



Induced OWA Operators in Neutrosophic Environment Applied in the Economic Assessment of Southeast Asian Countries

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Abstract

The ordered weighted averaging (OWA) operator is a fundamental tool in decision-making processes, particularly under conditions of uncertainty, by aggregating inputs through a reordering mechanism based on predefined weights. Despite its utility, the classical OWA operator is limited in addressing complex decision scenarios characterized by uncertainty, indeterminacy, and inconsistency. This study introduces two innovative aggregation operators, the induced ordered weighted averaging operator under a neutrosophic environment ($IOWA_N$) and its generalized form ($GLOWA_N$), which integrate the ordering flexibility of induced OWA with the expressive power of neutrosophic sets. The proposed operators advance existing models by (i) enabling simultaneous aggregation of truth, indeterminacy, and falsity information, (ii) incorporating application-driven inducing functions for dynamic ordering, and (iii) offering a unified framework encompassing arithmetic, geometric, harmonic, quadratic, and extreme-case induced operators. We formally define the operators, establish their mathematical properties, and present several novel extensions, including interval-valued, bipolar, probabilistic, entropy-based, and time-dynamic neutrosophic versions. An illustrative case study of the economic assessment of Southeast Asian countries was performed using a methodology based on ($GLOWA_N$) Operators and results show that rankings align with real-world economic data. Comparative analyses highlight its superior performance in modeling intricate decision dynamics. The proposed algorithms are effective in a group decision-making environment within uncertain domains, solving problems of uncertainty, complexity, and multidimensional information, including sustainable development, policy formulation, and healthcare decision analysis.

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1 Introduction

Decision-making often involves uncertainty due to vague, incomplete, or imprecise information. Fuzzy set [47–49] theory provides the mathematical framework to handle this uncertainty by allowing elements to partially belong to a set. The introduction of Intuitionist [3] and neutrosophic [37] theories helps further to solve vagueness and indeterminacy by considering non-membership and indeterminacy values as well. Many decision-making methodologies and aggregation operators in multi-criteria decision-making (MCDM) under uncertain environments have evolved significantly, which are used to combine multiple criteria or opinions into a single decision metric, ensuring consistency and rationality in decision outcomes. Many researchers advance the multi-criteria decision-making (MCDM) using fuzzy and neutrosophic frameworks to handle uncertainty in complex problems. Methods such as vector similarity measures of type-2 hesitant fuzzy sets, generalized Dice measures of single-valued neutrosophic sets, and temporal intuitionistic fuzzy sets with soft sets have improved decision accuracy [15, 26, 27]. Additional tools, including r,s,t-spherical hesitant fuzzy set correlations and soft covered ideals in semigroups, further support structured and reliable MCDM modeling [28, 29].

Recent advances in neutrosophic theory have introduced a variety of decision methodologies to better handle indeterminacy in decision-making. Significant contributions include the development of N-valued neutrosophic trapezoidal numbers with similarity measures for MCDM [11], the NPOWA operator for aggregating probabilistic neutrosophic information [35], and comprehensive analyses of the journal Neutrosophic Sets and Systems scientific progress [34]. Further methodological expansions include octagonal neutrosophic numbers and their α – cut representations [33], interval cut set operations for interval-valued neutrosophic sets [32], and energy-based neutrosophic soft sets for enhanced MCDM applications [6]. Additional advancements encompass Q-neutrosophic soft expert sets [5] and optimization models such as minimum spanning trees under trapezoidal fuzzy neutrosophic environments [7], collectively demonstrating the expanding versatility of neutrosophic frameworks.

In real-world decision-making, aggregating information from diverse sources under uncertainty is a fundamental task. Classical aggregation operators, such as the ordered weighted averaging (OWA) operator [41], have provided a flexible means of modeling human preferences by reordering input values and applying predefined weights. However, OWA operators are limited in effectively handling uncertain and imprecise information, especially when the data includes indeterminacy or conflicting evidence. To address these limitations, the induced OWA (IOWA) operator [44] was proposed, introducing an inducing variable to reorder the inputs, enabling context-sensitive aggregation. While IOWA improves flexibility and robustness over the standard OWA, it still falls short in environments characterized by deep uncertainty and vagueness. Yager [42] introduced induced aggregation operators, sparking numerous applications and enhancements. Building upon this, Merigó [19] proposed a unified model that

bridges the gap between weighted average and induced OWA operators, offering more flexibility in aggregation scenarios. After the development of IOWA, further OWA operator families were introduced, like induced generalized OWA operators [24], uncertain induced aggregation operators by Merigó [23]. Many recent studies have advanced multi-criteria decision-making by integrating fuzzy, neutrosophic sets, and OWA operators. D'Urso et al. [12] proposed a fuzzy OWA-based framework, while Agahi [2] introduced the FCWOWA operator for more flexible aggregation, and Shambayati et al. [36] applied neutrosophic optimization to circular economy supply chains, highlighting the practical value of these approaches.

The concept of induced aggregation operators has played a transformative role in advancing multi-criteria decision-making (MCDM) and fuzzy logic-based systems. Beginning with Yager's [42] foundational work on induced aggregation operators, scholars have since developed generalized and hybrid models such as the Induced Generalized OWA [24] and linguistic extensions [40]. Merigó and collaborators introduced several key models, such as the unified weighted average and IOWA [21], Minkowski distance-based IOWA [22], and induced linguistic operators [23], demonstrating their utility across decision-making and classification problems. Applications have included extensions to VIKOR [18], hesitant fuzzy environments [46], and competitive behavior modeling [17]. Furthermore, researchers have shown the practical relevance of these models in real-world contexts [8] (Wei et al., 2020), including corporate social responsibility [4], systems design [38], and negotiation [9, 10]. The evolution of these models reflects a growing emphasis on nuanced, flexible, and context-aware aggregation strategies, enhancing the robustness and interpretability of decision-support tools. Induced and generalized aggregation operators have gained significant attention in recent years for their flexibility in addressing complex decision-making problems. Building upon the foundational work on ordered weighted averaging (OWA) operators [43, 45], recent studies have explored advanced formulations, such as distance-induced and entropy-based variants, to enhance their applicability. For example, Gong et al. [13] proposed a generalized distance-induced OWA operator for evaluating indicator systems in China, while Zheng et al. [50] introduced a fuzzy induced OWA operator based on entropy and probability. In machine learning and feature selection, Janani et al. [14] employed OWA-based ensemble techniques to improve classification accuracy. Similarly, Merigó and Casanovas [20] integrated induced operators with the Dempster-Shafer framework. While Rahman [30, 31] extended induced Einstein aggregation operators under complex fuzzy and Intuitionistic environments and demonstrated with group decision methodology. Recent developments also include graph-induced rank aggregation [16] and multi-level induced OWA models in DEA frameworks [25], demonstrating the growing versatility and robustness of these operators across various domains.

Despite significant developments in fuzzy, intuitionistic, and neutrosophic decision-making models, existing aggregation operators still face notable limitations when dealing with highly uncertain and inconsistent information. Classical OWA operators are unable to represent indeterminacy, while induced OWA operators (IOWA) improve flexibility but remain restricted to crisp or fuzzy environments. In many real-world problems such as national economic assessment, policy evaluation, sustainable development, and risk-based decision-making, information is often incomplete,

conflicting, or partially reliable. Such data cannot be effectively modeled using single-valued membership functions. These limitations motivate the need for a more expressive and robust aggregation mechanism that can simultaneously consider truth, indeterminacy, and falsity information, while also retaining the context-sensitive ordering capability of induced operators. Neutrosophic sets provide a richer representation of uncertainty, but existing neutrosophic OWA-based models do not incorporate dynamic inducing variables or generalized aggregation behaviors. The proposed $IOWA_N$ and $GLOWA_N$ operators address these gaps by combining the strength of neutrosophic modeling with the flexibility of induced ordering. Our approach provides several advantages:

- **Enhanced Uncertainty Handling:** The operators aggregate three-dimensional neutrosophic information (T, I, F), capturing inconsistency and indeterminacy not addressed by classical OWA or fuzzy aggregation operators.
- **Dynamic and Context-Sensitive Ordering:** Through an inducing function, the method prioritizes values based on application-driven significance, improving reliability and interpretability.
- **Generalized Aggregation Framework:** The $GLOWA_N$ operator unifies multiple families of aggregation (arithmetic, geometric, harmonic, quadratic, and extreme-case) under a single structure, improving adaptability across diverse decision scenarios.
- **Practical Relevance:** The improved operators enable more accurate modeling of complex, uncertain environments, as demonstrated in the Southeast Asian economic assessment case study.

Therefore, this study introduces $IOWA_N$ and $GLOWA_N$ as essential extensions that overcome existing limitations and provide a powerful framework for decision-making in environments characterized by high uncertainty and multi-dimensional information.

In the proposed $IOWA_N$ and $GLOWA_N$ operators, the inducing function plays a central role in determining the order in which neutrosophic values are aggregated. Unlike classical OWA operators that sort values solely based on their numerical magnitude, the inducing function allows the ordering to be driven by an application-specific criterion. For a neutrosophic value $x_i = (T_i, I_i, F_i)$, the inducing function $f(x_i)$ may reflect reliability, confidence, preference, risk, or any domain-related priority. This ensures that alternatives with higher significance in the decision context are placed earlier in the aggregation process. The inducing function thus provides dynamic and context-aware ordering, enabling the operator to:

- emphasize alternatives with higher truth membership or lower falsity,
- penalize alternatives with inconsistent or indeterminate information,
- incorporates expert- or criterion-specific importance into the aggregation process,
- enhances robustness in environments where information is uncertain or conflicting.

