LEARNING AND TEACHING MATHEMATICS: INTERPRETING STUDENT TEACHERS' VOICES

SANDRA RUTH SCHUCK

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CERTIFICATE

I certify that this thesis has not already been submitted for any degree and is not being submitted as part of candidature for any other degree.

I also certify that the thesis has been written by me and that any help that I have received in preparing this thesis, and all sources used, have been acknowledged in this thesis.

Signature of Candidate

Production Note:
Signature removed prior to publication.

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PREFACE

The following publications and conference papers developed from aspects of the research contained in this thesis:

Schuck, S. (1996). Reflections on the dilemmas and tensions in mathematics education courses for student teachers. *Asia-Pacific Journal of Teacher Education (formerly the South Pacific Jnl of Teacher Education) 24(1)* pp. 75-82.

Schuck, S. (1996). Chains in primary teacher mathematics education courses: An analysis of powerful constraints. *Mathematics Education Research Journal, 8(2)* (in press).

Schuck, S. (1996). Probing understanding of what it means to be a primary school teacher of mathematics. In S. Groves, J. Mousley, I. Robottom and R. Tytler (Eds.) *Contemporary Approaches to Research in Mathematics, Science and Environmental Education* (in press). Geelong, Victoria: Centre for Studies in Mathematics, Science and Environmental Education, Deakin University.

Schuck, S. (1995). The role of mathematics education courses in primary teacher education programs. In R. Hunting, G. Fitzsimons, P. Clarkson, & A. Bishop (Eds.), *Regional Collaboration in Mathematics Education 1995: Proceedings of ICMI International Conference* (pp. 511-620). Melbourne: ICMI.

Schuck, S. (1995). Forging links and breaking chains in primary teacher education: negotiating powerful ideas. In B. Atweh, S. Flavel (Eds.), *MERGA-18, Galtha: Proceedings of the Eighteenth Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 465-470). Darwin: MERGA.

Schuck, S. (1995). Teaching and Learning Mathematics - Student Teachers' Perspectives. In S. Groves and R. Tytler (Eds.) *Contemporary Approaches to Research in Mathematics and Science Education*, pp. 169-173. Geelong, Victoria: Centre for Studies in Mathematics, Science and Environmental Education, Deakin University.

Schuck, S. (1995). *Using a research metaphor to stimulate mathematical thought.* Paper presented at the Mathematical Association of NSW Post-Secondary Mini-Conference, Sydney, March.

Schuck, S. (1994) *Mathematics education for student teachers: Disciplining the pedagogy.* Paper presented at the Australian Association for Research in Education Conference, Newcastle, November.

Schuck, S. (1993). Attitudes and beliefs of first year student primary teachers. Poster presented at the *Mathematics Education Research Group of Australasia 16th Annual Conference (MERGA 16)*, Brisbane, July.

Schuck, S. (1993). Attitudes and beliefs of pre-service primary teachers and their effect on learning. Paper presented to the *Special Interest Group on Adult Learning, MERGA 16*, Brisbane, July.

Schuck, S. (1993) Learning and teaching mathematics - Students' perspectives. Poster presentation at *DORP Poster session*, UTS (Broadway), November.

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ABSTRACT

This research study has investigated the beliefs that prospective primary school teachers hold about the epistemology of mathematics, and the teaching and learning of mathematics. In particular, it considered the following questions:

- What beliefs and attitudes about mathematics and mathematics education do first year primary school student teachers bring into their tertiary education?
- Are any of the students' beliefs about mathematics and mathematics education similar to the beliefs of the teacher educators in mathematics education and how do students interact with first year mathematics education subjects in the teacher education course?
- How do students' attitudes and beliefs influence their success in learning new mathematics at this stage of their lives?
- How do students' beliefs and attitudes affect their ideas on good practice in the teaching of mathematics in the primary school?

The research design was qualitative, using a case study investigation of 50 students in their first year of a teacher education course. The students' passage through the first year mathematics education subjects provided valuable insights into their beliefs, principally by means of interviews and open-ended questionnaires. The study was designed to have pedagogical outcomes for the students, by embedding the collection and interpretation of data in the teaching and learning of their course.

My personal perspective throughout this research has been that mathematics is a socio-cultural phenomenon, and that the learning of mathematics is achieved through the mediation of language, social interaction and culture. This perspective of mathematics and the learning of mathematics has influenced the choice of methodology and the research questions asked.

Results indicated that students often held two or more philosophies of mathematics and moved between these philosophies, depending on context. Further, students generally considered that the characteristics of a good teacher included being supportive and enthusiastic. Good pedagogy was believed to incorporate practical activities

demonstrating relevance, and providing "fun" for pupils. However, an alarming result was that having higher order knowledge about mathematics was often seen by the students as being a disadvantage for a teacher, principally because students believed such teachers would be less empathetic to struggling pupils.

These beliefs affected students' interactions with the first year university mathematics education subjects, as their beliefs about the importance of subject matter knowledge were at variance with the beliefs of the teacher educators. This dissonance led to devaluing of the mathematics education subjects by some of the students.

The study has led to the conclusion that a number of the students' beliefs about mathematics, and the teaching and learning of mathematics, should not be left unchallenged. Those beliefs dealing with ideas on good pedagogy should be strengthened, while beliefs about the nature of mathematics and the value of subject matter knowledge should be made more transparent and addressed. On the other side of the coin, teacher educators need to acknowledge the differences in the beliefs that student teachers and teacher educators might hold, and to consider ways of making mathematics education courses more relevant and meaningful for students.

1. PERSONAL PERSPECTIVES

1.1 Introduction

My thesis is predicated upon a personal understanding that mathematics is culturally and socially constructed. Such a view has consequences for the way that mathematics is admitted a place in the primary school curriculum and the way in which it is taught. The concern of the study presented here is to both investigate the first year of a preservice primary courses in mathematics education and more pointedly to enquire into the range and variety of prospective primary school teachers' beliefs and experiences as they progress through the mathematics education curriculum[1].

The thesis asks and seeks to answer questions regarding the beliefs of prospective primary school teachers about mathematics and the learning and teaching of mathematics. The chapter introduces the study and explains the theoretical framework on which it is based. A justification for the investigation is provided.

1.2 The value of the study

The study's major contribution to our understanding of mathematics education today is the theorising which arises from interacting with the images held by student teachers in a number of areas. These images comprise their views on mathematics as a study, and the ways in which it should be taught in our primary schools and learnt in our teacher education courses.

> *I like rote work because it worked for me.* (Nita)

> *[Mathematics is] mainly calculations and things like that ...* (Aaron)

> *I'd describe maths as the calculation of certain things to do with numbers and objects and how you can work out certain formulas and methods, simplify, how to count and subtract and things like that.* (Maria)

[1] Throughout this thesis mathematics education is taken to be the study of the pedagogical issues involved in the teaching and learning of mathematics.

I sort of felt okay [about having to teach maths in primary school] because I thought it was primary maths so it can't be that hard ... (Maria)

I was looking forward to [studying maths in this course at university] because I was wondering how you would tell us, teach us to teach maths in primary school now and see if it's any different to how I was taught. (Terry)

... just the basic facts, whether, you know, it's long division or that sort of thing, I can do it but whether or not I can do it in front of thirty kids and understand where they're at, um that's another story [laughs]. *And I think that's one thing that I wish to gain from the course.* (Joanne)

The way I was taught maths as far as I can recall was ... by repetition, ... the idea was to memorise, not really to make sense of what we were learning, ... I could never really make out why we were learning maths. My attitudes towards teaching probably would have been influenced by these views so much that I would try and make it fun, because kids love to learn by playing games or whatever ... (Maria)

The above students have completed their first year in a Bachelor of Teaching in Primary Teaching or Bachelor of Education in Teacher Librarianship program at an Australian university. The Bachelor of Teaching in Primary Teaching program is for the initial preparation of primary school teachers and is a three year course with an option of gaining a Bachelor of Education degree after a fourth year. The Bachelor of Education in Teacher Librarianship is a four year course which prepares students for teaching both in the school library and in the mainstream classroom. In the first year of their studies, all these students are required to enrol for an orientation course which covers the major learning areas encompassed by the primary school syllabus. In the State of New South Wales, these are known as the "Key Learning Areas". In their second semester, students study the first of a four subject sequence in mathematics education, as mathematics is one of the Key Learning Areas.

Clearly, from the reported student voices, prospective primary school teachers have very definite ideas about the nature of mathematics. They have clear memories of their learning of mathematics in the past, and they hold specific requirements for a mathematics education course in their teacher preparation.

The thesis examines both the students' views and their interactions with the mathematics education subjects that they study in the first year of their teacher education program. Of interest is the nexus between the students' attitudes and beliefs about mathematics and mathematics education, the participating teacher educators' beliefs about the nature of mathematics and mathematics education, and the learning about mathematics and mathematical pedagogy which occurs during the students' first year of study (see Figure 1.1).

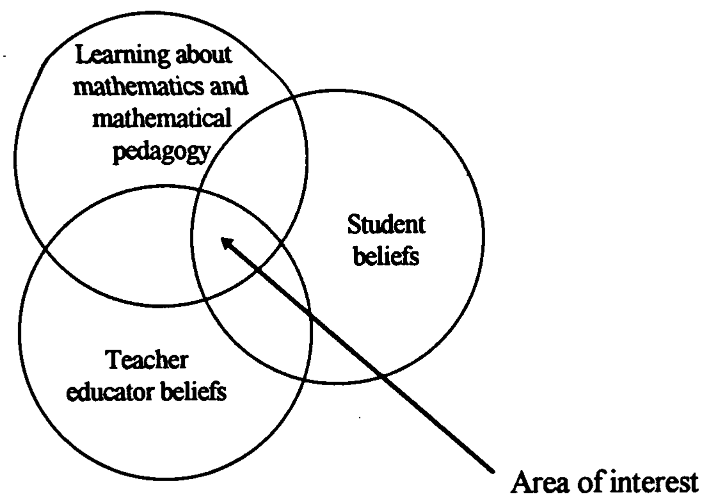

Figure 1.1: Focus of the research study

A significant argument which will be developed in the study is that prospective primary school teachers entering teacher education courses have developed attitudes and beliefs about mathematics education as a result of their perceived past experiences. Their conceptions of the nature of mathematics influence their success at both learning and teaching mathematics. Similarly, beliefs about what defines good pedagogical practice will affect students' own practices both in learning and teaching mathematics. In the chapters that follow I will give a brief overview of common conceptualisations about the nature of mathematics and how these are thought to influence the teaching of mathematics in primary schools. I shall also examine the role of teacher education courses in mathematics and further, shall focus on the beliefs of prospective primary school teachers about the teaching and learning of mathematics.

1.3 The research questions

This study investigates prospective teachers' beliefs and attitudes and poses the questions:

- What beliefs and attitudes about mathematics and mathematics education do first year primary school student teachers bring into their tertiary education?
- Are any of the students' beliefs about mathematics and mathematics education similar to the beliefs of the teacher educators in mathematics education and how do students interact with first year mathematics education subjects in the teacher education course?
- How do students' attitudes and beliefs influence their success in learning new mathematics at this stage of their lives?
- How do students' beliefs and attitudes affect their ideas on good practice in the teaching of mathematics in primary school?

My brief, then, is to examine what beliefs and attitudes students bring into their first year of a teacher education course and how they construct the subject matter of two first year subjects in mathematics education as a result of the lenses through which they see these subjects. Also critical to this discussion, is the way in which the teacher educators themselves perceive these initial mathematics education subjects.

It has been suggested that teacher education in mathematics can be far more effective if it takes into account the student teacher's own educational history, particularly his or her beliefs about mathematics learning, mathematics teaching, and mathematics as a discipline. The present study's purpose is to "unpack" student teachers' beliefs about mathematics and the pedagogy of mathematics, and also to investigate the interaction between these beliefs and students' constructions as they progress through the mathematics education subjects in their first year of teacher education. Teacher educators' beliefs are also of relevance here as their beliefs explain why the subjects have the structure and mathematical content that they do have. An examination of the possible match or mismatch between the students' requirements for subjects of this nature, and the teacher educators' views on what is needed to develop good teaching practice in mathematics, will follow.

1.4 The role of self in the research

At this point it is fruitful to foreshadow the style of writing to be used in this thesis. The thesis is written in the first person because to use the third person would create the false impression that the author is not present and active in the construction of the text. My perspectives are of consequence to the action being portrayed. I am working within a socio-cultural framework. My lens, therefore, will reflect a world view that has been determined by my own experiences and by the social context in which I live. This is not necessarily the same context as the reader's and so a multiplicity of readings of this work are possible. Consequently, my reading and interpretation will be prefaced by comments indicating that these are my constructions and that it is quite possible that others will bring different readings to the subject matter.

In the light of what has just been discussed in the last paragraph, it is necessary for me to be explicit about my experiences and beliefs in order to further clarify the lens through which I am viewing this study. I am a lecturer in mathematics education with a background both in mathematics and education. Throughout my teaching career I have interacted with many primary school teachers and prospective primary school teachers who have indicated to me their dislike of, and apprehension about, mathematics. It has consequently become of great importance to me, to intervene in some way, so that this scenario can be moderated or changed. An anecdote of my experiences in this area will demonstrate the consequent dilemma with which I am faced.

In the first year of our new Bachelor of Teaching in Primary Teaching and Bachelor of Education in Teacher Librarianship programs, I was much gratified to see the enthusiasm with which students undertook the orientation subject in mathematics. Journal entries made by students indicated that they had enjoyed the subject, were surprised at how accessible and interesting the mathematics was, in contrast with their memories of drill and practice, and many felt that their confidence had improved in the area.

However, in their second semester, students met new mathematics and new approaches to mathematics. The mathematics reflected the historical and cultural origins of the topics being studied. The mathematics was not tertiary level work and could be appropriate for extension classes in primary mathematics. However, it neither gave a direct reflection of what was in the NSW primary school syllabus for K - 6, nor did it mirror what students remembered as, or regarded as appropriate, primary school mathematics. Consequently, students responded to the subject matter with consternation. It was at this stage that it became obvious to me that the influence of students' beliefs about mathematics had to be examined and considered, and possibly challenged, if a different type of experience from the one students had undergone throughout school, was to be effective. It was from such a genesis that this study came into being.

To reiterate then, the study is conducted within a socio-cultural theoretical framework. It is my belief that students construct their own knowledge and do so through the mediation of language and social interaction. Context and culture are seen as important parts of the individual's development of knowledge. This has important implications for the teaching of mathematics and the teaching of mathematics education. A discussion of the questions which arise from such implications follows.

If mathematics is viewed as culture free, then is it also value free? If, on the one hand, mathematics consists of indisputable truths that need to be passed from one generation to another, does this imply that there is no need for choice or concern about the suitability of the given mathematics for different people? On the other hand, if these so-called unarguable facts are seen to have been generated by particular cultures in particular times, is the implication that different cultures create different mathematics and that what is suitable mathematics for one cultural group might not be suitable mathematics for another? In this thesis I consider such questions and discuss their meaning for the particular context being studied.

1.5 A plan of the thesis

Central to my belief that mathematics is socially and culturally created, is a belief in the importance of being aware of the history of the mathematics we study, and the importance of individuals being helped to make choices about the kind of mathematics they learn. This debate is taken up in Chapter 2, in which the diversity of views about the nature of mathematics is examined, and following from this, the differing visions about the nature of mathematics education are discussed. In this discussion, once again mathematics education is taken to be the study of the pedagogy attached to the learning and teaching of mathematics.

As a result of my conceptualisation of mathematics and its generation, several difficulties arise for me. Firstly, what mathematics do I choose to explore with my students, and secondly, how do I challenge their conceptualisations of mathematics and the way it is taught and learnt. Chapter 3 considers the aims of teacher education courses, such as the one with which I am engaged. Because of my belief that the history of a particular culture will determine its mathematics and mathematics education, I have, in Chapter 3, embarked on a brief study of the history of mathematics education in Australia, and I have shown that the particular nature of this history has led to a particular kind of mathematical experience occurring in our schools.

All students, as a result of experiences that they have had throughout their lives, prior to coming to university, bring with them a large amount of baggage in terms of beliefs. In particular, the beliefs held about mathematics and about teaching will influence their passage through the teacher education course and through their careers as teachers. Chapter 4 examines some of the literature on beliefs about mathematics and mathematics teaching, as well as beliefs about teaching in general.

The scene has now been set for the study itself. Recognising that the context of the study is fundamental to its significance, the first four chapters examine the context and the framework in which it is placed. Admitting various conceptualisations of mathematics to the discussion indicates that mathematics is not a single dimensioned subject but has many aspects to it and that differing views of these aspects will influence both the teaching and learning of the mathematics. It will also influence what mathematics is taught and learnt. The role of teacher education courses in mathematics education has a bearing on the mathematics education subjects examined within this thesis. These subjects reflect the influences of past and current developments in mathematics education, both here in Australia, and internationally. Finally, the latest research on beliefs, which will be widely discussed and referenced in the text, demonstrates the importance of beliefs in any educational context and so it is such beliefs that are the major focus of this study.

The methodology used is that of a qualitative case study, in which grounded theorising takes place. An examination of the characteristics of a qualitative paradigm and, in particular, of case study, takes place in Chapter 5. The strengths and limitations of such research are discussed and ways of minimising the limitations are considered. The development of my methodology is explained, with indications of how the design allowed the participants a central voice in the research.

Chapter 6 discusses the methods of analysis used to consider and interpret the data. A description of the qualitative computer software package used for analysis is given. It must be stressed however, that this software package is merely a tool to aid analysis and that the intellectual work is not done by the package, but remains my task. In Chapter 6 I foreshadow the importance of presenting the data in its raw form, in order that multiple readings of the text can be made. It is necessary to remember throughout the thesis that my constructions of the text occur as a result of my theoretical perspectives, and that alternative readings are possible.

The thesis then examines results emerging from this study and highlights the issues arising and also the silences in the text. The first of three results chapters, Chapter 7, considers the views and personal philosophies held by the student teachers about the nature of mathematics itself.

Chapter 8 considers the mathematics education subjects on offer to the participants in this study. It interrogates the reasons for the particular content being studied and the aims of these subjects in general. It also examines the views of the teacher educators and students regarding these subjects. Chapter 9, the final results chapter, discusses the beliefs of the student teachers about teaching mathematics and the nature of good pedagogy. Students' ideas on how they wish to behave as future teachers are also considered. In all these chapters annotated quotes are provided with indications of the possible implications.

Chapter 10 draws out the conclusions and implications of this study. It considers the dilemmas and paradoxes that exist for prospective teachers of mathematics in primary schools. The thesis concludes with an examination of the broader implications of this study both for teacher education in general, and mathematics education in particular.

1.6 Conclusion

The study will highlight the importance of student teacher beliefs about the nature of mathematics and mathematics learning and teaching, and suggest ways in which mathematics education courses in primary school teacher education programs can be effective in promoting an understanding and a use of mathematics that is dynamic, deep and rich. Emerging out of this research will be theory that will provide a useful framework for mathematics educators in teacher education programs.

In conclusion, it is worthwhile to note that the significance of this introductory chapter lies in the way that it makes explicit the researcher's position in the study. This position is at once both personal and public: while believing that the perspective that I bring to the study influences the research, I do not believe that the practices and experiences that I bring are so idiosyncratic that the research and its implications will not be of significance to other educators. Consequently, the chapter makes obvious the dual role of the researcher as interpreter and disseminator of valuable insights.

2. MATHEMATICS, EDUCATION AND MATHEMATICS EDUCATION

2.1 Introduction

This study is focused on the first year of a teacher education course and what prospective teachers bring to, and take from, the mathematics education subjects studied during this year. However, before examining students' interactions with the mathematics education subjects, it is necessary to make explicit the diversity of views on what mathematics actually comprises. Different ideas about the nature of mathematics lead to different emphases in the learning and teaching of mathematics. Hence the following chapter will present an overview of some of the different philosophies of mathematics that are presented in the literature, and offer an argument for a preferred conceptualisation of mathematics. It will also examine the various ways in which mathematics education is conceptualised and examine the question of whether mathematics education is substantively different from education in other disciplines and whether it is more than the sum of its derivative parts of mathematics and education.

As has been noted in Chapter 1, throughout this thesis mathematics education is taken to be the study of how mathematics is learnt and taught, that is to say the study of the pedagogy which attaches to mathematics.

In this chapter, I will consider the following questions in an attempt to clarify the issues mentioned above:

- What is mathematics?
- What are the theories of learning and teaching that influence mathematics education?
- What are the aims of mathematics education?

The question about the nature of mathematics has implications for mathematics education as it is the relationship between the mathematics and the learner that is the focus of study in mathematics education. Consequently the purpose of mathematics education will differ with various conceptualisations of mathematics.

A central issue of this study is the nexus between varying perceptions of what constitutes mathematics education. The subjects under examination in this study reflect a certain view of mathematics while the students enrolled in these subjects might display a different vision of mathematics. For this reason a discussion of varying conceptualisations of mathematics and of the purposes of mathematics education should clarify future discussion of this topic and set the background for the research questions of this thesis.

> *One's conceptions of what mathematics is affects one's conceptions of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential in it. ... The issue, then, is not, What is the best way to teach? but, What is mathematics really all about?* (Hersh, 1986, p. 13)

2.2 What is mathematics?

The above question is both a philosophical and a social one and has broad implications for discussions regarding the nature of mathematics education. The question has a social dimension in that many traditional views of mathematics empower certain groups in society. Thus, perpetuating those particular visions has resulted in what might be called a "mathematics for the elite". Burton (1996) suggests that much of what is taught in school classrooms as "immutable mathematics" is in fact a body of facts and strategies that were developed in a particular context, and as an expression of the social norms of particular times and places. Burton continues this argument by proposing that this socio-cultural view of mathematics does not receive general recognition, particularly among the larger non-mathematical community, in spite of increasing evidence that the teaching of the "immutable mathematics" described above has led to widespread marginalisation and failure for its learners. Burton propounds the view that the very socio-cultural nature of mathematics has led to a differentiation amongst its learners, of those who can engage with the traditional presentation of the subject, and those who are unable to do so. It is in this way, that a so-called "mathematics for the elite" can be seen to exist. Consequently, it is of great importance that the different conceptions of the nature of mathematics are made explicit and open to debate.

One such conception of mathematics, which has existed for over two thousand years, holds that mathematics is a body of infallible and unarguable truths. This view is described by Ernest (1991) as the *absolutist* view. This view can also be described as *foundationalist* because it attempts to place mathematics on a firm foundation by providing definite answers as to the nature of mathematics (Pateman, 1989). Within this view are the *logicist* and *formalist* philosophies. The *logicist* philosophy regards mathematics as part of logic, and all concepts of mathematics are thought to be reducible to logical concepts, with mathematical truths proved by means of logical rules and axioms. *Logicism* is connected to the philosophy of *realism*, in that there is a mathematical world external to the minds of humans, and logic is applied to the mathematical facts of reality to determine new facts.

The *formalist* view regards mathematics as a game without meaning outside the boundaries of the game. In this view, mathematics is independent of reality, with rules that are developed in the mind and which need to have inner consistency, but do not need to relate to the outside world. *Formalism* seems closely related to another stream of philosophical thinking, *idealism*, which, at its most extreme, "*either denies the existence of any external reality at all or denies humans direct access to knowledge of that reality*" (Pateman, 1989, p. 17).

In a skilful argument Ernest rejects the absolutist conceptualisation of mathematics by showing that contradictions occur when we assume that all mathematics can be derived from the laws of logic and certain given truths and that these truths are a priori knowledge, independent of any empirical observations. Ernest quotes Lakatos (1962) who showed that all mathematics depends on some assumptions and, in order to prove these assumptions, other assumptions are needed, leading to an infinite regress.

Another traditional view which originated with the Greek philosopher, Plato, is named after him. The *Platonist* conceptualisation of mathematics stems from the so-called objects of mathematics. These objects exist independently of our knowledge of them. Mathematics describes the relationships between the objects and structures of mathematics, the behaviours of natural phenomena, their interrelationships, and it, too, is independent of humankind. Again, this leads to the idea of mathematics existing before human intervention, and to its being discovered not created. Ernest (1989, p. 250) describes Platonism as follows:

> *... there is the Platonist view of mathematics as a static but unified body of certain knowledge. Mathematics is discovered, not created.*

The philosophy of *Platonism* is connected to the philosophy of *realism* as it seeks to describe the ideals or models for mathematical realities already in existence, and, as such, is certain, and independent of human intervention.

Ernest (1991) points out two major weaknesses of Platonism:
Firstly, if mathematics is independent of humanity, how do we get to know it? If, on the one hand, this is by intuition, then each person will bring a different interpretation to mathematics and this cannot be so if mathematics is unchanging and immutable. On the other hand, if it is by logic, then we are forced to argue in an absolutist manner about the existence of basic truths and correct reasoning and how do we judge if our reasoning is correct? This argument has already been rejected in the above. The second weakness is that Platonism only considers one facet of mathematics and ignores the dynamic processes that are involved in its constructive, computational nature, or in its utility.

Once again Ernest has built up a case for refuting this vision of mathematics. It is interesting to note, however, that many great mathematicians, such as Cantor and Gödel, appear to find this philosophy of mathematics to be utterly plausible. In fact this conception of mathematics has been very influential over the last two thousand years, and underlies much of the mathematical experience that gives those who believe in the socio-cultural nature of mathematics, cause for concern in mathematics education today (see the discussion of Burton's view on p. 11). For example, it is felt by many mathematics educators that the dualistic vision of mathematics proposed by Platonism has created alienation and anxiety among those for whom this image is meaningless; that the static nature of the Platonic conceptualisation has led to the disempowerment of many learners and that the lack of humanity in this view of mathematics has led to the underparticipation in mathematical endeavours of certain sectors of the population (Ernest, 1991).

Both the *absolutist* and *Platonist* conceptualisations of mathematics have been popular over the past two millennia. However, as can be seen from the above discussion, there is a view that these conceptualisations have caused social imbalance and inequity by marginalising all but a few. This view has led to the perception, among certain groups of mathematics educators, that more appropriate, empowering and successful learning experiences for students of mathematics involve having a new and different epistemological basis for mathematics. This view of mathematics needs to incorporate a multicultural, gender inclusive approach in which mathematics is responsive to the needs of the society which it serves.

A view of mathematics that seems to encourage a more equitable framework for mathematical education is one suggested by Wittgenstein. Wittgenstein shuns systematic explanations and so his work is not clearly laid out in an orderly fashion. Watson (1989) explains his view by saying that Wittgenstein saw mathematics, not as a product of the physical world, but as a way of organising our constructions of that world to fit with our social environment.

> ... *Mathematics uses a wide variety of signing systems in accomplishing the appropriation of the material by the social in a particular way, in particular contexts, and in serving the interests of particular sections of a particular social order. By studying the way that various signing systems are used in mathematics we can assemble a picture of mathematics as a social phenomenon.* (Watson, 1989, p. 18)

Mathematics is constructed by people through their use of it. In return their social lives are transformed by the mathematics. Hence it can be seen that this view emphasises the social aspects of mathematics; mathematics is demystified and not seen as elitist, but as an attempt for all people to make sense of their environment. An important aspect of Wittgenstein's approach is his saying *"the mathematician is an inventor not a discoverer"* (Wittgenstein, 1978, paragraph 168), a refutation of the Platonist view discussed above. Watson points out that in many ways Wittgenstein's view is a counter-intuitive one for most people, when many conclusions seem to be waiting to be discovered by us in our attempts to make sense of mathematics. However, it would appear to me, that this is a result of our upbringing in Platonist classrooms, where mathematical conclusions were given to us as pre-existing results and where agreements by society have led to certain conventions that, over time, appear to have always existed. Conventions are rarely presented as such in the classroom. The distinction between what is an arbitrary decision, accepted by convention, and what is a logical consequence of accepted assumptions is not usually pointed out.

Ernest (1991) clarifies Wittgenstein's approach further by showing that the latter believed that use of language implies acceptance of rules and

> *... the 'truths' of mathematics and logic depend for their acceptance on the linguistic rules of use of terms and grammar, as well [as] on the rules governing proofs. These underlying rules confer certainty on the 'truths', for they cannot be false without breaking the rules, which would be flying in the face of accepted use. Thus it is the linguistic rules underlying the 'truths' of mathematics and logic that ensure that they cannot be falsified.* (Ernest, 1991, p. 32)

These linguistic conventions stem from our socio-linguistic traditions and practices. Hence the mathematics based on these conventions is not unchanging and static as it changes as the language changes, which occurs frequently in practice.

An appealing view of mathematics that is also refuted by Wittgenstein is the *Empiricist* theory. This holds that mathematics grows naturally out of the physical situations which it represents. For example seeing two birds and then another three birds would seem to be an obvious example of the given number fact that 2 + 3 = 5. However, when the process of addition is analysed, it can be seen that we are using many rules and conventions that are human made, not givens of the situation. For example if we were using a different number base, as other societies have done in ancient times, we might get a very different answer (using the binary system as the computer does, we would say 10 + 11 = 101). Also if we used a different definition of addition, perhaps pairing each bird in the first group with each bird in the second group we would get 6 as the answer. Because we are so used to calculations with these agreements upon symbols and conventions, it becomes difficult to see them as culturally determined rather than as natural descriptions of our environment.

Another criticism of empiricism is that so much of mathematics is abstract and does not grow out of observations of the physical world, but instead develops out of other mathematical concepts.

Lakatos has developed another philosophy of mathematics in which mathematical practice is paramount and mathematics is what humans do, with all the concomitant errors and imperfections. As mathematicians are fallible, proofs and concepts are always open to revision and change as new challenges and interpretations occur. Mathematics is very much a human activity and needs to be viewed in conjunction with its past development and its integration into other areas.

> *Mathematical activity is human activity. Certain aspects of this activity - as of any human activity - can be studied by psychology, others by history. Heuristic is not primarily interested in these aspects.* [sic] *But mathematical activity produces mathematics. Mathematics, this product of human activity, 'alienates itself' from the human activity which has been producing it. It becomes a living, growing organism, that acquires a certain autonomy from the activity which has produced it; it develops its own autonomous laws of growth, its own dialectic. ... The activity of human mathematicians as it appears in history, is only a fumbling realisation of the wonderful dialectic of mathematical ideas. But any mathematician, if he has talent, spark, genius, communicates with, feels the sweep of, and obeys this dialectic of ideas.* (Lakatos, 1976, p. 146)

Lakatos's theory *(quasi-empiricism)* is incomplete as Lakatos died before his work was finished. The theory has great potential, although I believe there are difficulties inherent in the idea that the mathematics that grows out of human activity appears to become independent of the human contribution. To my mind, the above quote appears to paint a picture of humans striving to understand and communicate with pre-existing ideas. However, it is the stress on the fallibility of mathematics, with the attendant suggestion of a human, and by implication therefore, a social component to mathematics, that suggests the contribution Lakatos has provided to the discussion about the social construction of mathematics.

Lakatos's theory does need further development. Ernest (1991) discusses its weaknesses in several areas: aspects of discussion of the nature of mathematics are missing - the strength of the certainty of mathematical knowledge; the nature of the objects of mathematics; the applications of mathematics and the justification for bringing history of mathematics into this theory. However, Ernest goes on to say that the above are all omissions rather than inconsistencies and that no obvious flaw in Lakatos's argument is apparent.

Both Lakatos's work on quasi- empiricism and Wittgenstein's view of mathematics have been used as a basis for the ideas of *social constructivism* as delineated by Ernest (1991). Social constructivism proposes that mathematics is a social construction. The construction of mathematical knowledge goes through a cyclical process which involves individuals in subjectively constructing mathematical knowledge; this subjective knowledge is then opened to public debate in a written form or by discussion and if this knowledge is acceptable in the public arena, according to agreed upon criteria, it becomes "objective knowledge" (Ernest, 1991, p. 43), that is knowledge which is socially accepted. The objective knowledge is then reconstructed by the individual as an inner subjective representation and so the cycle continues.

So it can be seen in this vision of mathematics that knowledge is constructed; that social acceptance makes knowledge objective; and that the role of language or linguistic forms is essential in the construction of knowledge. This view of mathematics appears plausible, given the way mathematical knowledge has changed over the years, in different societies and cultures. It implies that the truths of mathematics are variable and dynamic and socially determined.

Dengate and Lerman (1995) suggest that the various rationalist and fallibilist philosophies have contributed significantly to the development of ontologies which debate the absolute nature of knowledge. They suggest further that these philosophies have encouraged theories of learning which reflect the above view of mathematics.

2.3 What are the theories of learning and teaching that influence mathematics education?

The fallibilist view of mathematics seems related to constructivist theories of learning in all disciplines. Such theories are predicated on the notion of the thinking individual constructing personal meaning via some sort of mental activity, which encompasses problem solving (Dengate and Lerman, 1995). However, the interactions between social and individual agents in the development of human knowledge is a matter of significant debate. Piaget believed the individual to be the central factor in meaning-making. Developing from, and inspired by, the work of Piaget, *radical constructivism* built upon the basic tenets that all knowledge is constructed by the individual, the new knowledge is either accommodated by new constructions or assimilated into pre-existing schemas and that language is a basis for finding a fit between different individuals' constructions. In this philosophy of learning, language and communication are secondary to the constructions of the individual. Thought is seen to be more primary in this than language. This is in direct contrast with the work of Vygotsky, who placed communication and social life at the centre of learning (Lerman, 1996). In radical constructivism, coming to know is an adaptive process in that individuals develop ways of thinking and acting which allow them to make sense of their experience, and this is regarded as knowledge. Knowledge is not some given that can be transmitted to the individual. It has to be constructed by each individual, so it is personal and private. In radical constructivism the best one can hope for in interactions with others and with the environment is a "fit" of experiences and interpretations rather than a "match".

Lerman (1993, 1996) has criticised aspects of the radical constructivist position. In particular, he has focused on the role of language and of social life. Lerman expresses concern that the philosophy underlying radical constructivism leads to solipsism, and that the problem of intersubjectivity is unresolved by radical constructivists. As each person's constructions are private and based upon their existing schemas, and a teacher can only interpret these constructions through his or her own constructions, the whole idea of intersubjective agreement becomes complex and problematic. These limitations of radical constructivism, in terms of social context and language, have led to modifications which include the role of social influences in learning and the role of language.

Bauersfeld (1992) was one of the constructivists who modified radical constructivism to incorporate the social context. Some of the convictions that he shares with other constructivists are that

> *Mathematising is a practice based on social conventions rather than the applying of an universally applicable set of eternal truths; ... this holds for mathematics itself.*
>
> *... Representations are individual constructs, emerging through social interaction as a viable balance between the person's actual interests and her realised constraints, rather than an internal one-to-one mapping of something pre-given or a fitting re-construction of "the" world.* (Bauersfeld, 1992, p. 21)

Lerman (1993, 1996) rejects the notion that constructivists can modify their theory by simply adding in a dimension which addresses the social aspects. Lerman argues that knowledge is socially determined and we are socially situated as human beings. Understandings will depend on what we bring to the learning situation, and are socio-cultural in nature. It is my opinion that this view accords to a large extent with Ernest's view of social construction (Ernest, 1991) although Lerman himself expresses concern at ideas of social constructivism. Lerman feels that what he describes as a socio-cultural theory of learning cannot be merged with constructivism due to fundamental disagreement about the role of language. He believes that the two views come from radically different interpretations of human consciousness, the one a metaphysical, individualistic view, the other an essentially socio/historico/cultural view (Lerman, 1994). He indicates that social constructivism is an attempt to overcome the shortcomings of radical constructivism without changing the underlying theoretical framework. However, it is possible that Ernest's use of the term "social constructivism" is being misinterpreted when it is construed as a modification of the radical constructivist position. This misinterpretation arises out of the acceptance of a theory of learning, also labelled social constructivism, which takes a constructivist position and includes social and cultural aspects in the theory. However, my interpretation of Ernest's "social constructivism" is that mathematics is a social construction in that it is socially determined and developed. In his introduction of social constructivism (Ernest, 1991, p. 42) Ernest acknowledges the contributions of Lakatos and Wittgenstein but does not mention constructivist perspectives at all. In fact, Ernest's conceptualisation of social constructivism seems to reflect the same concerns about radical constructivism that Lerman expresses. Ernest also feels there is substantial agreement between social constructivism and Vygotsky's social theory of the mind, which Lerman suggests as a more productive approach for mathematics education. Ernest's view of social construction of mathematics appears to overcome some of the limitations of radical constructivism specified by Lerman, while at the same time rejecting the more traditional views of mathematics and to my mind is a coherent and meaningful framework.

Further, it appears that there is considerable agreement between Ernest's view of social construction and the view discussed by Dengate and Lerman (1995) in which meaning is carried within social interactions through "*acculturation*", and discourse is situated historically and culturally rather than reflecting any objective reality. Accordingly, this view suggests that knowledge is "*neither real nor construed, but rather that it is situated by people in context and that communication drives cognition*" (Dengate and Lerman, 1995, p. 31).

Constructivist views and socio-cultural views of what mathematics is and how it is learnt are currently dominant among mathematics educators and, as will be demonstrated in what follows, there are particular theories of teaching which are consonant with these views of learning. These views have a significance far beyond a theoretical discussion about mathematics. The change in conceptualisations of mathematics from those of an immutable and invariant mathematics, an absolutist mathematics, to those conceptualisations that see mathematics as socially responsive, fallible and variable has implications for education. The view of mathematics as socially responsive and fallible has inherent in it a role in determining who does mathematics and what sort of mathematics they do. Further, the literature on teachers' beliefs and philosophies (Thompson, 1992; Ernest, 1989; Pateman, 1989) suggests that there are clear links between teachers' conceptualisations of mathematics and the way that they teach mathematics. Dengate and Lerman (1995) propose that

> *It may be fruitful and conceptually clear for mathematics teachers who are reflecting on the orientation of their instructional approach to consider whether their major focus is on mathematical content, individual learners or classroom discourse with the attendant metaphors of the teacher as a transmitter, facilitator or mediator* (p. 33).

Pateman (1989) suggests that a teacher's conceptualisation of mathematics will have an influence on the teacher's views on how mathematics should be taught. He does, however, add the caveat, that actual teaching practice is quite often independent of beliefs and intentions. Nevertheless, a teacher's view of reality and consequently of the position of mathematics in that "reality" is linked to that teacher's focus in teaching. Pateman considers what the characteristics of classrooms operating under various philosophies will look like: for *absolutist* teachers, their presentation of mathematics will demonstrate its well known and previously determined nature. The possibility of mathematical discovery or invention will be denied. Pateman suggests that a teacher believing in the *logicist* philosophy of mathematics will have the general aim of presenting mathematics in a concise and accurate manner, with problems being solved as applications of the mathematics rather than as the *raison d'etre* for developing the mathematics. "... *the things of mathematics may well be abstracted from experience, but the emphasis will be on developing deductive chains with each step clearly stated*" (Pateman, 1989, pp. 24-25). Pateman further suggests that the *formalist* teacher will stress the learning of rules;

> *the teacher is advised to present material in a clear way and then to have the students practise that material in order to master the particular rules of the game. That the material presented may be used to solve problems drawn from our experience is a kind of fortunate accident for the formalist teacher. Applications are always regarded as something to throw the mathematics at after that mathematics has been learned.* (Pateman, 1989, p. 26)

Pateman continues by discussing the classroom approach likely to be found with *radical constructivist* teachers: in their classrooms the importance of experience will be emphasised, but so also will be the need for individual reflection. The teacher who believes in the *socio-cultural* nature of mathematics will consider the interaction of individual and society and emphasise the importance of communication. Cultural location will be paramount.

It is interesting to consider the conceptualisation of mathematics clearly supported by the *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991). It should be noted that this conceptualisation is an example of the way in which people in an Australian context have responded to a particular view of mathematics. The *Statement* describes mathematics in terms of processes and stresses that

> *Mathematical knowledge is not empirical knowledge in that its source is not physical reality; rather its source is patterns and relationships created in the mind.* (Australian Education Council, 1991, p. 4)

It goes on to say that mathematical investigation is creative and inventive. The influence of Lakatos can be seen in statements such as

> *Mathematics provides powerful, precise and concise methods of representing patterns and relationships. These mathematical representations are then treated as objects in their own right ...* (Australian Education Council, 1991, p. 4)

While the *National Statement* is welcomed by many mathematics educators, it must be noted that there are a number of mathematics educators who do not agree with its views. For mathematics educators who see mathematics as a socio-cultural phenomenon, the emphasis on the processes of mathematics and on the social context of mathematics is generally applauded, although there are some who feel that the *Statement* is too conservative and that it seems to leave classroom teachers out of its construction and purpose (Ellerton and Clements, 1994). On the other hand, those who hold Platonist views decry the loss of skills that students will experience and see the study of mathematics in terms of its cultural and historical position as avoidance of the central issues and content of mathematics. However, this dispute is not central to the discussion in this chapter and so will not be addressed, except to note that it does exist.

The conceptualisations of mathematics and the concomitant views on mathematics education that exist in such diversity have led to a variety of views on the appropriate aims for mathematics education. This shall be discussed in the next section.

2.4 The aims of mathematics education

Firstly it is necessary to examine what, in fact, mathematics education is. Is it merely the study of education in a particular discipline or is it something more than this? How might the field of mathematics education best be portrayed?

Booker (1993) suggests that mathematics education is far more than just a sum of its two parts. The goals of mathematics education include not only the learning of subject matter content that is appropriate, and provision of suitable pedagogy for teachers, but an understanding of the mathematical process or a coming to know what the "doing of mathematics" is all about. The last is what Ball (1988a) has called "knowledge about mathematics" and describes this as follows:

> *Knowledge about mathematics includes understandings about the nature of knowledge in the discipline - where it comes from, how it changes, and how truth is established; the relative centrality of different ideas, as well as what is conventional or socially agreed upon in mathematics versus what is necessary or logical.* (Ball, 1988a, p. 4)

Mathematics educators cannot achieve these goals by being expert mathematicians with little understanding of how people learn, or expert educators with a patchy understanding of the processes, context and content of mathematics. Consequently the challenge of mathematics education is "*good mathematics – taught well ...*" (Lappan & Even, 1989, p. 2).

It has already been argued that the purpose of mathematics will differ according to who is asked and what view they have of mathematics. Ernest (1991) initially distinguishes three major interest groups, although he later expands on this. I shall use Ernest's initial distinction as I find it useful in this context. The groups are the educators, mathematicians and the representatives of industry and society. Their underlying conceptualisations of mathematics, and of education in general, may differ and these will dictate their views of the purposes of mathematics education. Some of the different views can be seen as:

- Interest in social reform; the empowering of the individual, and the individual's role in learning mathematics, by giving him/her access to mathematical discourse.
- The transmission of the central ideas of mathematics, with a view to cultivating notions of mathematical elegance and logic.
- The meeting of society's needs from an economic and industrial point of view; hence for this group the purpose of mathematics education is to provide a utilitarian framework, to teach mathematics that is first and foremost, useful. The point of view of this group has caused a shift in requirements for mathematics. Whereas in earlier times, a great need might have existed for people who could perform a large variety of mathematical calculations, the development of electronic devices to do this quickly and accurately has made this skill far less important. Modern views of mathematical requirements now incorporate a need to solve problems, to be able to use and interpret results critically and thoughtfully and to work on mathematics cooperatively.

A current debate on the role of subject matter content is also ongoing. Subject matter content, as well as the processes of mathematics, is seen to be of vital importance to most mathematics educators. This will be discussed further in Chapter 3 on the role of teacher education courses in mathematics education.

The *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991) gives an outline of the goals for mathematics education in schools. These are widely accepted by the mathematics education community and are sufficiently open to be descriptive rather than prescriptive. The goals are in keeping with goals contained in policy documents from other countries, such as the United Kingdom's "Cockcroft Report" *Mathematics Counts* (Cockcroft, 1982) and the American *Curriculum and Evaluation*

Standards for School Mathematics (National Council of Teachers of Mathematics, 1989). Both of these significant documents emphasise a view of mathematics which is dynamic and responsive to social factors and both have similar goals to those described below.

The goals for mathematics education in Australian schools as given by the *National Statement* (pp. 11 -15) can be summarised in the following ways:

> For students to develop confidence and competence in dealing with commonly occurring mathematics;
>
> For students to develop positive attitudes towards mathematics;
>
> For students to be able to work on mathematical problems collaboratively and individually;
>
> For students to be able to communicate mathematically;
>
> For students to be able to use appropriate tools and technology;
>
> For students to experience a variety of mathematical processes.

The actual goals, then, are:

As a result of learning mathematics in school all students should:

- *realise that mathematics is relevant to them personally and to their community;*
- *gain pleasure from mathematics and appreciate its fascination and power;*
- *realise that mathematics is an activity requiring the observation, representation and application of patterns;*
- *acquire the mathematical knowledge, ways of thinking and confidence to use mathematics to:*

    conduct everyday affairs such as monetary exchanges, planning and organising events, and measuring;

    make individual and collaborative decisions at the personal, civic and vocational levels;

    engage in the mathematical study needed for further education and employment;

- *develop skills in presenting and interpreting mathematical arguments;*
- *possess sufficient command of mathematical expressions, representations and technology to:*

interpret information ... in which mathematics is used;
continue to learn mathematics independently and collaboratively;
communicate mathematically to a range of audiences;

- *appreciate:*
  that mathematics is a dynamic field with its roots in many cultures;
  its relationship to social and technological change. (p. 15)

It can be seen that these goals derive from a view of mathematics as an everyday, dynamic, socially determined, problem solving activity; an activity that takes account of cultural and social differences, that endeavours to empower the individual, and seeks to be accessible to all people.

2.5 Conclusion

The above chapter has considered the background to the central study in so far as it has examined briefly some aspects of the nature of mathematics, the purposes of mathematics education, the differences in world views that prospective teachers might bring to their study of mathematics, and the learning and teaching theories influencing mathematics education.

It is important to be aware of the dominant views of mathematics, as these views will determine the type of mathematics taught in the schools and so govern the role of mathematics education in teacher education programs. This chapter has discussed a variety of conceptualisations of the nature of mathematics and has shown that the one most dominant among the mathematics educators today is a vision of a dynamic and human mathematics that helps us to make sense of our world and the relationships within it. If such a vision is held, a shift in the methods of teaching mathematics is clearly indicated.

The idea that mathematics is socially constructed leads to the view that mathematics is both personal and social, and that all learning is constructed as a result of social and cultural experiences. The role of language is important in this as is the fact that mathematics is constantly being adapted to the needs of the society which it serves.

Finally, the above views of the nature of mathematics and the way it is learnt have implications for the aims of mathematics education. No longer is it valid to use a transmission model of teaching in which information is poured into the empty container of the learner; knowing a large number of mathematical facts and procedures is no longer regarded as knowing mathematics. To know mathematics is to know about its nature and the processes of doing mathematics; to develop ways of making sense of mathematics; to use mathematics to make sense of one's world; to develop new ways of solving new problems and to be able to communicate with others mathematically, using the language of mathematics and its syntax and concepts. The goals of the *National Statement* clearly show this sort of emphasis for the learning of school mathematics.

The implications of viewing mathematics as socially constructed, and the resultant goals for mathematics education, will be examined further in Chapter 3, in which the purposes of a teacher education course in mathematics education are discussed.

3. THE ROLE OF MATHEMATICS EDUCATION COURSES IN INITIAL PRIMARY TEACHER EDUCATION PROGRAMS

3.1 Introduction

The previous chapter looked at the nature of mathematics and considered the diversity of views that have been held about mathematics over the years. A view widely accepted by those interested in the pedagogy of mathematics is that mathematics is a dynamic, everyday, culturally and socially determined activity which includes a study of the relationships between objects in our world and considers our attempts to make sense of these relationships.

The dominant view of mathematics in any society will inevitably affect the role of the teacher education courses dealing with mathematics education. In countries and ages where mathematics is seen as a set of procedures and rules, not open to negotiation, but rather as an absolutist subject, the teacher education courses will focus on the most efficient ways of teaching those rules and procedures. In countries where the more common view of mathematics incorporates a culturally embedded, variable conceptualisation, the emphasis of teacher education courses will be on sharing this view of mathematics with prospective teachers.

The following chapter will give a brief overview of the history of mathematics education, examining the changes in its role as changes in views about mathematics and learning occurred. It shall then look at the literature on mathematics education courses in teacher education programs for primary teachers, as they exist at present, and attempt to analyse the role of such courses. In particular the chapter will examine the following questions:

- How has mathematics education changed over the ages?
- What do primary school teachers need to know about mathematics in order to teach it?
- What is the role of mathematics education courses in initial primary teacher education programs?
- What are the constraints that may prevent or hinder the successful execution of this role?

- How important is subject matter knowledge for prospective primary school teachers?

The position which I will take in this study is that mathematics is socially and culturally embedded and is a dynamic and interactive phenomenon. This is not to say that I view subject content knowledge as unimportant but rather that the subject content should be viewed in the context in which it is, or was, developed.

3.2 The history of mathematics education

Over the centuries mathematics has been an essential part of education. Booker (1993) discusses how the "six gentlemanly arts" of the Zhou period of about 200 B.C. incorporated a clearly organised view of mathematics, with exact instructions as to what the pupils, who were all male, should learn at various ages.

The procedures and rules that still dominate in so many mathematics classrooms today existed for many years in the mathematics education paradigm. The primary focus over the years was, and perhaps still is for many teachers, to hand on the self evident truths of mathematics to the next generation, in as efficient a way as possible. (Of course these truths might well have been different in different cultures, periods and societies as can be seen in the oral traditions of Indian mathematics, Ancient Greek emphasis on geometry and the mysticism of Hebrew numbers). The mathematics that was generally considered important until extremely recently was of an absolutist nature, as discussed in the previous chapter. Only in the last few decades did the popular view of mathematics start changing and broadly-based acknowledgment that mathematics is dynamic and socially determined is relatively recent.

Mathematics education also incorporates views about the nature of learning. A vision of learning held for thousands of years was that the mind was a "tabula rasa"; a blank tablet on which the wisdom of the elders could be inscribed. Consequently many studies were done on the best way of transferring knowledge to the student by the expert. As understanding grew of the importance of interactions between individuals, and of the influence of the context in which the learning took place, a new type of educational practice developed. In this more socially based education, language was particularly valued as being significant in the negotiation of meaning.

However, the above two aspects of mathematics education are only part of the picture. As has been discussed in the last chapter, mathematics education is a distinct field in which the special challenges of learning mathematics are viewed as being different from those of learning in general. Further, mathematics education is also more than simply a focus on the mathematics that is to be learnt, but examines as well, the interaction between the mathematics and the learner.

The field of mathematics education is regarded as being only a couple of hundred years old. This is in sharp contrast to both the discipline of mathematics, which is regarded as being several millennia in age, and the role of education which was debated in Ancient Greek times. Aristotle, himself, wrote

> *In modern times there are opposing views about the practice of education. There is no general agreement about what the young should learn either in relation to virtue or in relation to the best in life; nor is it clear whether their education ought to be directed more towards the intellect than towards the character of the soul. The problem has been complicated by what we see happening before our eyes, and it is not certain whether training should be directed at things useful in life, or at those conducive to virtue, or at nonessentials...* (Aristotle quoted by Higginson, 1989, pp. 6-7)

The following brief review of the history of mathematics education has been undertaken here because it has such significant consequences for the way mathematics is taught; both for its influence on the mathematics curriculum and on the attitudes and beliefs that students of mathematics in primary teacher education courses bring with them to their studies.

Mathematics education began primarily in the universities of Europe in the 19th century (Kilpatrick, 1992). The field developed as mathematicians and educators started examining the problem of what mathematics should be taught in school, how, and to whom. The field of mathematics education has, of course, been influenced by movements in the areas of mathematics and education as discussed above. However, the field of mathematics education has struggled to develop its own identity by examining issues that are peculiar to this area.

Throughout the 19th century universities prepared teachers of mathematics for secondary schools by having them study **mathematics** but very little emphasis was directed at the actual **teaching of mathematics**. It was only towards the end of that century that teachers began to get instruction in strategies and procedures for teaching mathematics to secondary school students.

Elementary school teachers were generally prepared in separate institutions, which were not regarded as institutions of higher learning but were regarded as being training institutions, which academically were, at best, of an equal standard to secondary schools. The limited education of the teachers in mathematics led to a heavy reliance on textbooks for the teaching of arithmetic.

When public schooling for the whole population was introduced it was felt that secondary teachers needed more professional preparation, as the education in mathematical content that they had received previously, did not address any pedagogical issues that they would encounter in their future teaching. At the same time, demands for the upgrading of the teacher preparation for elementary school teachers led to a change of status for the educational institutions involved in primary school teacher education. It was at this stage, in the late 19th century, that mathematics education evolved as a field of study.

While this background to the history of mathematics education was common to most English speaking countries, there are special characteristics of Australian history that have contributed to the unique nature of aspects of the Australian curriculum and situation in mathematics education.

In Australia the country's colonial beginnings were very important in influencing the nature of mathematics education. Further to this, the fact that Australia was a penal colony set its course apart from that of the United States of America. The administrators attempted to mimic the English education system as closely as possible. In fact, Clements, Grimison and Ellerton (1989) maintain that this adherence to British models of education was strictly maintained up to the middle of the 20th century.

> *From the beginning, then, Australian elementary schools adopted curricula similar to those used in charity schools at home.* [sic] *There was little, if any, recognition of the peculiarly harsh conditions, or of the backgrounds, skills, and needs of the children. The purpose of the schools was to "civilise", to render the "destitute" respectable. By 1840 the English charity school curriculum was accepted without question in the schools established in each of the Australian colonies - the main purpose of schooling being to provide a moral training and instruction in reading, writing and arithmetic .... The establishment of these schools took place despite the fact that the curricula borrowed from home were usually inappropriate.* (Clements et al, 1989, p. 53)

In the 1860s a payment-by-results system was introduced in Britain and consequently in Australia. This led to school inspectors testing whether primary school children could answer standardised questions, formulated in England. In turn, this led to teachers preparing children for such questions; questions which were often relatively meaningless[1] in the Australian environment and which could only be answered with a great deal of rote learning. However, the rote learning was thought to be quite desirable as it disciplined the mind and gave a training in "*exact and accurate thought*" (Gladman, 1877, quoted in Clements et al, 1989, p. 62). Primary schools were supposed to provide a basic curriculum, including arithmetic developed in a distant country. No further aims existed for primary education.

By the early twentieth century, the payments-by-results system had been abolished and both primary and secondary curricula had changed. However, little change took place in the mathematics classroom.

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[1] It is hard to imagine what meaning a young Aboriginal child, (or any other child, for that matter) could attach to such questions as the following (used by a Victorian inspector in 1877):
If 249.804 bushels of oats last 804.573 horses for 7.4 days, how many horses would 347.147 bushels feed for the same time? (Quoted by Griffiths in Clements et al, 1989, p. 63)

> *Mathematics classrooms were assumed to be places where a rigid, academic approach was needed, where it was desirable that children be drilled in basic number facts and in the acquisition and memorisation of mere factual knowledge and routine skills.* (Clements et al, 1989, p.67)

It is only in the last three decades that most mathematics educators believe that mathematics education has matured, and possibly come of age (Kilpatrick, 1992). In particular, the development of mathematics education in Australia is thought to have been given a nudge forward by developments in the 1970s (Clements et al, 1989). These include the beginning of much discussion and collaboration among mathematics educators in all states and territories of Australia; the improved initial and post-initial education in mathematics for teachers; and an awareness of the need to improve the access of **all** groups of people in Australia to mathematics.

Many mathematics educators of the 1990s see this awareness about accessibility and responsibility to encourage all students, as a call for further change in mathematics education. In order for this change to occur, it is necessary to analyse what makes primary school teaching of mathematics effective and encouraging of change in a positive direction. If it is possible to identify the areas that determine quality mathematics teaching, then mathematics education courses in teacher education programs have a clear indication of an appropriate curriculum. The following section takes up this discussion.

3.3 What do primary school teachers need to know about mathematics and mathematics teaching?

Over the years research has examined the role of the teacher in effective learning. Early researchers classified teachers according to various characteristics such as enthusiasm, strictness and knowledge about subject matter (eg Hart, 1934).

A 1960s American study, the National Longitudinal Study of Mathematical Abilities (Begle and Geeslin, 1972) attempted to identify teacher characteristics associated with student achievement in mathematics. It resulted in the surprising conclusion that no single teacher characteristic proved to be "*consistently and significantly correlated with student achievement*" (Begle and Geeslin, 1972). The underlying assumptions providing the conceptual framework for this study were not questioned or criticised and so subject matter knowledge and attitudes of teachers came to be regarded as being of little importance in teaching.

Studies in the late 1970s then considered what generic teacher behaviours might bring to the teaching of mathematics. These included questioning, and use of praise, as well as qualities such as enthusiasm and clarity. Many of these studies focused on primary school teachers as achievement in mathematics at this level was considered to be extremely important. Ball (1988b) describes these studies and quotes various researchers who showed that orderliness, routine and teacher control were regarded as being particularly applicable to classes in primary school mathematics. She goes on to show that this picture of "good teaching" stemmed from a view of mathematics that stressed the gaining of a body of skills through drill and practice.

Fennema, Carpenter and Peterson (1989) relate how this approach to teaching mathematics led to the consensus in the United States of America that reform in mathematics teaching was badly needed. Students were found to be relatively successful at performing low order computation but seemed to be far less successful at higher order cognitive skills such as problem solving. As advances in technology led to a lesser demand for such computational skills and to a greater need for conceptual understanding and problem solving skills, the 1980s saw an overwhelming belief that great change was necessary in what mathematics was taught, and how it should be taught. Equally, in this period of social change, **who** should be taught mathematics became an issue of importance.

The emphasis on understanding of mathematics is not new to the mathematical education scene. In the 1960s, the so-called "new maths" emphasised understanding of the structures and connectedness of the important ideas of mathematics. However, this conceptualisation of mathematics education failed, possibly because of its characterisation of understanding of mathematics. This view of understanding corresponded to ideas of Thiele (1941, quoted in Fennema et al, 1989) and Van Engen

(1953, quoted in Fennema et al, 1989). Van Engen (quoted in Fennema et al, p. 196) said that

> *the pupil who understands is in possession of the cause and effect relationships, the logical implications and the sequence of thought that unite two or more statements by means of the bonds of logic.*

This definition describes relationships based on the structures of the mathematics; it makes no mention of the position of the learner. The "new maths" movement was devised by mathematicians with little mediation by educators. Further, in implementing the "new maths", few teachers attempted to relate the ideas of the "new maths" to the informal knowledge of the child. The inservice teacher education could also be said to have been inadequate to the task of radically changing teachers' classroom practices.

In an attempt to rectify the problems created by the "new maths" movement, the seventies saw a "back to basics" movement which led to the type of teaching described by Ball (1988b) on p. 34 of this chapter. It could be said that this movement was a reaction to the "new maths" and was led by policy makers, responding to public outrage rather than educational arguments. However, from the 1960s some mathematics educators began to investigate further the notion of understanding. Research by Ausubel, Piaget, Bruner and others considered children's learning and suggested that knowledge about this learning be incorporated into any development of mathematics programs, along with the ideas about the structure of mathematics. It seemed that understanding needed to take the learner into account and the following definition of understanding by Hiebert, (1986, quoted in Fennema et al, 1989) accords with this view. Understanding is

> *the process of <u>creating</u> relationships between pieces of knowledge. Students understand something as they recognise how it relates to other things they already know.* (Quoted in Fennema et al, p. 196)

In other words, it is the learner and his or her relationship with the knowledge that is crucial, rather than the relationships inherent in the mathematics itself.

However, focussing on pupils and their relationships with knowledge does not give a complete view of mathematics education either, as the role of the teacher is also vital in determining to what extent learning takes place. Fennema et al (1989) describe a curriculum project, *Developing Mathematical Processes* (DMP) developed by Romberg, Harvey, Moser and Montgomery in 1974 which attempted to take into consideration both the structures of mathematics and how children learn. However, what this project did not examine was the belief structures of the teachers. As many of the teachers did not share the beliefs and objectives of the authors of the project, these teachers would wittingly or unwittingly modify the activities so that the underlying focus on mathematical understanding was no longer there. Any program in mathematics education must take into account the mathematics to be taught, the way children learn and the belief system and knowledge of the teacher.

Current research on how children learn provides a clearer indication of how concepts are acquired and also attends to the role of the teacher. The American publication *Professional Standards for Teaching Mathematics*, developed by the National Council of Teachers of Mathematics in 1991, suggests that teachers are expected to have a range of knowledge about mathematical concepts and procedures, the processes of mathematics and the nature of mathematics. They also need to be able to use this knowledge in making decisions in their classrooms as to how best to help their students learn mathematics. Shulman (1986a) uses the phrase "pedagogical knowledge" which he describes as

> *the understanding of how particular topics, principles, strategies, and the like in specific subject areas are comprehended or typically misconstrued, are learned and likely to be forgotten. Such knowledge includes the categories within which similar problem types or conceptions can be classified (what are the ten most frequently encountered types of algebra word problems? least well-grasped grammatical constructions?), and the psychology of learning them.* (p. 26)

This knowledge includes knowledge of what the learner brings to the situation in terms of concepts, procedures and misconceptions, and knowledge of the stages of understanding that the child may pass through in mastering the topic.

As can be seen from the above requirements, the teaching of mathematics is a complex and demanding task, particularly in the context of primary schools where teachers have a responsibility to be knowledgable about a wide range of learning areas. Teachers need to have a clear vision of the nature of mathematics and a strong understanding of how children learn and what informal mathematical knowledge they bring with them to school. Consequently it is necessary for teachers to be decision makers and reflective practitioners rather than unquestioning deliverers of a set curriculum. It has also been noted in Chapter 2 that teachers have their own belief systems that influence their perceptions and actions in the classroom. Teachers need to be aware of the effect of these beliefs on their behaviour in the classroom. These requirements of the task of teaching, and characteristics of the primary school teacher, have great bearing on the role of teacher education courses in mathematics education.

3.4 The role of mathematics education courses in initial primary teacher education programs.

This section will consider the curriculum[2] of initial primary mathematics education courses viewed as desirable by many teacher educators, and will also consider the curriculum recommended by the Discipline Review of Teacher Education in Mathematics and Science (Department of Employment, Education and Training, 1989). Additionally, it will examine some of the literature on this topic. This analysis will then act as a touchstone for the discussion of the mathematics education subject studied in this thesis.

It should be noted that what are named as mathematics education courses in this thesis are elsewhere called mathematics method courses or mathematics curriculum courses. These are all courses which concentrate on the field of mathematics education with its

[2] Curriculum, here, embodies the syllabus and the reasonings which lie behind it, as well as the context in which it is conducted.

multiplicity of perspectives. In particular, all of these courses have as a focus of their study, the interaction between the learner and the mathematics.

The different conceptualisations of mathematics, some of which have been presented earlier, have led to differing emphases on what should be taught in mathematics education courses. I have argued in earlier sections of this chapter that at different stages of the history of mathematics education both in Australia and in other English speaking countries, initial teacher education courses have emphasised subject matter knowledge rather than pedagogical content knowledge; taught strategies and procedures that emphasised rote learning; and offered techniques for implementing set curricula efficiently. In the 1960s and 1970s, prospective primary teachers were taught how to become effective technicians in mathematics teaching as it was felt that the techniques of teaching mathematics would result in desirable outcomes in mathematics classrooms.

The 1980s saw the development of the "problem solving movement" which emphasised the processes of mathematics (for example, inquiry, investigation and estimation) as being more important than the content. However, although many teachers accepted the basic tenets of this movement, their teaching did not appear to change much. A survey by Clarke (1984) indicated that mathematics lessons in Victoria were still very much in the "chalk and talk" tradition, with much use of textbooks and teacher exposition. Sullivan, Bourke and Scott (1995) confirm that this way of teaching still exists in Australia in the 1990s. They suggest an alternative way of teaching, which, while totally in keeping with the philosophy of the problem solving movement, would nevertheless be regarded as innovative by many teachers in Australia. The above results are also corroborated by American studies of both primary and secondary schools which paint much the same picture with its emphasis on rules, procedures and memorisation (Cobb, Wood, Yackel & Perlwitz, 1992; Cohen & Ball, 1990; Peterson, 1988; Stodolsky, 1988; Burns, 1986; Goodlad, 1984). Consequently, in an attempt to help prospective teachers move away from the traditional notions of mathematics as a highly procedural, tightly sequenced body of knowledge, many initial courses in mathematics education currently emphasise process and tend to minimise the mathematical content contained in their curricula.

Also, in recent times, the importance of teacher attitudes and beliefs has been highlighted. This, too, has resulted in a change of emphasis in initial mathematics education courses. In an American study, Teacher Education and Learning to Teach, conducted by the National Center for Research on Teacher Education (Floden, McDiarmid and Wiemers, 1990), methods instructors at three universities were interviewed about their syllabi for their methods courses. All three of them felt that their priority was to promote positive attitudes towards mathematics rather than assist students develop understanding of what it means to do mathematics. Responsibility for teaching mathematical content was not seen as theirs, and the instructors either assumed that their students had sufficient knowledge of the mathematical content of the primary school curriculum or they would teach content only when it became apparent that it was not known by their students.

