

Assessment of the Structural Integrity of Timber Bridges Using Dynamic Approach

by

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Doctor of Philosophy**



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ABSTRACT

In this study, a systematic approach was adopted to investigate, numerically and experimentally, localised defects and/or damage in timber bridges, such as rot, using modal based damage detection techniques. An existing damage detection method namely damage index (DI) method that utilises modal strain energy before and after damaged state was adopted. One contribution of this study was to modify the DI method by an additional step of normalising the modal curvature, which would minimise the dominance of higher modes.

In the numerical models, a comparative study of the effects of numerical integration techniques used in a damage detection process was carried out. The results show that when mode shape curvature integrations use the rectangular rule for the numerical integration, it yields better results than the trapezoidal rule.

In the numerical examples using a finite element model of timber beam, the modified DI (MDI) methods were found to perform better than its original form for locating single and multiple damage scenarios. For the DI methods, two types of formulations were adopted and modified, and they are denoted as modified damage index I (MDI-I) and modified damage index II (MDI-II). Another modal based damage detection method, namely changes in flexibility (CIF), was adopted for locating damage. It was found that the CIF method performed reasonably well for single damage but not multiple damage scenarios.

As part of the study, the modified damage index methods were utilised for evaluating severity of damage. For the MDI-I method, the formulation was not derived to evaluate damage severity directly. Instead, a hybrid of the MDI-I and CIF methods (HMC), was proposed for evaluating severity of damage in terms of loss of 'I' (moment of inertia). Using three levels of damage, i.e. light (L), medium (M) and severe (S), the HMC method is able to predict the medium and severe damage quite well, but it is less efficient for light damage scenarios. For the MDI-II method, further manipulation of the algorithm can predict the severity of damage in terms of loss of 'I'. This method is able to predict the medium and severe damage quite well but is not as good for the light damage. Both methods, HMC and MDI-II, for predicting severity of damage, required some adjustment using a weighting factor in order to obtain reasonable results.

An experimental modal analysis (EMA) test program of timber beams was undertaken. This was done to verify the robustness of the modified damage index methods for detecting location and estimating severity of damage. The laboratory investigation was conducted on the corresponding changes of modal parameters due to loss of section. The MDI methods were used to detect location of damage and to evaluate the severity of damage in the test beams. A mode shape reconstruction technique was utilised to enhance the capability of the damage detection algorithms with limited number of sensors. The test results and analysis show that location of damage is quite accurately estimated with the available sensors. The methods demonstrate that they are less mode dependant and can detect damage with a higher degree of confidence. The MDI methods also show that they are able to predict the severe damage well, but it is less accurate for the medium damage and not as good for light damage.

The damage index II (DI-II) method extended to plate-like structures (DI-II-P) was adopted and evaluated for detecting damage. Based on finite element analysis (FEA) results of a laboratory timber bridge, the DI-II-P method which utilises two-dimensional (2-D) mode shape curvature was employed to detect location of damage. The results show that the method based on 2-D mode shape curvature is able to locate damage quite well, numerically. A supplementary work using the DI-II-P method in a timber plate model was carried out. The results also show that the method was able to predict the damage location well.

A process of updating a laboratory timber bridge, analytically, is presented. A finite element model was developed and updated with experimental modal data. Material properties of timber beam (girders) and plywood (deck) as well as the screw connection between deck and girder were experimentally investigated. These test results were then used for the finite element modelling. The model has been developed sequentially starting with a preliminary model having very simple features. It followed by the advanced model calibrated with the experimental modal data employing a global objective function, consisting of errors of natural frequencies and modal assurance criterion. The calibrated finite element model shows a good correlation to the experimental model with minor adjustments to the real material properties and boundary conditions. The calibrated model can reasonably be used to study the damaged behaviour of the laboratory timber bridge.

