# Modelling and Control of Unmanned Ground Vehicles

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## CERTIFICATE OF AUTHORSHIP/ORIGINALITY

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of the requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and in the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Thanh Hung Tran

# Abstract

The thesis focuses on issues of vehicle modelling incorporating wheel-terrain interaction and low-level control design taking into account uncertainties and input time delay. Addressing these issues is of significant importance in achieving persistent autonomy for outdoor UGVs, especially when navigating on unprepared terrains.

The test-bed vehicle used for this research is retrofitted from an all-terrain 20-hp, 0.5-tonne vehicle. Its driveline system consists of an internal combustion engine, continuous variable transmission (CVT), gearbox, differential, chains, and eight wheels. The vehicle is driven in the skid-steering mode, which is popular for many off-road land-vehicle platforms.

In this thesis, a comprehensive approach is proposed for modelling the driveline. The approach considers the difference in speed between two outputs of the differential and the turning mechanism of the vehicle. It describes dynamics of all components in the vehicle driveline in an integrated manner with the vehicle motion. Given a pattern of the throttle position, left and right braking efforts as the inputs, the dynamic behaviour of the wheels and other components of the UGV can be predicted.

For controlling the vehicle at the low level, PID controllers are firstly used for all actuators. As many components of the vehicle exhibit nonlinearities and time delay, the large overshoots encountered in the outputs can lead to undesirable vehicle behaviours. To alleviate the problem, a novel control approach is proposed for suppression of overshoots resulting from PID control. Sliding mode control (SMC) is employed, for this, with time delay compensated by using an output predictor. As a result, the proposed approach can improve significantly system robustness and reduce substantially step response overshoot. Notably, the design is generic in that it can be applied for many dynamic processes.

Knowledge of the interaction between the UGV and the terrain plays an important role in increasing its autonomy and securing the safety for off-road locomotion. In this regard, vehicle kinematic equations are combined with the theory of terramechanics for dynamic modelling of the interaction between the vehicle wheels and a variety of terrain types. Also, a fast algorithm is developed to enable online implementation. The novel interaction model takes into account the relationship between normal stresses, shear stresses, and shear displacement of the terrain that is in contact with the wheels in deriving the three-dimensional reaction forces.

Finally, all modelling and control algorithms are integrated into a unique simulator for emulating the vehicle mobility characteristics. In particular, the wheel's slip and rolling resistance can also be derived to provide useful information for closed-loop control when the UGV is navigating in an unknown environment. The simulator, as a tool for analysing the vehicle mobility, is helpful for further research on relevant topics such as traction control, safe and effective locomotion.

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# List of Symbols

| Symbols | Nomenclature | Unit |
|---|---|---|
| $\alpha_i$ , $\beta_i$ | Angles of shear stress on the $i^{th}$ wheel in standard spherical coordinates | rad |
| $\chi$ | Ground slope angle | rad |
| $\delta$ | Damping ratio | - |
| $\delta_1$ | Lower limit of damping ratio | - |
| $\delta_2$ | Higher limit of damping ratio | - |
| $\hat{\delta}$ | Approximation of damping ratio | - |
| $\varepsilon$ | Error threshold in search algorithm | % |
| $\phi$ | Terrain internal friction angle | rad |
| $\gamma_i$ | Angle between slip velocity on the $i^{th}$ wheel and its tangential component | rad |
| $\eta$ | SMC parameter | - |
| $\varphi_i$ | Elevation angle of shear stress in new spherical coordinates | rad |
| $\lambda$ | SMC parameter | - |
| $\theta$ | Angle of wheel contact with terrain | rad |
| $\theta_1$ | Wheel entry angle at first contact point | rad |
| $\theta_{1_i}$ | The $i^{th}$ wheel entry angle | rad |
| $\theta_2$ | Wheel exit angle at last contact point | rad |
| $\theta_{2_i}$ | The $i^{th}$ wheel exit angle | rad |
| $\theta_b$ | Brake actuator position | % |
| $\theta_{c1_i}, \theta_{c2_i}$ | Intersection points between shear stress under the $i^{th}$ wheel and its linearisation | rad |
| $\theta_e$ | Engine throttle position | % |
| $\theta_i$ | The $i^{th}$ wheel contact angle | rad |
| $\theta_m$ | Maximum stress point | rad |