> *The methods course has responsibility for developing a positive view of mathematics, but not for appreciating mathematics as a field of inquiry. The primary responsibility of the methods course is to help teachers acquire methods for teaching some of the topics in the current school curriculum and to learn how to develop similar methods for other topics. The criteria promoted for choosing these topics is based in student engagement, not in the mathematical content itself. Although these are subject matter methods instructors, they emphasize learning and affect, not content.* (Floden et al, 1990, pp. 10-11)

The situation described above is also found in some Australian universities. I make this claim on the basis of my examination of subject descriptions from three major Australian universities. At these universities, the initial courses in mathematics for prospective primary school teachers stress the importance of positive attitudes towards mathematics and mathematics teaching; the teacher educators agree on the significance of helping student teachers acquire strategies for teaching mathematics effectively and for learning what children know, and how they come to know, in mathematics. The processes of mathematical inquiry are also regarded as important. However the mathematics education subjects at these universities do not seem to emphasise the importance of subject content knowledge.

It is instructive at this point, however, to consider the somewhat different views, of an influential, government instigated discipline review of mathematics education courses for teacher education programs in Australia. The Discipline Review of Teacher Education in Mathematics and Science (Department of Employment, Education and Training (DEET), 1989), otherwise known as the "Speedy Report", indicates that many students entering initial primary programs have only a superficial knowledge of mathematics content although over fifty per cent have studied at least one mathematics subject up to year 12 level. The report continues by suggesting that

> *... students need to be mathematically competent and have higher order mathematical knowledge to give them confidence to deal with the mathematical development of children ....*
> *It is proposed that the equivalent of 50 per cent of the 144 hours be devoted to specific mathematics content which should cover such topics as number systems and number theory; computation and estimation; measurement and geometry; statistics and probability; patterns, functions and relationships; and basic concepts of algebra. These concepts need to be explicit, they need to be assessed, and the minimum level of competence which should be achieved by all graduating students needs to be defined and used as a quality control.* (DEET, 1989, pp. 23-24)

The remaining time is recommended to be spent on curriculum and pedagogy for primary school children. It can seen that the Speedy Report endorses the inclusion of explicit subject content matter in initial primary teacher education courses, an endorsement that is at variance with the beliefs of many mathematics educators in initial primary courses. In the study concerning the response of universities to the Speedy Report, (Whitehead, Symington, Mackay and Vincent, 1993) it was found that the recommendations contained in the Report were not universally accepted. For example, the notion of explicit studies in mathematics in early childhood programs was not accepted by many, as they felt that the prime focus of their studies was the development of children, rather than teaching the content of mathematics.

A further perspective on teacher knowledge is that suggested by Shulman (1986b). Shulman suggests that a useful way of thinking about content knowledge is to distinguish between three categories of content knowledge; subject matter knowledge, pedagogical content knowledge and curricular knowledge. Pedagogical content knowledge has already been described to some extent on pp. 36-37. More discussion on what each of these categories comprises follows here:

- **subject matter content knowledge:** refers to the amount of knowledge and the organisation of the knowledge about the actual subject matter. This goes beyond knowledge of facts or concepts to understanding the underlying structures of the subject, and is unique to each subject area.

> *Teachers must not only be capable of defining for students the accepted truths in a domain. They must also be able to explain why a particular proposition is deemed warranted, why it is worth knowing, and how it relates to other propositions, both within the discipline and without, both in theory and in practice.* (Shulman, 1986b, p. 9)

- **pedagogical content knowledge:** subject matter knowledge which considers those aspects of content most appropriate to teaching of the subject matter. This form of knowledge includes an understanding of what makes a topic easy or difficult to learn, knowledge of common misconceptions and preconceptions, and access to strategies for helping students construct the subject matter in a meaningful manner. The quote by Shulman (1986a) on pp. 36-37 further clarifies this description.
- **Curricular knowledge:** the teacher should possess understanding of what comprises the curriculum, what alternatives are available for instruction, what materials are associated with the curriculum, as well as other aspects of curriculum knowledge.

Further to the current debate about the importance of subject matter content knowledge, it is recognised by many mathematics educators, and is certainly argued in this thesis, that the links between content and pedagogy need to be examined. Some content matter is more suitable for allowing exploration of strong mathematical ideas than is other content matter; and some content matter is more accessible to children. The Speedy Report indicates what fundamental mathematical concepts should be included in an initial primary teacher mathematics curriculum. The Speedy Report is of interest in this thesis because of the support it gives to the rationale for the mathematics education subject being studied in this research. This issue is revisited in Chapter 8.

The Report also stresses the need for explicit links between the mathematics and the mathematics education studies in the teacher education program. These links are regarded as important in helping students to see their studies in mathematics education as coherent and in helping them to become aware of the relationships between their understandings of concepts and the understandings of their pupils. Developing strategies for helping students learn mathematics are also mentioned as a necessary part of initial teacher education.

Other influences on contemporary thinking regarding mathematics and teacher education are apparent in the current textbooks available for teacher education courses. Consequently, it is enlightening to consider what a recent mathematics education textbook suggests as appropriate preparation of prospective primary school teachers. A recently published Australian book on primary teacher education (Perry & Conroy, 1994) asks that prospective teachers "participate" in the book, rather than merely read it. Written from a constructivist perspective, the goals of this text for prospective and practising teachers are to engage the "participator" (Perry & Conroy, 1994) in an

> *...active, problematic, socially interactive collection of experiences in which you will be both the teacher and the learner and, during which you will construct your own understandings of how to facilitate the learning of mathematics by children in their ... primary years. .... the material in the book is aimed at stimulating your thinking and that of your colleagues to the extent that the answers will suggest themselves to you.* (Perry & Conroy, 1994, p. 3)

This text can be seen to emulate the goals of mathematics education as suggested by those with a vision of mathematics that is socially constructed and a vision of learning that is active and participatory.

To encapsulate the general trends of mathematics education courses, it could be said that one of the major goals of mathematics education is to change the nature of mathematics teaching and learning in school. In spite of changes in views about the nature of mathematics and the nature of learning, many classrooms still present a vision of mathematics as a rigid and procedurally based subject, not dissimilar to the 19th century mathematics described by Clements et al in an earlier paragraph. Although there may be debate about the importance of subject content matter, few teacher educators would argue about the vision of mathematics that is considered desirable in the primary school classroom; a vision of a dynamic, socially determined, living mathematics, which is enquiry based.

3.5 The constraints on mathematics education courses

It can be seen that the goals of mathematics education courses in teacher education programs are generally agreed to be:

- development of prospective teachers' knowledge of how children learn mathematics;
- growth of awareness of the informal mathematical knowledge that children possess;
- exposure to instructional methods that will promote conceptual understanding of mathematics.

Two other goals that this thesis is arguing as essential are:

- the growth of understanding of the nature of mathematics and the development of a strong subject matter knowledge for prospective primary school teachers;
- an awareness for these student teachers of their own beliefs and attitudes about mathematics and mathematics teaching.

It is generally agreed (Floden, McDiarmid & Wiemers, 1990; Schram, Wilcox, Lanier and Lappan, 1988; Ball, 1988c; Entwistle, 1984) that tertiary programs in general, and mathematics education courses in particular, are faced with a number of constraints that might prevent the realisation of the above goals.

Entwistle (1984) discusses the incongruity occurring in institutions of higher education, where lecturers generally espouse goals of critical thinking for their students but teach and assess for conformity and acquisition of factual knowledge. Allied to this is the problem discussed by Ball (1988c). Ball describes how many teacher education courses teach students about children's learning but lecturers are not influenced in their own teaching by this view.

> *Rarely do they treat teacher education students as learners who actively construct understandings about specific subject matter and its pedagogy. Instead of taking what they already know and believe into account, teacher educators tend to view prospective teachers as simply lacking particular knowledge and skills.* (Ball, 1988c, p. 2)

This argument is continued by McDiarmid (1989) who suggests that teacher educators generally do not challenge student teachers' beliefs:

> *Rather than challenging students' initial beliefs, teacher educators tend to focus on issues that they and their students already agree on ... As a consequence, most prospective teachers complete their teacher education programs without having examined, much less questioned, their most fundamental beliefs about teaching, learners, learning, subject matter, and the role of context. I would argue, in fact, that teacher education students rarely become aware of the assumptions under which they operate. Instead, they either reconfigure ideas and information they encounter to fit their initial beliefs and understandings - or they simply reject what doesn't fit.* (pp. 4-5)

Ball (1988c) mentions other constraints occurring from prospective students' images of what mathematics and mathematics teaching really are, based on their experiences in the mathematics classroom during their school lives. These experiences have persuaded many students that mathematics is a set of unrelated rules and procedures to be learnt, that the mathematics taught to them has been chosen in a quite arbitrary fashion and that teachers need to give clear directions and provide many opportunities for drill and practice. The situation is generally not improved during student practice teaching sessions, where a reinforcement of this vision often occurs.

Other constraints exist due to the limitations of students' knowledge of the subject matter of mathematics and a strong belief shared by teachers, administrators and parents that mathematics in primary school is purely arithmetic (Schram et al, 1988). It is interesting to note here that there is surprisingly little difference in the beliefs of students with majors in mathematics and those who do not have much, or any, tertiary mathematics. This similarity in beliefs is probably due to the kind of experiences that the mathematics majors undergo; experiences which once again affirm their view of mathematics as procedurally based (Ball, 1988a).

3.6 Conclusion

This chapter has considered the history of mathematics education, particularly in the Australian context, and has traced the change in requirements for mathematics education courses in teacher education programs.

The chapter has also examined the role of the teacher in a primary school mathematics classroom and has suggested that teachers should be:

- aware of the nature of mathematics and mathematical processes;
- knowledgeable about how children learn and what influence the social and cultural context has upon this learning;
- able to analyse their own beliefs about mathematics, pedagogy and mathematics education and ascertain the influence of these beliefs on their teaching.

If the role of the teacher is agreed to be as described here, then the goals of teacher education programs should be to prepare prospective teachers for this role. This implies that mathematics education courses in teacher education programs for primary school teaching should:

- help student teachers develop understanding of how children learn;
- raise student teachers' awareness of the variety of appropriate teaching strategies that will help children develop conceptual understanding of mathematics;

- encourage student teachers to have positive attitudes to the learning and teaching of mathematics and to mathematics itself.

I also believe that it is necessary for mathematics education courses to provide student teachers with an opportunity to strengthen and develop their understanding of mathematics, its nature, processes and content. As I embark on a voyage through this study I am convinced that prospective primary teachers need to have a strong subject content matter understanding as well as a clear idea of the processes of mathematics, the pedagogical content matter described by Shulman (1986a; 1986b).

Finally, it is necessary for student teachers to be aware of the weight of the attitudes and beliefs that they bring to their learning and teaching. The following chapters will examine beliefs in greater detail, both as discussed in the relevant literature, and as displayed in my research study of student teachers.

4. THE AFFECTIVE COMPONENT OF LEARNING MATHEMATICS AND LEARNING TO TEACH MATHEMATICS

4.1 Introduction

A picture has been painted so far in which the background shows various visions of mathematics and mathematics teaching. However the foreground is still incomplete and this cannot be completed until the domain of affect is examined and reviewed. The investigation in this research examines prospective primary school teachers' beliefs about mathematics and mathematics education. Consequently it is productive to review the literature on beliefs and the affective component of learning mathematics.

Much of the research on mathematics education in the years leading up to the 1980s did not focus on the place of beliefs and attitudes in the study of learning and teaching mathematics. This is felt by McLeod (1992) to have been a result of the lack of a theoretical framework for the affective domain, and a lack of suitable methodology for its study. In recent times, however, a great deal of research on attitudes towards mathematics has occurred, much of it using a quantitative framework in which questionnaires and attitude scales are used. Beliefs, too, have been examined to a greater extent as researchers have come to realise that the beliefs that students and teachers have about the teaching and learning of mathematics, and the nature of mathematics, influence that teaching and learning. The call for reform in mathematics education, proposed, both in Australia in the *National Statement on Mathematics* (Australian Education Council, 1991), and in the USA in the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989) has led to an increase in interest in the affective domain of learning and teaching mathematics, and a recognition of the importance of this area. While earlier research acknowledged the importance of cognitive factors, it is now recognised that affective factors also need to be studied, in order to gain a fuller understanding of the nature of learning and teaching mathematics.

Schoenfeld (1994) suggests that there is a close relationship between the nature of a research community and the methods that the community employs in its research. In mathematics education, for example, this can be taken to mean that the kind of research methods that are evolving in the area are reflective of the trends in the field. So when the behaviourists dominated the field of mathematics education research, the methodology used reflected their interests. Affective factors were not considered relevant to behaviourist studies. Further, the 1960s saw an accountability movement arise in some states of the United States of America. This led to a proliferation of studies of competencies. Schoenfeld cites such studies in which teachers' competencies were being examined. Competency of a teacher was defined by the marks that the students in that teacher's class gained for standard procedural tests. Consequently, teachers who were good disciplinarians, enforcing drill and rote learning, scored highly on the competency tests. However, these same teachers were themselves quite often lacking in conceptual understanding of the mathematics.

However, from the 1980s the "ownership" of the fields of inquiry changed, in that research became driven more extensively by the interests of the researchers themselves, rather than by external interests and legislation. This led to a change in the focus of inquiry and descriptive studies of areas such as beliefs became common. Studies of teachers' beliefs approached the investigation from a different direction. The value of multiple perspectives and approaches also became evident. The change in focus of the field led to a change in focus of the methodology because the topics of research required new ways of investigation. The complexity of constructs such as beliefs was acknowledged, and a consideration of methods which capture such complex phenomena was called for.

These studies have attracted criticisms from process-product proponents arguing that student achievement is not given much attention in such studies. However, Schoenfeld suggests that it is important to consider the different kinds of studies in order to evaluate the contribution of such studies to the body of knowledge about affect. In this way, the richness of the content is not lost, but the use of multiple sources of data and analysis provide triangulation.

Beliefs are often seen as part of the cognitive domain and their interaction with attitudes is consequently sometimes neglected. In this chapter I shall take the view proposed by McLeod (1992), that beliefs lie at the intersection of the cognitive and affective domains. Hence, in McLeod's theoretical framework, while beliefs are mainly cognitive in nature, the role that they play in the affective domain is critical. Therefore, they are seen as part of the affective domain, along with attitudes and emotions. Beliefs are developed over a relatively long period of time, while at the other extreme, emotions appear and disappear quite quickly. Therefore, we can think of beliefs, attitudes and emotions as lying on a continuum representing increasing levels of affective response and decreasing levels of cognitive response. Consequently, any study of either beliefs or attitudes, which does not take cognisance of the other, must necessarily be incomplete.

McLeod (1994) suggests that links between research on affective issues in mathematics learning and teaching, and research on those topics relating to improved practice in mathematics education are essential if research in the area is to be of any value. In the research which follows, these sort of links have been cultivated and considered.

In this chapter the following questions are examined in relation to beliefs and attitudes:

- What is meant by the affective component of learning and teaching?
- What are common beliefs held by primary school teachers and prospective teachers about mathematics and the teaching and learning of mathematics?
- How does the whole area of affect influence teachers and prospective teachers in their teaching and learning of mathematics?

4.2 The affective component of learning and teaching mathematics

As has been noted by McLeod (1992, 1994) (see p. 47 of this chapter), a theoretical framework on affect is essential if discussion of beliefs and attitudes is to be coherent, and of value in research on the learning and teaching of mathematics. McLeod (1992) summarises a theoretical framework first proposed by Mandler (1984; 1989). In this theoretical framework, a person experiences an affective response when a plan of action is obstructed in some way. The affective response is interpreted in the context in which it occurs. Consequently, beliefs that a person might have about a particular topic, and the environment in which the "blockage" takes place, along with cultural factors, will influence how the affect is translated. For example, if a person has a belief that a mathematics problem is either right or wrong and that it should be solved within a short time period, then not being able to solve the problem immediately, or getting a different answer to others, will produce an affective response which will be interpreted as frustration, disappointment or anger. On the other hand, solving what is perceived as a difficult problem will lead to emotions of joy and elation. The emotion is immediate and shortlived. However, if the situation leading to that emotion is repeated regularly, the intensity of the emotion dies down and the affect becomes one of a set of attitudes to a situation, which are more stable and less intense. Hence affect has three components of importance in it: the very stable aspect of belief which is not high intensity; the somewhat less stable aspect of attitude which has more intensity attached to it and the unstable aspect of emotion which has a great intensity to it.

The interaction between the three aspects of affect is of great importance when analysing the importance of beliefs and attitudes. The three components of affect influence each other and have implications for teaching and learning mathematics. Students hold a set of beliefs about mathematics and about themselves which influence the development of their affective responses to mathematics. Further, as the sort of blockages discussed above will occur frequently in the learning of mathematics, both positive and negative emotions will be experienced by the learner. Finally, if such situations are encountered repeatedly, positive and negative attitudes towards mathematics will develop.

Because emotion is very unstable it is not easily measured. Consequently, not as much research has been done on this aspect of affect as on the others. Constructivist researchers are developing a constructivist framework for research on emotions and this may well lead to an increase in studies on this aspect. Certainly, the current trends in research to consider a small number of subjects and study them in depth makes research of such an unstable phenomenon more possible than is the case in large-scale studies using surveys.

A conceptualisation of the affective domain, such as the one discussed above, is useful in providing some structure for research in this area. McLeod (1992) suggested a framework incorporating the relationships between beliefs, attitudes and emotions. Southwell (1995) suggests that this framework be extended to include values. Southwell suggests that values differ from beliefs in terms of the consistency with which an individual lives according to the values. A person might believe certain notions but not put them into practice; whereas if the belief has been accepted as a value, it is integrated into that person's way of living. I suggest that the notions of espoused beliefs and beliefs-in-practice are equivalent to Southwell's notions of beliefs and values. The distinction between these two ideas is an important one in studies of teachers' beliefs about teaching mathematics and prospective teachers' beliefs about the learning and teaching of mathematics. As will be shown in my study, prospective teachers hold a number of beliefs which are in fact, quite different from the values that they display. Likewise, a study of the beliefs of prospective elementary school teachers from the United States by Foss and Kleinsasser (1994), shows a similar disparity between beliefs and values. Foss and Kleinsasser found that although students espoused a variety of strategies for good teaching, such as the use of manipulatives, they did not tend to use these strategies in their teaching.

The above framework considers the relationship between beliefs, attitudes and emotions, and also considers values. The next section shall review research on beliefs, particularly in the field of teacher education and the learning and teaching of mathematics in particular. Firstly, however, it is necessary to determine how the literature defines beliefs and to make explicit the interpretation of the concept as it is used in this thesis.

The concept of beliefs has not been investigated to a large extent in educational literature. Reasons for this could be the interactions between beliefs and knowledge which make distinctions complex; the difficulty in developing definitions which take into account all aspects of beliefs; and the debatable value of searching for definitive characteristics of beliefs and knowledge. Consequently most researchers have taken as their starting point the assumption that their readers know what beliefs are.

Thompson (1992) has discussed the difference between beliefs and knowledge: she suggests that beliefs can be distinguished from knowledge in a number of ways. The most relevant features of beliefs, for this research, are the following:

- Not all beliefs are held with the same degree of conviction.
- Unlike knowledge, beliefs are accepted as being open to dispute. Knowledge is regarded by many as having truth or certainty, an alternative interpretation being that it is widely accepted by consensus. Beliefs, however, can be independent of consensus.
- General procedures for evaluating the validity of knowledge are in existence, whereas beliefs are characterised by a lack of agreement on how they are to be judged. Beliefs are not held up to the same sort of scrutiny as is knowledge.

However, it should be noted that beliefs can become knowledge and vice versa. A belief for which later evidence arises can be accepted as knowledge. Similarly, some features of knowledge are dependent on the theories of the day, and with the passage of time, can fall out of favour and be relegated to the position of beliefs.

In the present study, prospective primary teachers' beliefs about mathematics education are under investigation. The area of student teachers' beliefs is a relatively new topic for examination. However, there is a growing body of research which indicates that beliefs influence both the way that pupils approach the learning of mathematics and the way teachers teach mathematics (Spangler, 1992; Dougherty, 1990; Thompson, 1984; Lerman, 1983). These beliefs span many different areas, namely, beliefs about teaching and learning in general; beliefs about mathematics; and beliefs about the teaching and learning of mathematics.

4.3 Research on beliefs

This section shall review the literature dealing with beliefs about a variety of issues associated with teaching:

- beliefs of prospective teachers about teaching and learning in general;
- beliefs about mathematics;
- beliefs about mathematics education;
- the relationship between beliefs about mathematics, mathematics teaching and teaching practice;
- intervention by teacher education programs.

4.3.1 Prospective teachers' beliefs about teaching and learning

Holt-Reynolds (1994; 1991a;1991b) has described the beliefs of prospective teachers about teaching as strong beliefs, based to a large part on personal histories, and has further stated that these beliefs interact forcefully with student teachers' experiences of learning to become teachers. The results have a certain generalisability about them but it should be noted that Holt-Reynolds' work has been with prospective secondary school teachers enrolled in a content area literacy course. However, similar findings have been found in studies of prospective primary school teachers of mathematics (Ball, 1989; 1988c; Schram, Wilcox, Lanier and Lappan, 1988). In particular, Holt-Reynolds suggests a number of hypotheses:

- Student teachers' prior knowledge mediates their learning in their teacher education courses.
- Student teachers "*invent self-as-teacher based on what self-as-student believes*" (Holt-Reynolds, 1994, p. 3), that is, prospective teachers constantly evaluate pedagogical ideas in terms of their own anticipated reaction to the ideas, had they been students.
- Student teachers' beliefs remain implicit unless they are openly challenged and discussed in teacher education courses.

Other beliefs that Holt-Reynolds has found to be consistently held by prospective teachers are beliefs that learning is directly related to effort and interest, and understanding, which students agree is of importance, is also a function of interest, effort and awareness of applicability. A teacher should be enthusiastic, organised and tell students what they need to know and why they need to know it. Prospective teachers have constructed ideas of what it means to learn, from their own limited experiences.

When examining the implications that beliefs about learning and teaching might have on the efficacy of teacher education courses, Holt-Reynolds suggests that prospective teachers do not generally understand, or perhaps even notice, the underlying rationales that their lecturers at university have, for giving a particular structure to their subjects. It should be added that the academics might not necessarily make the rationale explicit either. Holt-Reynolds feels that this is because academics do not appreciate the power and tenacity of the beliefs that student teachers bring to their courses. Student teachers do not question the nature of their own beliefs or attempt to reconstruct them, but rather see them as frameworks into which their teacher education courses must fit. Those parts of the education course that cannot be assimilated into the framework are usually discarded. Consequently the challenge of teacher education becomes the task of making explicit prospective teachers' beliefs, and engaging these beliefs in an effort to expose their limitations and strengths.

Holt-Reynolds' viewpoint is reinforced by Britzman (1986) who suggests that a utilitarian approach to teacher education is required by prospective teachers, who see teaching methods as ends rather than means. Student teachers require practical methods, recipes for classroom actions and perceive those teacher education courses which do not provide such things as being irrelevant and out of touch with the "normal" classroom.

A further complication exists in the case of the preparation of primary school teachers of mathematics. There are a large number of beliefs about mathematics and mathematics teaching and learning which will also be part of the implicit set of beliefs of the prospective teacher. Consequently, not only do beliefs about teaching and learning, in general, have to be made explicit but so do beliefs in the specific area of mathematics education.

4.3.2 Beliefs about mathematics

Mathematics can be conceptualised in a variety of ways. In Chapter 2 I discussed some of the diverse range of philosophies of mathematics. These philosophies are essentially beliefs about mathematics. Among the philosophies that were discussed in Chapter 2 were the *absolutist*, *Platonist* and *fallibilist* philosophies. The philosophies are linked to particular conceptualisations that are held about mathematics. A model that has been used in this research is proposed by Ernest (1989). He describes three common conceptualisations of mathematics as follows:

> *First of all, there is the instrumentalist view that mathematics is an accumulation of facts, rules and skills to be used in the pursuance of some external end. Thus mathematics is a set of unrelated but utilitarian rules and facts. Secondly, there is the Platonist view of mathematics as a static but unified body of certain knowledge. Mathematics is discovered, not created. Thirdly, there is the problem-solving view of mathematics as a dynamic, continually expanding field of human creation and invention, a cultural product. Mathematics is a process of inquiry and coming to know, not a finished product, for its results remain open to revision.* (p. 250)

The problem solving view has socio-cultural implications and seems to lie midway between *realist* and *idealist* views of mathematics. This agrees with the views held by Wittengenstein (1978) and Lakatos (1976) who suggested that mathematics is a human activity, which is fallible and has meaning in a particular setting. The *problem solving* view indicates that there is a sociological perspective of mathematics.

Foss and Kleinsasser (1996) reported that prospective primary school teachers saw mathematics as being a set of computational skills and arithmetic, that were important in helping with everyday life tasks such as balancing a cheque book. This could be seen to be allied to the first view proposed by Ernest (1989, p. 250), in which mathematics is seen as a set of unrelated but utilitarian rules and facts.

It is probable that people's conceptualisations of mathematics will influence their beliefs about mathematics education, in particular, their learning of mathematics. Stodolsky (1985) describes how beliefs about mathematics will lead to certain behaviours in the mathematics classroom, that differ in nature from behaviour in the social studies classroom. In social studies classes, pupils were more likely to work in groups, and work on tasks that developed higher-order skills, while in mathematics lessons, pupils were more likely to sit alone and work from a textbook. Of course, the teacher's conceptualisations of mathematics are also of importance here.

4.3.3 Beliefs about mathematics education

Skemp (1978) suggested that there were two types of learning of mathematics that could occur:

- instrumental learning in which the learner focuses on the rules and procedures but does not develop any conceptual understanding of the mathematics, and
- relational understanding in which the learner constructs the underlying conceptual framework and develops understanding which is richer and deeper than instrumental understanding.

The two ways of learning mathematics would be likely to stem from different conceptualisations of mathematics itself; instrumental learning seems closely linked to Ernest's instrumentalist conceptualisation and relational learning to the so-called Platonist view, although it is not in disagreement with the problem solving view described by Ernest.

Further links between beliefs about mathematics and mathematics education are presented below. A model of mathematics teaching proposed by Kuhs and Ball, is presented in a review by Thompson (1992, p. 136) that is based on a synthesis of the literature on beliefs in mathematics education, teacher education and research on teaching and learning. Kuhs and Ball identify a minimum of four distinctive views on how mathematics should be taught.

1. ***Learner-focused:*** *mathematics teaching that focuses on the learner's personal construction of mathematical knowledge;*
2. ***Content-focused with an emphasis on conceptual understanding:*** *mathematics teaching that is driven by the content itself but emphasizes conceptual understanding;*
3. ***Content focused with an emphasis on performance:*** *Mathematics teaching that emphasizes student performance and mastery of mathematical rules and procedures, and*
4. ***Classroom - focused:*** *mathematics teaching based on knowledge about effective classrooms.* (Kuhs and Ball, quoted in Thompson, 1992, p. 136)

Each of these develops out of a particular conception of mathematics and of mathematics learning. Kuhs and Ball state that the first, the learner-focused view derives from a constructivist view of mathematics learning as it centres on student involvement in **doing** mathematics. It consequently is likely to be adopted by teachers who see mathematics as a problem solving process, as delineated by Ernest (1989).

The second view of mathematics teaching, that of content focus with emphasis on understanding would correspond to the Platonist view described earlier. The third view follows naturally from an instrumentalist view of mathematics. It is the model of teaching that is most often criticised by educators. The fourth conviction assumes that the curriculum is set and it is the classroom activity that will determine the success or failure of the learning.

In the study of prospective primary school teachers conducted by Foss and Kleinsasser (1996), interviews with student teachers revealed the belief that teaching mathematics successfully in the primary school depended on the teacher's personality and on the classroom ethos. If the teacher was patient and enthusiastic, and learning was fun and interesting, then children would learn all the mathematics required. In other words, they found that prospective primary school teachers seemed to possess the fourth view suggested by Kuhs and Ball in Thompson (1992), that the classroom activity and ethos was the most important factor in promoting learning. The model suggested by Kuhs and Ball does not mention the role of the teacher in the teaching of mathematics, although this did seem to be an important factor for the Foss and Kleinsasser study. In the latter study, the teacher's role was central in two ways: firstly, in setting up activities that were fun and obtained a classroom atmosphere that was conducive to learning, and secondly, in "setting the tone" of the class by showing enthusiasm and patience. No mention seemed to made by the student teachers in the Foss and Kleinsasser study of the role of the pupil in the learning of mathematics.

4.3.4 Relationship between beliefs about mathematics, mathematics teaching and teaching practice

The relationships between beliefs about mathematics, mathematics teaching and teaching practice are not totally clear. Some studies show definite links between beliefs and practice (Thompson, 1984) while others report discrepancies between the two (Cooney, 1985). The fact that inconsistencies are found may indicate that other factors are at work in determining the teacher's practices. There is a tension between beliefs, practices and the social context. Cooney (1985) describes a student teacher who, as a result of experiencing tensions between his views of teaching mathematics by problem solving and the realities of his teaching situation, very quickly modified his beliefs on mathematics teaching. The distinction between beliefs and values may be relevant here.

In the Foss and Kleinsasser study (1996), student teachers' firm and unchanging views of mathematics provided obstacles to the success of their methods course, in changing their beliefs and actions to do with teaching mathematics. When observed on practicum, student teachers still emphasised memorisation and drill and practice. They also used inadequately prepared games which lacked clear links to the mathematics they were aiming to teach. This supports the Thompson study (1984) which suggested that beliefs about mathematics are indeed influential in determining teaching practice.

For further investigation it would be of interest to see whether certain beliefs are modified more readily than others. For example, does an instrumentalist view of mathematics enjoy more acceptance in the classroom, thus leading to modification in beliefs about process-oriented views such as the problem solving view of mathematics and the learner focused teaching model?

It would also appear that views are readily changed when difficulties with implementing these views occur. For example, Foss and Kleinsasser (1994) found that student teachers agreed in a survey that manipulatives are important. However, lack of confidence about how to use the manipulatives, or ignorance as to when the manipulatives should be introduced led to few student teachers actually using such manipulatives while on teaching practice. Espoused beliefs, mirroring the teacher educator's beliefs on good practice, very quickly give way to actual beliefs (values), personally derived, concerning structure, discipline and order in a classroom. This is not inconsistent with the findings of Cooney's research (1985), discussed above.

These inconsistencies could also be due to the way data is collected on teachers' beliefs. When questionnaires are used, some of the respondents answer these questionnaires on beliefs in terms of a commitment to an abstract ideal and what they believe should be done, rather than in terms of actual practice. This may be more likely in the case of student teachers than experienced teachers.

It would seem likely that lack of sufficient background in mathematics leads to discrepancy between beliefs and practice. This could be of great significance for teacher education courses. Borko, Eisenhart, Brown, Underhill, Jones and Agard (1992) address this matter in their study of the inconsistency between belief and practice of a student teacher. While maintaining that conceptual understanding of a topic was vital, attempts by the student teacher, in a primary school classroom, to explain the conceptual underpinnings of fraction division to her pupils, led the student into an embarrassing situation as she realised that she did not understand the process of fraction division herself. Explanations were postponed to a future date (never reached) and the student teacher reverted to the "safe" ground of teaching procedures on fraction operations. It can be seen that this relationship between beliefs and practice is not a simple one, and the interplay between the two is dynamic, each playing its part in modifying the other.

As discussed in Chapter 2, pp. 20-22, Pateman (1989) and others suggest that teachers' personal philosophies will influence their practice in the classroom. Further, school mathematics has been strongly influenced by two of the foundationalist philosophies, the logicist and the formalist, and their influence can be seen in most school textbooks.

4.3.5 Intervention by teacher education programs

It would appear that prospective teachers' beliefs are not easily changed during their progress through teacher education programs. However, a North American study found that changes had occurred in students' beliefs about mathematics, the structure of mathematics classes and the process of learning mathematics (Schram, Wilcox, Lanier and Lappan, 1988) as a result of their participation in a ten-week long mathematics course.

In the Australian setting, Perry, Geoghegan, Howe and Owens (1995) offer a one-semester elective subject in mathematics for prospective primary school teachers, which uses an approach centred on an *experiential learning cycle* involving the stages of experiencing, discussing, generalising and applying. A series of norms for social interaction are developed interactively with the students. In reports on the results (Perry, Geoghegan, Howe & Owens, 1995, and Perry, Geoghegan, Owens & Howe, 1995) the authors suggest that the learning opportunities created helped students both to develop their own mathematical backgrounds, and gain insights into the role of the mathematics teacher in the classroom. Common beliefs about the nature of cooperative learning and the role of the teacher were successfully challenged by the subject.

4.4 Implications of the beliefs literature for the present study

As the importance of the area of affect in mathematics education has become apparent, a great many studies of teachers' beliefs about the learning and teaching of mathematics have been undertaken. So it may be seen that my research study reported here is part of a growing movement to understand the teaching and learning of mathematics as deserving of attention in both the cognitive and affective areas. This study intends to bring an Australian perspective to the topic, as many of the current studies are from the United Kingdom and United States of America. It also intends to provide a fresh look at some of the themes that arise, and to examine in detail, aspects of prospective teachers' beliefs that have not been the focus of other research. Some aspects of the study reaffirm results of other research and other aspects will focus on areas that have not previously been examined.

The methodology used in this study also contributes to the development of the field, as much of what has been done previously used a quantitative framework in contrast to the qualitative framework used here. The methodology used in my study has been chosen because of the amount of detail that it allows the data to yield. However, it must be recognised that because different methodologies seek answers to different questions, some disparity may occur in results using different paradigms. For an example of the disparity in results that can arise due to different research paradigms, a study by Foss and Kleinsasser (1994) on prospective teachers' beliefs used both quantitative and qualitative data collection methods in order to gain triangulation of results. The primary objectives of the paper cited here were to discuss the triangulation of data collection and analysis, and compare and explain the conflicting results that arose from the study, as an outcome of using the two different research paradigms. The triangulation in this case resulted in outcomes that were either in agreement, or were contradictory and inconsistent. There was agreement in the data collected by the different methods, regarding students' views of mathematics as arithmetic. However, inconsistencies appeared where prospective teachers' responses to statements on pedagogy were not reflected in their own activities. The authors concluded that the results obtained by qualitative research allowed a richer, deeper picture to emerge and a study of the contradictions provided by the two paradigms enhanced the understanding of the process of learning to teach mathematics.

It should also be pointed out, that for all parts of my study, my own set of beliefs and theoretical dispositions will lead to interpretations of data that are personal constructions albeit well defended ones. Having a social-cultural approach to learning also influences the way the study is undertaken.

The success of teacher education courses in mathematics can be judged on a number of criteria: Do students find the courses to be of value; do they apply the principles upheld in the courses to their learning and teaching; do they emerge with improved attitudes towards mathematics and the teaching of mathematics?

From the review of the studies conducted above, it is clear that if student teachers have a particular set of beliefs about teaching and learning; mathematics and mathematics education, then these beliefs will determine how effective they find the teacher education courses on mathematics; and the attitudes of the students towards mathematics will also be heavily influenced by these beliefs.

Consequently, questions that this area of research asks are of value to teacher education programs as they provide the impetus for teacher educators to examine their teaching. These are questions such as the following:

- What are primary student teacher beliefs about mathematics and mathematics teaching at the initial stages of their teacher education program?
- How do these beliefs influence students' success in learning new mathematics at this stage of their lives, and in learning how to teach mathematics to primary school children?
- Does the teacher education program intervene, in any way, in students' conceptions and attitudes?

These are the questions that this research study seeks to answer.

4.5 Conclusion

This chapter has examined the affective component of learning and teaching mathematics. A theoretical framework for research on beliefs has been adopted which suggests that beliefs are at the intersection of the cognitive and affective domains. The interaction between beliefs, attitudes and emotions was examined, as it appeared in relevant research.

From the understandings developed in the literature it has become apparent that the area of research into beliefs is a complex one, and requires appropriate methodology for its study. The dynamic interplay between attitudes and beliefs is an important area for research. Similarly, the distinction between beliefs and values should be noted, as these play an important part in my research study.

The beliefs of student teachers have been shown to have certain common characteristics, regardless of the level of teaching for which students are being prepared, or of the discipline which students are studying. Most prospective teachers appear to believe that their personalities and the interest value of the work is of primary importance in promoting learning. Teachers need to be enthusiastic and supportive and the topic needs to be interesting and have applicability. Further, the studies reviewed indicate that student teachers do not appear to pay much attention to the role of the pupil in the learning process.

The areas of philosophies of mathematics, and other beliefs about mathematics have also been subjects of research. It has been shown in various studies that both teachers' and students' conceptions of mathematics are instrumental in determining the way mathematics is taught and learnt. In teacher education courses various attempts have been made to intervene in the beliefs that students hold about the teaching and learning of mathematics. These interventions have met with varying degrees of success.

This research study has as its focus the beliefs of prospective primary school teachers during their first year in a teacher education program. Having knowledge of these beliefs is essential if teacher educators are to succeed at helping students to become successful users and teachers of mathematics. The methodology chosen for this study has been selected because it is considered to be the most appropriate way of examining students' beliefs. The following chapter discusses the methodology in detail and examines its suitability for this study.

In conclusion, there is an acknowledgment that no description of mathematics teaching and learning is adequate and complete unless it includes consideration of the beliefs and intentions of teachers and students (Fenstermacher, 1980). This study considers the beliefs and intentions of student teachers, so that their passage through a teacher education program is better understood.

5. THE STUDY: DESIGN, METHODOLOGY AND DATA COLLECTION

5.1 Introduction

As has been discussed in Chapter 1, this research seeks answers to a number of questions in the belief that the derived knowledge will not only add to the understanding of the particular student group involved, but will lead to the emergence and construction of theory and practice that will provide a useful framework for other mathematics educators in teacher education programs. In addition, it will enable me to examine and interrogate my own behaviours and insights in order that I might continuously improve my practice in the area.

The following four questions form the conceptual framework for the thesis:

- What beliefs and attitudes about mathematics and mathematics education do first year primary school student teachers bring into their tertiary education?
- How do these beliefs and attitudes influence the way that the students view the content and approaches used in mathematics education subjects in teacher education programs?
- How do students' attitudes and beliefs influence their success in learning new mathematics at this stage of their lives?
- How do these beliefs and attitudes affect students' ideas on good practice in the teaching of mathematics in primary school?

In order to develop an understanding of the answers to the research questions, a variety of data collection methods were used. Within all these phases the goal was to examine what beliefs and attitudes students bring to their first year of an initial primary teacher education program at an Australian university, and how they construct the subject matter of the two first year subjects in mathematics education offered at that university. Also of interest were the ways in which these constructions are mediated by prior experience. The investigation was concerned with deriving perceptions of the relevance of the mathematics education subjects, asking what students gain from them, and if these subjects are in any way influential in changing students' ideas of teaching mathematics.

A qualitative research methodology was used in this study. The reasons for choosing this methodology will be examined in this chapter. The theoretical underpinnings for qualitative research using case study and interpretive methods will be discussed to show how this methodology was appropriate for the study considered here. The focus of the chapter will not only be directed towards the general nature of the methodology but also towards the specific design I have used, and my reasons for developing the study within this framework. A thorough description of the participants will also be given.

In Chapter 1, I have discussed the compelling reasons for engaging in this study. My awareness of the views held by prospective primary school teachers, about the learning and teaching of mathematics, was a major factor in initiating the research. The belief that these views would have a major influence on the student teachers' interactions with the mathematics education subjects, offered as part of their teacher education program, influenced the choice of student group for the study. It became important to conduct the study during the students' first year of a teacher education degree, in order to investigate students' beliefs and attitudes before any extensive interaction with the teacher education course had occurred. Another reason for conducting the study in the students' first year was to encourage students to examine and make explicit their beliefs. The first year of the degree is one in which ideas are gathered, examined, negotiated and used as starting points for the teaching and learning of powerful mathematical-pedagogical concepts.

5.2 Methodology used

The study under discussion can be viewed as employing ethnographic methods. The term *ethnography* comes to educational research from anthropology. Spradley's important work *The Ethnographic Interview* (1979) defines ethnography as "*the work of describing a culture*" (p. 3). He uses the word "culture" to refer to "*the acquired knowledge that people use to interpret experience and generate social behaviour*" (p. 5). Describing a culture can mean, therefore, describing the indigenous people of an area; description of the life of urban blacks; the illumination of the habits and thoughts of poverty stricken inhabitants of a tenement building; or, as I have chosen, the description and pursuit of understanding of the culture of prospective primary school teachers. Referring to Spradley's meaning of the word "culture", I will show that student teachers do share a body of acquired knowledge which influences and shapes their views and behaviours. They share a common language about nurturing and caring for their future pupils. They share a past in which their mathematical experiences have led them to develop views about mathematics and mathematics education. They interpret much of what they experience in their teacher education courses through the images they possess of the work of teachers and of the characteristics of good teachers.

Wiersma (1986) suggests that a definition which would describe educational ethnographic research is as follows: "*The process of providing scientific descriptions of educational systems, processes, and phenomena within their specific contexts*" (p. 233).

I argue that a combination of these two descriptions of ethnographic research is relevant in this study. As maintained above, the prospective primary school teacher cohort is in fact part of a culture of prospective teachers. The culture has a particular language, using phrases and jargon that expresses the cultural norm of nurturing pupils and motivating them by certain accepted behaviours; their attitudes towards the study of mathematics are also part of the cultural view; and their reactions to mathematics education courses are typical of the accepted cultural behaviour. Hence this thesis certainly examines a particular culture. It also investigates educational processes and phenomena within a specific context.

The essential feature of an ethnographic study, according to Spradley, is concern with the meaning and actions of the people we seek to understand. An ethnographer's task is to tell the story of the people in a particular culture, from their perspective. Spradley emphasises that "*rather than studying people, ethnography means learning from people*" (p. 3). To achieve this goal, ethnographers immerse themselves in the culture in an attempt to infer meaning from the behaviour and language of the culture.

In this research study, my background as teacher educator allows me to immerse myself in the prospective primary school teacher culture. By observing their actions and reactions concerning mathematics and associated pedagogy, I am able to construct an understanding of the norms and accumulated knowledge of the culture. It should be acknowledged, however, that, for many, ethnographic studies imply deep engagement, on the part of the researchers, in the daily linked life of the culture in question. In this sense, my study can be said to be employing the tools and procedures of ethnographic scholarship, if not the intense engagement. Further, I have been able, in ethnographic terms, to access "native informants" to provide me with direct information about the culture. It has been noted by qualitative researchers, such as McCracken (1988), that immersion in, and familiarity with, the culture may have both disadvantages and advantages. On the one hand, the researcher might develop a variety of preconceptions and might have her powers of observation and analysis dulled by the intimacy she enjoys with the culture; on the other hand, it allows her greater insights and understanding of the culture. Consequently, it is my task as someone close to the culture, to use that intimacy to my advantage, analytically speaking, and to minimise the difficulties that immersion in a culture can possibly create. One might describe my role as a critical friend to the culture.

The study can also be seen as *field research* in that it is conducted in the educational context under examination. The student teachers themselves, are the source of the data, and it is their beliefs and attitudes that are the primary focus of the study. The research is essentially an attempt to describe and interpret, in as accurate a manner as possible, a particular student culture with respect to mathematics education. It is recognised that the context of the study is an important factor in the interpretation of data. However, descriptions of student teachers from various sources indicate that similar student teacher cultures exist in most teacher education programs in English speaking countries such as Australia, the United Kingdom and the United States (Pateman, 1989; Ernest, 1989; Foss & Kleinsasser, 1996). The recognisability of the study allows its results to be generalised to other contexts (institutions or countries). Consequently, although this case study was set up as "*a 'bounded' system ... within which issues are indicated, discovered or studied so that a tolerably full understanding of the case is possible*" (Adelman, Jenkins and Kemmis, 1983, p. 3), it allows generalisation to similar cases. The so-called "shock of recognition" (Adelman, Jenkins and Kemmis, 1983, p. 4) is its passport to generalisability.

I believe that the research under discussion differs in one important way from a commonly understood description of ethnography.

> *The emphasis in ethnographic research is on describing the context in qualitative terms without superimposing the researcher's ... own value system on the situation. Thus ethnographic research takes a qualitative - phenomenological approach.*
> (Wiersma, 1986, p. 234)

Wiersma glosses over a difficulty that exists in ethnographic research. It is recognised that it is impossible to describe and interpret qualitative data without any subjectivity being present. To ensure that the researcher's value system is not superimposed on the situation would require the researcher to be totally uninvolved in the phenomenon being studied. Even then, the impartial observer would carry a value system that cannot be divorced from any interpretations made by the observer. Wiersma also stresses that the purpose of ethnographic research is description and he neglects an equally important aspect of ethnography, that of interpretation. To describe a situation without interpreting it through the lens of the researcher's experiences and beliefs is to give a very barren and incomplete picture of the phenomenon under examination. It does not acknowledge that any discussion of the phenomenon is seen through the eyes of the researcher and therefore cannot be a value-free and objective description. McCracken (1988), on the other hand, suggests that the researcher's beliefs and experiences need to be examined in order to identify cultural categories and relationships that might be of importance in the study. Far from ignoring or distrusting the personal experience of the investigator, he suggests that a detailed appreciation of this experience is useful in making the investigator more sensitive to the issues being studied, and more aware of possible areas of investigation. This view accords with Strauss and Corbin's (1990) description of *theoretical sensitivity*. Theoretical sensitivity is the ability to give the data meaning and to detect relationships between various categories. Strauss and Corbin suggest that one of the sources of theoretical sensitivity is the professional experience of the researcher. While acknowledging that this kind of experience can limit vision, Strauss and Corbin suggest that professional knowledge allows the researcher to have a richer knowledge base and more insight into the study. The latter view accords with my belief that the experience and viewpoint or lens of the researcher cannot be ignored, and consequently should be used to good purpose rather than be denied. This issue is discussed in greater detail at the end of this chapter.

The study employs qualitative research methodology as it is felt that rich description is more illuminative in this case than statistical data such as that gained from the use of Likert scales investigating attitudes and beliefs. The study recognises that qualitative and quantitative methodologies seek to answer different, although often complementary, questions. The purpose of this study is not to discover how many, and what kinds of people share a particular view about mathematics, but rather to gain access to the cultural categories and assumptions by which prospective primary school teachers construct their vision of teaching and learning mathematics.

> *The quantitative researcher uses a lens that brings a narrow strip of the field of vision into very precise focus. The qualitative researcher uses a lens that permits a much less precise vision of a much broader strip.* (McCracken, 1988, p. 16)

While much work on affective issues in the learning of mathematics has been characterised by its emphasis on definitions of terms, concern with measurement issues and dependence on quantitative methods, McLeod (1992) suggests that this emphasis could be complemented and enhanced by an alternative paradigm for research on affect, which emphasises theoretical issues and uses qualitative methods and interviews. The latter methodology is one which I have felt was appropriate for my study. It recognises the complexity and variability of the phenomenon being researched. It seeks to understand the phenomenon *in situ* and to provide a theory of the ways in which beliefs and attitudes interact to construct a means of viewing the learning and teaching of mathematics. As further confirmation of my choice of methodology, it is interesting to note that in the study of the beliefs, conceptions and practices of prospective teachers undertaken by Foss and Kleinsasser (1994), discussed in Chapter 4, p. 60, conflicting results appeared when data was collected by quantitative and qualitative means. In this study, Foss and Kleinsasser show that the contrasting research paradigms, initially used to provide triangulation of data and analysis, led to three types of results: convergent, inconsistent and contradictory results. Convergence occurred when students saw mathematics as being procedural, and these beliefs were seen to be exhibited in the students' teaching. Inconsistent and contradictory results occurred when students espoused a view of mathematics that incorporated investigatory work but their implementations of such teaching were rare and problematic. The quantitative data indicated that students were changing in their ideas about teaching during their teacher education course, and were more knowledgable about both methods and content after studying methods subject in mathematics. However, the interviews, observations, field notes and video tapes that constituted the qualitative data showed the prospective teachers' failure to demonstrate evidence of such change in their beliefs and actions. Foss and Kleinsasser concluded that the area of teachers' beliefs and conceptualisations is an extremely complex one and cannot be easily described using statistical evidence.

5.3 The participating students

As explained earlier, the study focuses on a group of students, all within their first year in an initial primary school teacher education program at an Australian university in 1993. The students were all members of two tutorial groups out of a total of four groups in the first year cohort. The numbers varied throughout the year as students left the course, joined other groups or came to these groups from elsewhere. There were approximately 100 students originally in the first year of the primary teacher education course. Fifty of these were in the two groups participating in the study. Of these fifty students, only forty remained at the end of the year. A further two students joined the group in second semester, bringing the year-end total up to 42.

These two groups were initially chosen because in the first semester I was both the researcher and their lecturer. This made the student body more accessible, in order that questions could be posed at appropriate moments, and it also made it possible to build up a positive relationship with the students. The situation also increased the opportunities for me to inform my own practice as an action researcher, as I was able to respond promptly to issues that emerged.

It is important to note that the first semester subject was a pass-fail one, in which criteria for passing were dependent on the submission of required work rather than on the grading of that work. Students were required to submit two assignments and maintain reflective journals as requirements for passing the subject. In order to gain a pass grade in this subject, students had to attend all classes and submit the two assignments and the reflective journal. No grades were given for the quality of the work; assignments deemed to be incomplete, or lacking in some way, were required to be resubmitted. A failing grade could only be obtained by non-submission of work or non-attendance of workshop sessions in the subject. As a researcher, I did not feel that I was abusing a position of power as lecturer: indeed, I believed that the students' engagement in the research study would increase the benefits they obtained from the mathematics education subjects. Throughout this research, a very important component has been the encouragement to students to reflect about their teaching and learning of mathematics and to make explicit and challenge the beliefs that they have in this area. I consequently believed that to engage in this research was beneficial to the students. Indeed, I have put forward a case at a State mini-conference for teachers at the post-secondary level, for using a research metaphor in the teaching of prospective primary school teachers, in order to help stimulate mathematical thought by students (Schuck, 1995), quite independently of any research outcomes that might be obtained by the teacher educator.

During the second semester of the study the participating groups were taught by another teacher educator. Here the subject was graded. This would have made it more problematic for the participants had I been both teacher and researcher. Permission to use the material gathered in the first semester was sought when students were no longer in my class, thus allowing students the freedom to refuse to become involved if they so desired, without fear of some sort of retribution when being graded[1]. By the second semester, many of the students were well known to me, as I was to them, and I had a positive relationship with them, which was helpful in allowing me access to their views. The fact that I did not teach the students in that semester was also helpful to me, in that I managed to achieve some sort of distance from the students during that time.

Very soon after the commencement of the study the initial cohort of fifty students became one fewer in number. Of the forty nine students now in the two tutorial groups, 32 students had graduated from secondary school in the last three years before entering this program, and the

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[1] It should be noted that the university at which the study took place has rigorous ethics procedures to prevent abuse of any such power relationships. At the beginning of the study I applied for and obtained approval from the university Ethics Committee. Please see Appendix A for letters of permission used with participating students.

remaining 17 students had left secondary school over a span of years ranging from 1958 to 1989. (See Table 5.1)

The university entry system operating in Australia, in 1993, nominated all students who had gained their Higher School Certificates (HSC) in 1992 as potential Category A students. All other students were nominated as potential Category B students. For the purposes of my research, this division was not helpful to me, and I have chosen to group all students whose last year at school was 1990, 1991 or 1992 as "recent school-leavers" whereas all students who had left school prior to 1990 were classified as "mature age" entrants. Using this distinction, approximately 65% of the students were recent school-leavers while the remaining 35% were mature age entrants. Table 5.1 shows the breakdown according to gender and category of these students.

Categories of Entrants

Gender	Recent School Leavers	Mature Age Entrants	Total
Female	28	14	42
Male	4	3	7
Total	32	17	49

Table 5.1: Category and gender of students. N=49.

Table 5.2 shows the educational background of the students in terms of highest level of school which the student had reached, and highest mathematical level reached. "Other" in the "level of schooling" row refers to students who had either completed their school studies elsewhere, or had done so at a time before the present nomenclature was introduced. One student has been placed in this category because her educational background is unknown.

In the state of New South Wales, there are six years of high school available to students, culminating in the Higher School Certificate (HSC). These years are years seven to twelve. Students are able to leave school from the end of year ten. The highest level of mathematics studied falls into various categories: students whose final year of study in mathematics was year 10; those who started on the HSC course in mathematics but did not continue beyond the end of year 11; those who studied the 2 Unit Mathematics in Society subject, a mathematics subject for those who wished to have a more applied study in mathematics and who did not intend to study mathematics at the tertiary level; 2 Unit Mathematics which is the lowest level of mathematics generally required for tertiary study, and 3 Unit Mathematics, for which a greater degree of proficiency in mathematics is required. No-one in this group had studied 4 Unit Mathematics, the highest level available at the time.

	Level of Schooling			
Highest level of mathematics	**Other/ Unknown**	**Year 10**	**Year 11**	**Year 12**
Other	3			
Year 10	1	1	2	
Year 11				2
2U, Mathematics in Society				10
2U				20
3U				10

Table 5.2: Highest level of schooling and of maths. N=49

There is also remarkably little ethnic diversity among the teacher education students participating in the study; they certainly do not reflect the composition of the Australian tertiary student population in general. Fewer than five per cent of the group could be said to be of a non English speaking background; these included one Japanese and one Sri Lankan student.

A large part of one of the groups participating in the study was a group of students who intended to become teacher librarians. Much of their four year Bachelor of Education in Teacher Librarianship course was common to the general Bachelor of Teaching in Primary Teaching degree in which the other students were enrolled; their first year of study differed only in one or two subjects. However, most of them did not intend to teach in an ordinary primary school classroom but intended instead to become teacher librarians. Fourteen out of the total 49 students fell into this category.

Finally the students could be categorised according to whether they felt they had been able to get good grades at school mathematics and whether they had believed themselves to be successful at mathematics; not always the same question as results will show. Table 5.3 displays results across a matrix of grades and beliefs about success. Note that as this information was collected later in the study than the above details about level of schooling, gender and category, the number of participants had varied and at this stage of data collection N = 32.

	Grades Obtained for Mathematics			
Response to Statement "I am Good at Mathematics"	Good grades	Satisfactory grades	Poor grades	Total
Strongly agree	2			2
Agree	11	4		15
Not sure	4	4		8
Disagree	1	4	2	7
Strongly disagree				
Total	18	12	2	32

Table 5.3: Grades at school and beliefs about success at mathematics. N=32

Students were asked to respond to the question about their grades in mathematics at school by choosing one of three categories: good, satisfactory or poor to describe their grades. They were then asked to respond to the statement "I believe that I am good at mathematics" along a five position continuum ranging from strongly agree to strongly disagree. Table 5.3 shows the results. Finally it is interesting to note that of the 49 students initially in the study, only one chose to study mathematics as her elective discipline[2].

As has been mentioned earlier, the two groups were initially chosen because of my access to them as both researcher and lecturer in the first part of the study, and because they were not in my classes during the second part of the study, when my position as lecturer could have influenced their responses. However, although I did not set out to choose students who would be representative of the whole student teacher population, a study of the above demographic data reveals a fairly typical picture of the prospective primary school teacher, both within the participating university and within the primary student teacher population as a whole, in Australia and other western countries such as the United States of America (Foss & Kleinsasser, 1994; Ball, 1988d)[3]. The higher number of mature age students was due to the fact that the teacher librarian cohort tends to have more mature age students than the Bachelor of Teaching in Primary Teaching program.

5.4 Data collection

The data were collected in four distinct phases over the period of approximately one year. The different methods of data collection, and the different types of data collected, had the dual purpose of serving as triangulation and of providing extra information where required.

The word "triangulation" derives from surveying, in which it is used to indicate that there are two or more fixed views or sightings of a phenomenon, coming from different angles. In qualitative research triangulation is used to describe the attempts to ensure that data collected

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[2] Students were required to choose one of a number of subjects, as a major course of study within their program. The study of this subject was not professionally oriented but aimed at enriching the student's knowledge of a particular discipline.

[3] Foss and Kleinsasser describe the prospective elementary teachers in their study as follows: n=22, one male and 21 females. Eleven students were over the age of 24 and eleven were between the ages of 18 and 24. The average age of the students was 26. Most of the students were married, raising children or maintaining employment while enrolled in college. This would be typical of the students in my study (precise data is not available).

In Ball's study 19 students participated, ten elementary prospective teachers, nine secondary student teachers. One was an African American, one was of Asian ethnicity and the others were Caucasian. Fourteen of the students were between 19 and 21 years old. Only one of the elementary students was male. Ball claims that the students in her study reflect the population of teacher education students (presumably in North America, although this is not stated) in terms of characteristics of ethnicity, gender and age. No information as to whether any of the students spoke English as a second language was provided.

in different ways will give a coherent reading of a story. Delamont (1992) describes three main types of triangulation:

- Triangulation between methods - the collection of data by a variety of methods;
- Triangulation between investigators - the collection of data and studying of the phenomenon by more than one person;
- Triangulation within method - attempting to collect several types of data within the one method.

Triangulation can also occur at the analysis stage by scrutinising the data.

It will be shown later in this chapter, that triangulation between methods, between investigators and within method were all used in this study.

The study uses a grounded theory paradigm in which successive stages of data collection grew out of the previous phases. The term "grounded theory" was first used by Glaser and Strauss (1967) to describe a strategy for developing substantive theory. With grounded theorising, the theory is grounded in the data and emerges from the data. It is termed grounded theory "*because of its emphasis on the generation of theory and the data in which that theory is grounded*" (Glaser, quoted in Strauss, 1987). The strength of grounded theorising is the basing of any emerging theory in the data from which it arose. This allows the theory to reflect "reality". According to Strauss and Corbin (1990, p. 9) "*Formulating theoretical interpretations of data grounded in reality provides a powerful means both for understanding the world 'out there' and for developing action strategies that will allow for some measure of control over it*". The characteristic of grounded theory is that it is inductively derived from the study of a particular phenomenon. An area of study is researched and the theory pertinent to that area is allowed to emerge. This theory is then systematically verified through further data collection and analysis. The data collection, analysis and theory building are thus inextricably linked (Strauss and Corbin, 1990).

In grounded theorising the data are carefully and rigorously analysed and are collected and coded using methods of "constant comparison" (McCracken, 1988). When using constant comparison, the data are collected and examined for key issues that might become categories of study. Further data are then collected which contain incidents which link to these previous categories of data. New categories are also explored. The focus is on the organisation of all the ideas that emerge from analysis of the data. As the data are collected they are analysed, narrowed and refined so that the next phase of data collection can reflect this refinement in the questions asked.

In this study, in each of the four phases described, data were collected and promptly analysed to get an overview of the important issues arising, and then these issues were further treated in the next collection stage. Consequently the theory grew out of the data and influenced the direction of future data collection.

The research study is distinctive in that it incorporates grounded theorising, thick description, and interpretation which provides a rich and deep understanding of first year student teachers' beliefs and attitudes in the area of the learning and teaching of mathematics.

The four phases of data collection are listed below. Full description of each of the phases follows:

- **Phase one**: data collected in the first semester, 1993, by three different methods. The first set of data were interviews formulated and conducted by the students themselves. Both questions and responses to the questions comprised the data. Journal entries on how the student teachers believed learning occurred were also included in this phase. Finally, a third set of data on the students' experiences and beliefs about the practicum were part of the phase one data.
- **Phase two:** data collected by means of an open ended questionnaire given to students towards the end of the second semester, 1993.
- **Phase three:** in-depth interviews with eight students comprised the phase three data. These interviews took place after the close of the second semester of the students' first year of the teacher education course, or at the beginning of the following year.
- **Phase four:** in-depth interviews with the four teacher educators responsible for the mathematics education subjects. These took place in May 1994.

5.4.1 Phase one: paired interviews

Of particular interest to me were the areas of mathematics learning and teaching that the students felt were problematic or worthy of examination. My goal of learning about these areas was achieved by having the students themselves construct the questions on issues concerning mathematics and mathematics pedagogy so that the questions reflected those areas about which they were concerned or in which they had an interest. Effectively, then, the questions themselves became a source of data. The first phase of the study, consequently, was particularly noteworthy in that it gave the participants a voice in the construction of the research questions.

5.4.1.1 Data collection by paired interviews

As I have noted, the data collection for this part of the study involved students as both investigators and participants. The first phase of the study took place during semester one of the students' first year of the Bachelor of Teaching in Primary Teaching and Bachelor of Education in Teacher Librarianship programs. As has already been discussed in Chapter 1, students undertake an orientation subject in mathematics in their first semester which aims to improve attitudes to mathematics and to demonstrate changes in approaches to teaching mathematics in the primary school[4].

Early in semester one (March 1993), the first year students were each given the task of eliciting from another class member their current attitudes towards mathematics and mathematics teaching and how these have been affected or influenced by past mathematical experiences. Each student was asked to develop a set of five questions to ask a partner. The questions were to elicit information on attitudes and beliefs about the learning and teaching of mathematics. The task was given as an assignment in the orientation subject with the aim being to help students reflect on the importance of the affective component of learning mathematics and to assist students to become aware of their own stance in the area of beliefs and attitudes about mathematics[5]. At that stage, the objectives of the assignment were primarily to make explicit students' beliefs about learning and teaching mathematics so that these were open to further discussion later in the course. The data were then stored until the

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[4] See Appendix B for details of the subject outline for Primary Curriculum Orientation II - Mathematics, which describes the aims and content of this introductory subject.
[5] See Appendix B for a copy of the assignment for this phase of data collection.

second semester, when students were no longer in my classes. During the second semester, I requested permission from students to use this data in the present study. Those who chose not to give such permission had merely to withhold a letter authorising the use of their data. None of the students chose to do so. Appendix A contains an example of the permission letter.

Students were given approximately two weeks in which to develop and pose their questions. The timing was deliberate so that students had not had much exposure to the orientation course at the time of interviewing each other.

Students' choice of questions reflected their perspective on mathematics education. Consequently they were of great interest in the present study as they gave information about the questioner's attitudes as well as the respondent's views. Students chose partners using social grouping criteria. Each participant acted as both an interviewer and an interviewee, recording both questions and answers on audio tape. After the interview, each student wrote a precis of the pair's responses, including notes indicating the implications of both their own and their partner's attitudes for their future teaching. In this way students were, themselves, acting as researchers, reflecting on what questions were most important in the obtaining of information about attitudes on mathematics education and also describing the possible effects of these attitudes on future teaching experiences.

I matched the students' summaries against my analysis to provide an alternative review of the data. In this way *triangulation between investigators* (Delamont 1992) was provided as the students themselves were the initial investigators.

Other data collected during this initial phase comprised the following:
Approximately three weeks after the start of the students' first semester in the teacher education course, I requested participants to write notes in their reflective journals about the ways they believed children most effectively learnt mathematics. At this stage the data from the paired interviews had just been submitted and had not yet been read or analysed. I wished to ascertain what students' beliefs about learning were, and it was important to do this early in the course so that I could obtain their views before these had been greatly influenced by their

experiences in the teacher education program. These notes were then photocopied when the journals were submitted at the end of the semester.

The other questions I wished to have answered concerned students' educational backgrounds and also experiences on their first teaching practicum. Consequently, in the workshop directly after the first teaching practicum, I asked students to submit a note with some background information regarding their last year at school and highest level of mathematics, and also to answer three questions regarding the teaching practicum. These questions were:

- Question 1: What methods did your supervising teacher use to teach maths while you were on prac?
- Question 2: How appropriate did you feel these methods were?
- Question 3: If you had been a child in this class, would you have enjoyed learning maths?

From the answers to these questions, I wished to explore whether there was a fit between espoused beliefs about teaching, and opinions about the methods that had been observed on the practicum. The data on the professed beliefs came from the data collected in the paired interviews and the notes in the journals on effective teaching.

The notes on beliefs on teaching and learning, and the notes on the practicum were all regarded as phase one data as they were collected in the first semester, before students had interacted to any great extent with the teacher education course. All analysis was delayed until the second semester, when students were no longer in my classes. The data was then used to guide the next phase of data collection.

As this research uses a grounded theory paradigm, initial analysis of the data from phase one, and emerging theory were instrumental in the development of the next stage of the study, a questionnaire focusing on categories that emerged from the paired interviews and other data described above.

5.4.2 Phase two: the questionnaire

In this phase, data were collected from students in a written form. Data were collected by means of an open-ended questionnaire, and it is the use of the questionnaire in a qualitative framework that is the focus of this section. This data collection occurred later in the year than phase one, during the period October 1993 to February 1994.

The development of the questionnaire grew out of phase one, in which paired interviews were used to collect data, along with notes on beliefs about learning and notes on experiences during the first practicum. Questionnaires were distributed towards the end of the second semester of the participating students' first year in the teacher education program, approximately seven months after the paired interview data had been obtained. It was given at a relatively late stage of the year, as many of the questions concerned the students' experiences throughout the two mathematics education subjects offered in the first year of the teacher education program.

