

**A Non-destructive Damage Detection  
Method for Reinforced Concrete Structures  
Based on Modal Strain Energy**

by

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of the requirements for the degree of  
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# **CERTIFICATE OF AUTHORSHIP/ORIGINALITY**

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## ABSTRACT

In recent years, much research and development has been carried out on the use of vibration characteristics to detect structural damage in various types of structures. Being a type of the widely developed methods, the vibrations based damage detection methods, in particular modal based methods, are found to be promising in assessing the “health” condition of a structure in terms of locating and quantifying damage.

However, despite of advances in vibration based damage detection methods of numerical simulations and some laboratory experimental implementations, very limited progress has been reported in field applications. The main obstacles in field applications are associated with the uncertainties, such as measurement noise, processing errors and error due to limited measurement points. In addition, the complexity of civil structures and/or materials further undermines the effectiveness and reliability of developed damage detection methods. This is evident by the fact that much less research and development has been reported for timber or reinforced concrete structures in terms of damage detection. It is therefore necessary to investigate effects of measurement noise, processing errors and errors due to limited measurement points on damage detection and develop a new robust and reliable method to locate and quantify damage in reinforced concrete structures.

The aim of this PhD research work is to formulate a new non-destructive modal based damage detection method that is robust and reliable to evaluate the “state of health” for a reinforced concrete (RC) structure by investigating its capability to locate and to estimate severity of damage based on modal strain energy of a RC beam before and after damage. The new method is based on combination of mode shapes and mode shape curvatures from the undamaged and damaged states making the method more robust and less vulnerable to noise compared with other existing modal based damage detection methods that are either based only on mode shape or mode shape curvature.

Numerical studies that utilises finite element (FE) models of reinforced concrete beams were employed to investigate the effectiveness and the reliability of the proposed damage detection method. The FE model of a reinforced concrete beam was created

incorporating laboratory tested material properties for concrete and steel reinforcement and then validated both statically and dynamically. Damage scenarios that are commonly found in reinforced concrete structures (i.e. cracks and honeycomb) were numerically simulated. Using 41-node mode shape data, the damage detection results without noise pollution demonstrate that the proposed damage detection method is capable of locating the damage accurately. To evaluate impact of noise on the proposed new damage detection method, various magnitudes of white Gaussian noise were added to the time response data obtained from FE transient analysis. The noise-added data were then processed using the virtual experimental modal analysis to obtain the modal parameters with the same number of measurement points as experiments (9-nodes). The damage detection results under influence of noise illustrate that the proposed method is also able to identify the location of damage, although it occasionally generated false positives. In terms of damage severity estimation, the proposed damage detection method was able to estimate the severity of damage with reasonable accuracy without noise pollution. However, when noise is present, the proposed method is less reliable in estimating the severity of damage.

In this study, comparison has been made between the results of the proposed method and two existing popular methods with and without the noise influence. The comparison results prove that the proposed method is more robust and reliable than the two other methods in identifying the location of damage. An improvement to the proposed damage detection method (DI-NI) was introduced to enhance its robustness and reliability. The improved method was applied to detect damage in a number of numerical cases, both with and without noise present. The results show that the improved method does not significantly improve the accuracy of damage detection in the absence of noise, since the original results identifying the location of damage are already accurate. However, when noise is present, the results illustrate that the method is able to significantly improve reliability of damage detection, i.e. false positives are reduced, and location of damage is identified more precisely.

The experimental verification of the proposed damage detection methods for reinforced concrete structures was also performed on five reinforced concrete beams in the UTS Structures Laboratory. The experimental modal parameters such as natural frequencies and mode shapes were obtained by modal testing and experimental modal analysis. The

Cubic Spline interpolation technique was employed to reconstruct a finer mode shapes (9-to-41-nodes) from the measured data (9-node mode shape) in order to enhance the capability of the damage detection method to evaluate damage. Based on the findings of the numerical work, the improved version of the proposed damage detection method was utilised to detect damage using the experimental data. From the measured natural frequencies, it is observed that the percentage of drop in frequency for all modes is non-linearly changing as the inflicted damage becomes more severe but there is no clear evidence that frequency change can be clearly correlated to damage severity. For damage localisation, the improved method shows that it is capable of locating the damage, even though some false positives may be generated. Comparing to the two existing popular damage detection methods, the improved method is more accurate than the two other methods in identifying the locating damage.

In summary, a new modal strain energy based damage detection method has been successfully developed to overcome shortcomings of existing methods. The numerical and experimental evaluation and verification have shown that the new method, especially the improved version of the method, is reliable and effective in locating single and multiple damage scenarios. The new method is also successful in estimating severity of damage to a certain extent.

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# List of Publications Based on This Research