By reordering neutrosophic inputs based on meaningful criteria, the inducing function directly influences the final aggregated result, making $IOWA_N$ and $GLOWA_N$ significantly more flexible and realistic than traditional OWA or fuzzy aggregation methods.

The rest of the paper is structured as follows. In Sect. 2 showcase the preliminaries. In Sect. 3, we presented the formal definition of Induced OWA under a neutrosophic context ($IOWA_N$) and introduced the important results and extensions for $IOWA_N$. Section 4 presents a decision methodology, and a solved example with result discussion and comparison. Section 5 present a generalized induced OWA operator for neutrosophic sets and its types, a decision methodology using $G IOWA_N$ With an illustrative example of the economic assessment of Southeast Asian countries. Finally, the result discussion with comparison and sensitivity analysis is presented, and Sect. 6 concludes the study with future research directions.

2 Preliminaries

Definition 2.1 The ordered weighted averaging (OWA) operator [41] is a widely used aggregation operator that allows the integration of multiple criteria based on their ordered values. Given a set of n criteria $x_1, x_2, \dots, x_n$ with corresponding weights $w_1, w_2, \dots, w_n$, the OWA operator is defined as follows:

$$OWA(\mathbf{x}) = \sum_{i=1}^n w_i \cdot \gamma_i(\mathbf{x}) \quad (1)$$

where $\mathbf{x} = (x_1, x_2, \dots, x_n)$ represents the vector of inputs, and $\gamma_i(\mathbf{x})$ denotes the i -th largest element of the sorted vector $\mathbf{x}$, i.e., $\gamma_i(\mathbf{x})$ is the i -th largest value in the set $\{x_1, x_2, \dots, x_n\}$. The weights $w_1, w_2, \dots, w_n$ are assigned to these ordered values, and their sum is equal to 1, i.e., $\sum_{i=1}^n w_i = 1$. The OWA operator captures the degree of optimism or pessimism in the aggregation, based on the selected weight vector.

Definition 2.2 A neutrosophic set (NS) [37] is an extension of fuzzy set theory that models indeterminacy and uncertainty by using three membership functions: truth-membership, indeterminacy-membership, and falsity-membership. Let X be a universe of discourse. A neutrosophic set A is characterized by a triplet (T_A, I_A, F_A) where:

$$A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\} \quad (2)$$

Here: $T_A(x) \in [0, 1]$ is the truth-membership function, representing the degree to which element x belongs to A , $I_A(x) \in [0, 1]$ is the indeterminacy-membership function, representing the degree to which the truth value of x is indeterminate, $F_A(x) \in [0, 1]$ is the falsity-membership function, representing the degree to which element x does not belong to A . These functions satisfy the condition:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3 \quad (3)$$

This tripartite representation allows neutrosophic sets to handle complex, imprecise, and contradictory information more effectively than traditional fuzzy sets.

Definition 2.3 An induced operator [44] is an aggregation operator that transforms an input set of values based on a given ordering criterion and specific operator structure.

The induced operator generally modifies the classic OWA operator by introducing additional factors such as weights or probability distributions to account for specific requirements of the decision-making process. Formally, the induced OWA operator is defined as:

$$IOWA(\mathbf{x}) = \sum_{i=1}^n w_i \cdot f(\gamma_i(\mathbf{x})) \quad (4)$$

where f is a function applied to the ordered values $\gamma_i(\mathbf{x})$ to induce a transformation before performing the weighted sum. This function f can take various forms depending on the application context, and it is often used to introduce non-linearities or customized effects into the aggregation process. Induced operators extend the flexibility of classical OWA operators by allowing for a greater degree of control over the aggregation process.

3 Induced OWA Operator Under Neutrosophic Environment

In this section, we present the basic definition with an illustrative example of NIOWA.

Definition 3.1 Let $\mathcal{X} = \{x_1, x_2, \dots, x_n\}$ be a collection of neutrosophic values where each x_i is represented as a triplet:

$$x_i = (T_i, I_i, F_i)$$

with $T_i, I_i, F_i \in [0, 1]$, denoting the *truth-membership*, *indeterminacy-membership*, and *falsity-membership* degrees, respectively.

Now consider $f : \mathcal{X} \rightarrow \mathbb{R}$ be an *inducing function* that assigns a real value to each neutrosophic triplet, for instance, $f(x_i) = T_i - F_i$, or any other application-specific scoring function. Let $w = (w_1, w_2, \dots, w_n)$ be a weight vector such that $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$. Now, sort the neutrosophic values x_i in decreasing order based on the values of the inducing function $f(x_i)$, yielding a permutation π such that:

$$f(x_{\pi(1)}) \geq f(x_{\pi(2)}) \geq \dots \geq f(x_{\pi(n)})$$

Then, the induced OWA operator for a neutrosophic set is defined as:

$$IOWA_N(x_1, x_2, \dots, x_n) = \left(\sum_{j=1}^n w_j T_{\pi(j)}, \sum_{j=1}^n w_j I_{\pi(j)}, \sum_{j=1}^n w_j F_{\pi(j)} \right) \quad (5)$$

This operator provides an aggregated neutrosophic value by ordering the input values using an inducing function and applying a weighted average to each of the three components.

Example 1 Consider a set of three neutrosophic values:

$$x_1 = (0.9, 0.1, 0.05), \quad x_2 = (0.4, 0.3, 0.5), \quad x_3 = (0.7, 0.2, 0.1)$$

Let the inducing function be defined as:

$$f(x_i) = T_i - F_i$$

This gives:

$$f(x_1) = 0.85, \quad f(x_2) = -0.1, \quad f(x_3) = 0.6$$

Ordering the elements by decreasing $f(x_i)$:

$$x_{\pi(1)} = x_1, \quad x_{\pi(2)} = x_3, \quad x_{\pi(3)} = x_2$$

Let the weight vector be:

$$w = (0.5, 0.3, 0.2)$$

We compute the IOWA aggregation for a neutrosophic set using:

$$\text{IOWA}_N(x_1, x_2, x_3) = \left(\sum_{j=1}^3 w_j T_{\pi(j)}, \sum_{j=1}^3 w_j I_{\pi(j)}, \sum_{j=1}^3 w_j F_{\pi(j)} \right)$$

Computing each component:

$$T_{\text{agg}} = 0.5 \cdot 0.9 + 0.3 \cdot 0.7 + 0.2 \cdot 0.4 = 0.45 + 0.21 + 0.08 = 0.74$$

$$I_{\text{agg}} = 0.5 \cdot 0.1 + 0.3 \cdot 0.2 + 0.2 \cdot 0.3 = 0.05 + 0.06 + 0.06 = 0.17$$

$$F_{\text{agg}} = 0.5 \cdot 0.05 + 0.3 \cdot 0.1 + 0.2 \cdot 0.5 = 0.025 + 0.03 + 0.10 = 0.155$$

we get;

$$\text{IOWA}_N(x_1, x_2, x_3) = (0.74, 0.17, 0.155)$$

3.1 Some Important Results of IOWA_N

In this section, we present the basic properties, theorems, propositions, and remarks for IOWA_N .

Theorem 3.1 *If all inputs are equal, the result is equal to the common value.*

Proof Let all neutrosophic values be equal: $x_1 = x_2 = \dots = x_n = x = (T, I, F)$. Then the inducing function $f(x_i) = f(x)$ for all i , so the ordering remains unchanged. The IOWA aggregation becomes:

$$\text{IOWA}_N(x_1, \dots, x_n) = \left(\sum_{j=1}^n w_j T, \sum_{j=1}^n w_j I, \sum_{j=1}^n w_j F \right) = (T, I, F)$$

since $\sum_{j=1}^n w_j = 1$. $\square$

Theorem 3.2 *The aggregated result lies within the bounds of the individual components.*

Proof Let each neutrosophic value $x_i = (T_i, I_i, F_i) \in [0, 1]^3$, and let the weights $w_j \in [0, 1]$ with $\sum w_j = 1$. Since the IOWA operator is a weighted average of reordered values:

$$T_{\text{agg}} = \sum_{j=1}^n w_j T_{\pi(j)}, \quad I_{\text{agg}} = \sum_{j=1}^n w_j I_{\pi(j)}, \quad F_{\text{agg}} = \sum_{j=1}^n w_j F_{\pi(j)}$$

Each component is a convex combination of values in $[0, 1]$, hence:

$$T_{\text{agg}}, I_{\text{agg}}, F_{\text{agg}} \in [0, 1]$$

Thus, the aggregated neutrosophic value is bounded in $[0, 1]^3$. $\square$

Theorem 3.3 *If the inducing function yields equal values for all elements, the operator behaves like a standard OWA and is commutative.*

Proof Assume $f(x_1) = f(x_2) = \dots = f(x_n)$. Then the sorting step becomes irrelevant, and the IOWA operator reduces to:

$$\text{IOWA}_N(x_1, \dots, x_n) = \sum_{j=1}^n w_j x_j$$

which is symmetric with respect to the inputs. Thus, for any permutation σ , the result satisfies:

$$\text{IOWA}_N(x_1, \dots, x_n) = \text{IOWA}_N(x_{\sigma(1)}, \dots, x_{\sigma(n)})$$

Therefore, the operator is commutative under this condition. $\square$

Theorem 3.4 *If $x_i \leq x'_i$ componentwise and the inducing function preserves order, then the aggregation also satisfies:*

$$\text{IOWA}_N(x_1, \dots, x_n) \leq \text{IOWA}_N(x'_1, \dots, x'_n) \quad (6)$$

Proof Let $x_i = (T_i, I_i, F_i)$, $x'_i = (T'_i, I'_i, F'_i)$, and assume $x_i \leq x'_i$, i.e.,

$$T_i \leq T'_i, \quad I_i \leq I'_i, \quad F_i \leq F'_i \quad \forall i$$

Let the ordering induced by f be the same for both x_i and x'_i , i.e., π is unchanged. Then:

$$T_{\text{agg}} = \sum_{j=1}^n w_j T_{\pi(j)} \leq \sum_{j=1}^n w_j T'_{\pi(j)} = T'_{\text{agg}}$$

$$I_{\text{agg}} = \sum_{j=1}^n w_j I_{\pi(j)} \leq \sum_{j=1}^n w_j I'_{\pi(j)} = I'_{\text{agg}}$$

$$F_{\text{agg}} = \sum_{j=1}^n w_j F_{\pi(j)} \leq \sum_{j=1}^n w_j F'_{\pi(j)} = F'_{\text{agg}}$$

Hence,

$$\text{IOWA}_N(x_1, \dots, x_n) \leq \text{IOWA}_N(x'_1, \dots, x'_n)$$

□

Remark 3.1 The IOWA operator generalizes the classical OWA operator by allowing dynamic reordering based on an external inducing function.