5.4.2.1 Data collection by questionnaire

A distinctive feature of this research is the use of a questionnaire within a qualitative case study framework, in which both collection of data, through the questionnaire, and its subsequent analysis are accomplished using qualitative methods. The questionnaire in this instance is used almost as a written interview in that it probes understanding and beliefs and recognises the diversity of responses that might be given, rather than requiring answers which can be neatly placed along some sort of continuum or scale. It is composed mainly of open ended questions which have been structured by emergent issues which were embedded in the paired interviews and phase one notes. A copy of the questionnaire is presented in Appendix C. The analysis of data provided in the questionnaire will be discussed in the next chapter.

The questionnaire items evolved out of data collected in phase one. As students generated their own questions in phase one, both questions asked and answers given in this phase were of interest in the development of questions for phase two. As one of my major aims was to allow respondents to tell their story in their own terms, it was necessary for the questions in the questionnaire to reflect the prior interest shown by participants in various issues. Consequently, issues that appeared frequently in phase one were further cultivated in phase two and concepts from phase one that gave an indication of deeper underlying material were probed further in phase two. Relevant categories that had been identified in the literature review were also represented. Finally the issues that interested me, as researcher, were included. This process allowed me to examine further those areas in which silences had occurred in phase one, that is, issues that either I, as researcher, or the research literature, had identified as important, but which did not appear to have been viewed as such by the majority of the participants. The questionnaire allowed me to *triangulate between methods* (Delamont, 1992), by using written responses as well as interview data gained previously.

The items from which the questionnaire was compiled were divided into three sections: the first section asked students to choose the alternatives that most accurately described aspects of their educational background. These were closed questions.

The second section comprised a number of open-ended questions about the students' views on learning and teaching mathematics, and about their past experiences with mathematics. Consequently, questions about effective learning were asked in a somewhat more structured form than in the written notes from phase one - this was *triangulation within method* (Delamont, 1992), where different data were obtained using the same method. For example, questions were asked about the meaning of the word "fun" as derived from students' common statements that mathematics had to be fun; about the conceptual and procedural aspects of mathematics; another question asked students to describe a pupil who was successful at mathematics.

The third section dealt with open ended questions about the students' experiences within the first year mathematics education subjects and during their two practicums in the first year.

The data collected by questionnaire indicated to me that there might be some difference in responses according to age and other criteria, and these factors influenced the third phase of data collection, the in-depth interviewing of a small number of students.

5.4.3 Phase three: in-depth interviews with students

Eight students were selected for in-depth interviewing by means of *purposeful sampling* (Bogdan and Biklen, 1982). This type of sampling is one in which participants are chosen, not because they are representative of the general population, but rather because it is these particular participants that interest the researcher for one reason or another. As this phase grew out of two previous data collection phases, it became of interest to me to see whether responses would differ on certain issues as a result of either the age of the participant or the amount of reflection the participant had demonstrated in the earlier phases: I wished to investigate whether showing an ability to think carefully about past and present experiences would lead to different types of answers being given. I therefore required participants who were contrasted in terms of the length of time since they had been in secondary school, and I also wanted a range of students in terms of the amount of reflective ability they had shown in the previous stages of data collection, in particular in phase two. Both of these criteria for selection emerged as I started to analyse the data, and it became apparent that differences might appear in students' responses due to the variables of age, and ability to reflect about teaching and learning. It should also be noted that of the initial eight students approached to be interviewed, one did not appear to be interested in being a participant as she did not respond to my initial contact, and another student was consequently approached. Table 5.4 indicates how students were chosen for the in-depth interviews.

The primary purpose for the in-depth interviews was to provide triangulation for data obtained previously (*triangulation between methods*, Delamont, 1992) and also to provide opportunities for me to probe further where required, as the opportunity to further develop any particular response was enhanced by the methodology of phase three. The phase three interviews were conducted on a one to one basis in the period December 1993 to March 1994.

	Entry Category	
Range of reflection	**Recent school leavers**	**Mature age entrants**
Reflective	2	2
Not reflective	2	2

Table 5.4: Choice of students for in-depth interviews. N=8.

The test papers written by these eight students halfway through the semester (August 1993) have also been collected as an additional source of data about the students' beliefs and ways of organising their preparation for tests. Used together with the interviews, I was able to make links between their comments and the documentation available. I also had access to their marks for all assignments and tests and these assessments were useful when looking at comments made in the interviews that indicated that students had not valued a topic because they did not achieve high marks for that area.

A profile of each of the eight students, using pseudonyms, follows. These profiles are described in the order in which the students attended the interviews.

Nita: Female, mature age entry, highest year completed at school was year 10 in 1969. Highest level of mathematics at school was year 10 Advanced Mathematics. Also attended first year university in 1981. Seemed to be a reflective student in phases one and two[6]. Interviewed December 1993.

Aaron: Male, mature age entry, highest year completed at school was year 12 in 1989. Highest level of mathematics at school was 2 Unit Mathematics in Society in the HSC. Also obtained a Tertiary Preparation Certificate with Mathematics 1 and 2 at a college of Technical and Further Education (TAFE). Did not appear to engage in much reflection in phases one and two. Interviewed February 1994.

Maria: Female, recent school leaver, highest year completed at school was year 12 in 1992. Highest level of mathematics at school was Two Unit Mathematics in the HSC. Did not appear to engage in much reflection in phases one and two. Interviewed February 1994.

Gail: Female, mature age entry, highest year completed at school was Leaving Certificate in 1958 (equivalent to year 12). Highest level of mathematics at school was Mathematics 1 and 2 in the Leaving Certificate. Appeared to reflect deeply in phases one and two. Interviewed February 1994.

Mandy: Female, recent school leaver, highest year completed at school was year 12 in 1992. Highest level of mathematics at school was Three Unit Mathematics in the HSC. Seemed reflective in phases one and two. Interviewed February 1994.

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[6] See discussion of responses to phases one and two and examples of the students' work in Appendix D to assess the degree of reflective ability these students had shown.

Terry: Female, recent school leaver, highest year completed at school was year 12 in 1992. Highest level of mathematics was Two Unit Mathematics in the HSC. Seemed reflective in phases one and two. Interviewed March 1994.

Joanne: Female, recent school leaver, highest year completed at school was year 12 in 1992. Highest level of mathematics at school was Two Unit Mathematics in society in the HSC. Did not seem to be reflective in phases one and two. Interviewed March 1994.

Pip: Female, mature age entry, highest year completed at school was the equivalent of year 10 in 1969. Highest level of mathematics was year 10. Was not able to recall her past due to illness in recent years. Interviewed March 1994.

It can be seen that seven of the eight students were female and one was male. This choice was deliberate, in order to reflect the proportion of males and females in the entire first year group. Approximately one-eighth of the first year student population for primary teacher education in the participating university in 1993 was male.

5.4.3.1 Data collection by in-depth interviews

Questions for the interview were developed from the areas that had been raised by the students themselves in phase one, when certain topics or issues started emerging from the paired interviews. Some of these were investigated further in phase two, in the open questionnaires. In particular, I wanted to use the in-depth interviews to learn, not only what students believed about mathematics and the learning and teaching of mathematics, but also about their reactions to the mathematics education subjects they had recently experienced in their teacher education program. I was particularly interested in the second of these two subjects, entitled Mathematics Education 1. This subject is described in detail in Chapter 8. Discussion of the subject was a sensitive issue because lectures in the subject were given by one of my colleagues, and I felt that while students might not be prepared to commit criticisms of the subject to paper, they might be prepared to confide in me during an interview. Consequently, these interviews took place in the end of year break after the completion of the students' first year in the program. Students were approached to be interviewed, and as mentioned earlier, only one of the students contacted refused to be interviewed; the other students seemed quite happy to do so.

The interviews did have some formal structure in that a schedule of questions was prepared beforehand. However, this schedule was not strictly adhered to and students' responses quite often moved the interview into different areas. The interview schedule was prepared before the first interview, out of material ensuing from the previous phases of data collection. The first interview was then transcribed and the schedule modified as a result of further questions arising from the first interview. Further modifications took place after the next two interviews had taken place. Questions were rephrased and others added. In this way the interviews were used to develop my theoretical framework. In a sense the set of interviews could be seen as a continuous conversation rather than one discrete schedule repeated eight times.

The interviews were scheduled to take approximately one hour in time. Students were asked where they would like to meet me and all chose to meet in my office. The interviews were audio-taped, and after transcription, returned to the participants so that they could check I had transcribed their intent correctly. Actual time of interviews ranged from thirty-five minutes to ninety-five minutes.

The questions in the interviews covered a number of areas. The first set of questions dealt with the participant's conceptions of mathematics; the nature of mathematics and its uses. Another group of questions considered the participant's experiences with mathematics, and included questions on "good" or "bad" teachers of mathematics and why they were considered to be good or bad. The last section dealt with the students' beliefs about teaching mathematics and their interaction with the first year mathematics education subjects at university.

5.4.4 Phase four: in-depth interviews with teacher educators

Adding another dimension to this discussion is a different perspective; that of the participating **teacher educators**, their philosophies regarding the epistemology of mathematics, and what they see to be the role of the mathematics education subject sequence in the Bachelor of Teaching in Primary Teaching and Bachelor of Education in Teacher Librarianship degrees.

Four mathematics educators were interviewed. As one of the educators myself, I felt it was appropriate for me to answer the same schedule of questions that the other mathematics educators were asked. My responses were audio-taped and transcribed in the same way as the other responses, and analysed similarly.

The questions were developed from questions or comments that students had made in the previous phases, in particular in the in-depth interviews of phase three. They focussed on the educators' views of mathematics and mathematics teaching and how they saw the mathematics education subject sequence, in particular, Mathematics Education 1, as helping students develop the necessary skills to become teachers of mathematics.

For reasons of confidentiality, it is not possible to describe the individual teacher educators in any detail. However, it should be noted that apart from myself, a female mathematics educator with under ten years' experience at this university, the others all were male, in senior lecturer positions and all had been at this institution for over fifteen years. All four of the mathematics educators concerned had higher degrees in mathematics as well as qualifications in education.

5.4.4.1 Data collection in phase four: in-depth interviews with teacher educators

As mentioned above, interviews were developed out of previous data generated by students and concerned beliefs about mathematics and reasons for developing the mathematics sequence in a particular way. Consequently a structured interview was used and the schedule of questions was followed closely.

The interviews were conducted on an individual basis and audio-taped. After transcription, they were returned to the lecturers to ensure an accurate transcription had occurred. The data were then analysed.

In summary, then, the connections between the different phases of the collected data can be schematically represented as shown below in Figure 5.1.

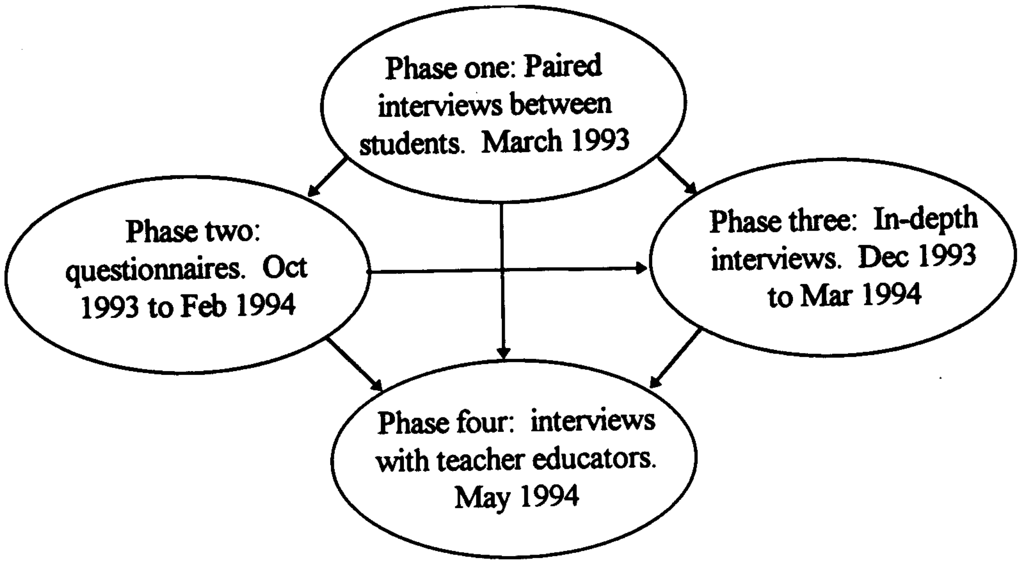

Figure 5.1: Relationships between the different phases of data collection.

5.5 Issues and dilemmas in using this methodology

A number of key issues arise in discussion of the chosen methodology. These issues deal with the interaction of the research and the participants; and with the advantages and disadvantages of the types of methodologies chosen. These are discussed, in order of importance.

5.5.1 Interactions with participants

One of the problems that arises in any case study is that of protection of the participants from possible consequences of the study. The participants have all had their names changed to maintain anonymity, but this is a very superficial solution to the stated problem: this anonymisation will only exist in terms of the wider readership of the study; those close to the study should be able to recognise participants from the descriptions of their behaviours and beliefs. In fact, if they were not recognisable to those familiar with the circumstances of the study, the question must arise of how authentic the description and interpretation can be. Consequently, the participants were asked to take responsibility for their stated viewpoints.

The ethical issue of whether participants should have to take such responsibility must surely arise with every case study. I have attempted to deal with this problem by asking participants to complete a contract in which they are asked for permission to publicise the results in a form that does not identify them in any way (see copy of contract in Appendix A). The participants were told the purposes of this research and told of the possible readership. I have asked participants to read transcriptions of their interviews with me, and to change any aspect with which they did not feel comfortable or which they felt did not reflect their intent at the time (see Appendix A for copy of this letter). In this way, I am making the processes used open to the participants and it is my hope that participation was executed in a thoughtful and reflective manner, with all possible consequences considered by the participants. It is also considered that the passing of time between data collection and publication of results will ensure the participants' anonymity to some degree. However, this issue of anonymity and responsibility for viewpoints is one which is of great concern to me as a qualitative researcher with conflicting aims of telling the participants' stories with as much authenticity as possible and at the same time, ensuring that participants are not recognised from their stories.

A related issue is one discussed by Howard (1995). He observes that the presence of the researcher and of the fieldwork will have an impact on the community being studied. The impact needs to be carefully monitored and the researcher should maintain careful watch that this impact is not deleterious to the participating community. This aspect of fieldwork, in turn has implications for the study and the independence of the results obtained. Further the development of trust and respect for one another as researcher and participant is essential if the participants are to be open and frank in their discussion. Consequently if the researcher is considered to have behaved in a way that could create unpleasant consequences for any of the participants, this trust is abused and all future researchers in that community will be affected. And so integrity of results may be in conflict with the integrity of the researcher. This issue is discussed again in more detail in Chapter 6.

5.5.2 Subjectivity

The problem of subjectivity is central to many discourses in interpretive research. Adelman, Jenkins and Kemmis (1983) propose that the reader brings a certain subjectivity to the reading of the case and the case consequently appeals to the reader's understanding and interpretation of the case. They submit that the researcher is not guaranteeing the veracity of the case's generalisations but rather offering a surrogate experience to the reader for interpretation.

Heshusius (1994) suggests that in the struggle to acknowledge the importance of subjectivity, most qualitative researchers see as vital the management of their subjectivity. She describes this struggle to control or restrain one's subjectivity as being of the same nature as positivistic concerns for a study to be objective in procedure. According to Heshusius, controlling your own subjectivity implies that you can distance the self from the phenomenon being studied, and that you can choose what aspects of self need to be regulated. She contends that this distancing of self is not possible and consequently, instead of controlling or restraining our subjectivity, we should rather concentrate on achieving *"participatory consciousness"* (Heshusius, 1994, p. 16) in which we reach out to the phenomenon being studied and integrate our selves with it. We stop being preoccupied with self and rather give all of our attention to the phenomenon. In this way, we are not judging our self for regulation purposes, but become aware of the self in a non-evaluatory manner. This "self-forgetfulness" allows the self to disintegrate and allows the researcher to become totally absorbed in the phenomenon. We no longer control the research but genuinely hear what the participants in the study have to tell us.

To a certain extent this study has achieved the objective of maintaining a participatory consciousness by allowing the students to phrase the initial questions out of which all further investigation grew. As the study was grounded in this initial data collection phase, the issues and questions which were discussed in later phases echoed the students' voices in the first phase. The study, consequently, is indeed listening to their questions and concerns, as well as attempting to answer my questions. I repeat the suggestion by Spradley that ethnography involves learning from the participants rather than studying them. It is in this spirit that the study was conducted.

While this study has foregrounded the voices of the students and teacher educators in relaying their stories, the task of interpretation is mine. In order that this interpretation should be as faithful as possible, I need to focus on the students and teacher educators rather than on my beliefs. This issue will be discussed further in Chapter 6, Section 6.2, p 95. However, my beliefs need to be open to the reader at the outset, and so these will be re-iterated.

As both researcher and teacher educator involved in the offering of the mathematics subject sequence that is discussed in this study, my beliefs about the nature of mathematics and of mathematics teaching inevitably affect the development and execution of the subject sequence. My belief is that mathematics is a socio-cultural phenomenon, and that different mathematics exists for different societies. To be able to "do" mathematics is to be able to make conjectures, investigate patterns, justify one's position and use mathematics to solve problems.

I did not always hold this view. In my early career, my underlying philosophy of mathematics was that of a logicist. I believed in the laws of logic governing mathematics and, as a teacher, explained carefully the steps of deduction used to derive any result. The change in my viewpoint came as a result of my reading; my reflecting about my own learning; and my observations of my students' learning. It appeared to me, that learning mathematics was highly dependent on the experiences and knowledge already possessed by the learner and that learning took place when activity occurred on the part of the learner, rather than the teacher.

My stated position has consequences for my presentation of mathematics education subject sequences in teacher education courses for prospective primary school teachers. I see this role to be one of developing a socio-cultural view of mathematics by implementing such a vision of mathematics. Consequently, the subject sequence in which I am involved is highly participatory and it emphasises use of the learning community and co-operative methods of learning. My role is to provide a rich environment in which investigations can take place, involving mathematical concepts that are deep and strong in connections. I observe group interactions and encourage the dissemination of information among the members of the group. I constantly work at resisting the giving of answers to the students or playing at being the expert: a difficult process to withstand as the students so often demand this role of me.

This is my position dealing with the methods of learning that I value and believe are effective. It should also be mentioned at this point, that I am not alone in my views on learning and teaching mathematics. As has been mentioned in Chapter 2, many other teacher educators believe that mathematics is learnt through socio-cultural means. I have to be vigilant, however, that I am not promoting this view by coercion, as the expert who has the answers, but rather that I allow students to experience this type of learning and come to their own conclusions about what mathematics is and how it can be taught. It is from this perspective that all interpretation of data is made; nevertheless every effort is made to ensure that the participants' stories are told from within their framework. For this reason I shall use the participants' quotes to illustrate any interpretations that I might make, in order to reduce the possibility that I am bringing my biases to the data. As much as possible, readers will be able to interpret for themselves the meaning of the data **from the data itself**, as it has been voiced by the participants. Nevertheless, it has to be remembered that even the selection of appropriate quotes is an interpretation of what I consider to be relevant.

5.5.3 Choice of methodologies

The following section analyses the advantages and disadvantages offered by the methodologies used in my research study.

5.5.3.1 Phase one: Implications of using students as investigators

Having the students pose the questions ensured that their perceptions of what was important in the affective area were heard. It also meant that respondents were more relaxed as they were talking to peers and so possibly were able to be more open about their experiences. Additionally, as mentioned before, the questions themselves provided a good source of data for the study. I believe that an additional advantage was that the process gave students the opportunity to benefit from the reflection about mathematics education that was necessary in order to pose appropriate questions, and that an increased awareness of other people's attitudes to mathematics was developed by the student interviewers.

5.5.3.2 Phase two: The merits of using a questionnaire in a qualitative study

One of the advantages of using a questionnaire over an interview was the ability to reach more participants. It is probable that students were more likely to answer a questionnaire as it could be done whenever there was some available time, and at a convenient location, without the presence of the researcher being necessary. A face-to-face interview would require either having to travel to a mutually agreed upon location, or allowing the researcher access to thestudents' homes. Also, many people feel more comfortable answering a relatively impersonal questionnaire rather than exposing their innermost thoughts in a face-to-face interview.

There are, of course, some disadvantages to collecting data by this means rather than by interviews. One such disadvantage is the difficulty in getting a high response rate as the questionnaire being discussed here cannot be answered quickly and does demand a great deal of thought from the respondent. The response rate for the questionnaire was approximately 76 per cent.

The second difficulty is in encouraging respondents to answer as fully and as deeply as they can. In an interview, the interviewer can probe further by asking appropriate questions, arising from the previous discussion. In a questionnaire, the lack of contact during the completion can lead to answers that are cursorily given. On the other hand, this in itself can provide valuable information about the respondent's position on a particular item. Further, the in-depth interviews of phase three allowed such questions to be probed further.

5.5.3.3 Phase three: Implications of using in-depth interviews with students

The advantage of using in-depth interviews was that these interviews allowed me to probe further when this was indicated by the data. The structure was more flexible than either of the structures used in the previous two phases as I was able to modify the questions or adapt them to the particular situation being experienced. Further, after each interview the questions could be modified to ensure that new areas of interest to me were covered. Another advantage was that I was able to obtain large amounts of expansive and contextual data without any difficulty or delay. In addition, the data obtained in the interviews was useful for discovering complex interconnections that might only have been hinted at in the earlier phases of data collection. It allowed me to verify theory that had emerged in the earlier stages by appropriate questioning. It also helped me to become aware of nuances that might not have been obvious in the earlier stages.

The disadvantage was that by my having more control over the collection of data, the students' stories might not be told. Instead responses to my questions could reflect my interests and biases in the area. To avoid this problem, I grounded the questions in the previous two phases, generally avoiding modification of the questions asked unless this would shed further light on areas the students had voiced as important.

5.5.3.4 Phase four: Implications of interviewing the teacher educators

The major issue here is the anonymity of the academic staff involved. This issue has already been debated to some extent in Section 5.5.1 on interaction with participants. In the earlier discussion, I considered the issue of whether a participant should be required to take responsibility for his or her responses or whether the data should be muted if it becomes identifiable.

It is a particularly acute problem in phase four because of the small number of people involved and because their positions can be easily identified. However, the teacher educators concerned were aware of these difficulties and agreed to the use of any data collected.

5.6 Conclusion

The chapter has given a detailed account of the study: its methodology, the participating students, and implications of using the stated paradigm. I have described how the research questions that are the basis of the research are best explored using a variety of qualitative methodologies.

The choice of methodology is not an easy one. As can be seen from the discussion in this chapter, there are a number of issues that need careful monitoring when using a qualitative framework and case study research. The chapter has outlined these issues and considered how to minimise the limitations and maximise the strengths of the given methodology. This is not to say that any other methodology would not have similar limitations and strengths, but rather to emphasise the necessity of being aware of these issues when conducting research and of being mindful of the necessity to consider ways of strengthening the study by avoiding any possible pitfalls.

One of the strengths of the given methodology is the way it allows the student teachers to be given a voice in the construction of the study. It is they who pose the questions that become the focus of further research. Further, the different methodologies used provide triangulation and it is this triangulation which adds to the rigour of the study.

The issues of rigour are further discussed in the next chapter, in which the analysis of the data is considered. In Chapter 6, a discussion of the methods of analysis using a qualitative computer software package will be examined, as well as a consideration of the aspects of a study that make it rigorous and robust.

The detailed picture of student teachers' affective responses to mathematics and its associated pedagogy, and the match or otherwise of these responses with those of participating teacher educators, will be productive in providing a framework for research on this area that delves deeply into the issues rather than merely presenting an overall but more superficial picture. Researchers in the area of affect in mathematics learning and teaching (Foss & Kleinsasser, 1994; McLeod, 1992) emphasise the contribution that can be made by the rich and informative data that are available using case studies and qualitative research frameworks.

6. ANALYSIS: LISTENING TO STUDENTS' VOICES

6.1 Introduction

The previous chapter described both the methodology, and the issues related to the methodology used in my research study. A full description of the methods of data collection was given. As has been discussed in Chapter 5, the analysis of the data began very early in the study, shortly after the first phase of data collection was completed. The time line on the next page (Figure 6.1) gives an indication of how the data collection and analysis of the data were linked.

As can be seen, the preliminary analysis of the data in phase one was used to focus my attention on any themes or emerging issues, in order that, in the next phase of data collection, questions could be posed about these matters. In this way, the data collection from phase one was instrumental in determining further questions in later phases of data collection, and the data collection, analysis and theorising were inextricably bound together to form a coherent whole.

In qualitative research, various issues may be considered important. One that is considered important in all discussions of analysis is that of the rigour of the research. This issue will therefore be considered here. Another issue that is pertinent to this research is the role of computers in the analysis of qualitative research. This issue arises as my research study uses a software tool for analysis and the implications of such usage therefore need to be made explicit. Consequently, this chapter will first consider these two matters. A detailed description of the methods of analysis will then be given.

6.2 Rigour in analysis of qualitative research

There has been much discussion in texts on qualitative research, about the difficulties of demonstrating rigour. Whereas validity and reliability are both definite requirements of quantitative studies, researchers have asked whether conventional conceptions of validity and reliability are possible within a qualitative framework (Sullivan, 1994, Merriam, 1988) or, indeed, whether they are desirable (Erickson, 1986; Lincoln & Guba, 1985).

	Phase 1	Phase 2	Phase 3	Phase 4
1993: Mar	First semester mathematics education subject begins. Student interviews and journal entries on learning			
May	Three week practicum. Notes on practicum 1			
Jun	First semester mathematics education subject ends			
Aug		Second semester mathematics education subject begins		
Sep		Three week prac		
Oct	Permission gained to use data. Initial analysis begins	Questionnaire given to students		
Nov		Some responses to questionnaire returned. Initial analysis begins. Second semester mathematics education subject ends	Initial preparation of interview schedule	
Dec		More responses returned. Initial analysis continues	In-depth interview with one student. Schedule modified	
1994: Jan		Responses returned		
Feb		Responses returned; analysis continues	In-depth interviews with 4 students	
Mar			In-depth interviews with 3 students	
April			Initial analysis	Interview schedule prepared
May				Interviews with 4 lecturers
1995: July to Nov	In depth analysis of data in all phases			

Figure 6.1: Time line of the study

Merriam (1988) suggests that rigour in qualitative studies derives from the researcher's presence, the nature of the interaction between the researcher and participants, the triangulation of data, the interpretation of perceptions and in rich, thick description. She suggests the concept of "dependability" to replace that of reliability; given the particular nature of the data collected, do the results make sense and are they consistent and trustworthy? While the existence of dependability in a qualitative study might be harder to establish than reliability in a quantitative study, this does not imply it is of lesser worth. In this study, both the "shock of recognition", discussed in Chapter 5, Section 5.2, p. 66, and the triangulation methods used, allow the reader to assess whether dependability is achieved.

Eisenhart and Howe (1992) argue that a definition of validity must be one that satisfies both quantitative and qualitative research designs. They suggest five general standards for validity in educational research (1992, pp. 657 - 663). I shall discuss each of these standards in the following section, and indicate how each standard has been met in my methodology.
The standards are:

- The fit between research questions, data collection procedures and analysis techniques;
- The effective application of specific data collection and analysis techniques;
- Alertness to and coherence of prior knowledge;
- Value constraints;
- Comprehensiveness.

1) *The fit between research questions, data collection procedures and analysis techniques:*
This standard suggests the research questions should drive the data collection and analysis techniques and the design should be cogently developed in line with the research questions. I have shown in Chapter 5 how the research questions led to the data collection techniques employed. The desire to allow students' questions and thoughts to influence the direction of further investigation led me to do an initial analysis of the phase one data, so that I could develop questions within my research framework, that emerged from the phase one data. The phase one data also influenced the data collection techniques used in the further phases of data collection (see Figure 5.1, p. 87). The methods of analysis used will be shown in this chapter to match the type of information required by my research questions.

2) *The effective application of specific data collection and analysis techniques*:
Eisenhart and Howe (1992) propose that the techniques should be competently and convincingly applied. There should be persuasive reasons for the specific use of particular techniques. Again, Chapter 5 has presented the arguments for each of the data collection techniques used and this chapter will present the case for the analysis techniques used.

3) *Alertness to and coherence of prior knowledge:*

It is essential for the assumptions and aims embedded in the execution of the study to be laid open and evaluated. I have exposed my beliefs and values in the area of mathematics education, both in Chapter 1 and Chapter 5, while Chapter 4 discussed the framework provided by the literature review on beliefs and attitudes. Continuing awareness of any prior knowledge will occur throughout the research.

4) *Value constraints:*

External value constraints:

External value constraints relate to the question of what the study is contributing - is it worthwhile? This means the findings of the study should be accessible to the community to which it is directed, in the case of this thesis, the community of teacher educators. Value-laden language also has to be avoided. This will be considered in the results and conclusions of this thesis.

Internal value constraints:

Internal value constraints refer to research ethics. By internal value, the authors of the standards refer to the way the research is conducted with respect to the participants in the study. The balance between the quality of the data and the principles of confidentiality and privacy is considered as part of this standard.

As has already been discussed in some depth in Chapter 5, great care was taken to ensure that the study was conducted ethically. Students were only invited to participate in the study when they were no longer in my classes, and both students and teacher educators were given opportunities to withdraw from the research study at any time without penalty. All names were changed to ensure confidentiality. The letters of permission in Appendix A contain examples of the procedures used to ensure that the research was conducted ethically. The issues involved in maintaining the balance between quality of data and ethics have been discussed in Chapter 5.

5) *Comprehensiveness:*

This standard includes issues already mentioned in the previous four standards. The study needs to have an overall clarity, coherence and competency in both theoretical and technical areas. It also needs to balance the value of the study against the ethical considerations involved in the study's publication. Some discussion of the issues involved in balancing contributions to the educational community against the ethical issues which arise, has occurred in Chapter 5 and will continue to engage me throughout the study.

Other researchers (Delamont, 1992, Marshall & Rossman, 1989) respond to the call for validity and reliability by suggesting other criteria to assess the trustworthiness of the qualitative study. These criteria are considered to be the characteristics which make a study rigorous in approach. As has been discussed in Chapter 5, Delamont (1992) suggests the use of different types of triangulation in data collection, namely triangulation within method, between methods and triangulation between investigators. Further details on triangulation were provided in the description of my research methodology in Chapter 5, Section 5.4, p. 75.

Marshall and Rossman (1989) recommend some criteria for establishing additional trustworthiness. These criteria include making data collection methods explicit, displaying negative instances of the findings and accounting for these; discussing biases; and making public strategies for data collection and analysis. With these criteria in mind, I shall describe in detail the processes of analysis used in my research, in order that these methods are made obvious to anyone wishing to duplicate or assess my research.

6.3 Using computer software to analyse qualitative data

Before embarking on a discussion of analytic methods used in this research study, I will develop the other issue raised in the introduction to this chapter: that of the use of computer software in qualitative studies.

The use of computer software for analysing qualitative data is becoming increasingly accepted and pertinent (Richards, 1995). There are obvious advantages in using a computer to handle large quantities of text in various ways. In this research project the qualitative computing software package NUD•IST (Non-Numerical Unstructured Data Indexing Searching and Theorising: a product of QSR Pty Ltd) has been used both to analyse and theorise. The software has been designed to facilitate the handling of qualitative data, the coding of such data and the consequent exploration of the material. Such exploration is considered essential to the process of identifying emerging relationships or themes.

The accepted procedures of analysis in any qualitative study can be expedited through the use of NUD•IST. These processes are:

- *affixing codes to a set of field notes drawn from observations or interviews.*
- *Noting reflections or other remarks in the margins.*
- *Sorting and sifting through these materials to identify similar phrases, relationships between variables, patterns, themes, distinct differences between subgroups, and common sequences.*
- *Isolating those patterns and processes, commonalities and differences, and taking them out to the field in the next wave of data collection.*
- *Gradually elaborating a small set of generalisations that cover the consistencies discerned in the database.*
- *Confronting those generalisations with a formalised body of knowledge in the form of constructs or theories.* (Miles and Huberman, 1994, p.9)

All of these processes can be accomplished with the aid of NUD•IST, although some are more readily achieved than others. Text can be stored, retrieved and coded using the software. An index system is constructed within NUD•IST, and represented as a hierarchical tree structure. The top of the tree is the most general code and subordinate levels become more specific. However, changing the hierarchy and combining levels or creating new levels to subsume the others does not create any difficulties in NUD•IST: consequently, the researcher continues to have flexibility about how to code the data at any point in the project, or how to change the indexing system.

Notes can be written, in the form of memos, which are dated and can be retrieved in reports, or analysed and coded themselves. However, I have found that the memo is not easily accessible when insights occur during the coding process, and I have had to write down those insights separately, entering those insights into the program at a later stage if further analysis of them is required.

A strength of NUD•IST is that the data and ideas are stored and handled in two different places in the software; the document and the index system. This feature allows the conceptual structure to be built and developed independently of the textual record (Richards, 1995). Accordingly, I was able to build up the indexing tree with categories derived from my reading of the text at an early stage of the study, without having to code the text with the indexing at that point.

It is the "sorting and sifting" through the data that is strongly aided by use of NUD•IST. The program allows the collection of common themes, or of extremes, or any other variation that might be required. The data from such sorts are collected and are accessible for further investigation. The operators available have been described as "*by far the most extensive and powerful set of code-based retrieval operators around*" (Weitzman and Miles, 1995, p. 248). It is the use of the operators such as "intersection", "matrix" and "near-to" that allow the formulation of theory to occur, as different sections of data can be compared, contrasted or structured in various ways. The strength of NUD•IST is the assistance it gives the researcher in theory emergence and theory construction as categories are evolved and developed (Richards & Richards, 1990).

It has been noted by many qualitative researchers that far more literature has been written on the collection of the data than on the subsequent analysis of it and the resultant theory-building (Richards & Richards 1990). Clear guidelines are available for analysis of quantitative data in the form of discussions of the appropriateness of statistical analyses that have been used to describe a particular situation. However the theory-building that emerges from the analysis of qualitative data can be personal and is dependent on individual creativity while at the same time needing to meet certain tenets related to the research questions being addressed. It must be sufficiently transparent to allow others to test the interpretation, yet it must also recognise the possibility of multiple readings.

NUD•IST is a package which helps the researcher in this area. Like many other qualitative data analysis packages, it allows easy access to relevant quotes, and can be used in searches for keywords and counts of particular occurrences, in both on-line and off-line documents. However, it should be emphasised that this facility would be of limited use if the accompanying theory building was not part of the process, as statistical packages can do these tasks just as ably. NUD•IST allows the researcher to delve more deeply into the substance of the data, to investigate categories that might not otherwise be obvious, and it encourages the formation of categories of greater delicacy than might be done without it. In this way, it assists theory to emerge and develop. However, the framework provided by NUD•IST acknowledges that the computer package is a tool which aids in the highlighting of categories, but the meaning making and construction of theory is done by the researcher not the software. So while NUD•IST is an extremely suitable data management tool, it does not replace the intellectual work of the user.

The question, however, arises of whether analysis which uses such software creates possibilities or constraints. This question shall be answered throughout this chapter, when a full description of my use of the software is provided.

6.4 Initial analysis

Initial analysis was completed immediately after each phase of data collection, to provide further information for my return to the field. See Figure 6.1, p. 96, for details of the timeline for the different phases.

6.4.1 Organisation of the analysis of data for phase one.

Phase one data were collected in March 1993, in the form of audio tapes containing interviews between pairs of students, and written summaries of the interviews including further discussion on what the data suggested to the students, as implications for their future lives as teachers. The other data collected at this time consisted of notes on how students believed pupils best learnt mathematics. In May after the student teachers' first practicum, data on student teachers' experiences on the practicum were collected. On receipt of all this data, the tapes were copied and the written summaries and notes were photocopied in readiness for future use. However, as the students were in my classes, I did not use the data, but stored them in preparation for the next semester. My interaction with the data at that stage, was purely in order to give feedback to students, as this goal was one of the aims of the data collection.

I waited until the second semester, 1993, to request permission to use the data in my research. As students were no longer in my classes, I felt that they were in a position to be able to refuse me access if they so desired. My intention was to dispose of any data from a student who refused me access. No student chose to do so. However, two students had left the course during the first semester and another two during the second semester, so their data had to be discarded as permission to use their data had not been obtained. As a result, the data from a total of 46 students could be used in phase one analysis.

After receiving permission from the participating students, data from the paired interviews were transcribed and put into a form that would be compatible with NUD•IST. Students' summaries, and notes on beliefs and the practicum, have also been put into a NUD•IST compatible form. The summaries have been matched against the analysis of the paired interviews done by myself, in order to provide an alternative review of the data. In this way *triangulation between investigators* (Delamont 1992) is provided as the students themselves were the initial investigators.

Initial analysis of the paired interviews data used the organisational framework provided by NUD•IST. It is interesting to note that the main contribution made by NUD•IST at this initial stage of the study was in the provision of a well thought out framework for categorising data in the first step of theory building. NUD•IST provides a way of creating an indexing system which can be seen as "*a collection of TREES of categories and the categories as NODES in the trees, linked by branches*" (NUDIST 1993, p. 4). See Figure 6.2 for a broad indication of how data is represented.

Note that apart from the heading of Root, the user is at liberty to name the categories as desired; consequently, the titles of the categories shown in Figure 6.2 are titles I have chosen to represent my categories, and are not fixed by NUD•IST.

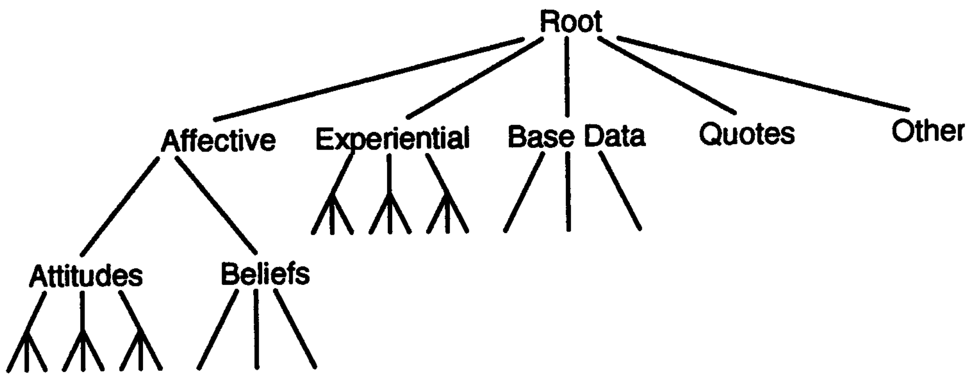

Figure 6.2 : Tree diagram of data

The indexing system for my research study was originally formulated by conceptualising the relationships between codes as an inverted hierarchical tree, with the most general categories at the top of the tree and the most specific categories at the bottom. I did this in the form of a diagram on paper, using the tree structure suggested by NUD•IST, as shown in Figure 6.2. It was this provision of a framework that has been so valuable for emerging theory. However, it should be noted that the hierarchical structure provided by the NUD•IST framework is somewhat limited conceptually. The data from this study would have benefited from a framework which allowed the base data to subsume all other data. From this the major categories could emerge and the minor categories could be seen as being to one side, as Figure 6.3 suggests. Consequently, ways of conceptualising the data, and interrelationships within the data, are influenced by the software package used to analyse the data: as NUD•IST does not have the capability of representing conceptual frameworks in any other way than as a linear and hierarchical structure, development of a conceptual framework is influenced by the limitations of the software

When data from the paired interviews had been transcribed and was ready for analysis, I decided not to code it in great detail but rather to get an idea of any underlying themes so that I would be able to use this information to prepare for my re-entry into the field and the next phase of data collection. Richards (1995) has suggested that it is critical to code as soon as data is obtained and that the first stage of coding can be rapid and general. First impressions are recorded, themes are located and early ideas are stored. This process is similar to the way that I initially analysed my first set of data. I created a tree structure to describe the themes, with general headings and levels moving down as they became more specific. Formulating this on paper was helpful to me, as the diagrams on NUD•IST suffer from a certain lack of clarity if a lot of detail is required.

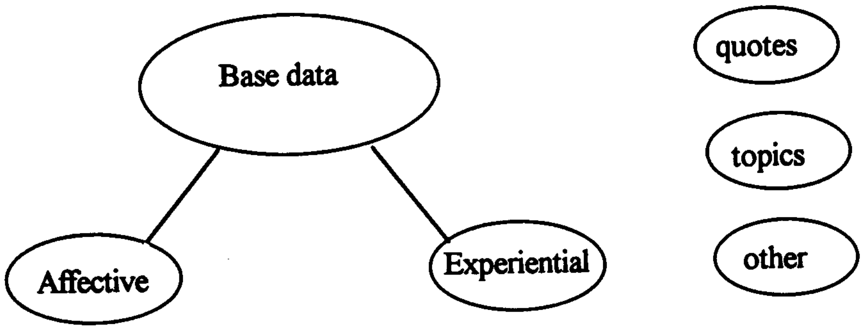

Figure 6.3: Conceptual representation of data.

From my initial coding of the data, certain themes became obvious. For example, students used the term "fun" frequently to describe their conceptions of good practice as mathematics teachers. Silences in the data also became apparent. The silences concerned issues or themes that either I, as researcher, or the research literature concerning this area, believed to be important, but which, in fact, were not seen by some, or all, of the participants as being of any consequence, and so were not discussed at all. For example, not much was said about the need to be knowledgable about mathematics, or about conceptual aspects of learning. I also considered those issues that were deemed to be important by only a small number of the participants. For example, a few students considered the seeing of connections and patterns to be important in mathematics education, but the data indicated that this view was not shared by the majority of students.

Thus, reading the text and formulating categories in the form of trees, as described above, enabled the identification of the commonalities and also silences in the data. This information helped to develop the next stage in my research study, by using the methods of grounded theorising. I used the issues of the first phase of the study to develop the questionnaire to be used in phase two of the study.

6.4.2 Organisation of the analysis of data for phase two.

Data collected using the questionnaire have not been analysed with a view to collecting statistical information. Within the qualitative framework being used in the study, categories were developed that appeared to be significant, either because they were judged to be common to the data collected from different respondents or because they indicated some issue of depth that was of interest or needed further investigation.

As with stage one, the first phase of coding involved reading the responses to the questionnaires and familiarising myself with the data in the questions so that this could help in the formulation of the interview schedule initially used in phase three. However, the entire process of data collection, analysis and posing of further questions was not a linear structure; the first interview schedule was developed in November 1993, when approximately two-thirds of the questionnaire responses had been received. The first interview itself, conducted in December 1993, was instrumental in suggesting changes to the interview schedule, as were later questionnaires, received throughout December, January and February. Consequently, the interaction between the analysis, data collection and generation of questions was complex, as indicated by Figure 6.1, p. 96.

The initial reading of questionnaires did not involve the use of NUD•IST but did require constant comparison with the categories and tree structure formulated in the first stage of analysis of phase one data. The purpose of this initial reading of phase two data was to help in the formulation of questions for the phase three interviews with students.

6.4.3 Organisation of the data analysis for phase three

As with the previous two phases, data were analysed using NUD•IST. Constant comparison was used to assess whether the categories initially developed needed modification, and whether new categories needed to be formed.

The initial analysis of this stage was once again accomplished by thorough reading of the text after it had been transcribed. Personally completing the transcriptions from tape allowed the analysis to begin with the process of transcribing. As I transcribed the text from the interviews, and then later read the text again and again, certain questions arose, all of which were recorded in a separate memo file. Some of these questions influenced the modification of the interview schedule and all the data from phase three were important in suggesting that a fourth phase of data be collected, from the teacher educators involved in the delivery of the mathematics education sequence in the Bachelor of Teaching and Bachelor of Education degrees. For example, a question that arose from the first interview concerned the constraints existing in classroom teaching. This issue was something that the first interviewee (Nita) had seen as being of some importance in influencing how she would teach. Consequently, a question on such constraints was introduced into the interview schedule. In the second interview, an insight into the constraints experienced by students in their first year of university was offered; such constraints were then investigated further in later interviews.

As has been shown in Figure 6.1, the first interview was conducted in December 1993, the next two interviews in early February 1994, and the remaining five interviews were conducted on the last day of February, or within the first week of March 1994. Consequently, the interview schedule had the opportunity for revision twice after its initial construction, in the periods between the interviews. Once again, therefore, the next data collection phase was grounded in the previous data.

The initial analysis of data from phase three was completed by May 1994, and it became apparent to me, that I needed to know what the participating teacher educators' rationales were for the structure of the Mathematics Education 1 subject, in order to assess the match between their rationales and the data from the students. Consequently, I developed a schedule of questions, and interviewed the four teacher educators concerned, in May 1994.

6.4.4 Organisation of the analysis of data for phase four

Data were collected from the four participating mathematics educators during May 1994. They were immediately transcribed and returned to the teacher educators for checking. No changes were made to the data by the teacher educators. Memos were made during the transcription, and stored in an insights document. The data were then stored for further analysis during the open coding stage.

6.5 Open coding

The next step in the analysis of the data was the precise coding of data. This was done using a combination of "open coding" techniques as suggested by Strauss and Corbin (1990) and coding or indexing of text (Miles and Huberman, 1994). As some time had elapsed since my initial development of the indexing system (see Figure 6.1, p. 96), I decided not to use the indexing already created but to start afresh, developing an indexing system as I coded the data. This process was a form of triangulation; after coding all the data from phase one, I had created an indexing system that was essentially the same as the original one developed, thus demonstrating the likelihood that the conceptual framework was an enduring one. Thus, my method of creating an indexing system differed from the earlier method, although the results were very similar.

6.5.1 Open coding of phase one

In this stage of analysis, I went through each document from phase one and coded each text unit. NUD•IST requires that the researcher chooses a text unit as the unit to be analysed; this unit can be a word, line, sentence, paragraph or document. I chose a paragraph to be my text unit, as I felt a paragraph would be large enough to get the context of what was being said, without being too broad to be useful. Each text unit was indexed by the ideas it seemed to contain. In the language of NUD•IST, a reference was added to the text unit at certain nodes in the index system. If a particular node in the index system did not exist, it was created when needed to refer to a text unit. In this way, the index system slowly built up. As I coded, I also recorded in a separate document, any insights I might be gaining.

At the end of each day of coding, I would "tidy up" the index system and assess which nodes belonged together and whether there was a major category that subsumed them. If so, I created the new category. NUD•IST allowed such changes without displacing the references to any of the text. I would also look at whether any nodes showed repetition; if so these were merged, again without losing any of the referent text units.

One of the difficulties that I encountered was that I had a number of different data sources in phase one. These were: the transcripts of the paired interviews, containing students' questions and answers, both of interest; the summaries of the interviews including any implications that students saw arising; the notes on beliefs about learning mathematics; and the student teachers' experiences on the practicum, which I incorporated with their beliefs data into one document for each pair of students. As a result, for this phase of data collection there were 75 documents, each falling under one of three different labels: interviews, summaries and beliefs. To gain comparisons, each document was numbered according to the pair of students contributing to it. For example, the students who interviewed each other in *Interview 15* would have their summaries in a document labelled *Summary 15*, and their beliefs entered in a document labelled *Beliefs 15*.

As I continued using NUD•IST and continued coding the data, constant comparison between emerging categories was executed. At the end of my coding of phase one data, I had a complex indexing system with four major categories and three supplementary ones. These categories then branched into various levels of nodes. In all, there were approximately 270 nodes in existence by the end of the open coding stage of phase one.

6.5.1.1 The indexing system for phase one.

The coding was executed according to a number of themes (see Figure 6.2, p. 104): details about the student were collected in a category called *base data*. The students' beliefs and attitudes were placed into an *affective* component category; and their experiences at school, or during the university mathematics subjects, were collected as an *experiential* category. Other categories at the top level were: a category collecting any *mathematical topics* the students had mentioned; one dealing with views of mathematics from a *student-as-parent* perspective; another category which indexed any *quotes* from the text that I had thought particularly noteworthy; and an *insights* category, which was created to allow the placement, on a temporary basis, of any insights that did not fit anywhere else. These top level categories then branched into sub-categories, which in turn, might have further branches. Figure 6.2, p. 104 shows the diagrammatic representation of some of the categories.

As can be seen, the coding was a massive task. Owing to the ease with which the system can be built on the computer, huge trees with vast numbers of nodes can be created. This is both an advantage and a disadvantage. It allows data to be coded very finely, so that subtle differences in words or meanings can be coded. However, it makes the coding process slow and laborious. Nevertheless, once the data has been indexed, it is better to have too much coding rather than too little, so that the data has the opportunity to speak to the researcher rather than be limited in what it has to offer. Delamont (1992) suggests that as many codes as possible should be generated, and that the data should be coded densely. The ease with which large numbers of codes can be handled in NUD•IST makes this suggestion easy to execute.

6.5.2 Open coding of phase two data

The next stage of analysis involved the open coding of the questionnaires. NUD•IST was again used to analyse the data, but this time in off-line documents. Rather than summarising the answers given in the questionnaire in on-line documents, all coding was done using the questionnaires as off-line documents. NUD•IST lends itself to situations in which the data are contained in off-line documents and the underlying framework of theory emergence and construction is not impeded by this aspect.

In the open coding stage, each question was used as a text unit, and indexed at any of the nodes which were relevant. The titles of the documents were developed to enable searches by person to be as easy as searches by theme.

Coding of the questions and responses in the questionnaire used in this study allowed the amalgamation of categories from this phase and the previous one; certain categories began appearing as important, through study and comparison, and theory thus emerged from the study. A number of new nodes were developed in this phase, for example, nodes about the characteristics of good teachers were expanded, as were methods that students had used in their learning. Other nodes, already in existence, were indexed by many of the documents in phase two, for example beliefs about good pedagogy involving practical work. By the end of phase two, the number of nodes had grown to 306, from approximately 270 in the previous phase. At this point, examination of the nodes suggested ways of reducing this number by thirteen to 293 nodes. This was done by merging nodes with similar characteristics or collecting all nodes that only indexed one or two documents into an "other" node. See Appendix E for examples of the nodes from phase two.

Throughout the open coding of phase two, insights continued to be written in a separate "insights" document, available to be introduced into NUD•IST for coding at a later stage. Any questions suggested by the data were placed in this insights document, for later checking, by using the operators in NUD•IST. An example of such a question was the following one: did people who showed ability at mathematics answer the questionnaire as thoroughly as those who had experienced problems with mathematics. It was in this "insights" document, formulated from memos written during open coding, that theoretical insights began to emerge.

6.5.3 Open coding of phase three data

The coding of phase three data was completed after all data had been collected, and after the open coding of phases one and two had taken place. This was done using the indexing system already set up on NUD•IST in the previous two phases. The interviews were introduced as on-line documents, with paragraphs as text units. I prepared the documents for analysis in NUD•IST by forming paragraphs as appropriate. My questions were not considered important in the analysis, but merely served as prompts to the participants, and so my questions were not coded.

At the beginning of this phase there were 293 nodes in existence. During this phase of analysis I merged the detailed nodes dealing with views of mathematics into one node; this can be done in NUD•IST without any indexing being lost. I also merged views on which skills were regarded as essential in mathematics into a *high-order skills* node and a *low-order skills* node. I further developed some new nodes dealing with the students' orientation to their mathematics education subjects at university; details about their entry to the Bachelor of Teaching and Bachelor of Education degrees; and other aspects of the university course. The node on orientation was used in subsequent categorising of responses to the questionnaire, as well as categorising responses to the in-depth interviews with students. As the interviews were taking place after the completion of the students' first year, I also added in a node on methods of learning, using Marton's and Säljö's (1984) classification of "*deep*" and "*surface*" learning. A few other nodes were added, including one on *students' ways of knowing*, adopting the term used by Belenky, Clinchy, Goldberger, & Tarule (1986).

At the end of the open coding of phase three data, I had just over 300 nodes in the index system. I also had a large number of memos written in a separate file. These captured any insights, questions or connections I had developed during the open coding process.

6.5.4 Open coding of phase four data.

The data collected from the participating teacher educators were a means of providing triangulation within method, as data were collected from both students and teacher educators using in-depth interviews, and the results of these provided information as to whether there was a match between the perceptions of the teacher educators and the students.

The existing categories were expanded to cater for the data collected from the teacher educators. To allow indexing of teacher educators' views on mathematics, the *views on mathematics* category was developed further by attaching "children" or subcategories to it: these categories were taken from Ernest (1989) and Pateman (1989). The categories were *instrumental*, which described a view of mathematics that emphasised the rules and procedures used to gain some goal; *Platonist*, to categorise those people who saw mathematics as a static but coherent mass of knowledge; and *problem solving*, to include those who saw mathematics as dynamic and fallible, and a product of the social and cultural environment in which it occurs (Ernest, 1989). A fourth category was *formalist*, to describe a view of mathematics as a game enacted according to certain rules (Pateman, 1989). These views have been described in greater detail in Chapter 2, Section 2.2, pp. 12-17 and Chapter 4, Section 4.3.2, pp. 54-55. These nodes were used to index the data from the teacher educators in phase four and then were used in the method of constant comparison, to see if any data from phases two or three could also be indexed at these nodes. After considering the data from phases one to three, another subcategory was introduced, that of *logicist*, as someone who sees mathematics as being indistinguishable from logic; logic is applied to mathematical facts to derive new facts (Pateman, 1989).

A few other nodes were added to the beliefs about good pedagogy, and to various other places in the indexing system. Beliefs about learning were expanded to contain the subcategories of *constructivism* and *socio-culturalism*. The category of *topics* was changed to *subject matter*, with subcategories of *topics* (with original indexing) and *processes*. The *processes* node was further divided into nodes describing the processes of mathematics such as *communication, justification, patterns*.

At the end of the open coding of phase four, approximately 330 nodes existed; these were then "tidied up" so that nodes that did not index many units were combined into more general nodes. Appendix E gives examples of the changes in nodes at the end of the open coding of phase four.

Constant comparison methods were then used to check that any nodes formed after the initial tree structure had been developed categorised any relevant data from earlier phases. Nodes from early phases that had not been utilised in later phases were collected into more general categories, and kept for later examination of so-called silences in the text.

6.6 Building theory

On completion of the open coding of all data, both the indexing and the memos were used to construct theory. By looking at questions written during the previous stages of analysis, various searches were indicated. Use of the operators offered in NUD•IST, for example "intersection" or "matrix" allowed hypotheses to emerge, and allowed emerging hypotheses to be tested. Consideration of the ages, or levels of entry of participants, led to the formulation of questions about the influence of age on beliefs, which in turn gave information about the types of past experiences that were associated with certain beliefs. Examination of the views of participants about mathematics led to the development of a theoretical framework concerning views about mathematics, beliefs about learning and teaching, and practice. It was also possible to analyse by person, across methods; this process enabled the data collected from different phases, but pertaining to one person, to be compared.

Consequently, analysis was done on several levels at this stage: analysis by person was done across all collection methods; this procedure enabled me to see whether any development in a person's ideas had occurred over the year; it also allowed examination of the types of responses encouraged by the different methods of data collection.

Another level of analysis was by themes: these reflected the questions that had arisen throughout the earlier coding. They included themes dealing with the use of particular words, and their meanings to the students, for example, "fun"; and also included themes related to students' visions of teaching and learning mathematics.

In examining the various themes that arose, my primary aim was to find text that was rich and deep which would then be used in the discussion. However, as a guide to ascertain how widespread a particular theme or comment was, I also did examine the numbers of participants whose comments had been placed in common categories. I considered the number of times a particular idea was discussed altogether; I also considered the number of mentions of a concept by one person in different documents and the number of documents indexed at a category or concept. It should be stressed that these counts do not have much statistical significance but merely gave an indication of how many participants had mentioned a particular concept, and how many times. From these counts, in a situation where very few students might express a certain idea, I am only able to say that those students who mentioned the concept often in a number of different documents, found that concept to be important. I can infer that the majority of students in such a case did not appear to find that concept important enough to mention in their data, but cannot use the numerical data as evidence of this statement. So the numerical information was only useful in that it indicated possible trends rather than providing a basis for statistical inference.

6.7 Conclusion

The chapter considered the issues that were central in the analysis of this research. These issues included discussion of the rigour of the research and the role that computer software played in the analysis of the study. I have endeavoured to give a full and detailed description of how the analysis was executed in order that readers might be able to replicate such analysis in similar studies and also so that the role of the computer software can be demonstrated to be merely a tool, and not the driving force in the research.

The next part of any discussion on analysis in a qualitative study involves the actual data. In order that readers may make interpretations and check the match of these interpretations against mine, it has been emphasised that the data needs to be available for the readers' appraisal. This is an important aspect of rigour in qualitative research. Accordingly, the following chapters give detailed examples of the data and the interpretations that I have made on the basis of the analysis of the data. The results of this analysis, using searches and operations within NUD•IST, are discussed fully in the next three chapters, in which theory can be seen to emanate and develop. A theoretical framework relating the results to the original research questions is considered after the results have been discussed.

The three ensuing chapters follow a framework that emerged out of the analysis of the data as major themes that arose from the study were those of :

- student teachers' conceptualisations of mathematics;
- student teachers' and teacher educators' views on the two first year subjects in mathematics education at the participating university;
- student teachers' beliefs about the teaching of mathematics in the primary school, and about good pedagogy.

These emerging issues provided the structure for the results chapters. Consequently, the first of these chapters, Chapter 7 considers student teachers' conceptualisations of mathematics; Chapter 8 considers the interactions of student teachers and teacher educators with the first year mathematics education subjects and Chapter 9 considers student teachers' views about the nature of good teaching in the primary school classroom.

7. "A BLACK AND WHITE SORT OF SUBJECT": STUDENT TEACHERS' CONCEPTUALISATIONS OF MATHEMATICS

7.1 Introduction

> *If you think of it in terms of numbers it's a standardised thing, you know, something is either 2 metres ... it's not very large or very small. So I guess it's a more, to me, a more structured black and white sort of subject to me.* (Gail, in phase three interview)

> *You know, each day, you are confronted with problems where you need the mathematical thinking and the logic and so that's probably one of the fundamental things - is the problem solving techniques and the logic that goes with that.* (Joanne, in phase three interview)

> *I'd describe maths as the calculation of certain things to do with numbers and objects and how (pause) how you can work out certain formulas and methods, simplify, how to count and subtract and things like that.* (Maria, in phase three interview)

> *I believed that maths was reasonable; logical. Any means by which students are encouraged to think logically through a problem would be beneficial.* (John, in his summary and discussion of his responses to the paired interview conducted in phase one)

Gail , Joanne, Maria and John are all participating students in my research study. The above quotes are examples of how they perceive mathematics. Each of the quotes seems to indicate a different philosophy of mathematics. These philosophies will be discussed in this chapter, using the descriptions of various philosophical views that were presented in Chapter 2.

It is believed by many researchers in the area of mathematics education that a person's philosophy of mathematics will influence the way they believe mathematics should be taught (Thompson, 1992; Ernest, 1991; Pateman, 1989; Hersh, 1986). This influence will be further discussed in Chapter 9, with respect to the data in my research. In this chapter I wish to explore the dynamic interplay between the different aspects of students' beliefs about mathematics. To examine students' beliefs about mathematics, I shall consider students' beliefs in a number of areas:

- their personal philosophies of mathematics;
- their feelings about, or perceptions of, mathematics;
- their views of the meaning of success in primary school mathematics;
- their views on what mathematical skills are important;
- their opinions on the relevance or importance of mathematics in general;
- their beliefs about who has mathematical ability.

Throughout this interrogation of the data, I shall seek to analyse the role of past experiences in the creation of the above beliefs. I shall also consider whether there is any interrelationship between the various beliefs held by individual students in the different categories listed above. Primarily, this chapter seeks to answer the following research question:

> "What beliefs and attitudes about mathematics and mathematics education do first year primary school student teachers bring into their tertiary education?"

7.2 Personal philosophies about mathematics

In coding the data from students and teacher educators, it became apparent that six major philosophical views about mathematics were held. In this chapter I shall focus only on those five philosophies held by student teachers; the philosophical positions of the teacher educators will be discussed in Chapter 8. I shall also discuss those positions about which student teachers were silent; that is, philosophies of mathematics that occupy prominence, both in the research literature, and in my own personal philosophy, but are not philosophies held by any of the students. I have distinguished seven positions that are relevant either because students are located in one, or more, of these positions, or because of the silence in the data from student teachers, regarding a particular position.

The seven positions can be located as belonging to one of two camps: *exogenic* and *endogenic* philosophies. The first position is based on a view of mathematics that considers mathematical knowledge to be composed of objective and absolute truths transmitted from the outside world, and considers the learning of mathematics to be subject-centred; the second camp views mathematical knowledge as constructed by individuals, hence dependent on the organism's processes, and the learning of mathematics is therefore seen as learner-centred and based on the experiences of the learner (Dengate & Lerman, 1995). In *exogenic* philosophies, mathematics is already determined; there is no place for creation; it is context free and value free. At the other extreme, that of *endogenic* philosophies, mathematics is created by humans, and has meaning only within a particular context. I have placed the seven relevant positions within these two categories as follows:

Exogenic philosophies of mathematics:

- The **instrumentalist** position: mathematics is a toolbox of disparate procedures, rules and formulas to be used to pursue some goal; there is no perceived coherence or structure within mathematics.
- The **Platonist** position: mathematics is a static body of knowledge that describes the external reality. It has internal coherence and structure but is independent of human endeavour.
- The **logicist** position: mathematics is logic, derived from certain basic axioms which are dictated by an external reality.
- The **formalist** position: mathematics is a game with mathematical objects as the focus and rules determining how these objects are used. There is little, or no,

connection between mathematics and an external reality and the usefulness of mathematics is irrelevant. Mathematics is an intellectual challenge.[1]

Endogenic philosophies of mathematics:

- The **problem-solving** view of mathematics: mathematics is dynamic, and fallible and is problem driven. Mathematics is created by humans to solve problems.
- The **constructivist** position: The individual constructs mathematics. Mathematics is a mental activity evolving from the world as **cognised,** rather than some external "reality". It is a psychological phenomenon.
- The **socio-cultural** position: mathematics is created within the context of a particular cultural group or society and reflects the cultural norms and needs of that culture. It is a sociological phenomenon.

Each of the student's quotes given at the beginning of this chapter indicates a particular view of mathematics: Gail's response emphasises the "black and white" nature of mathematics; an answer is either right or wrong. The structure of mathematics is clearly determined and is not negotiable. This response indicates a Platonist view of mathematics.

From Joanne's quote, it appears that Joanne sees mathematics as being relevant to the solution of problems; it is a way of making sense of the world. The logic and mathematical thinking exist to help us solve problems. This quote is an instance of a problem solving view where mathematics is seen as a process of inquiry.

Maria seems to have the most disjointed view of mathematics; to her mathematics is a set of strategies to calculate various amounts and to use a variety of formulas. Maria seems to hold an instrumentalist view of mathematics where a disconnected set of formulas and procedures are seen to exist in order to carry out various computations.

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[1] It should be noted that formalism is not strictly exogenic as it does not follow a model of knowledge that suggests that knowledge is determined by the outside world. However, its elements of absolutism make the placing of formalism in the exogenic category more appropriate than placement in the endogenic category. Formalists consider mathematics as static and absolute; once the rules are determined the game is fixed. Formalism is subject-centred, and focuses on the processes of mathematics not of the learner. Hence its placement in the exogenic category seems appropriate.

Finally, the quote by John indicates a logicist view of mathematics. The value of mathematics lies in the encouragement of logical thought. Mathematics is logical and, as such, is of value as a way for the individual to practise logical thought.

Of what consequence are these different views of mathematics? I shall show in this chapter that the view held by the student is strongly linked to the following four areas:

- the student's feelings about mathematics,
- the student's beliefs about the usefulness of mathematics,
- the student's attitude towards mathematics,
- the student's view of who can do mathematics.

Cobb (1994) has suggested that mathematics learning should be seen both as an individual construction and a process of enculturation. Similarly, my data suggest that each of the subcategories above is not exclusive; people may have a dominant philosophy of mathematics but may also hold other philosophical views, depending on context. This notion will be discussed later in the chapter.

It must be noted here, that philosophical questions about the nature of mathematics were not considered of great importance to the students, and in phase one, no students posed questions which dealt with this topic in any way. For most students, the nature of mathematics was a given, and therefore, the questions developed by students, for the paired interviews in phase one, did not focus on this aspect of mathematics. Consequently, data on the philosophical nature of mathematics were collected principally from the responses to the question I asked in phase three interviews: "If a Martian landed on Earth, how would you describe mathematics to this creature?".

The most commonly held philosophical view was the **instrumentalist** view. Eight students held this view exclusively. Another three students, whose dominant philosophy was not instrumentalist, made comments that indicated some aspects of instrumentalism in their thinking. Examples of their thinking will be given in the discussion on Platonists as, interestingly, all three students appeared to hold philosophies that were predominantly Platonist.