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## List of Notations

$\Delta$	change in the flexibility matrix
$\rho$	density
$\mu_{\beta_j}$	mean of $\beta_j$ values for all $j$ -th elements
$\sigma_{\beta_j}$	standard deviation of $\beta_j$ values for all $j$ -th elements
$\lambda_i$	eigenvalue for mode $i$ of undamaged beam
$\lambda_i^*$	eigenvalue for mode $i$ of damaged beam
$\omega$	natural frequency of system
$\omega_{Exp}$	natural frequency of experimental result
$\omega_{FE}$	natural frequency of finite element result
$\emptyset$	mode shape vector
$\emptyset_i$	mode shape or eigenvector of mode $i$
$\emptyset_{ij}$	mode shape vector of the $i^{th}$ mode and $j^{th}$ element of undamaged beam
$\emptyset_{ij}^*$	mode shape vector of the $i^{th}$ mode and $j^{th}$ element of damaged beam
$\emptyset_{ij}''$	mode curvature vector of the $i^{th}$ mode and $j^{th}$ element of undamaged beam
$\emptyset_{ij}^{*''}$	mode curvature vector of the $i^{th}$ mode and $j^{th}$ element of damaged beam
$\{\hat{\phi}\}_{ij}''$	normalised mode curvature vector of the $i^{th}$ mode and $j^{th}$ element of undamaged beam
$\{\hat{\phi}^*\}_{ij}''$	normalised mode curvature vector of the $i^{th}$ mode and $j^{th}$ element of damaged beam
$\delta$	deflection of the reinforced concrete beam
$\alpha_j$	damage severity estimator
$\varepsilon_{cu}$	ultimate strain of concrete
$\varepsilon_{tu}$	ultimate strain of reinforcing bars
$\varepsilon_{ts}$	tensile strain in the analysis step
*	denoting damaged case
1L3M5S7L	cumulative damage scenario of light damage at 1/8 span,

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	medium damage at 3/8 span, severe damage at 5/8 span, and light damage at 7/8 span
3-D	three-dimensional
41-nodes	41 measuring points taken from the MAF
9-nodes	9 measuring points taken from the VEMA
9-to-41-nodes	Reconstruct 9-nodes to 41-nodes using the Cubic Spine technique
ANN	artificial neural networks
$C$	system damping matrix
CFRP	carbon fibre reinforced polymer
CMSE	cross-modal strain energy
COMAC	coordinate modal assurance criterion
CWT	continuous wavelet transform
DD	damage detection
DDF	Digital Damage Fingerprints
$Denom_{ij}$	denominator of $\beta_{ij}$
$D.I._{compressive}$	compressive damage index of concrete
$D.I._{tensile}$	tensile damage index of concrete
DI-A	results of Damage Index A
DI-C	results of Damage Index C
DI-N	results of the proposed method
DI-NI	results of the improved damage detection method
DI-NO	results of the original damage detection method
DLV	damage locating vector
DOF	degree of freedom
DPD	Damage Parameters Database
DRC	damaged reinforced-concrete
DSD	dynamic steepest descent
$E_c$	modulus of elasticity of concrete
$E_j$	$j^{th}$ equivalent elemental modulus of elasticity of undamaged beam
$E_j^*$	$j^{th}$ equivalent elemental modulus of elasticity of damaged beam

*List of Notations*

$EI$	flexural stiffness
EMA	experimental modal analysis
EMD	empirical mode decomposition
$E_s$	Modulus of elasticity of reinforcement
EXPASS	Expansion Pass
$F$	system force vector
$f_c'$	compressive strength of concrete
$f_{ct}$	tensile strength of concrete
FE	finite element
FEA	finite element analysis
FEM	finite element model
FFT	fast Fourier transform
FDPI	Frequency Direct Parameter Identification
FRF	frequency response function
FSD	fussy steepest descent
$f_y$	yield strength of reinforcement
GFI	global flexibility index
HB	honeycomb damage in the bottom of RC beam
HHT	Hilbert-Huang transform
HT	honeycomb damage at the top of RC beam
I	moment of inertia
IMFs	intrinsic mode functions
ISPPR	Intelligent Signal Processing and Pattern Recognition
$K$	system stiffness matrix
$K_j$	elemental stiffness matrix for $j^{th}$ element
$K_{jo}$	baseline stiffness contribution of the same element due to undamaged geometry of structure
$K_{jo}^*$	baseline stiffness contribution of the same element due to damaged geometry of structure
$L$	lower triangular matrix
L (for damage)	light damage
LVDT	linear variable differential transformer
$M$	system mass matrix

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M (for damage)	medium damage
$M_j$	mass of $j^{th}$ element of undamaged beam
$M_j^*$	mass of $j^{th}$ element of damaged beam
MAC	modal assurance criterion
<i>MACerror</i>	MAC error between FE and experimental models
MAF	Modal Analysis Function
MDI	modified damage index
MIMO	multiple-input-multiple-output
MLP	multi-layer perceptron
MSE	modal strain energy
$MSE_{ij}$	modal Strain Energy for the Euler-Bernoulli beam model of the $j^{th}$ element of $i^{th}$ mode before the occurrence of damage
$MSE_{ij}^*$	modal Strain Energy for the Euler-Bernoulli beam model of the $j^{th}$ element of $i^{th}$ mode after the occurrence of damage
MSEC	modal strain energy change
MSECR	modal strain energy change ratio
$N_{2fc}$	number of complete cycles to failure for concrete
$N_{2fr}$	number of complete cycles to failure for reinforcing bars
NDE	non-destructive evaluation
NDE	non-destructive testing
<i>NError</i>	natural frequency difference between FE and experimental models
<i>Num<sub>ij</sub></i>	numerator of $\beta_{ij}$
<i>OF</i>	objective function
P	Static load on reinforced concrete beams
RC	reinforced concrete
S (for damage)	severe damage
SDOF	single degree of freedom
SHM	structural health monitoring
SIMO	single-input-multiple-output
SISO	single-input-single-output

*List of Notations*

SWT	stationary wavelet transform
TSD	tunable steepest descent
VEMA	Virtual Experimental Modal Analysis
$Z$	system displacement vector
$\dot{Z}$	system velocity vector
$\ddot{Z}$	system acceleration vector
$Z_j$	damage location index