Remark 3.2 In decision-making problems where uncertainty, indeterminacy, and falsity are present, using IOWA under neutrosophic sets allows for more informative and robust aggregation than classical OWA or fuzzy models.

Remark 3.3 The choice of the inducing function f significantly affects the behavior of the IOWA operator. For instance, selecting $f(x) = T_i - F_i$ prioritizes alternatives with higher truth-membership and lower falsity.

Proposition 3.1 *The IOWA operator under neutrosophic sets is idempotent if all weights are zero except one.*

Proof Let $w_j = 1$ for some $j = k$ and $w_i = 0$ for all $i \neq k$. Then the aggregation becomes:

$$\text{IOWA}_N(x_1, \dots, x_n) = x_{\pi(k)}$$

Since $x_{\pi(k)} \in \{x_1, \dots, x_n\}$, the result is one of the original neutrosophic values. Thus, the operator is idempotent under this weighting scheme. □

Proposition 3.2 *If all the weights are equal, the IOWA operator behaves like the arithmetic mean operator applied to reordered neutrosophic values.*

Proof Let $w_j = \frac{1}{n}$ for all j . Then the IOWA operator becomes:

$$\text{IOWA}_N(x_1, \dots, x_n) = \frac{1}{n} \sum_{j=1}^n x_{\pi(j)}$$

This is equivalent to computing the mean of the reordered values, hence the operator reduces to the arithmetic mean over the reordered neutrosophic tuples. □

Proposition 3.3 *If the inducing function f is strictly monotonic and all inputs are distinct, the ordering induced on inputs is unique.*

Proof Let $x_i \neq x_j$ for $i \neq j$, and f be strictly increasing. Then $f(x_i) \neq f(x_j)$ for all $i \neq j$, implying that the sorted order π is unique. □

3.2 Some Extensions of the Induced OWA Operator Under Neutrosophic Set

Let us denote a neutrosophic number as $x_i = (T_i, I_i, F_i)$, where T_i , I_i , and F_i are the truth-membership, indeterminacy-membership, and falsity-membership values, respectively, each in $[0, 1]$. Now consider $I = (i_1, i_2, \dots, i_n)$ be a permutation of $\{1, 2, \dots, n\}$ such that the inducing variable r_{i_j} satisfies $r_{i_1} \geq r_{i_2} \geq \dots \geq r_{i_n}$. Then, the general Induced OWA (IOWA) operator under a neutrosophic environment is given by:

$$\text{IOWA}_\omega^N(x_1, \dots, x_n) = \sum_{j=1}^n \omega_j \cdot x_{i_j}, \quad \text{with } x_{i_j} = (T_{i_j}, I_{i_j}, F_{i_j})$$

This yields the neutrosophic result:

$$\text{IOWA}_\omega^N = \left(\sum_{j=1}^n \omega_j T_{i_j}, \sum_{j=1}^n \omega_j I_{i_j}, \sum_{j=1}^n \omega_j F_{i_j} \right)$$

We now define extensions of this operator:

Definition 3.2 (Interval-Valued Neutrosophic IOWA (IVN-IOWA))

Let $x_i = ([T_i^L, T_i^U], [I_i^L, I_i^U], [F_i^L, F_i^U])$, then:

$$\text{IVN-IOWA}_\omega = \left(\left[\sum_{j=1}^n \omega_j T_{i_j}^L, \sum_{j=1}^n \omega_j T_{i_j}^U \right], \left[\sum_{j=1}^n \omega_j I_{i_j}^L, \sum_{j=1}^n \omega_j I_{i_j}^U \right], \left[\sum_{j=1}^n \omega_j F_{i_j}^L, \sum_{j=1}^n \omega_j F_{i_j}^U \right] \right) \quad (7)$$

Definition 3.3 (Bipolar Neutrosophic IOWA (BN-IOWA)) For bipolar neutrosophic numbers $x_i = (T_i^+, I_i^+, F_i^+; T_i^-, I_i^-, F_i^-)$, the aggregation is:

$$\text{BN-IOWA}_\omega = \left(\sum_{j=1}^n \omega_j T_{i_j}^+, \sum_{j=1}^n \omega_j I_{i_j}^+, \sum_{j=1}^n \omega_j F_{i_j}^+; \sum_{j=1}^n \omega_j T_{i_j}^-, \sum_{j=1}^n \omega_j I_{i_j}^-, \sum_{j=1}^n \omega_j F_{i_j}^- \right) \quad (8)$$

Definition 3.4 (Probabilistic Neutrosophic IOWA (PN-IOWA)) The Probabilistic Neutrosophic IOWA (PN-IOWA) operator is an extension of the IOWA operator, where the weights are determined using a convex combination of the probability values associated with each element and the original weights. Specifically, for each x_i associated with probability p_i such that $\sum p_i = 1$. The induced ordering is based on p_i , and the weights are redefined as $w_j = \alpha p_{i_j} + (1 - \alpha)w_{i_j}$, where $0 \leq \alpha \leq 1$ is a convex combination factor, and w_{i_j} represents the original weights. The reordering of probabilities is aligned with the induced ordering. Then, the PN-IOWA operator is defined as:

$$\text{PN-IOWA} = \left(\sum_{j=1}^n w_j T_{i_j}, \sum_{j=1}^n w_j I_{i_j}, \sum_{j=1}^n w_j F_{i_j} \right) \quad (9)$$

where the weights w_j are determined using the convex combination of the original weights and the probability values.

Definition 3.5 (Dynamic Neutrosophic IOWA for Time-Series) Let $x_{it} = (T_{it}, I_{it}, F_{it})$ denote neutrosophic inputs at time t . Then, the time-aggregated dynamic IOWA operator over period T is:

$$\text{D-IOWA}_\omega = \left(\sum_{t=1}^T \alpha_t \sum_{j=1}^n \omega_j T_{ij}^{(t)}, \sum_{t=1}^T \alpha_t \sum_{j=1}^n \omega_j I_{ij}^{(t)}, \sum_{t=1}^T \alpha_t \sum_{j=1}^n \omega_j F_{ij}^{(t)} \right) \quad (10)$$

where α_t is the time-discounting or importance factor at time t .

Definition 3.6 (Entropy-Based Weighted Neutrosophic IOWA) Use an entropy-based weight generation scheme for neutrosophic components:

$$w_j = \frac{e_j^{-1}}{\sum_{k=1}^n e_k^{-1}}, \quad \text{where } e_j = -(T_j \ln T_j + I_j \ln I_j + F_j \ln F_j) \quad (11)$$

Then apply the standard IOWA operator using these w_j weights.

Definition 3.7 (Multi-Stage Neutrosophic IOWA (MSN-IOWA)) Suppose a two-layer decision-making structure with sub-criteria sets $C^{(1)}, C^{(2)}, \dots, C^{(m)}$. First, apply IOWA to each group:

$$y_k = \text{IOWA}^{(k)}(x_{k1}, x_{k2}, \dots, x_{kn_k}), \quad k = 1, 2, \dots, m \quad (12)$$

Then aggregate the stage-1 outputs y_k using a higher-level IOWA:

$$\text{MSN-IOWA} = \text{IOWA}^{(2)}(y_1, y_2, \dots, y_m) \quad (13)$$

4 Decision Methodology Using IOWA_N Operator

In this section, we propose a structured decision-making methodology based on the IOWA_N operator. This methodology enables the aggregation of expert opinions or multi-criteria evaluations where uncertainty, indeterminacy, and vagueness are explicitly handled. Figure 1 presents the decision methodology graphically.

4.1 Proposed Methodology

Step 1: Identify the decision alternatives $A = \{A_1, A_2, \dots, A_m\}$ and criteria $C = \{C_1, C_2, \dots, C_n\}$ to evaluate the alternatives. Assign neutrosophic values $x_{ij} = (T_{ij}, I_{ij}, F_{ij})$ for each alternative A_i under each criterion C_j .

Step 2: Choose an appropriate inducing function $f: \mathbb{N}^3 \rightarrow \mathbb{R}$ to determine the significance or relevance of each input based on a particular component, such as truth-membership T , indeterminacy I , or falsity F . A common choice is:

$$f(x_{ij}) = T_{ij} - F_{ij}$$

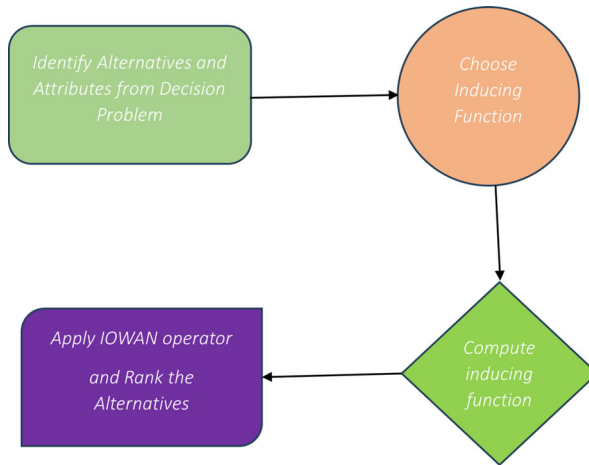


Fig. 1 Flow diagram of decision methodology using induced OWA neutrosophic operator

to emphasize reliable and less contradictory information.