The next viewpoint that was represented frequently was the **Platonist** view. Four students appeared to hold this view of mathematics exclusively. As has already been mentioned, another three students also showed instrumentalist views. These students are regarded as being predominantly Platonist, as the majority of their data indicates this view. The following discussion illustrates their positions.

A quote from Gail that illustrates her Platonist outlook is given at the beginning of the chapter. All her other data on her philosophy of mathematics indicate a Platonist viewpoint, in which she describes mathematics as being highly structured and absolutist in nature. The exception is her answer to a question from the questionnaire in phase two. This question asked students to describe what a primary school student, who is good at mathematics, knows or can do. Gail's response was the following:

> *That the child has a facility to (a) either manipulate numbers or (b) to reproduce correct answers.*
> *Children have been known to "be good at mathematics" yet when confronted with problems requiring the implementation of concepts, display an inability.*

The first part of her answer clearly indicates a view of mathematics that requires ability to do procedures without needing to have any conceptual understanding. This is an instrumentalist viewpoint. Her second paragraph qualifies the response: Gail indicates that she is aware of the paradox , commonly occurring at school, of students being regarded as good at mathematics if they can perform the necessary algorithm, without knowing how to apply that knowledge.

Pip is classified as a Platonist because she sees mathematics as being a highly structured body of knowledge, which must be learnt in stages. The following quote illustrates this classification:

Mathematics is a learnt thing at different stages and if you miss a stage it's very hard to do the next stage, you've got to have all the stages to put it together. (Interview, phase three)

Her comments reflecting instrumentalism are interesting because they could be interpreted as being part of a Platonist viewpoint: this point will be discussed at the end of this section, when a case study is made of Pip's philosophy.

Terry appears to be predominantly Platonist because:

[mathematics] is the understanding of numbers and how they can create different patterns ... (Interview, phase three)

It gives rise to discovery and better understanding of the world in which we live. (summary and discussion of paired interview, phase one)

Yet she too, answered the question about good primary school students in a way that indicated that being a good mathematics student does not necessarily imply an understanding of the underlying structure of maths:

A student may or may not actually understand, or just know the methods well.

These quotes illustrate how one context can elicit statements that would not be made in another context. To both Gail and Terry, being good at mathematics was interpreted as being good at mathematical procedures, rather than exhibiting any profound mathematical thinking.

Pip and Terry also appeared to hold other philosophies. Pip's case is discussed in detail at the end of this section. Terry could be regarded as also having some logicist views, as shown by the following quote:

> *It's [mathematics is] good for your brain to get you thinking, understanding, yeah, it's a good learning subject, um logic.* (Interview, phase three)

Few students were either logicists or formalists; this could be because these philosophical viewpoints imply a more sophisticated knowledge of mathematics than most of the students possessed, and also indicate a more intellectual approach to the study of mathematics than most of the students had experienced. Only one student expressed only **formalist** views:

> *Mathematics, to me, has been a subject that I can sit around and do for hours without getting bored, because it is so challenging. There are different things you can learn and different ways of learning it.* (Bernadette, in her summary and discussion of paired interview, phase one)

Two others, while having a different dominant philosophy, indicated aspects of formalism in some of their data:

> *I guess it is satisfying when you get something right ... and the challenge factor of it is obviously a good idea and just the whole techniques of rational [thought] and logic are good things to* have. (Joanne, interview, phase three)

It is interesting to note that Joanne's dominant philosophy is that of a problem solver (see quote at beginning of the chapter). Consequently her understanding of mathematics may be different from those in the absolutist areas. Pip was the other student indicating formalist views.

Two students made comments that expressed a logicist viewpoint, that is, belief in the logical nature of mathematics. Others sharing this view, as well as another, more dominant view, were Terry and Joanne.

> *..But I think probably the most important thing for me [in] mathematics is the logic and the rationale ...* (Joanne, Phase three interview)

The data from Terry, showing logicism, are shown on page 123.

Only two students expressed a **problem solving** view: Beryl and Joanne. Beryl expressed an awareness that many people had a Platonist view of mathematics, and this view concerned her:

> *You've got to make things imaginative and show it's not a set subject like so many people think it is; it can be taught in so many different ways ...* (Paired interview, phase one)

As has already been mentioned Joanne's data also had elements of logicism and formalism in it. However, it is possible that Joanne's perception of logic as being extremely valuable, stems from her problem solving view. By using logic she is able to construct mathematical models that will enable her to solve mathematical problems. Consequently, this view is not in contradiction with her dominant view of problem solving. This leads to a tentative hypothesis that problem solvers create mathematical models to solve problems; and it is in these mathematical models that rules of logic are important - hence problem solvers might well be logicists and formalists as well as holding the dominant philosophy of problem solving.

No students expressed either the **constructivist** or **socio-cultural** views of mathematics or made any allusion to these views whatsoever. It appeared that students had not considered mathematics to be either an individual construction or dependent on cultural norms, in any way. The relationships between the philosophical views that students did express, are represented in Figure 7.1. In Figure 7.1 the arrows represent the links between students' dominant philosophies and other views they might hold. Strongest links appeared to be between the Platonist and instrumentalist views: the three arrows indicate that three Platonists had some instrumental views. It is interesting to note that none of the students with instrumentalist views shared any other view; that only students in the Platonist and problem solving groups also shared aspects of other philosophies. I shall use Pip's case study to develop a hypothesis that has arisen as a result of these links: namely, that where Platonism is the dominant philosophy, expressions of other philosophies might well be expressions of Platonism instead.

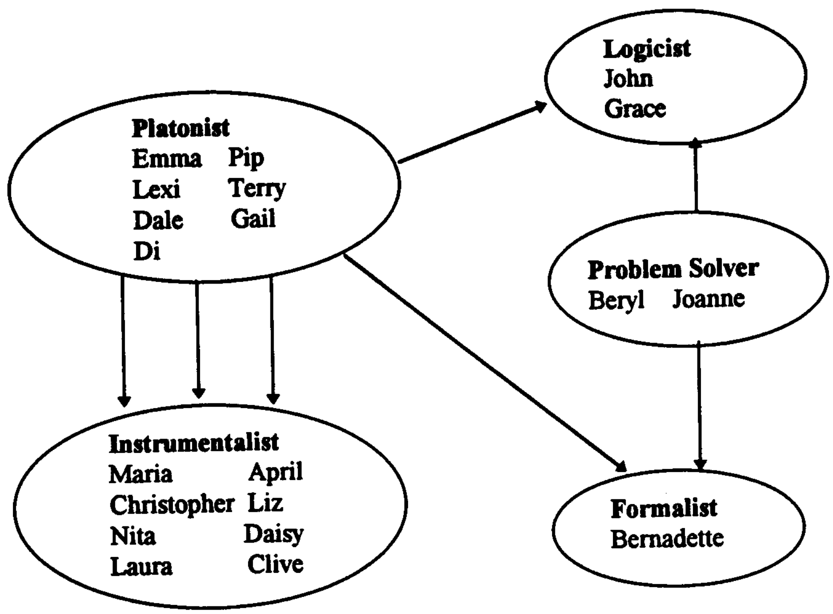

Figure 7.1: Relationships between philosophies

7.2.1 The case of Pip

Pip is a mature age female student who participated in all three phases of data collection from students. Pip had a "nervous breakdown" approximately eighteen months before her first year in the teacher education course. This "breakdown" resulted in a complete blackout of all previous experiences. As a result, Pip felt she was approaching the study of many subjects, including mathematics, as if she had never done them before. She was able to perform basic algorithms but the notion of new, flexible or creative work in mathematics disturbed her greatly. Pip was chosen for the phase three interview because she fitted the category of a mature age student who had not reflected much about her learning or teaching. The reasons for this lack of reflection became clear in the phase three interview when Pip discussed the difficulty she had in thinking about an experience, or writing in a reflective journal.

> *You've got to question and I don't know how to ask questions and I don't know which are the right questions to ask. ... I've got to get back to that level again*

where I question, I can't think of questions to ask. That's why the journals bothered me, I don't know what to write in the journals, I still don't know what to write. Twelve months later I still have no idea what we should write in the journals ... (phase three interview)

Pip was first and foremost a Platonist, as she believed strongly in the hierarchical nature of mathematics, and saw mathematics as a highly structured body of facts. A quote by her on p. 122 shows this belief. However, Pip also showed aspects of other philosophies in her data. For example, some of her data seemed to indicate that she was an instrumentalist - when talking about her experiences at a college of TAFE (Technical and Further Education), she explained how bewildering mathematical facts were to her:

It was really bamboozling. This is why I didn't take mathematics at TAFE, statistics or that, because I thought that there was no way, you know, that I could do it. (Interview, phase three)

However, seeing mathematics as a maze of confusing facts might not indicate an instrumentalist view at all, but rather a Platonist view. It could be the view of someone who believed that mathematics was cohesive and structured, but that she was unable to see the structure because of inability, or past experience; rather like an ant walking over a large framed picture, knowing the picture was one whole, but being unable to see that whole from the ant's perspective. I believe that Pip's opinion that mathematics was "bamboozling" could have stemmed from her view that mathematics was highly structured and that missing any part of the structure led to an inability to understand or execute any other aspect of it. Consequently, what appears to be an instrumentalist remark could have been a comment from a Platonist viewpoint, if the entire context were considered.

Another remark that Pip made, that could be considered instrumentalist if looked at in isolation, was one dealing with the learning of the topic of area: Pip was talking about how she had prepared for the entrance examination that mature age entrants were required to sit, to gain entry into the teacher education course:

> *So I went and got books and that and looking at area and stuff like that just sent me through the ceiling. I had no idea how to do it. So it's only certain things I can do, and ... area's not something I work out, even though I do sewing and that. I just place the pattern down, I know it's going to fit, I know how much to buy. You don't sort of take in the same context when you see the word area, you think "Oh my God, triangles".* (Phase three interview)

On the one level, Pip is saying that when she sees a topic in one context, she doesn't recognise it as being the same topic as in another context - showing a very good example of instrumentalism. At another level, this response again is that of a Platonist who is aware of her own deficiencies. It was Pip and not myself, as interviewer, who suggested that laying a pattern was also using knowledge of area. So while her emotional response was that area was about triangles, she had, in fact, made the connections for herself between the two examples she gave.

Pip also seemed to feel that the formalist approach was a desirable one but that it was out of her reach. Her views about the rigid structure and hierarchical nature of mathematics did not allow her to see mathematics as an entertaining pastime:

> *I think a lot of people like maths, a lot of people do it because they enjoy it, I would love to be able to do it because I enjoy it [laughs] ... I think problem solving and hypothesising and all that is excellent because it's like sitting down to a crossword, you can do the same thing in maths and you can ... you could get a lot of pleasure.*
>
> *I wish I could do it because I think it is exciting how this is the answer, and doing it because you want to do it, and you push it until you get the right answer.*
>
> *I've got this block! I think I have this block [laughs], I have this block that tells me it's maths, ohh! You know?* (Pip, in phase three interview)

Again, Pip's emotional reaction to mathematics highlighted only its rigidity and hierarchical nature, although she seemed aware that there were many other aspects to mathematics.

Pip could also be seen as a problem solver if the following quote is considered;

> *[mathematics is about how numbers interact with our lives] The ways we use it, the different ways it can be used to explore, because it can be very simpl; it can be very intent depending on the need.*

In this quote Pip moves away from the stages that she sees mathematics comprising, to a vision of the use of mathematics. This is the only time she talks about the dependency of the mathematics on the need for it. However, she is not suggesting that mathematics evolves in response to a problem but rather that solutions exist which will be chosen from a pre-existing body of facts, according to need - again a Platonist viewpoint.

A further interesting aspect of Pip's data is that she sees mathematics as being a language. However, where most views of language are strongly socio-cultural, it appears to me that Pip's view of a language is quite mechanistic. It comprises a list of phrases and terminology which all users are required to know. There is no indication of the language growing with its use, or changing according to context.

> *I still find it very hard to differentiate ... the different ... names and the changes and that. ... I got into trouble on prac teaching by not having mathematics language, but I don't know all the mathematics language. There is the language of mathematics and unless you're told what it is, you can't put it into practice and I kept saying, ... I didn't have "biggest" and "smallest" and I was saying the wrong things and I got into strife off the teacher for it. But as I said, I didn't have the language ...*
> *So that to me is something important that I'm going to have to learn, mathematics language, it's not just to be taken for granted that this is how you speak when you do mathematics ...*

> *We've got to be told that it's specifically the language. "The words that I am using are the words you should be using" ...*
> *Oh mathematics is definitely a language. ... And you have to know the language to explore it to its full extent.* (Phase three interview)

So Pip sees the language of mathematics as non-negotiable and independent of context. She wishes to know the exact language that must be used. This view is supported by her experiences on teaching practicum when a teacher criticised her loose use of some words. So language is seen by Pip as another part of the structure of mathematics and is just as absolutist in nature as other aspects. Perhaps "code" or "jargon" would be more appropriate words to use, to convey the meaning that Pip attaches to language.

The picture provided by this case study of Pip's philosophy(ies) of mathematics shows how the anxiety and apprehension, that many people experience when thinking about mathematics, can be due to views emphasising the rigid unyielding nature of a highly structured, absolutist subject. The implications of holding such a view can lead to a sort of paralysis in other areas, and an accentuation of feelings of inability and helplessness. I believe therefore, that for people like Pip, the Platonic view of mathematics is not an empowering one.

The suggestion, that a person's philosophy of mathematics will influence their feelings about mathematics, is investigated further in the next section. Pip's case study has shown an example of Platonism, in which other philosophies which appear in the data can be seen as manifestations of her Platonist outlook.

7.3 Feelings about mathematics

The students also held views about the nature of mathematics from an affective viewpoint rather than a philosophical one. Some of the students perceived mathematics as being boring or difficult; others found it interesting and useful. The following section considers the views or perceptions that students hold about mathematics.

The data about students' general perceptions of mathematics were collected throughout phases one to three; students discussed their feelings about mathematics in their paired interviews and summaries in phase one; in the questionnaires of phase two; and in the in-depth interviews of phase three.

Interesting links were found between philosophies of mathematics and views about the nature of mathematics from an affective viewpoint. For example, all students who held an **instrumentalist** view seemed to find mathematics frustrating and boring. The following quotes all illustrate this view:

> *I really didn't like maths when I was at school, the whole way through. I don't know why, I just think that it was one of those subjects that was difficult and boring and I think that it made that impression on a lot of people. I didn't find it very interesting, I just remember everybody saying, you know "5 x 2 = 10" just chanting and things like that.* (Liz as respondent in paired interview, phase one)

> *Maths can be very frustrating. As a teacher you need to be patient and bear with it. Understand that children need a while to comprehend a new system/way of going about problems. Once it "clicks", then "practise to make perfect" comes into it.* (Daisy, in her summary and discussion of her own responses in the paired interview, phase one)

Students also admitted that they could not see a use for mathematics, although they believed that others would be aware of such uses:

> *It must be useful for something - not somethin I, tied to a computer most of the time, would find handy to me.* (Nita, in phase three interview)

It also meant that they were unaware of any flexibility in the presentation of mathematical content and felt that all content had to be in the form of rules and procedures:

I thought that it was really odd that we were doing an essay in mathematics (laughs). (Nita, phase three interview)

Having a **Platonist** philosophy also appeared to have consequences for the way students felt about mathematics. For example, the perception that mathematics was independent of human thought and was absolutist in nature led to a perception of mathematics as being rigid and unyielding, and it was this view in many cases that appeared to lead to a nervousness or dislike of the subject. The following quotes illustrate this perception:

> *Security is very important, but I guess other subjects you've got a little more diversity of expression, and the way you tackle things. Maths to me was black and white. You were either right or wrong.*
> *In English [in the first year university course] we have to get up and perform ... and people laughed and responded, but mathematics I would never ... Because it's so black and white. There was only one thing, right or wrong. If you divide a number by three you get one answer. And what if you got the wrong answer! I mean, that would show you are so dumb!*
> *I still am confused that one can feel so threatened and intimidated by maths. That's still really disappointing because, um, ... I, I'm really surprised at that.* (Gail, in phase three interview)

and

> *Yeah, I think a lot of it ... um... is very... um ... yes, structured [interviewer suggests word], that's the word I was looking for and I feel the teachers didn't give much leniency and have a, you know, go on outside the borders ...*
> *The majority don't enjoy it because it's very boring.* (Emma as respondent in paired interview, phase one)

> *Because the "right answer" is so precise in maths, I always found classes in the subject more competitive than in the humanities. There was little room or*

> *consideration given for "half right", and no apparent room for creative thoughts.* (Dale in his summary and discussion of his responses in the paired interview, phase one)

Added to this was the difficulty that missing one part of mathematics led to confusion and a belief that it was impossible to continue. Pip describes this problem in her phase three interview:

> *Maths is a learnt thing, at different stages and if you miss a stage it's very hard to do the next stage, you've got to have all the stages to put it together.*
> *You study it because basically we're told we have to, but we don't know the underlying reason why, it's only later, and it becomes something that you don't even reflect on, because it is part of life. You've got to do it in your stages and if you mix your stages up, it makes it very hard.*

Only one student saw the Platonic nature of mathematics in a positive way:

> *Maths stimulates the mind and gives rise to discovery and better understanding of the world in which we live.* (Terry, in summary and discussion of her responses to paired interview, phase one)

Bernadette, who seemed **formalist** in her views, appeared to enjoy the "game" of mathematics. Her quote, given in the discussion on formalism, p. 123, indicates this enjoyment. It has also been shown in Pip's case study that Pip envied people who were able to see mathematics as an interesting intellectual challenge. Because many students see mathematics as being so vital, and yet so inaccessible, the notion of "playing with mathematics" seems like a pipedream.

Finally, both students who had a **problem solving** view of mathematics felt very positive towards mathematics. They felt the problem solving nature of mathematics made it interesting and useful. Consider the following quotes:

I mean, it's almost as if maths has two sides, the logic which is the underlying fundamental maths, or you can see it's a life skill as well and I guess you should be trying to teach that in every subject, not just in mathematics. And then you've got the very practical ... um, addition, subtraction, long division, multiplication, which are the means to solving the problem. (Joanne, in phase three interview)

It's definitely challenging because there are so many different ways that new problems come up and so you've got think of different ways to deal with them, so it's always different - you may give the same problem two years in a row and different people in your class may come up with different ways of solving it and you can't say that it's not right, so it's very, yeah, always different. (Beryl, as respondent in phase one paired interview)

From the above data it seems clear that holding a particular philosophy of mathematics has implications for your feelings about mathematics. Those students with absolutist philosophies of instrumentalism and Platonism, generally seemed to be intimidated by their visions of mathematics; either of rigidity and structure, or of its piecemeal nature which encouraged rote learning. Other visions of mathematics, if conceptualised, seemed out of reach. On the other hand, the two students who held fallibilist views seemed to feel positive about mathematics, and about their ability in mathematics.

7.4 The meaning of success at primary school mathematics

Students were also questioned as to their beliefs regarding the essence of primary school mathematics. The questionnaire of phase two was the vehicle for ascertaining this information. One of the questions in the questionnaire asked: "If a primary school student is good at mathematics what does that mean the student knows or can do?" The answers to this question were collected into three categories, which I arranged as a hierarchy, because of the increasing mathematical power that the views of success at mathematics offered. The categories, from lowest level of the hierarchy to highest are given on the next page:

- **Level 1** - A belief that students who are good at mathematics can ably perform procedures and routines; no understanding is necessary to ensure success at mathematics.
- **Level 2** - A belief that students who are good at mathematics have a sound understanding of the procedures and concepts involved in primary school mathematics.
- **Level 3** - A belief that students who are good at mathematics not only understand the concepts, but are able to apply them.

In the data, each level of the hierarchy subsumes the previous levels; student teachers who believed successful pupils understood mathematics, also stated that such pupils could competently perform procedures. Similarly, students who believed successful students could apply mathematics, also believed that these pupils could execute procedures and understand concepts.

Again silences existed in students' responses, in that all data fell into these three levels. Further levels indicating that successful pupils could identify patterns and form generalisations, or that successful pupils could use processes of justification or proof, did not arise from the data.

The links between the first category and the instrumentalist view of mathematics were strong: both emphasised the procedural aspect of mathematics, and an emphasis on rote-learning. In fact, the instrumentalist view contained the rote learning view within it; all students who held the view that successful pupils could do procedures without needing to understand the underlying concepts seemed to display an instrumentalist view of mathematics. However, students whose quotes belonged to the other two categories dealing with beliefs about successful pupils could hold instrumentalist, Platonist or problem solving views; the data did not imply that there was a one-to-one correspondence between the categories on beliefs about successful pupils and the categories on philosophies of mathematics. Figure 7.2 shows these interrelationships.

	Instrumentalist	Platonist	Problem solver
Level 1	Successful pupils know how to carry out procedures correctly		
Level 2	Successful pupils understand disjointed areas of mathematics and can do procedures	Successful pupils understand and can see the underlying structure of mathematics.	
Level 3	Successful pupils can apply mathematics to low level problems	Successful pupils search for existing solutions to problems - apply	Successful pupils pose problems and create solutions - apply

Figure 7.2: Interrelationship between philosophies of mathematics and beliefs about success in mathematics

Figure 7.2 illustrates the way in which the data on beliefs about successful students are linked to the data on philosophies of mathematics. Students with level 1 beliefs all fall into the instrumentalist category; students who have level 2 beliefs about successful pupils, (ie that pupils understand mathematics), are instrumentalist if they see pupils as understanding unrelated and disconnected sections of mathematics. They are Platonist if they see the pupil's understanding as an awareness of the logic and underlying structure of mathematics. Finally students who have level 3 beliefs are instrumentalist if they believe that pupils can apply procedures for the achievement of a low level goal; these students would see mathematics as a set of tools available to achieve certain ends. Students at level 3 are Platonists if they see mathematics as being a coherent and structured body of facts, that are predetermined. These facts can be applied to solve problems. The student at level 3, who is a problem solver, is a student who first poses a problem and then creates ways of solving it.

The three students whose predominant philosophies were either formalism (Bernadette) or logicism (John and Grace) all saw successful primary school students as those able to execute procedures, understand concepts and apply the mathematics to other areas. Consequently, these three students were placed in level 3 of the hierarchy. In the case of Bernadette, this again demonstrates how a particular context elicits a particular response, as Bernadette had not discussed in her other data, or even appeared to consider, the uses of mathematics. It was also surprising to me that the two logicists should be classified at level 3, as their views had been that the processes of thinking logically were of major benefit in developing thought rather than applying the mathematics to other contexts. However, in another way, if logicism and formalism are considered as sophisticated views of mathematics, then the classification of students as level 3 could be interpreted as seeing mathematics as a powerful tool, a view consistent with their philosophies. The following cell could consequently be added to Figure 7.2 above:

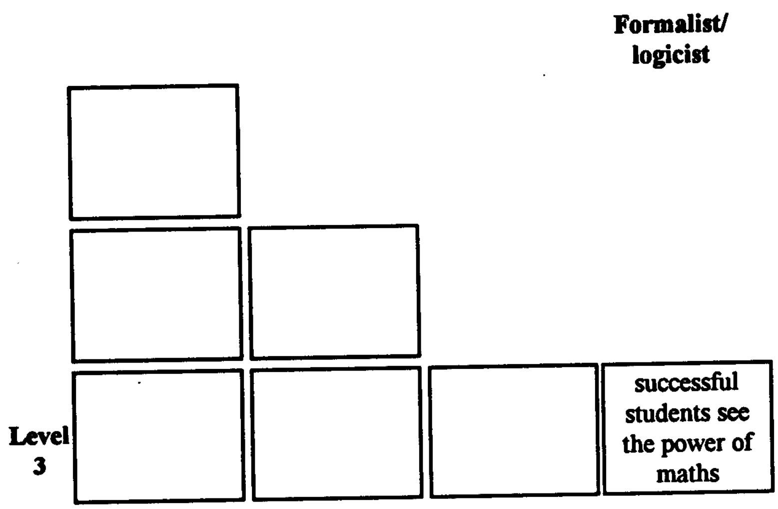

Figure 7.3: Modification of Figure 7.2

Some examples of quotes from the three different categories of beliefs about successful pupils are given below. The most likely philosophy of mathematics matching these quotes is also given. The data from which these quotes are taken were gathered from responses to the question:

"If a primary school student is good at mathematics, what does that mean the student knows or can do?"

Level 1 Responses: executing procedures: (all instrumentalist as they emphasise the doing of procedures without necessarily having a vision of the connectedness of mathematics):

> *The students know basic mathematics, have a good memory for drilled work and can achieve at their age level or better.* (Nita, phase two)

From my point of view, a student who is good at mathematics can do it. But that doesn't always mean they have a decent understanding of it. (April, phase two)

The student can competently complete certain mathematical tasks. He/she may not necessarily grasp the various concepts, having learnt by rote. (Laura, phase two)

The student can do adding, subtraction, basic multiplication, division and fractions without any difficulty. (Christopher, phase two)

Level 2 Responses: Executing procedures and understanding concepts:

It could mean the child understands the underlying meanings of the numbers and facts to be able to manipulate them (Agnes, phase two) - instrumentalist as she does not indicate any awareness of a coherence and structure of mathematics but emphasises the manipulation of disparate facts.

[The student] understands the concepts and procedures of a particular area - instrumentalist as disconnected areas of mathematics are known; connections to other areas are not mentioned. (Robyn, phase two)

[The student] can use principles and methods to get correct answers [and] understands underlying meaning - instrumentalist as emphasises the methods of getting answers, rather than the underlying structure. (Emma, phase two)

They may know the basic principles in mathematics, have a sound knowledge of the concepts , enjoy the subject (Lexi, phase two) - Platonist as the statement implies an awareness of a pre-existing basic structure of mathematics. "The basic principles" implies that these are predetermined.

Level 3 Responses:

It means that the student understands concepts and procedures and is able to use them in problem solving (Sally, phase two) - instrumentalist as does not imply any vision of mathematics as a connected body of facts, or as a dynamic field of knowledge. Various concepts and procedures exist and must be understood and used.

> *If a child is good at mathematics it means they are able to understand concepts of mathematics and relate them to practical applications and everyday situations* (Christine, phase two) - instrumentalist, as for Sally.

> *It means that they understand the fundamental concepts of mathematics, and can apply these concepts in both theoretical and practical situations. They must also be able to reason what they are doing* (Camilla, phase two) - Platonist as is aware of the existence of a basic structure of mathematics, which incorporates reasoning. The definite article implies that there is a basic structure containing fundamental concepts.

> *To be good at mathematics, the student must know the concept behind the procedures and be able to apply these concepts to a range of new situations* (Beryl, phase two) - problem solver as will use the mathematics to advance knowledge in new areas. Beryl has been identified as a problem solver in earlier discussion.

It should be noted, however, that a difficulty exists in trying to ascertain a person's philosophy from only one response and so other information (if available) about the person's dominant philosophy was used to confirm the philosophy given here. Of the 30 students from the original cohort submitting questionnaires, six students gave answers which could be considered as level 1 responses; seven responses were classified as belonging to the level 2 category; 10 students gave responses dealing with applications of mathematics learnt. Six students did not answer this particular question. One student did not seem to fall into any of the categories as she did not answer the question directly. Her response is shown below:

It may mean the student has average ability. Or it may mean the student has major potential. It does not necessarily mean the child is bright or slow. (Alice)

Closely allied to beliefs about what kind of activity was undertaken by successful primary school mathematicians, were beliefs about the sorts of skills that comprised mathematics. The next section discusses data on this topic.

7.5 Beliefs about the skills comprising the study of mathematics

Data on mathematical skills were collected throughout the first three phases of data collection. Students saw the type of mathematics to be learnt or taught as an important issue, and so many asked questions about these skills in the phase one interviews. Some of the responses to questions in the phase two questionnaire were also pertinent. Many responses were gleaned from the phase three in-depth interviews, by questions probing the uses of mathematics.

In the analysis of the responses on skills, I divided data into two categories or levels: **low-order skills and higher-order skills**. Examples of each are shown below. I have called skills low-order if they comprise one-step reasoning, recall of facts, or are closely linked to procedural mathematics. This includes what I call "bread and butter" mathematics; that is, mathematics used to calculate one's change or add up the cost of a number of items in shopping; balancing a budget; reading a train timetable and measuring lengths of timber. Most of the mathematics used for such tasks involves one-step calculations or direct applications of formulas and procedures. Consequently, I have classified such mathematical skills as low-order.

The higher-order skills in mathematics comprise the processes of mathematics used in problem-solving; in justification of solutions; in the critical analysis of mathematics used in the media or for advertising; in the development of creative methods of solution; and in the search for patterns and the subsequent generalisations.

Most of the responses given by the participating students fell into the low-order skills category. The use of mathematics for daily life was a strong theme with the students; in fact, the mathematics seen by these students as essential, was usually arithmetic. The use of calculators was decried by some of the students, as they believed this usage replaced the essence of mathematical thought. Consider the following quotes:

> *Perhaps the most significant thing is that I belong to a generation who can survive without a calculator and I think that that's really significant because when I go shopping today at least I can tally up my grocery bill and be amazed that the kids at the till have to work out the price of two apples so I think that that's probably the most significant thing.* (Gail in phase one paired interview)

This view is important to Gail and is repeated in her phase three interview:

> *When I see shop assistants who can't ... who have to use a calculator to work out what two apples cost, I say to myself, well, I can't see the logic of ... [not learning tables by rote].*

Other quotes illustrate the "bread and butter" nature of the mathematical skills that students felt were the substance of mathematics.

> *I mean, some people study it for the interest of maths and how things work but um, normally you'd use it in everyday life and things like that, with accounts and stuff... Yeah, jobs and everyday life, just simply adding up something or working out a budget, all different ways.* (Maria in phase three interview)

> *[Mathematics is used] mainly [for]calculations and things like that..(pause) things that are going to be used for you to make ... make judgements, you know, to make approximations, on anything, not just length or anything like that but time and all that sort of stuff, even ... When you walk around the house; for um sort of timetables and do things like that, maths helps you there, just ... mainly calculations is what I perceive it as. Oh, only, most of the things that I can think of are calculations, like budgeting and things like that. Mainly practical, a lot of practical things.* (Aaron in phase three interview)

For counting, money uses, socially uh, when we buy things or exchange things, barter, um even just making phone calls, anything, you have to know numbers, you know, how much you have of something or what you need. (Mandy, phase three interview)

As a result of this view of the importance of basic procedures of arithmetic, students emphasised the need for the primary school mathematics curriculum to develop the basic skills of arithmetic, and for pupils to learn multiplication tables by rote, and to be able to perform all algorithms using pen and paper rather than calculators.

And mental arithmetic books, I haven't seen any of those around for a long time but I found those really good, where they couldn't use their calculators and they couldn't use ... had to use their mind and had a certain time to do it. (Aaron, phase three interview)
I always believed that the basics were the most important part of teaching mathematics. (Aaron in his summary and discussion of his responses to paired interview, phase one)

Yeah, I think that tables are important as a basis of maths, and a lot of maths are based on them and once you've sort of learnt them you don't forget them; I haven't. (Mandy, in phase three interview)

To get on in the world, particularly with money, you need to be able to understand at least the basics: multiplication, division, adding and subtraction. You need to be able to deal with money and just to deal with ordinary everyday things. (Sally in phase one paired interview)

It is absolutely necessary for maths to be taught in primary school so students can grasp the basic concepts of the subject. It is almost essential to have to know your "times tables". (Grace, in phase one paired interview)

[I consider the effective learning of primary school mathematics to be] a working knowledge of concepts and an introduction and mastery of basic skills eg addition, subtraction, multiplication and division. (Joanne in questionnaire, phase two)

The students considered that the execution of basic algorithms developed thinking and improved the mind. This, again, was in contrast to the use of calculators which made the mind "lazy".

I think in primary school I always thought it was important to learn how to do things on paper. To be using their minds. (Mandy, in phase three interview)

I think maths uses, like involves your mental capabilities so much. Whenever you are working out a sum or working out some calculations you're using your mental ability to work out the sum and that's using a particular type of thinking. It's promoting, it's involving your thought processes and that's important, especially for a young child, during primary school. You can start them off in a positive direction, it will help them towards their future and ... (Christopher in paired interview, phase one)

Few students saw higher-order skills as being the essence of mathematics; this could be because they were focussing on primary school mathematics, which most considered to be the basis for higher mathematics, and therefore, essential knowledge was that of basic skills, such as the four operations in arithmetic. However, a few did see mathematics as substantively comprising higher order skills:

I think it's the understanding of numbers and how they can create um, different patterns.
It's good to for ... it's good for your brain to get you thinking um, understanding, yeah, it's a good learning subject, um, logic. It makes you think and work out problems and that's good for children to be able to do that.
(Terry, phase three interview)

> *Oh, maths is definitely a language. Yes, I think it's ... and you have to know the language to be able to explore it to its full extent.* (Pip, phase three interview)

> *How would I describe it? Um, [pause] I mean, obviously maths is to do with numbers and to do with problems or problem solving. Numbers are a means by which we solve the problems. It's hard to just say what maths is as ... in essence. I'd say very generally it's problem solving ... and things like addition and subtraction and that sort of thing all are means by which problems are solved, numbers are a means by which problems are solved, same with algebra, I guess ...* (Joanne, phase three interview)

> *I feel that maths is an extremely important component of the primary school course. It teaches the students the basic, fundamental aspects of maths in the outside world. It also advances logical thinking and problem solving.* (Di, in her summary and discussion of her responses in the paired interview, phase one)

As can be seen from these responses, the view of higher order mathematical processes is somewhat limited. Students talk about logic and problem solving. Few other aspects of higher order processes and skills are mentioned. Such omissions are quite possibly due to students' past experiences in mathematics, in which only limited visions of higher order mathematical skills have been presented to them. This possibility will be investigated further, later in this chapter.

Some of the quotes did not fit neatly into one of the two categories suggested above as the context was important in determining what the student meant. These quotes demonstrate the difficulty, in some cases, of classifying mathematical skills as low order/higher order:

> *I'll either estimate or do it in my head. If I need to find out the exact amount I'll do it in my head but otherwise I just estimate it.* (Maria, phase three interview)

> The use of "estimate" could simply mean a wild guess, or it could involve mathematical processes of approximation by using sophisticated mental computational techniques.

Another example in which the context would determine what kind of skill was being discussed was the following:

> *I believe that written methods are necessary to give children a basic knowledge of mathematics functions.* (Bonny, in notes on how students best learnt mathematics, phase one)
> Is the word "function" being used loosely to describe any sort of mathematics or does it mean a relationship between variables?

The above quotes illustrate the complexity of analysis of text and the necessity to consider the context in which statements are made.

7.6 Beliefs about the relevance and importance of mathematics

> *You know we're not talking about sewing, we're talking about maths. That's a foundation subject for the rest of their lives, so I think it's very crucial.* (Gail, phase three interview)

The importance of mathematics in everyday life appeared to be an issue in which many students showed interest. In the phase one interviews a great many questions were posed about this topic. Mathematics permeated the lives of the students.

> *... because society can't exist without it, our whole negotiation of price, buying, our medical world, every aspect of life is actually touched by mathematics.* (Gail in phase three interview)

> *Maths is the bones of the skeleton. Without maths the child cannot accumulate other knowledge. We need maths to bind together the everyday things ... without*

maths we probably would fall apart. (Pip in response, paired one interview, phase one)

The perception of many students was that knowledge of mathematics was crucial for effective day to day living. However, many of the responses to questions, asked either in the paired interviews of phase one or the in-depth interviews of phase three , indicated that the sort of knowledge required was that of basic skills; the "bread and butter" mathematics discussed in the previous section. The following quotes comprise questions asked by students in phase one interviews and responses given both in phase one and phase three interviews:

> *Mathematics is very useful in our every day life and often children are unaware of the benefits that mathematics skills provide.* (Adi in summary and discussion of her responses to paired interview, phase one)

> *Aah, yes I think it's sort of necessary. It's used in everyday life. I think it's vital to do, and I think it's necessary to start at a young age.* (Terry, in phase three interview)

> *Well, I used to think maths was a useful and interesting subject especially in the junior years because the concepts that were taught were the basic skills such as fractions and arithmetic which often are essential for everyday life.* (Terry in phase one paired interview)

> Question: *Do you believe maths plays a major role in society?*
> Answer: *Yes, I do feel that maths plays a major role in society as today most places require the simple knowledge of how to add up and how also to use computers.* (Question by Ruth and answer by Dawn in paired interview, phase one)

> Question: *Why is maths important?*

Answer: *I think that knowledge of maths is an essential part of life because it's used in that and it can be used for counting measures, telling time, multiplying, everything ... you can't manage going through life without knowing some mathematics.* (Question by Thea and answer by Mandy in paired interview, phase one)

Question: *When do you consider maths to be important?*
Answer: *Everyday, I think we use maths in some way to calculate just about something everyday so I mean, just using money or just calculating things ... we use it everyday.* (Question by Mandy and answer by Thea in paired interview, phase one)

All the quotes given above illustrate a view of mathematics as comprising low-order skills, and it is these skills that are considered as fundamental to life. As a result of this view, primary mathematics is seen as being far more vital than secondary school mathematics - it is in primary school that all the "fundamental mathematics" is learnt; this is the mathematics that is then used in everyday life.

I think a lot depends on whether you're um, how much you need to learn the mathematics. Like if you've got to a stage where the thing that you're having trouble with doesn't really matter to you, or you think it doesn't matter, then you're not really going to try and get over it.
I was in primary school and just about everything counted then, you couldn't really get much further without learning the next bit. (Aaron in phase three interview)

... I have a positive attitude towards maths because I think it is a really essential subject for the primary school curriculum and it helps you learn basic skills that need to be used throughout life ... (Terry in response in paired interview, phase one)

But primary school maths is the basis of everything. It's the most applied maths you ever learn so it's vital that you learn it then and learn it correctly. (Amy in response in paired interview, phase one)

In contrast, high school mathematics is seen as being fairly irrelevant:

... a lot of maths doesn't seem to apply. I mean, there's maths that lives forever ... that a lot ... I've never used calculus since I've left school so ... and we used logarithm tables and we had little old thin books and ... but I mean, they're still using logarithms but how do you use that when you leave school? (Nita, phase three interview)

The things like calculus and algebra, I didn't understand how I would need these in my later life, unless you were going to do a maths degree or anything. (Doreen, response in paired interview, phase one)

Having studied 3/4 Unit Maths in senior school I know that most of what I learnt will be totally irrelevant in my future career unless I decide to head towards the area of maths in my tertiary studies, which at this point in time is unlikely. (Grace in phase one interview)

I think that primary maths is very relevant to your life but I think that once you get to high school, maths there is so advanced that you would just never need it. (Amy , in response in paired interview, phase one)

Having done two unit maths last year, I don't think I will ever use calculus, but all my basic operations (+ - x /) are very important in everyday life. The practical applications of maths learnt in the primary school are endless and need to be taught thoroughly to provide a good understanding. (Bernadette, in her summary and discussion of responses to paired interview, phase one)

So basic arithmetic and other low order skills are seen as essential, whereas the processes of higher mathematics, for example, of proof, investigation, verification and deduction are not seen as necessary and are not considered as useful skills for everyday living. This could be as a result of an antiquated secondary school curriculum, or perhaps because the processes of mathematics discussed above are not made explicit to students at high school.

Another function for primary mathematics is to provide the foundation for high school mathematics:

> Question: *Do you think mathematics is essential to the primary school curriculum? Why?*
> Answer: *Yes, I think it is ... and I think fairly obviously it is because the maths is used in the study of the subjects that later on ... all the technical things ... the technical things you want to do after you leave school. So you need to have a very good grounding in maths, even to go on to high school and the basic things that are taught at primary school are very important to that so I think it's very essential.* (Question by Terry and answer by Clive in paired interview, phase one)

> Question: *Obviously maths is a necessary subject for primary school students. Do you think that more emphasis should be placed on teaching maths or less?*
> Answer: *Probably more emphasis on mathematics because the maths we do in primary school is important because we need it. It's like the fundamentals that you need when you go to high school and everything.* (Question by Bonny and answer by Camilla in paired interview, phase one)

> Question: *Do you think that maths is a necessary part of the primary school curriculum and why?*
> Answer: *Yes, because I think if the children don't get a basic idea of maths and the fundamentals of maths in primary school once they hit high school that's it, they're lost. So primary school makes a lot of difference to how you cope when*

you get into high school. (Question by Camilla and answer by Bridget in paired interview, phase one)

Question: *What is the importance of maths in the primary school classroom?*
Answer: *It's very important because it's the beginning of maths for children. They go along to high school and they are just assumed to know so many basic things like multiplication and division and things like that, that others just assume they must know that. And after high school they are things that are built on and on and on, and as primary school is the basis, it's the start and so you must get it right then. Otherwise everything else is going to be aborted.*
(Question by Bernadette and answer by Beryl in paired interview, phase one)

More information on the usefulness of mathematics was obtained from two questions in the phase two questionnaire (see Appendix C). These questions asked the following:

"In the interviews, students talked about "the game of school" where what was learnt in school did not seem to have much bearing on what was known outside the school.
Based on your experiences of school mathematics, please would you comment on this view."

and

"Something else that many students said in the interviews was that they felt that mathematics that was not useful should not be taught. What do you feel about this?"

The responses to these questions provided illumination on students' past experiences and the role these experiences played in influencing their ideas on what should be taught in school mathematics. Some interesting contrasts between the responses to the two questions arose in certain cases. Most students indicated in their responses to the second of the two questions, that they believed that **all** mathematics was useful, even if they, personally, could not see the value themselves. This belief agrees with the view, expressed above, that mathematics occurs in all areas of life and is part of the fabric of everyday life.

The following quotes are responses to the second question. They indicate a view that all mathematics is useful, even if it is not clear how:

"Something else that many students said in the interviews was that they felt that mathematics that was not useful should not be taught. What do you feel about this?"

> *It depends on what is considered useful and what's not. Also, the information that is not considered useful by some, may be very relevant to others. I don't think we can judge that.* (Bridget)

> *All mathematics can be useful. It depends on what career you will take up and at primary school who knows what who wants to do when they get older! It is important then to get a sound base of knowledge and extend that in high school. Even if you don't use it at the time, that knowledge can be used later in life - uni, tech, work etc.* (Daisy)

> *I think all mathematics is useful to some extent and I think, as it is now, it should be essential to year 10 and then a choice given.* (Terry)

> *Mathematics that is "not useful" doesn't exist. All mathematics is "useful", even though the results of its use may not be a 20 storey building or the correct change at the milk bar.* (John)

> *All mathematics is useful in some way. We just have to find it.* (Nita)

> *I believe it is wrong to say that some mathematics is useless. Because all mathematics is relevant in some way. Some of the children in schools will become mathematicians and engineers and it would be rather ignorant for*

teachers to ignore or leave out components of mathematics as it will definitely be very important for some children. (Christopher)

The belief that all mathematics is useful indicated for some students, a way of improving the learning of mathematics. As uses exist for all mathematics, teachers merely need to make sure that students are aware of these uses, and can see the relevance of all mathematical topics.

Most mathematics is useful but as students we just weren't told what it was useful for. So all mathematics should be taught, students should just know what they are being taught and what it does. (Camilla)

Mathematics should be taught, but in an interesting way that highlights the relationship (if any) with real life. (Dale)

Only one student conceded that while all mathematics might **not** be useful, the intellectual stimulation and enjoyment provided by the topic were its justification for existence.

A lot of mathematics that is not useful is enjoyable to a number of students. These students shouldn't be deprived of challenging and interesting activities [that they would be] if these areas were not taught. (Sally)

Another student was dubious about the value of interesting mathematics:

I quite agree with this[that mathematics that is not useful should not be taught]. I think it is more important to spend time on mathematics that children will need and use, than other mathematics. However, some mathematics that isn't useful is interesting and may have a place. (Beryl)

A number of students agreed with the view voiced in the question, that is, that all mathematics that was not useful should not be taught:

I agree strongly with this. Mathematics that is not useful has no point. I believe mathematics that should be learnt should be relevant to later life and experience - what is the point of learning concepts that are of no use once learnt? (Christine)

Agree. What's the point of learning something that you're not going to use! Remembering formulas only to forget them in a couple of months. (Robyn)

This is true. Children need to see what they are learning as relevant and useful so that they involve themselves in the learning process. (Emma)

I think that some of the mathematics should be left to the further education where it is specifically going to be used. (Aaron)

Of the 30 respondents to the questionnaire, eight agreed that mathematics that was not useful should not be taught, 16 believed all school mathematics should be taught (of whom one believed that the mathematics should be taught because of its interest, and the others felt that all mathematics was useful) and two were equivocal about the issue. Four students did not answer the question.

Interesting contrasts were provided by the answers to the first question (see below), for those people who believed that all mathematics was useful. Many of these students painted visions of their past experiences in mathematics in which mathematics was perceived as useless and irrelevant. These visions appeared to lead to a disenchantment with mathematics. Other students described mathematics subjects taken at school which demonstrated the relevance of mathematics, and these students felt far more positive about their experiences. Consequently, having a deep seated belief that all mathematics was useful, and should be taught, seemed at odds with past experiences. The conviction that the relevance of all mathematics must be shown to the students seems to be the unifying link between these two questions. Students feel that the usefulness of mathematics must be portrayed in order for the learning of mathematics to be a positive experience.

Responses to the first question illustrate the contrast between the answers to the two questions; the first question is repeated before responses to it are given:
"In the interviews, students talked about 'the game of school' where what was learnt in school did not seem to have much bearing on what was known outside the school. Based on your experiences of school mathematics, please would you comment on this view."

> *I found this[maths at school had no bearing to our outside lives] to be true in that what we learnt did not seem relevant to anything outside of school. This needs to change for more effective learning of mathematics.* (Bridget)

Yet Bridget had earlier said that what seemed irrelevant to one person might be considered relevant by someone else. So espoused beliefs are in conflict with past experiences. The distinction between values and beliefs made by Southwell (1995) is of significance here (see Chapter 4, p. 50).

And Christopher also seemed to have beliefs and values that were in conflict. While stating that teachers should not leave out any mathematics as it might be important for some of the students in the class, he had this to say about his own experiences:

> *One problem with my mathematics experience during primary and high school was that the work we completed did not seem to bear much relevance to real life in society. The mathematics should have been made more relevant and the questions should have related to society more often.*

Similarly, Terry's answers to the two questions were in contrast to each other. While believing that all mathematics is useful to some extent, her experiences were different:

Yes, school mathematics hardly ever related concepts learnt to outside school experiences - and I think it should - this would be much more effective in learning and understanding mathematics.

Daisy's response to the first question may clarify these inconsistencies. While stating, as her answer to the second question, that all mathematics can be useful, depending on the student's aspirations, she suggests in her response to the first question, that the purpose of the mathematics taught must be made clear:

I agree [that the mathematics taught at school did not have much bearing on life in the outside world] to the extent that in primary I wasn't taught the basics so then it didn't make sense in the outside world. I was only taught the procedures and the question that arose every year was <u>why?</u> Why are we doing this? What is the purpose of doing this?

On the other hand, those students who had been satisfied with their past experiences, and had found them enjoyable and beneficial, showed consistency in their responses to the two questions: Those who had found mathematics to be relevant, agreed, in answer to the second question, that only useful mathematics should be taught. Christine is an example; she felt strongly that only useful mathematics should be taught. She found that her experiences of 2 Unit Mathematics in Society (MIS) were practical and relevant, and consequently enjoyable and worthwhile. It is not surprising therefore, that she felt this model of mathematics teaching should be used.

I agreed with this view for my mathematics, up until year 12 where I took up Mathematics in Society. Until then I had always been in Advanced Mathematics or 2 Unit. MIS however, was fun and experience related. We had terrific excursions which motivated me to learn more and essentially I enjoyed mathematics for the first time ever.

Whereas Robyn's less than satisfactory experiences reinforced her beliefs that mathematics needed to be relevant:

> *Agree[that mathematics at school did not have much bearing on what was known outside school]! Year 12 2 Unit Mathematics is so irrelevant! I now believe I would have enjoyed Mathematics in Society more where the maths related to everyday life more!*

Sally's perceptions of her past experiences are consistent with her beliefs about the usefulness of mathematics:

> *Much of the mathematics learnt at school, particularly at secondary school level, didn't have much bearing on the outside world, but I enjoyed most of it.*

The fact that Sally enjoyed her past experiences in mathematics, while perceiving them as irrelevant to everyday life, has led to her holding a different view about mathematics from the other students.

One of the major difficulties that students will experience if they believe that mathematics is extremely important, yet fail to actually see its relevance themselves, is that mathematics becomes an elusive and mystical experience at which not everyone can succeed. Their view of mathematical power becomes restricted to low order skills as this is the only area where relevance is obvious. The result is a marginalised group of people who are disempowered by their views of mathematics as they do not have any access to the potential power of mathematics.

> *Pip seems to acknowledge the importance of mathematics in a child's education, but the thought of complex maths frightens her somewhat.* (Daisy about Pip in her summary and discussion of Pip's responses to paired interview, phase one)

> *Yes, it's important but on a personal level I just don't like maths at all, I never have, I never ever will. But it is important to be taught in education.* (Renee in phase one interview)

Further discussion, on students' visions of who was mathematically empowered and who was not, follows in the next section, which investigates the type of person students considered able to do mathematics.

7.7 People who can do mathematics

A clear distinction arose in the phase one data between "words" and "numbers" people. Three students seemed to emphasise this distinction and posed questions asking whether their interviewee belonged to one or the other group. In response to this question, two students replied that they belonged to both groups, and did not seem to share their interviewer's beliefs about this division. However, in other phase one interviews, many students spontaneously suggested that they were not "natural mathematics people", without prompts about the distinction.

The following are questions or answers from the paired interviews of phase one:

> Question: *Would you regard yourself as a word person or numbers person?*
> Answer: *Actually I'd be both I think because although I really, really enjoyed English at school I was fairly good at maths too so I sort of enjoyed both and so I find myself as both a word person and a number person.* (Question by Gail and answer by Adi in paired interview, phase one)

On the other hand, some students saw a clear division between mathematics and humanities people:

> *Well, I've always thought that there were maths people in the world, like I divided everyone up, see there are maths people and there also English and science people and I ... I decided, you know long ago, that I was an English and science person and maths just wasn't for me. So that's probably affected me because otherwise, specially in high school, I just put my effort into English and science and maths was just a huge burden ...* (Thea, phase one paired interview)

As can be seen, students saw their own perception, of the existence of two groups of people, as being detrimental to their mathematical experiences at school. Others saw mathematics as being for an intellectual elite, which students who were not "mathematically inclined" could not hope to join:

> *... and it was a subject that we, as children, felt was difficult. Maths was difficult, it wasn't a subject for the ordinary student. It was [for] somebody that was really good at something, ... or had a higher intellect, that would do well in the subject. Students with an average intellect, I mean most of us considered ourselves average intellect, weren't really able to grasp the subject.* (Bianca, phase one paired interview)

Another group of students defined themselves as "not being mathematical" because they struggled with mathematics, and did not enjoy it as a result of the difficulties they experienced:

> *Yes, I did find maths to be a difficult and tiresome subject as I'm not a mathematical person. But with all the exercises we were given it was very tiresome and so you got quite bored and because I was quite slow and most of the others could finish before me so I never actually got to finish and so I had to go home and do what I hadn't yet done in class.* (Maria, in phase one paired interview)

They also saw people who were "mathematics people" as enjoying all sorts of experiences that they, themselves, had found boring or difficult. For example, Pip talks about the historical aspects of mathematics that were studied in the first year subject:

> *Yeah, I didn't want to learn the history of maths and all that. If I'd done that I would have been a mathematician [laughs], made that an elective, you know, sort of thing, I just ...* (Pip, phase three interview)

This belief fits with the view of mathematics as being independent of human endeavour. The historical aspects become irrelevant when mathematics is static and unchangeable. Mathematicians were also seen as people who liked numbers and this was also confused with accountancy:

> *I guess it's the pure mathematician who loves numbers for the sake of numbers.* (Gail, phase three)
>
> *No, not overly, they weren't overly ... they weren't accountants or anything.* (Mandy, in answer to whether her parents were good at mathematics, phase three interview)

Most students believed that mathematical ability was something you were born with.

> *I think you've got it or you haven't got it.* (Gail, Phase three interview)
>
> *It's a bit like lateral thinking. I think it's something that you either have or you haven't got.* (Nita, phase three interview)

The students' views of who is good at mathematics has implications, both for their learning, and for their teaching. If only certain people can do mathematics, there is little point in spending much time or energy trying to develop any sort of mathematical power. Low order skills become very important, as these are the only skills likely to be attainable. Further, this view is divisive in that students classify people into the people who can do mathematics and those who cannot.

7.8 Conclusion

This chapter has painted a picture of the beliefs held by student teachers about mathematics. The majority of students appear to be instrumentalist or Platonist in philosophy. Most students see mathematics as being important, but difficult and boring; the essence of mathematics appears to be low order or basic skills. A silence exists as regards socio-cultural or constructivist views of mathematics; and as regards the higher processes of mathematics, such as reasoning, justifying, generalising and showing critical thought. To the majority of students, mathematics is about arithmetic and the procedures used to competently obtain correct answers. These so-called "basic skills" are seen as the essential curriculum of primary school mathematics.

Juxtaposed alongside these beliefs are the feelings that higher order mathematical processes are irrelevant, and that not all students can do mathematics. At the same time, calculators deprive students of the ability to think for themselves. Although many students were disenchanted with their school experiences, both at primary and high school, ironically, the aspects of mathematics which they seem to value at present are precisely those which led to disenchantment.

The following chapter continues the story of these students. The beliefs discussed in Chapter 7 are quite different from the beliefs of the teacher educators and the proponents of the university mathematics education subjects. Chapter 8 develops the discussion on the interaction between the students and the mathematics education subjects in their first year of university. The tensions and conflicts between the students' beliefs and the teacher educators' beliefs will be revealed and discussed.

It has been shown in Chapter 7 that students hold a variety of interrelated beliefs about mathematics. These beliefs influence the way students learn mathematics at university, and will teach mathematics in the primary school. Chapter 8 considers the learning of mathematics at university and Chapter 9 discusses students' ideas about their future teaching of primary school mathematics.

8. STUDENTS, TEACHER EDUCATORS AND THE FIRST YEAR MATHEMATICS EDUCATION SUBJECTS

8.1 Introduction

In the previous chapter students' beliefs about mathematics were examined from a number of perspectives. Students' personal philosophies were analysed to assess the influence of such philosophies on other beliefs. Links were found between exogenic philosophies and negative perceptions of mathematics, and between endogenic philosophies and more open views of mathematics. Beliefs about the substance of mathematics generally centred around low-order skills used in every day living. Silences existed in the data regarding the possibility of mathematics being socio-cultural in nature or comprising processes of justification, critical analysis and investigation.

Armed with these beliefs, student teachers enter their teacher education course. Their beliefs about mathematics lead to certain expectations about the nature of their mathematics education subjects in the teacher education course. Their orientation to university study also places them in certain positions as regards the mathematics education subjects.

This chapter will examine the interactions that occur, during the first year subjects, and with the teacher educators involved in the offering of the subjects. In particular, this chapter will focus on the following:

- The structure of the first year subjects and the history of their development;
- The philosophies and related beliefs of the teacher educators involved in the offering of these subjects;
- The reactions of the students to these subjects, together with students' orientations to study, expectations of the subjects and approaches to studying mathematics.

In particular, this chapter will be investigating the following research questions:

Are any of the students' beliefs about mathematics and mathematics education similar to the beliefs of the teacher educators in mathematics education, and how do students interact with first year mathematics education subjects in the teacher education course?

How do students' attitudes and beliefs influence their success in learning new mathematics at this stage of their lives?

8.2 The first year mathematics education subjects

The first year mathematics education subjects, in the Bachelor of Teaching in Primary Teaching and the Bachelor of Education in Teacher Librarianship degrees at the participating university, comprise two one-semester subjects. In the first semester students have two hours of mathematics education per fortnight, in a subject that is oriented to the primary school. It is part of an inter-disciplinary subject on current approaches to teaching in the primary school, and is one component of four in this subject. As has been noted in Chapter 1, the aims of the mathematics component of the curriculum orientation subject are to improve attitudes to mathematics; to broaden students' views of the scope of primary school mathematics; and to demonstrate or model some of the current approaches to teaching in the primary school. Consequently, at the time of the study, students had an introductory session and then did five two hour workshops, meeting fortnightly to do these. The workshops included a selection of activities on a different topic each fortnight, to indicate the diversity of tasks available on the topic, and the subject requirements included maintenance of a reflective journal, in which entries about the workshops and past experiences with the workshop topics were written. The emphasis in this subject was not on conceptual development, but on the challenging of beliefs and attitudes.

The year of the study, 1993, was the second year that the primary curriculum orientation subject had been in existence and changes had been made to the subject, as a result of subject evaluations by the students. However, as the students had indicated that, in general, the subject was highly valued, the changes made were minor ones.

The workshops dealt with the following topics: An introduction to the K-6 syllabus together with a video showing current approaches to teaching in primary schools; calculators in primary school; LOGO in primary school; space (or informal geometry); measurement; and probability.

Apart from the introductory lecture, workshops were participatory and co-operative; and most of them used a problem solving approach in which discovery and exploration were encouraged; games dealing with various concepts were used, and discussion within the group was of importance. Many of the activities were suitable for primary school pupils at various levels; and all dealt with topics from the primary school curriculum.

As has been noted in Chapter 5, assessment in this first semester was on a pass-fail basis, based on attendance at workshops, and submission of assigned work. The assigned work consisted of three tasks: the paired interviews together with summaries of the implications of the pair's attitudes and beliefs; submission of the reflective journal which had entries from before and after each workshop; and a summary of an article from a professional journal, dealing with one of the workshop topics. No grades were assigned and the task was given a pass grade if it had been completed in a satisfactory manner. Incomplete or unsatisfactory submissions were returned to the student for improvement, and then resubmitted. The subject outline for this orientation subject can be found in Appendix B.

In the second semester, the students started the first of four mathematics education subjects, Mathematics Education 1. Two hours a week of class time was devoted to this subject. In Mathematics Education 1, students covered three major areas of curriculum. The first dealt with learning theories in the area of mathematics education; the next section dealt with the historical development of our number systems and a discussion of the characteristics of some ancient numeration systems; and the last section dealt with topics from number theory. The topics from number theory were all done in the form of workshops with activities to be done co-operatively in groups, with the use of concrete aids where needed. Again, a problem solving approach was used and a variety of mathematical models were suggested to develop new concepts. In this subject, emphasis was on concept development, building awareness of the socio-cultural nature of mathematics, and the seeking of links between different areas of mathematics.

Assessment for Mathematics Education 1 in 1993 comprised four items. The first item was an essay requiring students to research two historical occurrences in mathematics and write an essay describing them, together with some discussion of how they could be used in the primary mathematics classroom. The next was a mid-semester test on the learning theories and ancient numeration systems. Once the number theory section of Mathematics Education 1 had begun, students were required to prepare a group presentation dealing with one of a number of topics from number theory, and present this to the class. Finally an end of semester test examined topics covered in number theory, including the group presentations. Students were given a grade at the end of the semester, which was an aggregate of all the assessments.

As with the orientation subject in the first semester, Mathematics Education 1 had been developed in current format in 1992, and at the time of the study, this was the second year that it was being run. Students had evaluated the subject after the first year and some substantial changes were made as a result of the evaluations; these changes included one of the prescribed texts and some of the content.

The subjects were originally developed as part of a restructuring of the Bachelor of Teaching in Primary Teaching and the Bachelor of Education in Teacher Librarianship degrees. The mathematics education subjects were developed soon after the Discipline Review of Teacher Education in Mathematics and Science (Department of Employment, Education and Training (DEET), 1989) was published, and consequently were influenced by this report. Other documents of the time, that were influential in modifying the nature of the mathematics education subjects, were the National Statement on Mathematics in Australian Schools (Australian Education Council, 1990) and Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989).

From the Discipline Review came statements such as the following:

... primary ... teaching needs teachers of mathematics who themselves have learned mathematics by constructing their own knowledge through discovery, exploration and problem solving in relevant and supportive environments. To be skilled in the mechanics is no longer sufficient. To be skilled in applying mathematical knowledge across the total range of real life situations is imperative. (DEET, p. 17)

The ideal of modelling desirable practices was therefore reinforced by the Review. Further, the Review made a number of recommendations for mathematics education subjects in teacher education programs for prospective primary school teachers. They suggested that such subjects should:

enable students to understand the fundamental concepts which underlie the major strands of the mathematics curriculum K-10 and thus recognise the linkages between early mathematical ideas and higher order mathematical knowledge;
assist students to develop an understanding of the nature and history of mathematical activity;
allocate sufficient time for students to become mathematically competent and to reflect on their own learning and performance;
ensure that students are mathematically competent before graduation;
model good inclusive teaching practice and reflect technological change;
engender enthusiasm in students for mathematics. (DEET, p. 23)

All of these recommendations were reflected by the aims of the mathematics education subjects under discussion. The orientation course, as has been stated earlier, had a primary aim of improving attitudes to mathematics and encouraging students to reflect on their own learning and experiences. The Mathematics Education 1 subject had the aims of developing links between areas of higher order mathematics and primary school mathematics; showing the connectedness of mathematics; revealing some aspects of the history of mathematics in order to demonstrate cultural influences on the development of mathematics; encouraging students to gain some understanding of the nature of mathematical activity, both by examining such activity as it has occurred in the past, and by engaging, themselves, in mathematical activity involving problem solving and cooperative learning. It also aimed to increase students' competency in mathematics and to help students become aware of current theories about the learning of mathematics. A subject outline stating the objectives and content of Mathematics Education 1 can be found in Appendix B.

Specific mathematics content was suggested by the Review and much of this content was placed in the Mathematics Education 1 subject, as academic staff involved in the development of the sequence of mathematics education subjects wished to challenge students' notions about the nature of mathematics, early in the course. Consequently, the topics of number theory and patterns, suggested by the Review as topics to be covered (DEET, p. 24), were introduced into Mathematics Education 1. Another topic recommended by the Review, number systems, was one that had been a part of the former mathematics education subject, and so this was continued.

The above discussion on the rationale for the structure of the two first year subjects explains how the two subjects evolved in general terms. Of course, each of the academics involved in the development and offering of the subjects, also had personal views about the merits, or otherwise, of various aspects of the two subjects. These will be examined further in the next section, in which the beliefs of the teacher educators involved with these subjects are considered and discussed.

8.3 Beliefs of teacher educators about mathematics and mathematics education for prospective primary school teachers

To place the students' views about the mathematics education subjects into context, it did not appear sufficient to merely describe the subjects themselves. The intent and execution of the subjects is highly dependent on the teacher educators involved. Consequently, the beliefs of the four teacher educators are of major importance in setting the context for the students' views of, and experiences with, the first year mathematics education subjects.

The four teacher educators were interviewed in phase four. Three of the academics concerned were males at senior lecturer level, all of whom had been at the participating institution for at least fifteen years. The fourth teacher educator was myself, a female at lecturer level, who, at the time of the study, had been at the institution for eight years. All four teacher educators have post-graduate degrees in mathematics, as well as post-graduate teaching qualifications. The male teacher educators have all been given pseudonyms to maintain confidentiality; these pseudonyms being George, James and Simon. I shall refer to myself as Sandy and report on my data as if I were an observer, external to the generation of this data. In this way, I am attempting to keep some distance between my position as researcher, and my position as teacher educator. No further description will be given of the men, to preserve anonymity as far as is possible.

8.3.1 Philosophies of mathematics

The teacher educators were asked about their philosophies in a question in the phase four interviews, in which they were asked to explain what mathematics is. George and James both indicated how difficult they found that question to be. The information about their philosophies indicated that there were similarities between the data of George, James and Sandy. Simon's data seemed to differ in some respects. This was of interest with regards to the interactions with the students, as Simon was their lecturer in semester two.

George, James and Sandy all appeared to hold fallibilist views of mathematics; they noted that mathematics changed, depending on context, and in the case of Sandy and James, there was an awareness of the central position of culture in determining mathematics. George seemed to hold a position more closely aligned with constructivism, in that he saw mathematics as being a way of thinking about the world. To illustrate this claim I shall briefly give the expressed views of the teacher educators, one at a time.

> *I see it [mathematics] as ... in terms of what it does, what influence it has on people. I see it as being ... something which is so pervasive and I suppose I see it too as a way of thinking, more than anything else, more than a particular thing. It's a way of thinking.* (George, phase four)

George sees mathematics as being something that exists in the mind rather than out in the external world. However, it is a way of making sense of the perceived world, and consequently, is all-pervasive. Therefore, this data suggests that George is constructivist in nature. He shows what philosophically could be called an **idealist** position; that knowledge occurs in the mind rather than in some external reality (Pateman, 1989). However, to distinguish this from the idealist position that formalism holds, constructivism does relate to what the individual **perceives** as the reality; whereas, in formalism, links to "reality" are of little, or no, importance.