| Symbols | Nomenclature | Unit |
|---|---|---|
| $\theta_{m_i}$ | Maximum stress point on the $i^{th}$ wheel | rad |
| $\theta_{x1_i}, \theta_{x2_i}$ | Intersection points between the normal stress distribution under the $i^{th}$ wheel and its linearisation | rad |
| $\theta_M$ | Motor position (throttle control) | % |
| $\rho$ | Azimuth angle between vehicle frame and earth coordinates | rad |
| $\sigma$ | Normal stress under a wheel | kPa |
| $\sigma_1$ | Normal stress in front region | kPa |
| $\sigma_{1_i}$ | Normal stress in front region under the $i^{th}$ wheel | kPa |
| $\hat{\sigma}_{1_i}$ | Linear approximation of normal stress in front region under the $i^{th}$ wheel | kPa |
| $\sigma_{1X_i}$ | Longitudinal component of normal stress in front region under the $i^{th}$ wheel | kPa |
| $\sigma_{1Z_i}$ | Vertical component of normal stress in front region under the $i^{th}$ wheel | kPa |
| $\hat{\sigma}_{1X_i}$ | Linear approximation of the longitudinal component of normal stress in front region under the $i^{th}$ wheel | kPa |
| $\hat{\sigma}_{1Z_i}$ | Linear approximation of the vertical component of normal stress in front region under the $i^{th}$ wheel | kPa |
| $\sigma_2$ | Normal stress in rear region | kPa |
| $\sigma_{2_i}$ | Normal stress in rear region under the $i^{th}$ wheel | kPa |
| $\hat{\sigma}_{2_i}$ | Linear approximation of normal stress in rear region under the $i^{th}$ wheel | kPa |
| $\sigma_{2X_i}$ | Longitudinal component of normal stress in rear region under the $i^{th}$ wheel | kPa |

| **Symbols** | **Nomenclature** | **Unit** |
|---|---|---|
| $\sigma_{2Z_i}$ | Vertical component of normal stress in rear region under the $i^{th}$ wheel | kPa |
| $\hat{\sigma}_{2X_i}$ | Linear approximation of longitudinal component of normal stress in rear region under the $i^{th}$ wheel | kPa |
| $\hat{\sigma}_{2Z_i}$ | Linear approximation of vertical component of normal stress in rear region under the $i^{th}$ wheel | kPa |
| $\sigma_i$ | Normal stress under the $i^{th}$ wheel | kPa |
| $\sigma_{X_i}$ | Longitudinal component of normal stress under the $i^{th}$ wheel | kPa |
| $\sigma_{Y_i}$ | Lateral component of normal stress under the $i^{th}$ wheel | kPa |
| $\sigma_{Z_i}$ | Vertical component of normal stress under the $i^{th}$ wheel | kPa |
| $\tau$ | Shear stress | kPa |
| $\tau_1$ | Shear stress in front region | kPa |
| $\tau_{1_i}$ | Shear stress in front region on the $i^{th}$ wheel | kPa |
| $\hat{\tau}_{1_i}$ | Linear approximation of shear stress in front region on the $i^{th}$ wheel | kPa |
| $\tau_{1t_i}$ | Tangential component of shear stress in front region on the $i^{th}$ wheel | kPa |
| $\hat{\tau}_{1t_i}$ | Linear approximation of tangential shear stress in front region on the $i^{th}$ wheel | kPa |
| $\tau_{1X_i}$ | Longitudinal component of shear stress in front region on the $i^{th}$ wheel | kPa |
| $\tau_{1Y_i}$ | Lateral component of shear stress in front region on the $i^{th}$ wheel | kPa |
| $\tau_{1Z_i}$ | Vertical component of shear stress in front region on the $i^{th}$ wheel | kPa |