Step 3: For each set of neutrosophic values $\{x_{1j}, x_{2j}, \dots, x_{mj}\}$ under criterion C_j , compute $f(x_{ij})$ and sort the values according to the descending order of $f(x_{ij})$. Let π_j be the permutation that sorts the inputs accordingly.

Step 4: Let $w = (w_1, w_2, \dots, w_m)$ be the weight vector such that $w_i \in [0, 1]$ and $\sum w_i = 1$. Apply the $IOWA_N$ operator as:

$$IOWA_N j = \left(\sum_{i=1}^m w_i T_{\pi_j(i)}, \sum_{i=1}^m w_i I_{\pi_j(i)}, \sum_{i=1}^m w_i F_{\pi_j(i)} \right)$$

This gives a single aggregated neutrosophic score for each criterion C_j .

Step 5: Aggregate the truth, indeterminacy, and falsity value of each criterion to calculate the final neutrosophic value.

4.2 Illustrative Example

In this case study, a company aims to select the best supplier among four alternatives A_1, A_2, A_3, A_4 based on three criteria: Cost (C_1), Quality (C_2), and Delivery Time (C_3). Each criterion is evaluated using neutrosophic numbers to represent degrees of truth (T), indeterminacy (I), and falsity (F). An inducing function $f(x)$, reflecting risk perception, is provided for each alternative.

Step 1: Neutrosophic decision matrix

Alternative	C ₁	C ₂	C ₃
A ₁	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.2)	(0.8, 0.1, 0.1)
A ₂	(0.9, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.6, 0.2, 0.2)
A ₃	(0.6, 0.3, 0.2)	(0.8, 0.1, 0.2)	(0.7, 0.2, 0.1)
A ₄	(0.8, 0.1, 0.2)	(0.6, 0.2, 0.2)	(0.9, 0.1, 0.1)

Step 2: Inducing variable $f(x)$

$$f(A_1) = 0.6, \quad f(A_2) = 0.8, \quad f(A_3) = 0.5, \quad f(A_4) = 0.7$$

Step 3: Reordering based on inducing function

$$\text{Order: } A_2, A_4, A_1, A_3$$

Step 4: Apply $IOWA_N$ for each criterion Let the induced OWA weights be

$$w = (0.4, 0.3, 0.2, 0.1)$$

Then we calculate the $IOWA_N$

For C_1 :

$$T_{agg}^{(1)} = 0.4 \cdot 0.9 + 0.3 \cdot 0.8 + 0.2 \cdot 0.7 + 0.1 \cdot 0.6 = 0.81$$

$$I_{agg}^{(1)} = 0.4 \cdot 0.1 + 0.3 \cdot 0.1 + 0.2 \cdot 0.2 + 0.1 \cdot 0.3 = 0.14$$

$$F_{agg}^{(1)} = 0.4 \cdot 0.1 + 0.3 \cdot 0.2 + 0.2 \cdot 0.1 + 0.1 \cdot 0.2 = 0.14$$

For C_2 :

$$T_{agg}^{(2)} = 0.4 \cdot 0.7 + 0.3 \cdot 0.6 + 0.2 \cdot 0.6 + 0.1 \cdot 0.8 = 0.67$$

$$I_{agg}^{(2)} = 0.4 \cdot 0.2 + 0.3 \cdot 0.2 + 0.2 \cdot 0.3 + 0.1 \cdot 0.1 = 0.21$$

$$F_{agg}^{(2)} = 0.4 \cdot 0.1 + 0.3 \cdot 0.2 + 0.2 \cdot 0.2 + 0.1 \cdot 0.2 = 0.17$$

For C_3 :

$$T_{agg}^{(3)} = 0.4 \cdot 0.6 + 0.3 \cdot 0.9 + 0.2 \cdot 0.8 + 0.1 \cdot 0.7 = 0.74$$

$$I_{agg}^{(3)} = 0.4 \cdot 0.2 + 0.3 \cdot 0.1 + 0.2 \cdot 0.1 + 0.1 \cdot 0.2 = 0.15$$

$$F_{agg}^{(3)} = 0.4 \cdot 0.2 + 0.3 \cdot 0.1 + 0.2 \cdot 0.1 + 0.1 \cdot 0.1 = 0.14$$

Step 5: Final aggregation across criteria

$$T_{final} = \frac{0.81 + 0.67 + 0.74}{3} = 0.74, \quad I_{final} = \frac{0.14 + 0.21 + 0.15}{3} = 0.17, \quad F_{final} = \frac{0.14 + 0.17 + 0.14}{3} = 0.15$$

Based on the $IOWA_N$ operator, the final neutrosophic score is:

$$(0.74, 0.17, 0.15)$$

The illustrative case study demonstrates the practical application and effectiveness of the proposed $IOWA_N$ operator in a decision-making environment characterized by uncertainty and indeterminacy. The results indicate that the selected supplier (corresponding to the aggregated neutrosophic score of $(0.74, 0.17, 0.15)$) offers a high degree of reliability with minimal falsity and moderate indeterminacy. This outcome supports the effectiveness of the $IOWA_N$ operator in handling complex evaluations where ambiguity and partial knowledge are inherent.

4.3 Comparison of OWA, Induced OWA, and $IOWA_N$

To better understand the advancement introduced by the induced ordered weighted averaging operator for neutrosophic sets, $IOWA_N$. We present a comparative analysis (Table 1) against the classical OWA and IOWA operators. The comparison focuses on their structural form, capability to handle uncertainty, flexibility in aggregation, and suitability for real-world decision-making scenarios.

From Table 1, it is evident that the proposed $IOWA_N$ operator outperforms both classical OWA and standard IOWA in multiple dimensions, especially in its ability to manage uncertainty and indeterminacy.

Table 1 Comparison of OWA, IOWA, and $IOWA_N$

Feature	OWA	IOWA	$IOWA_N$
Input Type	Crisp / Fuzzy values	Crisp values with inducing variable	Neutrosophic numbers (T, I, F)
Ordering Basis	Input values (descending)	Inducing variable	Inducing variable
Uncertainty Handling	via fuzzy sets	Limited	via neutrosophic sets
Aggregation Formula	$\sum_{j=1}^n w_j x_{(j)}$	$\sum_{j=1}^n w_j x_{i_j}$	$\sum w_j T_{i_j}, \sum w_j I_{i_j}, \sum w_j F_{i_j}$
Weight Dependency	Position-based	Induced ordering	Induced ordering with uncertainty
Flexibility in Modeling	Moderate	High	Very high
Information Loss	crisp inputs only	no indeterminacy	captures indeterminacy

- **OWA** aggregates inputs based on descending order of their magnitude and applies position-based weights. However, it lacks the flexibility to incorporate additional information like reliability or probability of values. Its inability to represent indeterminacy limits its effectiveness in uncertain environments.
- **IOWA** improves upon OWA by allowing the ordering of elements to be influenced by an inducing variable, which adds contextual flexibility. Despite this improvement, IOWA still operates on crisp or fuzzy values and does not explicitly model the indeterminacy present in real-world data.
- $IOWA_N$, by contrast, integrates the neutrosophic set theory, which distinctly captures three dimensions of information: truth (T), indeterminacy (I), and falsity (F). This triad enables robust modeling of highly uncertain, inconsistent, or incomplete information. It also supports interval-valued and probabilistic extensions, making it highly adaptable for complex decision-making environments such as risk assessment, medical diagnosis, or multi-criteria evaluations.

In summary, the $IOWA_N$ operator presents a natural generalization of both OWA and IOWA operators, offering superior expressiveness and adaptability. Its strength lies in handling imprecise, indeterminate, and inconsistent data characteristics common in real-world decision-making problems.

5 Generalized Induced OWA Operator for Neutrosophic Sets

Definition 5.1 (Generalized Induced OWA Operator for Neutrosophic Sets ($GIOWA_N$))

Let $X = \{x_1, x_2, \dots, x_n\}$ be a collection of neutrosophic values, where each $x_i = \langle T_i, I_i, F_i \rangle \in [0, 1]^3$, and let $f(x_i) \in \mathbb{R}$ be an inducing function.

1. Sort the elements in descending order based on $f(x_i)$ to obtain the permutation π such that:

$$f(x_{\pi(1)}) \geq f(x_{\pi(2)}) \geq \dots \geq f(x_{\pi(n)})$$

2. Let $W = \{w_1, w_2, \dots, w_n\}$ be the associated weight vector with $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$.