As well as holding the view that mathematics is a way of thinking, George is also aware of the changing nature of mathematics:

> *Well, as I said before, mathematics is a way of thinking. I suppose that's what I meant by having them [the students] become better thinkers: being able to see mathematics where it exists and to be continually searching for mathematics, and to be aware of its pervasiveness and its effect and influence on people. It also helps, I think, for us to show them the evolution of number systems, because mathematics is also evolving all the time, too.*

One of the other teacher educators, James, explained his visions of mathematics, pointing out that it was a Western vision of mathematics that he held; this showed an awareness, on James' part, of other kinds of mathematics existing in other societies. He gave examples to illustrate this belief, clearly showing that he held a view of maths that was socio-cultural in nature.

> *I suppose there are ... various ways I can look at that. There's the received body of mathematical processes and understandings that have come to us, that we're familiar with, because it's mathematics that has been developed and passed on to us and we use those mathematics in all kinds of applications and different ways ...: by and large, that's the mathematics of the Western world. We developed that from the Greek mathematics. There was input from the Arab*

world, that mediated input, in terms of numeration, from the Hindus, so there are various inputs into that.
But by and large, that mathematics would be Western mathematics with its approach to proof, ... it's so important to be able to prove propositions and so on, although people can do mathematics without that same kind of understanding of the necessity for proof.

James went on to discuss the Indian mathematician, Ramunajan, who while being a very gifted mathematician, did not use the idea of proof, as accepted in Western society. Ramunajan's colleague and mentor, Hardy, found that Ramunajan did not prove results, as he would have done.

So I'm quite aware that there are different ways of doing mathematics and it's real mathematics, say in Ramunajan's case, he was obviously doing real mathematics but he wasn't looking at it in the same way as Hardy was ...
So there are different ways we can do mathematics, but for me, the way I've understood and used mathematics is very much in that Western tradition. So, what I understand by mathematics, as I've said, is what I've received in that Western tradition but I suppose I also understand that there are other ways that you come to mathematics and do mathematics.

Sandy started off by expressing what appeared to be a problem-solving view: that mathematics existed to solve problems in our environment:

To me, mathematics is a study of ... how, making sense of the environment ... of one's world. And all the means by which we can help to interact with our world more easily. So it's the study of patterns which allows us to generalise, it's the process of problem solving, it's communication, it's using ... mathematics is used to help us interact with our environment.

She noted in later text how mathematics was a reflection of the problems and needs of a particular society and evolved with the needs of that society; again this shows a perception of mathematics as being socio-cultural.

the ... ancient numeration systems ... are really a study of our number system from a different angle, looking at how it's culturally based. Looking at how the needs of society led to particular types of number systems being developed, looking at how there is no one right system that was discovered by us. It was invented and created by people as they went along, because the particular thing they had invented made more sense than what had gone before. So it's looking at that; it's this interaction with the environment, it is creative, different cultural aspects [exist] so it's culturally driven, it's dynamic, it keeps changing...

So all of the above three teacher educators saw mathematics as being an expression of the society or culture in which it was situated, and they all believed that it was dynamic and fallible.

However, as with the data of Joanne, who seemed to hold mainly problem solving views of mathematics (see Section 7.2, p. 123), data from George and James also showed elements of formalism in it. This could confirm the hypothesis that those who see mathematics as a human endeavour, use mathematical models to solve problems, and so see the rules and objects in these models as a necessary part of mathematics.

When talking about clock arithmetic, one of the topics in Mathematics Education 1, George expressed the view that this topic was worthwhile because:

I think it's important that they get, the students do get an idea of a mathematical system, that you can have a system which has operations, if you like, and entities, and rules for operating with the entities and then seeing what happens in that system. That's after all what the natural number system is, it is only a system, anyway. You've got numbers as entities, you've got operations, and clock arithmetic's just a different kind of system. It's just like matrices is a mathematical system.
It's important to see the kind of thinking involved in operating within a well-defined system, so that's what I see as its value.

Here George shows that he sees aspects of mathematics as dealing with objects, and rules to manipulate those objects; a formalist view. However, formalism, when in conjunction with another philosophy of mathematics, can be seen as a view that mathematical systems are models representing an interaction with the environment, and it is the notion of mathematical systems that therefore has value. It seems that it is when formalism is the dominant, or only philosophy, that it represents thought which is independent of any applications to external reality.

James shows a similar view when talking about the value of clock arithmetic in Mathematics Education 1:

> *Well, now this is real mathematics [laughs]. You could put another kind of spin onto what mathematics is about, where you can look at it in its own little formal system in a microcosm. Where you look at absolutely wacky things like four plus three equals two, and in our conventional arithmetic that doesn't make sense at all, but we can set up a set of rules where this is consistent and makes sense. So we can look at a kind of mathematics which is different from any other mathematics that a lot of them [the students] would ever have done.*

However, James did not seem to feel it was sufficient to study mathematical systems merely for the reasons given above; he also felt that there were some applications of clock arithmetic that could be useful, other than knowledge of how to manipulate a mathematical system:

> *Though it's also a very nice little topic in that it doesn't just have to be abstract formulations, we can show some very nice illustrations of how this kind of mathematics does actually occur in real life, in cyclical representations and it's not by any means a useless kind of thing.*

However, the main reason for teaching the topic of clock arithmetic was to show a different kind of mathematical system, rather than because it was of use in the student teachers' lives.

But I'd see it as a nice little topic to leave in, in the sort of context that I've been talking about, where this can open up their horizons about what mathematics is ...

So while James and George are clearly of the belief that mathematics is a human endeavour which evolves constantly, and is used to interact with the environment, they also share beliefs that mathematics is the study of a set of rules and mathematical objects that can be manipulated according to some design, which is not necessarily reflected by the society or the external environment. These beliefs, together with the data on Simon, discussed in the next section, explain why certain topics, such as clock arithmetic, are included in the Mathematics Education 1 curriculum. This somewhat sophisticated view of mathematics will be shown to create certain tensions with some of the students studying Mathematics Education 1.

It is also interesting to note that this view of mathematics, as being analogous to a some sort of intellectual game, is held by people who are highly competent at mathematics. They do not restrict their view of mathematics only to mathematics with obvious applications; they enjoy the examination of mathematical systems that are well defined and rigorous. They are interested in mathematics as an intellectual exercise. Only Bernadette, of the students, seemed to view mathematics in this way (Section 7.2, p. 123); Pip wished she could see mathematics as an intellectual pursuit but her emotional interaction with mathematics did not allow this view (Section 7.2.1, p. 127); and Joanne enjoyed the challenge when feelings of success were involved in such a study of mathematics (Section 7.2, p. 123). The view that mathematics contains an element of intellectual "play" may well be what sets apart those people who are successful at mathematics from those people who are not so successful.

8.3.2 Simon's case study

Simon displays a somewhat different view from the other lecturers, in that he is first and foremost, a formalist. Consider the following quote:

How would I describe mathematics? It's a multiplicity of little games that result from sets of rules. So within a particular environment, taking some set of characters like numbers, objects of some kind, make up some rules which must be obeyed and then you investigate all the possible results that might come from those rules.
At the same time you have to take into account that it has practical applications, but that's not in mathematics, that's more arithmetic.

Clearly, the applications of mathematics are quite secondary in importance: what does matter is that the rules are obeyed and the structure observed. So mathematics is still a human endeavour, but it is an intellectual challenge rather than an attempt to make sense of the environment.

It just turns out that some of the games that have been invented are useful in practical situations and many of them we don't know uses for yet, but a lot of them probably have uses ...
and I'm not suggesting that they weren't born out of necessity. I think the idea of counting and so on came from necessity, but then as we saw what could flow from them we now develop mathematics in a more pure way.
We develop mathematics and then sometimes it's useful and sometimes it's not, it doesn't really matter.

So for Simon, mathematics is principally an intellectual game, and any applications of mathematics that develop are usually by chance, and are considered to be arithmetical rather than mathematical, and are not the reason for the existence of the mathematics. This view differs from the dominant views held by the other three teacher educators, who all see mathematics as being very pervasive in people's lives, and who either see mathematics as a way of thinking and of making sense of life, or see it as being a way of responding to the cultural needs of a particular group or society. In both cases mathematics has evolved out of need rather than purely as an intellectual activity.

This leads to another area in which beliefs are held, that is crucial in determining the interrelationships between the students, teacher educators and the first year mathematics education subjects. The beliefs that are relevant here are those held by the teacher educators about the purpose of the mathematics education subjects in teacher education courses.

8.3.3 Teacher educators' thoughts about the purpose of mathematics education subjects

The teacher educators were asked, in their phase four interviews, what they considered the purposes of the mathematics education sequence to be. The following are their responses to this question:

> *the prime purpose of it [the mathematics education sequence] is to improve the quality of students' mathematical thinking.*
> *I know they're methods subjects in a sense, and they're intended to help students become good teachers of mathematics but I think in a way, that's an end point, I think our beginning has got to be to do things in such a way which will help our students become better thinkers in terms of mathematics. The quality of their teaching will just flow from that.* (George, phase four interview)

This agrees with George's view that mathematics is a way of thinking. To help the students become successful teachers of mathematics, it is necessary to help them see the power of mathematics by improving their mathematical thought.

George, therefore, believes that the students' mathematical background should be strong and coherent. Consequently, he believes that the students should be taught some explicit mathematics, as well as mathematics methodology. He talks about this explicit mathematics in the following quote:

> *But we put more [explicit mathematics] in because of the Discipline Review, and it also fits in with our desire to make ... to improve ... not just, not just to work on the methods of teaching, on the students' methods of teaching maths in schools, but in strengthening the students' maths knowledge and maths background.*

George showed concern that teacher education students did not share his view that to be effective teachers of mathematics they needed to have a strong mathematics background and be able to think mathematically:

> *They [the students] say, you know, they say to you "Will we be teaching this?" and this is one of the things we have to do - is to broaden their horizons, I suppose, because their horizons seem to start at kindergarten and finish at grade 6 and if we can't do something about broadening it now then I think when they get into schools and they start teaching, say they're teaching fourth class, their horizons may even narrow more, more than broaden, it'll narrow down to fourth class, maybe ...*
>
> *Our problem, of course, is that our students have got constructions of knowledge that might not in our view be satisfactory and that is one of our biggest tasks, to try to break them down, to modify them, to make the students realise, indeed, that some of the things they believe are on very shaky foundations.*

On the other hand, George also believed that one of the most important aims of the mathematics education sequence was to improve attitudes. This often came into conflict with his belief that students should be helped to think mathematically, as the latter might require the challenging of student discourses on mathematics.

> *but ... I've often thought that if we could do nothing else in three years, forget the content of what we do, forget how ... what we teach them, forget how we teach them to do, to teach certain things, if we did nothing else but created attitudes which are positive to the learning of mathematics we would ... the rest would flow.*
>
> *Now if we can get the attitudes changed and the approaches changed, then these reasonably intelligent students will pick up the rest, so maybe we should be promoting a total change in the approach.*

This dilemma is shared by James and Sandy. Both wish to share some of their interest and belief in the magic of mathematics with their students, and improve attitudes to the study of mathematics. However, both also believe that students' background knowledge is of great importance and so should be developed. These aims will be shown as being in conflict, in Section 8.4.1 on students' beliefs about the purpose of the mathematics education sequence. Students do not share the view that background knowledge is of great importance.

James had the following to say to express his belief about the purpose of the mathematics education sequence:

> *What I'd like to see our graduates able to do is to help their pupils understand mathematics, be able to do mathematics, to be interested, even fascinated by mathematics, to help their pupils see the relevance of mathematics.*

Yet he too, expresses the view that a deep understanding is essential, for the concepts of mathematics that would be taught by the prospective primary school teachers.

> *One of the very problematical things there is how important is it for our students to really understand and be able to do the mathematics, even at relatively simple levels, to really know what they are on about. And I do have a feeling that it's important for our students to really understand those concepts.*

So James, too, is expressing the dual aims, of promoting interest in, and enjoyment of, mathematics, while at the same time believing that a thorough understanding of mathematics is important. He justifies the latter belief as follows:

> *If the teacher doesn't really know, doesn't really understand a particular concept, they are not very well placed in that classroom. Must be in a little bit of a fog if um ..., and I can imagine a teacher giving rather ill directed kind of activities if that teacher doesn't really understand the point of ... [the concept with which they are working]*
> *So I see that as being important, that we, as far as we can, require our students to work towards building those concepts for themselves and relate that, as far as*

possible, to how that will help them teach the children and the sort of activities you want the children to do and so on. Those things are tied together, I think.

Yet, at the same time, James expresses the dilemma discussed above, of improving attitudes, while encouraging the development of mathematical thought. Assessment is one of the contributors to the dilemma:

> *Yes, it's a dilemma, isn't it, because we want to improve their attitudes and we want them to like doing mathematics, hopefully find it interesting or fascinating ... and get rid of their anxieties and so on ...*
> *So it is a dilemma, and maybe it's just, in some ways, a matter of de-emphasising the sort of heavy assessment side of it, but still, only in terms of examinations, say, ...*
> *I don't think we should back away from trying to ensure that their own maths background is as strong as it can be, given the time available.*

Sandy also expresses the view that mathematical thinking is important, as well as the improvement of attitudes and the development of confidence by the students. She too, is aware of the difficulties these two aims pose:

> *The overall purpose of the maths education subjects, I think, is first and foremost to help prospective teachers learn how to teach mathematics in the classroom.*
> *So that means that we want them to first of all have a good understanding of the mathematics that they're going to teach, not just the procedures and rules that they've learnt up to this point, but to understand the concepts underlying those things.*
> *We also would like them to be able to learn effective ways of teaching mathematics, which means that we have to fight against their conceptions of mathematics just being rule driven and therefore if you teach the rules and everybody learns the rules off by heart, then they know it.*
> *So it's a matter of teaching them the processes of mathematics: communication, problem solving, generalisation, searching for patterns, inductive reasoning,*

deductive reasoning, and once they have the idea of the processes of mathematics then hopefully they will be able to use those processes in their classroom environment.

This quote shows that, like George and James, Sandy believes that before students can be effective teachers, they have to have a strong understanding of mathematics. She also wishes the students to approach mathematics with understanding.

So the whole idea of the subjects is to give them a view of mathematics, which is probably different [to the ones they are holding at present] and to give them this view so that ... and to give them sufficient confidence to use it so that they can go out and teach mathematics in a different way from the way that they were probably taught, in using modern approaches and using modern ideas about the nature of maths.
And I think this is actually one of the biggest problems that we've got: trying to sell the idea that although we are not actually doing the content of the K-6 syllabus, what we are doing is helping to prepare them to be teachers of that K-6 syllabus. That's the challenge, I think.

From the above quote, it can be seen that Sandy is aware of the difficulties that exist in developing students' mathematical thinking.

Simon also believes that it is important for the students to have a strong mathematical background. The purpose of the mathematics education sequence is:

to fit these students to teach maths effectively in primary school.
And then within that, they need to have an understanding of what mathematics is ..., where it comes from, and they also need to have their mathematics improved, many of them. So I think we want to give them mathematical skills and understanding of some knowledge, as well as "how to teach" skills, education skills.

However, unlike the other three teacher educators, Simon does not mention the improvement of attitudes or the need to instil confidence in the students. His major aim is to equip students to teach in primary schools, with a strong mathematical background and some knowledge of the basic principles for teaching:

> *[the subjects contain content on] how you would teach mathematics and why teach mathematics, and what mathematics is, and a little better understanding of the maths they already have.*
>
> *There's a lot that we have to get through and we do have to visit a whole lot of sections of the syllabus because they have often just forgotten all about it and we need to visit it because they need to have some reminder of it before they start teaching, when they go to prac.*
>
> *At the same time it's absurd for us to say, well, we should cover every bit of the syllabus because they won't be able to teach it otherwise. If we give them basic principles for looking at content and analysing it and setting it up so that it can be learnt by the kids, then if we've missed a complete topic, they should still be able to do it by the basic principles.*

It appears that, for Simon, the mathematics education subjects are unproblematic as he does not mention any dilemmas arising from his views on the importance of background knowledge. From the data, it appears that he does not believe there is any conflict in the views held by the teacher educators and the students. He is committed to developing the students' mathematical background as best he can and believes the mathematics education subjects fulfil this aim. To illustrate these views, consider the answer he gave when asked the following question, in the phase four interview: "How do you respond to the criticism that students make, that the mathematics that they do in Mathematics Education 1, is not relevant for teaching as it does not cover what is in the K-6 syllabus?"

> *Well, I think that's total nonsense. For a start, I always say I am not here to teach the K-6 syllabus that the Department of Education in the limited area of NSW has come up with. That's not my job, I want to see them able to be teachers in other systems, in other countries, anywhere.*

So I don't want to be bound by that syllabus anyway and most of what's in maths[education 1], if not all, is relevant to that syllabus, not in the sense that it's necessarily in there, ..., but the teacher who teaches the syllabus is teaching it for a purpose. And one of the purposes is to cover other material that's in later on, in the secondary school and it's just as important for the sixth class teacher to know why they're teaching a particular thing in terms of year 7 material, as it is to just teach their year 6 material

So, unlike the other three teacher educators, Simon seems to have confidence that what is being taught is unproblematic, and he seems to have an assured and untroubled view of his role in the offering of the mathematics education subject sequence.

A study of the perceived constraints in mathematics education will provide further information on the issues that the teacher educators see as problematic. The next section discusses these constraints.

8.3.4 Constraints in teacher education in mathematics

A number of issues arose as perceived constraints, in the interviews of phase four. Some issues were a direct response to the question: "What do you think are the biggest difficulties or constraints that face us in the offering of the mathematics education subject sequence?"

The constraints that the teacher educators mentioned were varied, and dealt with some of the problems discussed above: the conflict between developing positive attitudes and improving mathematical background; the difficulties associated with assessment; and the perception that students did not see the value of some of the content. Other constraints were also mentioned. The following extracts from interviews deal first with the issues already discussed in the previous section, and then go on to demonstrate some other difficulties teacher educators find in mathematics education.

George was concerned that students' orientation to the subjects was somewhat narrow, and did not allow students to assess the true value of any mathematics done, unless it was in the K-6 syllabus. Some of this concern has been illustrated in the extract from George's data, given in Section 8.3.3, p. 174. George expanded on this concern:

> *I think it's always a great worry that our students, the students we have now who are supposedly better quality students, are still making these kinds of comments: "Will we be teaching this?" rather than "What's the relevance?"*
> *If they asked you what the relevance of it was, that's a good question, and you can discuss that, but when they ask you "Will we be teaching this?" and you have to say no, they might say "Well, what's the point?", which is a pity.*

George also saw the necessity of convincing the students of the value of studying the history of mathematics, and he believed this could be problematic.

> *I think our main task is to convince our students that those reasons [reasons for studying various topics] are valid. And that's always a continuing worry that we have ... we can talk to ourselves and nod our heads to each other, because we know, because we might agree on those things, but getting it across to students is often the difficult task.*
> *They seem to have a desire to write down anything you say because it might be in the exam. It's difficult just to get them to think so they can think and reflect on what you've said ... and talk to you.*

As well as concerns about students' orientation to the subjects, George saw two other major constraints concerning the students, but existing for the teacher educators:

> *... two things [constraints or difficulties], I suppose. I think that attitude is one, and the other one [is that] I think that a lot of our students have not been taught for understanding in their own teaching. They'll tell you that. In fact many of them will tell you that it doesn't matter either, ... they won't say it in an accusatory way of their teachers. They'll say "Well, so what, you know, what does it matter. I know how to do this anyway, what does it matter if I don't know why". Which is a pretty worrying thing from the point of view of the teaching ... I had some very bright students say that to me, "Why? Why should we worry?"*

So I suppose attitudes like that, I think, but also, the attitudes of others who perhaps are just not, don't look upon themselves as being mathematically very competent and ... that strikes a more serious worry, I think.

James saw assessment as being in conflict with the development of positive attitudes. This concern was mentioned in an earlier quote in Section 8.3.3, p. 176 and is discussed further in the following quote:

The emphasis on, or as they perceive it, the emphasis on assessment can really affect the way they view what they are doing and increase their anxiety and all that, and I think that's something that's a bit of a problem in what we are doing with the students.
... yet when we insist on them learning the hard things, being able to do it and showing that they can do it, then that, even though that, in itself, has a worthwhile aim, it then can start to contradict the other side where we're trying to improve their attitude and get rid of their anxieties and so on. So it is a dilemma ... in terms of those different aims that we've got in that course.

Like George, Sandy saw the students' orientations to the course as being problematic, and, like him, also saw the students' past experiences as a constraint on their learning. Attitudes were also considered a major constraint:

A number of problems - first of all the very, very strongly held beliefs that students come into the course with, about maths as a particular type of subject which is very rule dominated, formal, procedural. Second of all, their attitudes which generally are negative because of this belief and because of the experiences they've had in the past.
[Another problem is] The fact that they haven't really been exposed to very much of what we're talking about here, so it's difficult for them to build up a picture of that and so it's easy for them to reject it. The fact that they have such a strongly vocational orientation and what they want are the ten golden rules of teaching maths, not to be developing their own maths so their motivation is not

good either. And of course the lack of time is always a problem, although we are quite lucky with the amount of time that we have.

She suggests, that it is necessary for the teacher educators to interrupt student discourses in order to make the mathematics education subjects of more value:

> *I think that we have to try and challenge their beliefs about maths first of all. If you believe that it's a series of facts and we've therefore got to have covered all the facts in the K-6 syllabus, then you are dissatisfied with what we offer.*
> *If you can see the processes of maths, then of course, the processes that they're learning in this course are just as applicable to what they are going to be teaching as they are to this course.*

George, James and Sandy are concerned about similar issues, which deal with their perceptions of students' attitudes and orientations and the dilemmas created when students' affective domains are placed alongside the cognitive demands of the mathematics course. Simon, once again, seems to have a different set of concerns, which deal more with procedural matters such as the structure of the course and the demands on the academic staff:

> *I think we have far too many little subjects. We should have bigger subjects and in two hours a week, we don't really get to know the students very well ... If we had, say a three [hour subject] and two fours[four hours a week], [and one subject] each year, I've always felt that that would be better and I think the other thing is that we only get them for ten weeks [in a semester], so the amount of time and the frequency of seeing them is pretty devastating. The first couple of weeks we can't give them much in terms of assigned work then suddenly we're away at prac, can't give them anything there, and then the last couple of weeks, if all their assignments are due then, we can't ever mark them, the workload's too great.*
> *I suppose the greatest problem is that none of us have time any more to do the course justice and we're being forced into reducing the number of assignments and relying on examinations, and not sitting down and having individual*

consultations with the students who have problems and things like that. Basically we're not doing what we are here to do, we're told we have to do other things.

Simon appears to be satisfied with the content of the mathematics education subjects, and the students' reactions to them, but is concerned about the lack of time to build relationships with the students, and about the difficulties in assessing the students' work in the best possible way. He sees the major constraints in the mathematics education subjects as arising from the large workload demands on the teacher educators.

The above views indicate the positions that the teacher educators bring to the delivery of the mathematics education subjects. While all four of the teacher educators have been involved in these subjects at some point, at the time of the study I was the teacher educator responsible for the orientation subject in first semester, and the participating students were in my classes. Simon was the teacher educator for those classes with participating students in the second semester, when the students studied Mathematics Education 1. Consequently, while the views of the other teacher educators were of importance in the development and modification of Mathematics Education 1, Simon's views hold particular importance for the study. My views are also of importance as I was responsible for the orientation subject for the participating students. The interactions of students' views and teacher educators' views will be discussed in the next section.

8.4 Students' responses to mathematics education in a teacher education course

The students' views will be considered in a number of areas. These are areas that the data have suggested are important in this discussion. The first of these areas is that of students' orientation to their studies at university. Gibbs, Morgan and Taylor (1984) identify orientations to studying at university that explain why the student has chosen to study. My data show that these orientations have implications for the way subjects are evaluated by the students, and are also linked to the students' conceptions of learning and to their approaches to study.

Gibbs et al (1984, pp. 171-174) suggest that there are four major orientations to study at university and that each orientation can be seen as intrinsic or extrinsic:

- **academic:** intrinsic - an interest in the subject matter as an intellectual challenge; extrinsic - getting good grades and choosing a subject because the student was good at it at high school, rather than because the student had a particular interest in that subject.
- **vocational:** intrinsic - interest in the course as a preparation for the student's future career; extrinsic - choosing the course because the qualification is necessary for the student's future career.
- **personal:** intrinsic - the studies are for personal development; extrinsic - the studies compensate for missed opportunities, or prove the student is able to succeed.
- **social:** the course is irrelevant to reasons for being at university.

I shall show, through the students' data, that all the students in this study had a strong intrinsic vocational orientation, that is, they were there to learn how to become teachers. This has implications for their passage through the course. Gibbs et al (1984) found that students in their study, who were enrolled in a degree in Hotel and Catering Administration, and who could be seen to fall into the intrinsic vocational orientation

> *were critical of any parts of it [the course] that they thought were irrelevant to their future careers. They tended to place emphasis on the practical side of the course and to like the industrial year best of all. Since their interest was in becoming trained, students with this orientation tended to work hard on the course while they could see its relevance to their chosen career.* (p.173)

The results of a vocational orientation, as found by Gibbs et al (1984), agree strongly with my data, and this will be shown in the following section. Another interesting aspect of orientation to university courses, is the junction or disjunction of the students' orientations and the teacher educators' orientations, and with the teacher educators' perceptions of student orientations. All the above teacher educators have been shown to have an academic orientation, themselves, as they enjoyed the intellectual challenge of the work. However, they also showed that they saw their roles as helping with students' vocational orientations, as all four of the academics believed that the purpose of the course was to help students become good teachers. I shall show that a major cause of tension in the mathematics education subjects is that the teacher educators and students held different visions of what a good teacher of mathematics in the primary school knows and does.

As can be seen from the above data on the beliefs of the academics, the teacher educators believed that students needed to be knowledgable about content in a number of areas. These areas have been described by Shulman (1986b) as follows:

- subject matter content knowledge
- pedagogical content knowledge
- curricular knowledge

These areas have been more fully described in Section 3.4, p. 41 on teacher education. In considering the students' views on what the purpose of their teacher education course was, it will be shown that most students did not see knowledge in all the above areas as being necessary.

8.4.1 Students' views of the purposes of mathematics education subjects

Data for this section was collected through all three phases of student data collection. In phase one, some of the students discussed the purpose of the mathematics education subjects, in their interviews with each other and in their notes on how mathematics is best learnt . The phase two questionnaire asked the question: "What are your expectations for a mathematics education course at university - what do you hope to get out of it?" (See Appendix C). In phase three students were asked what they had envisaged, at the start of their teacher education studies, as the purposes of the mathematics education subjects. It should be noted that the responses to the phase two question would have occurred near the end of the first year of study, while the phase one and phase three responses referred to students' ideas very early in their university studies (although phase three data was collected at the end of the first year, students were asked to think back to the beginning of their teacher education studies, and reconstruct their expectations at that point).

Immediately the students' responses are considered, it can be seen that there is a disjunction in the views held by students and teacher educators. While both groups agree that the purpose of the teacher education course in mathematics is to help the students to become good teachers of mathematics at the primary level, their perceptions of what constitutes good teaching differ. The following data illustrate the different views on the purposes of teacher education mathematics subjects. Seven major purposes of the mathematics education subjects were suggested by students and teacher educators. These were:

- developing correct vocabulary;
- learning how to teach the basics;
- improving the student teachers' attitudes;
- developing understanding of primary school concepts;
- learning how to teach well;
- learning the new methods of teaching mathematics;
- improving students' background knowledge.

Each of these will be discussed below with illustrative quotes.

8.4.1.1 Developing correct vocabulary

Four students saw the purpose of the mathematics education subjects as to help them develop the correct vocabulary for teaching mathematics. Some of Pip's concern about language has already been discussed in Section 7.2.1, pp. 128-129. She felt that she was not able to communicate correctly with pupils on her teaching practicum, and this view was reinforced by the teacher who criticised her loose use of certain terms. Pip stated that she believed the teacher educators should indicate specifically when they were using mathematically correct terms so that the student teachers could write these terms down and use them when required. For Pip, a clear distinction needed to be made between the use of everyday language and the mathematical register.

So that to me is something important that I'm really going to have to learn, maths language, and it has to be pointed out that it is maths language, it's not just to be taken for granted that this is how you speak when you do maths ...

Gail, too, found the actual phrases used in teaching mathematics to be of utmost importance. She illustrated this in a discussion about an experience that had occurred on the practicum:

instead of me saying three times three is nine, she [the supervising teacher] said no, I must say three lots of three is nine. And I thought to myself "These are the rules that I really want to get to know. What are the words that you use so that the child understands ... what phrases do you use that unlocks a child's blank, you know, a blockage of understanding?" That's what I want to get out of mathematics.

Aaron also indicated that it was the vocabulary that he was hoping to learn in the mathematics education course:

I was sort of excited about having to learn how to teach maths to children. Like I'm only used to teaching maths to high school students and adults. They'd be able to relate to my terms and the same terms that they use are the same as I use, but how to relate it to children of primary school age? I'm looking forward to learning how to explain it so that they understand what I'm trying to get across because I'm not used to using the same vocabulary and the same examples ...

Interestingly, all three of the above students are mature age students and this is the only area in which their views constitute the majority. It would appear that mature age students' long break from schooling has led them to believe that, along with methods and content, the language of mathematics has also changed. Terry is the fourth student who mentions phrases and vocabulary as important; she is not a mature age student and her interest seems a little broader than that of the other three. Her desire is a more general one of presenting the mathematics clearly:

Just to be able to explain it in a way that they will understand. Even if I might understand it myself when I'm just reading it, um, I think that'll be hard to teach. I'd have to really think about how I was going to present that so the children would understand it.

Yeah, how to actually say it. Because we have to learn to ....

It should also be noted, at this point, that Terry, Gail and Pip have all been classified as predominantly Platonists in chapter 7 (there is insufficient data to classify Aaron), and their view of a static and highly structured mathematics accords with the placing of such importance on knowledge of the correct phrases and terminology as the keys to understanding mathematics.

None of the teacher educators expressed any views on the importance of mathematical vocabulary; although James did indicate that he thought the issue of language was important. However, by this he meant the communication, by individuals, of their understanding and meaning making, rather than the learning of various key phrases.

8.4.1.2 Learning how to teach the basics

Eleven students, five of whom were mature age students, felt that the purpose of the mathematics education subjects was to help them learn how to teach the basics; that is, the mathematics that they believed was the core of the primary school curriculum. Most felt confident about their ability to do this mathematics themselves, but were unsure as to how they would teach the basics to their pupils.

I thought I'd be starting with the basis, you know, the basics of addition, how you could teach them addition, different ways of doing it, and building up through the way you would do it in the school, you know, that sort of thing. (Pip, phase three interview)

I think that some of the basic skills I would have ... which I just take for granted, I would have a great deal of difficulty teaching. And so I think that that's a very *essential thing, just the basic facts, whether, you know, it's long division or that sort of thing, I can do it but whether or not I can do it in front of thirty kids and understand where they're at, um that's another story [laughs].* (Joanne, phase three interview)

A clear picture emerges, of what students believe a good teacher does when teaching mathematics to a class. The teacher stands in front of the class and explains carefully and painstakingly the different steps involved in building up a basic skill. The language used and the sequence of explanation is very important. Knowledge about the context in which this mathematics is to be taught is far less important and is only hinted at.

A source of tension in Mathematics Education 1 can be seen if the above data are considered together with the fact that, of the teacher educators, only Simon mentioned the necessity of teaching the students how to teach basic skills. Simon suggested that the K-6 curriculum is visited in order to refresh students' memories, but he does not suggest that the students are taught how to teach basic skills in the ways envisaged in the above quotes. The other three teacher educators do not mention learning to teach basic skills at all, when discussing the purpose of the mathematics education subjects. Although these skills are, in fact, discussed in later mathematics education subjects, the teacher educators believe that, at this stage, extending students' background knowledge and modelling good teaching practices will help students to learn how to teach.

8.4.1.3 Improving the student teachers' attitudes

It has been shown in the section on teacher educators above, that George, James and Sandy believed one of the purposes of the mathematics education sequence was to improve student teachers' attitudes to mathematics (see Section 8.3.3). Nine of the students, three of whom were mature age, also held this view. The students were aware that their attitudes to mathematics had been somewhat negative in the past, and they hoped that the mathematics education sequence would improve their attitudes, as they believed that to be good teachers of mathematics, they needed positive attitudes.

... because I wasn't sure how to teach maths in a really enjoyable way that's what I wanted to find out. (Terry, phase three)

When we started in that first semester, oh we were all keen, this is going to be something wonderful, new, (laughs), this was going to change all that way we'd thought of maths and approached maths teaching. (Gail, phase three)

I've got a very good attitude towards it but there are some areas such as fractions and things that I felt were boring, going through primary school. So I think if I looked into that more and tried to develop better ways of learning, that might improve my attitude. (Lexi, paired interview, phase one)

It is relevant to note here that while George, James and Sandy did see attitude improvement as important, Simon did not mention this as a goal. This could have had implications for the students' interactions with the Mathematics Education 1 subject, as Simon was the students' lecturer for this subject.

8.4.1.4 Developing understanding of primary school concepts

Many students felt that they adequately understood the concepts that they would have to teach in primary school. However, eight students, two of whom were mature age, mentioned that this was a goal of their passage through the mathematics education subjects. All of the teacher educators had stressed that understanding of primary school mathematics was a priority in their subjects; again a disjunction might have occurred because of the different views on how that understanding should be achieved. The teacher educators believed that this would be achieved by extracting the principles from the primary school mathematics and studying these, in a variety of ways. The student teachers believed that the actual primary school mathematics should be taught to them, in much the same way as it had been taught to them at school.

I'm not very strong in mathematics. I'd do it [study it in the teacher education course] just so I can build up a strength ...

Question: *Are there any particular areas now that you might have done at primary school which you feel a bit nervous about teaching?*

Answer: *Oh, long division, I haven't done that in a quite a while. .... I just hope that um, it doesn't get skipped over very briefly, it's something you'd hate to miss out and not understand it when you have to try explain it. So long as everyone understands it themselves.* (Maria in phase three interview)

[What I want from the mathematics education subjects is] a clearer understanding and revision of my own mathematics knowledge. Then being confident enough to teach it. (Alice, questionnaire, phase two, question 24)

I hope to be able to understand the fundamental concepts behind the mathematics I will be teaching. (Beryl, questionnaire, phase two, question 24)

8.4.1.5 Learning how to teach well

The purpose most commonly envisaged for the mathematics education subjects was a broad one; to help students learn how to teach well. Nineteen students, six of whom were mature age, expressed this desire. This indicates strongly the intrinsic vocational orientation that these students hold towards their studies at university. They are there because they wish to learn how to become good teachers. The four teacher educators also felt that this was a primary purpose of the mathematics education subjects.

For Maria, learning to teach well, meant learning how to explain to pupils:

What I[would] find the most daunting would be trying to explain the maths to them, trying ... For me to learn how to teach them the maths which is the hard part, because sometimes I work out in my head my own little way of doing it but trying to actually explain it is another thing. (Phase three interview)

Nita had a similar goal:

I don't know really what I was expecting. I think I was expecting to learn how to stand up there and put it across.

Aaron also wanted to know how to explain concepts to pupils:

... it was one of the things that was troubling me, "how the hell am I going to teach those kids those things; I wouldn't have a clue"

For Gail, teaching well was the ability to structure the mathematics into well defined stages:

I think that it is important to me to be able to teach maths as I see in this ... with this ability to break it down into stages

Mandy wanted to learn to teach well so that she could teach in the way she had been taught:

I would say I wanted to learn how to approach it, for the children, and them learning how to enjoy it, and being able to teach it in ways that they could understand it and apply themselves and enjoy it the way I did, I think.

So a variety of different images of learning to teach well emerge. A common characteristic is that the teacher plays a major role in explaining and sequencing the work for the pupils; and the learning is seen as very much teacher-directed.

8.4.1.6 Learning the new methods of teaching mathematics

A number of students saw the changes in mathematics education as being sufficiently large to warrant their study in mathematics education subjects at university. Nine students, including four mature age students, saw learning new methods as a purpose of the mathematics education sequence.

I was looking forward to it because I was wondering how you would tell us, or teach us, to teach maths in primary school now, and see if it's any different to how I was taught. I was interested to see the difference. (Terry, phase three)

... it's just the methods have changed so drastically, I mean I can't help my kids with their homework and they're at high school, so I have to learn the methods that they're learning and then help them. (Nita, phase three)

In fact, I'm really excited about this creative side to maths, and I can't wait to see lots of innovative lessons. Perhaps because my attitudes to "chalk/talk" maths classes is ambivalent, I'm more prepared to welcome new teaching styles. (Dale, summary in phase one)

Of the teacher educators, James, Sandy and Simon mentioned that they wished to make students aware of the current approaches to mathematics education.

8.4.1.7 Improving students' background knowledge

Improving students' background knowledge was a major purpose of the mathematics education subject sequence, as far as the teacher educators were concerned. As discussed earlier in this chapter, the teacher educators felt that in order to prepare students to be good teachers of mathematics, they had to help students increase their background knowledge for all the topics they studied. However, the students did not see this as a priority at all; their vocational orientation led them to require the learning of methods and of primary school content, but did not persuade them that background knowledge would improve their future teaching. This conflict in views on the purpose of mathematics education subjects led to students rejecting some of the content of the Mathematics Education 1 subject.

Only two students mentioned that the learning of background content in mathematics had any value. Maria and Mandy both saw background knowledge as useful:

It's all going to help me and I need to know and it might even help my maths a little bit, just studying the background. (Maria, phase three interview)

I thought having background knowledge as a teacher is very important because if you get a child asking you a question and you don't know it, it doesn't look too good, specially if it's not difficult. (Mandy, phase three interview)

In the questionnaire one of the questions asked "...which would you regard as more important for a student teacher - to have background knowledge in a particular area of mathematics or to have a sound knowledge of the methods required to effectively use that mathematics? Why do you believe this?" (See Appendix C for copy of the questionnaire).

The responses to this question were more commonly in support of the knowledge of how to apply mathematics than of having background knowledge in that area. Only two students indicated that they thought background knowledge was more important; another seven felt both were important although generally the knowledge of applications was considered more important.

The above results indicate that there is a substantial chasm between the beliefs of the student teachers and the teacher educators as regards the purpose of Mathematics Education 1. Teacher educators see the development of background knowledge as extremely important; student teachers do not seem to value such knowledge. The following section shall show how the students' orientations to Mathematics Education 1 and their beliefs about its purpose led them to interpret the Mathematics Education 1 subject in a particular way.

8.4.2 Students' orientations to study

The above quotes on the purposes of the mathematics education subjects in the teacher education course clearly show that the students all hold an intrinsic vocational orientation to their studies. They are not interested in the subject matter *per se*, and other aspects of university life are of secondary importance to their major goal of learning how to teach.

All the purposes discussed by students in the above section demonstrate a desire to teach well; even the desire to have improved attitudes to mathematics arises because students believe that they cannot be good teachers without having a positive attitude to mathematics.

> *Maybe in primary school I had a negative attitude but now I think I've got a quite positive attitude and part of becoming a teacher, I suppose, is you have to commit yourself to becoming positive..., a teacher in a positive way. So it may affect my attitudes but I've just got to think about that and try and convert negative attitudes to positive attitudes, I suppose.* (Eileen, phase one paired interview)

As a result of this orientation the students appear to interpret all their experiences through one of two filters: either by assessing the subject from the viewpoint of "self-as-primary-school-pupil", or by assessing the subject from a viewpoint of "self-as-teacher" (Holt-Reynolds, 1991b). Holt-Reynolds (1991b) has shown in her study of nine student teachers that each decision made about professional practice, by the student teachers, was achieved by assessing how they would have reacted to the particular practice had they been students in a class that was implementing that practice. The students in Holt-Reynolds' study were all undergraduate students with previous course work in educational psychology and/or multicultural education and some experience of tutoring individual students in their subject matter majors of English and mathematics. The students were all taking a course in content area reading, writing and discussion for secondary teachers. Holt-Reynolds (1991b) names the selves that the student teacher assumes in her study, as "Self-as-Student" and "Self-as-Teacher". The student teacher decides whether Self-as-Teacher would benefit from implementing a particular practice, by listening to the responses of Self-as-Student who undergoes the imagined activity. In this way the student teachers decide whether or not a particular practice has value for them.

My data indicates that, for prospective primary school teachers, this inner conversation is strongly allied to the students' orientation to studies. As the students in my study are so strongly vocationally oriented, all subjects appear to be judged for their value in preparing the student to become a good teacher. In order to make this judgement in the mathematics education subject sequence, the students would assess the mathematics content from the viewpoint of self-as-primary-school-pupil, and this assessment allowed self-as-teacher to make a decision as to the value of the topic. If self-as-teacher would use that topic in her/his classroom then the topic is assessed as worthy of time and effort and is regarded as a useful topic to study. If self-as-teacher cannot envisage teaching that topic, the topic is regarded as worthless, and discarded. Self-as-teacher makes the decision about the topic by consulting self-as-primary-school-pupil. If self-as-primary-school-pupil can cope with the demands of the work, and the topic agrees with self-as-teacher's notions of what mathematics should be taught, the topic is given the validity that it requires, in order to be learnt by the student teacher. Consequently, I propose that for primary school student teachers, there are three "selves" that are consulted: self-as-primary-school-pupil; self-as-teacher; and self-as student-learning-to-teach. From the discussion above, it can be seen that self-as-primary-school-pupil tests the content in order to advise self-as-teacher. If self-as-teacher is convinced of the worth of the content, then self-as-student-learning-to-teach finds value in the content in the mathematics education subject.

The following quotes all illustrate the constant use of the filters of self-as-primary-school-student, self-as-teacher and self-as-student-learning-to-teach. Maria, in the phase three interview, was asked whether she would have studied the mathematics education subjects if they had not been compulsory. The conversation continued (the boldface is added):

> *Oh yeah, I'd definitely do it, ... because I'm not very strong in maths I'd do it just so I can build up a strength. Because I mean that's something that they really need to know and you want to teach it well, especially mathematics.*
> Question: *And did you feel that you got any help in that direction in the first year mathematics, Mathematics Education 1 in particular?*
> Answer: *Um, oh I learnt a few new little skills and ideas but* ***I didn't really use them on prac, I mainly used the syllabus, like that really helped more, my*** *[supervising] teacher went by the syllabus so I used that but um, some things ...yeah, some things I found interesting.*

Maria first says that she felt it was important for her to build up her mathematics (and it should be noted that Maria was one of the two students who felt background knowledge had value for student teachers. See quote p. 194). Yet when asked whether the Mathematics Education 1 subject had helped her achieve that aim, she answers a different question: was the subject content useful on the teaching practicum? So Maria appears to see building up subject content knowledge as purely developing content knowledge of the primary school syllabus.

Terry, in her phase three interview, discussed studying the ancient numeration systems in Mathematics Education 1, in the second semester. It should be noted here that the ancient numeration systems were taught to students for a variety of reasons: to help students see what made a numeration system efficient in a particular context; to show how mathematics developed in a socio-cultural context and was dependent on the needs of the society in which it was situated; and to isolate the underlying principles of our place value system. Further, study of different numeration systems was recommended in the Discipline Review (DEET, 1989, p. 24). However, Terry did not seem aware of these reasons:

> *And second semester I didn't think was as practical, we did all those different numeral systems and I just really didn't see where they were used as much. But I didn't think it was ... I didn't think primary school ... I don't think I would have understood it, when I was in primary school.*
>
> *I don't know, I don't know why we did that. I didn't know why or I didn't really see the point. I didn't mind it, because I'd never done that, but I just didn't think ... I didn't think, I wouldn't do it with my primary school class.*

Terry evaluated the worth of the topic by assessing it as self-as-teacher. To do this she first put herself into her self-as-primary-school-pupil position and decided she would not have understood the concepts. This then linked to her assessment as self-as-teacher: it was not a useful topic. Terry seems quite unaware of any other reason for studying the topic in the mathematics education sequence. It should be noted here, that Simon's position as teacher educator might have had some influence on Terry's apparent lack of understanding as to why the topic was being covered. As Simon was a formalist, he might not have seen it as necessary to convince the students of the value of the topic, but might have regarded the intellectual challenge inherent in the study as sufficient reason for studying the systems. Gail illustrates this point when she discusses the students' confusion as to why they were studying various topics, and she explains why they did not question the lecturer about the rationale for studying these topics:

> *We really didn't know why we were doing it, and so a lot of the good was lost, it wasn't being reinforced, it wasn't being consolidated, it wasn't being tied in with the base ten or with children. There was not an atmosphere of being able to ask questions ...*
> *One had to be very bold to speak forth, and then one runs the risk of antagonising a lecturer and we all know what the penalties of that are, so everyone, I found, started to just go in and just sit. There was no response, there was no communication and it became to me, a performance...*

Pip, too, saw Simon's teaching as a performance. She seemed daunted by the ease with which Simon went through processes of solving problems, and she did not feel confident enough to question the rationale for these problems:

> *In class he was a bit too much of a showman, and because we had to move so fast, we haven't got time to do a lot of things. You see, he was excellent to watch in class...*
> *but I didn't question, there again I'm still that submissive old ..., I'm just learning, as I said, to question things. I should have, but I never thought about it.*
> *I mean, to watch him work in class was, I thought ... I would stop writing there again because he explained things so simply ... he did it so well on the board. It was all showmanship. It had me intrigued that he could do all these ... so it really wasn't... but it was over my head...*
> *Yeah, that he could do it all, that he had this much knowledge. He could do it, could do a sum this way and this way and ... without having to stop and think about it. It was excellent how he knew all this information and that, but to watch*

him work out columns, now I asked him a couple of times to go over things and he would, he'd do it again but I still didn't ...
I still have no idea, so to me that was a total waste of time.

Further, the students were reinforced in their belief as self-as-teacher, that studying the ancient numeration systems was not of value, as it could not be used in the school. Teachers on the practicum dismissed any lessons students might have prepared on the ancient numeration systems, as irrelevant and wasteful of time:

> *...we were actually doing all the Egyptian [numeration system] and everything when we went out on our first prac and I thought this will be terrific you know, we really can tell the children all about the history of mathematics and they can understand it all more and they can get really excited about it too.*
> *And of course, the teacher said "Oh, are you doing that rubbish? That's just absolutely ludicrous, you know. We'll get on with this, there's too much to cover in the curriculum without all that" and so that was squelched ...*
> *That reinforces something in my mind, it reinforces that what we're doing at university is so totally unrelated to what happens in reality in schools, where do you tie them both in? That's what it reinforces.* (Gail, phase three interview)

There appears to be a mismatch between the beliefs about mathematics of the teacher educators and the supervising teachers. As a result, the student's self-as-teacher receives confirmation that the topic is not going to be useful, as "real teachers" have indicated that it has little value in the primary classroom and self-as-teacher seems to derive valued information from the supervising teacher as well as from self-as-primary-school-pupil. This confirmation for self-as-teacher, together with the fact that the topic is not evaluated for its usefulness in any other context but that of the primary school classroom, leads to a perception by self-as-student-learning-to-teach that the topic is without any value.

Pip, too, assessed the study of the ancient number systems, from her position as self-as-teacher and saw it as worthless. As self-as-student-learning-to-teach, she was very concerned that valuable time was being "wasted" on this topic, that should have been spent on learning to teach the basics.

> *To me to spend a whole semester basically doing what we did on the history of it [mathematics], to me was a waste of time, because we know the subjects are crammed up and there's so much you've got to fit into it to take out [when you teach]. To me that could be extension work or ... you know ... but to have a whole semester on the history [laughs] ... I mean you know you do your different ... Roman, and the numerals of all... I know that, I know the Mayan and all that too, but to the extent that we went into it. I don't know if they [primary school pupils] do it to that extent, so to me that was pointless, you could learn a bit of it, I mean all the kids know the Roman numerals, I've got a maths book at home that's got the Mayan in it, just an exercise, just a kid's thing that you can photocopy and that has the Mayan in it, but it was only an extra worksheet for extension work or things like that. A lot of it I found was a waste. I really needed, I wanted to rather do the nitty gritty.*
>
> *I'm worried that if we skim over a lot of the basic skills, with this superfluous stuff, that we're not going to come out, we're going to come out with a very cloudy idea of everything.*

(It should be noted that only 25 per cent of the Mathematics Education 1 class time was actually spent studying the ancient numeration systems and our number system.)

On the other hand, some topics were seen as worthwhile by self-as-student-learning-to-teach, because self-as-teacher could see value in them. A study of number patterns was one of these for Terry:

> *The patterns were actually not too bad because I thought that they were more practical. I thought, you know you see patterns in things and I thought that was useful for children to be able to look at something and see a pattern ... I think that's useful.*

The primary orientation course in the first semester was seen in a positive light by nearly all the students as its value to self-as-teacher was obvious:

I think the first one [mathematics subject] was very important because it put us back into the learning frame of mind and it was a good way to be introduced to maths again. Um, and I think it was very important because it equipped us with some practical activities that you could teach in the classroom and you could feel confident about teaching with them because you'd just done them and I think that that was good. (Joanne, phase three interview)

Yet another aspect of the students' constant evaluation of topics from the perspective of the primary school pupil is that the prospective teachers, and, possibly, their supervising teachers, have a very narrow view of the mathematics that could or should be done by primary school pupils. Students have a limited view of self-as-primary-school-student: their past experiences have closed their minds to the possibilities that might exist in primary school mathematics, and their successes and failures in primary school mathematics have encouraged them to place ceilings on the sorts of activities that might be done in the primary school classroom. The selves-as-primary-school-pupils are, in fact, projections into the past by the students themselves. Their experiences in the primary school classroom have led them to accept as authentic activities, that self-as-teacher would teach, either activities similar to their past experiences, or activities that promise to be highly enjoyable, regardless of whether learning occurs or not. As the above quotes illustrate, where the mathematics education topics in the university course are not seen to be **directly** related to the classroom, they are regarded as irrelevant. The filters that the student teachers use create conflicts between their aims for the mathematics education sequence, and the aims of the teacher educators. Background knowledge is generally judged by self-as-primary-school-pupil to be too hard and not particularly relevant, and so self-as-teacher receives confirmation that such content will not be taught in the primary school classroom. As a result, student teachers are bewildered about the reasons for the learning of such content in their teacher education course. When these feelings about such content are juxtaposed with the desires of the teacher educators to improve and strengthen students' background knowledge, conflict in aims is inevitable.

Holding such orientations to the study of mathematics education has implications for the way students study the topics in the mathematics education sequence. Where topics are seen as of little worth, it appears that students revert to ways of learning them that emphasise instrumental methods of rote learning to pass examinations. It is not perceived as worthwhile for students to put the necessary time and effort into the understanding of the subjects. If they can get by with the memorisation of a few rules, this is seen as expedient as the students do not believe they will ever need this knowledge again.

8.4.3 Strategies for learning new mathematics

Students were asked about their strategies for studying mathematics at university, in the first year mathematics education subject, in phase two and in phase three. In the phase two questionnaire, one of the questions asks: "What do you feel are your most effective and successful strategies for learning new mathematics concepts at university?" (See questionnaire in Appendix C, q. 21). In phase three interviews students were asked how they had prepared themselves for the assessment tasks in Mathematics Education 1.

As indicated in the above section, where students saw the value of the topic they were studying, they made some effort to understand it and to investigate it in depth. The value of a topic was judged from the perspective of the self-as-teacher. It was judged to be worth studying if it was perceived as a topic that would help the student become a better teacher of mathematics at primary school level. The following extracts illustrate how students studied Mathematics Education 1, by first filtering the content through their selves as primary school pupil and as teacher, and then by choosing the most appropriate learning strategy available to them for the particular assessment. To some extent, when the material was not considered useful, motivation came from extrinsic vocational orientations; the qualification was needed and so the subject was studied sufficiently to achieve a passing grade. It should also be noted that students did not always have a very large repertoire of appropriate learning strategies for studying mathematics content. Much of their learning in the past comprised only rote learning and reproduction of facts and formulas; other learning strategies were often lacking.

Maria in her phase three interview discussed preparation for the final test in Mathematics Education 1:

> *[I used the] class notes, and material that we've been given, the workshops. Went through the workshops, made sure I understood that, went through the textbooks, just used all the material and sort of elaborated.*
>
> *Oh yeah, just read through the notes over and over and memorise it and highlight certain bits.*

Terry mentioned the constraints on her learning at university when talking about how she had prepared for tests in Mathematics Education 1.

> *I read the readings we had to do. All the essential readings I read and I don't think I read the ones that were ... I read the recommended ones but not the optional ones.*
>
> *I just didn't have enough time, I think. If we were given them in class I probably would have read them because I liked making sure that I'd go over everything we were given.*

But Terry saw that the topics had value for her teaching and so prepared herself quite thoroughly:

> *Because I knew that it would be needed, not just for the exam, but to do that topic and it might help me to understand that topic more or have suggestions on how to teach it or what children find hard in that topic or whatever they had to say about it.*

The way that Terry prepared indicated that she was interested in getting a general understanding of the material covered in class. Marton and Säljö (1984) classified such learning as **deep learning**:

> *Not details, mainly I'd read a section and then I'd try and pick out the main points, what's he trying to say, you know, what's the writer trying to tell me their*

view of and that's what I write down. Not usually the detail or ... if it's a formula or something, I'll write down all the steps but not if it's just general telling about something ...

I read the information we were given, either the workshops or the actual notes we made in class and I usually wrote ... rewrote them or summarised them, and then did examples and worked them out and made sure I understood them ...

On the other hand, Aaron considered most of what was done in Mathematics Education 1 to be a waste of time for self-as-student-learning-to-teach:

Then I got to Maths Ed 1. "God what is this crap?" I just didn't, I didn't find anything in Maths Ed 1 really ... I found just about everything else really irrelevant, I couldn't ... I felt maybe it might be something we'd get taught in Maths Ed 5 or Maths Ed 6 or something, because I thought most of it would have been things you use in, as extension activities so I didn't think it was very practical.

and he had quite different methods of dealing with the work. His methods appear to be those of **surface learning** (Marton & Säljö, 1984):

Well, I didn't have many lecture notes, I didn't sort of find much was said in the lectures, to write down so I got a lot of other people's notes and read theirs.

I just, I ... because I failed the first exam, I think I missed out by half a mark or something and then we got the results back for the other assignments and I mean I was just scraping through and I thought "God, I've got to pass this exam" and then I thought: "!?! it, I'm not going to spend, you know, too much time on it" No, I thought it was enough to understand enough to pass. That's pretty well the way we worked, I think. None of us were really that concerned about getting high marks, we just wanted to pass, which is, I suppose the wrong attitude, but at that stage of the year that's really all we felt like.*

Aaron is exhibiting extrinsic motivation and this affects the way he studies the subject. The reason that it is extrinsic rather than intrinsic is because self-as-student-learning-to-teach cannot see the value of the work when judging it as self-as-primary-school-pupil and self-as-teacher.

> *Especially for us, because a lot of us had trouble understanding it, and I thought, you know, if we don't understand it, how are these kids going to understand it. Then we got to this stuff and we just didn't understand it, nothing, nothing clicked and we thought "Oh, what the hell, none of us are going to teach this. We're not going to teach stuff we don't know"*

Some of the students confessed to feeling flawed when faced with new ways of learning. Having been used to a model of classroom interaction in mathematics, in which the teacher stands at the front of the classroom and transmits information to passive students, they were not happy with an alternative model presented in the first year subjects at university. The participating teacher educators all believed that learning occurs when students are active participants, striving to make meaning for themselves. Consequently, most of the topics were presented in the form of workshops which required collaboration, discussion and the assumption of responsibility, by the students, for their own learning. Teacher educators saw themselves as facilitators, guides and mediators, but tried not to act in the role the students expected of them; that of expert, always ready to provide answers, which could then be written down and memorised for assessment purposes. While most of the students conceded that this participative way of learning was an appropriate way of teaching primary school pupils (see Section 9.4), many of them felt that their past experiences prevented them from benefiting from this way of learning. Pip, in particular, found the participative, "hands-on" workshops were alien to her:

> *... even though it was simplistic and it was hands on, it wasn't my sort of maths that I could understand. This is what I'm saying, the hands on was foreign to me, totally foreign to me, that we did here.*
> *Because [in] lectures I take my notes, in the workshops you sort of forget to take your notes, that's another thing, you forget to note take during the tutorials, you really do, because you work on what you're doing but you forget to write all the other information of how you came by your answer. So that makes it very hard.*

Joanne also found the methods of learning difficult, as they were so different from her past experiences, and from her expectations for the subject:

> *I find it very frustrating when we're in maths and all the lecturer wants to do is practical workshop activities because I don't think that that's what it's all about. I think that they're very important and when you do something, you do understand it more, it fixes in your mind and that sort of thing but I don't think that that should be the only thing that we're doing.*
> *... because our generation hasn't been brought up with that all the way through, it's hard just to jump into that now and so we've been used to the pen and paper [method of] working it out and it's very much easier for us to want to go back to that and so it's hard to know....*

Joanne continues, by discussing the dilemma that she perceives the teacher educators to be experiencing, the dilemma of offering students conflict or comfort. This has been discussed by the teacher educators to some extent in an earlier section of this chapter (see Section 8.3.3, pp. 174-177)

> *You're [as teacher educators] in a situation where you need to decide whether it's more important for students to work in the hands on approach as they should be teaching perhaps, or whether it's more important for students to work how they feel comfortable. And I think that you could get a compromise of both.*

Joanne then discusses ways of improving the subject:

> *Yeah, [you should discuss] why these things were being taught and the rationale behind it and perhaps going back to the basic skills that you actually used and some sort of written, you'd write it down as you were doing it, because when then when you're coming to revise, when you come to the end of semester you've got in your resource file everything that you've done and you know about it rather than just a whole lot of workshops and activities that you then have to analyse.*

In the above quote, Joanne makes two points which indicate her perspective, which is primarily a problem solving one (see Section 7.2, p. 119): firstly, that students need to know why they are being taught the topics in the Mathematics Education 1 curriculum, and secondly, they need to be made aware of how these topics relate or link to primary school basic skills. Again, it is possible that Simon did not emphasise either of these points, as he might not have seen this as necessary due to his formalist philosophy and apparent unawareness of student attitudes.

For those who saw the links between the mathematics done in the Mathematics Education 1 subject, and that taught in primary school, the mathematics education subject seemed to tie previous notions together. Daisy was one of these students:

> *Now, when we are doing mathematics at uni[versity] on base 10 for example, I have finally placed together all the mathematics I have learnt over the years. I now know why. I understand why we did the procedural part and what for. It makes a lot of sense now - it all fits in like a jigsaw puzzle.* (phase two questionnaire, q. 8)

Others found the use of concrete material helped them to clarify their understanding of a particular topic:

> *Now of all the things I've established in that first year, I would think I picked up that, an understanding of what it means to work in a different base, because we worked with paddle pop sticks. As simple as it is, that was terribly convincing for me. I understood, I handled, I experienced.* (Gail, phase three interview)

Beatrice also found the new methods of learning were methods that suited her. It is interesting to note that Gail, Beatrice and Pip are all mature age students, yet their perceptions of what type of lessons benefited them most differed.

> *Orientation mathematics was worthwhile because I was able to learn mathematics the way that is the most relevant for me ie by exploring and*

manipulating. It also required interaction between students, allowed for discussion, debate and if desired, further exploration. (Beatrice, phase two questionnaire, q. 26)

8.5 Conclusion

This chapter has examined the first year mathematics education subjects in a teacher education course. It has traced the development of the subjects and considered the influences on the subject of the participating teacher educators' beliefs and philosophies on mathematics. The student teachers' orientations to mathematics education were then analysed and shown to affect their passage through these subjects.

Both the teacher educators, and the current literature consulted on mathematics education, indicated that there is a need for student teachers:

- to be made aware of the historical and cultural influences on mathematics;
- to improve their attitudes towards mathematics;
- to strengthen their background knowledge in mathematics;
- to learn how to teach mathematics well.

The student teachers, on the other hand, saw the purpose of the mathematics education subject sequence as to teach them how to teach; this one major aim encompassed various other goals, such as learning the correct vocabulary, or learning how to teach the basics of primary school mathematics. They did not share those aims of the teacher educators dealing with the history of mathematics or the importance of background knowledge.

It has been shown, in this chapter, that students have a strong intrinsic vocational orientation to the study of mathematics. They are interested in studying the mathematics that, in their minds, will enable them to become good teachers. This orientation has led to the student teachers assessing the value of the first year mathematics education subjects for self-as-student-learning-to-teach through the filter of self-as-primary-school-pupil and self-as-teacher. These filters influence students' assessment of the worth of the subjects, and determine, to some extent, the way that they interact with the subjects.

It has been shown, too, that the teacher educator's beliefs and goals are influential in determining how the subject is received through the filters discussed above. Simon does not appear to believe that he has to explain the rationale for the topics presented in the mathematics education subjects, and he does not seem aware of the difficulties this poses for students. His love of mathematics as an intellectual challenge leads to the creation of an environment which does not seem to favour the way most of the students learn.

A new player was seen to enter the field in the form of the supervising teacher. When faced with a conflict in the values and beliefs of the teacher educators and the supervising teachers, self-as-student-learning-to-teach has to decide who is "authentic". Self-as-teacher advises self-as-student-learning-to-teach to model practices against those of the supervising teacher, as this is the teacher in the "real" classroom, and so is presumably the "real" teacher.

The students' past experiences also play a part in influencing the way students interact with the mathematics education subjects. Strategies of learning that require passive reception of knowledge transmitted by the expert-teacher are preferred in many cases to active, participatory methods of learning. Paradoxically, while stating that such "transmissive" methods are preferred, students discuss occasions where such methods are used as being confusing and difficult to understand (see the comments by Gail and Pip on Simon's "performance", pp. 198-199). Further if self-as-student-learning-to-teach has judged a topic to be of little value for future teaching, the student resorts to surface approaches to learning.

The implications of all these results will be discussed in detail in Chapter 10. However, before doing so, it is necessary to complete the picture of the students in their first year of a teacher education course. This requires knowledge of how the students believe self-as-teacher should behave. Discussion of students' beliefs about the characteristics of a good teacher, and the goals for self-as-teacher are discussed in the next chapter.

9. "SOUR FACED CREATURES": STUDENTS' VIEWS ON TEACHERS OF MATHEMATICS AND ON TEACHING MATHEMATICS

9.1 Introduction

> *Our mathematics teachers were fairly negative in their outlook and very dull and sour faced ... those kinds of teachers were story book kinds of teachers: it's like having a stepsister looking ugly and disgusting; it's the same thing with the mathematics teacher - you expect the mathematics teacher to be very straight laced and with no sense of humour at all.* (Bianca, paired interview, phase one)

Bianca describes the views that many of the students hold about mathematics teachers, based on their school experiences. Students have a wealth of ideas about the sort of primary school teachers of mathematics they, themselves, wish to be, and these are moulded by their experiences and beliefs about teachers of mathematics.

In the last two chapters I have considered students' views on mathematics, and on the first year mathematics education subjects in the teacher education course. The previous chapter showed that the participating students have a very strong intrinsic vocational orientation to their studies at university and have a major goal of learning how to teach well. All topics dealt with, in the mathematics education sequence, are judged through the filters of self-as-student-learning-to-teach, self-as-primary-school-pupil and self-as-teacher. Self-as-student-learning-to-teach judges whether the methods of the teacher education course are valid, and guides the way assessment tasks are handled; self-as-primary-school-pupil decides whether the work to be done in the mathematics education subject is at a manageable standard for a primary school pupil; self-as-teacher pictures giving a lesson on this topic and checks this mental image to see if it matches the picture already held, of self-as-teacher at work in the classroom. The students are also very much influenced by the actions and beliefs of their supervising teachers during their two teaching practicums in the schools in the first year of their course. Students have very clear ideas of what a good teacher of mathematics in the primary school does; and students have also built images for themselves of the appropriate actions and characteristics of self-as-teacher; these are the activities and attributes that they wish to exhibit when they teach in the future.

This chapter will consider the conceptions that student teachers hold of teachers of mathematics in the primary school. It will discuss the views held by the participating students, on the characteristics of good teachers of mathematics, and of good pedagogy. It will also examine students' visions of themselves as teachers. These visions will be compared with students' experiences of learning mathematics at school and possible links between the beliefs and experiences will be considered. In particular, the chapter will address the last of the research questions:

"How do students' beliefs and attitudes affect their ideas on good practice in the teaching of mathematics in the primary school?"

As has been discussed on pp. 56-57 of Chapter 4, Thompson (1992) quotes Kuhs and Ball (1986) as identifying four major views on how mathematics should be taught:

- *learner-focused: mathematics teaching that focuses on the learner's personal construction of mathematical knowledge;*
- *content-focused with an emphasis on conceptual understanding: mathematics teaching that is driven by the content itself but emphasises conceptual understanding;*
- *content-focused with an emphasis on performance: mathematics teaching that emphasises student performance and mastery of mathematical rules and procedures;*
- *classroom focused: mathematics teaching based on knowledge about effective classrooms.* (Kuhs and Ball, quoted in Thompson, p. 136)

Thompson describes the Kuhs and Ball model further by suggesting that there are links between the views on how mathematics should be taught and the philosophical positions of the teacher. These links are discussed in greater detail in Chapter 4, pp. 56-57. My data show that espoused views on how mathematics should be taught might not necessarily be consistent with actual practice. It might be more accurate to say that the links exist between the teacher's philosophical position and actual practice than to claim links between the teacher's philosophical view and views on how mathematics should be taught. These links will be discussed in further detail at the end of this chapter.

