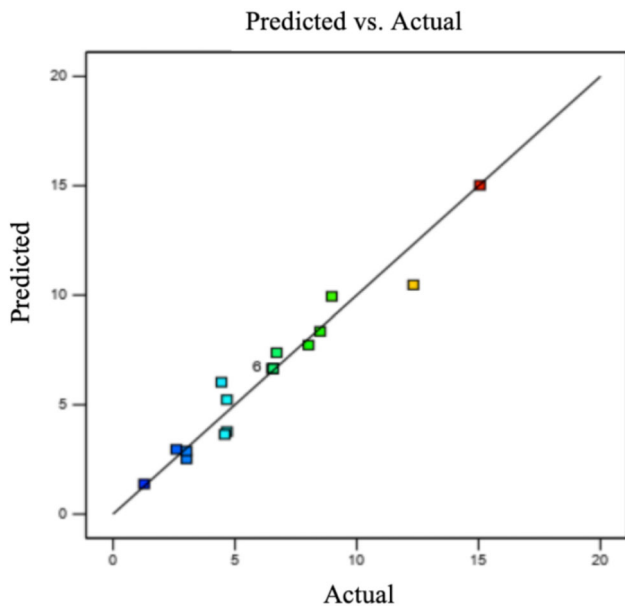


Fig. 14 Comparison of the predicted and actual values of the Nu (average) based on model outcomes

the absence of a magnetic field, allows for unrestricted convective currents, and the increased ϕ improves the fluid's heat transfer capabilities. Conversely, Nu (average) is minimized when $Ha = 100$ and $\phi = 0$, where the strong magnetic field suppresses natural convection through Lorentz forces, reducing heat transfer, and the absence of nanoparticles offers no thermal enhancement. The 2D desirability plot in Fig. 13(b) reveals the most desirable combinations of Ha and ϕ for optimizing Nu (average), indicating the influence of both parameters on heat transfer efficiency. Moreover, Fig. 13(c) provides a 3D surface plot, further demonstrating the impact of Ha and ϕ on Nu (average). As Ha increases, the magnetic field's suppression of convection becomes more pronounced, leading to reduced heat transfer. However, as ϕ increases, the presence of nanoparticles compensates for this suppression by enhancing the fluid's thermal properties, resulting in a more balanced heat transfer performance.

Figure 14 illustrates the comparison between predicted and actual values of the Nu (average). Ideally, in such diagrams, data points should be scattered around a 45° line, which would indicate a perfect match between predicted and experimental results across the entire data range. The observed data in the figure shows that the predicted values closely follow the slope of this line, demonstrating that the model's accuracy is reasonably high. The outcomes generated by the RSM show a strong agreement with experimental data, reinforcing the reliability of the model as depicted in Fig. 14. The scatter distribution suggests that the CCD model effectively captures the heat transfer behavior, allowing for precise predictions of the heat transfer rate.

Figure 15 provides a visualization of the desirability ramps from the numerical optimization process, which aims to determine the most economically viable conditions for thermal transport. The optimization process considers the following input ranges: Ra between 10^4 and 10^6 , Ha between 0 and 100, and ϕ from 0 to 0.04. The primary objective of this optimization is to maximize Nu (average), representing the overall heat transfer efficiency. The optimal conditions for thermal transport are found at $Ra = 10^6$ (indicating maximum buoyancy-driven convection), $Ha = 0$ (reflecting the absence of magnetic field effects), and $\phi = 0.04$ (corresponding to the maximum nanoparticle concentration). The optimization search, based on 20 initial starting points, identifies an optimal Nu (average) of 15.02 under these conditions.

4.5 Artificial neural network

An artificial neural network (ANN) is a computational model inspired by the human brain's network of neurons. It consists of interconnected layers: an input layer that receives data, one or more hidden layers that process the information through weighted connections, and an output layer that delivers the result. Each connection has an associated weight that adjusts as the network learns through a process called backpropagation, where errors are minimized using optimization algorithms. In this study, a feed-forward backpropagation neural network was employed, as this type of network is well-suited for solving finite input–output mapping problems. The Nu (average) is predicted using RSM techniques, which incorporate three input variables (Ra , Ha , and ϕ) within the ranges of $10^4 \leq Ra \leq 10^6$, $0 \leq Ha \leq 100$ and $0 \leq \phi \leq 0.04$, while keeping other parameters constant. The same dataset used in the RSM analysis is also employed in the ANN model to assess the behavior of Nu (average).