The bridge model was then used to verify the numerical results for detecting damage. The bridge was inflicted with various damage scenarios with loss of section similar to the timber beam models. The limited number of data was expanded using the 2-D cubic spline. Using the reconstructed data for detecting damage yields better results than just using 'as is' data. Using the undamaged and damaged modal data, the DI-II-P method was employed to detect the location of damage. The results of using the first nine modes showed that generally the severe damage is able to be located by the method. It performs reasonably well for the medium damage but does not perform as good in the light damage scenarios. However, in some cases the method can present some problems in identifying severe damage, which may be due to lack of normalisation of mode shape curvature. Complementary work was undertaken using the method on a timber plate, experimentally. The results showed that the damage detection process in the timber plate is less efficient compared to the laboratory timber bridge.

A comprehensive comparative study was carried out based on the results of the numerical and experimental investigation of damage detection on timber beam, laboratory timber bridge and timber plate. For the timber beam, both damage detection methods, MDI-I and MDI-II, were capable of detecting medium and severe damage in the numerical and experimental studies. However, the light damage was not identified well using the experimental data in the presence of noise. To estimate damage severity in the timber beam, the HMC method performed well for the medium and severe damage. The method did not work well in estimating severity of light damage. Similar conclusions can be drawn in using the MDI-II method to estimate the damage severity. The results of applying the DI-II-P method (using 9 modes) to locate damage in the laboratory timber bridge showed that numerical and experimental data are capable of detecting all severe damage for damage cases with less than three damage locations. While for light and medium damage, the experimental data did not work well as compared to the numerical one. For the timber plate (a complementary work), the numerical and experimental results also showed that they are able to detect the severe damage well. However, there were serious false positives appearing in the light damage cases in the experimental results.

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Sadhu! Sadhu! Sadhu!

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7. Li, J., Samali, B., Crews, K., **Choi, F.** & Shestha, R. 2005, 'Theoretical and experimental studies on assessment of bridges using simple dynamic procedures', *Australian Structural Engineering Conference 2005: Structural Engineering - Preserving and Building into the Future*, 11-14 September, Newcastle City Hall, Australia: Structural College of EA, (published on CDROM).
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List of Notations

$\bar{\beta}_{ij}$	damage indicator for the modified DI methods
$\bar{\alpha}_j$	severity estimator of modified DI-II method for all j -th element
$\bar{\phi}_i''$	normalised curvature vector for undamaged state for modified DI methods
\bar{Z}_j	normalised $\bar{\beta}_{ij}$ into the standard normal space
ν	Poisson's ratio (Chapters 2 to 4)
$v(x)$	deflected shape (Chapter 5)
Φ	eigenvector matrix
Ω	eigenvalue matrix
Δ	Difference of S_u and S_d
ζ	damping ratio
ν (for DI-II-P)	Poisson's ratio of an isotropic plate (for Chapter 5)
ρ , DENS	density
μ_{β_j}	mean of β_j values for all j -th element
σ_{β_j}	standard deviation of β_j values for all j -th element
$(\delta_j)_{max}$	maximum absolute value of elements δ_{ij}
(3.375m, 0.9m)	convention for damage at position of 3.375m along the span length and 0.9m across the width for laboratory timber bridge and timber plate
ω_{dk}	damped natural frequency of mode k
$(EI)_j$	Stiffness of j -th element
λ_i	square of ω_i or pole value for mode i
ω_i	circular frequency of mode i
ϕ_i	mode shape or eigenvector of mode i
(i modes)	i modes used in the damage detection algorithms
δ_{ij}	i -th column of Δ
β_{ij}	damage indicator for the DI methods