9.2 Characteristics of good primary school teachers of mathematics

Students discussed the characteristics of good primary school teachers of mathematics in all three phases of data collection. In the first phase, some of the students asked questions such as: "Describe your best teacher/worst teacher and say why this person was so good/bad". In the phase two questionnaire, one of the questions asked students to describe the characteristics of a good primary school teacher of mathematics (see questionnaire in Appendix C, q. 13). Another question on the questionnaire (q. 17) asked students to choose the most important characteristics of a teacher, that would lead to a productive classroom. This question asked students to indicate the amount of time that should be spent on each of a variety of pedagogical principles. These principles included management of students' learning; student support; providing mathematical challenges; providing opportunities for development of mathematical knowledge; and any other principle that the student wished to include. Any responses, in which the suggested percentage of time for one principle was considerably larger than those percentages suggested for other areas, was coded at that attribute. Phase three interviews also dealt with characteristics of good teachers, by asking students to describe any teacher who had stood out in their past experience, as being particularly good, and to analyse what characteristics had made this teacher so good.

The attributes most often listed as desirable for a teacher were:

- to be supportive of pupils' endeavours;
- to have positive attitudes about mathematics;
- to possess good communication skills.

Other characteristics that were mentioned were:

- to use a variety of teaching methods;
- to exhibit good management skills;

- to be knowledgable about mathematics;
- to be knowledgable about pedagogy;
- to be knowledgable about children.

However, the last five characteristics were mentioned far more infrequently than the first three; and in fact, there were indications that being knowledgable about mathematics was considered by many to be detrimental to the quality of the teaching. This point will be discussed towards the end of this section. Each of the above characteristics will be discussed with illustrative examples.

9.2.1 Being supportive of pupils

A good teacher was, above all, caring, nurturing and supportive. This characteristic was mentioned over forty times in the students' data, and appeared in more than one-quarter of the students' documents[1]. Students mentioned this attribute often in the phase one interviews, and referred to their experiences at school, as justification for holding the view that being caring and supportive was very important. In the responses to the question on the questionnaire (see questionnaire in Appendix C, q. 17) asking students to indicate what proportion of a teacher's time should be devoted to pupil support, nine students chose to emphasise the importance of this attribute over other attributes.

The following extracts from the data illustrate the ways in which support for pupils is suggested:

> *Also, mathematics teachers need to be caring and understanding towards their pupils. They can't just shrug off a child's problem, they've got to be patient and bear with them. The child must constructively learn - mathematics must make sense to him or her, not just to the teacher. Teachers need to realise this.* (Daisy in phase two questionnaire, q. 13)

> *A good teacher is;*

[1] As has been noted in chapter 6, the numerical evidence has little statistical meaning but merely indicates general trends.

- innovative

- PATIENT

- understanding.

(Joanne, phase two questionnaire, q. 13; her emphasis)

They should be:

caring towards their pupils;

fun and likeable;

not too strict;

not discouraging towards children who are slower at picking up new ideas.

(April, phase two questionnaire, q.13)

Accessibility - the teacher and his/her willingness to repeat certain procedures is vital. The child will confidently attempt the work if fully aware of the teacher's eagerness to repeat lessons, be it in class or outside class periods. (Laura, summary of paired interviews, phase one)

Students' past experiences were mentioned often. Some students had been fortunate and had been taught mathematics by supportive teachers in the past, which reinforced their belief in the importance of supportive and sympathetic teachers. Other students had found the mathematics classroom[2] to be an unhappy and tense place, due to the attitude of the teacher. Bianca's quote, at the beginning of the chapter, illustrates the view she holds, of sour faced teachers. Daisy, Aaron and Adam reflect on their experience in different ways:

Through my schooling I have learnt that every student needs personal attention at some stage to check on their progress and to help them. I know that students are eager to learn. If you are not there to help them when they're eager, they'll become frustrated. They'll begin to lose concentration, they'll lack faith in themselves and they will ultimately learn to hate maths. Fear will then result

[2] It is often the case that although, in the primary school, the same teacher takes the class for all subjects, when mathematics lessons begin, the atmosphere of the classroom changes perceptibly.

because they can't solve the problems given to them. (Daisy, summary of paired interviews, phase one)

> *If the teacher can be relaxed and encourage you in which ever way you're going, you know, I think that's great. I was lucky I had a teacher who could do that* (Aaron, phase three interview).

> *Yes, I do think I would have learnt maths better if I had been taught it in a different way, as part of co-operative learning and group activities and by being encouraged to ask questions and being made to feel comfortable to ask questions about maths problems I didn't understand and it would have made maths a lot more fun. I would have been a lot more enthusiastic about learning, and not just think of maths as something to get out of the way because I wasn't very good at it.* (Adam, paired interview, phase one)

A supportive teacher would also encourage group work, as working with others is seen as a means of support for the individual pupil. Group work is seen as a way of removing some of the tension from the mathematics classroom.

> *... so I think as far as my views on maths teaching go, I think it's important for a teacher to build confidence in a child's attitudes to maths, and make them logical and apply maths procedures to every day life. And it should be more of a group thing because I also remember doing my mentals out of my mental book. I had to sit there and we weren't allowed to do them together, we had to sit there by ourselves and of course I was hopeless at that too so I think there should be more group involvement in maths and more of the group scene.* (Thea, paired interview, phase one)

So a good teacher is one who is supportive of the pupils' endeavours in the following ways: by being understanding and encouraging; by being aware of the different paces at which children need to go; by being happy to repeatedly explain a concept to a child; and by encouraging children to work co-operatively.

9.2.2 Holding positive attitudes to mathematics

Students generally thought that having a positive and enthusiastic view of mathematics was very important in teaching. This belief was mentioned throughout the three phases of data collection. Students referred to their past experiences and related how attitudes of their teachers had been important factors in their schooling in mathematics. As has been shown earlier, in Section 8.4.1.3, pp. 189-190, students believe that it is essential for them to improve their own attitudes to mathematics if they are to be effective teachers of mathematics in the primary school. Data on the holding of positive attitudes occurred frequently; again over forty times, and the need for the teacher to have positive attitudes to mathematics was mentioned in approximately one-quarter of all the students' documents, from phase one to phase three.

The following quotes all show the importance of good attitudes, as perceived by student teachers.

> *[A good teacher is] keen and motivated. The sort of person who can sort of suck the kids into their own enthusiasm, I think. You've really got to like it otherwise if you don't like a subject you're not going to impart any enthusiasm for the subject anyway. I suppose just mainly keen and motivated and very well prepared. But that applies to every subject.* (Nita, phase three interview)

Having a good attitude to mathematics seemed to solve a variety of problems experienced in mathematics lessons. If the teacher has a positive attitude to the subject, this attitude helps to make mathematics more palatable for the pupils. As a result, pupils develop a positive attitude to the subject:

> *If they [the teachers] have a very good attitude they will make it more interesting. If they are interested in the subject then they will liven it up and make it known that they like the subject and enjoy it. It's more likely to catch on with the children than if they don't. Then they'll [the pupils] catch on and enjoy*

it too and that brings about a better attitude for the children. (Bev in paired interview, phase one)

I suppose the way of teaching maths, like my mathematics teacher, if they have got a positive attitude to the subject and they really, really enjoy the teaching of maths, have real enthusiasm for it and the kids can see that, then that's going to rub off on them. Yes, yes that's the heart of that question. I'd say that the teacher's attitude to the subject is mirrored in the students' attitude to it. (Colin, paired interview, phase one)

Once the children have a positive attitude to mathematics, they will be prepared to spend more time doing mathematics and so will not experience problems with it:

If the teacher's attitude is positive or excited the child will like maths and go home and practise it and practise it and have fun doing it so they will have a very good ability in doing mathematics. (Bernadette, paired interview, phase one)

The idea that teacher's attitude is strongly linked to pupil's ability is shared by a number of students:

The teacher's attitude is vitally important as it is reflected in each individual child's attitude and ability. A teacher's job involves improving both, so the teacher must have a positive attitude towards maths. (Beryl, summary of paired interviews, phase one)

From my reflections I realise that as a teacher you do have to give each student the opportunity to make maths a positive experience and one way of doing this is by giving each student the resources to meet their needs, but also by being positive as an educator, that maths is not something that has to be a chore, but can be a fun and worthwhile learning experience. (Bronwen in her summary of the paired interviews, phase one)

> *I also see the enthusiasm of the maths teacher as the key to facilitating learning, as experienced by me in my senior years of high school.* (Doreen, summary of paired interviews, phase one)

However, Camilla warns that the positive attitude should be genuine, and not assumed:

> *They should be enthusiastic about mathematics but the enthusiasm should be genuine and the activities should reflect their enthusiasm.* (Camilla, questionnaire, phase two, q 13)

The above extracts from the data all show the belief that having a positive attitude is an essential attribute of a good teacher of mathematics in the primary school. Possession of such an attitude is thought to inspire pupils to become enthusiastic themselves, which in turn leads to more time being spent on the subject, with the result that ability in mathematics improves. The difficulty is that the attitude must be a genuine one, and many of the student teachers do not have a positive attitude to mathematics. This point shall be discussed further in the section on students' visions of self-as-teacher (Sections 9.6.3 and 9.6.7).

9.2.3 Having good communication skills

Good communication skills were thought to be an important part of being a good teacher. Data dealing with communication in good teaching appeared about twenty times. The data on this attribute were found throughout the three phases. Students discussed the ability to explain clearly in their phase one interviews and summaries, and responded to the questions about good teachers in phases two and three by indicating that this skill was important. The following quotes indicate the ways in which this attribute was mentioned.

Some of the students described good communication as the ability to present difficult concepts clearly, so that these concepts were not confusing to the learner:

[The characteristic of a good teacher is the] ability to represent mathematics principles in a way easily understood, or preferably seen or explored by pupils. (Clive, in phase two questionnaire, q. 13)

[A good teacher] encourages experimentation. Explains clearly and ensures understanding. (Robyn, in phase two questionnaire, q.13)

[A good teacher is] one who explains concepts thoroughly and checks understanding and works through numerous examples. (Grace, in phase two questionnaire, q.13)

It is noteworthy that, to Clive, good communication is not only the ability to explain clearly but also to structure activities so that the pupils can gain understanding of concepts by exploration.

Others felt that good communication involved the passing on of enthusiasm for the subject and presenting the material in an interesting way:

... that they are able to communicate to the children that mathematics can be interesting and relevant. (Bridget, in phase two questionnaire, q.13)

I believe maths should be taught with clear instructions and so that it is fun for the children to learn. (Dawn, in summary of paired interview, phase one)

Robyn had more to say about the value of communication in this area:

[A good teacher] provokes interest, challenge, conversation. (phase two questionnaire, q.13)

Experiences in the past seemed to be influential in determining the importance of any of the attributes. Both Agnes and Aaron felt communication skills were important for a good teacher, and that the presence, or lack, of such skills in their teachers during their own schooling had affected them. It is also worth noting, at this point, how students gained insight into each other's views through the methodology of phase one. This emerges from the data:

> *Aaron when reflecting on secondary school also made a general comment on the teachers he encountered there. He said that on average most teachers were good and that this depended on the way the teachers could communicate to the students.* (Agnes about Aaron, in the summary of the paired interviews, phase one)

> *I have realised that a teacher's attitude does affect those they teach as I had a teacher in secondary school who was a lovely lady but not a very good teacher. This teacher had difficulty in communicating what she knew to the class and so the class as a whole weren't very confident.* (Agnes about herself in the summary of the paired interviews, phase one)

The remaining characteristics mentioned as being desirable for a good teacher appeared far less frequently in the students' data. For these five characteristics, fewer than ten mentions were made of any of them, throughout all the students' data. This indicates that the majority of students did not perceive these characteristics as vital for a good teacher. Of course, it is possible that for some students, these characteristics were extremely important, but the infrequent mention of these characteristics indicates that such students would have been few in number.

9.2.4 Using a variety of teaching methods

Eight students felt that showing versatility in teaching methods was important. All instances of quotes on this attribute appeared as responses to the question on characteristics of a good teacher (q. 13) in the questionnaire of phase two. Students did not mention the need to use variety in teaching, in the phase one interviews, or other data of phase one. It appears that awareness of the need for variety evolved during students' first year in the teacher education course. Another possibility is that this awareness is an effect of the progressive focussing of the methodology.

The quotes showing ideas on versatility are given below:

> *[A good teacher is] one with a positive attitude towards mathematics. With various ideas about how to present particular concepts. One who employs different games, activities and group work and most of all one who can stimulate their students and let them make their own observations and conclusions about particular concepts - yet guide them to the right ones.* (Christine, q. 13, questionnaire, phase two)

> *[Characteristics of good teachers are] relating mathematics to their use in everyday life, explaining, not just teaching the facts. A variety of methods taught for the one concept, variety in lessons, practical application.* (Terry, q.13, questionnaire, phase two)

> *Teachers need to show interest, have lots of energy, be funny, have a lot of vitality and most of all be courageous and take challenges. Do different things constantly, don't stick to the same old boring routine - change styles when teaching mathematics and be flexible.* (Daisy, q. 13, questionnaire, phase two)

> *[A good teacher is] someone who is versatile and able to teach many varied mathematics lessons - from drill to experiments.* (Nita, q. 13, questionnaire, phase two)

These quotes indicate a belief that if a teacher can vary the methods of teaching a topic, this will enhance the learning process for the children, as well as making it more interesting.

9.2.5 Possessing management skills

Eight student documents indicated that students thought that the ability to effectively manage the learning in the primary school classroom was an important characteristic possessed by a good teacher. Interestingly, this data arose mainly in the phase two questionnaires: no-one mentioned this early in the year, in the phase one data collection, and only two (Nita and Aaron) mentioned the need for management at the end of the year, in the phase three interviews. The data arose mainly from the question on the questionnaire of phase two (see Appendix C, q. 17) which specifically asked students to allocate time allowances to a variety of attributes, one of which was management of student learning. Five of the eight students, who are indexed as thinking that management skills are important, are classified in this way, because they suggested a larger proportion of time be spent on management than on other areas. One of these students also answered the question on the characteristics of good teachers (q13 of questionnaire) by saying:

> *A good primary teacher allows children to explore concepts and ideas while providing experiences that will enhance their learning by being relevant, practical and interesting.* (Sally, questionnaire, q 13)

Another student suggested that a good teacher was able to "*create a good learning environment*". (Alice, questionnaire q 13)

It appears, from the above data, that classroom management was not a characteristic that students had considered important until this was suggested to them in the questionnaire (q.17). Further, it should be remembered that no student mentioned this attribute in the phase one interviews, which occurred at the beginning of their teacher education studies, before any teaching experience had occurred. The phase two questionnaires were answered after two practicum experiences had been completed. These practicum experiences might have indicated to students the need for some sort of classroom management. However, if this suggestion is true, then it is surprising that discussion of the need for classroom management did not arise in greater detail in the phase three interviews, which were also after the two first year practicum experiences. It appears that there are a number of possible reasons why, for most students, classroom management was not considered as worthy of discussion. These could be because the level of teaching that they had experienced on the practicum did not expose such a need; or perhaps because the whole issue of classroom management was not considered to be specifically a mathematics teaching issue, and so was not considered appropriate for discussion in this study; or perhaps because the need for classroom management in a primary school classroom is not as explicit as it is in a secondary school classroom. It is also possible that classroom management is not presented as being problematic by teacher educators in the educational studies area, or viewed as being an issue by first year students. The shift in view of the teacher as performer to teacher as manager is a more sophisticated understanding of the act of teaching.

9.2.6 Being knowledgable about mathematics

Only five students felt that a good teacher should be knowledgable about mathematics, and all data on this emerged from the question regarding characteristics of a good teacher (q. 13 on the questionnaire - see Appendix C). All five students also believed that a teacher needed to be enthusiastic and/or supportive of pupils. Their quotes are given below:

> *[A good teacher has the following characteristics:]*
> *1. Sound knowledge of mathematics*
> *2. Enjoys mathematics and is willing to make it enjoyable to the students*
> *3. Provides working material for students - involving models, equipment etc.*
> (Laura, q. 13, questionnaire, phase two)

> *[A good teacher is]*
> *innovative*
> *patient*
> *understanding*
> *knowledgable. Mathematics requires knowledge, a teacher should not wing it.*
> (Joanne, q.13, questionnaire, phase two)

> *They have to know what they are doing. Be positive and look like they are enjoying the subject. Understand the underlying uses of the principles.*
> (Emma, q.13, questionnaire, phase two)

[A good teacher shows]

- enthusiasm

- belief in students' abilities

- [ability to]create a good learning environment

- reasonably sound knowledge.

(Alice, q. 13, questionnaire, phase two)

[A good teacher has a] good understanding of knowledge of mathematics and the way it can be used;

is able to take risks;

is willing to take children outside the classroom.

(Bernadette, q. 13, questionnaire, phase two)

It can be seen from the above data, that the need for teachers to be knowledgable about mathematics did not appear to be important to students in phase one; the early stages of their university studies. No student discussed this need at any point of their paired interviews, or in their summaries about the implications of their past experiences and attitudes. It is only in the questionnaire that a few students mention the need to be knowledgable; and as can be seen in comparison with earlier categories on characteristics of good teachers, the number of students who see knowledge of mathematics as an important characteristic is very small. This situation may reflect the students' perceptions of their own weaknesses or vulnerabilities (see Section 7.7). It is likely that if students perceive themselves to be uncertain in deeper knowledge, they are not going to cite this sort of knowledge as a requisite condition for a teacher.

9.2.7 Being knowledgable about pedagogy

Three students showed awareness of the need for pedagogical content knowledge (Shulman, 1986b), although not necessarily as Shulman has described such knowledge (see Chapter 3, p. 41). Each student's data came from a different phase of the data collection. The quotes and their sources are shown below:

Emma felt that teachers had to be quite knowledgable about the uses and value of the concepts studied:

> *For teachers to introduce new concepts they have to find beforehand the correct way to help children understand how they can be of use. A great deal of thought and investigation must firstly be used by teachers to decide whether the students will benefit from the new concepts they want to introduce.* (Emma, summary of paired interviews, phase one)

Aaron believed that it was necessary for teachers to find out what games they could use in their teaching:

> *But I suppose there are a lot of other games but the teacher's got to know them to start with. And I think it depends a lot on your class, like if you've got a class who just won't sit still and do a game like that then you gotta find another way.* (Aaron, phase three interview)

Mandy thought it was necessary for the teacher to know how to make the mathematics interesting for the students, as well as ensuring understanding:

> *A good primary school teacher of mathematics should be understanding towards all children of different abilities and should know the underlying concepts and methods that teach mathematics in an effective non-boring way that the children can understand.* (Mandy, q. 15, questionnaire, phase two)

9.2.8 Being knowledgable about children

Aaron and Gail were the only students who indicated that a good teacher should have some knowledge of his/her pupils. Aaron's quote in the above section illustrates this awareness; another quote by him is given below:

> *[A good teacher has] confidence, ability to explain in different ways, knowledge of students.* (Aaron, q.13, questionnaire, phase two)

Gail discussed the difficulties of student practicum teaching, where the student teacher did not have an opportunity to get to know the pupils well:

> *As a regular teacher I guess you've got a greater understanding of what a child knows and what he doesn't know and I think that the first - term is perhaps a long time - but well, maybe the first term is really establishing that understanding of the basis on which you're going to build the rest of the year.*
> (phase three interview)

The above data show that, to the student teachers, the major characteristics of a good teacher are the following: the ability to be supportive of pupils; to possess a positive attitude towards mathematics themselves; and to be able to communicate well. Other characteristics that were seen by only a few students as important were: being able to use a variety of teaching methods; ability to manage the learning environment effectively; and being knowledgable about mathematics, pedagogy and pupils. These data seem to indicate that, to most students, the ability to be a good teacher cannot be learnt, to a large extent, as it is dependent on the teacher having a suitable personality and a good attitude. The data also indicate that the teacher is at the centre of the students' thinking rather than the learner. As being knowledgable about mathematics, pedagogy or pupils is not seen as an important characteristic of a good teacher, it would appear that the major value of teacher education courses in mathematics, is in teaching the students how to explain well. Subject matter knowledge and the ability to do mathematics are often seen as liabilities for good teachers. This alarming situation is discussed in the next section.

9.3 Teachers who can do mathematics

Many students saw the ability to do mathematics as a disadvantage for primary school teachers. Such students believed that being able to do mathematics oneself would make it difficult to understand students' problems and so would interfere with the most important characteristic of a good teacher; being supportive of pupils' efforts. Many of the students who felt this way cited examples from their past schooling which justified their view. Data on this belief appeared in all three phases of student data.

Pip talked about the advantages of having little background knowledge, in the phase three interview.

> *I won't come in with any extra baggage that I have picked up all the way through high school and this is ... why I think I click with little kids, with the kids, because I'm still on their level, with a lot of things ...*
> *We [mature age students] don't have the HSC knowledge to fall back on. [But] it didn't bother me because I thought I could handle ..., I know I can do up to the level of maths that primary can do.*
> *I know just from personal experience I don't ... because of my lack of knowledge I don't get agro [aggressive/angry] if they don't get it right. I can sit down for hours and help them work a sum out and show the different ways of working it out without losing my temper, because I don't have this ... baggage of extra knowledge on me. Because I didn't have this knowledge, is why I was told I was stupid. "You should know how to do it, you're an idiot." But I didn't have the knowledge to do it ... so you know, I think it's an extra plus that I've got, to take into a classroom.*

So Pip sees her lack of knowledge as an advantage to her teaching. Rene shares Pip's view about lack of understanding being an advantage in teaching:

> *I feel that my lack of understanding in particular areas at primary school will aid my teaching of that area. For example I will have a better knowledge of teaching fractions and of its practical applications. This will enable me to understand and react appropriately to the problem the students may have.*
> (Summary of paired interview, phase one)

Aaron relates how he battled with mathematics at school, when he had a teacher who was a brilliant mathematician. It appears that students like Aaron believe that their difficulties with a teacher arose because of the teacher's ability in mathematics rather than because of other characteristics of the teacher. It is interesting to note that Aaron's comments about his teacher at school are very similar to Pip's comments about Simon (see Chapter 8, p. 198). While both Pip and Aaron express awe at the ability of the teacher, they acknowledge that they were not able to understand the teacher's explanations.

> *He was a genius at maths, like he could do anything. Especially when you're in year 8, you see this guy who can do these amazing things with mathematics and you think "God, who is this guy?"*
> *But he just couldn't ... I suppose he was frustrated with us dills who just sat there but ... he was brilliant at maths, he just couldn't get it across, unless you were brilliant as well, then he got through to you with his method but uh (shakes his head) ...* (Phase three interview)

Other students also gave examples of teachers who were good at mathematics and seemed to be poor teachers as a result of this ability:

> *I think that background knowledge is important but the knowledge of teaching procedure is more important. Many people are good at mathematics but because it has always come easily to them, are not so good at explaining it to others.* (Sally, q.11 in phase two questionnaire)

Some students asked questions, in the paired interviews of phase one, indicating that the questioner had doubts about the value of being confident about mathematics:

> *At primary school you were confident ... will that make it easier for you to teach and empathise with students that have less ability than you or wont [it?] ...* (Dale asking John in paired interview, phase one)

This theme was picked up by other students in the paired interviews, when their partners seemed to have a high ability at mathematics:

Clive's attitude towards mathematics reflects the way he will teach mathematics in the future. Maths is an important subject to him and therefore he will teach it enthusiastically which often influences the students' attitudes towards the subject. ***Although*** *Clive did well in this subject he realises the importance of developing these skills and will use a variety of methods to ensure the students' understanding.* (Terry, in summary of paired interview with Clive, phase one. The boldface is mine)

Dale also expresses this reservation about John's ability at mathematics and its implications for his teaching (the emphasis is mine):

His experiences in this subject at junior school were generally very positive, apparently as a result of his competence as a mathematician.
John considers some of the new innovations in maths teaching useful, but overall he seems to prefer more 'classic' lessons, centred around individual students using workbooks or similar resources. He feels that the brighter students will automatically enjoy maths, but will use some new techniques to interest and involve those of lesser ability. John is well aware of the need to empathise with struggling students, ***although occasionally he might find this difficult.*** (phase one summary)

The opinion, that being good at mathematics was not necessarily helpful for teaching mathematics in primary school, was expressed by students who were good at mathematics, as well as those who were not:

Because I picked it up so easily, I think that will make it hard for me to teach it in a way, because I have trouble understanding how hard it is for some kids to pick it up. So I'll really have to concentrate on giving everyone a fair go, trying to explain as best as I can so really it's giving them a fair opportunity to understand the work in their own time. (Cara, in paired interview, phase one)

Those who suffered from a lack of confidence in their own ability in mathematics, comforted themselves that it was not all that important to be able to do mathematics (the emphasis is mine):

> *I'll think "Oh God, I've got to teach maths now" and get all uptight about it. I've got to say it's not hard, instead of thinking it's gonna be hard. It is, it's attitude and so I'll have to ...* ***It doesn't matter if you're not good at it ...*** (Bronwen, in paired interview, phase one)

However, there were students who did not agree with this view and felt that their knowledge and ability were advantages:

> *I guess my ability to understand a lot of the maths throughout school enabled me to enjoy it more easily, whereas people who struggle with maths don't often see the advantages of knowing mathematics.* (Mandy in the summary of paired interviews, phase one)

And another student indicated that lack of ability in mathematics was, in fact, a disadvantage for teaching:

> *As a result of these experiences, April expresses her reluctance to teach maths in a primary school. It could be said that this may be due to a lack of confidence in her own mathematical aptitude.* (Barbara about April, in summary of paired interview, phase one)

In general, however, it appears that most of the students do not value knowledge of mathematics, neither as a purpose of mathematics education courses at university, nor as a characteristic of a good teacher. In fact, they seem to feel that such knowledge is a disadvantage to teaching in the primary school.

9.4 Views on good pedagogy

Students discussed their beliefs about good pedagogy throughout the study. Many students compared their past experiences to those they felt would have been more effective in learning mathematics. As a result of these experiences, students seemed to form some clear notions of what constituted good pedagogy. Students' notions of good pedagogy agreed with many of the principles recommended by the teacher educators. These principles appeared to develop, not only from students' past experiences, but also from their experiences during the teacher education course, and their reading of the literature and observation in primary school classrooms. Several pedagogical principles were suggested by students. The three principles most commonly suggested were (in order of perceived importance):

- using practical methods of teaching;
- illustrating the relevance of the mathematics;
- making sure mathematics lessons are fun.

These three principles for good pedagogy were mentioned very often in student documents from all three phases. Each principle was discussed in over half of the student documents, and in a variety of contexts: teaching in a practical way was mentioned 145 times; the demonstration of relevance 117 times; and the need to provide lessons that were fun 80 times. These principles were clearly regarded as very important in the teaching of mathematics.

Other principles that were regarded as part of good pedagogy were:

- using variety in teaching;
- teaching in a child-centred way;
- ensuring teaching was at the correct level for each child;
- ensuring lessons were interesting;
- covering theoretical aspects of mathematics;
- providing challenges for pupils;
- showing the connections between mathematical topics and other areas (not mentioned in paired interviews or summaries of phase one);
- providing drill for pupils (only mentioned in phase two and three data).

A minority of students mentioned the following three principles as part of good pedagogy:

- demonstrating how to do the work (only mentioned in phase two and three data);
- making use of questioning techniques (not mentioned in the paired interviews or summaries of phase one);
- introducing innovation into lessons (only mentioned in four documents);

It is worth considering at which stage of the year principles such as the above three arose. It would appear that students did not consider the use of demonstration and questioning at the beginning of their first year subjects. However, these principles arose after students had been exposed both to the teacher education subjects and to the practicum. It is possible that both these phenomena influenced the emergence of ideas about demonstration and questioning. The use of questioning is discussed as a teaching strategy in some of the education studies that students undertake in the first year. It is also likely that students had some experience of demonstration in mathematics lessons, both from Simon in Mathematics Education 1 and from their supervising teachers during the two practicums.

Two other aspects of pedagogy were discussed, primarily in the questionnaire (see Appendix C, q. 7), in a question which asked students to discuss the balance that should exist, between procedural and conceptual knowledge, in order for effective learning to take place. The questionnaire introduced the concepts of procedural and conceptual knowledge, and then asked students to determine the balance they believed each aspect of knowledge should contribute. This led to the following principles being suggested:

- emphasising conceptual aspects of content matter;
- teaching procedural aspects of mathematics.

Conceptual aspects were considered to be more important than procedural.

Students' views on good pedagogy parallelled the teacher educators' views in many ways, but an exception to this match was the challenging of beliefs about learning and about mathematics. The teacher educators had mentioned this principle as part of good pedagogy, but this notion was not mentioned at all by students.

9.5 Students' voices on good pedagogy

Examples of the ways students perceived the principles listed above are given for each of the principles. Many of the quotes came from a source of data that has not been used up to this point. Students were required to write journal entries on how they believed pupils best learnt mathematics (see Chapter 5, p. 78-79): because these were responses on a particular topic, they were not as open-ended as other data collection methods. Consequently, it is principally in the discussion of good pedagogy that the data from these documents appear relevant. I have called these journal entries "Beliefs documents" in the quotes.

9.5.1 Using practical methods of teaching

Students were suggesting a variety of practices when they discussed using practical methods of teaching. Some of the different meanings are illustrated in the quotes below. Some of the students believed that teaching in a practical way implied use of materials, and "hands-on" activities, which encouraged exploration and discovery:

> *I think the best way to teach maths is to give children materials and tasks to do, allow them to do the tasks and then regroup to discuss their findings and then the teacher can link their experiences to maths concepts. Basically teach with practical materials and exploration.* (Adi, Beliefs document, phase one)

Others saw the value of practical methods as helping pupils to apply mathematical concepts to the solution of problems:

> *I think children best learn maths through practical methods of learning. This is because it is "easy" to explain the basics to children but to make sure they really understand and have grasped the concept you are teaching, they need to be able to apply what they know to examples.* (Agnes, Beliefs document, phase one)

I also believe that the teaching of mathematics should not be something that you can't see and can't relate to, but something concrete and related to the everyday lives of the children. (Bronwen, Beliefs document, phase one)

The above examples suggest that practical mathematics is applied mathematics; but as Daisy shows, applied mathematics can simply be the creation of word problems:

I would also bring practical situations into maths so problems are easier to understand eg Suzy went to the shops and bought some chips, carrots, ice cream and a hat. What did this add up to if the mean (average) was $16.95? (Beliefs document, phase one)

However, other students discussed problem solving in a real life context as opposed to solving problems to practise a skill:

Colin acknowledges the importance of practical maths. He believes that maths can be made more interesting by relating it to real life situations. Practical maths provides more of an incentive for students to solve problems, as opposed to working out problems for the pure sake of it. By making maths more practical, the students are more motivated to learn because they realise the importance of maths in everyday life. (Christopher about Colin in summary of paired interview, phase one)

Some of the students thought that a combination of theory and practical activities was the best way of learning:

I think that children learn maths to the best of their ability through written and practical methods and that fun and interesting practical activities then enforce this knowledge and make maths more enjoyable. (Bonny, Beliefs document, phase one)

Beryl warns that practical work without substance might have undesirable consequences. Like Bonny and others she believes that a mix of practical work and theory are needed, but she is the only one who mentions the difficulties inherent in merely having fun.

> *Yet too much practical field work may lead the students to believe that maths is fun - but that is all. Its importance may diminish with the amount of practical work involved. I believe that the practical work should be there to demonstrate the theory, to support it, so the children have a wider understanding of the applications.* (Summary of paired interviews, phase one)

Practical work is also seen as more motivating and enjoyable, and encouraging social development:

> *Maths can be made much more enjoyable and less monotonous by putting more emphasis on practical work. Practical work would make the students more enthusiastic and interested, which is very important for their mental development. Group work would also improve their social skills and attitude to mathematics.* (Christopher, Beliefs document, phase one)

Students acknowledged being influenced by the video, *Getting the most out of maths,* which they had seen in the introductory mathematics orientation session:

> *The video which we watched in the first maths workshop showed a practical demonstration of volume. The children were working in small groups but with their hands. That's how I think kids get the most out of maths, by looking at how it can fit into practical everyday uses instead of pointless textbook exercises.* (Rene, Beliefs document, phase one)

Amy's quote below is noteworthy because she seems to be one of the few students who suggests the practical work precedes the theoretical; most other students indicate that the concept should be learnt first and then practised and applied in a practical way. Amy's view agrees with the problem solving view that the mathematics develops out of the experience or problem:

Children learn maths best through practical experiences with a theoretical follow up. The practical lets the children see how maths is related to real life and in physical terms. The practical side of maths is usually more interesting than straight equation learning, so the children learn more from the experience. (Amy, Beliefs document, phase one)

Doreen also seems to hold the view that the practical situation will give rise to the concept that needs to be taught:

I think the best way is to make the learning situation as practical as possible. For younger children, shops, weighing objects, measuring objects can often be incorporated as "play-time". I would try to introduce each maths lesson with a practical activity that involves the concepts to be taught.

In general, the students do not appear to disagree with the current literature on how pupils learn, in that they see learning taking place when pupils are actively involved in the learning process themselves, and are able to see applications of the mathematics that they are learning.

I think maths has got to be a lot of practical exercises and stuff that the kids can get involved in and, if it's too much out of a textbook, a lot of the kids just don't catch on as quickly. (Bridget, paired interview, phase one)

If I was in primary school and we did workshops in primary school it would have made me then have to ask or understand, whereas if you are just sitting back, there's no need to understand because you're not doing anything, or you can hide it. (Terry, phase three interview)

Further, some of the students felt that the lack of practical methods in their own learning as school students had contributed to their lack of understanding and enjoyment of mathematics at school:

She felt that if more practical teaching methods had been employed in her maths lessons, she would have enjoyed maths much more and would have been motivated to learn maths. This would have helped her to be prepared for high school maths more than what she was. (Bonny about Camilla in the summary of paired interview, phase one)

It is interesting to note, however, that many students held reservations about learning using practical methods, when discussing their experiences in the first year mathematics education subjects at university (see Section 8.4.3, pp. 205-206).

9.5.2 Illustrating the relevance of the mathematics

Another concept that was considered very important in good pedagogy was the demonstration of relevance of mathematical concepts learnt. This is closely linked with the practical teaching principle discussed above, in which practical teaching is considered to be teaching in which knowledge is applied.

Children best learn maths when they can see the relevance of the subject to everyday things. They also learn better if they can see how something works and can be made entertaining. (Sally, Beliefs document, phase one)

I believe that children learn maths best when they are shown how certain mathematical concepts work in reality and in everyday life. (April, Beliefs document, phase one)

I believe children best learn and understand maths if they can see a practical application of the theory. If they can see how it relates to their environment and that it actually has a purpose then they have, I believe, a much better chance of retaining the information. (Beatrice, Beliefs document, phase one)

The views in these quotes, and many others like them, agree with the views expressed by the students, in Chapter 7 (pp. 153-156), where they discuss their feelings about mathematics and its usefulness. Most of the students believe that mathematics must be shown to be relevant, and so good pedagogy includes illustrating this relevance to pupils. Colin explains the importance of relevance in depth.

> *Well, I think mathematics is a subject where if you make the activities more related to real life and real life situations and interests that the kids have, then it's going to make it more interesting ... Yeah, so for example [for] a particular problem in mathematics, the children can much more easily relate to something that they are going to find in their lives. And so therefore they will have more of an incentive to work it out if it's something that is useful, rather than the only use for this is well, up in our heads or I guess, a mark out of ten for these questions.* (Colin, in paired interview, phase one)

9.5.3 Making sure mathematics lessons are fun

Ball (1988d) and Foss and Kleinsasser (1996) record similar results in their studies of American preservice elementary teachers' beliefs about teaching mathematics. These results were that student teachers believed that "making mathematics fun" was central to the learning process. Further, they found that students expected that lessons that were enjoyable would succeed in teaching mathematics, regardless of the content or context of the lessons. These findings certainly correspond with my data, which indicate the importance that students place on providing fun, as a pedagogical principle. This result appears to be generalisable across two different countries with three different student cohorts, in that the data that I have gathered seem to be consonant with the data produced in the other studies.

Students throughout my study talked about the need to make mathematics fun. As a result of the importance this statement appeared to have in the phase one data, a question on the phase two questionnaire probed further what might be meant by fun (see questionnaire in Appendix C, q. 12). The question asked: "Many students stated in the interviews that they believed mathematics should be fun. What is your view? Please explain first what you understand by the expression 'mathematics should be fun'."

The quotes below are taken both from the phase one data, in which this concept of fun was originally mentioned, and then from responses to the above question in the phase two data.

> *I think children learn maths best by having it made enjoyable to them. If maths is presented through activities, games and other group and individual workshops, children are likely to develop an enthusiasm for maths and hopefully then curiosity will increase.* (Barbara, Beliefs document, phase one)

> *The best way to teach and for children to learn maths is to create an enjoyable atmosphere so as to make maths "fun".* (Daisy, Beliefs document, phase one)

> *If you teach students in a way that creates games and activities they seem to remember it more. They enjoy it a lot more which improves their attitudes towards maths and helps them learn more. Also just to make the lesson more exciting for them, it could help for them to learn that way too.* (Lexi, paired interview, phase one)

Again, the video shown in the first session of the orientation course was mentioned by a number of students, as they had been struck by the apparent enjoyment of the pupils in the video:

> *Like it could be approached in a different way, to make it seem more fun, like in that video [Getting the most out of maths - shown in first lecture 1/3/93] where they did the games with the beads and stuff [laughs] and you could do it that kind of way - the message gets across just as well if not better but you don't have a negative approach to it which would make a big difference.* (Di, in paired interview, phase one)

Bridget explains how her past experiences have led to this emphasis on having fun in mathematics:

Our conception of the way in which we were taught maths seems to be that it was hard, dull and boring and that this needs to be rectified in order to make maths as interesting and as much fun as possible. Thus ensuring that our future students gain a better understanding of the basics of maths and a positive attitude to the learning of it. (Summary of paired interview, phase one)

Different students had different conceptions of what "fun" was, though: to some it was the playing of games, in contrast with their school experiences of sitting in silence, completing textbook exercises; others had a more profound meaning for fun. Contrast the two quotes given below by Christine and Gail. Note that Gail is a mature age student who was at school over thirty years ago, while Christine is a recent school leaver:

Maths needs to be fun, and made able to relate to. I believe if maths is taught in a fun, practical sense, by using maths-related games and group work it would be far more beneficial in the long run. (Christine, summary of phase one paired interview)

Well, I think it's more than important, I think it's actually essential and fun, meaning, I said this in my questionnaire, that fun to me isn't just frivolity and laughter, fun is that freedom to understand, to conceive something, to grasp it. That to me is fun, I love that. (Gail, phase three interview)

In the questionnaire (q. 12), Gail explained further what "fun" meant to her:

- *That there should be involvement and participation*
- *Being fun does not mean to me a constant flow of tricks and puzzles*
- *A child (AND A UNI STUDENT) can only have "fun" when s/he is understanding and achieving - there is no fun in being overwhelmed and failing*
- *Fun implies to me a teacher who realises that a concept must be presented in a variety of ways in order that people of all levels grasp that concept*

- *Fun means - never being humiliated for being wrong or not knowing how to do a question!*

So to Gail, fun occurs when the classroom environment is supportive and encouraging of learning.

Aaron did not view fun as being as important as the opportunity to see relevance in the mathematics:

> *I think [mathematics should be] interesting and relevant more than fun. Mathematics would be more enjoyable because the students can see it as useful.* (q. 12, questionnaire, phase two)

Agnes saw fun as being the antidote to her experiences in the mathematics classroom.

> *What I believe the statement suggests is that mathematics should be able to be experienced and enjoyed. And that it doesn't have to live up to its expectations [in] society of being boring and completely "black and white". Children should be able to explore and discover with mathematics - something I did not get an opportunity to do.* (q. 12, questionnaire, phase two)

Lexi viewed fun in a similar way to Agnes:

> *Mathematics should be fun and enjoyable. I believe the student teachers were referring to mathematics not being difficult, useless and boring but enjoyable, challenging and interesting.*
> *Children who enjoy mathematics and think it is "fun" will continue to attempt mathematics in future years.* (q. 12, questionnaire, phase two)

Fun appeared to be required to prevent mathematics from being boring; most students seemed to have found mathematics to be very dry and boring in their school days. This feeling was expressed repeatedly:

Saying that "mathematics should be fun" indicates that a child should be keen to learn and not try to avoid it because it is boring. Mathematics should be taught in a variety of ways and should involve using concrete materials wherever possible. Fun activities should be incorporated into the lessons to keep the children interested. (Mandy, q.12, questionnaire, phase two)

So fun should be incorporated into lessons as a remedy for the dull, boring experience usually provided (according to students' perceptions), and for most of the students this is done by introducing games, relevance, and group work, in the hope that this will make lessons more interesting. Concrete materials and exploration are also suggested to enhance lessons. However, only Gail seems to believe that understanding mathematical concepts provides fun.

The beliefs that students hold about the above three pedagogical principles are similar to some of the beliefs of the teacher educators: good pedagogy incorporates teaching in a way that encourages the school students' active participation in learning, through use of concrete materials, applications to students' environments, and the emphasis on the relevance of the learnt material. The learning process should be made enjoyable and encourage co-operative learning. However, the teacher education students only appear to share some of the beliefs of the teacher educators about good pedagogy; a silence appears to exist in the students' data, about the links between the use of practical, hands-on methods and the extraction of mathematical principles from the practical activity. The practical pedagogy suggested by the students appears to have a major goal of making the learning process fun, rather than promoting learning; and it is the having of fun that is seen as leading to learning, rather than the practical activities themselves. Consequently, a priority develops when planning any mathematical activity for teaching: the activity should first and foremost be enjoyable. The mathematical content of the activity is not seen as being as important as the activity's relevance and enjoyability.

The other principles for good pedagogy were not mentioned as often as the above three principles. This seems to indicate that they were not considered by the majority of the student teachers to be as important as the above three principles, although they may well have been considered important by individual students. Each of the next set of pedagogical principles will be discussed below with extracts from the students' data to indicate how they arose.

9.5.4 Using variety in teaching

The most commonly stated principle in the next group of pedagogical principles was that variety was important in mathematics lessons. Just under one third of the students' documents contained references to variety and it was mentioned 65 times in those documents. This agrees with students' notions of good teachers as being those who are versatile in their teaching, as discussed in the previous section (pp. 220-221). The discussion of variety appeared in all phases of data collection, in an assortment of contexts.

Colin shows an awareness of the complexity of the nature of children's learning and responds to the question on how children best learn (Beliefs document) by discussing the importance of context: Colin's beliefs about the context of learning are shared by the teacher educators and can also be found in the current literature:

> *I believe the best way for children to learn maths is not confined to any one way or style of teaching. There are so many variables that to say one style of teaching works best is not possible; eg time of day, teacher personality, the children as individuals themselves, the children's and teacher's attitudes. I would guess that using a variety of methods would work the best; e.g. formal teaching, small group activity, hands-on experimentation etc. Also some children may respond better to some ways of teaching and other children to others so using a variety of methods would suit children's differing needs.* (Beliefs document, phase one)

Terry sees the merits of using variety in teaching because she does not think that any one method is better than the others:

I think the best way to teach maths is in groups, the class as a whole and individually. I think it is necessary to provide this variety. It is not possible to choose the best way to teach maths because the ways mentioned are effective, however not as effective if they are not combined with different methods to teach. Although I think group work or work in pairs is a great way to teach because it involves interaction with other students which requires co-operation, it is essential for the students to work individually so they do not rely on other students but are able to complete the work themselves. The combined teaching methods could give variability which I think the children would enjoy because it would create interest as you would use different approaches to the topic. This non-uniformity type of teaching could be beneficial to the students. (Beliefs document, phase one)

The majority of students, though, suggested using a variety of teaching methods as they felt this would alleviate the boredom that use of just one method might create:

Well, I think that you should be enthusiastic with a class, make sure they [are] getting help as well, and let them not be afraid [to] ask questions and for help, a variety of different teaching techniques could be used and I think that would be a good idea to make it not so boring. (Bev, paired interview, phase one)

One way to make maths more interesting is to use different methods, maybe one day use a book or a text and then other days you explain to the children. (Agnes, paired interview, phase one)

As with other pedagogical principles, past experiences led students to seek different methods of learning from those they had experienced at school:

My attitude now to maths is that it can be fun and it is not difficult; different approaches need to be used to the ones I experienced as a child to make maths come alive. (Alice, in summary of paired interview, phase one)

Maria suggested that use of a variety of methods would keep children interested and alert:

> *I wouldn't sort of use the same methods all the time, I'd sort of change it so, not only to keep the children on their toes, but to not get them so relaxed with a certain method, so I'd be using group work and peer work, peer teaching and have games and things like that.* (phase three interview)

In general, then, only a few students suggested using a variety of methods because of the differing contexts in which learning might take place. Most students seemed to believe that variety would make the learning more interesting and alleviate the boredom often felt in a mathematics lesson.

9.5.5 Teaching in a child-centred way

Child-centred teaching is implicit in the pedagogical principle of teaching in a practical way, as practical learning included the idea of exploration and discovery. It was also mentioned specifically by a number of students as a pedagogical principle in its own right. Discussion of pedagogy that could be classified as child-centred appeared 63 times in the students' data.

As well as discovery and exploration, child-centred learning included allowances for children to work at different paces:

> *But I believe that you can let the children learn maths at their own pace and [the teacher should] provide the resources so as they can explore and progress if they need to do so.* (Bronwen, Beliefs document, phase one)

Like Colin, in the section on using a variety of methods, John is aware that children learn in different ways:

> *To vary activity and method may maintain child interest and enthusiasm. Children each have different talents therefore different activities may provide*

> *scope for these different abilities to come out.* (John, Beliefs document, phase one)

Lexi holds a vision of child-centred pedagogy which is similar to the views of the teacher educators:

> *I believe children learn maths best by being involved in their own learning. They learn most effectively by using their own assumptions and exploring different alternatives. An inquisitive student questions all types of questions and answers and eventually resolves the question and would understand the concept better. A student who has been told the answer is not able to fully comprehend how the answer was obtained.* (Belief document, phase one)

Joanne mentioned the constraints that might prevent teachers from using a child-centred approach:

> *In different situations, different things might be better, and for each child there is a better way of doing things, but in a classroom you can't say, " Well, Jimmy needs this so Jimmy can have that, but Sally needs this so Sally can have that".* (phase three interview)

The data on child-centred pedagogy show that students have thought carefully about teaching and have developed ideas on pedagogy that are similar to those of the teacher educators. As these ideas appear throughout the three phases of data collection, they appear to have been influenced by a variety of sources. However, a paradox exists in that, although child-centred teaching is regarded as important, knowledge about the children is not seen as very important: I have shown in the section about good teachers that only two students felt a good teacher should have some knowledge about pupils (see Section 9.2.8, pp. 225-226).

9.5.6 Ensuring teaching was at the correct level for each child

Teaching at the correct level is closely linked to the idea of child-centred pedagogy, and could be seen as one aspect of child-centred pedagogy. Some students emphasised this aspect only:

> *Some students have a natural talent to pick up on concepts before they are explained fully and have the ability to race ahead. I feel some students are great at just listening to the teaching and pick up the concepts this way. But then there are the children who need to sit and talk to a friend about the work. Different children learn at different speeds and in different ways.* (Cara, Beliefs document, phase one)

Again, past experiences led to students seeing the importance of teaching at the correct level:

> *In year 9 and 10 I was in advanced maths and I don't think I should have been there. I got through, but I didn't really, I think I would have been better off in the intermediate class, sort of not excelling, but being a lot more confident than struggling in that advanced class.* (Aaron, phase three interview).

Another aspect of teaching at the correct level was using different techniques for pupils at different levels:

> *I think it depends on the classes. With the higher ability classes I would say they would, may be more interested in learning from books, but with the lower ability classes I would say they would enjoy more practical because the smarter people in maths always want to become lawyers and doctors and they really want to get to know everything and really achieve well, whereas the lower people just want to pass the time in maths and if you make it fun for them they might learn to enjoy it.* (Mandy, phase three)

As can be seen from the above, teaching at the correct level is closely linked to the use of child-centred methods and is also connected to the idea that using a variety of methods is beneficial to the pupils.

9.5.7 Ensuring lessons were interesting

The idea that lessons should be interesting is closely allied with the idea that lessons should be fun. Students are concerned that their classrooms should be happy, interesting and exciting places. Again, the belief emerges from the data, that if lessons are interesting, then, as with fun, learning will occur.

> *I believe that mathematics needs to be made more interesting so as that children enjoy maths and want to learn more about mathematics.* (Ruth, Beliefs document, phase one)

> *Things that you want children to learn should not be made into a chore, but instead made interesting and, when possible, fun.* (Tina, summary of paired interview, phase one)

9.5.8 Covering theoretical aspects of mathematics

Forty mentions were made about the value of teaching the theory underlying a concept. Theory was seen as providing the understanding:

> *I believe children best learn maths through a combination of theory for understanding and practical exercises for realisation to mathematical relationships.* (Christine, Beliefs document, phase one)

As has been mentioned earlier (p. 235) theory is generally seen as preceding the practical:

> *Actually teaching the children maths by doing work of a theoretical nature is important. However, this learning should be followed up by practical*

approaches to maths as well as a number of group activities. (Eileen, Beliefs document, phase one)

All students mentioning the need to cover theory also felt that practical aspects of each topic should be taught. A balance between theory and practical work was required. This is in contrast to those students who suggested practical teaching, and did not mention theory at all (see pp. 233-234). This is consistent with the theme occurring throughout the data, which suggests that intellectual concerns, and cognitive demands, are not nearly as important as enjoyable, practical activities, and that to make cognitive areas more palatable, there must be a balance of practical work and theory.

9.5.9 Providing challenges for pupils

Students mentioned the need for the work to be challenging thirty eight times, in 23 of the student documents. Many discussed this in their phase one interviews, and those who did could have been influenced by their reading of the NSW Mathematics K-6 Curriculum document which suggested that "Mathematics learning is more effective when it is interesting, enjoyable and challenging"(NSW Dept of Education, 1989, p. 4).

> *You've got to make it interesting for the class or for the students, you got to, like, make it relevant and it's got to be fun as well, and it's got to be clear at the time, make it interesting so the children are challenged in mathematics.* (Helen, paired interview, phase one)
>
> *It should be challenging and they should enjoy solving problems and they should just enjoy doing it, probably have a better attitude towards it which will help them in later life.* (Thea, paired interview, phase one)

A contrast is shown when experiences on the practicum are discussed (see below).

> *So I often sidled up to those children [who were having difficulties] and I guess in some ways I have more affinity to them because the really good ones are too threatening to me. You know, they'd be way ahead of me. So I was happy to leave it to them just to achieve the examples I had worked out beforehand and I*

knew [that I had] got it right. But the weaker ones who had trouble ... I really loved working with them. (Gail, phase three interview)

Although students believe that it is necessary for their future pupils to be challenged, they are concerned that they will not be able to cope with the mathematical demands of providing these challenges. This point will be discussed further in Section 9.6, p.256 on self-as-teacher.

9.5.10 Showing the connections between mathematical topics and other areas

Data on connections emerged from the beliefs documents, and phases two and three. No student mentioned the importance of seeing the connections in mathematics, in the paired interviews and summaries of phase one. In the beliefs documents of phase one, and in the questionnaires of phase two, students stressed the connections between the mathematics and real life.

> *I believe that the best way for children to learn maths would be through hands-on experience, putting what the child has learnt into real life experiences.* (Robyn, Beliefs document, phase one)

> *[A good primary school student] can apply these concepts in both theoretical and practical situations.* (Camilla, questionnaire, q. 15)

Adi is one of the few who suggests connections between the mathematical activities and the mathematics itself:

> *I think the best way to teach maths is to give children materials and tasks to do, allow them to do the tasks and then regroup to discuss their findings and then the teacher can link their experiences to maths concepts. Basically teach with practical materials and exploration.* (Beliefs document, phase one)

Mandy felt that pupils should be able to see connections between the ways of learning in mathematics and in other subjects:

I'd like them to have learnt something and remember how they learnt it so that they can apply the ways that they've learnt to other subjects so that if they learnt something in an enjoyable way and it really stuck in their minds then they could use that way of doing it in some other aspect. (phase three interview)

No student mentioned the need to show connections between the various areas of mathematics, or between mathematics and other subjects. This indicates an instrumentalist way of thinking about mathematics (see Chapter 7, p. 118). People holding instrumentalist views see mathematics as a kit of various and unrelated procedures, and do not see the connections between the different areas of mathematics. The silence from students about showing connections between areas of mathematics seems to indicate the instrumentalist view, as such connections are seldom seen in this view, and only the applications are perceived.

9.5.11 Providing drill for pupils

The question of whether drill is a desirable pedagogical technique is debated throughout the study. In the first phase of data collection, students talked about their negative memories of learning tables, using methods of drill and rote learning. No-one in that first phase was enthusiastic about the use of drill. However, some responses in the second phase showed tentative approval for drill. A question on the questionnaire (q. 10, see Appendix C) asked students the following: "In the taped interviews many students mentioned doing a lot of drill in primary school mathematics. Do you see drill as being different from practice? If so, in what way? Do you think drill has a place in the learning of primary school mathematics?" Many of the students felt that drill was needed to a certain extent, although most students tended to qualify their remarks about drill, by suggesting that it was not used exclusively.

Contrast the experiences discussed in phase one paired interviews and phase two questionnaires, with students' views about using drill in the phase two questionnaire;

I found it was frustrating when going over things again and again and again in class and having to write things like mentals on the board over and stuff that I didn't like.

Yeah, I hated mentals because I could do them relatively well, but as you said it was [only] one activity to do ... that was really boring and they were all really the same and no imagination. (Cara, paired interview, phase one)

I really didn't like maths when I was at school, the whole way through. I don't know why, I just think that it was one of those subjects that was difficult and boring and I think that it made that impression on a lot of people. I didn't find it very interesting, I just remember everybody saying, you know "5 x 2 = 10" just chanting and things like that, I don't think that's the right way to go about it. (Liz, paired interview, phase one)

I feel that this attitude falls back on her primary teachers who did not use a very interesting or creative way to teach the subject. Liz felt that her teachers basically stayed within the traditional style of drilling and chanting the times tables, making maths boring and repetitive for her and her classmates. (Emma about Liz, in summary of paired interview, phase one)

Maria explained that although her maths abilities weren't so good, that her teacher also played an important role in determining her attitudes. Maria thought maths to be tedious and rather boring at school for the fact that she was often set boring exercises, time after time. (Christine about Maria, in summary of paired interview, phase one)

Some students commented in phase two about the role that drill had played in providing negative experiences for them in the mathematics classroom:

When I was at school, [mathematics] was boring, tedious and basically consisted of lots of drill. If there were more activities the children could take part in, mathematics would be more fun. (April, q. 12, phase two questionnaire)

It is interesting to note that in the phase one data, not one student recommended the use of drill as a principle of good pedagogy. Drill was only mentioned in a perjorative sense, as a reason for finding mathematics dull and routine. However, after students had been on teaching practicum in the first year, many of them commented on the use of drill in the classroom in which they had been placed. This observation of drill appeared to be influential in suggesting that drill did have a place as a sound pedagogical principle. Self-as-teacher was again being influenced by the model provided by the supervising teacher on the practicum.

> *My teacher had a 1/3 composite class: for year one she had them reciting different number patterns - even, odd, months in the year, counting by 5's and 10's etc. For third grade they mostly worked from their maths textbook.* (April, Beliefs document, phase one)

Gail, in reply to the question on the questionnaire that asked "Do you think that drill has a place in the learning of primary school mathematics?" replied: "*It must have, because I have seen it in each class I observed during prac*".

And Nita, talking about the methods used on the practicum, commented that:

> *The drilled work certainly improved their mathematics. They had some practical lessons and had obviously grabbed the concepts.* (q. 23, phase two questionnaire)

On the other hand, students were once again part of a triangular dilemma in which, while self-as-teacher was receiving messages from the supervising teachers about the value of drill, self-as-student-learning-to-teach was being influenced by the teacher educators, who wished to show students a different image of mathematics, as a subject that did not merely consist of rote learning and drilling of arithmetic tables. At the same time, making the whole situation more complex, self-as-primary-school-student evaluated previous experiences with drill, and had mixed feelings about the value of these experiences.

> *I would probably have been bored with tables drill and exercises, but the group work, ... would have provided variety and make it more interesting.* (Sally, assessing the classroom activities she observed on the practicum, q. 23, phase two questionnaire)

> *But I like rote work because it worked for me, I would be quite happy to use that, but I would temper it with other stuff.* (Nita, phase three interview)

Those students, who still remembered their tables, felt that the end had justified the means:

> *Although it was rote learning, I think tables are a good thing to learn that way, because that's what I did and I still remember mine and I knew them very well. I think in some cases, rote learning is very practical, especially for tables.* (Terry, phase three interview)

While others felt there was no justification for use of drill:

> *Drill is different [from practice] in that the child/student just continues to chant a particular way, not paying attention to the answer ie times tables. It has no place [in the learning of primary school mathematics].* (Laura, q. 10, phase two questionnaire)

And Gail confessed to being perplexed about the whole issue:

> *I think we are creatures who once things are put to memory, those things go down very deep and I think in terms of tables, now this might be contrary to everything I've said, I don't know, but in terms of things like tables and formulas, there is a point where one memorises and then one later maybe understand.* (Phase three interview)

Where students did recommend the use of drill, it was generally only as one of a variety of pedagogical principles:

> *It has some place [in the learning of primary school mathematics] but not used solely as a form of teaching mathematics concepts.* (Bridget, q. 10, phase two questionnaire)

Students' feelings about the place of drill in the primary school classroom have been shown to be slightly schizophrenic; on the one hand, drill was what made mathematics boring and tedious when they were at school; on the other hand, those who can remember their tables are pleased that they persevered with learning them. The supervising teachers that the students observe during practicum, all seem to emphasise use of drill, and again there is a mixed reaction to these observations: some feel that drill must be appropriate as it is being used; others compare the lessons with their negative experiences and so reject the use of drill. Adding to the complexity of the situation is the position held by the teacher educators, who wish to show students that there is far more to primary school mathematics than the drill that the students remember, and so play down the role of drill in learning mathematics. Consequently, the students' dilemma as regards the use of drill is not easily resolved.

The final three pedagogical principles of demonstrating the work, using questioning and introducing innovation are not mentioned often in the data. They do not appear to be major principles that students see as highly important. An example of each principle will be given below to illustrate the context in which they appeared in the data:

9.5.12 Demonstrating how to do the work

> *You'd try and explain them on the board and you'd use examples as well, and make sure you do use examples on the board and go through the examples. I always found that very useful.* (Maria, phase three interview)

9.5.13 Making use of questioning techniques

> *I would teach maths in a positive manner, as I would teach any subject. I would try to help children develop a positive attitude towards it too, and encourage them to ask questions and question each other's ideas.* (Liz, Beliefs document, phase one)

9.5.14 Introducing innovation into lessons

> *Maths can be taken from boredom to achievement by innovation and involvement. High expectations which may be hard to meet.* (John, summary of paired interview, phase one)

The whole range of ideas, discussed above, indicate the pedagogical principles that various students see as being important for good teaching. While many of these principles are principles upheld by the teacher educators, there are also some principles suggested by the teacher educators which are not mentioned by any of the students, for example, the challenging of beliefs.

The students' views on the characteristics of good teachers, and on the principles of good pedagogy, appeared to be heavily influenced by their past experiences, their observations of teachers in the primary school, and their learning in the mathematics education subjects and other subjects in the tertiary program. All of their beliefs were influential in determining what characteristics and skills they felt they would need when analysing their roles as future teachers. The following section discusses the beliefs that students hold about self-as-teacher.

9.6 Self-as-Teacher: Aspirations and Ambitions

The thesis has considered the past experiences that student teachers have had in mathematics classrooms, as well as their beliefs about what made those experiences positive or negative. Section 9.2 considered the students' views about the characteristics of good teachers. The following discussion considers the data that provide insights into the other side of the coin, that is, the vision that students hold of themselves as future teachers. This issue was seen as extremely important by students entering the teacher education course: identifying and developing the characteristics that they believed were essential for their successful lives as teachers. Students had strong images of what self-as-teacher should be like. These images were discussed most often in the paired interviews, as many of the questions that students posed, probed the characteristics that their partners envisaged for self-as-teacher. Further, students were asked, in the assignment on paired interviews, to write summaries of the paired interviews, and indicate the implications for their future teaching. This led to many students discussing either their own characteristics as self-as-teacher, or those of their partner. Consequently, the major part of the data on self-as-teacher emerged from phase one, in particular, from the paired interviews, and summaries of these interviews. This meant, therefore, that the teacher education course might not have had a great influence on these data, as the course had only been underway for two weeks at the time of the interviews. An influence that students did note in developing their selves-as-teachers was the influence of their past experiences in mathematics at school.

For some, the idea that past experiences would ensure that students would teach in a completely different way from the one in which they had been taught, was mentioned frequently in the paired interviews' data of phase one. It also arose in the phase three data.

> *So you drew on your, say, negative or adverse history of the liking or hating of maths and you basically turned it around for yourself and created a more positive attitude towards it?* (Laura in paired interview with Bianca, phase one)

> Question: *Maybe some of your negative experiences will help shape how you teach it?*
> Answer: *Yeah, I will try to make those negatives which I experienced into positive influences, reflecting towards my maths experiences.* (Question by Colin and answer by Christopher, paired interview, phase one)

I think it [my awareness of my struggles at school] will make me even more careful of how I teach it, and make sure I keep careful watch and if I see any struggling, because I know how I felt when I was doing that myself. So I think from my own experience, that will affect the way I teach. (Terry, phase three interview)

I do feel that my teachers' attitudes towards maths influenced my current attitude toward mathematics, for [example] my bad experience with maths where I was humiliated in front of the class in year 5 for laughing at a girl who couldn't do a problem, and then getting even a larger problem to do and not being able to it. I know that that's not the way to make kids learn. (Adam in paired interview, phase one)

And Bev had this to say about her experiences:

My past mathematical experiences have greatly influenced my attitude towards maths. I feel the knowledge of these negative experiences will actually benefit me in my teaching of the subject. For example, I know exactly how to go about making the subject unenjoyable! (Summary, phase one)

And John described Dale's experiences as follows:

A lack of identification with such an environment may result in a tendency [by Dale] to lean away from it in the future. (Summary, phase one)

On the other hand, some of the students recalled very pleasant experiences at school, and they too, acknowledged that these experiences would be influential in the practices they adopted.

Well, because I enjoyed the practical methods and found that they helped me to learn, I'll be influenced by those, to use those kind of methods when I do teach maths, when I start teaching. I'll still also use the written methods because I *found that they also did give me the basis of the maths but I think I'll be using both and I think it's also important that you help the kids who lag behind in mathematics.* (Bonny, paired interview, phase one)

Doreen believes she is looking forward to teaching maths and particularly to using her own experiences as a resource. For example, techniques and concrete resources that she enjoyed learning with are more likely to appear in her future teaching than those which she didn't easily learn from. (Joanne about Doreen, summary , phase one)

Many students had built up an image of self-as-teacher, either based on experiences which had been positive for them, or experiences which they did not want to emulate. A number of students talked about the positive role models which their teachers had provided for them.

I really like it. The main reason why I enjoyed it, I think my enjoyment of it would really encourage me to teach other children to enjoy it and the ways that I found were best, when taught to me, would probably be the ways that I think children should learn. (Tina, paired interview, phase one)

Fortunately in sixth grade, Eileen was taught by a teacher whom she now sees as a type of role model. Eileen can now realise the effect an understanding teacher can have on the ability to learn. (Lexi about Eileen, summary of paired interview, phase one)

I hope that my teaching practices will be as successful and enjoyable as my primary school experiences, as I will always remember and respect my teachers for introducing me to maths the way they did. (Mandy, summary, phase one)

The above text shows that past experiences and past teachers appear to have been an important influence in determining the nature of self-as-teacher for the students. Based on these experiences, a number of characteristics were commonly chosen as desirable for the students in their role as self-as-teacher. Most often mentioned was the desire for self-as-teacher to ensure that pupils had fun in the mathematics class. This occurred in over 70 per cent of the paired interviews and summaries documents. The desire to have a fun-filled classroom is not so surprising in the light of previous data, which showed that a good teacher was one who had a supportive atmosphere in the classroom, and good pedagogy included having fun in mathematics lessons.