| Symbols | Nomenclature | Unit |
|---|---|---|
| $\hat{\tau}_{1X_i}$ | Linear approximation of longitudinal component of shear stress in front region on the $i^{th}$ wheel | kPa |
| $\hat{\tau}_{1Y_i}$ | Linear approximation of lateral component shear stress in front region on the $i^{th}$ wheel | kPa |
| $\hat{\tau}_{1Z_i}$ | Linear approximation of vertical component shear stress in front region on the $i^{th}$ wheel | kPa |
| $\tau_2$ | Shear stress in rear region | kPa |
| $\tau_{2_i}$ | Shear stress in rear region on the $i^{th}$ wheel | kPa |
| $\hat{\tau}_{2_i}$ | Linear approximation of shear stress in rear region on the $i^{th}$ wheel | kPa |
| $\tau_{2t_i}$ | Tangential component of shear stress in rear region on the $i^{th}$ wheel | kPa |
| $\hat{\tau}_{2t_i}$ | Linear approximation of tangential shear stress in rear region on the $i^{th}$ wheel | kPa |
| $\tau_{2X_i}$ | Longitudinal component of shear stress in rear region on the $i^{th}$ wheel | kPa |
| $\tau_{2Y_i}$ | Lateral component of shear stress in rear region on the $i^{th}$ wheel | kPa |
| $\tau_{2Z_i}$ | Vertical component of shear stress in rear region on the $i^{th}$ wheel | kPa |
| $\hat{\tau}_{2X_i}$ | Linear approximation of longitudinal component of shear stress in rear region on the $i^{th}$ wheel | kPa |
| $\hat{\tau}_{2Y_i}$ | Linear approximation of lateral component shear stress in rear region on the $i^{th}$ wheel | kPa |
| $\hat{\tau}_{2Z_i}$ | Linear approximation of vertical component of shear stress in rear region on the $i^{th}$ wheel | kPa |

| Symbols | Nomenclature | Unit |
|---|---|---|
| $\tau_e$ | Engine time constant | s |
| $\tau_i$ | Shear stress on the $i^{th}$ wheel | kPa |
| $\tau_m$ | Motor time constant | s |
| $\tau_{t_i}$ | Tangential component of the shear stress on the $i^{th}$ wheel | kPa |
| $\tau_{X_i}$ | Longitudinal component of shear stress on the $i^{th}$ wheel | kPa |
| $\tau_{Y_i}$ | Lateral component of shear stress on the $i^{th}$ wheel | kPa |
| $\tau_{Z_i}$ | Vertical component of shear stress on the $i^{th}$ wheel | kPa |
| $\omega_e$ | Engine rotational speed | RPM |
| $\omega_c$ | CVT output speed | RPM |
| $\omega_d$ | Rotational speed of differential's case | RPM |
| $\omega_{dL}$ | Differential's output speed on left side | RPM |
| $\omega_{dR}$ | Differential's output speed on right side | RPM |
| $\omega_G$ | Gearbox output speed | RPM |
| $\omega_n$ | Natural frequency | rad/s |
| $\omega_{n1}$ | Lower limit of natural frequency | rad/s |
| $\omega_{n2}$ | Higher limit of natural frequency | rad/s |
| $\hat{\omega}_n$ | Approximation of natural frequency | rad/s |
| $\omega_i$ | Speed of the $i^{th}$ wheel | rad/s |
| $\omega_w$ | Wheel rotational speed | rad/s |
| $\omega_{wL}$, $\omega_L$ | Left wheel rotational speed | rad/s |
| $\omega_{wR}$, $\omega_R$ | Right wheel rotational speed | rad/s |
| $\Omega$ | Vehicle turning rate | rad/s |
| $a$ | Longitudinal distance between successive wheel axles | m |