β_{ijk}	damage indicator for the DI-II-P method
α_j	severity estimator of DI-II method for all j -th element
δ_k	damping factor of mode k
ν_{LR}	Poisson's ratio of applied stress in longitudinal direction and accompanying lateral strain in radial direction
ν_{LT}	Poisson's ratio of applied stress in longitudinal direction and accompanying lateral strain in tangential direction
ν_{RL}	Poisson's ratio of applied stress in radial direction and accompanying lateral strain in longitudinal direction
ν_{RT}	Poisson's ratio of applied stress in radial direction and accompanying lateral strain in tangential direction
ν_{TL}	Poisson's ratio of applied stress in tangential direction and accompanying lateral strain in longitudinal direction
ν_{TR}	Poisson's ratio of applied stress in tangential direction and accompanying lateral strain in radial direction
σ_x	stress in x direction
ε_x	strain in x direction
*	denoting damaged case
1-D	one-dimensional
2-D	two-dimensional
2M4S5S6S	cumulative damage scenario of medium damage at 2/8 span, severe damage at 4/8, 5/8 and 6/8 span
3-D	three-dimensional
A (for damage)	small damage inflicted in timber plate
A (for DI-II-P)	area of the plate surface (for Chapter 5)
ADC	analogue-to-digital converter
B (for damage)	large damage inflicted in timber plate
c	damping coefficient
c_{cr}	critical damping coefficient
CIF	changes in flexibility
COMAC	coordinate modal assurance criterion
D (for DI-II-P)	flexural rigidity of a plate (for Chapter 5)
DD	damage detection

$Denom_{ij}$	denominator of β_{ij}
DFA	dynamic frequency analysis
DI	damage index
DI-I	damage index I
DI-II	damage index II
DI-II-P	damage index II method for plate-like structures
DOF	degree of freedom
DSP	digital signal processing
E, MOE	modulus of elasticity
EI	stiffness
E_L, E_R, E_T	MOE in directions of longitudinal, radial and tangential, respectively
EX, EY, EZ	MOE in global directions of X, Y and Z, respectively, for FE models
$f(\omega)$	input signal with respect to frequency ω
FE	finite element
FEA	finite element analysis
FEM	finite element model
FFT	fast Fourier transform
FIFO	file-input-file-output
F_{ij}	fractional modal strain energy for undamaged beam
f_{ij}	$\int_j \left(\frac{\partial^2 \phi_i}{\partial x^2} \right)^2 dx \bigg/ \int_0^L \left(\frac{\partial^2 \phi_i}{\partial x^2} \right)^2 dx$, ratio of integrations for square second derivative of ϕ_i for j -th element of mode i and the overall structure
F_{ijk}	fractional modal strain energy of subregion jk for undamaged beam
f_{ijk}	$\frac{\int_{b_k}^{b_{k+1}} \int_{a_j}^{a_{j+1}} \left(\frac{\partial^2 \phi_i}{\partial x^2} \right)^2 + \left(\frac{\partial^2 \phi_i}{\partial y^2} \right)^2 + 2\nu \left(\frac{\partial^2 \phi_i}{\partial x^2} \right) \left(\frac{\partial^2 \phi_i}{\partial y^2} \right) + 2(1-\nu) \left(\frac{\partial^2 \phi_i}{\partial x \partial y} \right)^2 dx dy}{\int_0^b \int_0^a \left(\frac{\partial^2 \phi_i}{\partial x^2} \right)^2 + \left(\frac{\partial^2 \phi_i}{\partial y^2} \right)^2 + 2\nu \left(\frac{\partial^2 \phi_i}{\partial x^2} \right) \left(\frac{\partial^2 \phi_i}{\partial y^2} \right) + 2(1-\nu) \left(\frac{\partial^2 \phi_i}{\partial x \partial y} \right)^2 dx dy}$, ratio of integrations for second derivatives of ϕ_i for subregion jk of mode i and the overall structure