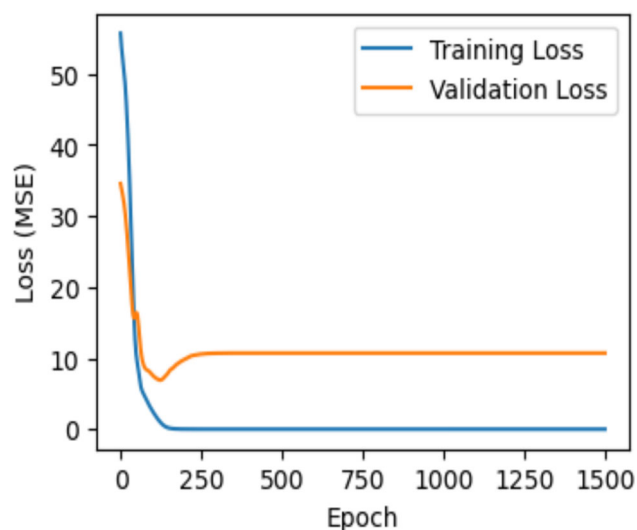
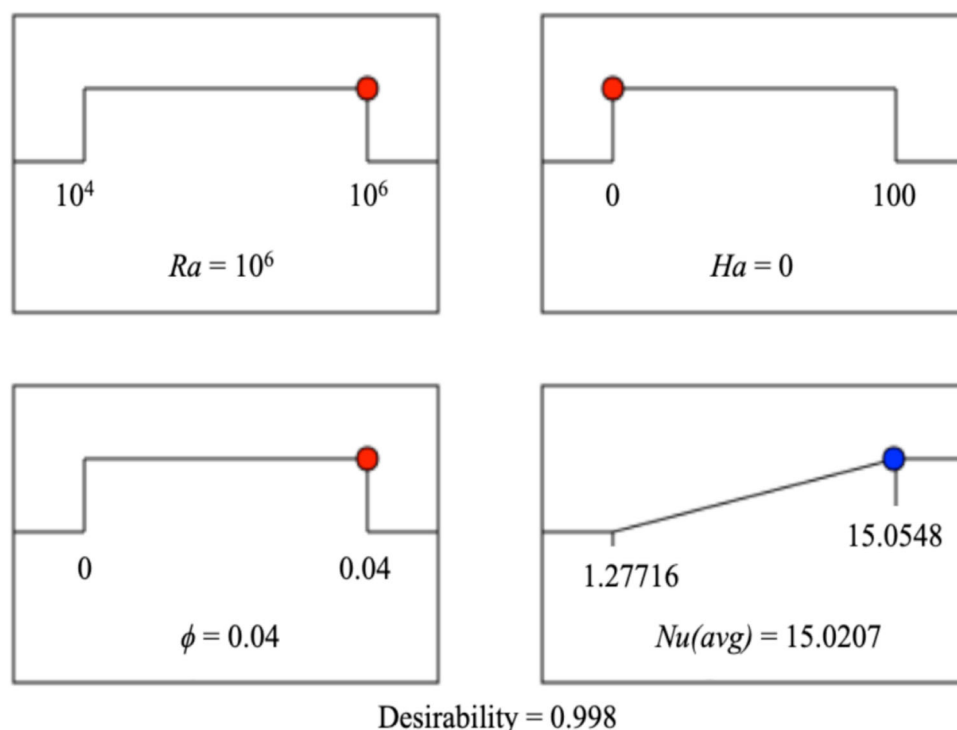
The ANN is trained using the Levenberg–Marquardt backpropagation technique, which provides fast convergence during training. The rectified linear unit (ReLU) is chosen as the activation function the input layer and $\tanh(x)$ for hidden layers due to its efficiency in handling non-linearities, while ReLU is applied in the output layer. The hyperbolic tangent (Tanh) function is employed for non-linear activation, as described by the formula.

$$ReLU(x) = \max(0, x) \quad (9)$$

$$\tanh(x) = \frac{2}{1 + e^{-2x}} - 1 \quad (10)$$

The performance of the model was evaluated using the mean squared error (MSE) as a loss function, defined as:

$$MSE = \frac{1}{m} \left(\sum_{i=1}^m |Nu_{avg} - \widehat{Nu}_{avg}| \right) \quad (11)$$

Fig. 15 Desirability ramp for the Nu (average)**Fig. 16** ANN loss (MSE) plot

where Nu_{avg} represents the predicted Nu (average) from the ANN mode, $\widehat{Nu}_{avg}$ stands for the simulated (actual) Nu (average), and m is the total number of observations.