Then, the Generalized Induced OWA ($GIOWA_N$) aggregation under parameter $\lambda \neq 0$ is defined as:

$$GIOWA_N^\lambda(X) = \left(\left(\sum_{j=1}^n w_j T_{\pi(j)}^\lambda \right)^{1/\lambda}, \left(\sum_{j=1}^n w_j I_{\pi(j)}^\lambda \right)^{1/\lambda}, \left(\sum_{j=1}^n w_j F_{\pi(j)}^\lambda \right)^{1/\lambda} \right) \quad (14)$$

By changing the values of λ , we get different families of the Generalized Induced OWA Operator under a neutrosophic environment. The following (Table 2) shows the cases and further definitions with examples that illustrate the concept, and a complete summary is presented in Table 3. We define the following families of $GIOWA_N$ operators:

Table 2 Families of Generalized Induced Neutrosophic OWA Operators

Aggregation Type	Parameter λ	Name of Operator
Arithmetic Mean	$\lambda = 1$	Induced Neutrosophic OWA (IOWA _N)
Harmonic Mean	$\lambda = -1$	Harmonic GIOWA _N
Quadratic Mean	$\lambda = 2$	Quadratic GIOWA _N
Geometric Mean	$\lambda \rightarrow 0$	Geometric GIOWA _N

Definition 5.2 (Arithmetic GIOWA_N (A-GIOWA_N)) When all weights are equal, $w_i = \frac{1}{n}$ for all i , and $\lambda = 1$, the operator reduces to the Arithmetic GIOWA_N, given by:

$$\text{A-GIOWA}_N(X) = \left(\frac{1}{n} \sum_{i=1}^n T_{\pi(i)}, \frac{1}{n} \sum_{i=1}^n I_{\pi(i)}, \frac{1}{n} \sum_{i=1}^n F_{\pi(i)} \right) \quad (15)$$

x_i	T_i	I_i	F_i
x_1	0.7	0.2	0.1
x_2	0.6	0.3	0.1
x_3	0.8	0.1	0.1

Example: With inducing values $u = (0.8, 0.9, 0.7)$, reordering by u gives $x_{(1)} = x_2, x_{(2)} = x_1, x_{(3)} = x_3$.

Table 3 Summary of various GIOWA_N operators

Operator	Definition
GIOWA _N ^{λ}	$\left(\sum_{i=1}^n w_i (T_{\pi(i)})^\lambda \right)^{1/\lambda}, \left(\sum_{i=1}^n w_i (I_{\pi(i)})^\lambda \right)^{1/\lambda}, \left(\sum_{i=1}^n w_i (F_{\pi(i)})^\lambda \right)^{1/\lambda}$
Geometric GIOWA _N	$\prod_{i=1}^n (T_{\pi(i)})^{w_i}, \prod_{i=1}^n (I_{\pi(i)})^{w_i}, \prod_{i=1}^n (F_{\pi(i)})^{w_i}$
Arithmetic GIOWA _N	$\frac{1}{n} \sum_{i=1}^n T_{\pi(i)}, \frac{1}{n} \sum_{i=1}^n I_{\pi(i)}, \frac{1}{n} \sum_{i=1}^n F_{\pi(i)}$
Quadratic GIOWA _N	$\left(\sum_{i=1}^n \frac{i^2}{\sum_{j=1}^n j^2} (T_{\pi(i)})^\lambda \right)^{1/\lambda}, \left(\sum_{i=1}^n \frac{i^2}{\sum_{j=1}^n j^2} (I_{\pi(i)})^\lambda \right)^{1/\lambda}, \left(\sum_{i=1}^n \frac{i^2}{\sum_{j=1}^n j^2} (F_{\pi(i)})^\lambda \right)^{1/\lambda}$
Harmonic GIOWA _N	$\left(\sum_{i=1}^n \frac{1/i}{\sum_{j=1}^n 1/j} (T_{\pi(i)})^\lambda \right)^{1/\lambda}, \left(\sum_{i=1}^n \frac{1/i}{\sum_{j=1}^n 1/j} (I_{\pi(i)})^\lambda \right)^{1/\lambda}, \left(\sum_{i=1}^n \frac{1/i}{\sum_{j=1}^n 1/j} (F_{\pi(i)})^\lambda \right)^{1/\lambda}$
Min-GIOWA _N	$(T_{\pi(1)}, I_{\pi(1)}, F_{\pi(1)})$
Max-GIOWA _N	$(T_{\pi(n)}, I_{\pi(n)}, F_{\pi(n)})$

Thus,

$$\text{A-GIOWA}_N(X) = \left(\frac{0.6 + 0.7 + 0.8}{3}, \frac{0.3 + 0.2 + 0.1}{3}, \frac{0.1 + 0.1 + 0.1}{3} \right) = (0.7, 0.2, 0.1)$$

Definition 5.3 (Harmonic GIOWA_N (H-GIOWA_N)) If weights are assigned proportional to the reciprocal of the position index, as follows;

$$w_i = \frac{1/i}{\sum_{j=1}^n 1/j},$$

then Harmonic GIOWA_N is defined as:

$$\text{H-GIOWA}_N(X) = \left(\left(\sum_{i=1}^n \frac{1/i}{\sum_{j=1}^n 1/j} (T_{\pi(i)})^\lambda \right)^{1/\lambda}, \left(\sum_{i=1}^n \frac{1/i}{\sum_{j=1}^n 1/j} (I_{\pi(i)})^\lambda \right)^{1/\lambda}, \left(\sum_{i=1}^n \frac{1/i}{\sum_{j=1}^n 1/j} (F_{\pi(i)})^\lambda \right)^{1/\lambda} \right) \quad (16)$$

Example:

$$\sum_{j=1}^3 \frac{1}{j} = 1 + 0.5 + 0.333 \approx 1.833$$

thus,

$$w_1 \approx 0.545, \quad w_2 \approx 0.273, \quad w_3 \approx 0.182$$

Thus,

$$\text{H-GIOWA}_N(X) = (0.545(0.6) + 0.273(0.7) + 0.182(0.8), 0.545(0.3) + 0.273(0.2) + 0.182(0.1), 0.1)$$

Computing:

$$T \approx 0.664, \quad I \approx 0.237, \quad F = 0.1$$

Thus,

$$\text{H-GIOWA}_N(X) = (0.664, 0.237, 0.1)$$

Definition 5.4 (Quadratic GIOWA_N (Q-GIOWA_N)) If weights are assigned proportional to the square of the position index, as follows:

$$w_i = \frac{i^2}{\sum_{j=1}^n j^2},$$

then Quadratic GIOWA_N is defined as:

$$\text{Q-GIOWA}_N(X) = \left(\left(\sum_{i=1}^n \frac{i^2}{\sum_{j=1}^n j^2} (T_{\pi(i)})^\lambda \right)^{1/\lambda}, \left(\sum_{i=1}^n \frac{i^2}{\sum_{j=1}^n j^2} (I_{\pi(i)})^\lambda \right)^{1/\lambda}, \left(\sum_{i=1}^n \frac{i^2}{\sum_{j=1}^n j^2} (F_{\pi(i)})^\lambda \right)^{1/\lambda} \right) \quad (17)$$

Example:

$$\sum_{j=1}^3 j^2 = 1 + 4 + 9 = 14$$

thus,

$$w_1 = \frac{1}{14}, \quad w_2 = \frac{4}{14}, \quad w_3 = \frac{9}{14}$$

Thus,

$$\text{Q-GIOWA}_N(X) = \left(\frac{1}{14}(0.6) + \frac{4}{14}(0.7) + \frac{9}{14}(0.8), \frac{1}{14}(0.3) + \frac{4}{14}(0.2) + \frac{9}{14}(0.1), 0.1 \right)$$

Computing:

$$T \approx 0.757, \quad I \approx 0.143, \quad F = 0.1$$

Thus,

$$\text{Q-GIOWA}_N(X) = (0.757, 0.143, 0.1)$$

Definition 5.5 (Geometric GIOWA_N Operator) When $\lambda \rightarrow 0$, the GIOWA_N^λ operator converges to the Geometric GIOWA_N operator, defined as:

$$\text{GIOWA}_N^{\text{geo}}(X) = \left(\prod_{i=1}^n (T_{\pi(i)})^{w_i}, \prod_{i=1}^n (I_{\pi(i)})^{w_i}, \prod_{i=1}^n (F_{\pi(i)})^{w_i} \right) \quad (18)$$

Definition 5.6 (Min- GIOWA_N Operator) If the weight vector is defined by $w_1 = 1$ and $w_i = 0$ for all $i \neq 1$, focusing solely on the first reordered element, the Min- GIOWA_N operator is defined as:

$$\text{Min-GIOWA}_N(X) = (T_{\pi(1)}, I_{\pi(1)}, F_{\pi(1)}) \quad (19)$$

Definition 5.7 (Max- GIOWA_N Operator) If the weight vector is defined by $w_n = 1$ and $w_i = 0$ for all $i \neq n$, focusing solely on the last reordered element, the Max- GIOWA_N operator is defined as:

$$\text{Max-GIOWA}_N(X) = (T_{\pi(n)}, I_{\pi(n)}, F_{\pi(n)}) \quad (20)$$

The GIOWA_N operators are a versatile family of aggregation tools used in multi-criteria decision-making, offering flexibility in handling uncertainty and preferences. These operators include variations such as the GIOWA_N^λ , which adjusts criteria values with a power parameter λ to capture varying importance, and the Geometric GIOWA_N , which uses the geometric mean to emphasize multiplicative relationships among criteria. The Arithmetic GIOWA_N aggregates criteria linearly, ideal for additive relationships, while the Quadratic GIOWA_N gives more weight to larger values, focusing on extreme values. In contrast, the Harmonic GIOWA_N operator emphasizes smaller values, “penalizing” larger ones. Finally, the Min- GIOWA_N and Max- GIOWA_N focus on the worst and best-case outcomes, respectively. Each operator caters to different

decision-making scenarios, allowing for tailored aggregation based on the decision-maker's preferences and the nature of the problem.

5.1 Decision Methodology Using $GLOWA_N$ Operator

This section presents a decision-making methodology based on the generalized induced ordered weighted averaging ($GLOWA_N$) operator for Neutrosophic Sets, specifically designed for multiple experts in a decision-making process. Figure 2 presents the decision methodology graphically.