Other characteristics and roles envisaged for self-as-teacher were the following (in order of frequency of appearance in the data):

- being able to demonstrate the relevance of the mathematics to the pupils
- being an enthusiastic teacher
- giving interesting lessons
- being encouraging
- knowing how to use practical methods of teaching
- using resources
- being able to improve the attitudes of the pupils
- being able to improve self-as-teacher's own attitude
- using child centred methods
- being able to teach for understanding
- being aware of problems that pupils were experiencing
- catering for all pupils

Another set of characteristics were mentioned less than ten times for each characteristic, indicating that only a few students regarded these characteristics as important. Briefly, these characteristics, in order of frequency of appearance in data, were:

- showing flexibility in teaching
- encouraging co-operative methods of learning
- exhibiting patience in teaching
- ensuring that theory was taught
- being aware of gender issues
- being a good communicator
- being knowledgable about mathematics

A category also existed into which quotes were placed, which indicated themes that were mentioned in only one document. These will be discussed in Section 9.6.13 (pp. 278-281).

9.6.1 Ensuring pupils have fun

As has been shown in Section 9.5.3, (pp. 238-243) students believe that an important aspect of good pedagogy is to ensure that pupils have fun. Consequently, it is not surprising that a major aspiration for self-as-teacher is to provide a classroom that is filled with fun.

> *Well, when I am a teacher, maths is going to be fun and I'm going to teach it in more of a practical way with games and things that kids can relate to rather than just having set tasks and saying do it.* (Christine, paired interview, phase one)

> *When I do teach, I want to be there for that "eagerness" and eliminate that fear of mathematics. MATHS IS FUN if you teach it that way. Relating maths to practical situations inspires young children and their love for maths will grow.* (Daisy, summary of paired interview, phase one)

A difficulty for self-as-teacher, though, is knowing how to make the lessons fun; most past experiences are not helpful in this regard.

> *I would try and make maths learning as fun as possible and as visual and tangible as I possibly could. For fractions, for example, you would use graphs or models to represent the operations/concepts so the children could see it. Unfortunately, I am not sure as to how many primary maths concepts lend themselves to this idea.* (Clive, Beliefs document, phase one)

Emma discussed how she would make it fun for pupils:

[I would] try to play games with them, games you know, maths in games and with those they enjoy it, they find a way to enjoy it, but also try and teach them how important it is outside the sums. In society it is important as it is used in most occupations, to show that children that the things that they do are important and then put your imagination into it and they find it fun and important. (paired interview, phase one)

So games and relevance are the means of providing fun. Imagination is also important. However, Emma, too, indicated that she would like to find out how to make mathematics fun for pupils:

But I feel that both Liz and I do want to find a more creative way of teaching maths as we missed out on that during our primary schooling. We would like to give children the chance to actually enjoy maths in an interesting and fun way. (Emma, discussing the implications of both her own and Liz's experiences in the summary of paired interview, phase one)

Alice also talked about the influence of her past experiences on her desire to provide fun for pupils:

My attitudes towards teaching probably have been influenced by these views [of mathematics as a dull and drill filled subject] so much that I would try and make it fun, because kids love to learn by playing games or whatever. So I'd really try and involve the kids by group activities, and just making it fun and interesting and something to look forward to and not something to dread and to have to memorise. (paired interview, phase one)

To conclude the discussion on this aspiration of self-as-teacher, Aaron describes his view of future teaching:

After the tape was switched off, Aaron went on to say that he saw teaching as much of a game and that he didn't want the children to take him too seriously,

> *they all needed to laugh. As long he could make lots of jokes then the children could laugh and he could laugh. This was very important to him and that he really liked being with children.* (At end of phase three interview)

It appears from much of the data that have been presented above, that most students wish to have classrooms which are filled with laughter and enjoyment. Many students are not quite sure how to achieve this aim in a mathematics classroom, and perhaps have doubts whether mathematics can be presented in an enjoyable way. However, the use of games, jokes and practical work is perceived as being the means of having fun in the mathematics classroom.

9.6.2 Being able to demonstrate the relevance of the mathematics to the pupils

Relevance has been shown to occupy an important position in students' views. In Chapter 7 (pp. 152-155), students stressed that the relevance of the mathematics learnt needed to be emphasised. Further, in the current chapter, one of the major principles for good pedagogy was thought to be the illustration of the relevance of the mathematics taught (see Section 9.5.2). Consequently, students see an important attribute of self-as-teacher as the ability to demonstrate the relevance of the mathematics taught.

> *Now it [mathematics] appears more relevant and important and we can't predict outcomes without the use of maths. So I think that's how I would use it with kids, to give them all the information that they need, that they can then apply to whatever field of work or whatever field of study they want to go into, to make it interesting and make them feel that they have achieved something.* (Beatrice, paired interview, phase one)

> *As a result of this interview [the paired interview of phase one] I also decided that it would be great to relate the teaching and doing of maths to real life incidents. It would make the learning of maths a lot easier and it also would be encouraging, for some students, to see ways [in which] what they are learning could be used in society.* (Cara, summary of paired interview, phase one)

Aaron had clear ideas about the value of showing relevance to pupils. This has been mentioned in earlier discussion (p. 241). The quote below illustrates why Aaron thinks self-as-teacher should emphasise the relevance of the mathematics, and how this should be done:

> *I'd use examples that they can relate to, it's no good saying to a child that if you learn angles or something or whatever then it will help you build a road or something like that, that doesn't mean anything to kids. Whereas if you teach them that there's so many cups in a litre or something like that then you can say "well you know if you're going to eat so much ice cream ...", just relate it to things that they can understand and they sort of appear important. You don't trivialise anything by making it something that they think "God, who cares?".* (phase three interview)

Students also made decisions about self-as-teacher based on opinions of self-as-primary-school-pupil. Alice described how self-as-primary-school-pupil would have liked to have been taught:

> *I would have liked to have been taught in a way, as I mentioned throughout this interview, that was fun and involved games and showed ways in which things like geometry or your times table or division could be used in daily life.* (Alice, paired interview, phase one)

9.6.3 Being an enthusiastic teacher

Students saw one of the most important characteristics of a good teacher as being that the teacher was enthusiastic about the teaching of mathematics (see pp. 216-218). Consequently, this was an attribute that many students wished self-as-teacher to possess. Past experiences were of interest here, as students often did not feel enthusiastic about mathematics as a result of their experiences at school. Students in this situation felt that they would need to develop positive attitudes before they started teaching so that they could be enthusiastic when teaching primary school mathematics. This issue is discussed further in Section 9.6.7 on attitude change (p. 269). Other students felt positive about their past experiences and, to them, being enthusiastic about their teaching of mathematics was unproblematic.

> *I also feel that I am lucky to know that maths can be, and often is, enjoyable and I hope that this attitude will be conveyed to my future pupils.* (Joanne, summary of paired interview, phase one)

> *My hope is to take this positive attitude into my classroom and to pass this through to my students.* (Doreen, summary of paired interview, phase one)

> *Partly because I have always enjoyed maths and the challenges it offers, I hope that my teaching of children will enhance their attitudes by seeing their teacher enjoying the subject she is teaching.* (Mandy, summary of paired interview, phase one)

Students also seemed to feel that when their partners in the paired interviews had positive attitudes it implied that those students would be enthusiastic in their teaching of mathematics:

> *Colin can now translate this positive experience in high school maths to his future teachings in primary school. By having a positive attitude and constant enthusiasm, Colin can have a dramatic and positive impact on the way the pupils approach maths and all the subjects in general.* (Christopher about Colin, summary of paired interview, phase one)

> *My interviewee, Clive, has always enjoyed mathematics throughout his schooling because he was encouraged by his results. Therefore he has a positive attitude towards mathematics which he will take into the classroom when he teaches this subject.* (Terry about Clive, summary of paired interview, phase one)

Others were determined that they would display enthusiasm for their teaching in mathematics even when they did not enjoy part, or all of it.

> *In a classroom that I teach in, I will ensure that even in the topics that I don't enjoy as much, I will bring across an enthusiastic approach.* (Bernadette, summary of paired interview, phase one)

> *As just stated, I will approach my maths in a creative way so as to keep my students interested. I will try not to let my attitudes come through in my teaching of maths, so as not to give my students the same attitudes as I have in that particular area.* (Rene, summary of paired interview, phase one)

> *I would teach this subject with enthusiasm and use a variety of teaching methods such as group work, individual, one to one and games so the students will have a better chance to grasp and understand the concepts and learn in an enjoyable way.* (Terry about herself, summary of paired interview, phase one)

9.6.4 Giving interesting lessons

Strongly linked to the notion of lessons that were filled with fun is the idea of giving interesting lessons. For some students, adding interest to lessons is, perhaps, a more serious concept than having fun - it implies using practical, hands-on activities or showing the applications of the mathematics:

> *I am now very likely to incorporate many stimulating teaching methods: filling bottles to understand volume, ringing overseas to comprehend time differences, measuring the playground to grasp the concept of measurement.* (Gail, summary of paired interview, phase one)

> *My aim in my teaching of maths is to make maths challenging and thought provoking.* (Robyn, summary of paired interview, phase one)

> *You can't get back and sit behind desks and write, that's wrong, you've got to*

do a lot of practical things with graphs, especially with graphs and other things like that to make it more interesting for them so they get involved personally. (Beryl, paired interview, phase one)

For others, "fun" and "interesting" were interchangeable words:

Maria said that she would try to make maths fun and interesting for her students and she would go about this by introducing games to her teaching practices. A combination of theory and practical exercises would be the basis of her teaching practices. (Christine about Maria in summary of paired interview, phase one)

In general, self-as-teacher wanted the classroom to be a happy place, and this was achieved by having fun, being interesting and enthusiastic, and demonstrating the relevance of all that was done.

9.6.5 Being encouraging

Encouragement is linked to the notion that a good teacher is supportive of the pupils' endeavours. Many students remembered past experiences, in which teachers had not been very supportive of their endeavours and it appears as if it is largely as a reaction to these experiences that self-as-teacher wishes to encourage pupils' mathematical endeavours. Also, self-as-student-learning-to-teach consults self-as-primary-school-pupil about the learning of a new topic - the information gained is useful to self-as-teacher:

Having experienced so recently the bewilderment of tackling, for example, a calculator, I am more likely to be sensitive to the child who is confronted with difficult mathematical concepts. (Gail, paired interview, phase one)

Adam, who spoke earlier about being humiliated in a mathematics class (see p. 258) vowed not to let that happen in his class:

I feel my past experience in maths will influence the way I teach my students as I'll make sure that each of them knows that their answer is valued and that [I'll *teach] through group activities and cooperative learning. They'll learn from experience and what they are learning they can actually use in their life and [I'll make sure] that to make the environment in the classroom where, if they don't understand something they can feel comfortable to ask so that they don't get left behind.* (paired interview, phase one)

And Bonny also related her aspirations as self-as-teacher to her experiences as a child in primary school:

> *I tend to have a bias towards helping students who are behind in maths because I was once one of them.* (Summary of paired interview, phase one)

Adam discussed further his negative experiences when at school and spoke about his wish to prevent his students from undergoing the same sort of experiences. He will do this by being very encouraging to all students:

> *My attitudes towards maths were and are largely negative. However, I feel these attitudes are valuable in that I can learn from them, so my students do not experience the way I was taught maths at school. I hope to prevent negativity towards maths in my classroom, through creating an environment where students feel comfortable to ask questions and feel uninhibited in doing so.* (summary of paired interview, phase one)

9.6.6 Knowing how to use practical methods of teaching and using resources

These two attributes are similar enough in nature to be discussed together. Part of the knowledge about practical methods involves the use of resources with those methods. Students felt that knowledge about both these attributes was desirable for self-as-teacher. It is also noteworthy that using practical methods was regarded as the most important of pedagogical principles by the student teachers (see p. 231).

> *I would attempt in my teaching practices, to make my mathematics lessons experimental, allowing students to discover for themselves, with the use of*

teaching aids. Practical demonstrations, rather than traditional blackboard presentations, create class participation, and hands on experience, in the visualisation of mathematics, which can make it easier to understand. (Adi, summary of paired interview, phase one)

As I can see that the practical activities used to teach me maths were motivating, I will definitely incorporate them into my lessons when the time comes for me to teach. (Bonny, summary of paired interview, phase one)

I think I would attempt to bring maths out of the textbook context and show students the use of maths in our everyday lives - perhaps using visual and tactile aids. (Barbara, summary of paired interview, phase one)

9.6.7 Being able to improve the attitudes of both the pupils and self-as-teacher

As discussed earlier (see Section 9.6.3, p. 264), students believed that a most desirable attribute for self-as-teacher was to be enthusiastic. This was consistent with beliefs that good teachers had positive attitudes (see Section 9.2.2). One of the difficulties in exhibiting this characteristic, however, was that many students did not feel very positive about mathematics, and regarded their past experiences in mathematics as being unpleasant and tedious. Consequently, one of the priorities for self-as-teacher was to change attitude so that enthusiasm could be shown. How this was to be done was not completely clear, although some students did mention that this change of attitudes should be accomplished by the mathematics education sequence at university (see Section 8.4.1.3, p. 189-190). Another priority for self-as-teacher, mentioned as often as change of self-as-teacher's attitude, was the need to change the attitudes of the pupils to be taught. As the student teachers remembered their own school experiences in a negative light, they expected their pupils to feel much the same way, and so saw as essential, the changing of pupils' attitudes through their teaching.

In the following extracts from the text, students first speak of their desire to change their own attitude as self-as-teacher, and then this is followed by examples in which students talk about self-as-teacher changing the attitudes of the pupils they will teach.

Bronwen has already been quoted about her anxiety in teaching mathematics (p. 230). She continues:

> *Well, I'll will have to concentrate on that, not having had a positive attitude towards maths myself. I'll have to try hard not to let it scare me, I'm not very good at it so I don't want to teach it. I've got to view it in a positive way, not a chore.* (Bronwen, paired interview, phase one)

Others echoed this theme:

> *Well, I don't think I can go into teaching a maths class now with the same attitude that I had when I was at school because my attitude was a bit negative and as a teacher I've really got to have a positive attitude to everything that you do. So I guess that they didn't really shape my ... my experiences didn't shape my attitude, I've got to change my whole attitude to fit what I'm teaching, I guess.* (Rene, paired interview, phase one)

> *I would not let my previous experiences hinder the learning of maths for my students.* (Ruth, summary, phase one)

For all these students, then, self-as-teacher has to work at changing what is currently a negative attitude to mathematics, and mathematics teaching, into a positive attitude. This is seen as essential in order that pupils are not disadvantaged by having an unenthusiastic teacher. Students show a determination that self-as-teacher will be positive and enthusiastic about teaching mathematics.

What follows are the students' ideas about improving the attitudes of their pupils when they are teachers. Students are very aware of their own difficulties due to negative attitudes and are concerned that these difficulties are avoided with their pupils:

I don't want to produce a classroom of little Gails with the same bias as I've got. I would hope that at the end of the year they would be even ahead of me in my enjoyment of maths. You know, I would be able to instil in them things that I was struggling with, that we'd come through together and they'd actually ended up enjoying, you know. (Gail, phase three interview)

I hope that they will take away a positive outlook on maths in that you know, everything's possible. That just because they couldn't do something this year, that there's no reason why they can't do it next year. (Joanne, phase three interview)

I didn't want a child to have the same attitude towards maths that I had, I didn't want that. (Bianca, paired interview, phase one)

... but this means that with regard to my own teaching I'm probably more eager to encourage my students to have a good attitude towards it. (April, paired interview)

I also feel that maths is a subject that should be enjoyed and I feel that children need to gain this attitude early. Once a child thinks that they can't do something, they put up barriers that resist any attempt to teach them that this is not the case. (Sally, summary, phase one)

So students hope that self-as-teacher will be able to encourage future pupils to hold positive attitudes towards mathematics, in contrast to the attitudes the students themselves currently hold, or once held as pupils. How this attitude change is to come about was not visible in the data.

9.6.8 Using child centred methods

This method of teaching was mentioned in approximately 25% of the students' phase one documents. It is closely allied to using practical methods, as data categorised in this way included data on teaching using discovery and exploration . The central characteristic of data in this category was the desire for self-as-teacher to encourage participation and involvement by students.

> *First of all I'd motivate the class and give them more practical experience and let them problem solve by themselves so that they understand them fully, make sure that they don't do things which are maths lessons which keep going with exercises, pages and pages of exercises, yes sure give them a page of exercises to do but not just exercises set so that they go over the same things to do.* (Maria, paired interview, phase one)

> Question: *So as a teacher how would you encourage students like that to become involved?*
> Answer: *By using those new techniques which we saw on the video and also by thinking of other ways, asking them to tell me what the answer is, how they got to it and what would be another way of getting to the same answer.* (Question by Dale, and answer by John, paired interview, phase one)

> *Class involvement from all members will be an integral part of my teaching which will give opportunities to all students no matter of what standard they may be. The overly capable student will not be bored whilst waiting for the less able to catch on and the less confident will not feel intimidated by the good students. A mixed variety of maths will be taught.* (Rene, summary of paired interview, phase one)

9.6.9 Being able to teach for understanding

Just under 20% of the student documents from phase one mentioned the desire for self-as-teacher to teach for understanding. This is of interest as it is one of the few instances where the characteristic of self-as-teacher involves the cognitive side of learning. All characteristics discussed earlier in this chapter deal with affective issues, such as enthusiasm, encouragement, and ensuring that fun was had.

I have a positive attitude towards maths and that will come across in the way I teach it, because I think, you know, it's important, maths. So I want to teach it well and make sure the kids understand all the basics. (Clive, paired interview, phase one)

I would like to think that I had made them understand what it was that they were trying to learn and that they were able to use it in everyday ways. (Liz, summary of paired interview, phase one)

In my teaching I would utilise my experiences by: emphasising the relevance of maths in every day living; making sure students understand "why" a maths problem is solved a particular way; to make sense of the subject; allowing students to explore and enjoy maths, with the use of games, group activities and a supportive environment. (Alice, summary, phase one)

9.6.10 Being aware of problems that pupils experience

A number of students felt it was important for self-as-teacher to be aware of any problems that might be experienced by their pupils: this was usually a direct consequence of the students, themselves, having experienced problems in mathematics at school, and the teacher being quite unaware of these problems:

I will definitely try to find the time to identify the children who are having trouble and give them some extra help because it can really affect their self esteem and things like that if they are not doing well at school and feel that they don't have any help. (Bonny, paired interview, phase one)

To which Bridget added in her summary of her interview with Bonny:

The interview also revealed that because she herself was left behind, that Bonny would spend a lot of time in trying to make sure that none of her students fall behind or are left finding maths difficult. (Summary, phase one)

April referred to her own negative attitude to mathematics and continued:

> *In relating these attitudes to my teaching practices, I should be able to utilise them by attempting to understand how my prospective pupils feel about maths, whether they may enjoy it or find it difficult.* (Summary, phase one)

9.6.11 Catering for all pupils

A few students indicated that self-as-teacher would cater for all pupils: however, more students appeared concerned that they should be able to help the weaker pupils, than be able to offer all students the opportunity to work to their potential.

> *I think that when I do teach I will help all those who are finding it difficult and need that individual attention.* (Christopher, paired interview, phase one)

> *There is going to be a variety and range of abilities that you have to cater to. For example, you have to give the faster progressing student the resources to further extend their abilities so as they do not become bored and disinterested. Simultaneously giving the slower students the confidence and extra help to gain and achieve [is important].* (Bronwen, summary of paired interview, phase one)

The characteristics mentioned above are the major characteristics desired for self-as-teacher. As has been already noted, the emphasis of most of these attributes has been on the affective side of learning. Very little has been said about how students envisage teaching from a cognitive point of view. Students appear to believe that if self-as-teacher can be enthusiastic, encouraging and aware of children's problems, learning will occur.

9.6.12 Other characteristics of self-as-teacher

The second group of characteristics (listed on p. 260) were not mentioned very frequently, possibly indicating that few students saw them as important. The fact that few students mentioned these characteristics could, however, be due to a variety of reasons, such as the context in which the data arose; students' views on what is appropriate to discuss in the mathematics context; or other reasons not apparent to the researcher and reader. Hence, while it is possible that only a few students believed these characteristics were important, other explanations are possible for lack of data in these areas.

Of interest are the last two characteristics in the group: being a good communicator and being knowledgable about mathematics. These characteristics are of interest because they are mentioned very few times: Three mentions were made of the need to be a good communicator and two of the desire to be knowledgable about mathematics. As students had felt that one of the most important characteristics of a good teacher is that the teacher is able to communicate well, it is surprising that this data arose so infrequently in the self-as teacher area. There are a number of possible hypotheses that might go some way towards explaining this surprising result. However, I am unable to investigate these further from the data I have at hand.

The first hypothesis is that discussion about communication might not be seen by students as being in the "mathematical domain". As students appear to see mathematics as a written subject, in which little verbal communication takes place, perhaps discussion about the need to be a good communicator is not seen as appropriate in the context of this study.

Secondly, students might not see their communication skills as lacking in any way, and might well be reserving all discussion about self-as-teacher for thought about those areas of teaching mathematics that could be problematic for them. In the phase one interviews, from which most of the data about self-as-teacher emerged, students were encouraged to pose questions about aspects of mathematics and mathematics teaching and learning that appeared to be problematic - consequently, the issue of communication might not have arisen in the questions, and hence not been part of the discourse of phase one.

Thirdly, as students have indicated a desire to avoid being knowledgable about mathematics and see their role as being the providers of practical and enjoyable activities in mathematics, the need to communicate well might not arise.

The lack of attention to the other characteristic, the need to be knowledgable, is a consistent theme that has run throughout all the data and has been discussed both in Chapter 8 (see Section 8.4.1.7, pp. 193-194) and in the current chapter (Sections 9.2.6-9.2.8 and Section 9.3).

What follows are some extracts from the data indicating how the students that did espouse these characteristics for self-as-teacher, envisaged these attributes. The first two extracts focus on communication, but it is not clear from the data whether generic skills in communication are being discussed, or specifically skills in communicating mathematics:

> *So I'll really have to concentrate on giving everyone a fair go, trying to explain as best as I can so really it's giving them a fair opportunity to understand the work in their own time.* (Cara, summary, phase one)

> *However, I think that Aaron will master the "language" of the younger children and be able to communicate effectively to the class. I believe this because even though Aaron began his "maths experience" with what appears to be a negative influence (referring back to his memories of primary school) it seems that the positive ones he had later on outweighed these.* (Agnes about Aaron in summary, phase one)

In terms of being knowledgable as self-as-teacher, one student talked about the desire for pedagogical content knowledge and the other student talked about the need to be knowledgable about mathematics being related in a converse manner to her lack of understanding.

> *I also hope that I can be knowledgable enough to be able to use various teaching methods to help children understand and appreciate the value of maths and how important it is in everyday life.* (Aaron, summary of paired interview, phase one)

I feel that my lack of understanding in particular areas at primary school will aid my teaching of that area. For example I will have a better knowledge of teaching fractions and of its practical applications. This will enable me to understand and react appropriately to the problem the students may have. (Rene, summary, phase one)

It appears that Rene believes that she will be more knowledgable because of her lack of understanding. In this case, knowledge appears to refer more closely to her knowledge of how students feel when they do not understand a concept, than to knowledge either of mathematics or of how to teach mathematics effectively. No other student indicated that knowledge was an important attribute for self-as-teacher.

Finally, the category of "awareness of gender issues" deserves a mention, again because it signals a topic which both the literature on mathematics education, and I, as teacher educator, perceive as a major issue in the teaching of mathematics. However, it is only discussed five times in the students' data, and does not appear to be perceived as worthy of mention by the majority of students. (It might be of interest, at this point, to note that 86% of the original group of participants in the study are female). The following extracts from the data indicate how the students who discussed this issue, felt about it.

Eileen responded to a question by Lexi about the influence of gender issues on her past learning:

My school was coeducational and although I don't remember the boys discouraging me, I know of instances where they have, so perhaps if I was teaching, you can't separate the boys and the girls, you've just got to go and involve girls and boys in the activities. Like, really, you have to think about "is this going to isolate the boys, is this activity going to isolate the girls?" You go to a coeducational school, you've got to think about those kind of things and just think about what basics they are going to learn in the same way. (paired interview, phase one)

Sexism should also disappear as we strive to tell girls that maths can be fun, not hard and statistics prove that girls can do as well as if not better than boys.
(Nita, summary of interview, phase one)

In general, though, it appeared that few students perceived their problems in mathematics as having been related in any way to gender issues (given the caveat about frequency of data, discussed on pp. 274-275), and some who did mention gender issues in their data on teaching seemed to have been alerted to such issues by their reading on the matter. Others mentioned the perceptions of their teachers or mothers as having been influential in the development of their attitudes about mathematics.

I don't remember there being a division, these days there's a lot of talk about girls not having enough attention in maths classes but perhaps that's why I was never focused on for our answers, cos I was a girl and the boys were answering.
(Alice, paired interview, phase one)

Her mum has often told Joanne "I was never good at maths" implying that she didn't expect Joanne to be either. This seems to be a fairly common problem and girls can often be discouraged in this way to achieve in mathematics.
(Doreen about Joanne, summary, phase one)

9.6.13 Unusual attributes for self-as-teacher: Soft sounds

Some of the students mentioned desirable attributes for self-as-teacher that seemed unusual, or were not mentioned by any other students. These attributes are discussed below. All these attributes were collected into one category, indexed as "unusual"; attributes that had been suggested by one person only, were placed in this category. These are not silences in the data, as they were mentioned, albeit only once. I have consequently chosen to call these attributes "soft sounds". These "soft sounds" can be placed into two groups: ideas that developed out of the practices of the teacher education course, and ideas about self-as-teacher that concerned the pupil rather than the teacher.

9.6.13.1 Review and reflection

Adi was the only student to mention that review and reflection by teachers on their teaching practices was important. This is interesting, because Adi mentions this in her paired interview, just two weeks into the teacher education course, and no-one else mentions this attribute at any other stage of data collection. Yet most of the teacher educators, responsible for the Bachelor of Teaching in Primary Teaching and Bachelor of Education in Teacher Librarianship degrees, emphasised the importance of reflection, in the practice of a teacher, throughout the course.

> *I also have this attitude towards all primary curriculum subjects, that they can be taught most effectively, if we, in our teaching practices, constantly review and assess our performance and effectiveness as teachers.* (Adi, paired interview, phase one)

Lexi discussed a related issue, that of journal keeping, which had been discussed in the first session of orientation mathematics. She showed signs of having referred to the recommended readings on the subject, and this appeared to have influenced her view:

> *To overcome this problem [of having negative attitudes to mathematics], I think students should keep a personal journal. This practice proved successful at Giles Plains Primary School where teacher Peter O'Malley concluded "I have found them showing increasing confidence and ability" [quote from one of the recommended articles].* (summary of paired interview, phase one)

Again it is interesting that this is the only suggestion that journal keeping is desirable, throughout all the data collection phases, yet keeping a reflective journal is recommended in a number of the subjects in the teacher education course, throughout both semesters.

9.6.13.2 Ideas concerning pupils

Gail talks about the need to be fair as being a most important characteristic of self-as-teacher. She also is the only person who talks about helping children reach their potential:

> *I guess my aims as a teacher would be more personal aims in setting examples of fairness and discipline, of, I think this is an important one, enabling children to reach their potential. I find that is extremely urgent to me.* (phase three interview)

Joanne talked about the ability of children to develop at any stage of their schooling: this view is consistent with Joanne's dominant philosophical beliefs as a problem solver:

> *I think that that's one of the failings of my education is that you know, you get streamed as an average student, or as above average or whatever and that's all that people feel that they can do. You know, as much as they struggle to get out of that they'll always be in that stream; and I feel that's something that I want my kids to know is that that can always change, that if you're willing to work then it will change. I'd also like to see them equipped with the tools for reasoning and logic and problem solving and then also in the social skills.* (Phase three interview)

And Aaron is the only person who feels that intellectual struggles by pupils are acceptable:

> *If they struggle really, but as long as they're struggling because they want to know, more than they're struggling because I've set them work that they can't do, then I think that's alright.* (phase three interview)

These "soft sounds" in the data are of interest because so few students voice them. Yet many of the attributes that are mentioned so softly, are emphasised by the teacher educators involved in the course. This indicates, perhaps not surprisingly, that the impact of many years of schooling on students' beliefs is far more powerful than any new ideas suggested in the first year of the teacher education program. Self-as-primary-school-pupil is more dominant than self-as-student-learning-to-teach.

9.7 Conclusion

Kuhs and Ball (quoted in Thompson, 1992, p.136) have suggested that holding a particular philosophy of mathematics is linked to beliefs about pedagogy in mathematics (see Chapter 4, p. 56). Pateman (1989) has suggested that the classroom practices of people holding certain philosophies will correspond to those philosophies. I believe that Pateman's suggestion is the more accurate of the two: that is, it is the **practices** of the teachers that are linked to their primary philosophies, rather than their **beliefs** about pedagogy.

My data suggests that espoused beliefs about classroom practice do not necessarily match philosophies of mathematics although I am unable to tell from my data what beliefs-in-action would be like. Although the majority of the participating students appear to be either Platonists or instrumentalists (Ernest, 1989) (See Chapter 7, pp. 120-125), beliefs about what constitutes good teaching and good pedagogy, together with beliefs about the desired attributes of self-as-teacher indicate that a dominant view is that mathematics should be taught in a "classroom focused" way (Kuhs and Ball, quoted in Thompson, 1992, p.136). In such an approach, the emphasis is on the notion that classroom activity is organised according to perceptions of teacher effectiveness (Thompson, 1992). It is important for the teacher to create a classroom atmosphere that is encouraging, fun, and practical, and the learning of mathematics is thought to flow from this. This finding is contrary to Thompson's (1992) assertion that Platonists generally believe that mathematics should be taught in a content-focused way with an emphasis on conceptual understanding: the data in my study show that focussing on content is not a priority for many students; further that knowledge of mathematics is hardly valued at all. Similarly, the data show that while many students do have an instrumentalist view of mathematics, this does not imply that they believe teaching should focus on content. Thompson suggests that the role of a teacher with an instrumentalist perspective is to demonstrate, explain and structure the material in well-sequenced steps.

In this case, the role of the student is to listen carefully and then do exercises or problems that have been modelled by the teacher. While many students did emphasise the need to explain well, and the role of drill was debated, in general, students wished to move away from the procedural view of mathematics, and wished to emphasise practical activities, and discovery of content by the children.

It must be emphasised that these views are the espoused views of the students; it is quite possible that when students start teaching, these beliefs will be heavily modified and that the students will teach in the ways predicted by the Kuhs and Ball model. However, the data indicate that the links between espoused beliefs and philosophies of mathematics are not strong.

In this chapter the two reciprocal parts of the thesis are discussed: in the one section, the students' past experiences and the views deriving from these experiences are discussed. These views were also discussed in Chapters 7 and 8. The reciprocal part of the data was that data relating to the students' aspirations for their future teaching. The two sections can be seen to be closely linked although results from the one section do not necessarily imply results from the other.

The three selves of the student teacher: self-as-primary-school-pupil; self-as-teacher; and self-as-student-learning-to-teach continue to occupy a central role in the data. The principal findings of this chapter are that students do not wish to emphasise the content of mathematics in their teaching: either because of past experiences that were perceived as unpleasant by self-as-primary-school-pupil, or self-as-student-learning-to-teach; or because of lack of confidence in the teaching of mathematics by self-as-teacher. To be good teachers, students believe that they will devote their time and effort to making the classroom a comfortable, interesting and hands-on environment. The advocacy of practical methods seems to have the aim of giving pupils a fun-filled time; and it is the enjoyment of such fun that is believed to promote learning. The belief that teaching should be child-centred is, for some of the students, the way out of their difficulty of knowing little mathematics: if the **child** is to discover and develop results, then the teacher has no need to be knowledgable about the subject matter. This issue will be further discussed in the next chapter, in the overall conclusions drawn from this study.

10. DILEMMAS, PARADOXES AND SOME SUGGESTIONS FOR A WAY FORWARD: CONCLUSIONS OF THE STUDY

10.1 Introduction

The students have finished their discussions. Their ideas on mathematics, and on the teaching and learning of mathematics have been shared with you, the reader, and me, the researcher. The time to collect all these thoughts, which have developed from numerous conversations, and to construct some sort of synthesis of them, has come.

In this chapter I shall discuss the research questions that were central to this study and give a brief overview of how my chosen methodology has been used to explore these questions. I shall highlight the major points suggested by my review of the literature and then discuss the significant issues emerging from this research study. Implications of these results will then be discussed and a possible way forward suggested.

10.2 Research questions

In recent years, the links between the teacher's beliefs about mathematics and mathematical pedagogy, and the teacher's actual practice have been emphasised in various studies (for example, Sanders, 1994; Thompson, 1992; Ernest, 1989; Pateman, 1989). Fewer studies have examined the beliefs about mathematics and mathematical pedagogy that prospective primary school teachers bring into their teacher education courses (Foss & Kleinsasser, 1996; Malone, 1995; Ball, 1988d), and these studies have used a variety of research paradigms, ranging from quantitative studies using survey style instruments to qualitative studies using interviews and observations.

Just as pupils' knowledge and beliefs need to be examined by teachers, in order for effective teaching to occur, so too, do teacher educators need to be aware of the beliefs that student teachers bring to their tertiary studies (Holt-Reynolds, 1991b, Ball, 1988c). It consequently became quite apparent to me that teacher educators in mathematics should become aware of what beliefs prospective primary school teachers hold about mathematics and mathematical pedagogy, on entering the first year of teacher education studies. Teacher educators should also examine the interactions between these beliefs and the offerings of any mathematics education subject sequences in the teacher education course.

This study, therefore, investigated the beliefs about mathematics and the pedagogy of mathematics, of fifty student teachers throughout their first year of study in a primary teacher education course. It also examined the ways in which the students interacted with the first year mathematics education subjects. In particular, the study considered the following research questions:

- What beliefs and attitudes about mathematics and mathematics education do first year primary school student teachers bring into their tertiary education?
- Are any of the students' beliefs about mathematics and mathematics education similar to the beliefs of the teacher educators in mathematics education and how do students interact with first year mathematics education subjects in the teacher education course?
- How do students' attitudes and beliefs influence their success in learning new mathematics at this stage of their lives?
- How do students' beliefs and attitudes affect their ideas on good practice in the teaching of mathematics in the primary school?

The value of this study lies in the opportunities that it provides for teacher educators to help student teachers become confident and successful users and teachers of mathematics. The information gained in this study should encourage teacher educators to look at ways in which some of the students' beliefs - those that promote the use of powerful mathematics - can be reinforced and strengthened. It should also highlight those conceptions and beliefs which need to be interrupted, if student teachers are to become successful users and teachers of mathematics. Further, the study indicates areas in which the beliefs of teacher educators need to be examined, and possibly challenged. The study is also of help in demonstrating what factors might promote change, and what factors are not helpful in creating change. This is beneficial in pointing to possible ways forward.

10.3 Methodology of the study

In summary, the methodology used in this study was chosen because of the access it allowed to students' thoughts and beliefs. A qualitative framework was used as the study wished to gain insights into students' thinking. The first phase had several stages of data collection:

- students posed and answered questions about learning and teaching mathematics, in interviews with one another;
- they summarised the results of these interviews and discussed the implications of their findings;
- notes were written about the way students believed effective learning of mathematics took place;
- notes on the students' first practicum were collected.

As the students themselves were required to pose and answer the questions in the phase one interviews, this allowed them to indicate the areas of mathematics and mathematical pedagogy with which they were concerned, or which they saw as problematic. This meant that the **students** chose the themes for the study, that then became central to the research. In this way, I hoped to ensure the hearing of the students' voices rather than merely imposing my own set of concerns on them.

The second and third phases of data collection allowed me to follow those themes that had emerged in phase one data, and to probe further those issues that had been raised. I also used those phases to collect information about the students' interactions with the first year mathematics education subjects. A questionnaire was used in phase two as this allowed a broader scanning of the participants than in-depth interviews would have allowed, and was less invasive of the students' time and privacy. The questions were of an open-ended nature and the questionnaire served as a parallel to a written interview. Phase three comprised in-depth interviews with a sample of eight students, selected for this phase because they possessed particular characteristics of interest. This phase allowed me to probe any areas that had indicated some depth of data, in either phase one or phase two. It was in this phase that I was able, as researcher, to investigate by prompting, any areas that required further attention. Finally, phase four permitted me to interview the four teacher educators in depth, to ascertain their beliefs about mathematics and mathematics education. This was of use in ascertaining the similarities or differences in the views of teacher educators and students.

The data collected in all four phases were extremely rich and this richness has made possible the opportunity for the students to speak, and to tell the reader in their own words, what they believe and feel. The categories for examination and selection developed from the phase one interviews. Students indicated the importance of a particular issue by discussion in phase one, and it was these issues that were examined further in the later phases. Consequently, the theoretical framework of the study emerged from the students' views on what they perceived as important issues.

In any social research, interpretations of the data are, of necessity, personal to the researcher. However, where I have made interpretations of the data, the data are also presented so that readers can judge for themselves the reasonableness of the interpretations, and so that they are able to bring their own interpretations to the data. In this way, I claim that the data are shown in a manner sufficiently transparent to allow multiple readings. While I have immersed myself in the data, I have also taken care to allow the reader the opportunity to become immersed in them. I have also considered the silences in the text; my experiences as a teacher educator and my reading of the literature enabled me to recognise these silences.

10.4 Voices from the literature

Several important issues arise from the literature review, those being issues dealing with the philosophy of mathematics; the pedagogy common in many mathematics classrooms; and desirable ways of teaching mathematics. The role of belief systems is also discussed in the review of the literature. An outline of these issues is given below.

10.4.1 Philosophical views

Ernest (1991, 1989) and Pateman (1989) suggest a number of philosophies of mathematics that are prominent in various communities: Dengate and Lerman (1995) suggest that some of these can be grouped together as *exogenic* philosophies because they portray a vision of mathematics transmitted from the external world as certain, unchanging and dualistic. Others can be grouped together as *endogenic* philosophies because they incorporate a vision of mathematics that is created by humans, hence is dynamic and fallible and evolves in particular contexts. Ernest (1991) suggests that the views of mathematics held by teachers and others will have great implications for society. If mathematics is seen as context-free and value-free, then it is neutral: consequently, the nature of the mathematics learnt and taught is not part of the debate about access to mathematics for all; not about the marginalisation of various people in their interactions with mathematics; and not about the role of mathematics in the realms of power and wealth distribution. If however, mathematics is a socio-cultural construct, then questions of who chooses the mathematics to be taught; of what the nature of mathematics is; or of who does mathematics, become issues of importance in mathematics education.

It has been argued in this study that a common view currently held among teacher educators in mathematics is that mathematics is a socio-cultural construct and that issues of participation and access for all are regarded as of importance by many teacher educators. Further, while some teacher educators do not seem to emphasise the importance of subject matter (for example, in the study by Floden, McDiarmid and Wiemers, 1990, discussed in Chapter 3, p. 39), others stress that appropriate subject matter in mathematics does need to be taught to student teachers (Lappan & Even, 1989; Ball, 1988a). The view that subject matter knowledge is important, and that certain content matter in mathematics is more accessible and richer than other content, is endorsed by the Discipline Review of Teacher Education in Mathematics and Science (DEET, 1989), which suggests a variety of topics to be taught in a mathematics education sequence in a teacher education course for pre-service primary school teachers. Consequently, there is an emphasis, in certain parts of the research literature, on the need for strong, well connected and accessible mathematical content to be taught to student teachers.

10.4.2 Teaching and learning mathematics

The research literature also considers the pedagogy that most often occurs in a mathematics classroom (Pateman, 1989; Ball, 1988c; and others). This pedagogy highlights the practice of rote learning. The history of mathematics education shows the important role that rote learning has occupied in mathematics classrooms over the years. In the nineteenth century rote learning was considered desirable as it trained the mind and led to accurate thinking (Gladman, 1877, quoted in Clements, Grimison and Ellerton, 1989). In later years, conceptions of mathematics as a rigid and static subject led to a continuation of emphasis on rote learning.

It is only in relatively recent times that an acknowledgment of the importance of accessibility to mathematics for all students has been made by various teacher educators. Views on accessibility coupled with changing ideas on the epistemology of mathematics have led to the development of a reform movement in mathematics education. This reform movement promotes a view of mathematics very different from the absolutist view which, for many people, offers a restricted and limited vision of a rigid discipline emphasising low-order computation. Teacher educators within the reform movement focus on the development of higher order cognitive skills for all students and consider the relationship of the pupil to the knowledge, as opposed to considering only the mathematics itself.

In any attempts to reform the teaching of mathematics, the role of the teacher is absolutely vital. It is therefore necessary to consider three aspects of teaching and learning mathematics:

- the mathematics itself: what sort of mathematics is suitable for the primary school curriculum;
- the child: how the child learns and what informal mathematical knowledge is possessed by the child before entering the classroom;
- the teacher: what knowledge the teacher has about mathematics, and about how children learn mathematics, and what beliefs about mathematics, and the teaching and learning of mathematics are held by the teacher.

Teacher education courses throughout Australia reflect the various views on the nature of mathematics; and how mathematics should be taught. Teacher educators differ in their beliefs about the relative importance of the above three aspects of teaching and learning mathematics. Some teacher educators minimise the mathematics content in their curricula; others emphasise such content. Many innovative ways of changing student teachers' conceptions of mathematics, and of developing their mathematical background, have been noted in the literature (Perry, Geoghegan, Howe, & Owens, 1995; Perry & Conroy, 1994 , Schram, Wilcox, Lanier & Lappan, 1988), and these involve linking the pedagogy to the mathematics taught. However, there is little research on how beginning teachers, who have learnt mathematics at a tertiary institution in such an innovative way, put this learning into practice when teaching.

10.4.3 Beliefs about teaching mathematics

There are many difficulties that are involved in changing belief structures. Beliefs are considered by many to lie at the intersection of the cognitive and affective domains. Holt-Reynolds (1991b) suggests that student teachers' beliefs about teaching are so strong that they expect their teacher education courses to fit in with their existing beliefs, rather than attempting, themselves, to understand the underlying rationale for the structure and content of their teacher education courses. Hence it becomes imperative to engage and challenge the student teachers' beliefs. The literature on American prospective primary school teachers of mathematics (Foss and Kleinsasser, 1996; Ball, 1988d) indicates that student teachers believe their personalities and the way they structure the classroom activities are the fundamental factors influencing the way children learn. If the teachers are patient and enthusiastic, and the activity is fun and interesting, then learning will occur. The teacher is regarded as being extremely important in setting activities and creating an encouraging atmosphere. Not much attention is paid to the role of the pupil in learning. These are the beliefs that the literature indicates need challenging in teacher education courses.

My study indicates that these beliefs occur in the Australian context as well. Constraints mentioned above are voiced throughout my research. The students' interactions with the mathematics education subjects during their first year of the teacher education course confirm results found elsewhere and add to these, in somewhat surprising and, it must be said, alarming ways. The next section considers the major points emerging from my research.

10.5 Major findings of my research study and the implications of these findings

The results of this study developed a number of major themes related to:

- student teachers' beliefs about mathematics
- teacher educators' beliefs about mathematics
- views and experiences of students and teacher educators regarding teacher education in the area of mathematics
- student teachers' beliefs about the teaching of mathematics and about good pedagogy.

10.5.1 Student teachers' beliefs about mathematics

Students' philosophies of mathematics did not fit neatly into one category. The majority of students seemed to express views that were either instrumentalist in nature, or Platonist. If instrumentalist, they seemed to see mathematics as a toolkit of assorted techniques and procedures which were unrelated to each other. If Platonist, they saw mathematics as a rigid, ordered and sequenced body of mathematical truths and laws that existed independently of the human mind. Only a few students seemed to express other philosophies such as a problem solving view, in which mathematics was seen as developing in order to solve problems in a particular context. Of interest in this section were the following results:

10.5.1.1 Dominant philosophies

Many students did not seem to hold only one philosophy of mathematics; depending on the context, other views than the dominant view might be expressed. This appeared to happen in particular with two philosophies of mathematics: the Platonist and the problem solving viewpoints.

Those who were Platonists often made statements that indicated an instrumentalist viewpoint. This could be interpreted to mean that although they were aware that mathematics had a coherence to it, they personally could not see this coherence, and were forced to use whatever results they had access to, in a piecemeal and disjointed fashion.

These results and the tentative explanations I have offered for them, on the basis of the data provided, lead me to suggest that students who are weak in mathematics are disadvantaged by having Platonist philosophies of mathematics, as having such a viewpoint seems to set borders and limits to what these students believe they can achieve.

With the problem solving viewpoint, one of the two students also seemed to believe in the logical nature of mathematics: this would suggest that students who are true problem solvers, are empowered by such a view, and so use, as a set of tools in their problem solving, the logic and structure that they see existing in mathematics. In contrast to the Platonists discussed above, these students have access to the logic and power of mathematics, as they see the purpose of mathematics as being to serve them. Consequently, they are confident and successful users of mathematics as they have ownership of the mathematics. The same links between philosophies could be seen with the teacher educators, who, while principally holding a view of mathematics as fallible and dynamic, also believed in the logical nature of mathematics.

Further results dealing with philosophies of mathematics indicate that the students who held instrumentalist views found mathematics difficult and boring; not seeing the logic and coherence inherent in the subject appeared to make it a subject that had to be learnt by rote without any observation of existing patterns. Most students expressing Platonist views generally indicated that they too found mathematics to be a difficult and unpleasant subject; its perceived dualistic nature appeared to alienate them and provide a source of tension for them. Similarly the lack of access to the "complete picture" of mathematics that these students believed existed, seemed to create feelings of helplessness and demoralisation: a feeling that the power of mathematics did not exist to serve them.

It would appear from the above results that challenging students' philosophies of mathematics is important if students are to be successful and confident users of mathematics. Holding a fallibilist view seems to be more enabling for prospective primary school teachers than holding the more common absolutist viewpoints discussed above.

10.5.1.2 The scope of mathematics

Other major findings concerning students' beliefs about mathematics were that students felt that mathematics was extremely important and occupied a central position in the primary school curriculum. However, this importance was generally restricted to basic procedures and algorithms of arithmetic; few students saw mathematics as comprising anything else in primary school, and few students valued the mathematics done at secondary school, which went beyond the basic skills described above. Linked to this view was the point, emphasised often, that the relevance of the mathematics should be stressed in primary school. Again, however, it was the relevance of low-order skills that was mentioned in this capacity. The implications of these results are:

Firstly, that the mathematics taught in the primary school is unlikely to ever change in nature if the type of mathematics valued by student teachers is highly procedural. Student teachers who do not value higher order skills and concepts are unlikely to promote them in their classrooms.

Secondly, if mathematics is perceived as being about computation, then the use of calculators and computers is discouraged as these technological tools are seen as replacing the essential work and thought processes of the person doing mathematics. Consequently, such students will be concerned about continuing a tradition of mathematics teaching in which procedures, that they have found slow, tedious and boring in the past, will be elevated to the heights of essential mathematics.

Thirdly, there was a silence in the text regarding the question of what mathematics was considered relevant, and this question poses the following problems: is it possible to show the relevance of all mathematics at the levels at which most students learn mathematics? Should the historical development of mathematics be neglected because it has no immediate practical application? Should efforts to allow students to discover the beauty, consistency and elegance of certain mathematics be restricted only to an elite few? In my study, only one student indicated that if mathematics was interesting this was sufficient reason to study it. If student teachers stress that children must be made to see the relevance of the mathematics, does this limit mathematics to only "bread and butter" content as anything else is too abstract and does not relate to every day life?

Unless student teachers gain a different view of what mathematics actually is and can be, and are able to challenge their own notions of mathematics as merely arithmetic computations, the teacher education course will not be valued. It does not appear to emphasise the essential characteristics of mathematics as perceived by the students in terms of their beliefs and with their strong vocational orientation.

10.5.2 Teacher educators' beliefs about mathematics

Two points of significance for teacher education programs emerged from the teacher educators' interviews. Firstly, all four teacher educators shared an academic orientation, in that they enjoyed the intellectual challenge of the mathematics that they taught. Secondly, although the teacher educators seemed to have a dominant view of mathematics as being dependent on human endeavours, three of the four teacher educators expressed sentiments that indicated a formalist view. This was the dominant view expressed by one of the teacher educators, but the data from the others also showed elements of formalism. The significance of these results becomes clear if the content of the Mathematics Education 1 curriculum is considered. One of the topics in this curriculum is the topic of clock arithmetic. While the teacher educators, in the main, relished the idea of demonstrating mathematical systems to the students, the students did not generally value the inclusion of such topics. The teacher educators expressed views that there was merit in having students see the way in which a mathematical system could operate, given a few axioms and rules with which to work. Indeed, the Discipline Review (DEET, 1989) also recommended the inclusion of clock arithmetic and other topics in the teacher education curriculum, for similar reasons to those outlined here. However, when the students' requirements for a mathematics education course in teacher education are considered, a mismatch occurs. Students were eager to learn about any aspects of mathematics that they believed would improve their teaching of the subject in the primary school. However, they did not see the abstract nature of topics such as clock arithmetic as being in any way beneficial to their future teaching. It could be said, therefore, that teacher educators' views on what is essential curriculum for a mathematics education program need to be discussed with students and negotiation of the curriculum, by both students and teacher educators, should be encouraged. The teacher educators' love for mathematics should act as an motivation, rather than as a deterrent to the students. Consequently, mathematics that is both rich and relevant to the student teachers needs to be considered when the mathematics education curricula for teacher education courses are developed. Also, it is crucial that the teacher educators promote a love of mathematics in their students which allows the students to embrace a wide range of mathematical content as part of the essential curriculum for a person intending to be a primary school teacher.

10.5.3 Teacher education: Views and experiences of students and teacher educators

To explain the students' passage through the first year of the teacher education course in mathematics education, I have developed the constructs of self-as-student-learning-to-teach, self-as-primary-school-pupil and self-as-teacher. These three selves of each student teacher are active and interactive throughout the student's passage through the mathematics education subjects. Self-as-student-learning-to-teach has a strong vocational orientation to his or her studies, and sees the purpose of the teacher education course in mathematics education as being to teach self-as-student how to teach well. Consequently, any subject matter considered in the mathematics education subjects is tested by self-as-student against the other two selves in existence. If self-as-primary-school-pupil understands the work, or remembers doing such work in primary school, it is deemed suitable; if self-as-teacher can visualise teaching the subject matter, again it passes the test. However, if self-as-primary-school-pupil finds the subject matter too difficult, or self-as-teacher has problems in developing a mental image in which the subject matter is taught to a class, the subject matter is rejected by self-as-student-learning-to-teach. Further, self-as-teacher gains valued support from supervising teachers on the practicum, and often gives advice to self-as-student that conflicts with the advice offered by the teacher educators. The images that self-as-teacher holds about teaching are often reinforced by the supervising teacher's behaviours during the practicum, and these images are often at odds with images proposed by the teacher educators, for example, ideas concerning the role of drill in mathematics.

As discussed in Section 10.5.2, my research study also showed that the teacher educators in the study shared similar philosophies of mathematics, which were principally fallibilist in nature, although for one the dominant view was formalist. The views of the teacher educators had implications for the interactions between the students, the subjects and the teacher educators. In particular, having a formalist view affected such interactions. For the formalist teacher educator, the interest inherent in the mathematics was a far more motivational factor than the relevance of the mathematics. Consequently, many topics were presented by the teacher educator as topics that were of value because of their inherent elegance, beauty or logic. At the same time, students sought reasons for studying those topics, which would relate to the K-6 curriculum and their future teaching of this curriculum. When such reasons are not made clear to students, the result can be bafflement on their part as to why they are studying much of their mathematics education curriculum. It is likely that much of such confusion and alienation would be avoided if teacher educators examined their beliefs on the rationale for studying much of the mathematics in teacher education courses, and considered the needs of their students, which generally are to see relevance in the given topics. In this study, given the limited view of mathematics that most of the students held, when this view was juxtaposed alongside the formalist view of the teacher educator, some tension and conflict arose. A key implication for teacher educators is the importance of providing students with rationales for their mathematics education curriculum that acknowledge the student teachers' requirements for relevance and applications to their teaching.

Some of the purposes of the teacher education course appeared to be seen as important by both teacher educators and student teachers. All seemed to believe that the primary purpose of the mathematics education subjects was to help students teach well. Where teacher educators differed from student teachers, was in their opinions of what good teaching entailed. All the teacher educators believed that good teaching involved having a strong understanding of the background to the mathematics being taught in the primary school classroom; most of the student teachers did not see such knowledge as at all important. Further, possibly because most of the student teachers were principally either instrumentalist or Platonist, they saw the algorithms and procedures of primary mathematics as far more important subject matter to study in the teacher education course, than the development of mathematics in different societies and contexts.

Once again, an implication of this finding is that students need to be able to see the relevance of any mathematics they are taught in the mathematics education subjects, a theme that has already been woven into earlier sections of this chapter. Allied to this implication, is again the need for teacher educators to both examine and question their own beliefs, and to help students to challenge their beliefs about mathematics. Only if teacher educators consider their reasons for their practices in teacher education and student teachers examine their conceptions of the nature of mathematics, will it become possible for students to develop an appreciation for the mathematics offered in the mathematics education subjects.

Perhaps the most powerful result emerging from this section of the study is the fact that most student teachers did not seem to value any mathematical knowledge other than that required to effectively execute various primary school algorithms. This apparent disregard for mathematical background knowledge is heightened in the following section - student teachers' beliefs about teaching mathematics, and about good pedagogy.

10.5.4 Student teachers' beliefs about teaching mathematics and about good pedagogy

Like the students in the Foss and Kleinsasser (1996) study, the students in my research study believed that certain personal characteristics were fundamental for good teaching. A good teacher was seen as being supportive and enthusiastic. The other important characteristic of a good teacher was the ability to communicate well. Few students indicated that having some knowledge of either the subject content of mathematics, the pedagogy of mathematics, or the way that children learnt, was of any importance.

Following from the discussion about important characteristics for a good teacher was a somewhat disturbing conclusion arrived at by the student teachers. Not only was knowledge about mathematics, pedagogy or children unimportant, but also being knowledgable about mathematics was, in fact, a disadvantage which made the task of being a good teacher that much more difficult. Those students who were struggling with a particular concept felt that this struggle would be an asset in their teaching of the concept. Those students who were able to do mathematics without any difficulty felt this ability would make good teaching more difficult to achieve. Students drew experiences from their past in which they classified teachers who seemed to have difficulty communicating with them, or seemed to lack interest or sympathy for their pupils, as brilliant mathematicians who were, as a result of their brilliance, poor teachers. Having such teachers seemed to lead to the inevitable conclusion that a teacher could not be a good teacher and a brilliant mathematician at the same time. The students' experiences in the first year subject, Mathematics Education 1, reinforced this belief about teachers. Again the students were taught by someone they perceived to be a highly competent mathematician. It was this ability that was seen as making the teacher educator difficult to understand and the work difficult to do, rather than any other characteristics of either the teacher educator, the work, or the students' previous mathematical experiences. This perception reinforced the view that lack of knowledge and lack of ability in mathematics were beneficial for teaching mathematics in a primary school.

This perception of the students has a number of consequences. First of all, in their passage through the teacher education course, self-as-teacher does not value knowledge of mathematics and so self-as-student-learning-to teach does not attempt to gain that knowledge. Secondly, students are preparing themselves for classrooms that most probably will have what could be called an almost anti-intellectual atmosphere. The talented students in mathematics will be shunned by the teacher and the emphasis will be on helping only those students who have less knowledge and understanding than the teacher. Self-as-teacher's limited vision of the nature of mathematics will lead to more emphasis on the procedures that alienated the student teachers in the first place. Being knowledgable about mathematics, for self-as-teacher, will mean knowing how to do procedures and algorithms.

Throughout this thesis I have expressed my view that subject matter knowledge is important. However, it is not merely any kind of mathematical knowledge that is so essential. My results have shown that many students have a great deal of procedural knowledge, but that this does not enable them to apply mathematics to any but the most mundane of problems. It does not provide them with a means of justifying or verifying any results or of creating mathematics to solve problems that arise, which might be novel or different from those already met. I would argue that the kind of mathematical knowledge that student teachers need is knowledge about the nature of mathematics and what it means to know and do mathematics. However, as this kind of knowledge is not perceived as being of any value in the primary school classroom, student teachers are unlikely to seek to develop this area of their expertise. It is the task of the teacher educators to ensure that this kind of knowledge is valued, by placing it in the context of the primary school classroom.

Areas that were emphasised by the students as important aspects of good pedagogy were that mathematics should be taught in a practical way, the content matter should be relevant and the lessons should be fun. The emphasis in good pedagogy is on ensuring pupils are enjoying themselves and fun is generally not seen as being provided by intellectual challenge but rather by doing practical and relevant activities, which involve the use of games. The teacher is at the centre of the student teachers' thinking, rather than the learner, and it is the provision of fun-filled activities that is thought to create learning.

Students are exhibiting some knowledge of constructivist theories of learning when they suggest that good pedagogy involves practical methods of teaching (Section 9.5.1). Practical methods include hands-on exploration and discovery. However, when the superficial knowledge that students have about learning theory is coupled with their disregard for, and lack of knowledge about, mathematical concepts, a dangerous situation emerges. Learning is thought to develop out of the experience of having fun or of seeing the relevance of the activity, rather than out of any cognitive efforts of the pupils.

As part of their learning about good pedagogy in their various teacher education classes, student teachers are encouraged to "listen to their pupils"; not to "tell" the pupils; to act as facilitators, not as experts. This advice is translated in most students' minds, into a clear directive to let the pupils develop their mathematics without interference. For these students, this view of the respective roles of the mathematics teacher and the pupils justifies any perceived weaknesses or lack of knowledge in mathematics on the part of the teacher and confirms the validity of the notion of some students that they would be disadvantaged by being knowledgable about mathematics. Hence lack of knowledge about subject matter on the teacher's part is not seen to be a problem: whatever mathematics the pupils develop is thought to be good mathematics, as it is a product of their environment and context. So lessons are held during the practicum, or envisaged for the future, in which children are busy exploring and playing, but seldom is the mathematical content of the activity emphasised or developed. For the teacher educator, such results suggest the need for a critique of the assumptions on which teacher education programs are based.

Further, students' choices of pedagogy are made on the basis of the interest these choices will create. Few students indicated that they chose to use a variety of methods or practical activities because these might be more suited to a particular context or learning situation. Most choices of this nature, were made to alleviate the boredom that self-as-primary-school-pupil has experienced in mathematics lessons.

It seems to be that the gift of mathematics education that student teachers wish to give their pupils consists of brightly coloured paper, wrapped around an empty box: the pedagogy suggested as good pedagogy by the students, would be regarded as good by most mathematics educators, but the lack of knowledge about mathematics content, and the belief that knowledge of such content would not be beneficial to their teaching, implies that the pedagogy is somewhat hollow; that pupils engaged in mathematical activity in these student teachers' classrooms will be busy and happy, but not necessarily learning the kind of mathematics which will enable them to more fully and comprehensively engage mathematically with their world in a powerful and fruitful way.

10.6 Dilemmas and paradoxes

Arising from these results are a series of dilemmas and paradoxes concerning the learning and teaching of mathematics. Both teacher educators and student teachers experience various dilemmas and many dilemmas arise for student teachers as a result of conflicting views of self-as-teacher, self-as-student-learning-to-teach, and self-as-primary-school-pupil. These will now be considered.

Firstly, student teachers' beliefs about the pedagogy they wish to adopt in the mathematics classroom may be at variance with their beliefs about mathematics and mathematics learning and teaching. A series of issues arise for them:

- Self-as-teacher belief: *Enthusiasm is of prime importance in being a good teacher.* Yet most of the students did not feel particularly rhapsodic about mathematics, due to their past experiences in that subject. From where are these students to muster enthusiasm in order to teach a subject that they perceive as dull and difficult? Is the task of the teacher education course in mathematics education to develop this enthusiasm? If it is agreed that this is one of the tasks of the teacher education course, then teacher educators need to consider carefully their choices of mathematical experiences provided in their subjects.

- Self-as-teacher belief: *The teacher's role is to instruct the students in the procedures and algorithms of mathematics.* Yet many students in the teacher education course do not feel that they are successful users of this type of mathematics themselves. While repeatedly stating the importance of teaching "the basics", that is the multiplication tables, and algorithms and rules to which they themselves were subjected, students also admit that they do not feel confident about their mathematical ability, and indicate that their mathematical experiences in the past have not been particularly appropriate.

What kind of mathematics are prospective teachers going to need to know, in order to be the type of teachers they wish to be? Is the mathematics to be the rule-oriented, unconnected vision to which they were exposed in the past? They will feel comfortable teaching in this way as they have clear models of this type of pedagogy. However, it is precisely this conception of a procedure-based, rote-learned mathematics that alienated many of them in the past.

Related to this dilemma is another problem: while not valuing subject matter knowledge, student primary teachers are constrained by their lack of it. Many of them are terrified that their pupils will ask them questions about the conceptual underpinnings of a procedure; questions that they are convinced they will not be able to answer from their current knowledge base. However, they believe that children should be answered if such questions are posed.

- Self-as-teacher belief: *Good teaching involves encouraging the asking of questions by pupils, and the answering of these questions by the teacher.* How, then, are they to find out the answers? How can they instil understanding when they themselves may not fully understand?

Another aspect of good practice is recognition that each child is different and has the right to individual attention. Paradoxically, though, children who may be gifted in mathematics are shunned as pariahs.

- Self-as-teacher belief: *Good teachers always have the answers.* Gifted children challenge and threaten this notion.

Related to the above two dilemmas is the acceptance of child-centred education in which children actively construct their own knowledge.

- Self-as-teacher belief: *Good teaching is child-centred and allows the child to set the pace and direction of learning.* Yet the student teachers have spoken about experiencing insecurity when teaching mathematics, due to their perceptions of the dualistic nature of mathematics and to their frail content knowledge base. This insecurity has led to the student teachers' desire to be totally prepared, by working out a "script" before any lesson. Such a script has all answers to possible questions carefully worked out, and student teachers express great unwillingness to deviate from the script at any time. How then are they to deal with any straying from the script that must inevitably occur in a child-centred environment?

A paradox that has become apparent in this study, is the following:
Students teachers believe that good pedagogy is related to practical work involving exploration and discovery. Yet some of the students felt that this was not an appropriate way for them to learn in the mathematics education subjects at university. They expressed frustration at having to do a great deal of practical work and often mentioned how they preferred lectures by the "teacher-expert". However, when they did experience the latter kind of teaching they often did not feel they were learning much and found the content bewildering. It appears that some of the student teachers are rejecting ways of understanding and making meaning of the mathematics. Potentially this paradox lays the ground for a kind of pedagogical reproduction - teachers will reproduce inappropriate past practices because they are so entrenched in these less effective ways of learning.

Given these beliefs, what then do prospective student teachers see as the role of their teacher education courses in mathematics? Certainly, students believe such courses should be engendering enthusiasm for the subject of mathematics. Mathematics education courses should also be teaching prospective teachers the strategies and jargon that will show them how to explain procedures in different ways to their pupils. Mathematics education courses at university should also be helping the prospective teacher to carry out remedial work in the classroom. However, most students did not feel the same need to discuss a pedagogy for mathematically gifted children. Typically, this provoked an avoidance response - *"I hope I don't have anyone like that in my class"* - and student teachers indicated a deep seated belief that they themselves would not be able to do the mathematics needed to extend and stimulate gifted children.
Consequently any discussion of subject matter knowledge or pedagogical knowledge (Shulman, 1986a, p. 26) specific to the teaching of mathematically gifted and talented

pupils was dismissed as irrelevant, as students could not visualise a scenario in which they would either be experts at the work themselves, or be comfortable in allowing pupils to investigate mathematics in which they themselves did not feel competent. Such situations were to be avoided and hence did not need to be discussed in a mathematics education subject.

Entering the teacher education course with such dilemmas already in existence, it is a great relief for most students to interpret new-found learning as indicating that they will not be required to be expert teachers but rather be responsible for providing an environment that is secure and supportive and allows exploration and discovery - characteristics that most students feel confident they can provide. Their dilemmas appear to be resolved by choosing a path that requires little knowledge about mathematics or the learning of children.

Set against the above context of student teachers' perceived requirements for a teacher education course, the teacher educator's tensions become apparent. To engender enthusiasm for mathematics and mathematics teaching and learning means to avoid the feelings of frustration and failure that students have experienced in their past learning of mathematics. However, students are accustomed to reacting to difficult mathematics in exactly this way. If a problem cannot be solved immediately, past experience has taught them that it is not within their capabilities; if the mathematics that is encountered is different from their perception of what mathematics should be, then it is disregarded; and if the mathematics education subject is not seen to be teaching the skills that students require in order to enable them to clearly explain algorithms to their future pupils, then it is dismissed as irrelevant.

Consequently, what the teacher educator regards as good teaching in the tertiary situation is often met with lack of enthusiasm, frustration and disinterest, creating a tension for the teacher educator in terms of the conflicting goals of engendering enthusiasm and challenging student beliefs. Are students to be wooed into a positive but limited view of mathematics or are they to be exposed to a different and possibly more complex view of the discipline with the concomitant danger of distress and alienation?