Figure 16 illustrates the relationship between the loss function, specifically the mean squared error (MSE), and the number of epochs during the training process. The model was trained with an initial batch size of 20 over 64 epochs, using a neural network with a total of 5 layers. The training

duration was 14.53 s, performed on a machine, featuring an 8-core CPU, a 10-core GPU, and 8 GB of unified memory. The MSE value was calculated to be 2.096×10^{-14} , indicating a very low prediction error. The predictions generated by the ANN model are listed in Table 5, showing that the ANN predictions aligned closely with the actual data and outperform those obtained using RSM technique. These results indicate that the ANN provides a reliable and effective alternative to RSM for modeling and prediction tasks.

5 Conclusions

This research investigated the unsteady magnetohydrodynamic (MHD) flow of nanofluids containing Ag nanoparticles of blade shape in kerosene and HT across three types of wave numbers within a wavy octagonal enclosure. Using PARDISO, a stationary linear direct solver, precisely calculated outcomes were obtained. Furthermore, machine learning and statistical analysis are conducted employing RSM and ANN. The study explored the effects of various parameters, including the Ra , Ha , ϕ , and d_p , across three different λ . Using 2D and 3D RSM-based visualizations, the study analyzed how nanofluid influenced flow and thermal behavior, emphasizing crucial aspects. A regression framework was developed using the average Nusselt number and residuals to predict optimal conditions for enhancing heat transfer. The key findings are summarized as follows:

- For optimum heat transfer investigation, a trustworthy and efficient framework is presented by the hybrid approaches which incorporates ANN, RSM, and FEM-based CFD.
- As Ra increases, intensified buoyancy forces enhance natural convection, resulting in higher fluid flow and heat transfer rates, highlighting the importance of buoyancy-driven convection in the nanofluid.
- An increase in Ha suppresses fluid motion by generating a Lorentz force that opposes flow, leading to decayed streamline contours and isothermal lines, and reducing the overall heat transfer rate.
- A higher concentration of nanoparticles significantly enhances the heat transfer rate by improving the nanofluid's thermal conductivity and boosting energy transfer efficiency.
- The wavy geometry of the enclosure impacts flow patterns and heat transfer characteristics. Among the tested configurations, the domain with three waves ($\lambda = 3$) exhibits superior heat transfer rates compared to those with one or two waves, particularly as ϕ increases.
- Larger nanoparticles decrease heat transfer efficiency due to reduced surface area for interaction, whereas smaller nanoparticles enhance it through a higher surface area-to-volume ratio.
- The average Nu is precisely predicted based on a correlation generated using RSM, and the model's accuracy is verified by the coefficient of approximation ($R^2 = 95.07\%$).
- Superb prediction accuracy was obtained ($MSE = 2.096 \times 10^{-14}$, $R^2 = 95.07\%$), revealing that sophisticated ANN modeling executed far exceeding conventional statistical RSM strategies.
- The optimal average Nu of 15.0207 was acquired at $Ra = 10^6$, $Ha = 0$, $\phi = 0.04$.
- A statistical correlation was developed using CCD-based RSM techniques to predict the Nu (average) based on independent parameters. The ANN predictions matched the actual outcomes more accurately than the RSM predictions, indicating that ANN is a more effective alternative for computing the response function.

Finally, this study highlights the complex interplay of buoyancy forces, magnetic field effects, nanoparticle concentration, particle size, and enclosure geometry in determining the heat transfer performance of nanofluids in wavy cavities. These insights can be leveraged to optimize designs for heat exchangers and other thermal management systems, promoting enhanced efficiency in engineering applications.

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Data availability All data supporting the findings of this study are included in the manuscript. Additional data and materials may be available upon request from the corresponding author, subject to ethical considerations and privacy regulations.

Declarations

Conflict of interest The authors declare no competing interests.

AI Declaration The authors used AI-assisted technology (ChatGPT-3.5) for language editing and grammar checking.

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