| **Symbols** | **Nomenclature** | **Unit** |
|---|---|---|
| $a_X$ | Vehicle acceleration along longitudinal direction | m/s |
| $a_Y$ | Vehicle acceleration along lateral direction | m/s |
| $b$ | Wheel width | m |
| $b_e$ | Engine damping coefficient | Nms |
| $b_d$ | Differential damping coefficient | Nms |
| $b_w$ | Wheel damping coefficient | Nms |
| $b_{D,in}$ | Damping coefficient inside differential | Nms |
| $b_G$ | Gearbox damping coefficient | Nms |
| $c$ | Terrain cohesion parameter | kPa |
| $c_{r1}$, $c_{r2}$ | Wheel friction cofficients | $m.s^{-2}$, $s^{-1}$ |
| $d$ | Distance between the vehicle centroid and the centre of mass | m |
| $dA_i$ | Area increment around a contact point | $m^2$ |
| $dF_{X_i}$ | Total force increment along longitudinal direction acting on a very small contact area around a contact point | N |
| $dF_{Y_i}$ | Total force increment along lateral direction acting on a very small contact area around a contact point | N |
| $dF_{Z_i}$ | Total force increment along vertical direction acting on a very small contact area around a contact point | N |
| $e$ | Control error | |
| $e_{emf}$ | Motor back electromotive force | V |
| $e_o$ | PID error | |
| $f$ | Nonlinear function | - |
| $\hat{f}$ | Approximation of nonlinear function | - |
| $g$ | Gravitational acceleration | $m/s^2$ |

| Symbols | Nomenclature | Unit |
|---|---|---|
| $h$ | Height of the centre of mass above the ground | m |
| $i$ | Vehicle wheel slip | % |
| $i_{CVT}$ | CVT belt slip | % |
| $j$ | Shear displacement | m |
| $j_i$ | Shear displacement at a contact point on the $i^{th}$ wheel | m |
| $j_{X_i}$ | Shear displacement at a contact point on the $i^{th}$ wheel along longitudinal direction | m |
| $j_{Y_i}$ | Shear displacement at a contact point on the $i^{th}$ wheel along lateral direction | m |
| $j_{Z_i}$ | Shear displacement at a contact point on the $i^{th}$ wheel along vertical direction | m |
| $k$ | SMC parameter / time step | - |
| $k_{1_i}, k_{2_i}, c_{1_i}, c_{2_i}$ | Linearisation parameters for normal stress on the $i^{th}$ wheel | kPa/rad, kPa/rad, kPa, kPa |
| $k_{1X_i}, k_{2X_i}, c_{1X_i}, c_{2X_i}$ | Linearisation parameters for longitudinal component of normal stress on the $i^{th}$ wheel | kPa/rad, kPa/rad, kPa, kPa |
| $k_{1Z_i}, k_{2Z_i}, c_{1Z_i}, c_{2Z_i}$ | Linearisation parameters for lateral component of normal stress on the $i^{th}$ wheel | kPa/rad, kPa/rad, kPa, kPa |
| $k_{3_i}, k_{4_i}, c_{3_i}, c_{4_i}$ | Linearisation parameters for shear stress on the $i^{th}$ wheel | kPa/rad, kPa/rad, kPa, kPa |
| $k_{3t_i}, k_{4t_i}, c_{3t_i}, c_{4t_i}$ | Linearisation parameters for tangential component of shear stress on the $i^{th}$ wheel | kPa/rad, kPa/rad, kPa, kPa |
| $k_{3X_i}, k_{4X_i}, c_{3X_i}, c_{4X_i}$ | Linearisation parameters for longitudinal component of shear stress on the $i^{th}$ wheel | kPa/rad, kPa/rad, kPa, kPa |
| $k_{3Y_i}, k_{4Y_i}, c_{3Y_i}, c_{4Y_i}$ | Linearisation parameters for lateral component of shear stress on the $i^{th}$ wheel a | kPa/rad, kPa/rad, kPa, kPa |
| $k_{3Z_i}, k_{4Z_i}, c_{3Z_i}, c_{4Z_i}$ | Linearisation parameters for vertical component of shear stress on the $i^{th}$ wheel | kPa/rad, kPa/rad, kPa, kPa |