Step 1: Define the Decision Problem

The decision problem involves selecting the best alternative from a set of alternatives $A_1, A_2, \dots, A_m$ based on several criteria $C_1, C_2, \dots, C_n$. Each alternative is evaluated by multiple experts, $E_1, E_2, \dots, E_K$, on each criterion. The assessments are made using neutrosophic sets, where each element consists of three membership functions:

$T_{\pi(i)}$ (Truth membership), $I_{\pi(i)}$ (Indeterminacy membership), $F_{\pi(i)}$ (False membership).

These membership values represent the uncertainty associated with each expert's evaluation of the alternatives.

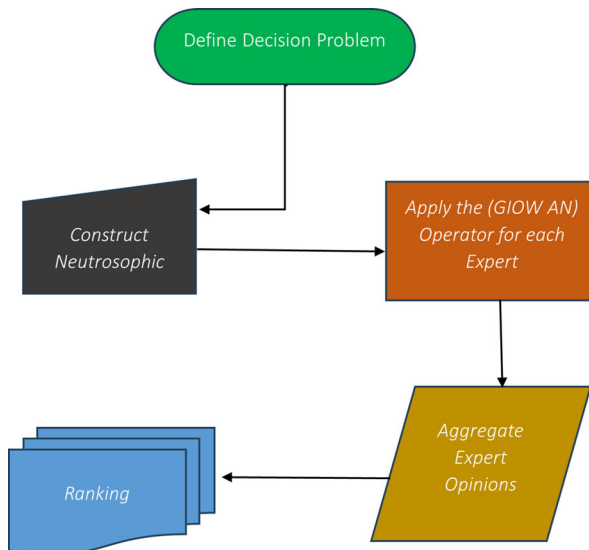


Fig. 2 Flow chart of decision methodology using generalized induced OWA neutrosophic operator

Step 2: Construct Neutrosophic Decision Matrices

For each expert E_k , construct a decision matrix $D^{(k)}$ representing the neutrosophic evaluations of all alternatives on all criteria. The matrix is defined as:

$$D^{(k)} = \left[\mu_{ij}^{(k)} \right]_{m \times n},$$

where $\mu_{ij}^{(k)} = (T_{\pi(i)}^{(k)}, I_{\pi(i)}^{(k)}, F_{\pi(i)}^{(k)})$ denotes the neutrosophic assessment of alternative A_i on criterion C_j by expert E_k .

Step 3: Apply the (GIOWA_N) Operator for Each Expert

For each expert, apply the GIOWA_N to the neutrosophic decision matrix. The GIOWA_N operator for each criterion is applied as:

$$GIOWA^{(k)}(T_{\pi(i)}, I_{\pi(i)}, F_{\pi(i)}) = \left(\sum_{i=1}^n w_i (T_{\pi(i)})^\lambda \right)^{1/\lambda}, \left(\sum_{i=1}^n w_i (I_{\pi(i)})^\lambda \right)^{1/\lambda}, \left(\sum_{i=1}^n w_i (F_{\pi(i)})^\lambda \right)^{1/\lambda} \quad (21)$$

where w_i are the weights assigned to each alternative, and λ is a parameter that controls the impact of extreme values on the aggregation.

Step 4: Aggregate Expert Opinions

After applying the GIOWA_N for each expert, aggregate the results across all experts. The aggregated truth, indeterminacy, and false membership values for each alternative are calculated as:

$$T_{\text{agg}} = \frac{1}{K} \sum_{k=1}^K T_{\text{agg}}^{(k)}, \quad I_{\text{agg}} = \frac{1}{K} \sum_{k=1}^K I_{\text{agg}}^{(k)}, \quad F_{\text{agg}} = \frac{1}{K} \sum_{k=1}^K F_{\text{agg}}^{(k)} \quad (22)$$

where K is the number of experts.

Step 5: Rank the Alternatives

Once the aggregated membership values T_{agg} , I_{agg} , F_{agg} for each alternative have been calculated, rank the alternatives based on a scoring function that combines the three membership aspects. For example, the score for each alternative A_i can be computed as:

$$S(A_i) = \alpha T_{\text{agg}}(A_i) - \beta I_{\text{agg}}(A_i) - \gamma F_{\text{agg}}(A_i) \quad (23)$$

where α, β, γ are weight parameters controlling the importance of truth, indeterminacy, and false membership. The alternatives are then ranked in descending order of their score $S(A_i)$. The alternative with the highest aggregated score is considered the

optimal choice. In cases where alternatives are close in score, further analysis can be done based on additional criteria or sensitivity analysis.

5.2 Illustrative Example

Now we present a solved case study to demonstrate the effectiveness of the proposed algorithm.

Step 1: Decision Problem A company wants to invest in Southeast Asian countries and evaluate them based on four economic criteria: Economic Growth (EG), Inflation Rate (IR), Unemployment Rate (UR), and Public Support (PS). The decision-making process involves the analysis of expert evaluations, where three experts provide assessments for each country with respect to each criterion. The experts use neutrosophic sets to represent their opinions, considering the truth, indeterminacy, and false memberships for each evaluation. These membership values are then aggregated using the $GIOWA_N$ to handle uncertainty and indeterminacy in expert opinions. The outcome of the decision-making process is to rank the countries based on their aggregated scores, identifying the most economically favorable countries in Southeast Asia.

Step 2: Construction of Neutrosophic Decision Matrices Below are the decision matrices $D^{(1)}$, $D^{(2)}$, and $D^{(3)}$ for all three experts $E^{(1)}$, $E^{(2)}$, and $E^{(3)}$. Each matrix represents the evaluations of the 16 Southeast Asian countries for the four criteria, using neutrosophic sets to denote the truth (T), indeterminacy (I), and false (F) membership values.

Country	C1=EG	C2=IR	C3=UR	C4=PS)
Singapore	(0.9, 0.1, 0.0)	(0.8, 0.2, 0.0)	(0.7, 0.3, 0.0)	(0.9, 0.1, 0.0)
Thailand	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.7, 0.2, 0.1)
Vietnam	(0.8, 0.2, 0.0)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.8, 0.2, 0.0)
Indonesia	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.6, 0.3, 0.1)
Malaysia	(0.8, 0.2, 0.0)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.7, 0.2, 0.1)
Philippines	(0.7, 0.3, 0.0)	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.7, 0.2, 0.1)
Myanmar	(0.5, 0.3, 0.2)	(0.4, 0.4, 0.2)	(0.3, 0.5, 0.2)	(0.6, 0.3, 0.1)
Cambodia	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.5, 0.4, 0.1)
Laos	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.5, 0.4, 0.1)
Brunei	(0.9, 0.1, 0.0)	(0.8, 0.2, 0.0)	(0.7, 0.3, 0.0)	(0.9, 0.1, 0.0)
Timor-Leste	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.3, 0.5, 0.2)	(0.6, 0.3, 0.1)
Bangladesh	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.7, 0.2, 0.1)
Nepal	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.3, 0.5, 0.2)	(0.6, 0.3, 0.1)
India	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.7, 0.2, 0.1)
Sri Lanka	(0.5, 0.4, 0.1)	(0.5, 0.3, 0.2)	(0.4, 0.4, 0.2)	(0.6, 0.3, 0.1)

$D^{(2)} =$	Country	C1=EG	C2=IR	C3=UR	C4=PS)
	Singapore	(0.8, 0.2, 0.0)	(0.7, 0.3, 0.0)	(0.6, 0.4, 0.0)	(0.8, 0.2, 0.0)
	Thailand	(0.7, 0.3, 0.0)	(0.6, 0.4, 0.0)	(0.5, 0.4, 0.1)	(0.7, 0.3, 0.0)
	Vietnam	(0.8, 0.2, 0.0)	(0.7, 0.3, 0.0)	(0.6, 0.3, 0.1)	(0.8, 0.2, 0.0)
	Indonesia	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.4, 0.4, 0.2)	(0.6, 0.3, 0.1)
	Malaysia	(0.7, 0.3, 0.0)	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.7, 0.2, 0.1)
	Philippines	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.4, 0.4, 0.2)	(0.7, 0.3, 0.0)
	Myanmar	(0.5, 0.4, 0.1)	(0.4, 0.4, 0.2)	(0.3, 0.5, 0.2)	(0.6, 0.3, 0.1)
	Cambodia	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.3, 0.5, 0.2)	(0.5, 0.4, 0.1)
	Laos	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.5, 0.4, 0.1)
	Brunei	(0.9, 0.1, 0.0)	(0.8, 0.2, 0.0)	(0.7, 0.3, 0.0)	(0.9, 0.1, 0.0)
	Timor-Leste	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.3, 0.5, 0.2)	(0.6, 0.3, 0.1)
	Bangladesh	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.7, 0.2, 0.1)
	Nepal	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.3, 0.5, 0.2)	(0.6, 0.3, 0.1)
	India	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.7, 0.2, 0.1)
	Sri Lanka	(0.5, 0.4, 0.1)	(0.5, 0.3, 0.2)	(0.4, 0.4, 0.2)	(0.6, 0.3, 0.1)