Exacerbating the situation are the students' brief encounters with the classroom on their teaching practicum, in which they see the so-called "real world of teaching" which reinforces their conceptions of teaching as telling and mathematics as rule bound. Many report that teachers have told them that what is being done in the university subject is impractical and irrelevant and given the world view of these teachers and their pupils, this is probably true. The status conferred on the student teacher during the practicum is highly dependent on the supervising teacher. Consequently, the student teacher must share the supervising teacher's view of mathematics or run the risk of having such status denied. Also part of this situation is the role played by pupils: pupils are not used to being active mathematical thinkers in these classrooms and the student teacher's attempts to implement any new ideas that might have arisen as a result of the university course will often end in failure. This same problem may not arise in other areas of the curriculum as primary school teachers tend to be more confident and knowledgeable about areas of the curriculum other than mathematics. It appears that teachers often lack the necessary pedagogical and subject matter knowledge, described by Shulman (1986a), in the area of mathematics in particular.

Another dilemma for the teacher educator involves ideas on learning. While believing whole heartedly in the strength of the learning community for aiding the learning of both student teachers in teacher education courses, and pupils in schools, the teacher educators do not believe this absolves the teacher from having a comprehensive background knowledge of the mathematics being learnt. The teacher educators believe that students at all stages benefit from discussion about mathematical ideas with peers; from justifying their results to peers and debating results with them; from making sense of others' ideas and challenging these ideas if they do not make sense; and from communicating mathematical ideas to their community in general. However, the teacher educators also believe that the teacher needs to have a thorough knowledge of the topic under discussion, in order to highlight the mathematical content that should emerge, to guide students back from fruitless ventures, and to be able to develop the sorts of activities that will encourage the exploration of powerful mathematical ideas.

The culture of the students studying the mathematics education subject is such that there is a strong pressure on the teacher educator to tell the students how to teach; to give them rules for teaching mathematics; and to show them the steps that make up various procedures that are, in the student teachers' eyes, the very essence of mathematics. Attempts to involve students in searching for patterns; making conjectures and then proving these (or at least justifying them); exploring the conceptual underpinnings to the primary mathematics they will teach; and investigating the connections between various areas of mathematics are often met with cries *of "Is this in the syllabus?"* or *"How will primary school children understand this when I find it so difficult?"* While idealising notions of good practice, the students constantly deny it in their own learning.

The teacher educator consistently has the dilemma of choosing between subject matter and approaches that educators in mathematics believe will lead to reform in mathematics teaching, or subject matter and approaches that the student teachers believe are essential preparation for their lives as teachers of mathematics in the primary school. The choice for the teacher educator becomes one of choosing between challenging and comforting the students.

In challenging student beliefs about what it means to do mathematics, and how to learn and teach mathematics, the teacher educator has to resist a socialisation process almost as strong as the one students will encounter on practicum and as beginning teachers. There is a recursive aspect to the whole scenario; teacher educators' ideas of good teaching in the tertiary area are exposed to a similar set of dilemmas and contradictions as are their students' ideas, in their learning to teach.

10.7 Looking forward

The findings of this study have various implications for student teachers and mathematics teacher educators. Some of the beliefs held by the student teachers are beliefs that should be reinforced and strengthened. These are the beliefs concerning good pedagogy. Students believe that good pedagogy involves practical work which is child-centred, collaborative and enjoyable. Teachers should be enthusiastic and supportive of their pupils. Such aspirations are in keeping with the ideas of most teacher educators.

What needs to be added to the mixture is a blend of knowledge about mathematics, pedagogy and children. The challenge for teacher education courses in mathematics education becomes a challenge to make intellectual stimulation in the mathematics education courses possible so that learning is engaging and satisfying, and so that subject matter knowledge becomes valued as part of the process of learning how to teach. The question is, how does such intellectual stimulation become part of the mathematics education course?

The first step is to challenge the beliefs of student teachers about mathematics and mathematics education. As long as students see mathematics as a rule bound and difficult subject, they will continue to shun any attempts to become knowledgable about mathematics. Students need to be exposed to a different vision of mathematics from the one that most of them have held all their lives. In fact, mere exposure to a new vision of mathematics is probably not, in itself, enough. The value of this "different kind of mathematics" must be clear. Students need to see what the paralysing consequences of holding restricted views of mathematics can be, and how empowering a broader view of mathematics is.

The difficulty in any attempts to encourage student teachers to challenge their beliefs about mathematics, is that their views are so pervasive throughout society. From governments, who aid and abet limited views, by stressing "a return to the basics" to those employers who believe that workers who can do long division using pen and paper are in some way superior to those who will use a calculator to find the answer, most members of society do not appreciate the value of attempts at reform in mathematics education. As student teachers will be testing their ideas and visions against those of their communities, changes in the community are essential if the socialisation of beginning teachers can be prevented from maintaining the status quo in the mathematics classroom. Further, students are highly influenced by the values of their supervising teachers on the practicum. Consequently, professional development courses for teachers should be offered by those who want to encourage reform in mathematics teaching. Similarly, the importance of courses for parents, such as the Family Mathematics programs, cannot be over-emphasised. If teachers and parents become supportive of changes in mathematics teaching, then student teachers will be more easily convinced of the value of reform.

Further, any new vision of mathematics needs to be demonstrated to be effective and needs to be seen as leading to the kind of classroom environment that self-as-teacher espouses. Student teachers wish, above all else, to be good teachers. Consequently if teaching a different kind of mathematics can be shown to display the characteristics that students believe good pedagogy involves, this could be a convincing argument for a different kind of teaching. My research has shown that student teachers value teaching that is practical, supportive of children's efforts and enjoyable for pupils. If student teachers learn that children find certain lessons supportive and enjoyable, this should strengthen the case for learning in that way. I believe such lessons should be ones in which children are encouraged to work collaboratively, to debate the meaning of their results, to value the mathematics underpinning the activity, to develop their own methods of finding solutions and to share these methods with others.

It also appears imperative to me, that if any change is to occur in the attitudes of prospective primary school students, that these students should be able to observe primary school teachers engaged, with their classes, in the sort of mathematics that promotes accessibility and power. Teacher education subjects should use videos with examples of such lessons wherever possible, and the teaching practicum needs to be monitored carefully. While it is not possible to place all students with supervising teachers who have a positive view of the kind of mathematics teaching espoused here, opportunities for discussion and debate among teacher educators, student teachers and supervising teachers should help to make explicit the advantages of, and constraints on, this type of teaching. Student teachers need to be aware that constraints do exist and they need to be able to develop their own methods of dealing with these constraints and of working within the existing framework, without having to surrender their ideas of teaching mathematics in an innovative and powerful manner. Again this should be an important part of the mathematics education subjects in a teacher education course.

The second step is to avoid those aspects of intervention by the teacher educators which are **not** effective. For example, the teacher educator appearing as expert mathematician can confirm or reinforce beliefs that students hold about their own inadequacy in mathematics; thus leading to a denial of the value of ability and knowledge in maths. It is more encouraging and positive to allow students to discover the strength of their own abilities, and those of their peers. Learning to test their conjectures and answers by having to justify them, themselves, will not only lead to a stronger vision of mathematics but will personally empower the students. Teacher educators also need to examine their own beliefs and consider whether the teacher education mathematics curricula that they develop do indeed allow students to become successful and thoughtful users of mathematics. The teacher educators also need to consider whether they are helping students to see the relevance of, and rationale for, the mathematics that they have chosen to teach students.

It might also be more productive to focus on pedagogical aspects of the subject sequence in a teacher education course, before considering mathematical content knowledge. As student teachers in this study have shown very strong vocational orientations, and have stressed the importance for themselves of seeing the relevance of the subject matter, the links between the mathematics covered in the teacher education course and the mathematics needed to teach in primary schools should be emphasised. Students need to be aware that the content knowledge covered in the first year of their teacher education course is strongly linked to the primary curriculum so that they are able to perceive it as relevant to their teacher education. A powerful way forward is to use the primary mathematics curriculum as a springboard for the teaching of new mathematics; mathematics that has as its starting point, primary school content, but then extends from there into richer and deeper mathematics. It should also be noted that the practicum experience is a major influence on the way the first year subjects are received by the student teachers. Students measure how well prepared their university courses have made them for teaching mathematics on the practicum, and they also use their supervising teachers' views of their university education as a gauge of how worthwhile the teacher education subjects are for the teaching of primary school mathematics. These factors again emphasise the need for the supervising teachers to share the reform view of mathematics.

Another important way of interrupting the current discourse of student teachers is by the use of case studies. Examining practices of teacher educators all over the world in this area helps students to see that they are not alone in their views; nor are the teacher educators being idiosyncratic in their approaches. Discussion and argument about different practices should allow students to become aware of their own beliefs and the possible ramifications of such beliefs. Such discussion might also be helpful in encouraging the teacher educators to examine their beliefs. Once students (and teacher educators) have exposed their own beliefs, they should be ready to consider alternatives. This should be done in a supportive environment where the difficulties inherent in change are understood and discussed with their community of student teachers.

The important question of whether there is a potential for change in beliefs and resultant attitudes underlies the entire study. However, given that the focus of this study is students in their first year of a teacher education program, it is doubtful that answers to this question can be supplied by this short interaction. Consequently, it would be of value for future research to continue to examine this question, for example, by expanding the time period over which students participate in a similar study.

10.8 Conclusion

This chapter has provided an overview of, and conclusions to, my research study into the beliefs of prospective primary school teachers during their first year of a teacher education course. The research questions driving this research were discussed and the methodology of the study reviewed. Conclusions and suggestions for ways forward were proposed.

The strengths of this study lie in the manner in which the methodology has provided a way to explore the research questions. By asking students to pose their own questions on the area of mathematics education, the study allowed the students to define the areas to be investigated. In this way, the methodology ensured that the students had a voice in the research. Further, the use of computer software for qualitative data analysis helped to pinpoint those parts of the text which could be used to offer insights into the students' thoughts. The methodology allowed the data to speak to the reader, often leading the reader to unexpected places, for example, where students talk of their lack of understanding being a strength for their future teaching (see Section 9.3, pp. 226-230). This has made easier the formation of conclusions, as the data were so rich in content.

Some caveats also have to be mentioned. As with any qualitative research, there are some difficulties in interpretation of the data. The researcher always has to "get under the skin" of the participants to ensure that the interpretation is grounded in the data. However, often a situation arises in which what is said could be interpreted in a few ways: for example, in this study, a student's statement about "guessing" can be interpreted either as evidence of low-order or of higher-order skills (see Chapter 7, pp. 143-144). It is for this reason that data has been presented in such detail: in order for readers to test their interpretations against mine.

Further, another characteristic of qualitative studies is that the context of any data is crucial: this limits the type of generalisation that can be made on the strength of any data. While it appears clear from this study that student teachers in many different contexts believe that personality and classroom ambience are the key factors in promoting learning, results about communication in this study appear to be different in different contexts. While student teachers believed that a good teacher communicates well, few of them felt that this was a major characteristic required for self-as-teacher (see Chapter 9, p. 275). This could be because of the context in which such discussions took place: perhaps views on communication for self-as-teacher were regarded as more appropriate for a study in general education than in mathematics education; and so, discussion of this did not arise in the particular context of self-as-teacher text. It must be stressed that the data cannot be divorced from the context in which they appear. This is both a strength and limitation of this type of research.

This chapter discussed the dilemmas that exist in the context of student teachers learning about mathematics and mathematics teaching. The existence of such dilemmas highlights the notion that there are no clear, absolute and indisputable answers, either in the study of students learning to teach mathematics, or in other areas of education. Groundwater Smith (1994) has argued that there is value in using dilemmas as a framework for the teacher education curriculum. She suggests that one of the roles of a teacher education course is to "unsettle" the student teacher. This is certainly considered to be the case here, where beliefs about mathematics education need to be challenged and examined. Further, I believe that this study had another role: that of unsettling the teacher educators involved in helping prospective primary teachers become effective teachers of mathematics. By hearing the voices of the students, as recorded here, teacher educators are presented with a clear indication that their beliefs, too, might need to be examined and challenged.

To enhance the teaching practices of prospective primary school teachers, both the beliefs that empower and those that constrain must be considered. An effective way of promoting change is to recognise, affirm, challenge, and then act on existing beliefs, rather than marginalising or ignoring them. Student teachers have indicated that they want their pupils to enjoy their mathematics lessons. They believe it is important to have a warm and supportive atmosphere in the classroom. Consequently, teacher educators need to ensure that the teaching experiences for student teachers include opportunities to work in primary schools in which intellectual challenges in mathematics are enjoyed by primary school pupils, and in which the pupils gain support from their community of peers.

Most importantly, however, student teachers, themselves, need to feel that mathematics is accessible to them. They need to be aware that the sort of mathematics being discussed is within their reach; and that being a knowledgable teacher of mathematics allows them choices and promotes good teaching. If they are able to see mathematics differently, and are able to enjoy the stimulation and encouragement offered, by working with a community of learners and by learning to trust their own thought processes, they will feel strengthened in their mathematical knowledge. This in turn, will encourage them to teach their pupils in an open, innovative and supportive way, and allow them to share with their pupils, the beliefs that they will have in the power of mathematics.

REFERENCES

Adelman, C., Jenkins, D. & Kemmis, S. (1983). Rethinking case study: Notes from the second Cambridge conference. In Deakin University Production Unit (Eds.), *Case study: An overview* (2nd ed.). Victoria: Deakin University.

Australian Education Council. (1991). *A national statement on mathematics for Australian schools.* Carlton, Victoria: Australian Education Council and Curriculum Corporation.

Ball, D.L. (1988a). *The subject matter preparation of prospective mathematics teachers: Challenging the myths* (Research Report 88-3). East Lansing: Michigan State University, National Center for Research on Teacher Education.

Ball, D.L. (1988b). *Research on teaching mathematics: Making subject matter knowledge part of the equation* (Research Report 88-2). East Lansing: Michigan State University, National Center for Research on Teacher Education.

Ball, D.L. (1988c). *Unlearning to teach mathematics* (Issue Paper 88-1). East Lansing: Michigan State University, National Center for Research on Teacher Education.

Ball, D. L. (1988d). *Knowledge and reasoning in mathematical pedagogy: Examining what prospective teachers bring to teacher education.* Doctoral Dissertation, Michigan State University, East Lansing, Michigan.

Ball, D.L. (1989). *Breaking with experience in learning to teach mathematics: The role of a preservice methods course* (Issue Paper 89-10). East Lansing: Michigan State University, National Center for Research on Teacher Education.

Bauersfeld, H. (1992). Integrating theories for mathematics education. *For the Learning of Mathematics, 12*(2), 19-28.

Begle, E.G. & Geeslin, W. (1972). *Teacher effectiveness in mathematics instruction* (National Longitudinal Study of Mathematical Abilities Reports: No 28). Washington, DC: Mathematical Association of America and the National Council of Teachers of Mathematics.

Belenky, M.F., Clinchy, B.M., Goldberger, N.R., & Tarule, J.M. (1986). *Women's ways of knowing: The development of self, voice, and mind.* New York: Basic Books.

Bogdan, R. & Biklen, S. K. (1982). *Qualitative research for education: An introduction to theory and methods.* Boston: Allyn & Bacon.

Booker, G. (1993). *Contexts for mathematics education.* Plenary Address to the Sixteenth Annual MERGA conference, Brisbane, July.

Borko, H., Eisenhart, M., Brown, C.A., Underhill, R., Jones, D., and Agard, P.C. (1992) Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education, 23*(3), 194-222.

Britzman, D.P. (1986). Cultural myths in the making of a teacher: Biography and social structure in teacher education. *Harvard Educational Review, 56*(4), 442-456.

Burns, M. (1986). Teaching "what to do" in arithmetic vs. teaching "what to do and why". *Educational Leadership 43*(7), 34-38.

Burton, L. (1996). Mathematics, and its learning, as narrative - A literacy for the twenty-first century. In D. Baker, J. Clay, & C. Fox (Eds.), *Challenging ways of knowing English, Mathematics and Science* (in press). London: Falmer Press.

Clarke, D. (1984). Secondary mathematics teaching: Towards a critical appraisal of current practice. *Vinculum, 21*(4), 16-21.

Clements, M.A., Grimison, L. A., & Ellerton, N. F. (1989). Colonialism and school mathematics in Australia 1788 - 1988. In N.F. Ellerton & M.A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 50-78). Geelong, Victoria: Deakin University Press.

Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher 23*(7), 13-20.

Cobb, P., Wood, T., Yackel, E. & Perlwitz, M. (1992). A follow-up assessment of a second-grade problem-centred mathematics project. *Educational Studies in Mathematics 23*(5), 483-504.

Cockcroft, W. (Chair) (1982). *Mathematics counts.* London: HMSO.

Cohen, D.K. & Ball, D.L. (1990). Policy and Practice: An overview. In D.K. Cohen, P.L. Peterson (Co-Directors), & S. Wilson, D. Ball, R. Putnam, R. Prawat, R. Heaton, J. Remillard, N. Wiemers, *Effects of state-level reform of elementary school mathematics curriculum on classroom practice* (Research Report 90-14). East Lansing: Michigan State University, National Center for Research on Teacher Education and Center for the Learning and Teaching of Elementary Subjects.

Cooney, T.J. (1985). A beginning teacher's view of problem solving. *Journal for Research in Mathematics Education, 16,* 324-336.

Delamont, S. (1992). *Fieldwork in educational settings: Methods, pitfalls and perspectives.* London: The Falmer Press.

Dengate, B. & Lerman, S. (1995) Learning theory in mathematics education: Using the wide angle lens and not just the microscope. *Mathematics Education Research Journal* 7(1), 26-36.

Department of Employment, Education and Training (DEET). (1989). *Discipline review of teacher education in mathematics and science* (Vol 1). Canberra: Australian Government Publishing Service.

Dougherty, B.J. (1990) Influences of teachers' cognitive/conceptual levels on problem-solving instruction. In G. Booker et al (Eds), *Proceedings of the Fourteenth International Conference for the Psychology of Mathematics Education.* (pp. 119-126). Oaxtepec, Mexico: International Group for the Psychology of Mathematics Education.

Eisenhart, M.A. & Howe, K.R. (1992). Validity in educational research. In M.D. LeCompte, W. L. Millroy & J. Preissle (Eds.), *The handbook of qualitative research in education* (pp. 643-680). San Diego, CA: Academic Press.

Ellerton, N. F. & Clements, M. A. (1994). *The national curriculum debacle.* West Perth: Meridian Press.

Entwistle, N. (1984). Contrasting perspectives on learning. In F. Marton, D. Hounsell, & N. Entwistle (Eds.), *The experience of learning* (pp 1-18). Edinburgh: Scottish Academic Press.

Erickson, F. (1986). Qualitative methods in research on teaching. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (3rd ed., pp. 119-161). New York: Macmillan.

Ernest, P. (1989). The impact of beliefs on the teaching of mathematics. In P. Ernest (Ed.), *Mathematics teaching: The state of the art* (pp. 249-254). Basingstoke: The Falmer Press.

Ernest, P. (1991). *The philosophy of mathematics education.* Hampshire, U.K.: The Falmer Press.

Fennema, E., Carpenter, T. P., & Peterson, P.L. (1989). Learning mathematics with understanding: Cognitively guided instruction. In J. Brophy (Ed.), *Advances in research on teaching* Vol 1 (pp. 195-221). Greenwich, Connecticut: JAI Press.

Fenstermacher, G.D. (1980). On learning to teach effectively from research on teacher effectiveness. In C. Denham & A. Lieberman (Eds.), *Time to learn* (pp 127-138). Washington, DC: National Institute of Education.

Floden, R.E., McDiarmid, G.W., & Wiemers, N. (1990). *Learning about mathematics in elementary methods courses* (Research Report 90-1). East Lansing: Michigan State University, National Center for Research on Teacher Education.

Foss, D.H. & Kleinsasser, R.C. (1994). *Investigating preservice teachers' beliefs, conceptions, and practices: Contrasting research paradigms.* Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, April.

Foss, D. H. & Kleinsasser, R. C. (1996). Preservice Elementary Teachers' Views of Pedagogical and Mathematical Content Knowledge. *Teaching and Teacher Education 12*(4) (in press).

Getting the most out of mathematics: teaching strategies for the classroom (video) (1990). South Melbourne: Thomas Nelson Australia.

Gibbs, G., Morgan, A., & Taylor, E. (1984). The world of the learner. In F. Marton, D. Hounsell, & N. Entwistle (Eds.), *The experience of learning* (pp. 165-188). Edinburgh: Scottish Academic Press.

Glaser, B. & Strauss, A. (1967). *The discovery of grounded theory.* Chicago: Aldine.

Goodlad, J. (1984). *A place called school: Prospects for the future.* New York: McGraw Hill.

Groundwater Smith, S. (1994). *Using dilemmas as a template for reflection.* Paper presented at the Annual Conference of the Australian Association for Research in Education, Newcastle, Australia, November.

Hersh, R. (1986). Some proposals for revising the philosophy of mathematics. In T.Tymoczko (Ed.), *New directions in the philosophy of mathematics* (pp. 9-28). Boston: Birkhauser.

Hart, F. (1934). *Teachers and teaching: By ten thousand high school seniors.* New York: Macmillan.

Heshusius, L. (1994). Freeing ourselves from objectivity: Managing subjectivity or turning toward a participatory mode of consciousness? *Educational Researcher, 23*(3), 15-22..

Higginson, W. (1989). Beware the Titanic deckchair: Remarks on desirable directions for change in mathematics education. In N.F. Ellerton & M.A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 4-17). Geelong, Victoria: Deakin University Press.

Holt-Reynolds, D. (1991a) *Practicing what we teach* (Research Report 91-5). East Lansing: Michigan State University, National Center for Research on Teacher Learning.

Holt-Reynolds, D. (1991b). *The dialogues of teacher education: Entering and influencing preservice teachers' internal conversations* (Research Report 91-4).). East Lansing: Michigan State University, National Center for Research on Teacher Learning

Holt-Reynolds, D. (1994). *Learning teaching, teaching teachers.* Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, April.

Howard, P. (1995). *Using conversational interviews and observations to access beliefs about mathematics.* Paper presented at Issues in Teacher Education in Mathematics Seminar, UTS, Sydney, March.

Kilpatrick, J. (1992). A history of research in mathematics education. In D.G. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 3-38). New York: MacMillan Publishing and National Council of Teachers of Mathematics.

Lakatos, I. (1962). Infinite regress and the foundations of mathematics. *Aristotelian Society Proceedings, Supplementary Volume No. 36,* (pp. 155-184).

Lakatos, I. (1976). *Proofs and refutations.* Cambridge, U.K.: Cambridge University Press.

Lappan, G. & Even, R. (1989). *Learning to teach: Constructing meaningful understanding of mathematical content* (Craft Paper 89-3). East Lansing: Michigan State University, National Center for Research on Teacher Education.

Lerman, S. (1983). Problem solving or knowledge centered: The influence of philosophy on mathematics teaching. *International Journal of Mathematical Education in Science and Technology, 14*(1), 59-66.

Lerman, S. (1993). *Constructivism and intersubjectivity: Parallel or shared meanings in mathematics?* Paper presented at the Constructivism: The Intersection of Disciplines Conference, Deakin University, Geelong, Victoria.

Lerman, S. (1994). Email communication in private correspondence.

Lerman, S. (1996). Intersubjectivity in mathematics learning: A challenge to the radical constructivist paradigm? *Journal for Research in Mathematics Education* 27(2), 133-150.

Lincoln, Y. and Guba, E. (1985). *Naturalistic inquiry.* Beverly Hills, CA: Sage Publications.

McCracken, G. (1988). *The Long Interview* (Sage University Paper Series on Qualitative Research Methods, Vol. 13). Newbury Park, CA: Sage.

McDiarmid, G.W. (1989). *Tilting at webs of belief: Field experiences as a means of breaking with experience* (Research Report 89-8). East Lansing: Michigan State University, National Center for Research on Teacher Education.

McLeod, D.B. (1992). Research on affect in mathematics education: A reconceptualization. In D. Grouws (Ed), *Handbook of research on mathematics teaching and learning* (pp. 575-596). New York: NCTM and Macmillan Publishing Co.

McLeod, D. (1994). Research on affect and mathematics learning in the JRME: 1970 to the present. *Journal for Research in Mathematics Education,* 25(6), 637-647

Malone, J. (1995). Determining preservice teachers' beliefs about their role in teaching mathematics. In A. Richards (Ed.), *FLAIR: Forging links and integrating resources, Proceedings of the 15th Biennial Conference of the Australian Association of Mathematics Teachers* (pp. 264-270), July, Darwin.

Mandler, G. (1984). *Mind and body: Psychology of emotion and stress.* New York: Norton.

Mandler, G. (1989). Affect and learning: Causes and consequences of emotional interactions. In D.B. McLeod & V.M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 3-19). New York: Springer-Verlag.

Marshall, C. & Rossman, G.B. (1989). *Designing Qualitative Research.* Newbury Park, CA: Sage Publications.

Marton, F. & Säljö, R. (1984). Approaches to learning. In F. Marton, D. Hounsell, & N. Entwistle (Eds.), *The Experience of Learning* (pp. 36-55). Edinburgh: Scottish Academic Press.

Merriam, S. B. (1988). *Case study research in education - A qualitative approach.* California: Jossey-Bass Inc.

Miles, M.B. & Huberman, A.M. (1994). *Qualitative Data Analysis* (2nd ed.) Thousand Oaks, CA: Sage Publications.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA, USA: NCTM.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics.* Reston, VA, USA: NCTM.

New South Wales Department of Education. (1989). *Mathematics K-6.* Sydney: NSW Department of Education.

NUDIST. (1993). *Computing and qualitative analysis with NUDIST:* Handout for workshop held at UTS, Sydney, April.

Pateman, N. (1989). *Teaching mathematics - A tantalising enterprise.* Geelong, Victoria: Deakin University.

Perry, B. & Conroy, J. (1994). *Early childhood and primary mathematics: A participative text for teachers.* Sydney: Harcourt Brace & Company.

Perry, B., Geoghegan, N., Howe, P., & Owens, K. (1995). An approach for developing the mathematics backgrounds of primary school teachers and other adult learners. In R. Hunting, G. Fitzsimons, P. Clarkson, & A. Bishop (Eds.), *Regional collaboration in mathematics education: Proceedings of the ICMI Regional Conference, Melbourne, April.* Melbourne: ICMI.

Perry, B., Geoghegan, N., Owens, K. & Howe, P.(1995) Cooperative learning and social constructivism in mathematics education. In B. Atweh & S. Flavel (Eds.), *MERGA 18 - Galtha: Proceedings of the eighteenth annual conference of the mathematics education research group of Australasia, Darwin, July.* Darwin: MERGA.

Peterson, P. (1988). Teaching for higher order thinking in mathematics: The challenge for the next decade. In D.A. Grouws and T.J. Cooney (Eds.), *Perspectives on research on effective mathematics teaching* (pp. 2-26). Hillsdale, NJ: Erlbaum.

Richards, L. (1995). *What did the technology do? Reflections on a three-year NUD•IST project.* Paper presented at the Inaugural NUD•IST Users' Conference, Sydney, July.

Richards, L. & Richards, T. (1990). *Critiquing qualitative computing: Grounded theory method vs code and retrieve techniques.* Paper presented at the Social Research Conference, Queensland.

Sanders, S.E. (1994). *Revealing teacher's notions about mathematics.* Paper presented at the Annual Conference of the Australian Association for Research in Education, Newcastle, Australia, November.

Schoenfeld, A. (1994) A discourse on methods. *Journal for Research in Mathematics Education 25*(6), 697-710.

Schram, P., Wilcox, S., Lanier, P. & Lappan, G. (1988). *Changing mathematical conceptions of preservice teachers: A content and pedagogical intervention* (Research Report 88-4). East Lansing: Michigan State University, National Center for Research on Teacher Education.

Schuck, S. (1995). *Using a research metaphor to stimulate mathematical thought.* Paper presented at the MANSW Post-Secondary Mini-Conference, Sydney, March.

Shulman, L.S. (1986a). Paradigms and research programs in the study of teaching: A contemporary perspective. In M.C. Wittrock (Ed.), *Handbook of research on teaching* (3rd ed., pp. 3-36). New York: Macmillan.

Shulman, L.S. (1986b). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Skemp, R. (1978). Relational understanding and instrumental understanding. *Arithmetic Teacher, 26*(3), 9-15.

Southwell, B. (1995). Towards a theoretical framework for research in beliefs and values in mathematics education. In B. Atweh & S. Flavel (Eds.), *MERGA-18: Galtha - Proceedings of the eighteenth annual conference of the Mathematics Education Group of Australasia* (pp. 477-483). Darwin: MERGA

Spangler, D.A. (1992). Assessing students' beliefs about mathematics. *Arithmetic Teacher 39*, November, 148-152.

Spradley, J. P. (1979). *The ethnographic interview.* Fort Worth: Holt, Rinehart and Winston.

Stodolsky, S. (1985). Telling math: Origins of math aversion and anxiety. *Educational Psychologist, 27*, 37-42.

Stodolsky, S. (1988). *The subject matters: Classroom activity in mathematics and social studies.* Chicago: University of Chicago.

Strauss, A.L. (1987). *Qualitative analysis for social scientists.* New York: Cambridge University Press.

Strauss, A.L. & Corbin, J. (1990). *Basics of qualitative research: Grounded theory procedures and techniques.* Newbury Park: Sage.

Sullivan, P. (1994). *Subjectivity and naturalistic descriptions.* Paper presented at the Contemporary Approaches to Research in Mathematics, Science and Environmental Education Symposium, Deakin University, Melbourne, December.

Sullivan, P., Bourke, D., & Scott, A. (1995). Open-ended tasks as stimuli for learning mathematics. In B. Atweh and S. Flavel (Eds.), *MERGA 18: Galtha - Proceedings of the eighteenth annual conference of the Mathematics Education Research Group of Australasia* (pp. 484-492). Darwin: MERGA.

Thompson, A.G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15,* 105-127.

Thompson, A.G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127 -146). New York: Macmillan Publishing Co.

Watson, H. (1989). A Wittgensteinian view of mathematics: Implications for teachers of mathematics. In N. F. Ellerton & M.A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 18-30). Geelong, Victoria: Deakin University.

Weitzman & Miles, M.B. (1995). *Computer programs for qualitative analysis.* Thousand Oaks, CA: Sage Publications.

Whitehead, G., Symington, D., Mackay, L., & Vincent, A. (1993). *The influence of discipline reviews on higher education - Review of teacher education in mathematics and science.* Canberra: Australian Government Publishing Services.

Wiersma, W. (1986). *Research methods in education: An introduction* (Fourth Ed.). Boston: Allyn and Bacon.

Wittgenstein, L. (1978). *Remarks on the foundations of mathematics* (3rd ed.). (G. von Wright, R. Rhees, & E. Anscombe, Trans.). Oxford: Basil Blackwell.

Appendix A : ETHICS APPROVALS

The following pages give examples of the documents relating to the ethical procedures used in this study. They are:

- A copy of the contract given to students requesting use of the taped interviews of phase one, conducted in semester 1, 1993
- The covering letter given to students with the questionnaire of phase two, October 1993
- A copy of the contract given to students requesting use of their responses to the phase two questionnaire, semester 2, 1993
- A copy of the contract given to students involved in the in-depth interviews of phase three
- A copy of the contract given to the teacher educators prior to the phase four interviews with them
- An example of the letter given to students asking them to check the transcriptions of their interviews of phase three.

CONSENT FORM

Permission to Use Taped Interview Data.

I ______________________________ agree to allow the data from my audio-taped interview conducted last semester in Primary Curriculum Orientation 2 - Mathematics to be used in the research project *Attitudes and Beliefs of Pre-Service Primary Teachers about Mathematics Education* being conducted by Sandra Schuck, *details of my position, faculty and university.*

I understand that the purpose of this study is to investigate and describe first year primary student teachers' attitudes and beliefs about mathematics education and to examine ways in which these students learn new mathematical material.

I am aware that the project is being undertaken by Sandra Schuck for her PhD thesis and that I am at liberty to contact Sandra Schuck or her supervisor, Susan Groundwater-Smith, (tel: xxx) if I have any concerns about the research. I also understand that I am free to withdraw my permission to use this data in the research project at any time I wish and without giving a reason and that this will not prejudice my future academic progress in any way.

I agree that I have been fully informed regarding the project. I realise that all information gathered will be confidential and that I will not be identifiable from any descriptions in the research.

I agree that the research data gathered from this project may be published in a form that does not identify me in any way.

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Signed by

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Witnessed by

Note:

This study has been approved by the University Human Research Ethics Committee. If you have any complaints or reservations about any aspect of your participation in this research you may contact the Ethics Committee through the Research Ethics Officer, (*name, telephone number*). Any complaint you make will be treated in confidence and investigated fully and you will be informed of the outcome.

Dear Student,

I am currently engaged in a research project which is investigating beliefs and attitudes that pre-service primary school teachers might have on the subject of mathematics education.

I would consequently appreciate it if you would respond to the questionnaire attached. You will see that I request your name at the top of the questionnaire - this will be merely so that I can follow up cases where the questionnaire has not been returned and also to match up your questionnaire with your interview data from first semester. **As soon as this has been done your name will be removed so that your response is quite anonymous.**

If for some reason you do not wish to engage in this questionnaire then please return it incomplete. This will ensure that there is no further contact about the matter.

I realise that, as a student, your life is a busy one, but I believe that this study is important as it will enable mathematics educators to better understand your feelings and beliefs and thus give better insight as to how to assist you in your learning of mathematics and your preparation to be a primary school teacher.

The answers will be used for my doctoral research about student attitudes and beliefs and also to modify the first year B.Teach. subject Mathematics Education 1.

Please feel free to contact me, or my supervisor, Susan Groundwater-Smith (contact number: xxx) if you wish to clarify these procedures.

Thank you for your cooperation
Yours sincerely

Sandy Schuck
Contact number:

Note:
This study has been approved by the University Human Research Ethics Committee. If you have any complaints or reservations about any aspect of your participation in this research you may contact the Ethics Committee through the Research Ethics Officer, *name (telephone number).*

CONSENT FORM

Questionnaire on Attitudes and Beliefs about Mathematics Education.

I _________________________________ agree to participate in the research project *Attitudes and Beliefs of Pre-Service Primary Teachers about Mathematics Education* being conducted by Sandra Schuck, *contact details provided.*

I understand that the purpose of this study is to investigate and describe first year primary student teachers' attitudes and beliefs about mathematics education and to examine ways in which these students learn new mathematical material.

I understand that participation in this research will involve the answering of a questionnaire on attitudes and beliefs about mathematics education and on my learning strategies in mathematics. This should take approximately 15 - 30 minutes to complete.

I am aware that the project is being undertaken by Sandra Schuck for her PhD thesis and that I am at liberty to contact Sandra Schuck or her supervisor, Susan Groundwater-Smith (contact number xxx), if I have any concerns about the research. I also understand that I am free to withdraw my participation from this research project at any time I wish and without giving a reason and that this will not prejudice my future academic progress in any way.

I agree that I have been fully informed regarding the project.

I agree that the research data gathered from this project may be published in a form that does not identify me in any way.

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Signed by

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Witnessed by

Note:

This study has been approved by the University Human Research Ethics Committee. If you have any complaints or reservations about any aspect of your participation in this research you may contact the Ethics Committee through the Research Ethics Officer, *name (telephone number)*. Any complaint you make will be treated in confidence and investigated fully and you will be informed of the outcome.

CONSENT FORM

Interview on Attitudes and Beliefs about Mathematics Education and Learning Strategies.

I ________________________________ agree to participate in the research project *Attitudes and Beliefs of Pre-Service Primary Teachers about Mathematics Education* being conducted by Sandra Schuck, Lecturer in Mathematics Education, *contact details provided.*

I understand that the purpose of this study is to investigate and describe first year primary student teachers' attitudes and beliefs about mathematics education and to examine ways in which these students learn new mathematical material.

I understand that participation in this research will involve an in-depth interview with Sandra Schuck in which my attitudes and beliefs about mathematics education and my learning strategies in mathematics will be discussed. The interview will be audio-taped and should take approximately one hour to complete. I will be shown a transcription of the interview when this has been completed and will be able to change anything which I feel is not accurate.

I am aware that the project is being undertaken by Sandra Schuck for her PhD thesis and that I am at liberty to contact Sandra Schuck or her supervisor, Susan Groundwater-Smith (contact number xxx), if I have any concerns about the research. I also understand that I am free to withdraw my participation from this research project at any time I wish and without giving a reason and that this will not prejudice my future academic progress in any way.

I agree that I have been fully informed regarding the project.

I agree that the research data gathered from this project may be published in a form that does not identify me in any way.

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Signed by

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Witnessed by

Note:

This study has been approved by the University Human Research Ethics Committee. If you have any complaints or reservations about any aspect of your participation in this research you may contact the Ethics Committee through the Research Ethics Officer, *name, (phone number).* Any complaint you make will be treated in confidence and investigated fully and you will be informed of the outcome.

CONSENT FORM FOR TEACHER EDUCATORS.

I .... agree to participate in the research project *Attitudes and Beliefs of Pre-service Primary Teachers about Mathematics Education* being conducted by Sandra Schuck, Lecturer in Mathematics Education, *contact details.*

I understand that the purpose of this study is to investigate and describe first year primary student teachers' attitudes and beliefs about mathematics education and to examine ways in which these students learn new mathematical material.

I am aware that the project is being undertaken by Sandra Schuck for her PhD thesis and that I am liberty to contact Sandra Schuck or her supervisor, Susan Groundwater Smith (contact number xxx), if I have any concerns about the research.

I agree that I have been fully informed regarding the project. I agree that the research data gathered from this project may be published in a form that does not identify me in any way.

....
(signed) (date)

REQUEST TO CHECK TRANSCRIPTION

Dear *name*

Enclosed please find a copy of the transcript of your interview with me, held 8 December 1993.

Please read through this and change anything that you believe is inaccurate, either because I did not transcribe your statements correctly or because you feel the message is contrary to your intentions. A red pen would be a good idea for your alterations.

Please note that I have changed any names mentioned in the interview in order to preserve your anonymity.

When you have finished examining the transcript could you please return it, sealed in the envelope in which it came, to my pigeon hole in the educational studies foyer. If unsure of where this is, ask *the secretary* or pop it under my office door. Please could you do this before the end of March.

Thanks once again for giving up your time so graciously. I appreciate this enormously.

Best wishes for this semester.

Sandy Schuck.

Appendix B : THE FIRST YEAR MATHEMATICS EDUCATION SUBJECTS

The following pages give examples of the documents relating to the first year mathematics education subjects offered at the participating university. These are:

- The subject outline for the first semester subject, Primary Curriculum Orientation II (Mathematics)
- The subject outline for the second semester subject Mathematics Education I
- A copy of the assignment given to students in the first semester to investigate their beliefs and attitudes (the basis of the phase one interviews)

PRIMARY CURRICULUM ORIENTATION II

MATHEMATICS

Semester 1, 1993

Pre-requisites: Nil

Class hours: One hour per week

Objectives:

1. To promote positive attitudes towards the study of mathematics.

2. To investigate new approaches to the learning of mathematics in the primary school.

3. To broaden understanding of the scope and nature of mathematics.

4. To illustrate the relevance of mathematics for the primary school teacher.

5. To investigate co-operative learning in the mathematics classroom.

Content:

1. The nature of the current mathematics syllabus document: its emphasis, approach and rationale.

2. Use of calculators in the primary school - activities, approaches, applications and justification for their use.

3. Topics in measurement - hands on activities developing various concepts including area, mass and volume.

4. Topics in space - introduction to geometry, group activities involving concrete materials. 2D and 3D geometry will be considered.

5. Probability - the teaching of this topic at the primary school level. Hands-on activities and investigations.

6. LOGO - an introduction to the use of turtle geometry in the mathematics classroom. The language of LOGO. Appropriate geometric activities.

Learning Experiences:

Introductory lecture discussion supplemented by video. Hands-on workshops held fortnightly. Reading of current literature. Reflection on attitudes and knowledge about mathematics. Interviewing of peers on attitudes to mathematics and mathematics teaching.

Course Requirements:

1. Full participation in class activities involving the use of equipment, together with attendance and contribution to discussion in all class periods.

2. Completion of interviews, journal entries and all class exercises in a satisfactory manner.

3. A satisfactory standard in literacy and numeracy must be exhibited in all submitted work.

Assessment:

The subject will be assessed on a Pass/Fail basis. To pass students will be required to satisfy **all** the following:

1. Attend **all** lectures/workshops. Students experiencing any problem in this area must see their lecturer and present appropriate documentation to explain their absence. **Failure to do so will result in failure of the subject.**

2. Submit an audio cassette with two interviews, together with written summaries, by due date (second workshop session). See separate handout. This will be assessed on a pass/fail basis.

3. Submit a journal which shows evidence of careful reflection about the mathematical topics studied and approaches used. These reflections should be written in an exercise book which must be submitted to the lecturer twice: 1) in the third maths workshop and 2) within the week after the final workshop. Students will be expected to write entries at the beginning and end of each workshop session.

4. Students are required to read an appropriate article on one of the topics studied and present a review of the article. A photocopy of the article must be submitted with the review, and the name, author and reference details must be clearly noted in the review. This is due within one week of the final workshop.

In order to gain a pass grade, a satisfactory standard is expected in each of submitted assignments: the interview assignment, journal and article review.

Late submission may result in the student failing the subject.

Mathematics Education 1

Semester 2, 1993

Number of class hours 2 hours per week

Prerequisites Primary Curriculum Orientation 2

Objectives

1. To develop an awareness of the importance of mathematics.

2. To develop understanding and appreciation of the principles and historical context of numeration systems.

3. To examine theories of mathematics learning and consider the selection of appropriate teaching/learning strategies and the use of resources and manipulative materials to support these strategies.

4. To extend students' competence in mathematics and raise awareness of the nature of the mathematical process through study of patterns and relations in selected areas of number theory.

Content

1. An introduction to the aims of mathematics education and methods of approach to the teaching and learning of mathematics; emphasis on learning theories and the importance of pupil involvement and the use of structured and unstructured materials; the classroom as a learning laboratory.

2. An examination of the historical development and principles underlying various systems of numeration; decimal numeration; non-decimal based systems.

3. A study of selected topics from number theory including prime and composite numbers, the fundamental theorem of arithmetic, greatest common divisor and least common multiple, clock arithmetic, and the integers, with emphasis on investigation and a search for pattern.

Learning Experiences

Lecture/discussion supplemented by videos; workshops; assigned exercises and reading for consolidation and extension; group presentations and independent literature reviews.

Course Requirements

1. Full participation in class activities involving the use of equipment, together with attendance and contribution to discussion in lectures.

2. Completion of class exercises and all other assigned work together with the compilation of a resource file.

3. Students must exhibit satisfactory standards in literacy and numeracy in all submitted work.

Assessment

In order to pass this subject students must reach a satisfactory standard in each of the following:

1. Assignments - a. essay due 26 August - 20%
   b. group presentation in October - 10%

2. Class test on learning theories, issues and numeration systems **to be held on Wednesday 22 September at 10.30 am** - 35%

3. Class test on topics from number theory - to be held in examination week - 35%

If a student is unable to meet any of the due dates an extension must be requested from the lecturer as soon as possible, preferably well in advance of the due date. Appropriate documentation may be required. Failure to consult with the lecturer in such cases may result in the student failing the assessable item.

Students are advised to acquaint themselves with the University's policy on plagiarism and cheating.

Resource File

Students are strongly advised to establish and carefully maintain a resource file to assist learning in this and future units and to serve as a resource when teaching. The file should be of durable type, systematically organised and containing lecture notes, assigned exercises, notes from readings and any other relevant material.

Basic Texts

The following books should be purchased as they will be used in mathematics education subjects throughout the course and will be needed for assignments and test preparation.

Bennett, A.B. & Nelson, L.T. (1992). *Mathematics for Elementary Teachers: A Conceptual Approach (3rd Edition).* U.S.A.: Wm. C. Brown.

Booker, G., Briggs, J., Davey, G and Nisbet, S. (1992). *Teaching Primary Mathematics.* Melbourne: Longman Cheshire.

Department of Education (NSW). (1989). *Mathematics K-6.* Sydney: Directorate of Studies.

PRIMARY CURRICULUM ORIENTATION II

MATHEMATICS

Assignment - Semester I, 1993

"In recent years, a great deal of attention has been focussed on the importance of attitudes in learning. Positive attitudes are believed to enhance not only the quality of learning, but also the degree to which learning and understanding become embedded in the real-life experiences of the individual.

There is a vital link between teachers' attitudes towards maths and those of their pupils. Children's attitudes reflect in part those of the teacher." [Basic Learning in Primary Schools Program: "Mathematics: Attitudes Count"; Commonwealth Schools Commission NSW Department of Education, No 3(d)]

PROJECT

Elicit from another class member what his/her current attitudes towards mathematics and mathematics teaching are and how these have been influenced or affected by his/her past mathematical experiences.

OUTLINE

1. Prepare a series of five interview questions;

2. Conduct a tape recorded interview with another member of the class; each of you to interview the other; both interviews should be on one tape;

3. Submit your tape recording along with a precis of the interview. This is to include your assessment of the attitudes of the interviewee and what implications these attitudes have for his/her teaching of mathematics;

4. As a result of the two interviews reflect on your attitudes to mathematics and mathematics teaching. How could your knowledge of these attitudes be utilised in your own teaching practices? Include these thoughts with your precis.

DUE DATE

This assignment is to be submitted by 4 pm on the 22 March, 1993. Students with difficulties in meeting the due date should contact their lecturer as soon as possible before the due date. Documentation may be required.

Appendix C : DATA COLLECTION INSTRUMENTS FOR PHASES TWO, THREE AND FOUR

The following data collection instruments are included in this appendix:

- The questionnnaire used in phase two. Note that students were required to answer on the questionnaire itself and ample space was left for these answers (although such space is not shown below).
- The three interview schedules used in phase three (original schedule modified twice). It should be remembered that the schedule was used as a guide only and I allowed the respondents to move the interviews into areas that were of interest to them. These areas were then added to the next schedule as modifications.
- The interview schedule used in phase four.

FIRST YEAR MATHEMATICS EDUCATION.
QUESTIONNAIRE.

NAME:

GROUP:

Questionnaire - October 1993.

Please could you answer the following questions as accurately as possible. Remember that there are no right answers. I am interested in **your** response.

For each of the following five questions, please could you circle the most appropriate response out of those given below.

1. The highest level of mathematics that I have done is:

   (a) Year 10

   (b) HSC - 2U Mathematics in Society

   (c) HSC - 2U

   (d) HSC - 3U

   (e) HSC - 4U

   (f) Other - please state.

   ____

2. My last year at school was:

   (a) before 1970

   (b) between 1970 and 1980 inclusive

   (c) between 1981 and 1990 inclusive

   (d) 1991

   (e) 1992

3. I am

   (a) female

   (b) male

4. My grades in school mathematics have generally been

   (a) good

   (b) poor

   (c) satisfactory.

5. **I believe** that I am good at mathematics

   (a) strongly agree
   (b) agree
   (c) not sure
   (d) disagree
   (e) strongly disagree

Please could you answer the following questions as fully as possible.
The following questions all refer to **mathematics** learning and teaching from your perspective as a prospective teacher.

Definitions:

Procedural knowledge: knowledge of the rules and procedures that need to be followed to carry out a mathematical task.
Conceptual knowledge: the underlying understanding of the concepts and structures of mathematics.

6. What do you consider effective learning of primary school mathematics to be?

7. What balance do you feel there should be between teaching of procedural and conceptual knowledge in order for effective learning (as you described it above) to take place? Give percentage of each kind of teaching.

8. What percentage of your mathematics learning at school do you regard as having been
procedural ____________________
conceptual ____________________

Comment on the above response

9. What is the most effective way of teaching the underlying structure of mathematics; that is, the concepts, reasoning and theories?

10. What do you believe is the role of practice in mathematics? How important is it in effective learning?

What are your views about the need for practice for different ability pupils. Do you believe that its role differs for different ability pupils?

In the taped interviews many students mentioned doing a lot of drill in primary school maths.
Do you see drill as being different from practice? If so, in what way?

Do you think that drill has a place in the learning of primary school mathematics?

11. I would like your opinion on which you would regard as more important for a student teacher - to have background knowledge in a particular area of mathematics or to have a sound knowledge of the methods required to effectively use that mathematics? Why do you believe this?

12. Many students stated in the interviews that they believed maths should be fun. What is your view? Please explain first what you understand by the expression "maths should be fun".

13. What do you believe are the characteristics of a good primary school teacher of mathematics?

14. What, if any, do you think are the characteristics of a bad primary school teacher of mathematics?

15. If a primary school student is good at mathematics what does that mean the student knows or can do?

16. As a teacher what do you believe is more important - the content of mathematics or the method of teaching it? Why?

17. As a teacher, what do you feel should be the relative amounts of time spent on each of the following when teaching maths. Give percentages of time you think would be appropriate. (Your answer should add to 100)

    management of students' learning ___________
    student support ___________________________
    providing mathematical challenges __________
    providing opportunities for development of mathematical knowledge
    ____
    other (please state) ____________________
    ____

18. In the interviews, students talked about "the game of school" where what was learnt in school did not seem to have much bearing on what was known outside school.

    Based on your experiences of school mathematics, please would you comment on this view.

19. Something else that many students said in the interviews was that they felt that maths that was not useful should not be taught. What do you feel about this?

20. Rank the following from most important to least important and explain your ranking:

    mathematics taught in primary school should be useful
    mathematics taught in primary school should be fun
    mathematics taught in primary school should be clearly understood
    mathematics taught in primary school should be relevant

The following questions should be answered from your position as a student of mathematics education at university.

21. What do you feel are your most effective and successful strategies for learning new mathematics concepts at university?

22. What do you feel are your least successful strategies for learning these new mathematics concepts?

23. Below is a list of teaching methods. Select those (as many as appropriate) which you feel describe the methods used by your supervisory teacher on teaching practicum II to teach mathematics. Circle the methods that were used.

    Use of: group work; textbook; worksheets; exercises; times table drill; investigations; games; problem solving; other (please explain)

    As a child would you like to have been taught mathematics in the way your supervising teacher taught it? Please explain.

    Do the methods you observed agree with your ideas on what teaching leads to effective learning of mathematics in primary school?

24. What are your expectations for a mathematics education course at university - what do you hope to get out of it?

25. What percentage of each type of knowledge do you feel is required in the Mathematics Education I subject studied thus far:

    procedural ________________________

    conceptual ________________________

26. Is there any aspect of either of the two maths subjects studied so far (Orientation maths and Maths Ed I) that you have found particularly worthwhile? If so, please describe this and explain why you thought it was so good.

27. Is there any aspect of either of the two maths subjects studied so far (Orientation maths and Maths Ed I) that you have thought was a complete waste of time? If so, please describe this aspect and explain why you feel this way.

Thank you very much for giving up your time.

Interview Questions - Phase III. Developed for 8 Dec 1993

1. What is mathematics - what does it mean to you? (If a Martian landed on Earth and wanted to know what this subject was, how would you explain it to the Martian?)

2. Do you believe maths should be studied at school? In this course?

3. Why do you believe we study maths at school? At uni in a B.Teach?

4. What is your present attitude to maths?

5. How do you think this developed?

6. Describe an ideal teacher of maths - what do you believe are best practices?

7. Can you describe any experiences that you remember from primary school maths? High school maths? Later?

8. What did you expect to learn in maths at uni? What should we be doing in this course in maths?

9. What did you feel were the strengths of Med I? Weaknesses?

10. What is the importance of challenges in maths?

11. The difficulty of the subject is a problem - How should we overcome this? (leave out? strategies to make it easier? etc)

12. Fun - did you have fun doing maths at school? Uni? What does this mean? Is fun important?

13. What are your goals for when you teach primary school maths?

Interview Questions - Phase III.
Modified as a result of interview 1.

1. What is mathematics - what does it mean to you? (If a Martian landed on Earth and wanted to know what this subject was, how would you explain it to the Martian?)

2. What are its uses? Are there any other reasons for studying maths?

3. What are your first memories of learning maths?
   What is your general memory of the atmosphere in your classroom when learning maths at primary school?
   At high school?
   Do you remember any incident/experience in learning maths that was especially good?
   Do you remember any unpleasant incidents?

Do you remember any lessons that were fun? What made them fun? Did you feel there were enough lessons like that?

4. Did you have any really good teachers for maths, either at primary or high school? What made you feel that they were good?

5. Did you have any teachers for maths that you felt were really bad? What were their characteristics that led to this impression?

6. What is your present attitude to maths?

7. How do you think this developed?

8. What are your goals for when you become a teacher?
How do you feel about having to teach maths then? What sort of methods would you use? How would you like to appear to the children?

9. How did you feel when you heard you had to study maths at uni? What did you hope would be covered in the maths course?

10. Do you feel that those things are being done?

11. What did you feel were the strengths of Med I? Weaknesses?

12. Do you remember times when you were challenged in maths? How did you feel? Do you feel it is good to have such experiences? Would you challenge pupils in your teaching?

13. What do you see as the constraints on you that might prevent you from teaching the way you would like to? What sort of teaching strategies might have to be changed because of these constraints?

14. Do you think there is a best way of teaching maths?

15. Some topics are very difficult conceptually - should they be left out the syllabus? If not, how should they be taught?

16. How did you study for Maths Ed 1 - what work did you do during the semester? In preparation for the exams? How did you go about completing the essay? What about the group assignment?

17. Which of the assessment tasks did you feel were worthwhile for your learning?

Interview Questions - Phase III. Developed 18 Feb 1994
(modified as a result of first three interviews)

1. What is mathematics - what does it mean to you? (If a Martian landed on Earth and wanted to know what this subject was, how would you explain it to the Martian?)

2. What are its uses? Are there any other reasons for studying maths, particularly at school? So what would you say is the point of teaching maths at school, what do you want your pupils to take away with them from your maths classes?

3. What are your first memories of learning maths?
What is your general memory of the atmosphere in your classroom when learning maths at primary school?
At high school?
Do you remember any incident/experience in learning maths that was especially good?
Do you remember any unpleasant incidents?
Any other good/bad incidents that you can remember?
Do you remember any lessons that were fun? What made them fun? Did you feel there were enough lessons like that?

4. Did you have any really good teachers for maths, either at primary or high school? What made you feel that they were good?

5. Did you have any teachers for maths that you felt were really bad? What were their characteristics that led to this impression?

6. What were your parents' attitudes to maths? Were they good at it? Did they help you with your maths at home? Were they supportive and encouraging in this area?

7. What is your present attitude to maths?

8. How do you think this developed?

9. What are your goals for when you become a teacher?
How do you feel about having to teach maths then? What sort of methods would you use? How would you like to appear to the children?

10. How did you feel when you heard you had to study maths at uni? What did you hope would be covered in the maths course? If the maths course was not compulsory would you still have done it? Why?

11. Do you feel that the things you expected in the maths course are being done?

12. What did you feel were the strengths of Med I? Weaknesses?

13. How could Med I be improved?
What about PCO II (maths)?

14. Do you remember times when you did something really difficult in maths? How did you feel? Did you cope with the topic/experience? Should such topics be left to later? Do you feel it is good to have such experiences? How did your experiences affect the way you want to teach?

15. What kind of things stop you from studying the way you would like to at uni? What might prevent you from teaching the way you want when you start work as a teacher?

16. Do you think there is a best way of teaching maths?

17. What were your strategies for learning maths at school?
Which worked - and in what way did they worked (marks, understanding)

18. How did you study for Maths Ed 1 - what work did you do during the semester? In preparation for the exams? How did you go about completing the essay? What about the group assignment?
Did you look at the details given or the overall message when preparing for the tests?

19. Which of the assessment tasks did you feel were worthwhile for your learning?

Interview Schedule for the developers and instructors in Maths Ed 1.

1. What is mathematics?

2. What do you see as the overall purpose of the mathematics education subjects in the B. Teach?

3. What do you believe is being taught and learnt in maths ed 1 in terms of content and processes.

4. In particular, why has the particular content of mathematics education 1 been chosen - what is the reason for including:
   - ancient numeration systems
   - different bases
   - patterns
   - clock arithmetic
   - integers
   - topics from number theory such as Fermat's theorem; sieve of Eratosthenes etc

5. What theories of learning maths are important and should be discussed with the students?

6. How do you believe our students best learn mathematics: how should we be teaching it to them?

7. What do you think are the biggest problems facing our success at teaching this course?

8. How do you respond to their criticism that the maths they do in M Ed 1 is not relevant for their teaching as it deals with topics not covered by the K-6 syllabus?

Appendix D : SOME EXAMPLES OF STUDENTS' RESPONSES

In Section 5.4.3 I discussed the selection of participants for the in-depth interviews. I used purposeful sampling in which one of the criteria was the amount of reflection the participant had demonstrated. I chose students so that I had a mix of reflective and unreflective participants. The following indicates the way in which students were selected.

To decide if a student fell into the category of "reflective thinker" I considered the data from phases one and two. Typically, I classified students as unreflective if their paired interviews were brief and did not go into any depth; and if their responses to the questionnaire were abrupt, incomplete or if some of the questions were not answered. I classified students as reflective if they discussed issues of substance in their paired interviews, or answered questions on the questionnaire carefully and with indications of having thought about the responses.

To give the reader an indication of the criteria I used to decide if students were reflective about their learning, I am including some typical responses. The responses are from Gail whom I had classified as reflective, and from Maria, whom I had classified as unreflective on the basis of her responses to the questionnaire.

Question 9:

What is the most effective way of teaching the underlying structure of mathematics; that is, the concepts, reasoning and theories?

Gail's response:

Through concrete examples and then consolidation through practice and rehearsal.

Maria's response:

No answer given.

Question 10 (part 2):

What are your views about the need for practice for different ability pupils? Do you believe that its role differs for different ability pupils?

Gail's response:

Yes, I think the bright children, being quick to initially grasp concepts, intuitively are able to apply them. Those who have difficulty grasping concepts only achieve that total grasping through much exposure.

Maria's response:

Practice should differ for individuals.

Question 12:

Many students stated in the interviews that they believed maths should be fun. What is your view? Please explain first what you understand by the expression "maths should be fun".

Gail's response:

- *That there should be involvement and participation.*
- *Being fun does not mean to me a constant flow of tricks and puzzles.*
- *A child (AND A UNI STUDENT) can only have "fun" when s/he is understanding and achieving - there is no fun in being overwhelmed and failing.*
- *Fun implies to me a teacher who realises that a concept must be presented in a variety of ways in order that people of all levels grasp that concept.*
- *Fun means - never being humiliated for being wrong or not knowing how to do a question.*

Maria's response:

No answer given.

Appendix E : EXAMPLES OF THE NODES AFTER ANALYSIS

Analysis by NUD•IST involves the formulation of nodes in a tree structure. Each document gets indexed at the nodes to which it refers. During the process of coding changes occur in the nodes as a result of refinement.

The following are examples of the lists of nodes that evolved using NUD•IST. The lists illustrate the changes that occurred after analysis of the data from each successive phase of data collection. I have included part of a report formulated in NUD•IST which describes the number of documents indexed by the various nodes. This particular report refers to the coding at the end of phase two.

The second example I have included here gives examples of nodes in existence at the end of phase four.

Q.S.R. NUD.IST Power version, revision 3.0.4 GUI.
Licensee: Sandra Schuck.

PROJECT: PHD, User SAndy Schuck, 10:53 am, 1 Sept, 1995.

****
(1) /affective component
*** No Definition
This node indexes 0 documents.
****
(1 1) /affective component/beliefs
*** No Definition
This node indexes 2 documents.
****
(1 1 1) /affective component/beliefs/learning
*** Definition:
beliefa about what learning of maths comprises; what it is
This node indexes 4 documents.
****
(1 1 2) /affective component/beliefs/teaching
*** Definition:
beliefs about the teaching of maths
This node indexes 7 documents.
****
(1 1 2 2) /affective component/beliefs/teaching/constraints
*** Definition:
Cut from node (1 1 3) .
This node indexes 6 documents.
****
(1 1 2 3) /affective component/beliefs/teaching/characteristics
*** Definition:
Copy of node (1 1 3) and its subtree.
This node indexes 1 document.
****
(1 1 2 3 1) /affective component/beliefs/teaching/characteristics/management
*** Definition:
a good teacher has effective classroom management
This node indexes 6 documents.
****
(1 1 2 3 2) /affective component/beliefs/teaching/characteristics/communication
*** Definition:
a good teacher can communicate clearly
This node indexes 16 documents.
****
(1 1 2 3 3) /affective component/beliefs/teaching/characteristics/confidence
*** No Definition
This node indexes 1 document.
****
(1 1 2 3 4) /affective component/beliefs/teaching/characteristics/pos_attitude
*** Definition:
a good teacher has a positve attitude towards maths
This node indexes 26 documents.
****
(1 1 2 3 5) /affective component/beliefs/teaching/characteristics/supportive

Q.S.R. NUD.IST Power version, revision 3.0.4 GUI.
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PROJECT: PHD, User SAndy Schuck, 3:37 pm, 11 Oct, 1995.

(1)	/affective component
(1 1)	/affective component/beliefs
(1 1 1)	/affective component/beliefs/learning
(1 1 1 1)	/affective component/beliefs/learning/constructivism
(1 1 1 2)	/affective component/beliefs/learning/socio-culturalism
(1 1 2)	/affective component/beliefs/teaching
(1 1 2 2)	/affective component/beliefs/teaching/constraints
(1 1 2 3)	/affective component/beliefs/teaching/characteristics
(1 1 2 3 1)	/affective component/beliefs/teaching/characteristics/management
(1 1 2 3 2)	/affective component/beliefs/teaching/characteristics/communication
(1 1 2 3 4)	/affective component/beliefs/teaching/characteristics/pos_attitude
(1 1 2 3 5)	/affective component/beliefs/teaching/characteristics/supportive
(1 1 2 3 6)	/affective component/beliefs/teaching/characteristics/versatile
(1 1 2 3 7)	/affective component/beliefs/teaching/characteristics/knowledgable
(1 1 2 3 9)	/affective component/beliefs/teaching/characteristics/other
(1 1 2 4)	/affective component/beliefs/teaching/poor-teacher
(1 1 2 4 1)	/affective component/beliefs/teaching/poor-teacher/unenthusiastic
(1 1 2 4 2)	/affective component/beliefs/teaching/poor-teacher/pen-and-paper
(1 1 2 4 3)	/affective component/beliefs/teaching/poor-teacher/threatening
(1 1 2 4 4)	/affective component/beliefs/teaching/poor-teacher/lack-of-clarity
(1 1 2 4 5)	/affective component/beliefs/teaching/poor-teacher/boring
(1 1 2 4 6)	/affective component/beliefs/teaching/poor-teacher/unaware
(1 1 2 4 7)	/affective component/beliefs/teaching/poor-teacher/isolated
(1 1 2 4 8)	/affective component/beliefs/teaching/poor-teacher/other
(1 1 2 7)	/affective component/beliefs/teaching/purpose
(1 1 2 7 1)	/affective component/beliefs/teaching/purpose/vocabulary
(1 1 2 7 2)	/affective component/beliefs/teaching/purpose/basics
(1 1 2 7 3)	/affective component/beliefs/teaching/purpose/attitude-improvement
(1 1 2 7 4)	/affective component/beliefs/teaching/purpose/understanding
(1 1 2 7 5)	/affective component/beliefs/teaching/purpose/teach well
(1 1 2 7 6)	/affective component/beliefs/teaching/purpose/learn-on-job
(1 1 2 7 7)	/affective component/beliefs/teaching/purpose/new methods
(1 1 2 7 8)	/affective component/beliefs/teaching/purpose/background-knowledge
(1 1 2 7 9)	/affective component/beliefs/teaching/purpose/other
(1 1 2 7 10)	/affective component/beliefs/teaching/purpose/processes
(1 1 2 8)	/affective component/beliefs/teaching/student-as-teacher
(1 1 2 8 1)	/affective component/beliefs/teaching/student-as-teacher/flexibility
(1 1 2 8 2)	/affective component/beliefs/teaching/student-as-teacher/enthusiasm
(1 1 2 8 3)	/affective component/beliefs/teaching/student-as-teacher/fun
(1 1 2 8 4)	/affective component/beliefs/teaching/student-as-teacher/encouragement
(1 1 2 8 5)	/affective component/beliefs/teaching/student-as-teacher/interesting
(1 1 2 8 6)	/affective component/beliefs/teaching/student-as-teacher/child-centred
(1 1 2 8 7)	/affective component/beliefs/teaching/student-as-teacher/practical
(1 1 2 8 8)	/affective component/beliefs/teaching/student-as-teacher/co-operation
(1 1 2 8 9)	/affective component/beliefs/teaching/student-as-teacher/relevance
(1 1 2 8 11)	/affective component/beliefs/teaching/student-as-teacher/communicate
(1 1 2 8 12)	/affective component/beliefs/teaching/student-as-teacher/resources
(1 1 2 8 13)	/affective component/beliefs/teaching/student-as-teacher/knowledgable
(1 1 2 8 14)	/affective component/beliefs/teaching/student-as-teacher/unusual