| Symbols | Nomenclature | Unit |
|---|---|---|
| $k_c$ , $k_\phi$ | Pressure-sinkage moduli parameters of a terrain | kN/m$^{n+1}$, kN/m$^{n+2}$ |
| $m$ | Vehicle's mass | kg |
| $m_w$ | Wheel's mass | kg |
| $n$ | Sinkage exponent parameter of terrain | - |
| $r$ | Wheel radius | m |
| $t_d$ | Time delay | s |
| $t_P$ | Peak time | s |
| $u$ | SMC output, PID input | % |
| $u_{eq}$ | Equivalent control output | % |
| $u_R$ | Robust control output | % |
| $x$ | Speed difference between the differential's case and its outputs | RPM |
| $(x_i, y_i)$ | Cartesian coordinates of a contact point on the $i^{th}$ wheel frame | m |
| $\mathbf{x}(t)$ | State variable vector | |
| $y$ | Process output | % |
| $y_d$ | Desired output (Reference or set-point) | % |
| $z$ | Wheel sinkage | m |
| $z_i$ | Sinkage of the $i^{th}$ wheel | m |
| **A**, **B** | System matrices in state space form | |
| $B$ | Half of the vehicle track | m |
| $B_m$ | Motor damping ratio | Nms |
| $D$ | Boundary of disturbance | |
| $F$ | Boundary of nonlinear function's approximation error | - |
| $F_R$ | Wheel rolling resistance | N |
| $F_{t,w}$ | Traction force at wheels | N |
| $F_{X_i}$ | Total reaction force acting on the $i^{th}$ wheel along longitudinal direction | N |

| Symbols | Nomenclature | Unit |
|---|---|---|
| $\hat{F}_{X_i}$ | Estimate of total reaction force acting on the $i^{th}$ wheel along longitudinal direction | N |
| $F_{Y_i}$ | Total reaction force acting on the $i^{th}$ wheel along lateral direction | N |
| $\hat{F}_{Y_i}$ | Estimate of total reaction force acting on the $i^{th}$ wheel along lateral direction | N |
| $F_{Z_i}$ | Total reaction force acting on the $i^{th}$ wheel along vertical direction | N |
| $\hat{F}_{Z_i}$ | Estimate of total reaction force acting on the $i^{th}$ wheel along vertical direction | N |
| $I$ | Motor armature current | A |
| $I_Z$ | Vehicle moment of inertia around Z axis | $kg.m^2$ |
| $J_e$ | Engine moment of inertia | $kg.m^2$ |
| $J_m$ | Motor rotor's moment of inertia | $kg.m^2s^{-2}$ |
| $J_w$ | Wheel moment of inertia | $kg.m^2$ |
| $K$ | Shear deformation modulus | m |
| $K_1$ | CVT gear ratio | - |
| $K_2$ | Gearbox gear ratio | - |
| $K_3$ | Chain system gear ratio | - |
| $K_a$ | Amplifier' voltage to current gain | $AV^{-1}$ |
| $K_e$ | Engine gain | N.m |
| $K_{em}=K_t$ | Motor electromotive force constant | $N.m.A^{-1}$ |
| $K_i$ | Actuator's current to force gain | $NA^{-1}$ |
| $K_m$ | Motor gain | - |
| $K_D$ | Derivative gain (D) | - |
| $K_I$ | Integral gain (I) | - |
| $K_G$ | Motor gear ratio | - |
| $K_P$ | Proportional gain (P) | - |
| $L_m$ | Motor electric inductance | H |
| $M_P$ | Percentage of overshoot | % |