$D^{(3)} =$	Country	C1=EG	C2=IR	C3=UR	C4=PS)
	Singapore	(0.85, 0.15, 0.0)	(0.75, 0.25, 0.0)	(0.65, 0.35, 0.0)	(0.85, 0.15, 0.0)
	Thailand	(0.75, 0.25, 0.0)	(0.65, 0.35, 0.0)	(0.55, 0.35, 0.1)	(0.75, 0.25, 0.0)
	Vietnam	(0.85, 0.15, 0.0)	(0.75, 0.25, 0.0)	(0.65, 0.30, 0.05)	(0.85, 0.15, 0.0)
	Indonesia	(0.65, 0.30, 0.05)	(0.55, 0.35, 0.1)	(0.45, 0.45, 0.1)	(0.65, 0.30, 0.05)
	Malaysia	(0.75, 0.25, 0.0)	(0.65, 0.30, 0.05)	(0.55, 0.40, 0.05)	(0.75, 0.25, 0.0)
	Philippines	(0.70, 0.30, 0.0)	(0.60, 0.35, 0.05)	(0.50, 0.40, 0.1)	(0.70, 0.30, 0.0)
	Myanmar	(0.55, 0.35, 0.1)	(0.45, 0.40, 0.15)	(0.35, 0.45, 0.2)	(0.55, 0.35, 0.1)
	Cambodia	(0.55, 0.35, 0.1)	(0.45, 0.40, 0.15)	(0.35, 0.45, 0.2)	(0.55, 0.35, 0.1)
	Laos	(0.65, 0.30, 0.05)	(0.55, 0.35, 0.1)	(0.45, 0.40, 0.15)	(0.65, 0.30, 0.05)
	Brunei	(0.90, 0.10, 0.0)	(0.80, 0.20, 0.0)	(0.70, 0.30, 0.0)	(0.90, 0.10, 0.0)
	Timor-Leste	(0.55, 0.35, 0.1)	(0.45, 0.40, 0.15)	(0.35, 0.45, 0.2)	(0.55, 0.35, 0.1)
	Bangladesh	(0.65, 0.30, 0.05)	(0.55, 0.35, 0.1)	(0.45, 0.40, 0.15)	(0.65, 0.30, 0.05)
	Nepal	(0.55, 0.35, 0.1)	(0.45, 0.40, 0.15)	(0.35, 0.45, 0.2)	(0.55, 0.35, 0.1)
	India	(0.75, 0.25, 0.0)	(0.65, 0.35, 0.0)	(0.55, 0.35, 0.1)	(0.75, 0.25, 0.0)
	Sri Lanka	(0.55, 0.35, 0.1)	(0.50, 0.40, 0.1)	(0.45, 0.40, 0.15)	(0.55, 0.35, 0.1)

Step 3: GIOWA_N calculation for each Expert To aggregate the neutrosophic evaluations for each alternative across the four criteria – Economic Growth (EG), Inflation Rate (IR), Unemployment Rate (UR), and Public Support (PS), the GIOWA_N is applied for each expert. We assume equal weights for each criterion and use $\lambda = 1$, which corresponds to the arithmetic mean:

$$\text{GIOWA}_\lambda(x_1, x_2, x_3, x_4) = \left(\sum_{i=1}^4 w_i \cdot x_i^\lambda \right)^{1/\lambda} \quad \text{with } w_i = \frac{1}{4} \quad (24)$$

To apply the GIOWA_N operator, we use an inducing function based on the reliability of the neutrosophic evaluations. The chosen function is:

$$f(x) = T - F \quad (25)$$

This function emphasizes high truth-membership and low falsity. The criteria for each expert are reordered based on descending values of f , and then the GIOWA_N operator is applied using equal weights. For one alternative, the complete calculations are presented below; consider the alternative as Vietnam. Table 4 below presents the calculated aggregation for all alternatives and experts. We calculate the inducing order for the alternative based on expert 1's opinion, and get the results as follows;

- C1 (EG): (0.8, 0.2, 0.0) $\Rightarrow f = 0.8$
- C2 (IR): (0.7, 0.2, 0.1) $\Rightarrow f = 0.6$
- C3 (UR): (0.6, 0.3, 0.1) $\Rightarrow f = 0.5$
- C4 (PS): (0.8, 0.2, 0.0) $\Rightarrow f = 0.8$

We get the induced orders as C1, C4, C2, C3.

$$T^{(1)} = \frac{0.8 + 0.8 + 0.7 + 0.6}{4} = 0.725, \quad I^{(1)} = 0.225, \quad F^{(1)} = 0.050$$

We calculate the inducing order for the alternative based on expert 2's opinion, and get the results as follows;

- C1 (EG): (0.8, 0.2, 0.0) $\Rightarrow f = 0.8$

Table 4 GIOWA_N aggregation of Countries for all 3 experts

Country	Expert 1			Expert 2			Expert 3		
	T_1	I_1	F_1	T_2	I_2	F_2	T_3	I_3	F_3
Singapore	0.825	0.175	0.000	0.775	0.2250	0.0000	0.775	0.225	0.000
Thailand	0.625	0.275	0.100	0.675	0.3000	0.0250	0.675	0.275	0.050
Vietnam	0.725	0.225	0.050	0.775	0.2125	0.0125	0.775	0.2125	0.0125
Indonesia	0.525	0.375	0.100	0.575	0.3500	0.0750	0.575	0.350	0.075
Malaysia	0.700	0.225	0.075	0.675	0.3000	0.0250	0.675	0.250	0.025
Philippines	0.625	0.300	0.075	0.625	0.3000	0.0500	0.625	0.2625	0.0275
Myanmar	0.450	0.375	0.175	0.450	0.3750	0.1250	0.450	0.3875	0.1625
Cambodia	0.500	0.400	0.150	0.425	0.4000	0.1500	0.450	0.3875	0.1625
Laos	0.500	0.400	0.150	0.500	0.4000	0.1250	0.550	0.3375	0.0875
Brunei	0.825	0.175	0.000	0.825	0.1750	0.0000	0.825	0.175	0.000
Timor-Leste	0.450	0.375	0.175	0.450	0.3750	0.1250	0.450	0.3875	0.1625
Bangladesh	0.550	0.350	0.100	0.550	0.3500	0.1000	0.550	0.3375	0.0875
Nepal	0.450	0.375	0.175	0.450	0.3750	0.1250	0.450	0.3875	0.1625
India	0.625	0.275	0.100	0.625	0.2750	0.1000	0.675	0.2375	0.0875
Sri Lanka	0.500	0.350	0.150	0.525	0.3000	0.1750	0.525	0.3375	0.1375

- C2 (IR): (0.7, 0.3, 0.0) $\Rightarrow f = 0.7$
- C3 (UR): (0.6, 0.3, 0.1) $\Rightarrow f = 0.5$
- C4 (PS): (0.8, 0.2, 0.0) $\Rightarrow f = 0.8$

We get the induced orders for expert 1 as C1, C4, C2, C3.

$$T^{(2)} = \frac{0.8 + 0.8 + 0.7 + 0.6}{4} = 0.725, \quad I^{(2)} = 0.2125, \quad F^{(2)} = 0.0125$$

We calculate the inducing order for the alternative based on expert 3's opinion, and get the results as follows;

- C1 (EG): (0.85, 0.15, 0.0) $\Rightarrow f = 0.85$
- C2 (IR): (0.75, 0.25, 0.0) $\Rightarrow f = 0.75$
- C3 (UR): (0.65, 0.30, 0.05) $\Rightarrow f = 0.60$
- C4 (PS): (0.85, 0.15, 0.0) $\Rightarrow f = 0.85$

We get the induced orders for expert 3 as C1, C4, C2, C3

$$T^{(3)} = \frac{0.85 + 0.85 + 0.75 + 0.65}{4} = 0.775, \quad I^{(3)} = 0.2125, \quad F^{(3)} = 0.0125$$

Step 4: Expert opinion aggregation The neutrosophic evaluations from all three experts were aggregated by computing the arithmetic mean for each component: truth-membership (T), indeterminacy-membership (I), and falsity-membership

Table 5 Final aggregated neutrosophic scores and rankings of Southeast Asian Countries

Rank	Country	T_{agg}	I_{agg}	F_{agg}	Score
1	Brunei	0.8250	0.1750	0.0000	0.7375
2	Singapore	0.7750	0.2250	0.0000	0.6625
3	Vietnam	0.7417	0.2292	0.0292	0.5979
4	India	0.6417	0.2625	0.0958	0.4823
5	Philippines	0.6250	0.2875	0.0508	0.4817
6	Malaysia	0.6667	0.2750	0.0583	0.4708
7	Thailand	0.6417	0.3083	0.0500	0.4375
8	Bangladesh	0.5500	0.3458	0.0958	0.4063
9	Indonesia	0.5583	0.3583	0.0833	0.3965
10	Sri Lanka	0.5167	0.3292	0.1542	0.3692
11	Timor-Leste	0.4500	0.3792	0.1542	0.3692
12	Nepal	0.4500	0.3792	0.1542	0.3692
13	Myanmar	0.4500	0.3792	0.1542	0.3692
14	Cambodia	0.4583	0.3958	0.1542	0.3626
15	Laos	0.5167	0.3792	0.1208	0.3271

(F) for every country. We compute the aggregated neutrosophic values as the mean across the three experts. Table 5 summarizes the aggregated values:

$$T_{\text{agg}} = \frac{0.725 + 0.775 + 0.725}{3} = 0.7417$$

$$I_{\text{agg}} = \frac{0.225 + 0.2125 + 0.250}{3} = 0.2292$$

$$F_{\text{agg}} = \frac{0.050 + 0.0125 + 0.025}{3} = 0.0292$$

Step 5: Score calculation and final rankings To rank the countries based on their overall economic favorability, a score function is applied:

$$S(A_i) = \alpha T_{\text{agg}}(A_i) - \beta I_{\text{agg}}(A_i) - \gamma F_{\text{agg}}(A_i)$$

where $\alpha = 1$, $\beta = 0.5$, and $\gamma = 1$. The calculated scores and final rankings are shown in the table below: We apply the score function:

$$S(A_i) = \alpha T_{\text{agg}} - \beta I_{\text{agg}} - \gamma F_{\text{agg}}, \quad \text{with } \alpha = 1, \beta = 0.5, \gamma = 1$$

$$S(\text{Vietnam}) = 0.7417 - 0.5 \times 0.2292 - 0.0292 = 0.7417 - 0.1146 - 0.0292 = 0.5979$$

Brunei and Singapore are the top-ranked countries based on generalized induced OWA aggregated for neutrosophic values, reflecting strong economic indicators with low uncertainty and falsity in expert assessments.