f_n	cyclic natural frequency
FRF	frequency response function
G	modulus of rigidity
$g(\omega)$	output signal with respect to frequency ω
g2Sg4Sg3Sg1M	cumulative damage scenario of medium damage at 1/8 span of girder 1, severe damage at 6/8 span of girder 2, 4/8 span of girder 4 and 2/8 span of girder 3
$G_{ff}(\omega)$	output auto-spectrum with respect to frequency ω
$G_{fg}(\omega)$	cross input-output spectrum with respect to frequency ω
gi	girder no. i
G_{LR}	G in longitudinal-radial plane
G_{LT}	G in longitudinal-tangential plane
G_{RT}	G in radial-tangential plane
GXY	shear modulus in XY plane for FE models
GXZ	shear modulus in XZ plane for FE models
GYZ	shear modulus in YZ plane for FE models
h	height
H_{ij}	FRF between the response of DOF i and reference DOF j
HMC	hybrid of MDI-I and CIF
HOT	higher order terms
I	moment of inertia
i, n	mode number
K	stiffness matrix
KX, kx	Stiffness of rigid link simulating screw connections in local x direction for laboratory timber bridge FE models
KY, ky	Stiffness of rigid link simulating screw connections in local y direction for laboratory timber bridge FE models
KZ, kz	Stiffness of rigid link simulating screw connections in local z direction for laboratory timber bridge FE models
L (for damage)	light damage for timber beam and laboratory timber bridge
L (for wood)	longitudinal direction
L, l, L (for measurement)	span length

List of Notations

L2(II)B-L3(IV)B-L3(V)A	cumulative damage scenario of small damage at location V of Line 3, large damage at location II of Line 2 and location IV of Line 3
L_b	length of timber beam
L_i	Line no. i
L_p	length of plywood
LSCE	least square complex exponential
LVDT	displacement transducer
M	mass matrix
m	mass per unit length
M (for damage)	medium damage for timber beam and laboratory timber bridge
MAC	modal assurance criterion
MAC_{error}	MAC error between FE and experimental models
MDI-I	modified damage index I
MDI-II	modified damage index II
MDOF	multiple degree of freedom
m_i	mode no. i
MIMO	multiple-input-multiple-output
MSE	modal strain energy
NDE	nondestructive evaluation
NDE	nondestructive testing
N_{Error}	natural frequency difference between FE and experimental models
NM, N	number of modes
Num_{ij}	numerator of β_{ij}
NUXY, NUXZ, NUYZ	Poisson's ratio in planes of XY, XZ and YZ, respectively, for FE models
N_x, N_y	subdivision in the directions of x and y
OF	objective function
P (for damage)	light damage for FE models of isotropic and orthotropic plates
q	an arbitrary distributed load

Q (for damage)	medium damage for FE models of isotropic and orthotropic plates
R (for damage)	severe damage for FE models of isotropic and orthotropic plates
R (for wood)	radial direction
r_{ijk}	residual value for mode k
S	stiffness matrix (Chapters 2 to 4) flexibility matrix (Chapter 5)
S (for damage)	severe damage for timber beam and laboratory timber bridge
S_d	damaged flexibility (for Chapter 5)
SDOF	single degree of freedom
SIMO	single-input-multiple-output
S_u	undamaged flexibility (for Chapter 5)
SX	Stiffness of rigid link simulating supports in global X direction for laboratory timber bridge FE models
SY	Stiffness of rigid link simulating supports in global Y direction for laboratory timber bridge FE models
SZ	Stiffness of rigid link simulating supports in global Z direction for laboratory timber bridge FE models
t (for DII-P)	thickness of a plate (for Chapter 5)
T (for wood)	tangential direction
T_b	thickness of timber beam
T_p	thickness of plywood
U	strain energy
U_i	modal strain energy of mode i
U_{ij}	modal strain energy of j -th element for mode i
W_b	width of timber beam
W_p	width of plywood
X	direction along the width of laboratory timber bridge
X_{Mr}	peak amplitude for half-power bandwidth method
Y	direction along the length of laboratory timber bridge
Z	out-of-plane direction of laboratory timber bridge

Z_j normalised β_{ij} into the standard normal space