| Symbols | Nomenclature | Unit |
|---|---|---|
| $M_R$ | Moment of turning resistance | N.m |
| $M_X$ | Rolling moment around X axis | N.m |
| $M_Y$ | Rolling moment around Y axis | N.m |
| $M_Z$ | Turning moment around Z axis | N.m |
| $\hat{M}_Z$ | Estimate of turning moment | N.m |
| $(N, E)$ | Position of the vehicle in earth coordinates | m, m |
| $R_m$ | Motor electric resistance | Ω |
| $T$ | Traction torque | N.m |
| $T_{bL}$ | Left brake torque | N.m |
| $T_{bR}$ | Right brake torque | N.m |
| $T_c$ | Load on CVT | N.m |
| $T_d$ | Load on differential's case | N.m |
| $T_{dL}$ | Load on differntial's left output | N.m |
| $T_{dR}$ | Load on differential's right output | N.m |
| $T_e$ | Engine generated torque | N.m |
| $T_{ec}$ | Load on engine | N.m |
| $T_{fric,e}$ | Engine friction torque | N.m |
| $T_{fric,D}$ | Differential friction torque | N.m |
| $T_{fric,G}$ | Gearbox friction torque | N.m |
| $T_i$ | Traction torque developed on the $i^{th}$ wheel | N.m |
| $\hat{T}_i$ | Estimate of traction torque developed on the $i^{th}$ wheel | N.m |
| $T_m$ | Motor torque | N.m |
| $T_{sL}$ | Total load torque on the left sun gear (differential) | N.m |
| $T_{sR}$ | Total load torque on the right sun gear (differential) | N.m |
| $T_w$ | Total load torque on all wheels | N.m |
| $T_{wL}$ | Load torque from left wheels | N.m |
| $T_{wR}$ | Load torque from right wheels | N.m |

| Symbols | Nomenclature | Unit |
|---|---|---|
| $T_G$ | Load torque on gearbox | N.m |
| $V$ | PID output (Process input, or Voltage) | V |
| $\mathbf{V}$ | Vehicle velocity vector | m/s |
| $V_{jX_i}$ | Longitudinal component of slip velocity of a contact point on the $i^{th}$ wheel | m/s |
| $V_{jY_i}$ | Lateral component of slip velocity of a contact point on the $i^{th}$ wheel | m/s |
| $V_{jZ_i}$ | Vertical component of slip velocity of a contact point on the $i^{th}$ wheel | m/s |
| $V_{t_i}$ | Tangential component of slip velocity of a contact point on the $i^{th}$ wheel | m/s |
| $V_L$ | Lyapunov function | - |
| $V_E$ | Vehicle velocity along east direction | m/s |
| $V_N$ | Vehicle velocity along north direction | m/s |
| $V_X$ | Vehicle longitudinal velocity | m/s |
| $V_Y$ | Vehicle lateral velocity | m/s |
| $S$ | Sliding surface | - |
| $S_{X_i}$ | Wheel slip ratio along longitudinal direction | % |
| $S_{Y_i}$ | Wheel slip ratio along lateral direction | % |
| $(X_i, Y_i, Z_i)$ | Cartesian coordinates of a contact point on the vehicle frame | m |
| $W$ | Vehicle weight | N |
| $W_i$ | Vertical load on the $i^{th}$ wheel | N |

# Abbreviations

| | |
|---|---|
| ALV | Autonomous Land Vehicle |
| CCD | Charge-Coupled Device |
| CVT | Continuous Variable Transmission |
| DARPA | Defense Advanced Research Projects Agency |
| FLIR | Forward Looking InfraRed |
| HMMWV | High-Mobility, Multipurpose, Wheeled Vehicle |
| LADAR | Light Detection And Ranging |
| MIMO | Multiple-Input Multiple-Output |
| MVEM | Mean Value Engine Model |
| SSV | Semiautonomous Surrogate Vehicle |
| SI | Spark Ignition |
| SMC | Sliding Mode Controller |
| PID | Proportional – Integral – Derivative |
| UGV | Unmanned Ground Vehicle |