5.3 Result Discussion and Comparison

This example was aimed to evaluate the economic performance of Southeast Asian countries, through 4 economic indicators such as: Economic Growth (EG), Inflation Rate (IR), Unemployment Rate (UR), and Public Support (PS). The expert opinions are integrated, and the uncertainty and indeterminacy of the decision-making process are handled based on neutrosophic sets. This approach enabled us to aggregate expert opinions in a more sophisticated way by considering not only truth values, but also indeterminacy and false values, by utilizing the $GIOWA_N$ operator to aggregate expert opinions. Figure 1 showcases the results graphically (Fig. 3).

A novelty to existing aggregations is that the neutrosophic values considered as the expert opinion and the criteria were rearranged based on an inducing function $f(x) = T - F$ and deviations (especially falsities) were minimized. This method provides a more consistent range of criteria and improves the decision process. For instance, Vietnam received an aggregate value (0.7417), which suggests that it obtained a good performance, and it had a relatively good economic performance according to the judgments of the experts. The lower level of indeterminacy and falsity values also indicated lower uncertainty, depicting that Vietnam is still a competitive economy in the Southeast Asian region. The research revealed that Brunei and Singapore were the countries that came on top, which was attributed mainly to their solid economic fundamentals, low inflation, and significant public support. Conversely, the economic

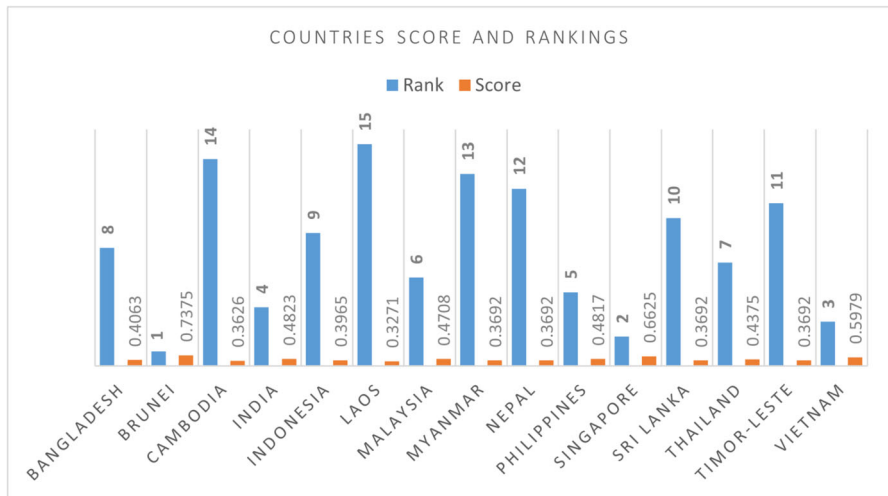


Fig. 3 Countries aggregated scores and final ranking

evaluations were of low quality for Laos, Myanmar, and Cambodia primarily because of greater uncertainty and indeterminacy. This ranking justifies the ground reality in terms of economy because countries with a stronger economy, such as Singapore and Brunei, may provide reliable expert estimates. This approach can be extended to any other domains or situations in which formal expert judgements need to be considered in complex, ambiguous, and uncertain environments. Below Table 6 presents the comparison with existing studies.

The proposed $IOWA_N$ and $GIOWA_N$ operators are compared with classical OWA, IOWA, and standard neutrosophic aggregation methods. The updated comparisons show that the proposed operators produce more stable rankings when indeterminacy or conflicting information is present. Furthermore, the generalized structure of $GIOWA_N$ offers greater adaptability, yielding results that remain consistent across different weighting and information conditions, unlike traditional operators that tend

Table 6 Comparison of $IOWA_N$ and $GIOWA_N$ with Existing Studies

Method	Uncertainty handling	Aggregation formula
OWA (Yager, 2003)	Fuzzy weighted	$\sum w_j x_j$
IOWA (Merig'o, 2012)	Inducing variable	$\sum w_j f(\gamma_j(x))$
Fuzzy OWA (D'Urso et al., 2024)	Fuzzy weighted	$\sum w_j x_j$
NPOWA (Saqlain et al., 2025)	Probabilistic ordered	$\sum_{j=1}^n \hat{p}_j N_{\pi(j)}$
$IOWA_N$ (Proposed)	Neutrosophic inducing	$\sum w_j T_{\pi(j)}, \sum w_j I_{\pi(j)}, \sum w_j F_{\pi(j)}$
$GIOWA_N$ (Proposed)	Generalized inducing	$\sum w_j T_{\pi(j)}^\lambda$

to fluctuate under high uncertainty. These expanded results confirm the enhanced robustness and flexibility of the proposed methodology.

5.4 Sensitivity Analysis

The Sensitivity analysis is performed to investigate how stable the rankings obtained by $GLOWA_N$ are for different situations. We studied the impact of the criterion weights' modifications (and of the small expert evaluation perturbations) on the ranking of the countries of Southeast Asia. To analyze sensitivity analysis for weight changes, we varied the weights of criteria (Economic Growth (EG), Inflation Rate (IR), Unemployment Rate (UR), and Public Support (PS)) while holding the expert evaluations constant. We observed that the weight on EG had the greatest impact. On the contrary, the transformation of the unemployment rate (UR) had a weaker impact on the ranking. To explore the sensitivity of the results to small changes in expert evaluations, in those situations, we replicated situations in which truth, indeterminacy, and falsity assessments by each expert on a country were altered by ± 0.05 . The results suggested that countries such as Brunei and Singapore were very stable, remaining top of the rankings despite such changes. On the other hand, countries like Laos and Cambodia, which held more variability in expert opinions, showed substantial year-to-year movement in rankings with incremental changes in expert feedback. Results of the sensitivity analysis show the dependability of $GLOWA_N$ in obtaining robust rankings for countries with steady economic growth. Countries with economic uncertainty as Myanmar and Laos, are provoked in ranking, in contrast to Brunei and Singapore, which are relatively stable regardless of a shift in the weight or expert evaluation. This shows that while the model is sensitive to changes in certain criteria weights and expert evaluations, it remains stable for countries with clear economic indicators.

5.5 Superiority and Limitations

Advantages The proposed $IOWA_N$ and $GLOWA_N$ operators provide improved handling of uncertainty by incorporating truth, indeterminacy, and falsity values simultaneously. This proves a robust representation than classical fuzzy or intuitionistic fuzzy OWA operators. The use of an inducing function allows dynamic and context-sensitive ordering, ensuring that the aggregation reflects the relative importance or reliability of the inputs. Another advantage is the high flexibility of the $GLOWA_N$ framework, which generalizes multiple aggregation types (arithmetic, geometric, harmonic, quadratic, and extreme cases) within a single structure. This makes the operators adaptable to different decision-making requirements. Additionally, the operators support several extensions such as interval-valued, bipolar, probabilistic, and entropy-based versions, making them suitable for complex multi-criteria and multi-expert environments. The case study further demonstrates their robustness and practicality.

Limitations Despite their strengths, the operators introduce higher computational complexity due to sorting operations and the need to aggregate three components for each

neutrosophic value. The choice of the inducing function also plays a critical role, as an unsuitable selection may lead to biased results.

The operators may be sensitive to parameter choices, especially in the generalized version, where λ affects how extreme values influence the aggregation. Furthermore, determining neutrosophic values and weights often requires expert knowledge, and the three-component structure (T, I, F) may be less intuitive for users unfamiliar with neutrosophic logic.

6 Conclusion

In this paper, we introduced the induced and generalized induced ordered weighted averaging operators under a neutrosophic set, i.e., $IOWA_N$, $GLOWA_N$, extending traditional aggregation frameworks to effectively address uncertainty, indeterminacy, and inconsistency in decision-making scenarios. By incorporating neutrosophic components, truth, indeterminacy, and falsity, the proposed operators allow for a richer representation of subjective judgments and incomplete knowledge. We explored their formal properties, demonstrated their advantages through illustrative examples (we introduced two separate decision methodologies for both $IOWA_N$, and $GLOWA_N$, respectively), and compared their performance with classical OWA and IOWA operators. The illustrative example was performed to assess the economic performance of Southeast Asian countries using four important economic indicators: Economic Growth (EG), Inflation Rate (IR), Unemployment Rate (UR), and Public Support (PS) for multiple experts. Expert evaluations were integrated, and the uncertainty and indeterminacy inherent in the decision-making process were addressed using neutrosophic sets. This method allowed us to consolidate expert opinions with greater sophistication, considering not only truth values but also indeterminacy and false values, and the $GLOWA_N$ operator was used to aggregate expert opinions.

The methodology offers key advantages, including the ability to handle multi-dimensional uncertainty and the use of an inducing function for context-aware ordering. The generalized $GLOWA_N$ framework also provides flexible aggregation behavior suitable for different decision contexts. However, the approach has some limitations, such as higher computational requirements, sensitivity to inducing functions and parameter choices, and the complexity of neutrosophic modeling for non-expert users. Future research may explore dynamic weighting schemes, entropy-based optimization, and integration into hybrid MCGDM models for deeper applicability across diverse domains.

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Declarations

Consent for Publication This article does not contain any studies with human participants or animals performed by any authors.

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